

CSCI 699 - ProbGen

Probabilistic and Generative Models

Willie Neiswanger

Lecture 11 - Predictive Uncertainty Quantification

Today

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Lecture: Predictive UQ models, including goals/setup/PGMs, Gaussian processes (GPs), and deep uncertainty models (BNNs, ensembles/dropout, etc.)

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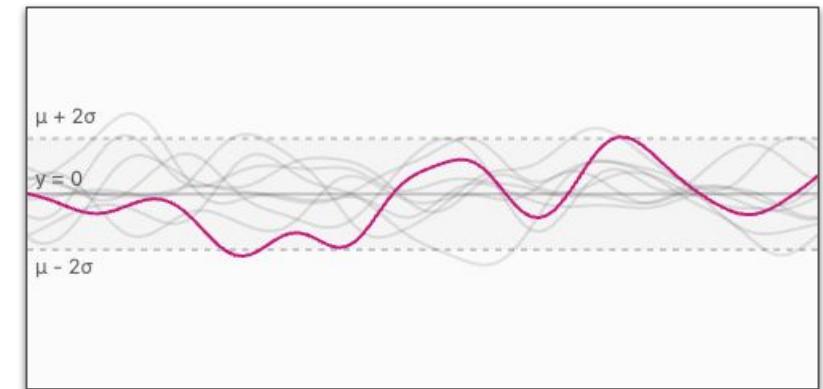
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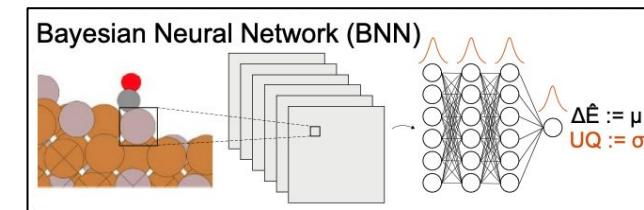
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A Visual Exploration of Gaussian Processes, distill.pub

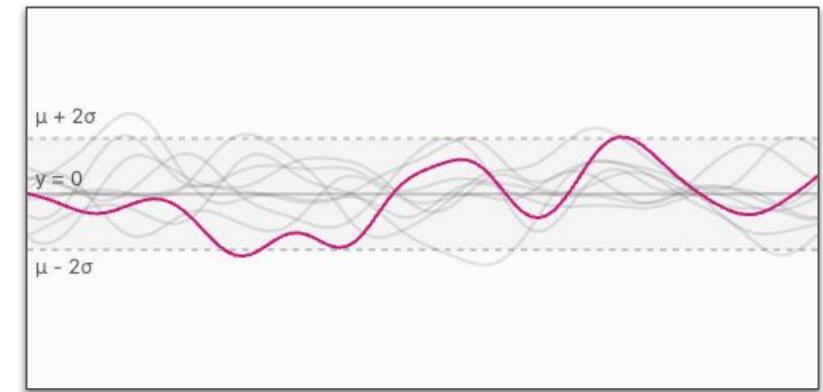


Tran, Neiswanger,
et al., MLST. 2020

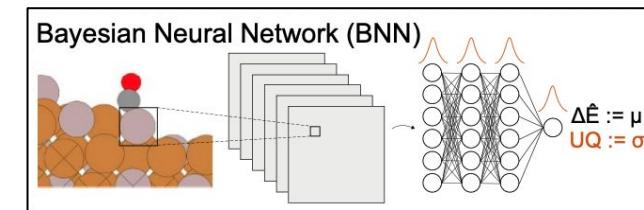
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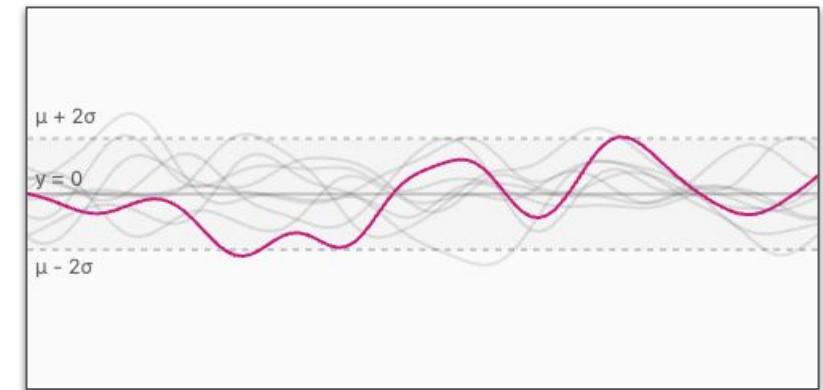


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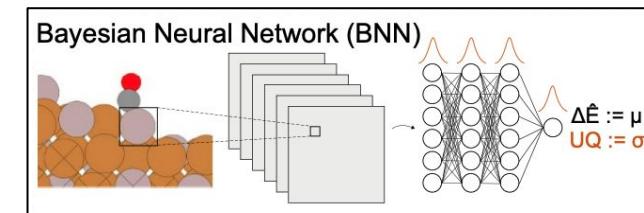
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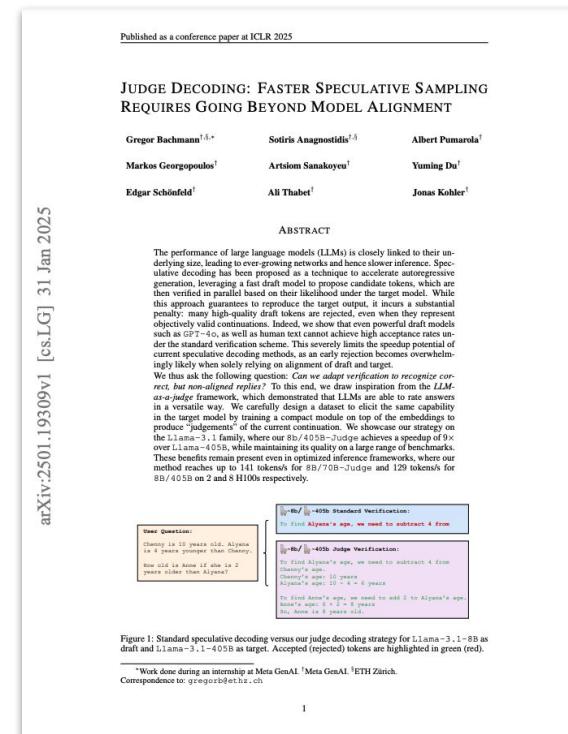
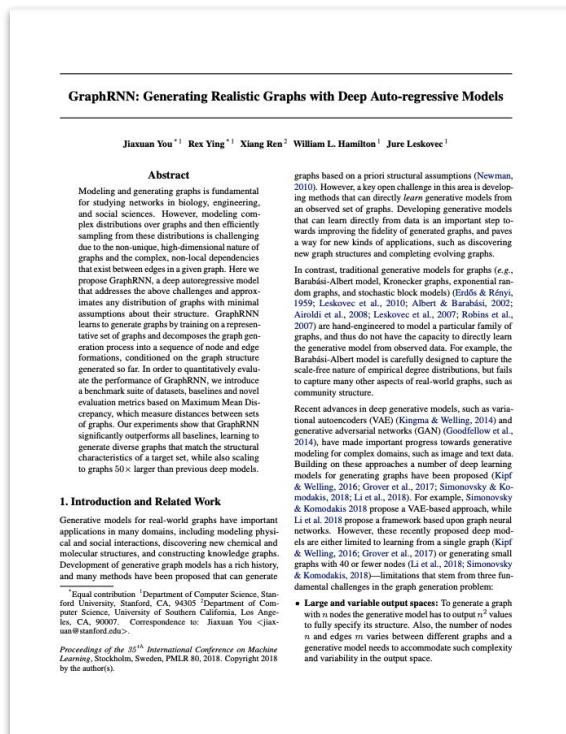
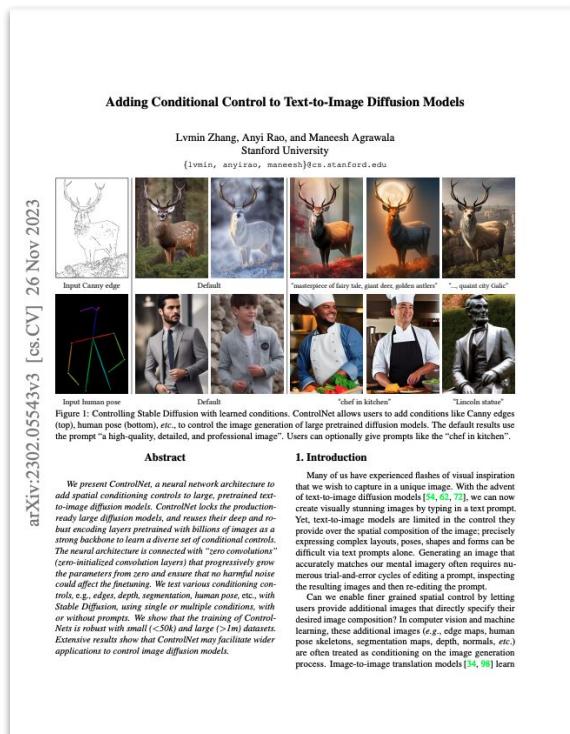
- Next batch of four paper presentations.



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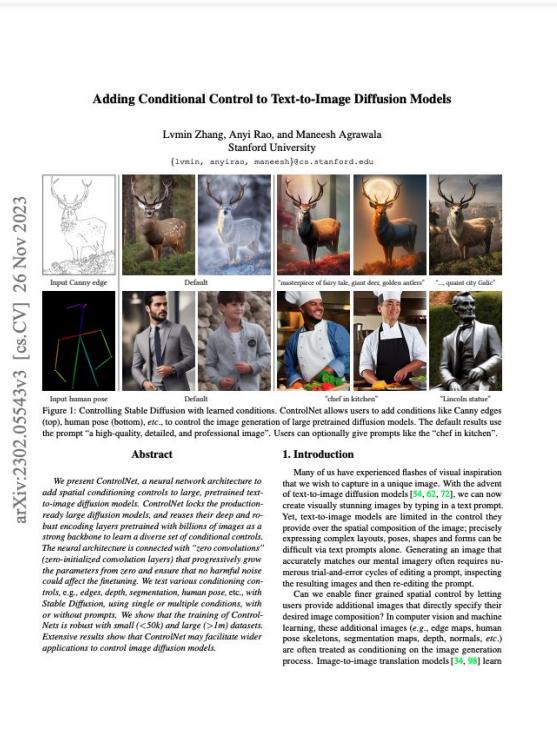
Today

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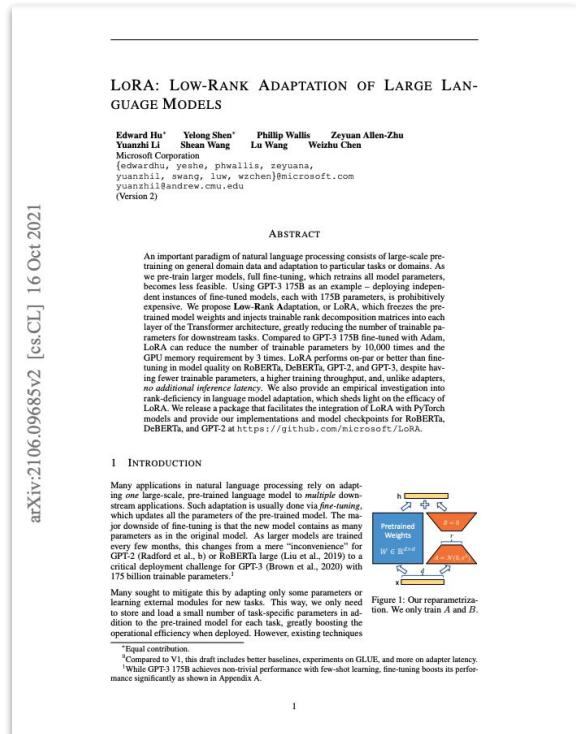
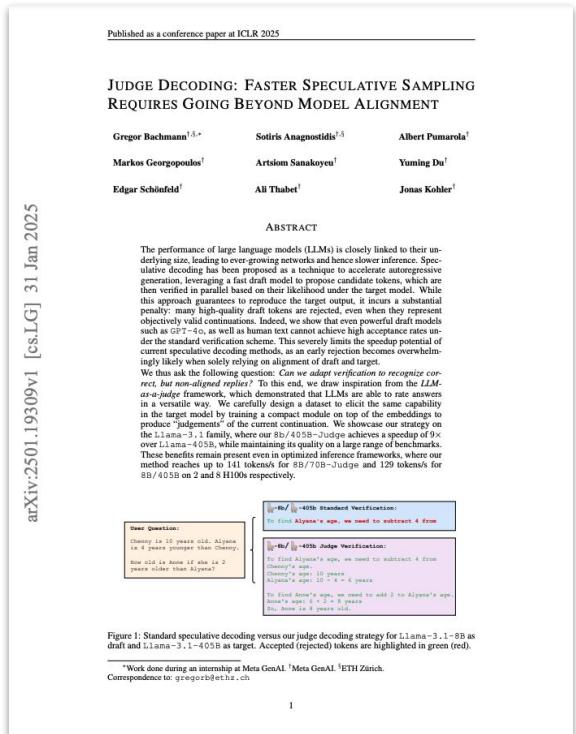
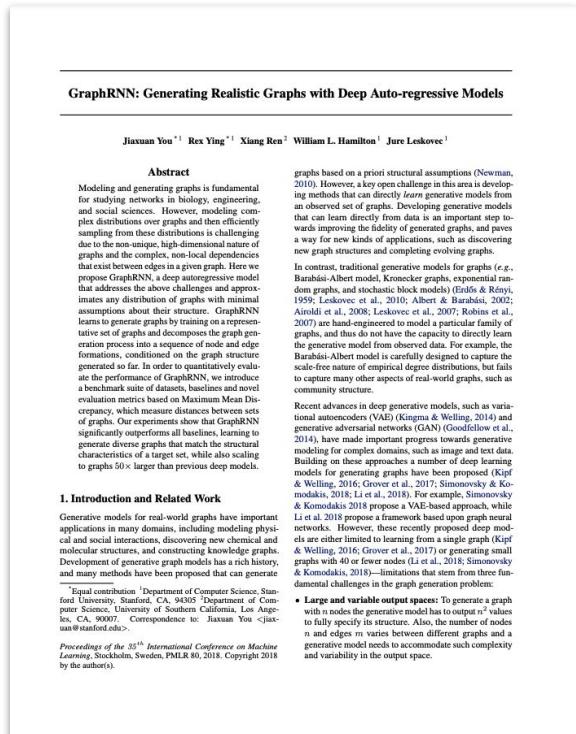


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arXiv:2302.05543v3 [cs.CV] 26 Nov 2023



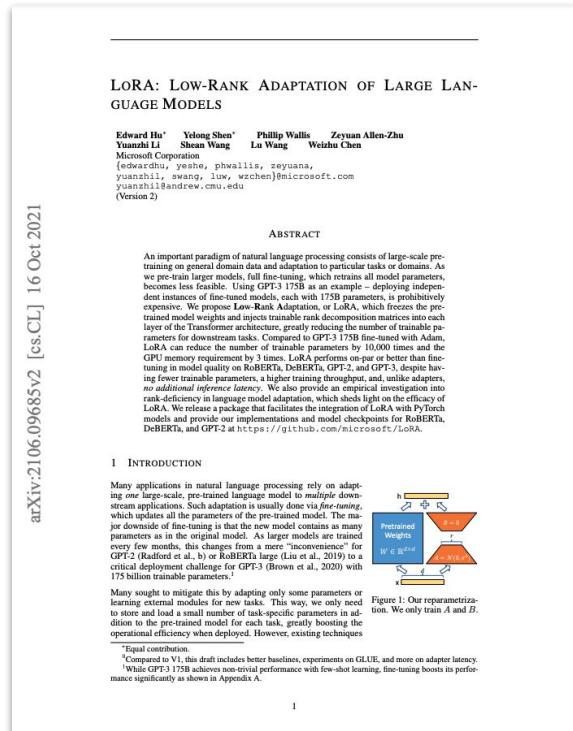
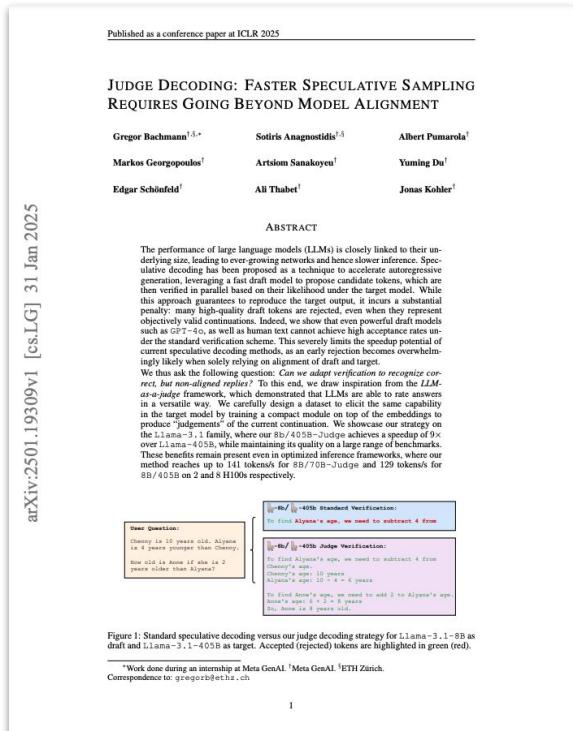
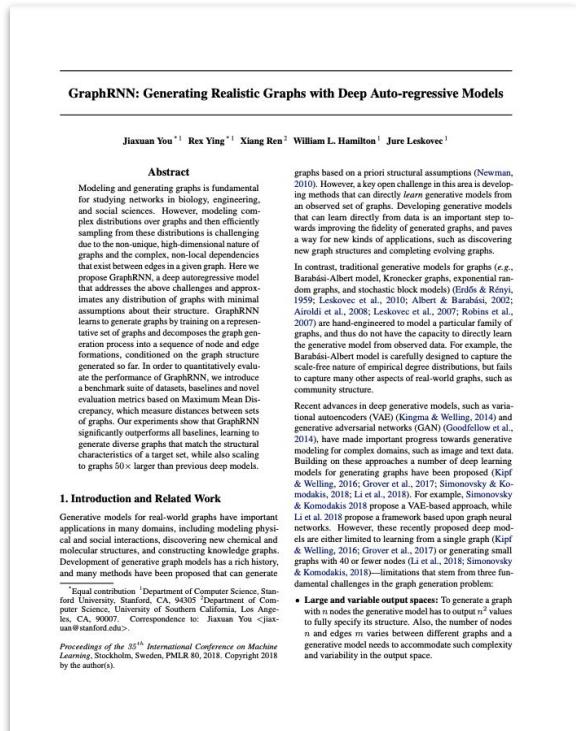
ControlNet – an important conditional gen paper

Today

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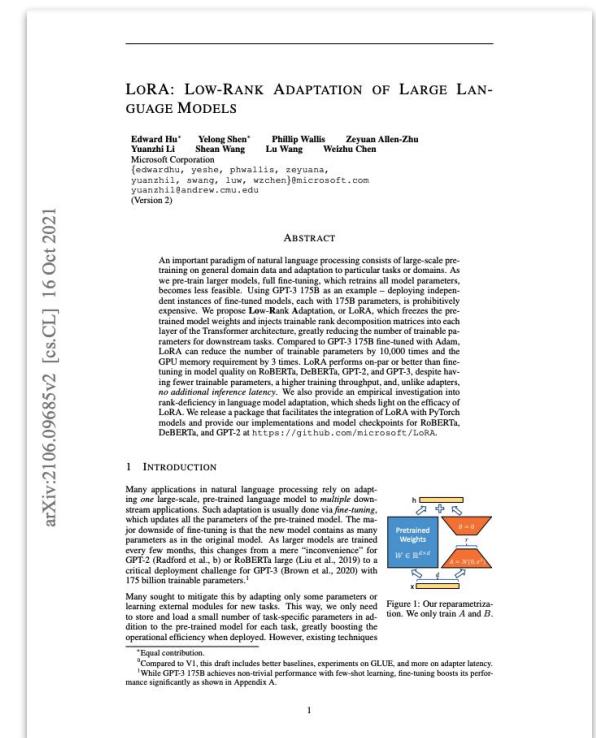
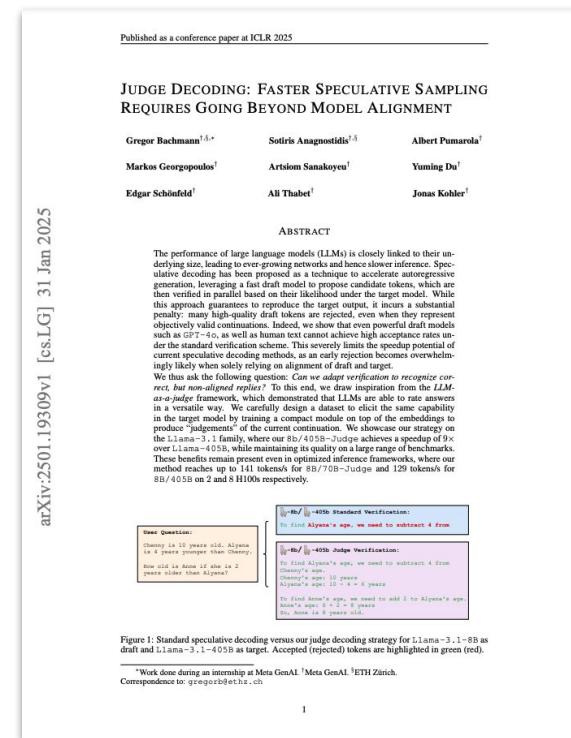
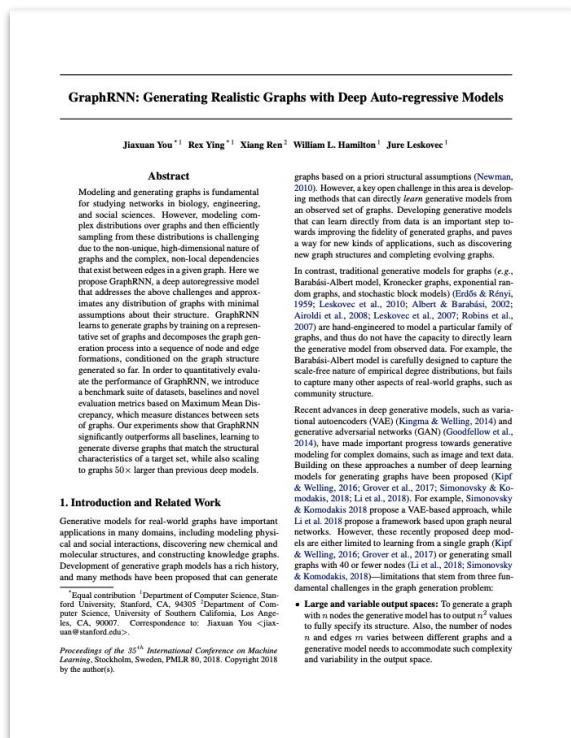
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GraphRNN: generate graphs via autoregression

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Speculative decoding and LLM-as-a-judge

Figure 1: Standard speculative decoding versus our judge decoding strategy for Llama-3-1-40B as draft and Llama-3-1-40B as target. Accepted (rejected) tokens are highlighted in green (red).

