COMP20003
Algorithms and Data Structures

Assignment #3 - Experimentation

Goh Xing Yang
1001969
October 29, 2020

1 Experimentation Data

Table 1: Statistics for Layout 0 (3 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	1	2	2	50	0.039493
100000	1	2	2	50	0.039493
1000000	1	2	2	50	0.039493
1500000	1	2	2	50	0.039493

Layout 0 only requires 2 expanded nodes in the Depth First Search (DFS) to achieve a winning solution so a budget of 10000 is sufficient, therefore, increasing the budget has no effect on the solution.

Table 2: Statistics for Layout 1 (4 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	1	3	3	71	0.042087
100000	1	3	3	71	0.042087
1000000	1	3	3	71	0.042087
1500000	1	3	3	71	0.042087

Layout 1 only requires 3 expanded nodes in the DFS to achieve a winning solution so a budget of 10000 is sufficient, therefore, increasing the budget has no effect on the solution.

Table 3: Statistics for Layout 2 (7 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	1	8	7	172	0.04055
100000	1	8	7	172	0.04055
1000000	1	8	7	172	0.04055
1500000	1	8	7	172	0.04055

Layout 2 only requires 7 expanded nodes in the DFS to achieve a winning solution so a budget of 10000 is sufficient, therefore, increasing the budget has no effect on the solution.

Table 4: Statistics for Layout 3 (17 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	1	10282	3541	77307	0.045804
100000	1	10282	3541	77307	0.045804
1000000	1	10282	3541	77307	0.045804
1500000	1	10282	3541	77307	0.045804

Layout 3 only requires 3541 expanded nodes in the DFS to achieve a winning solution so a budget of 10000 is sufficient, therefore, increasing the budget has no effect on the solution.

Table 5: Statistics for Layout 4 (32 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	1	2418	1065	27005	0.039436
100000	1	2418	1065	27005	0.039436
1000000	1	2418	1065	27005	0.039436
1500000	1	2418	1065	27005	0.039436

Layout 4 only requires 1065 expanded nodes in the DFS to achieve a winning solution so a budget of 10000 is sufficient, therefore, increasing the budget has no effect on the solution.

Table 6: Statistics for Layout 5 (36 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	4	26495	10000	167112	0.05984
100000	3	359818	100000	325157	0.307543
1000000	2	4488464	1000000	302302	3.307949
1500000	1	4898609	1090275	305827	3.565004

Winning solution found using 1.5M budget, with 1090275 expanded nodes required. Effects of the budget on the number of pegs left is plotted below:

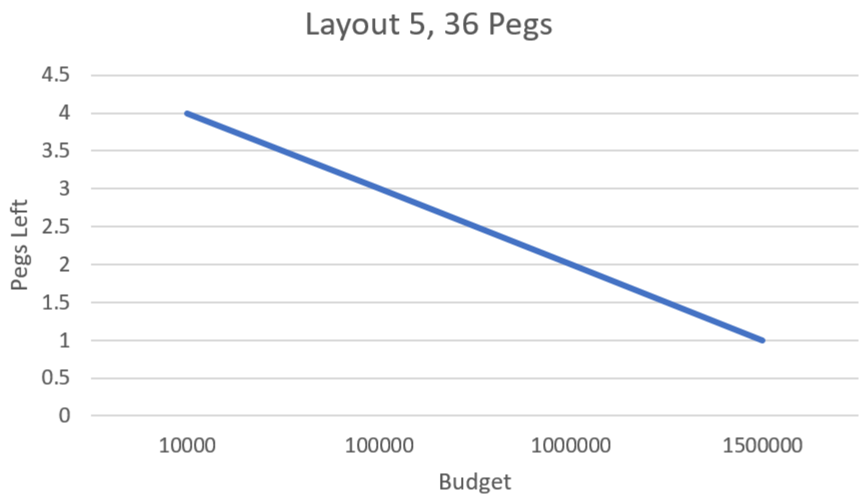


Figure 1: Budget vs number of pegs left for layout 5

Table 7: Statistics for Layout 6 (44 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	5	29368	10000	149485	0.066896
100000	4	374378	100000	324315	0.308342
1000000	3	4481233	1000000	316826	3.156305
1500000	3	7020668	1500000	298157	5.030905

Unable to find winning solution within 1.5M budget. Effects of the budget on the number of pegs left plotted is below:

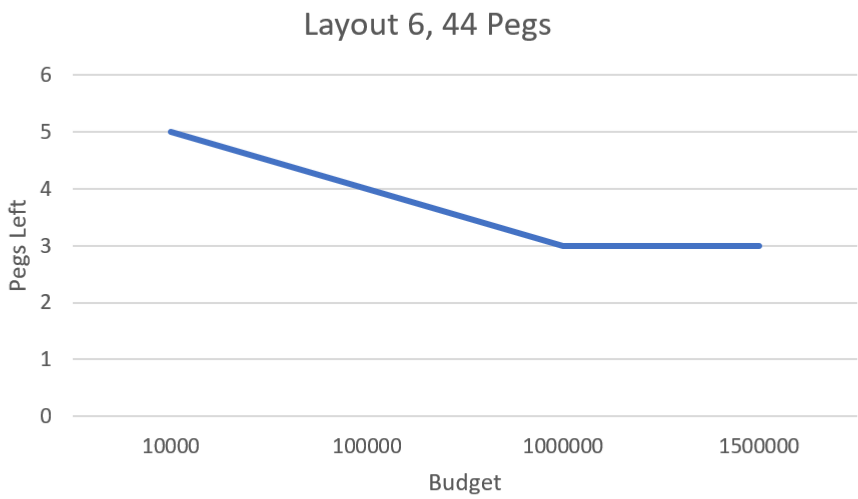


Figure 2: Budget vs number of pegs left for layout 6

Table 8: Statistics for Layout 7 (38 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	4	32469	10000	156071	0.064073
100000	2	386440	100000	314414	0.318051
1000000	2	4790308	1000000	297929	3.356498
1500000	2	7173504	1500000	278817	5.37987

Unable to find winning solution within 1.5M budget. Effects of the budget on the number of pegs left plotted is below:

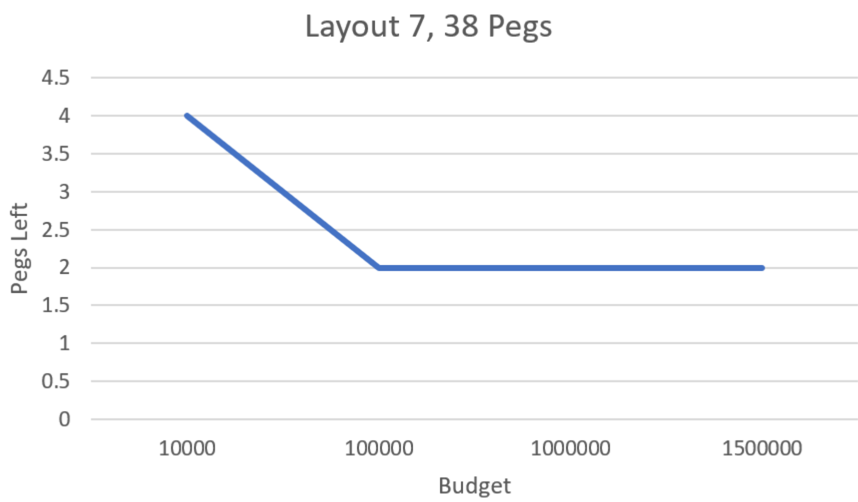


Figure 3: Budget vs number of pegs left for layout 7

Table 9: Statistics for Layout 8 (40 Pegs)

Budget	Pegs Left	Generated Nodes	Expanded Nodes	Expanded/Sec	Execution Time (Sec)
10000	6	27562	10000	167383	0.059743
100000	4	349921	100000	320955	0.31157
1000000	4	4073028	1000000	328191	3.046998
1500000	4	6361454	1500000	299354	5.010775

Unable to find winning solution within 1.5M budget. Effects of the budget on the number of pegs left plotted is below:

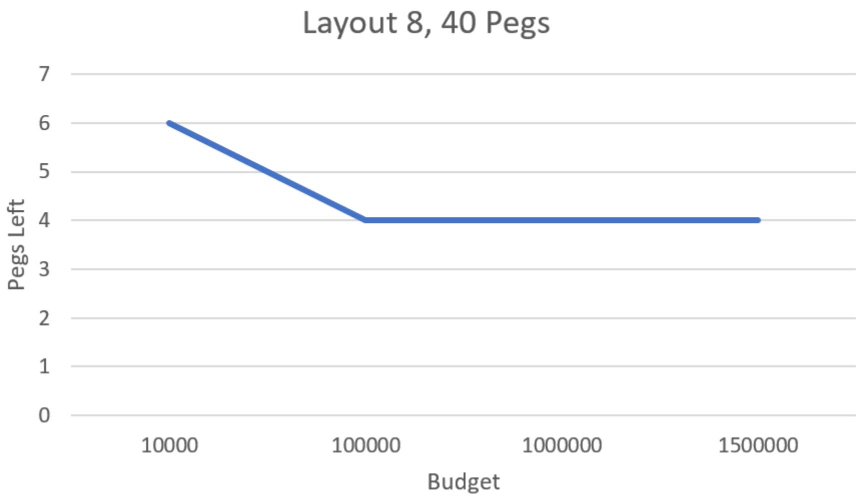
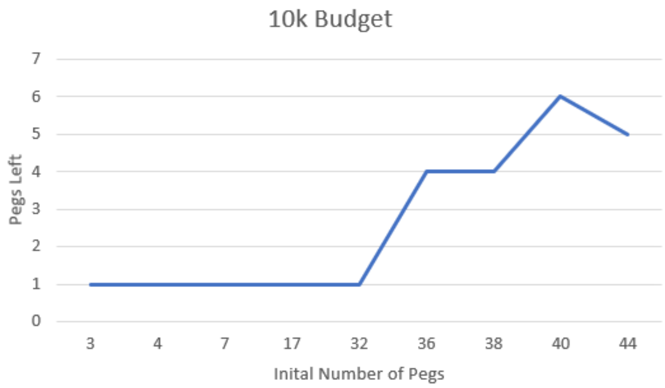
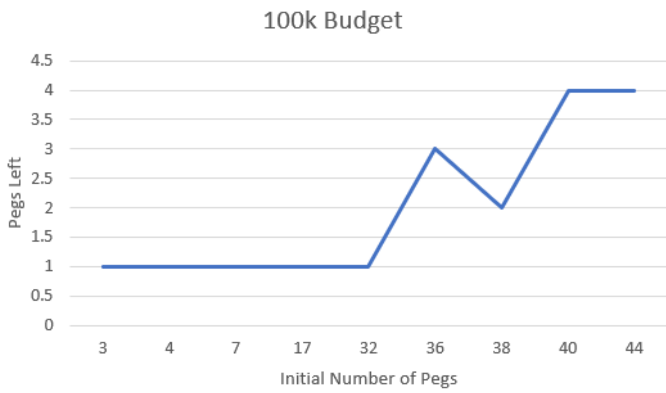