*Work done during an internship at Meta GenAI. ¹Meta GenAI. ²ETH Zürich.

Many applications in natural language processing rely on adapting one large-scale, pre-trained language model to multiple downstream applications. Such adaptation is usually done via *fine-tuning*, which updates the parameters of the pre-trained model. The downside of this approach is that the adapted model contains as many parameters as the original model. As larger models are trained every few months, this changes from a mere ‘inconvenience’ for GPT-3 to a critical challenge for large (Lin et al., 2019) to a critical development challenge for GPT-3 (Brown et al., 2020) with 175 billion trainable parameters.¹

Many ways to mitigate this by adapting only some parameters or learning external modules for new tasks. The way we need to do this is to *adapt* the pre-trained model for each task, greatly boosting the operational efficiency when deployed. However, existing techniques

¹Equal contribution.

²Compared to V1, this draft includes better baselines, experiments on GLUE, and more on adapter latency.

While GPT-3 175B achieves non-trivial performance with few-shot learning, fine-tuning boosts its performance significantly as shown in Appendix A.

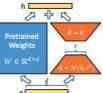
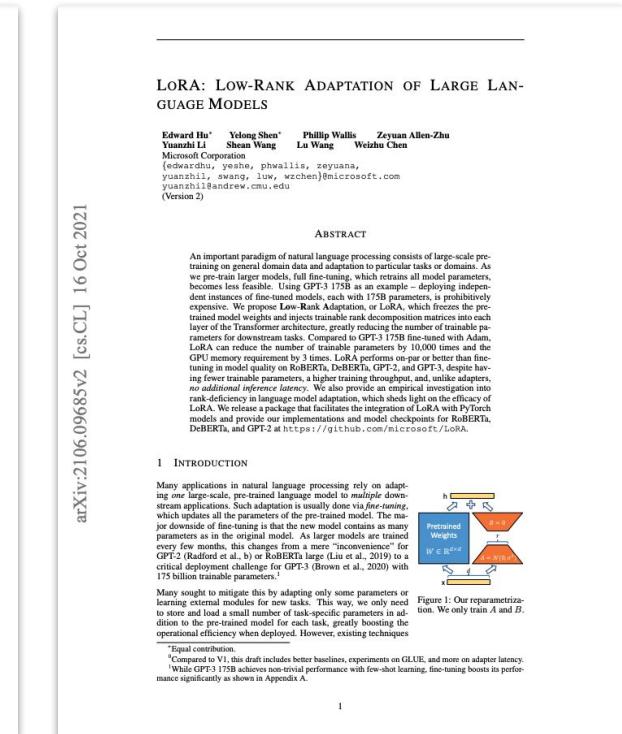
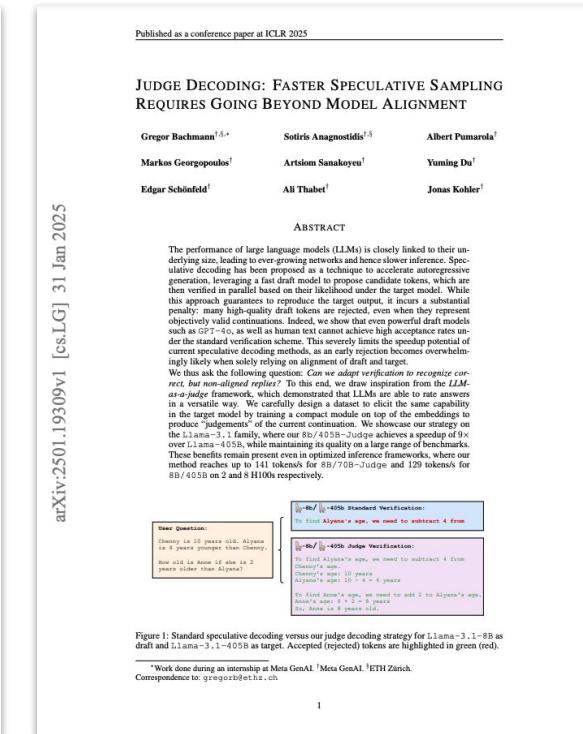
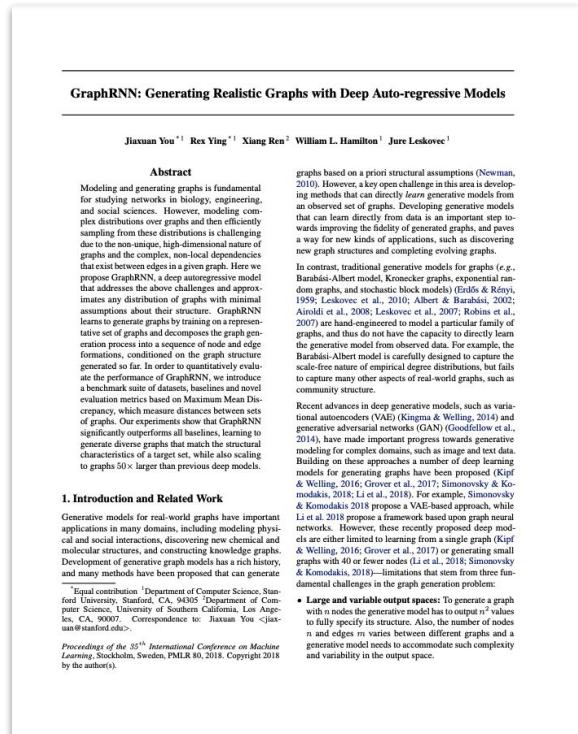


Figure 1: Our reparameterization.

Today

After: Fourth batch of four paper presentations.



LoRA – very popular efficient fine-tuning method



Figure 1: Our reparameterization scheme. Compared to V1, this draft includes better baselines, experiments on GLUE, and more on adapter latency. While GPT-3 175B achieves non-trivial performance with few-shot learning, fine-tuning boosts its performance significantly as shown in Appendix A.

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Quick Aside

Midway Reports and Grading:

- Thanks everyone for submitting your midway reports!
- We are going through these, and aim to give feedback / grades next week.

**Last Class: Final DGM – connections between:
Score-based generative models, diffusion models, and VAEs**

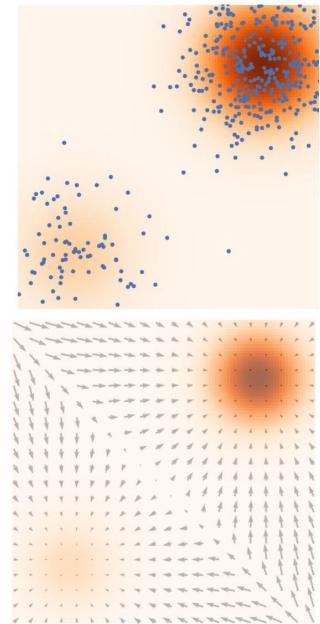
Review – Finished Score Matching

We learned about:

Review – Finished Score Matching

We learned about:

- Score-based generative models via *score matching* and *Langevin Monte Carlo*.
 - Learned by optimizing the Fisher divergence objective.
- Difficulties with sample coverages and score estimation.
- Extensions to noise-conditional score matching (NCSM).
 - Objective: weighted sum of Fisher divergences.
- Sampling via annealed LMC over multiple noise levels.

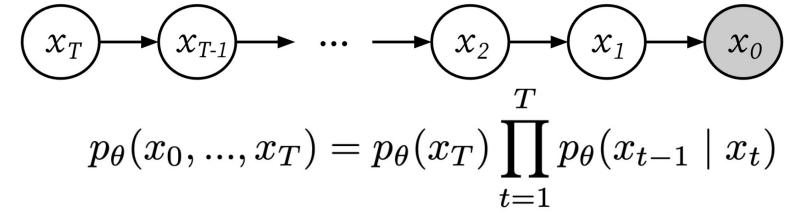


Review – Connections to DDPMs & VAEs

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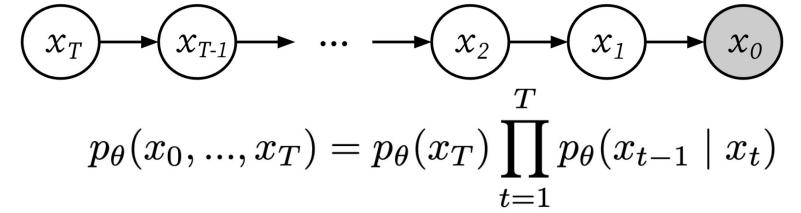
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- Shows optimizing the ELBO \Rightarrow equivalent to the “*sum of Fisher divergences*” loss from before!
 - And that sampling from the forward model \Rightarrow equivalent to Langevin dynamics on a learned score function!

DDPM Samples



Review – Connections to DDPMs & VAEs

- (1) DDPM shows that under a certain parameterization of the model, their algorithm is *equivalent to* denosing score matching.
 - Shows optimizing the ELBO \Rightarrow equivalent to the “*sum of Fisher divergences*” loss from before!
 - And that sampling from the forward model \Rightarrow equivalent to Langevin dynamics on a learned score function!
- (2) Shows that under this parameterization (along with a few additional tricks to better handle image data), sample quality is can be very good.
(Possibly better than most/all previous generative models up until this point).

Review – Explicit vs Implicit Probabilistic Generative Models

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Explicit

Defines a probability model

Then learns parameter of model.

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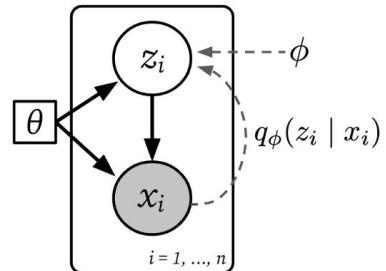
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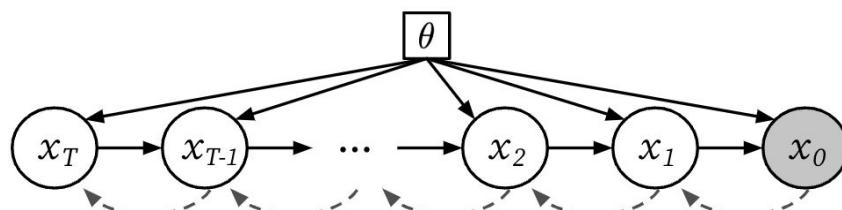
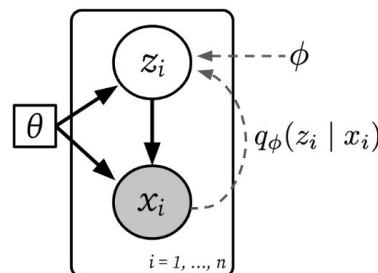
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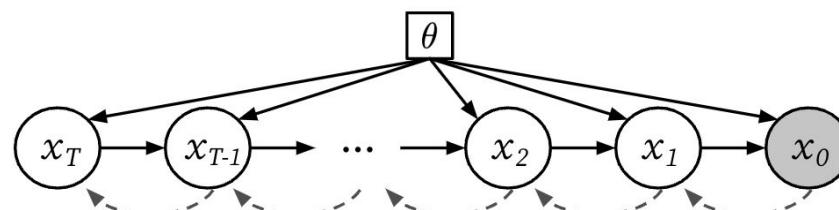
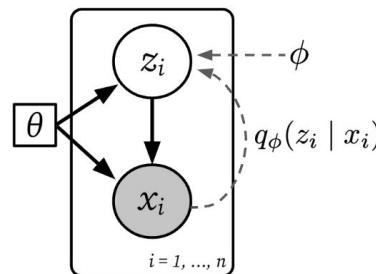
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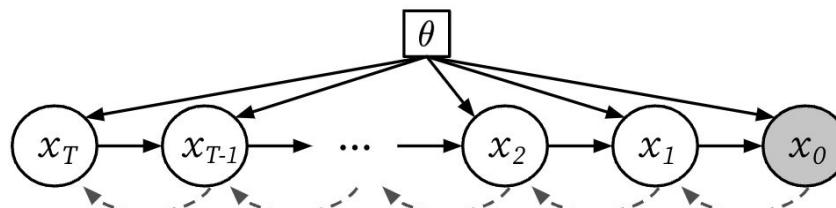
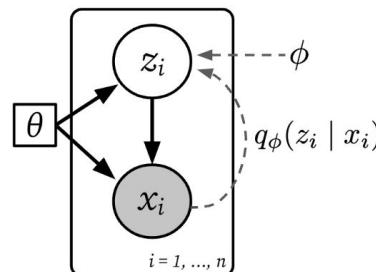
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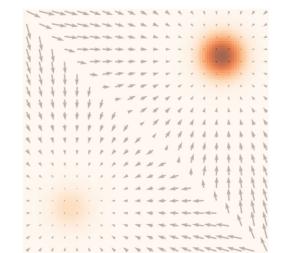
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Score-based Models

$$s(x) = \nabla_x \log p(x)$$



Source: Yang Song

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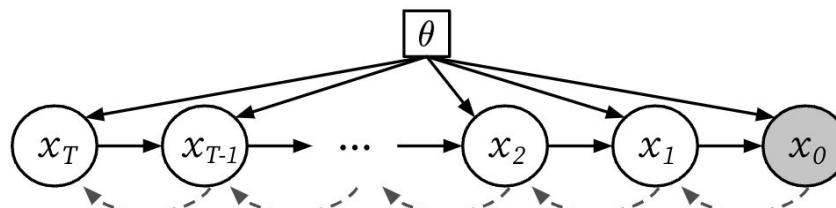
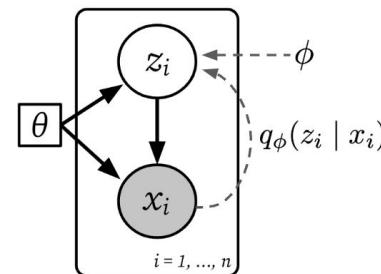
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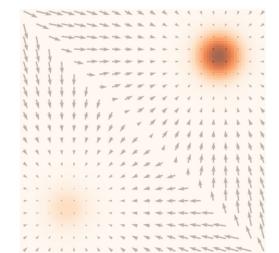
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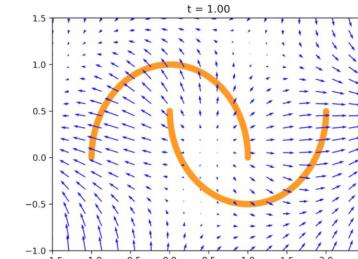
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Flow-matching

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Source: Yang Song



Velocity field

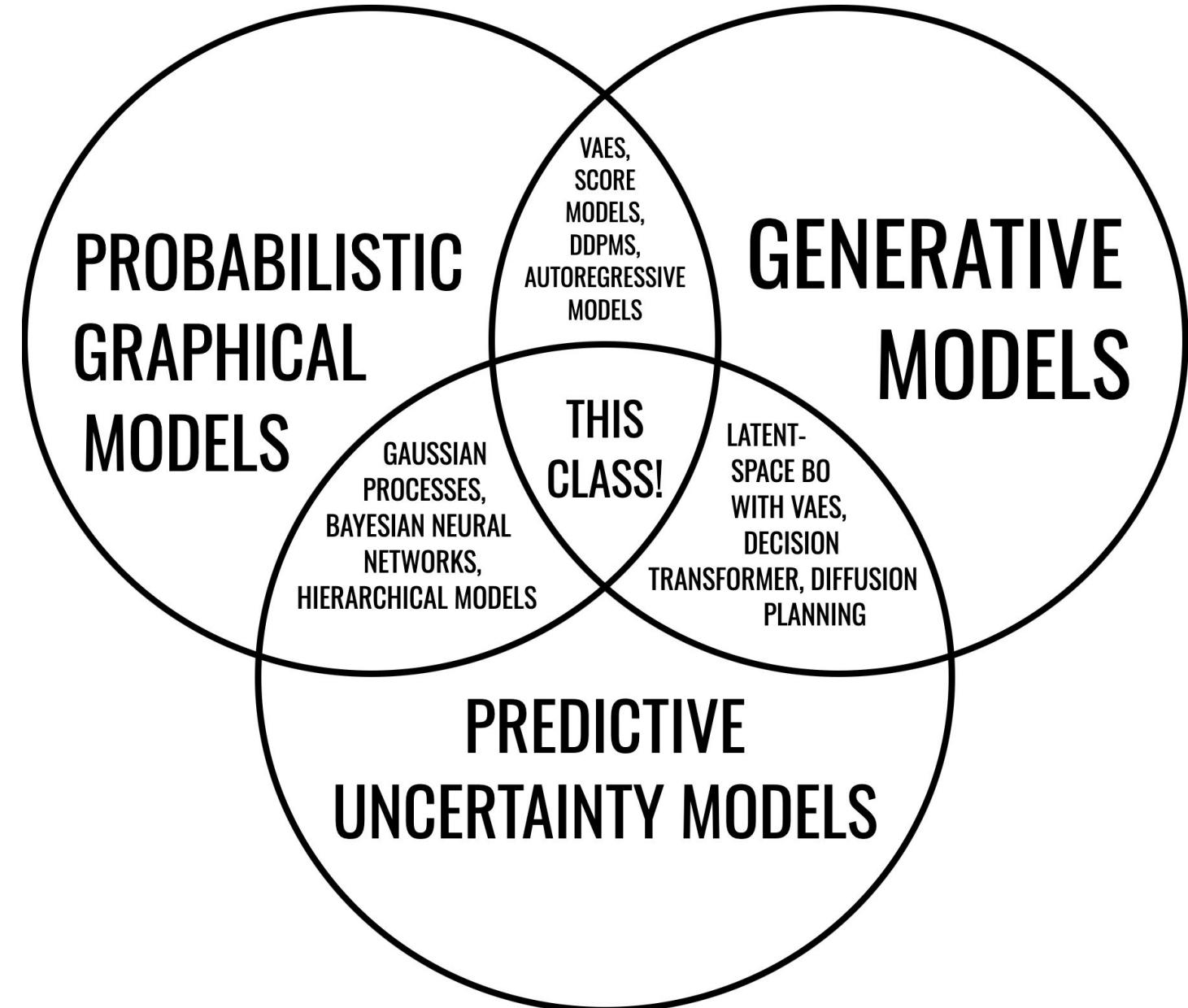
Source: Jakub Tomczak

Predictive Uncertainty Quantification

Class Outline

This course focuses on probabilistic models and their central role within modern machine learning and generative modeling.

This course is inspired by some previous classes...



Deep Generative Models vs Predictive UQ Models

At a high level...

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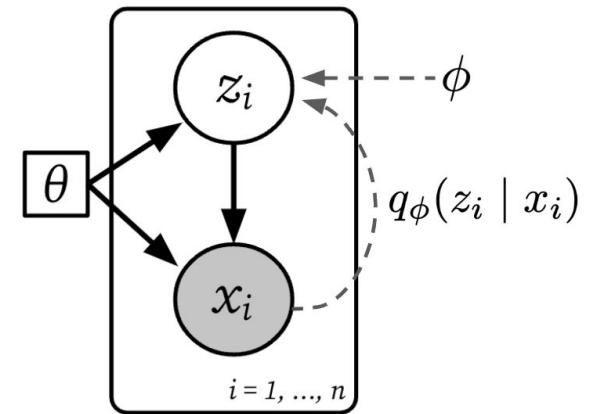
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Typically: defined as a joint PDF over observations and often latent variables – which we can describe via PGMs.



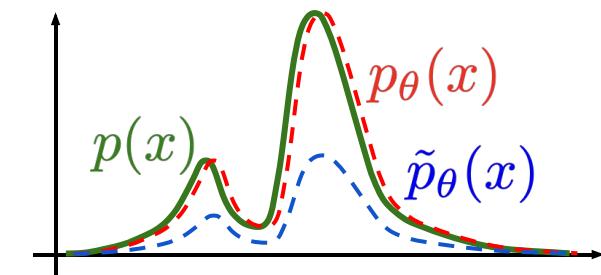
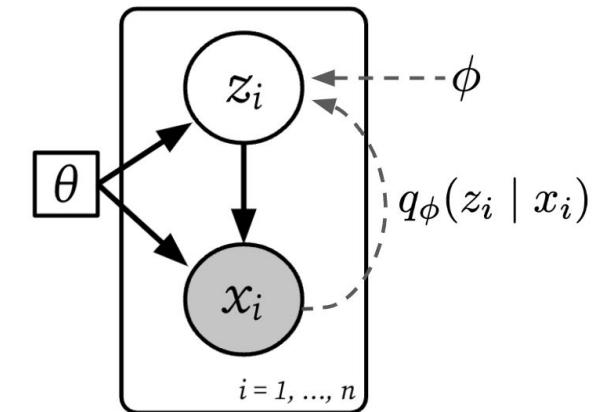
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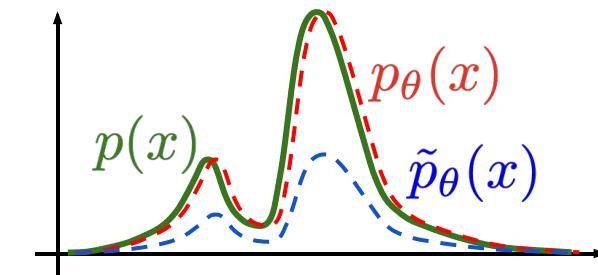
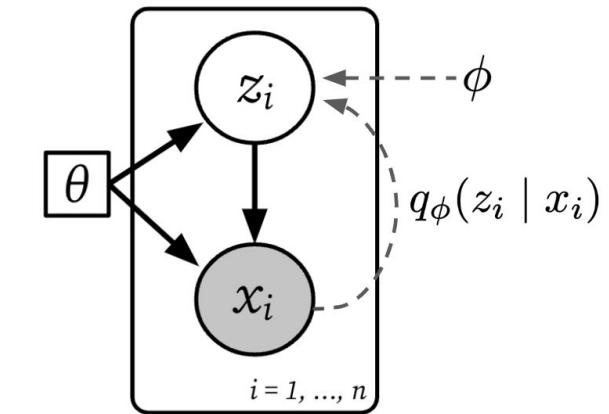
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And this was used in many tasks, e.g., inference queries and generative modeling.

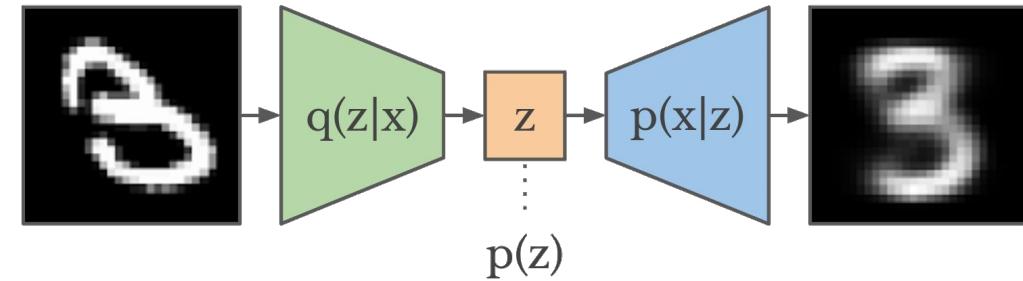
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(Though often supervised learning techniques are used – overall might be viewed as *self-supervised learning*)

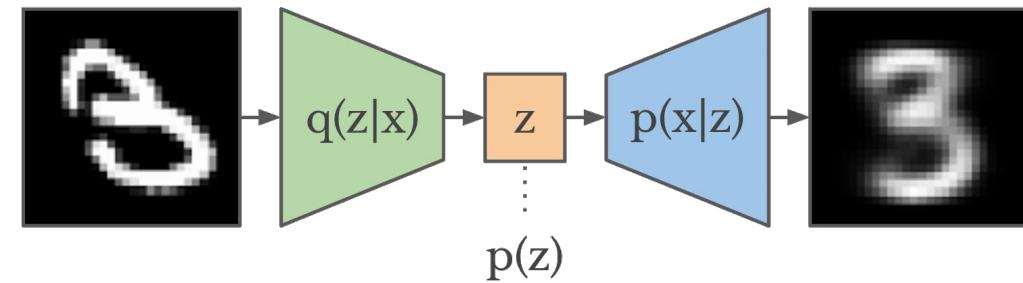


"Building Variational Auto-Encoders in TensorFlow", Danijar Hafner

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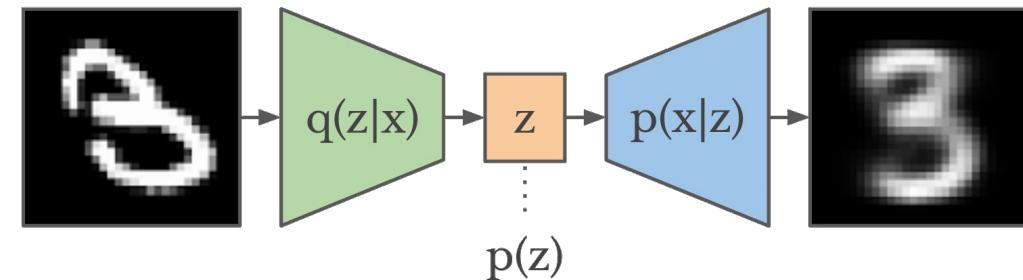
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- Try to learn a function mapping x to y .

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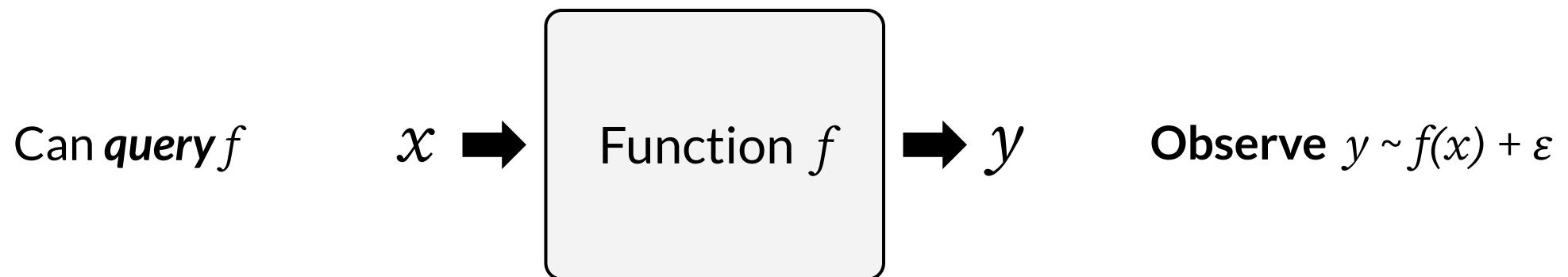
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In this lecture we will be focusing on probabilistic models for supervised learning.

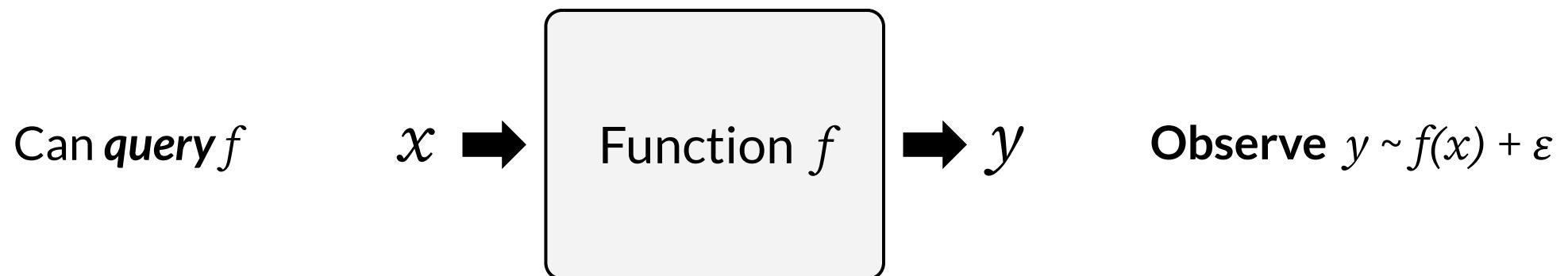
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And let's suppose we have a dataset of n (x, y) pairs:

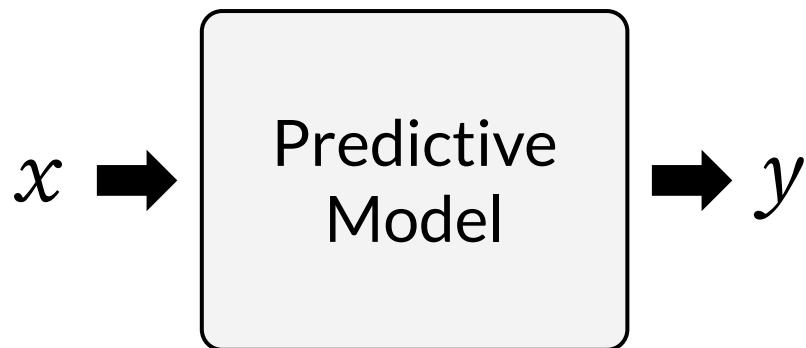
$$D_n = \begin{bmatrix} x_1, y_1 \\ x_2, y_2 \\ \vdots \\ x_n, y_n \end{bmatrix}$$

Deep Generative Models vs Predictive UQ Models

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(E.g., could be either regression or classification, depending on y).

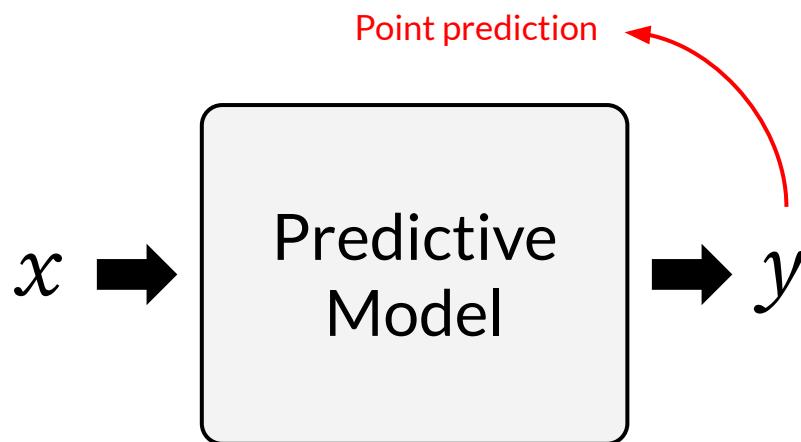
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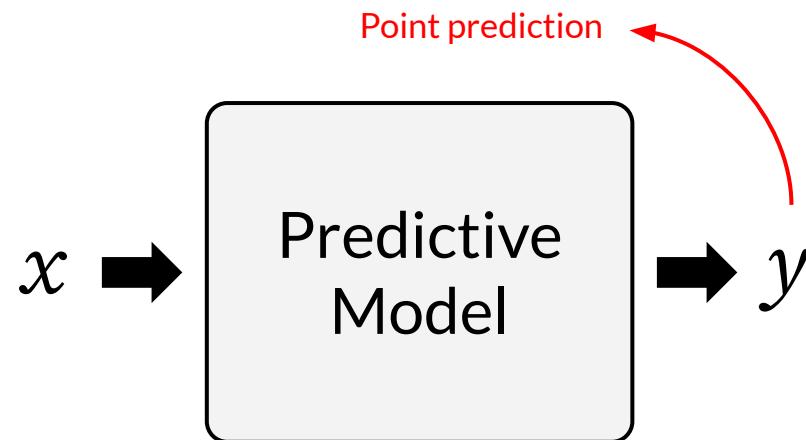
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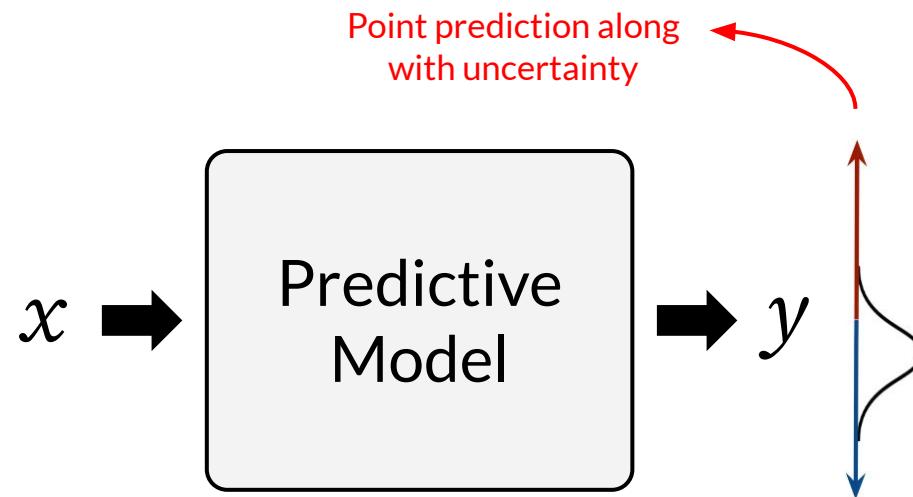
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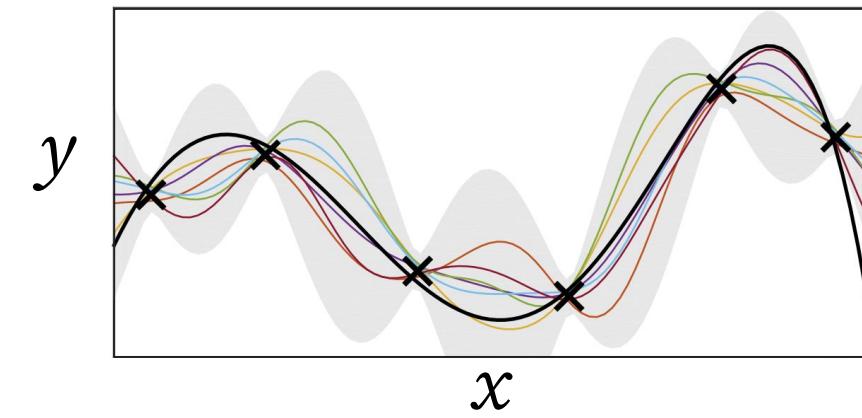
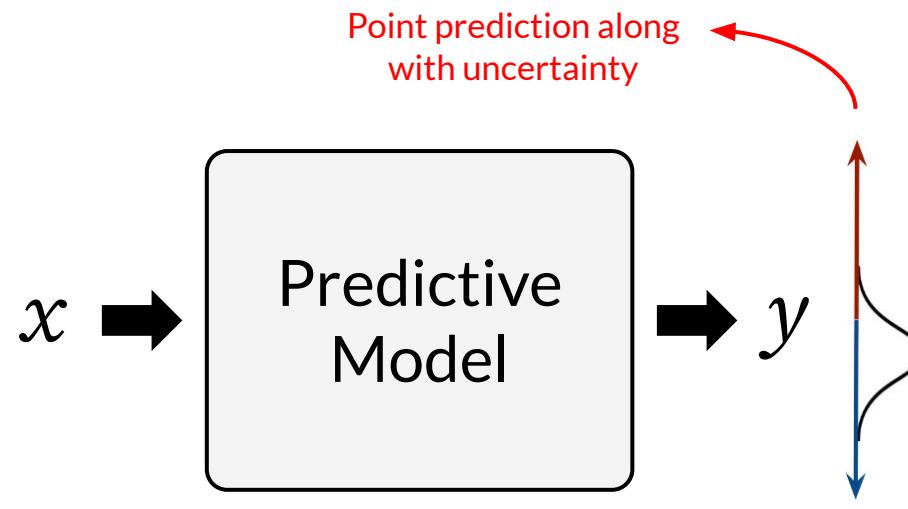
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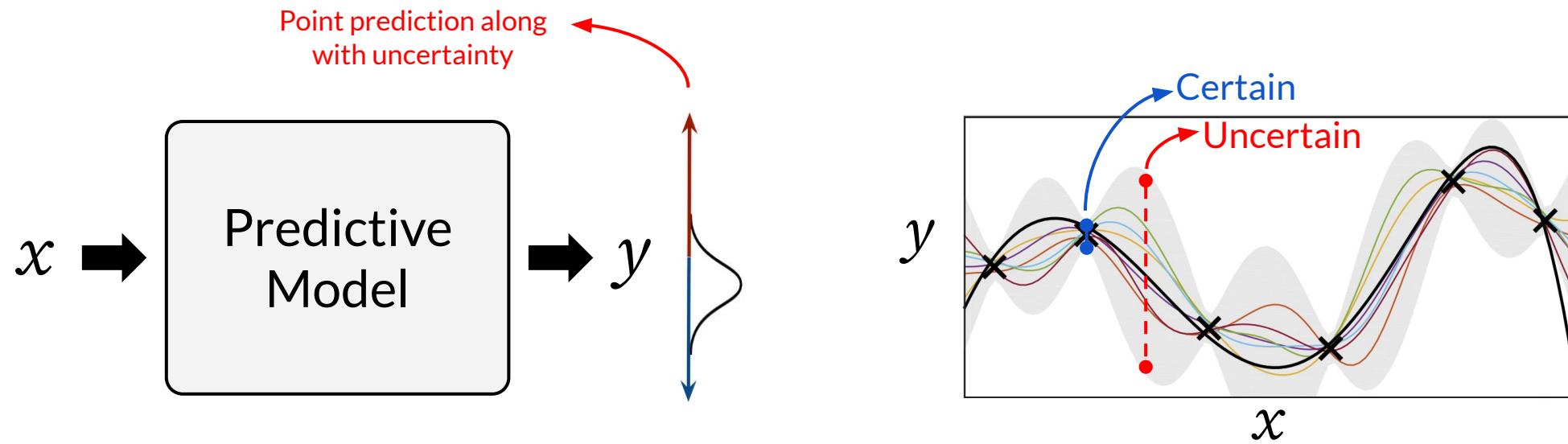
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Predictive UQ Models – Motivation

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- (4) Allows for better *ML + human interaction* and *automation* — we can estimate when AI should do a task and when a human should take over.

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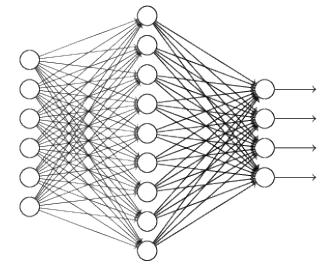
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Let me give a high-level intuition for how predictive UQ is used in decision making:

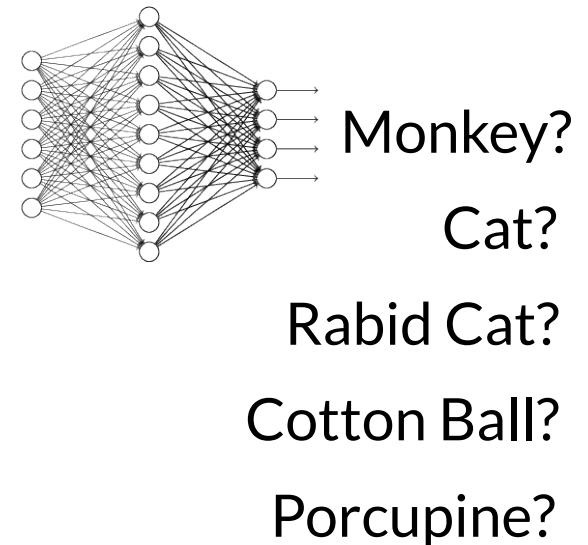
Predictive UQ Models – Motivation

Machine learning is based on predictions – typically point predictions.



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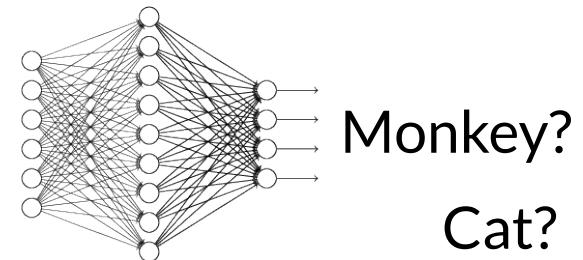
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Predictive UQ Models – Motivation

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Suppose we augment these predictions **with reliable uncertainties** (e.g., with a measure of confidence) ...

We can then balance our uncertainty about possible outcomes to make optimal choices (aka \Rightarrow *decision-making under uncertainty*).

This applies to both classical probabilistic models and probabilistic deep learning models.



"Should I pet it?"