Figure 4: Budget vs number of pegs left for layout 8

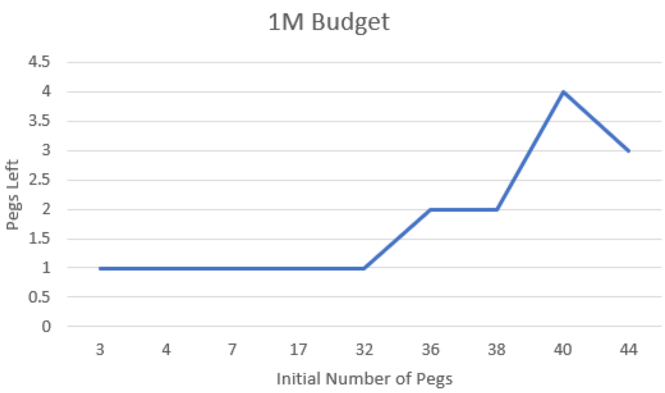
Compiling the data into figures and plotting the effects of the budget on the solution for various layouts:



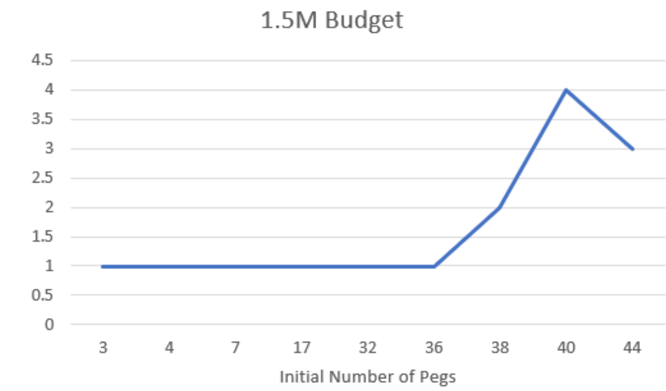
(a) Budget = 10k



(b) Budget = 100k



(c) Budget = 1M



(d) Budget = 1.5M

Figure 5: Initial number of pegs vs pegs left for various budgets

As the budget increases, the number of pegs left move towards 1, however, this convergence generally becomes much slower as the number of initial pegs increase. With a sufficient budget, all the board states (initial pegs) would converge towards one, however, with a high number of initial pegs this computation can quickly become infeasible in terms of time using current computing methods.

2 Discussion

For the first 5 layouts, a budget of 10000 is sufficient to converge to a winning solution, therefore, an increased budget does not change the number of generated nodes. An important note is that the DFS strategy is a brute force method that cycles between the left-right-up-down moves starting from (0,0) to (9,9). This search does not use any heuristics to guide the search towards the solution and will always generate and expand the same number of nodes when executed with the same input.

This DFS strategy means that the initial number of pegs is not the sole factor in determining the runtime, with the initial board layout position and number of permutations playing a large factor in the number of nodes expanded/generated. An example of this is between layout 3 and layout 4, with 17 and 32 pegs respectively. Although layout 3 has almost half the number of pegs as layout 4, it converges to the winning solution with only 1065 node expansions when compared to the 3541 node expansions required for layout 3, a factor of ≈ 3.32. This effect is also seen when comparing the best solution of layout 6 and layout 8 using a budget of 1.5M, where layout 6 contains 44 pegs initially and converges to 3 remaining pegs, however, layout 8 which contains 40 pegs only converges to 4 remaining pegs.

For optimisations, a board position that can be flipped or rotated into another board position are considered identical, so checking this condition which runs in $\theta(n^2)$ time before inserting unique board states into the hash-table and stack will reduce the number of node expansions required.