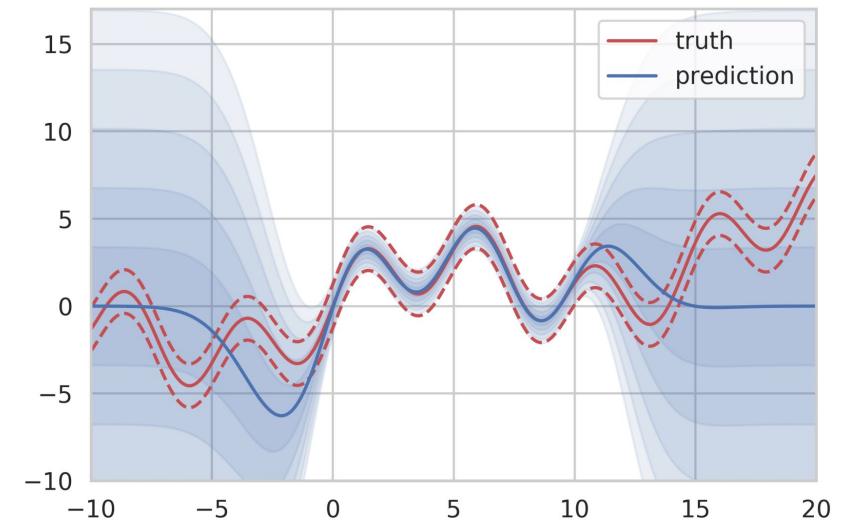
Predictive UQ Models – Decision Making Under Uncertainty

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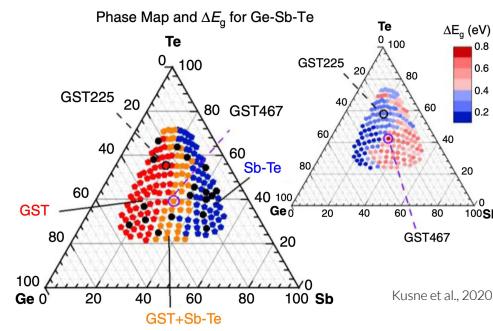
E.g., using probabilistic models for optimization, experimental design, and active learning.



Predictive UQ Models – Decision Making Under Uncertainty

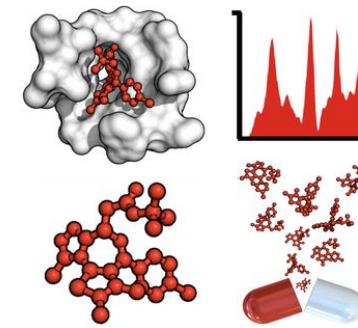
SCIENCE

Materials design



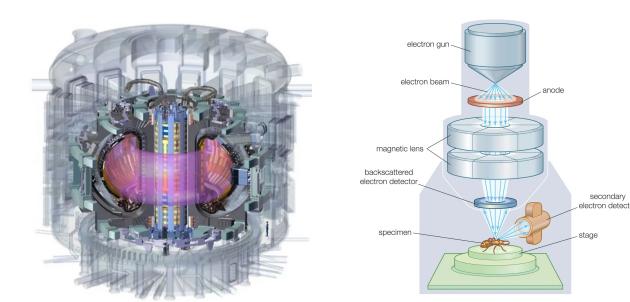
Materials design over large spaces of compounds.

Drug discovery



Target identification, lead optimization.

Large Science Machines

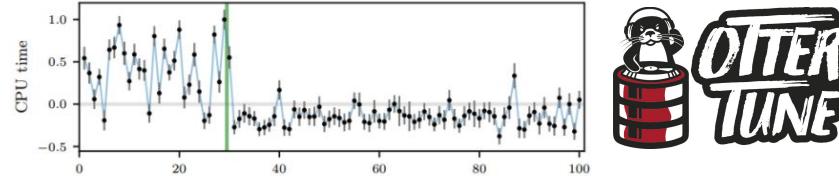


Optimize settings & controls for science use-cases.

Predictive UQ Models – Decision Making Under Uncertainty

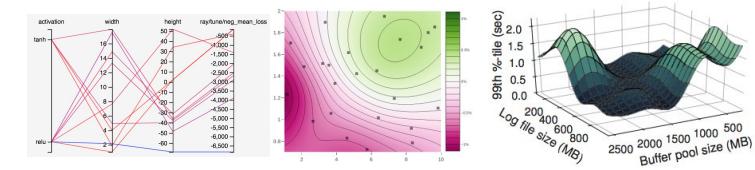
ENGINEERING

Computer Systems



Monitor and tune configurations for performance (latency, throughput).

Machine Learning



Find and assess new methods, models, architectures, hyperparameters.

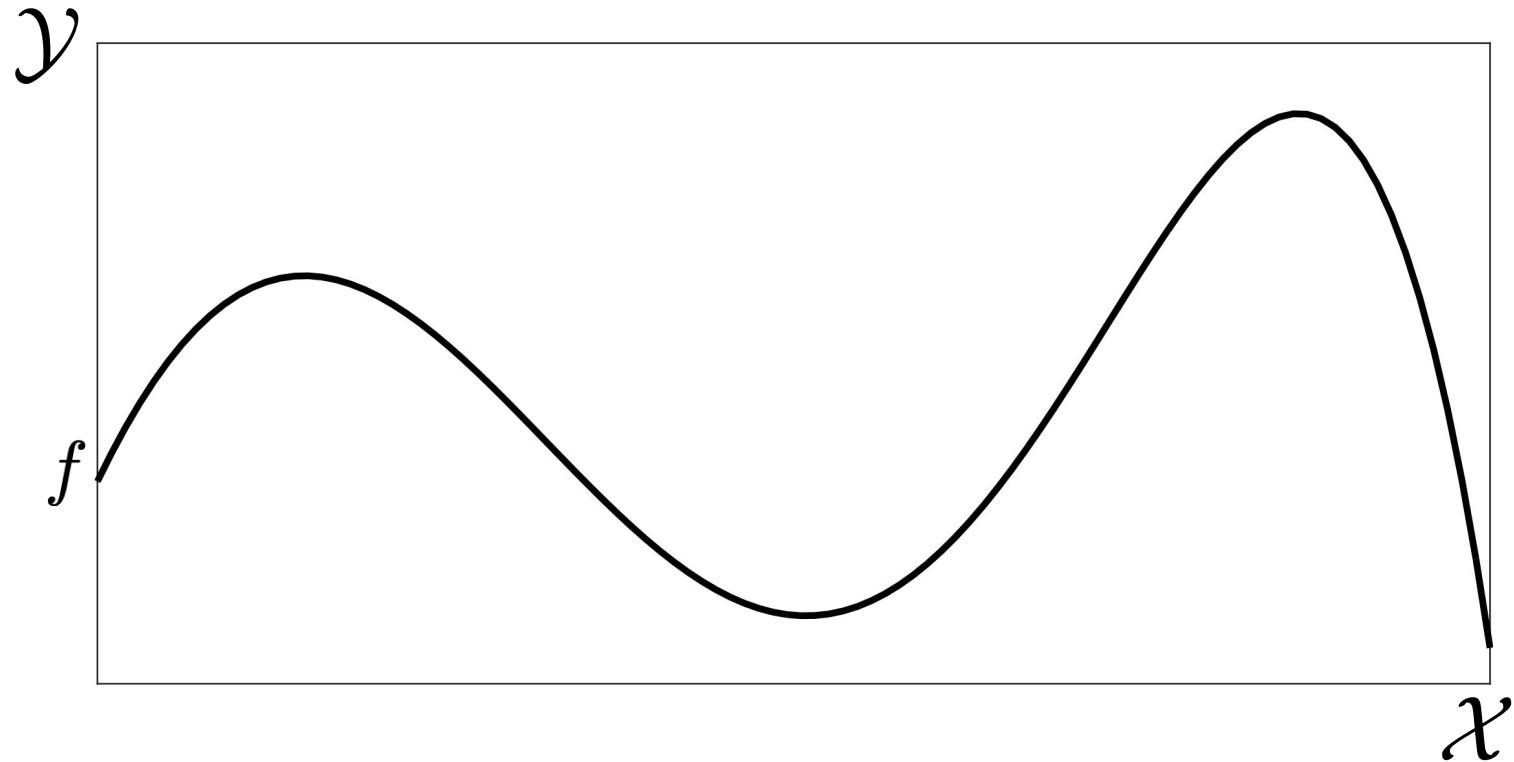
Predictive UQ Models – Background and Typical Setup

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Suppose we have an input space (*domain*) \mathcal{X} , output space \mathcal{Y} , and a function f .

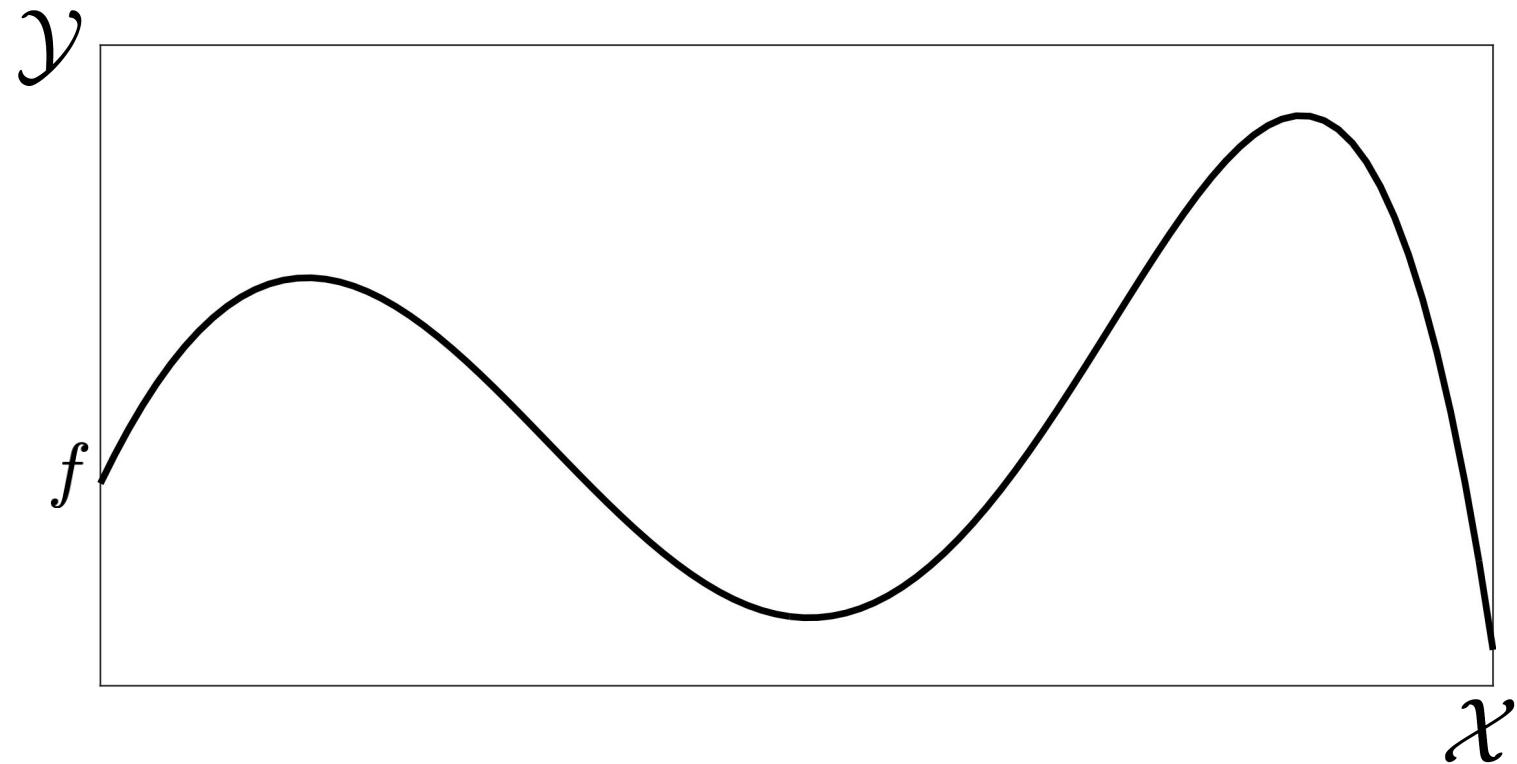
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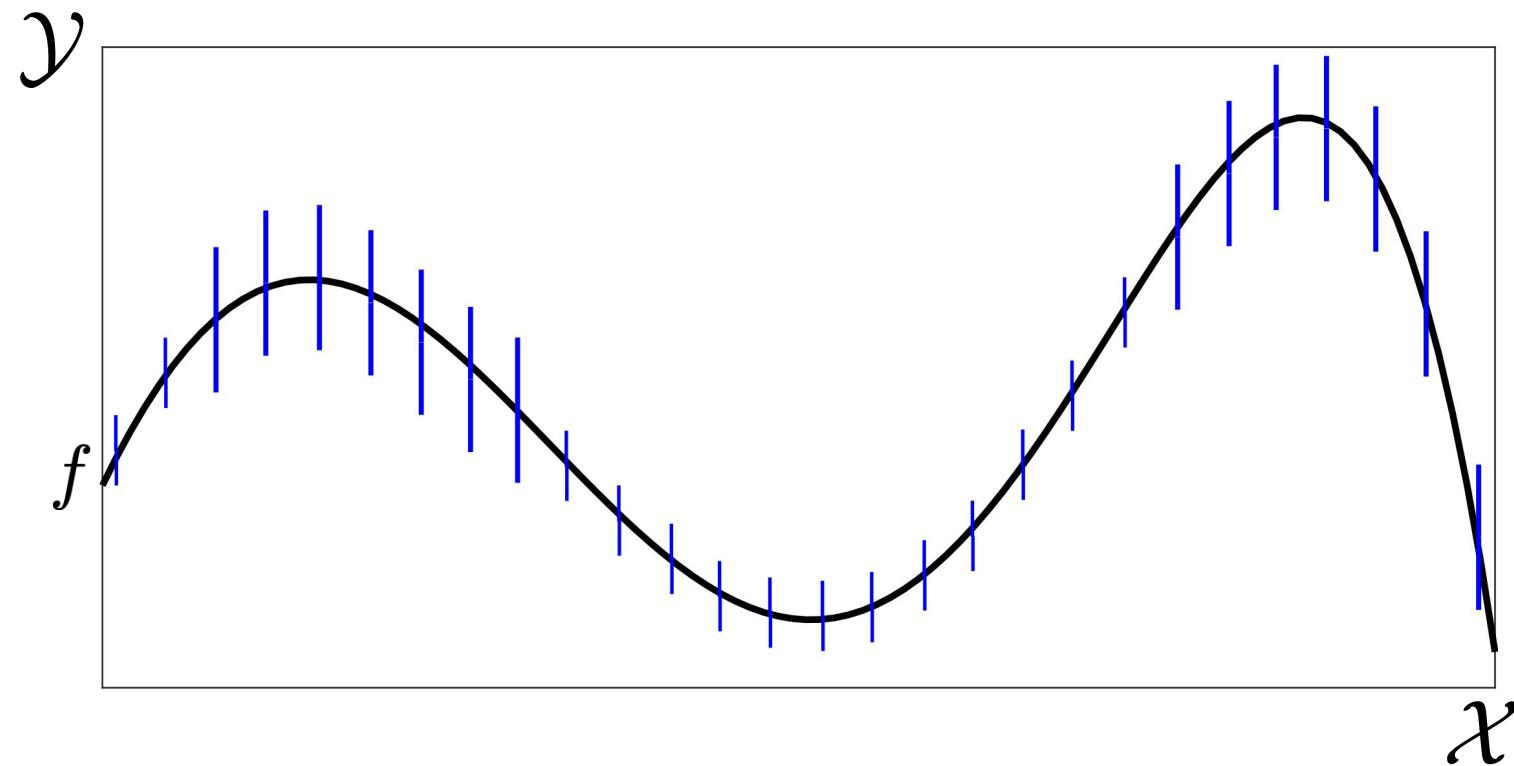
It is a **noisy function** \Rightarrow for each $x \in \mathcal{X}$, there is a distribution over $y \in \mathcal{Y}$.



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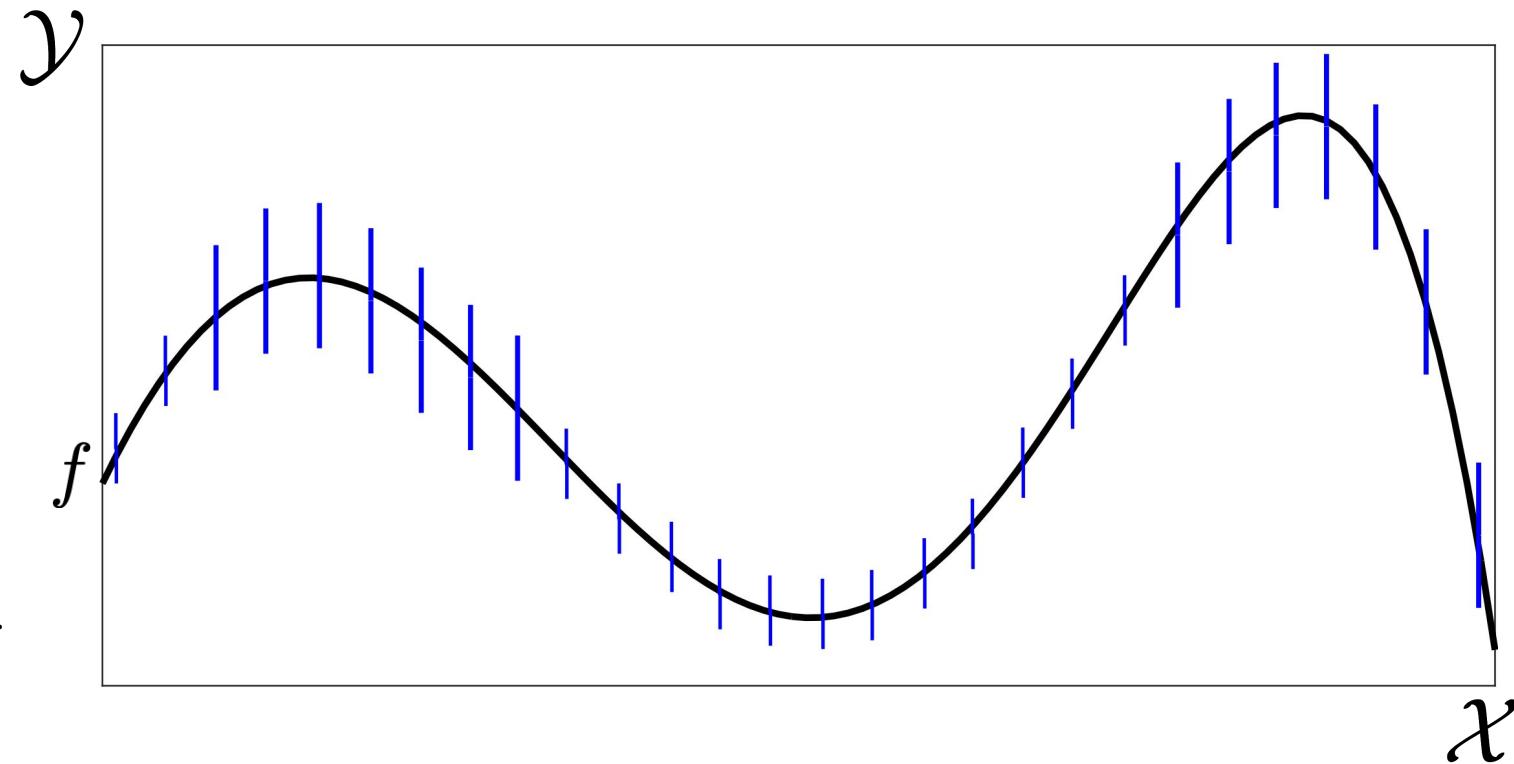
Often written

$$y \sim f(x) + \epsilon(x)$$

where:

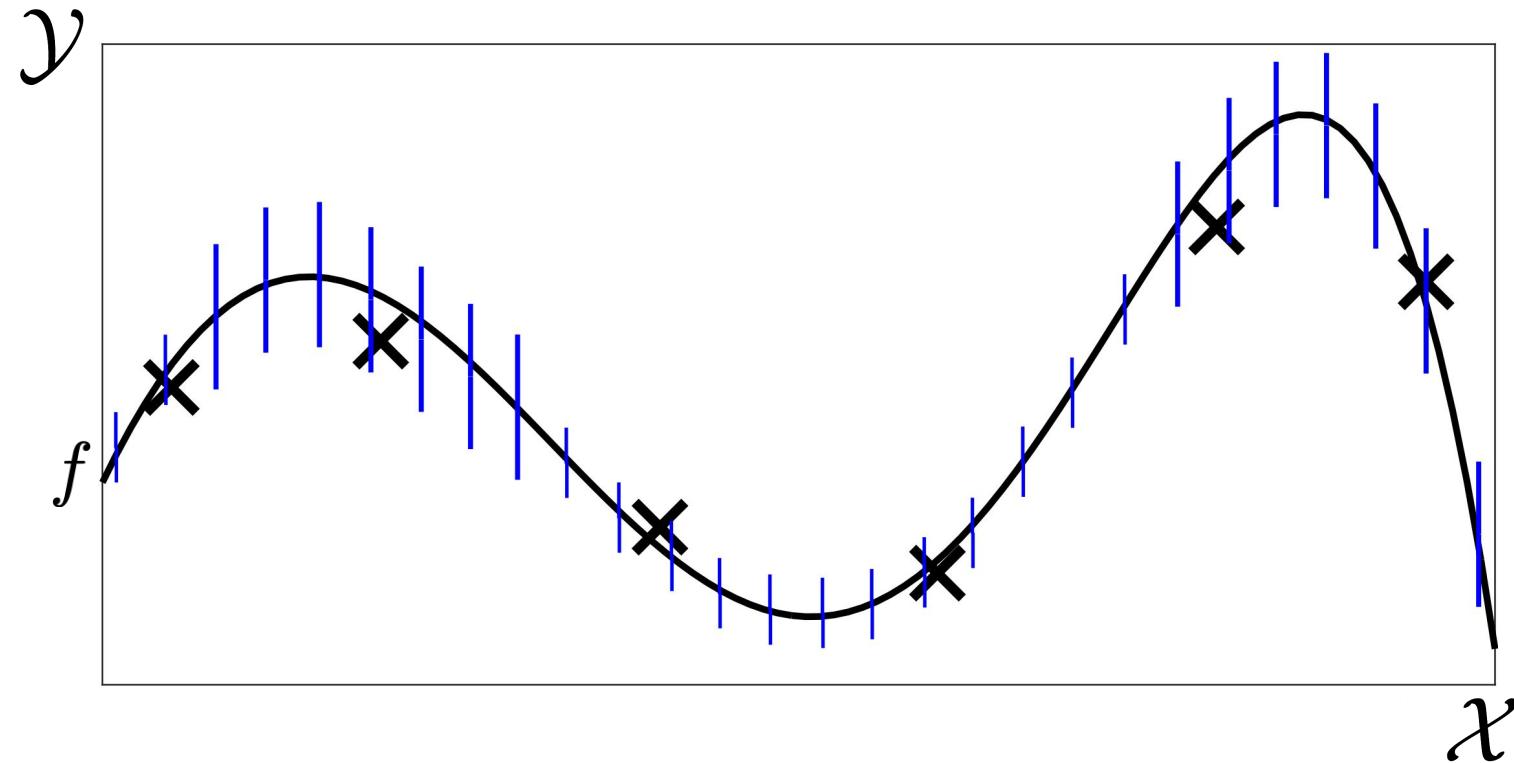
$f(x)$ Mean function
(deterministic)

$\epsilon(x)$ Zero-mean noise
(random variable).



Predictive UQ Models – Background and Typical Setup

Observations are sampled from this noisy function to yield the **dataset** $D_n = \begin{Bmatrix} x_1, y_1 \\ x_2, y_2 \\ \vdots \\ x_n, y_n \end{Bmatrix}$

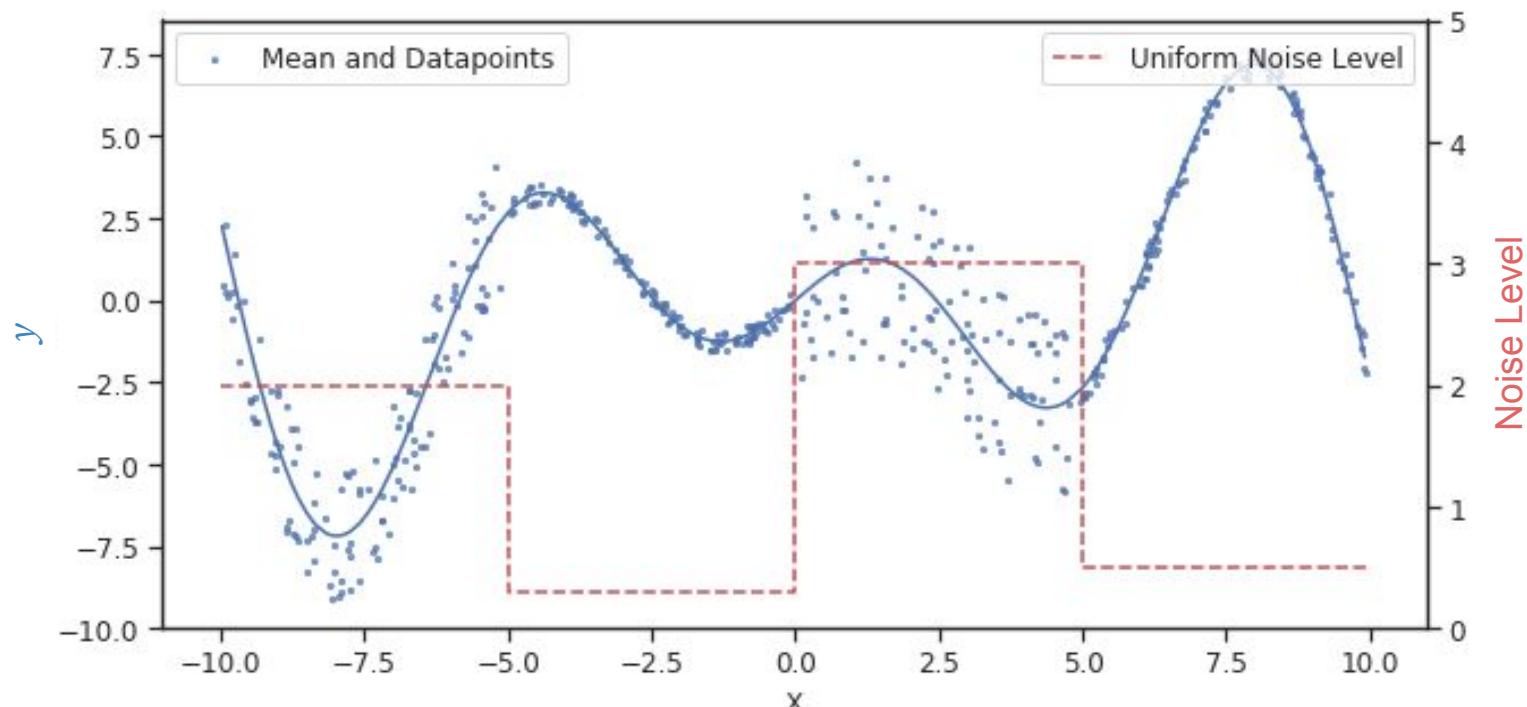


Predictive UQ Models – Background and Typical Setup

An aside:

Predictive UQ Models – Background and Typical Setup

An aside: noisy function may be **heteroscedastic**, i.e., noise non-uniform wrt inputs x .



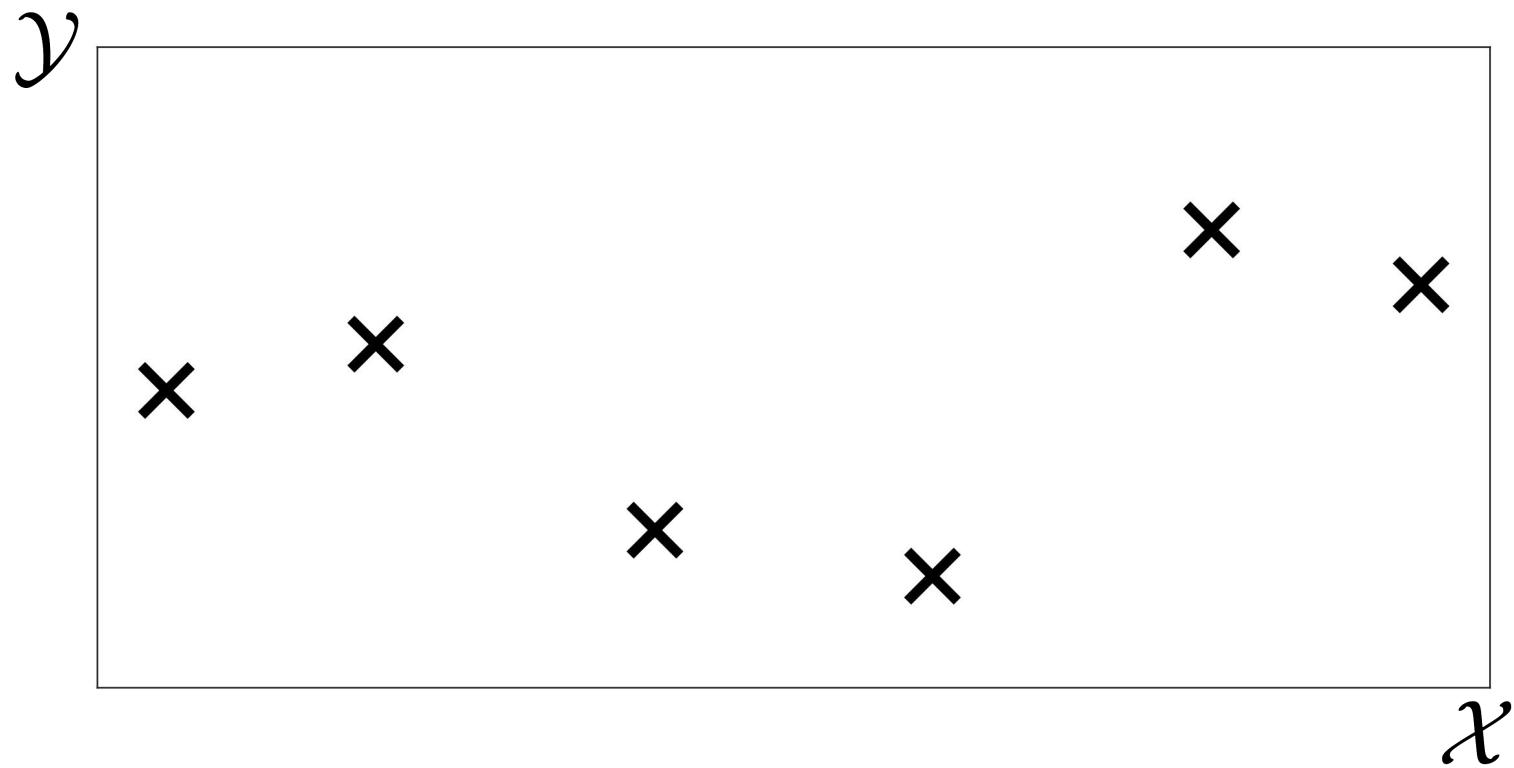
Source: Youngseog Chung

Predictive UQ Models – Background and Typical Setup

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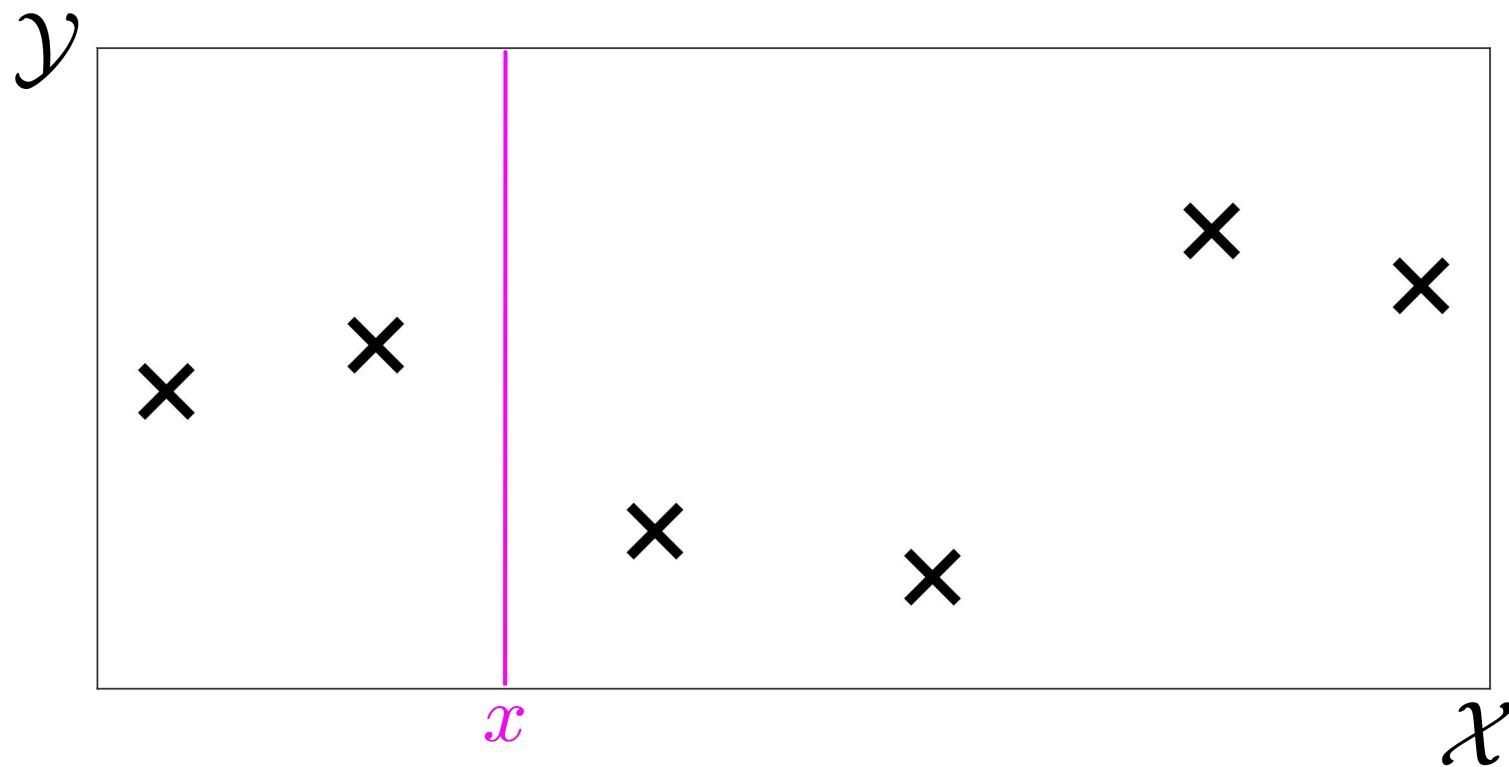
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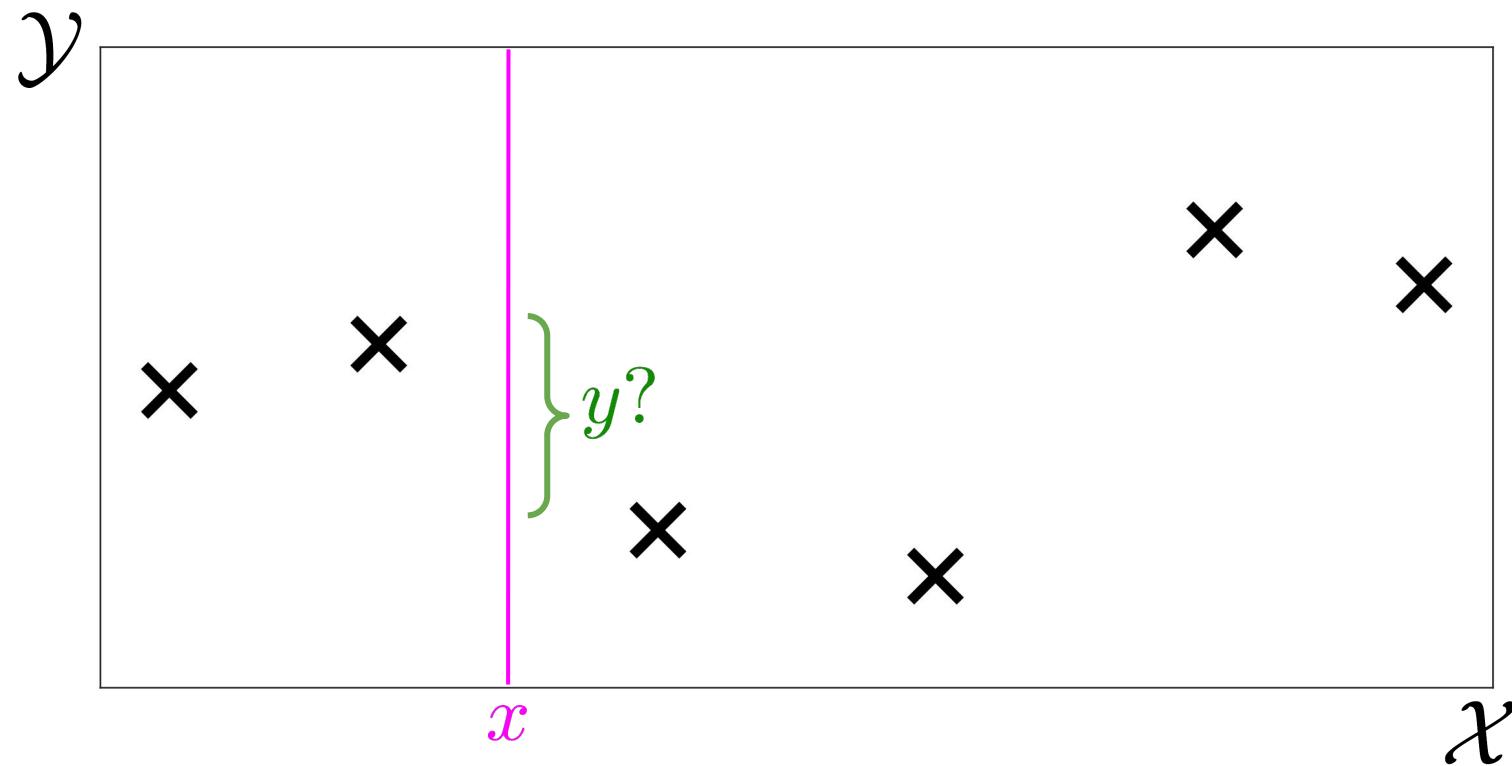
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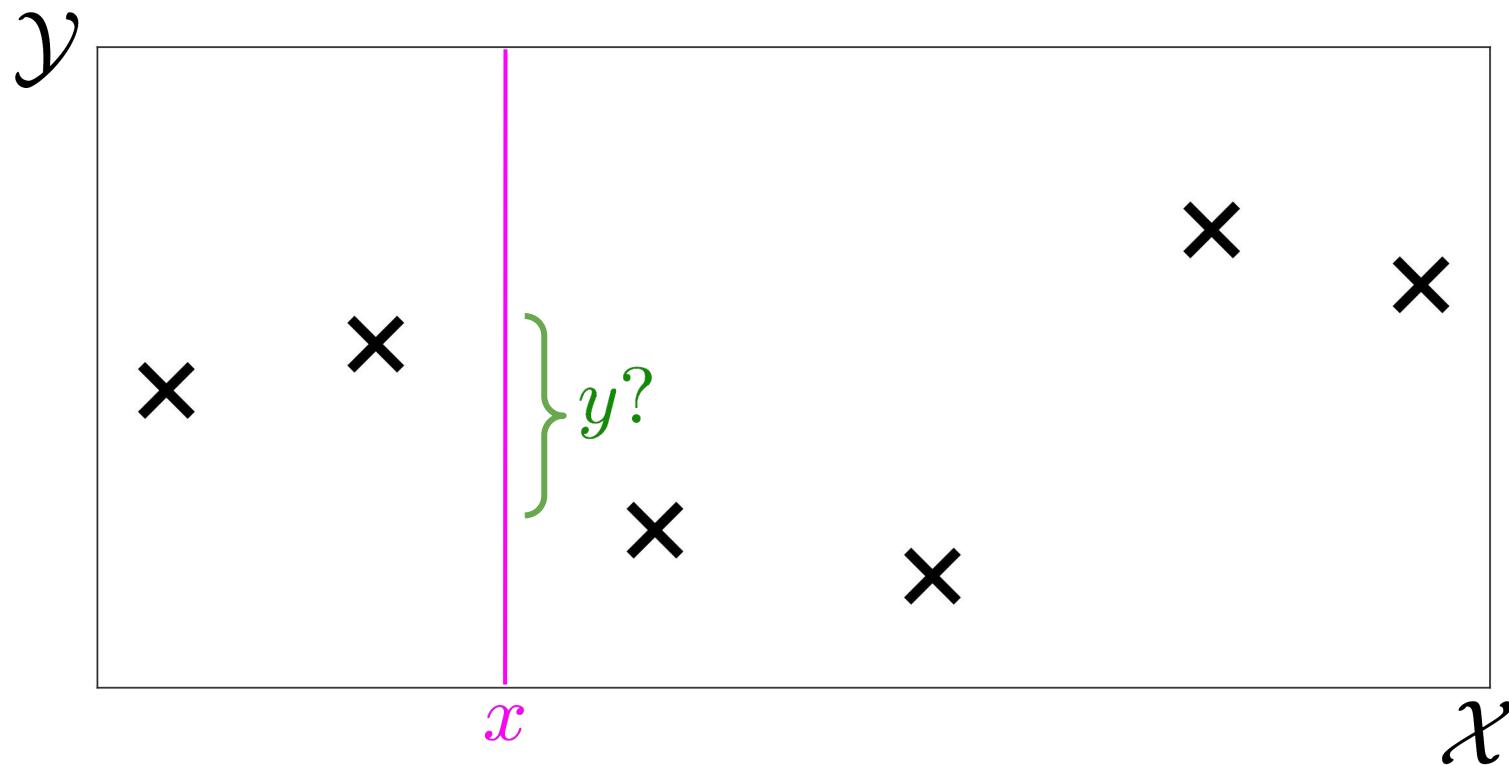


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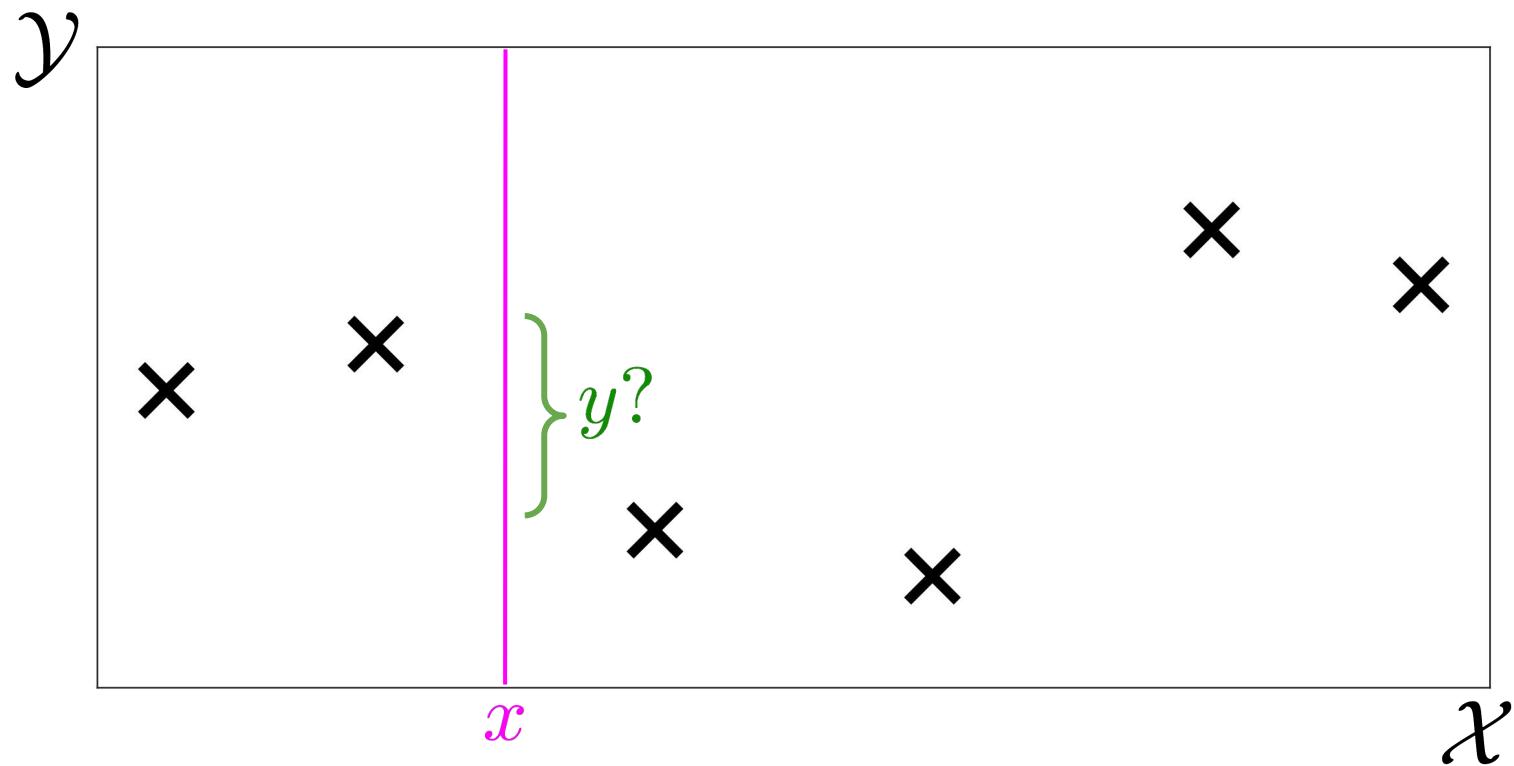
Intuitively, want to answer: for a given x , what are the probable y values?

Could return a distribution over y , or an interval over y , or samples, etc.



Predictive UQ Models – Background and Typical Setup

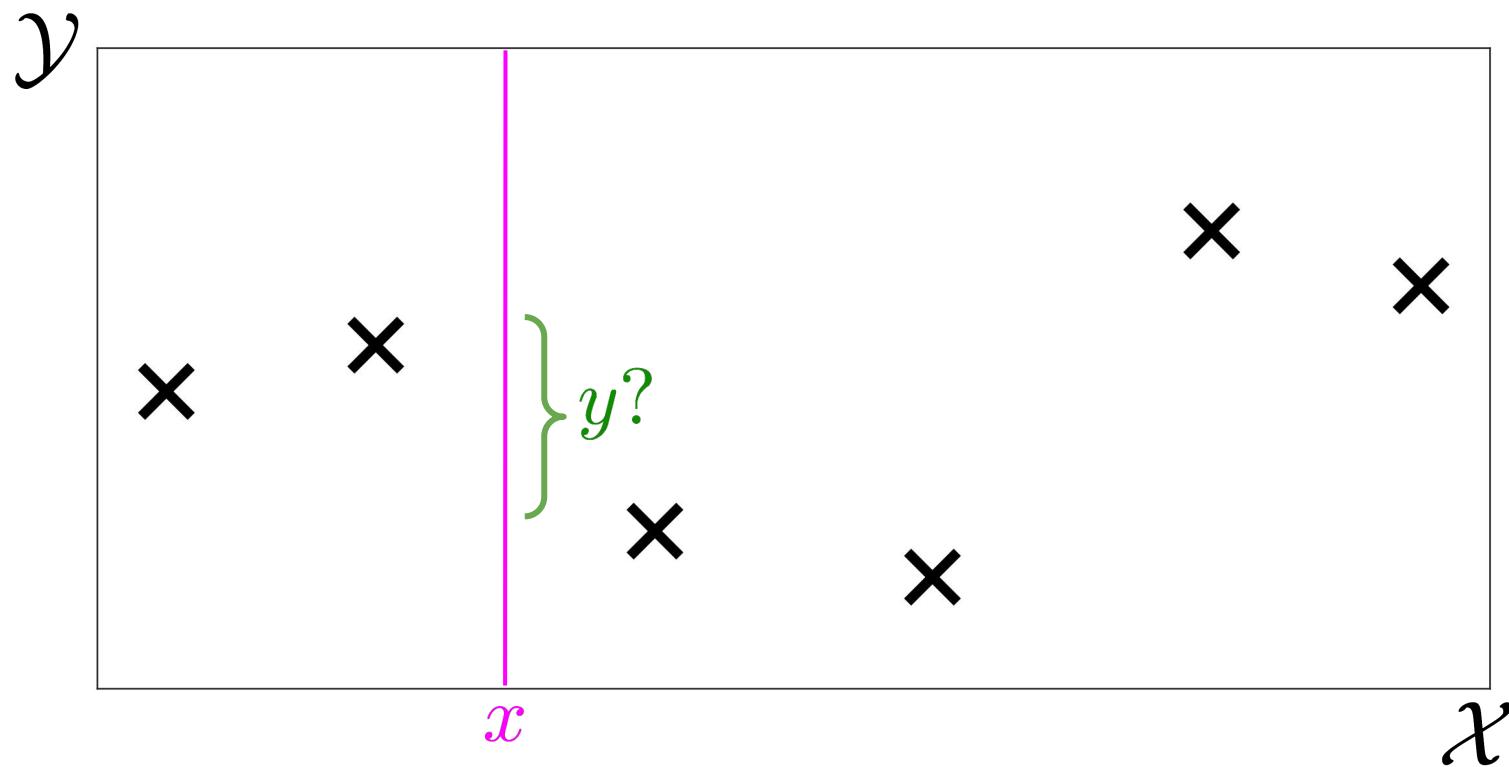
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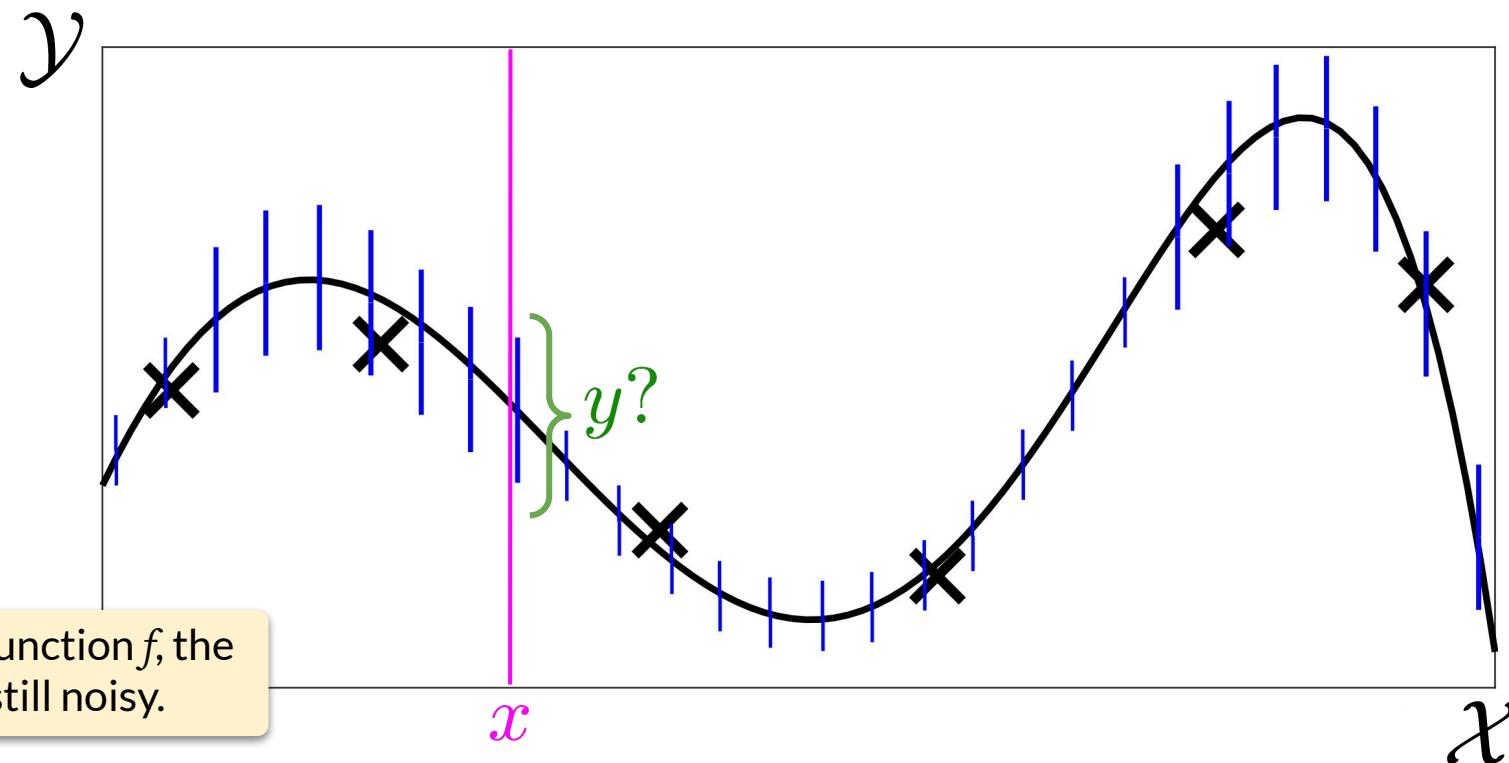
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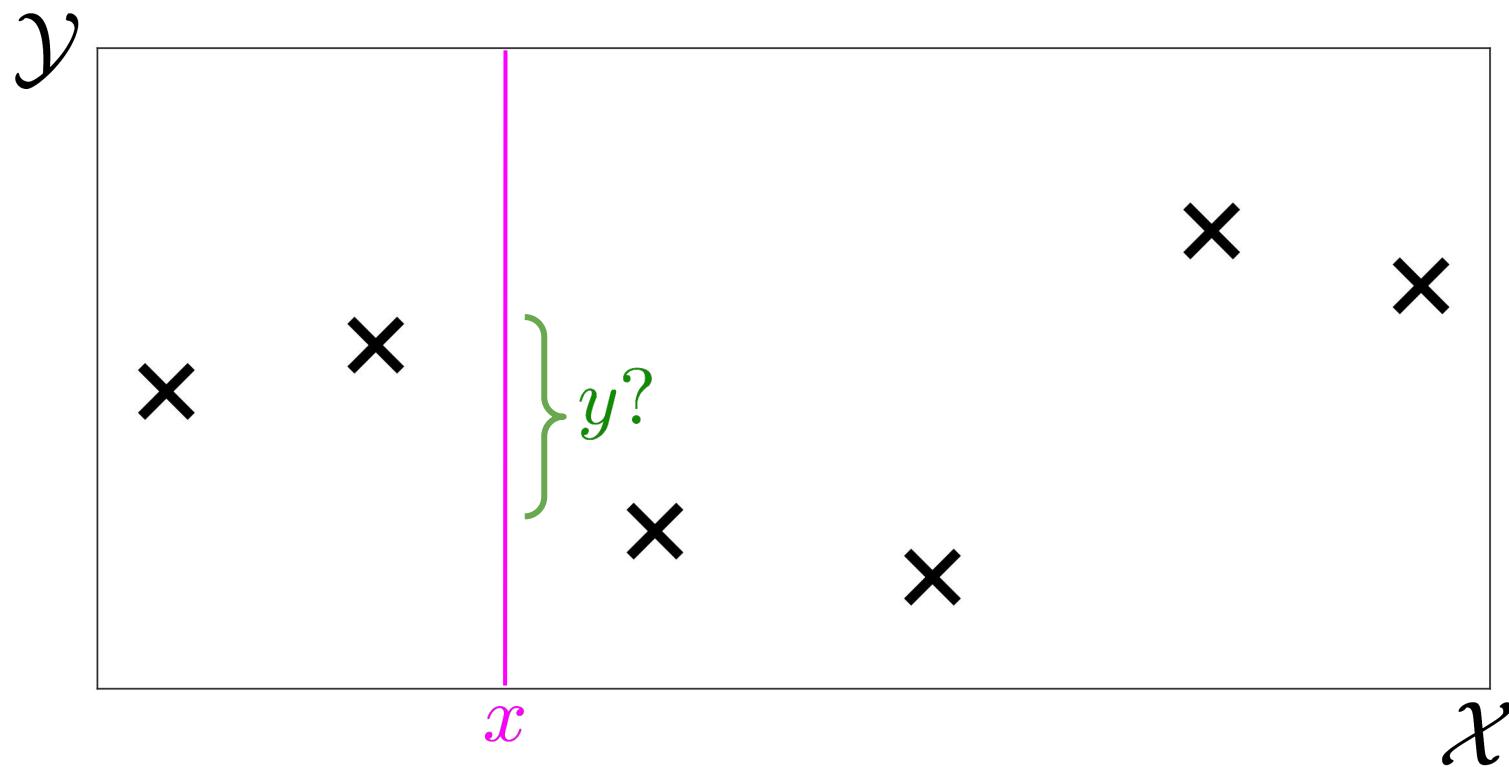
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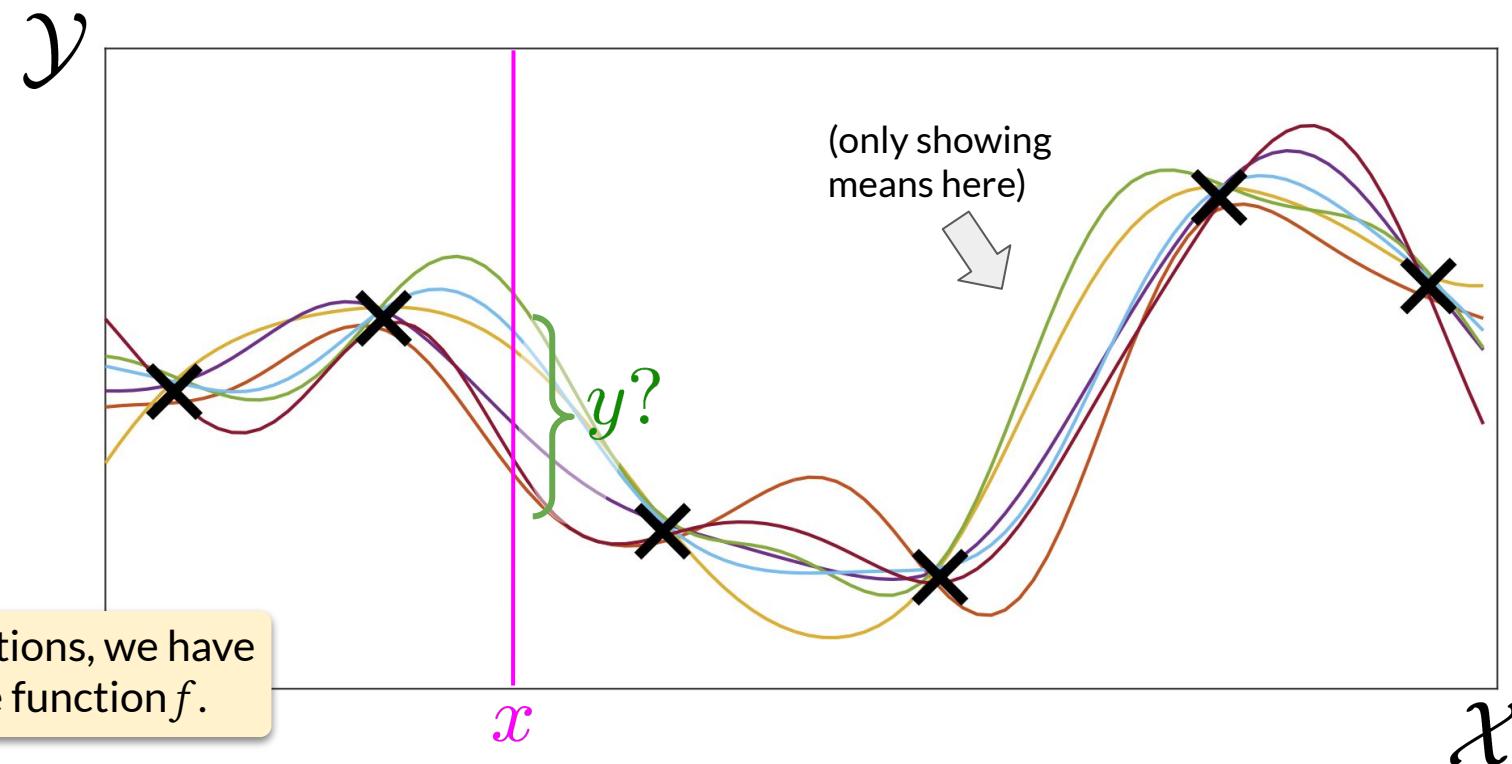
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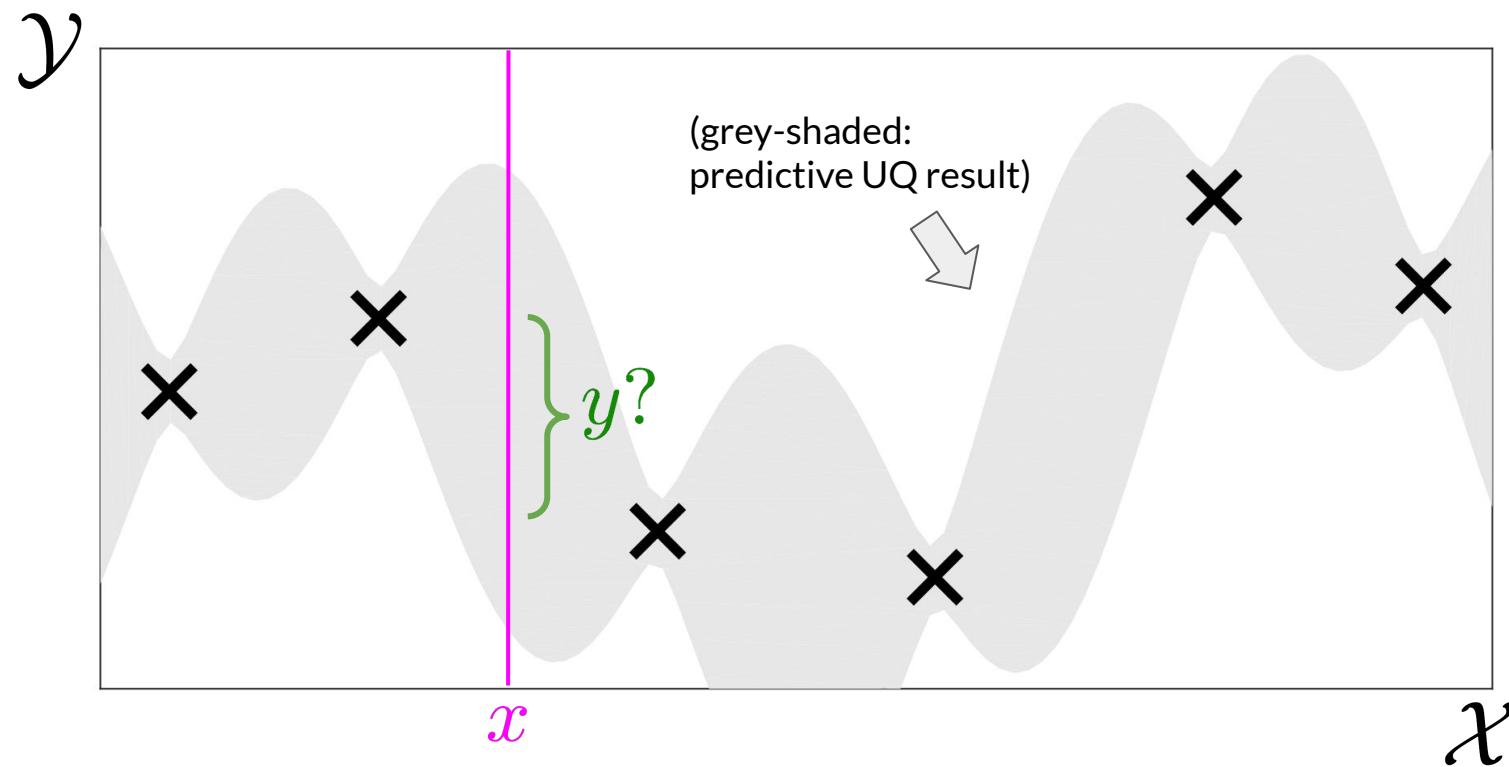
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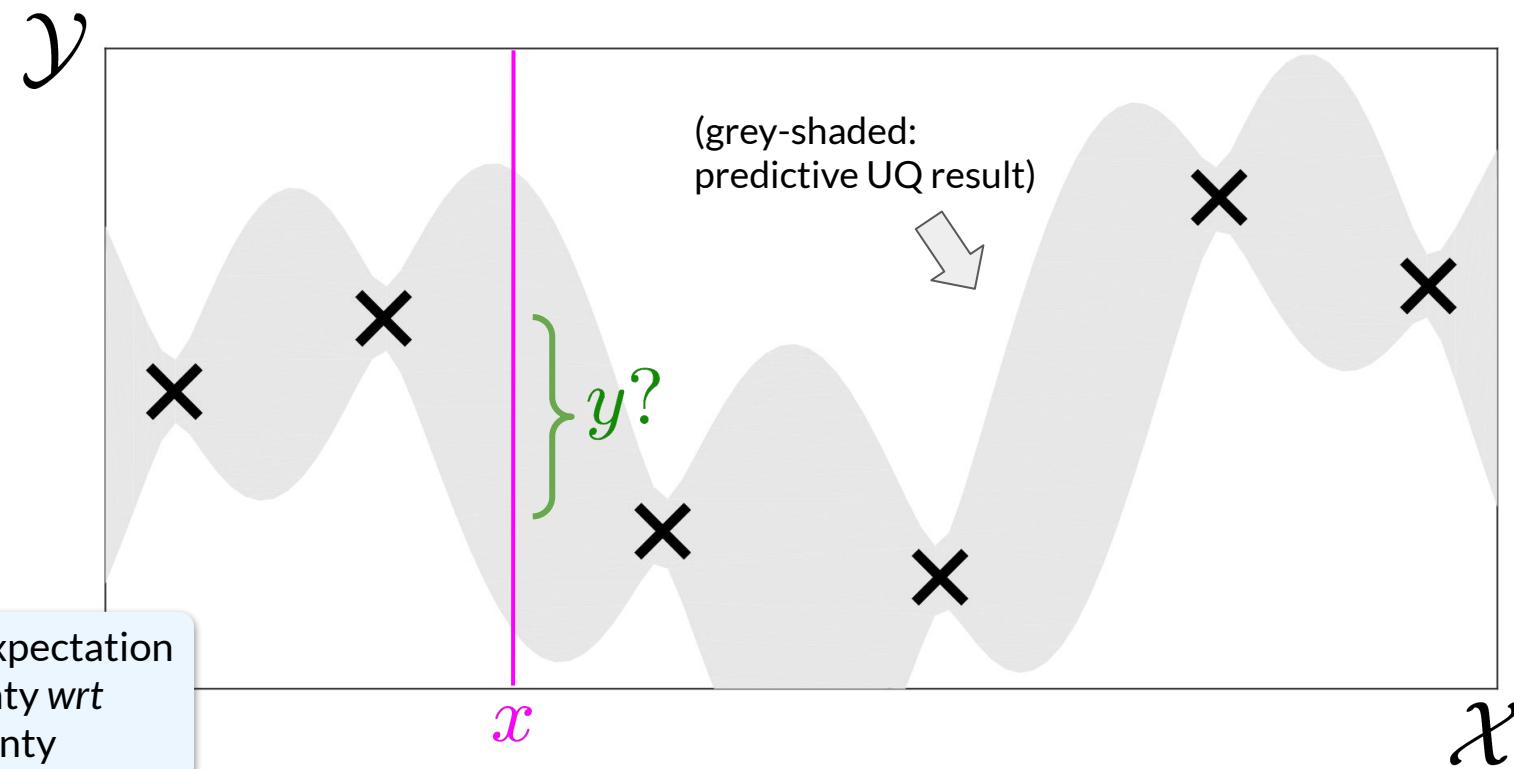
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Example: Bayesian linear model with homoscedastic Gaussian noise.

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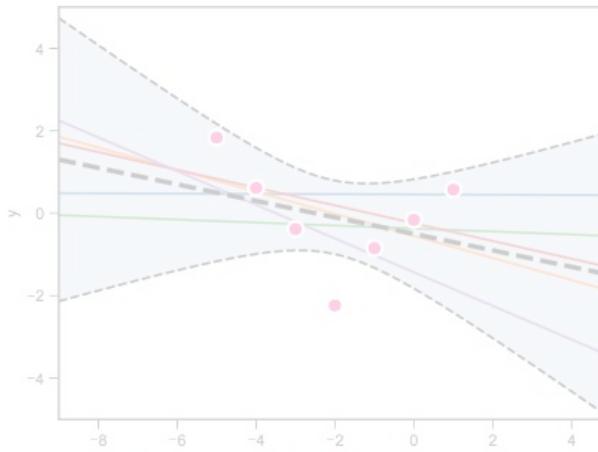
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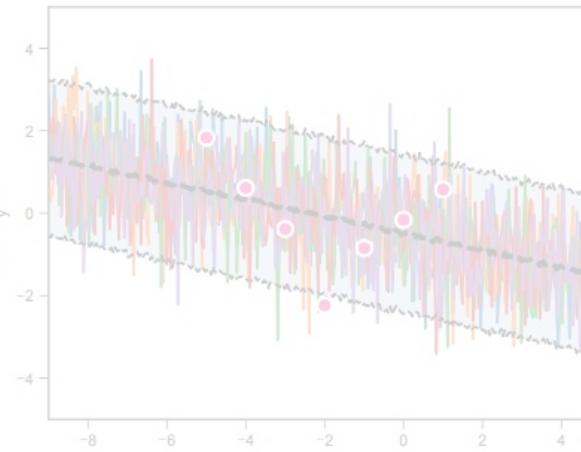
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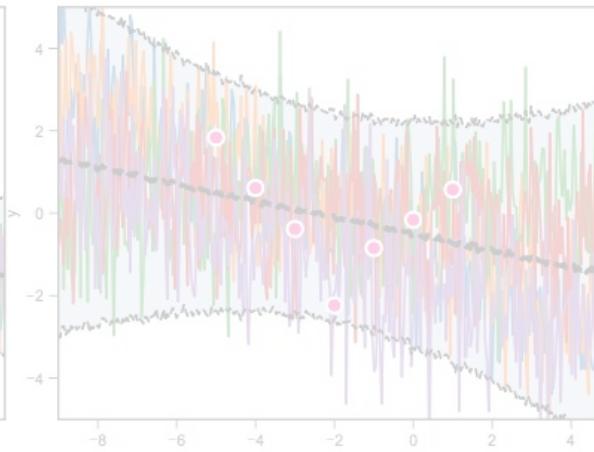
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(here: posterior distribution)*



*Aleatoric uncertainty shown for a single set of model parameters
(here: for MAP parameters)*

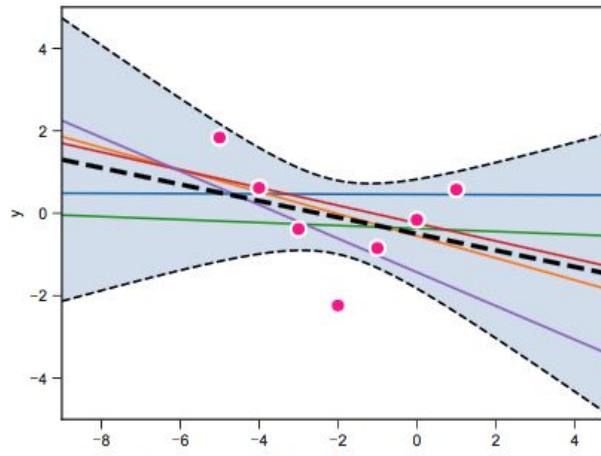


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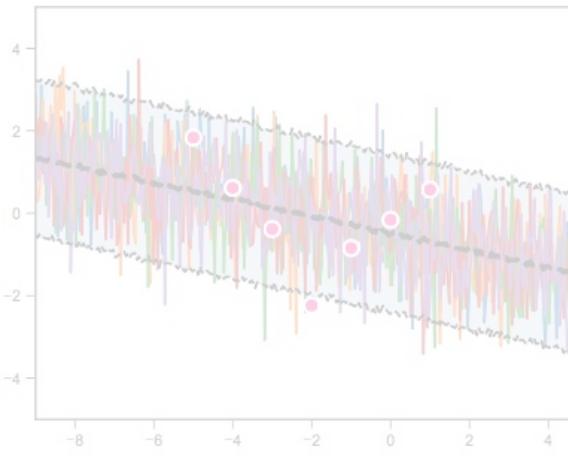
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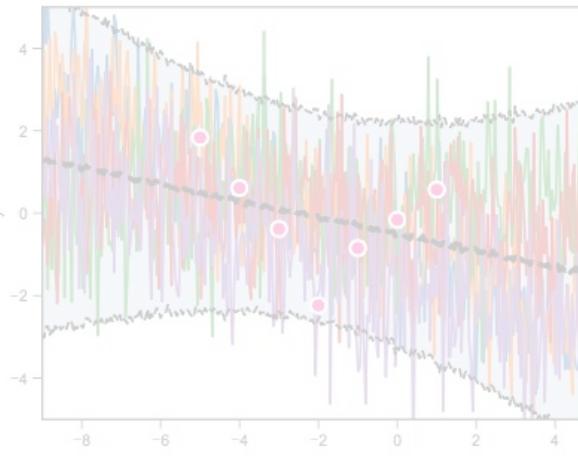
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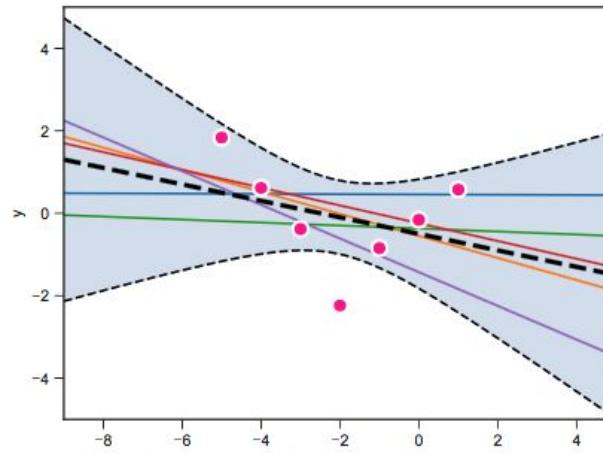


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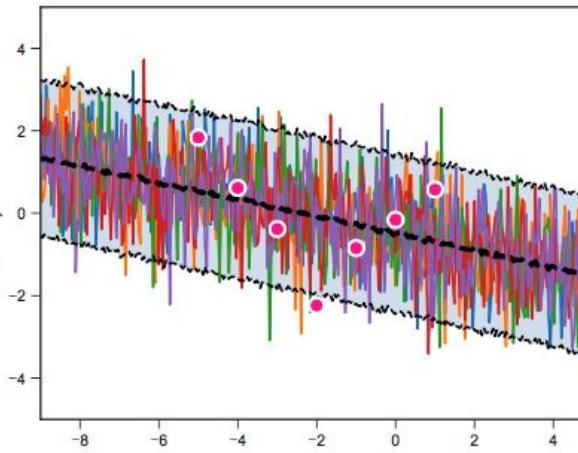
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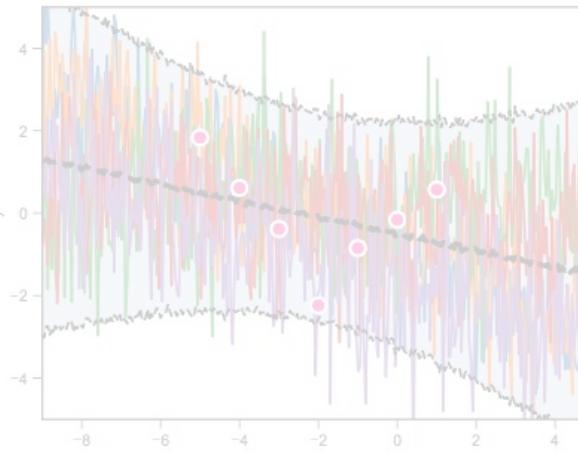
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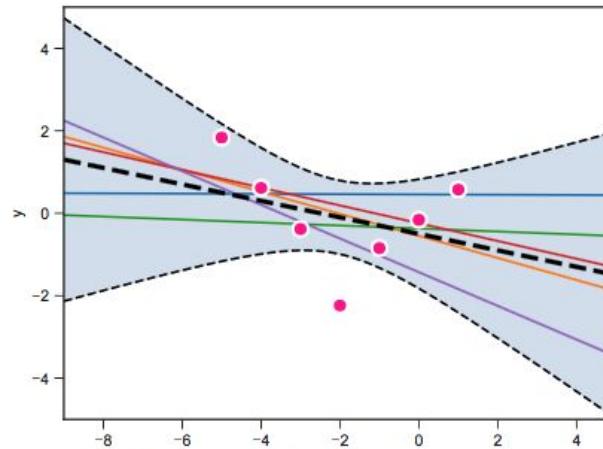


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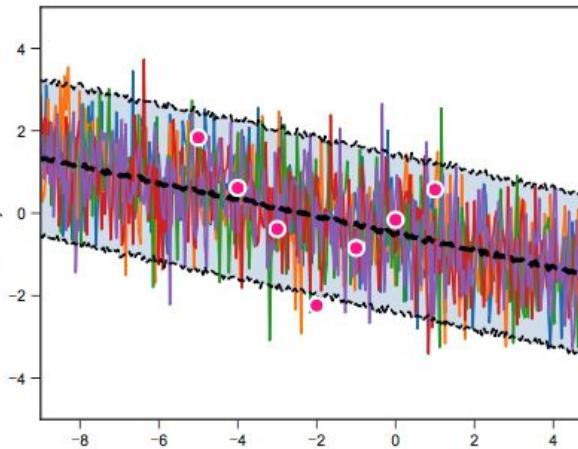
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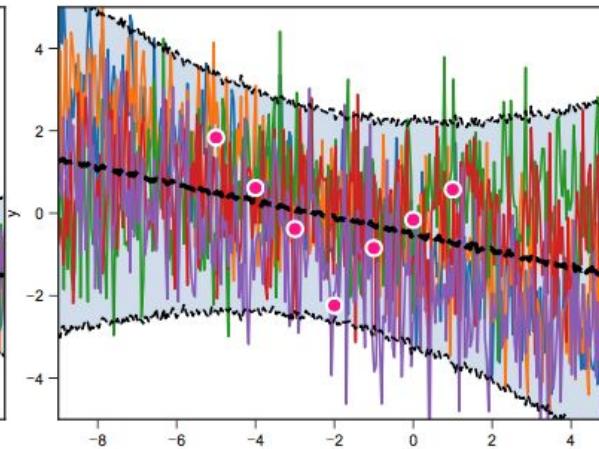
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A Note on PGMs for Predictive UQ Models

A quick note on the PGM structure for predictive UQ models.

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Suppose that we have a (probabilistic) predictive UQ model consisting of:

- Inputs x – input features
- Outputs y – output labels (assuming continuous / regression setting)
- Parameters θ – defining model from some family of models.

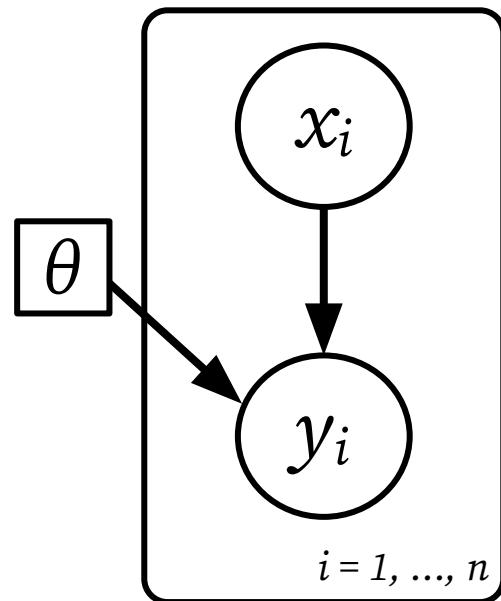
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Joint PDF over Inputs/Outputs



Generative Process

Prior: $p(x_i)$

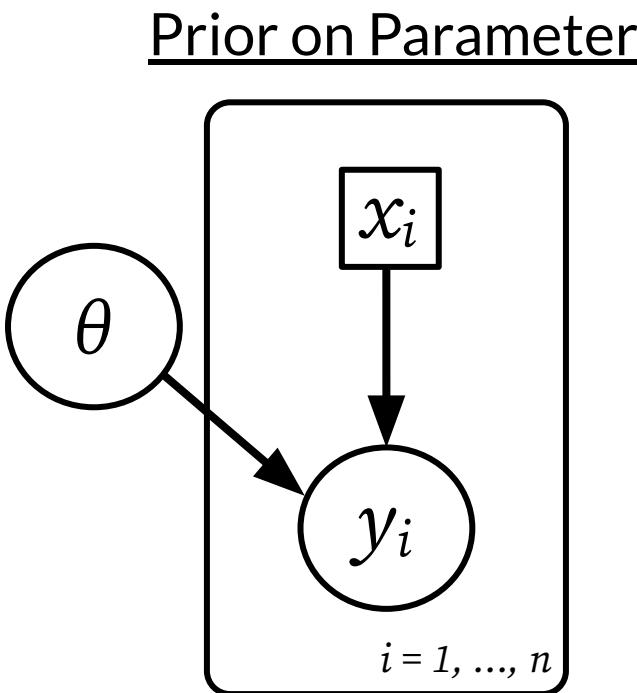
Likelihood: $p_\theta(y_i | x_i)$

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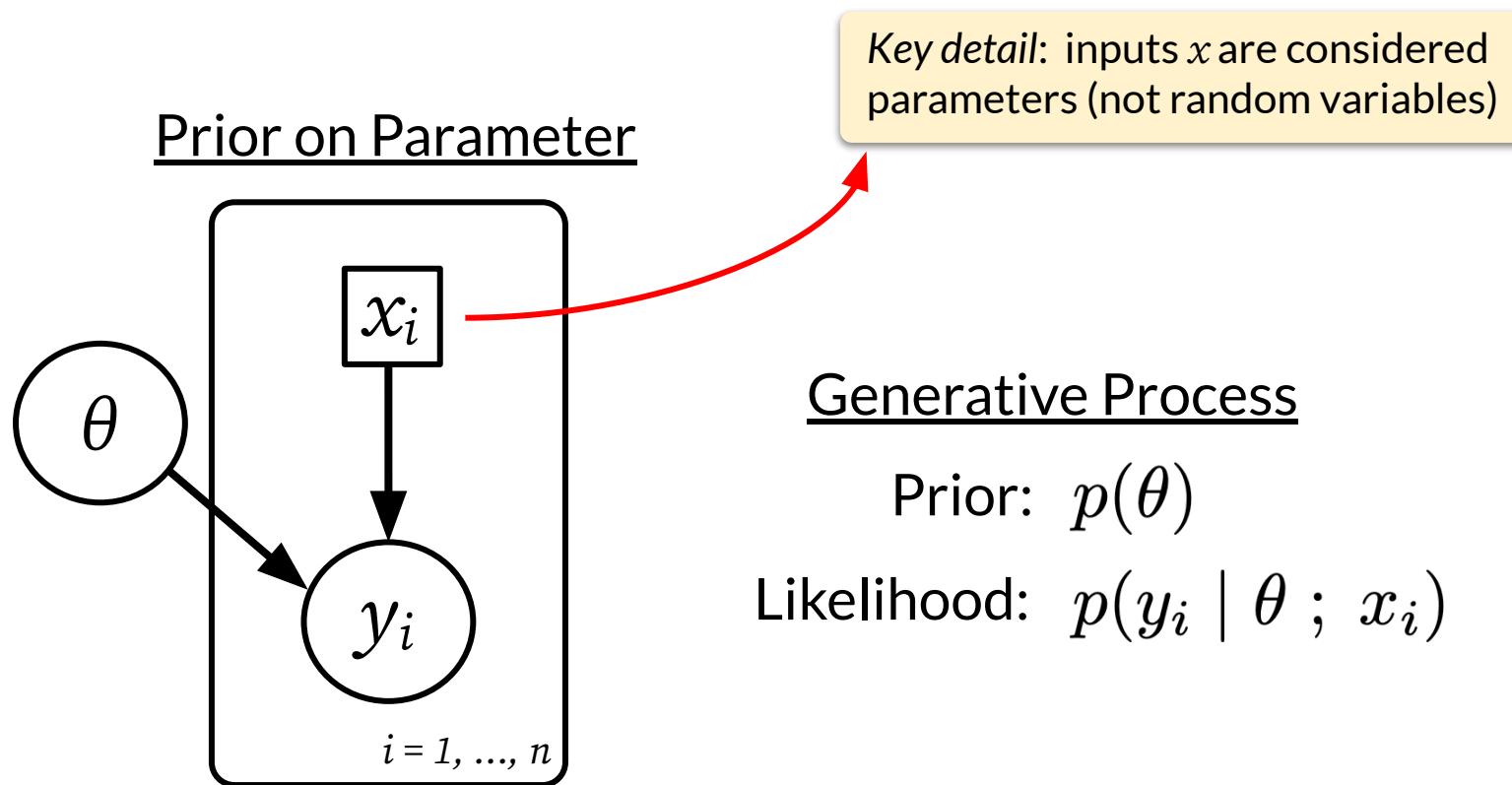
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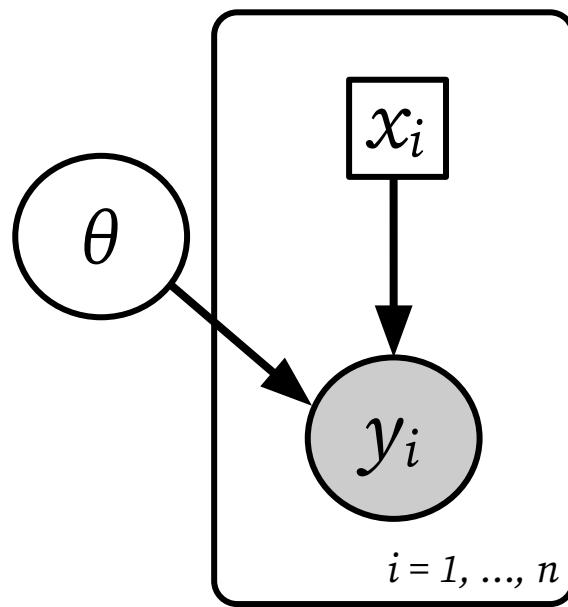
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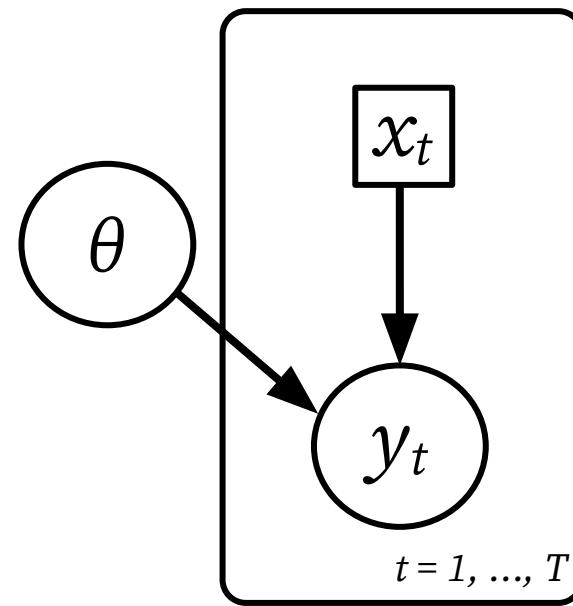
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Train



Test

Next Steps

In rest of lecture we will:

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- Go through a few popular *predictive UQ models*.
- Starting with **Gaussian Processes**.
- Then describing some **Deep Uncertainty Models** (neural network-based).
 - Including: some neural network + GP hybrid models.

Gaussian Processes

Gaussian Processes

A very popular type of predictive UQ model is called a **Gaussian Process (GP)**.

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Also referred to as a *Bayesian* or
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I aim to give a basic overview of GPs (a full overview could take a full lecture or more).
⇒ Later, I'll show some examples of models that combine GPs and neural networks.

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But primarily uses (multivariate) **Gaussian distribution** to model these functions !

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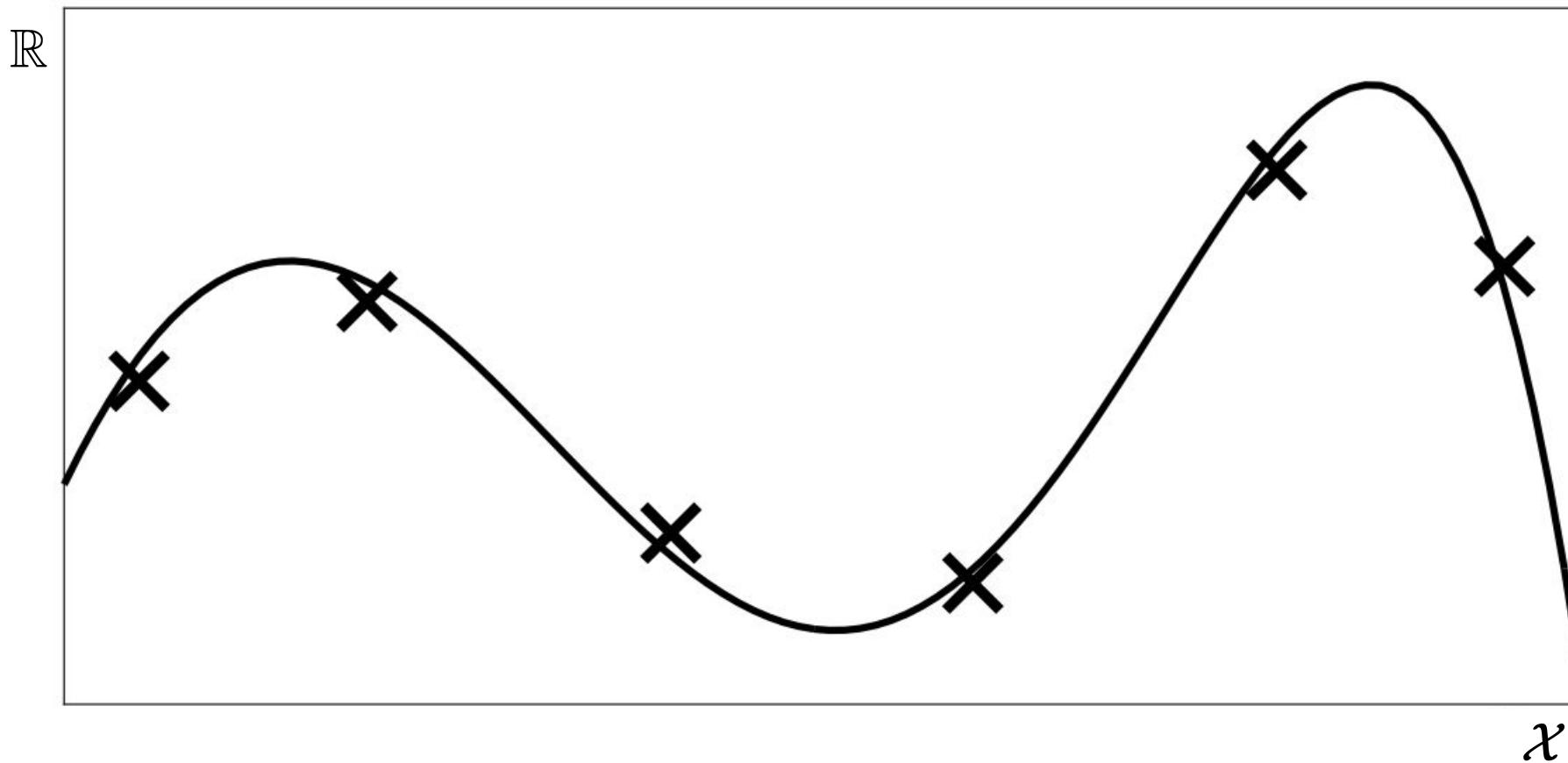
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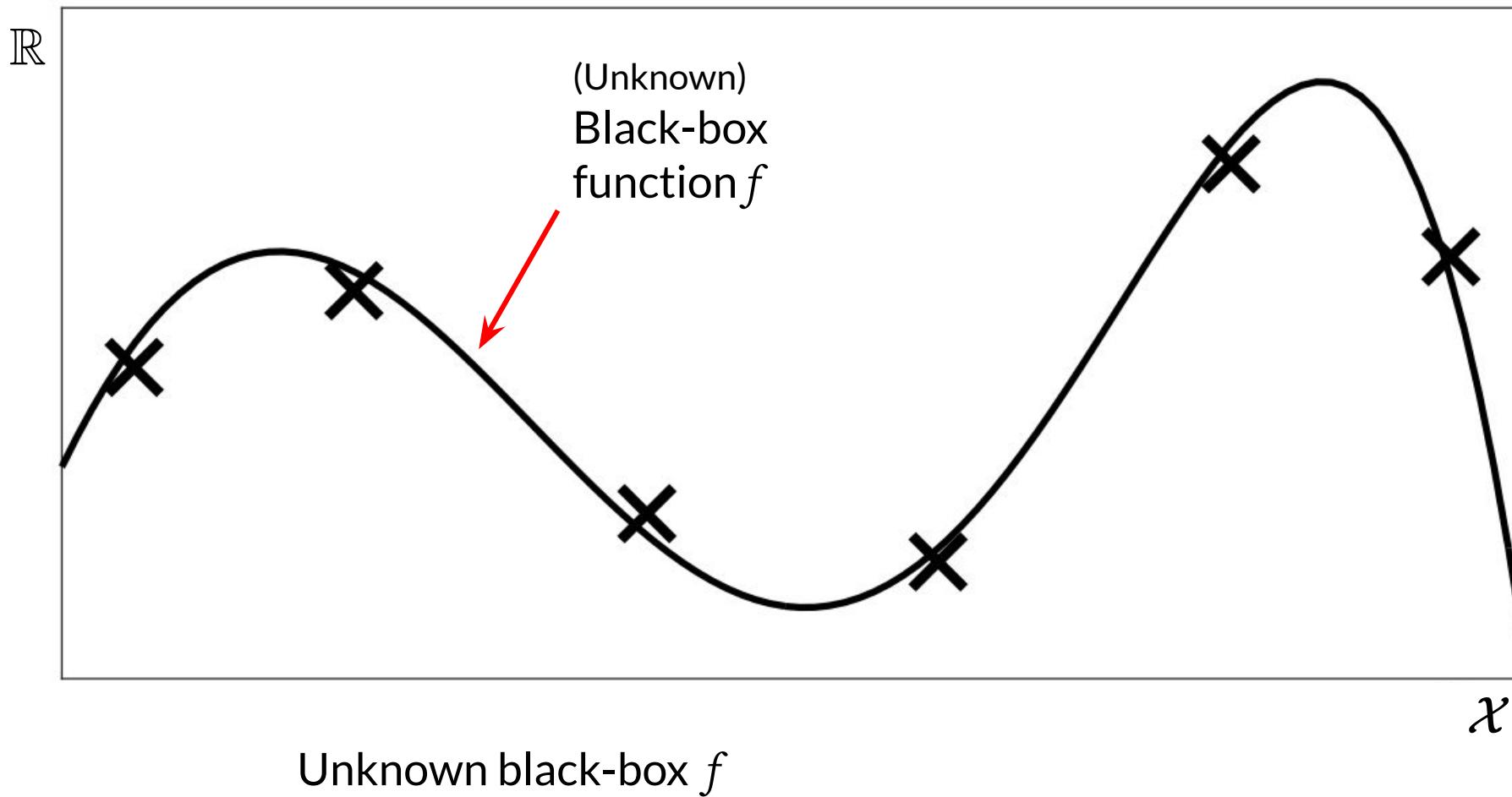
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Lets visualize this...

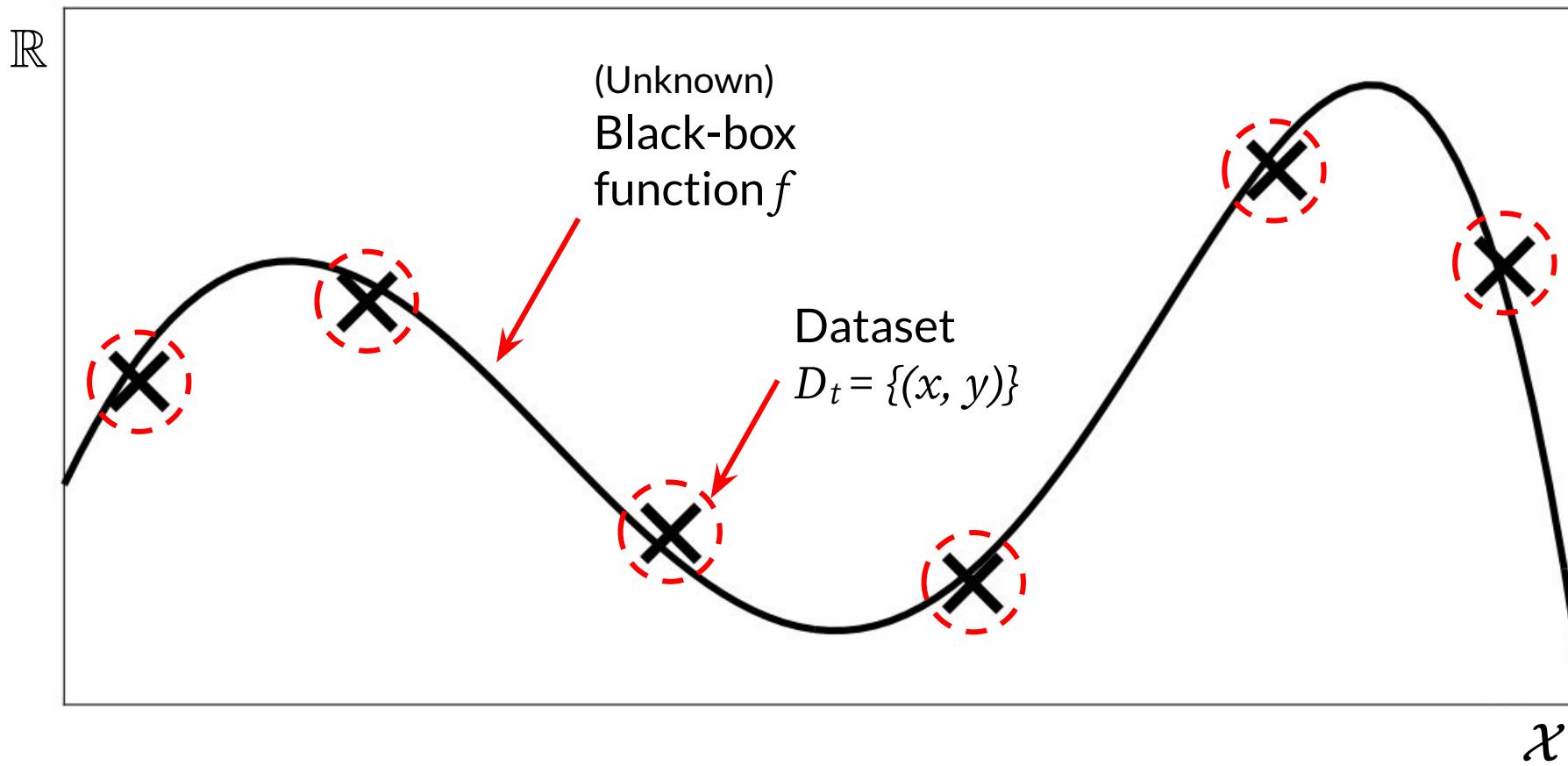
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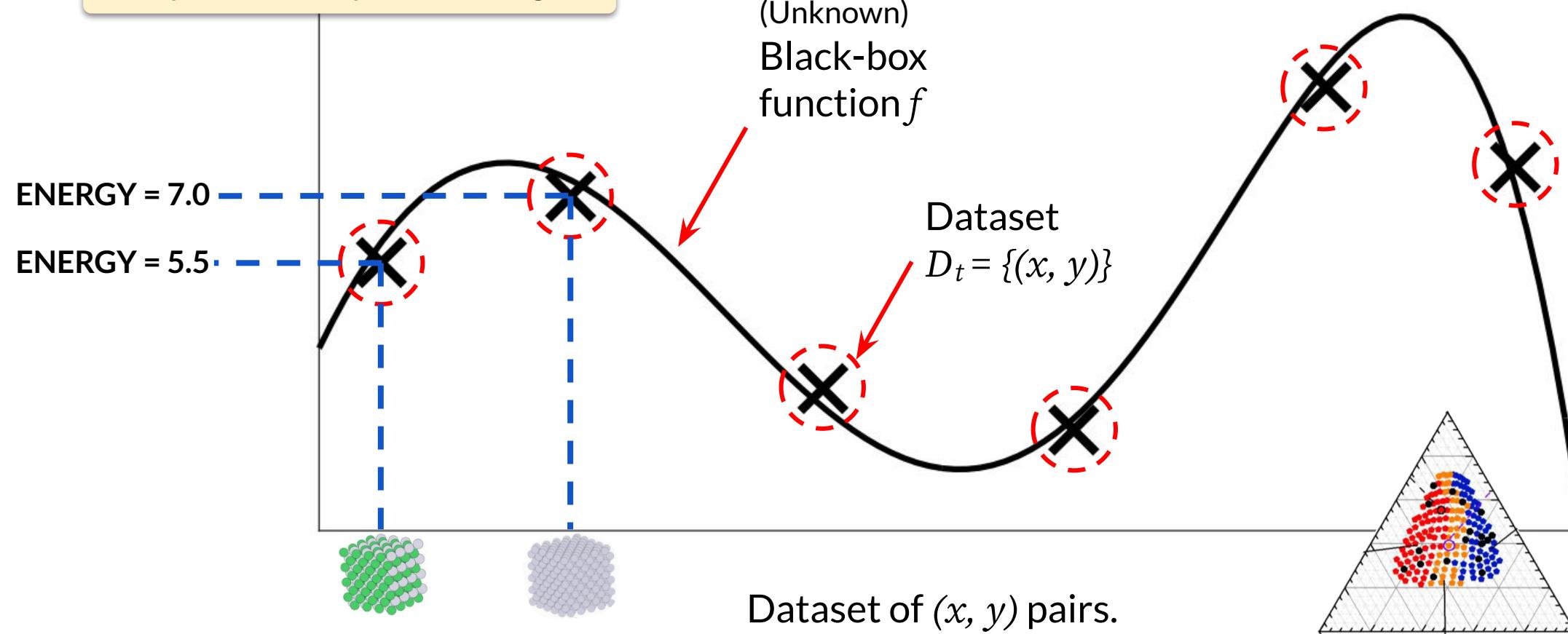


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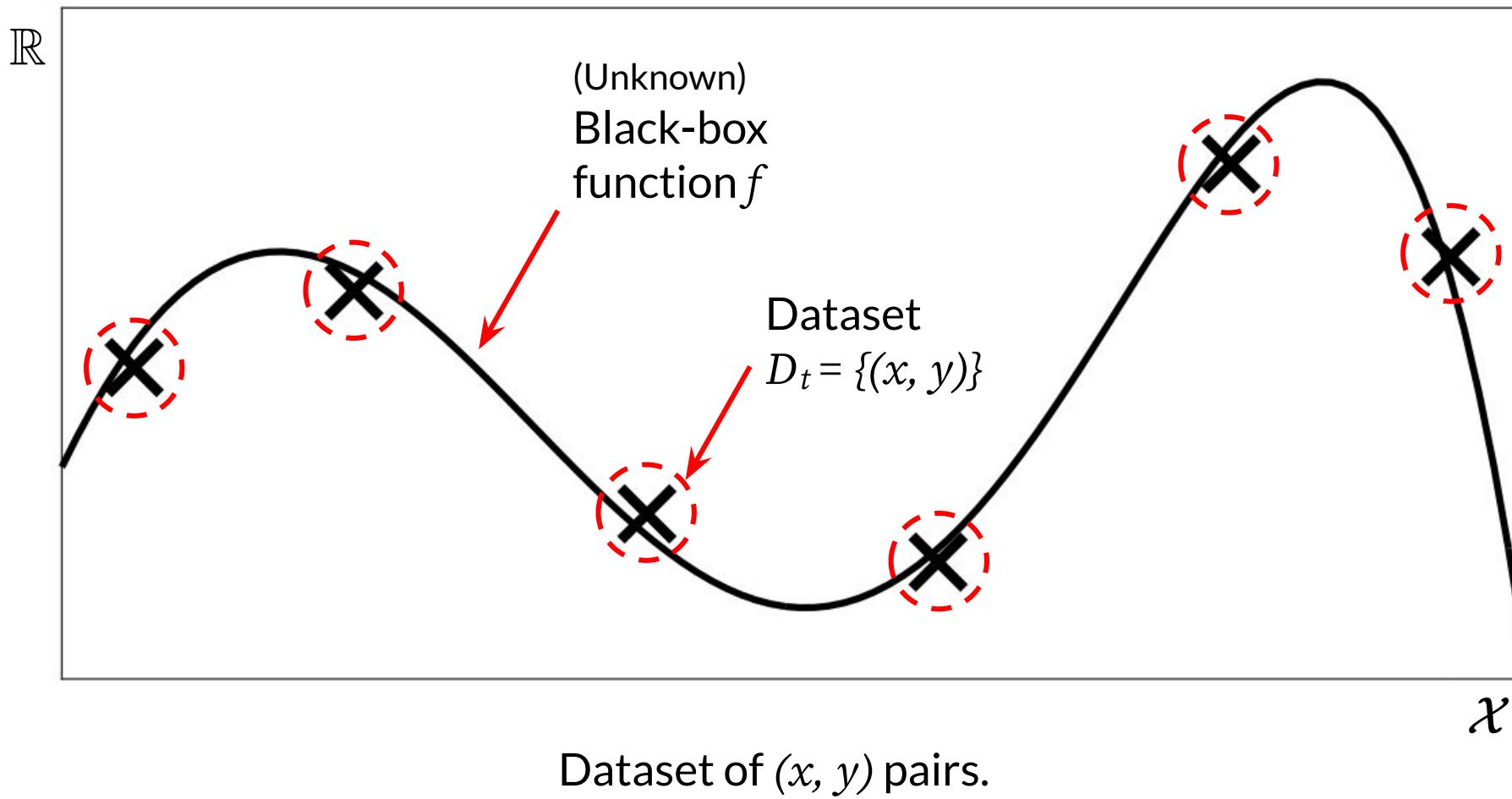


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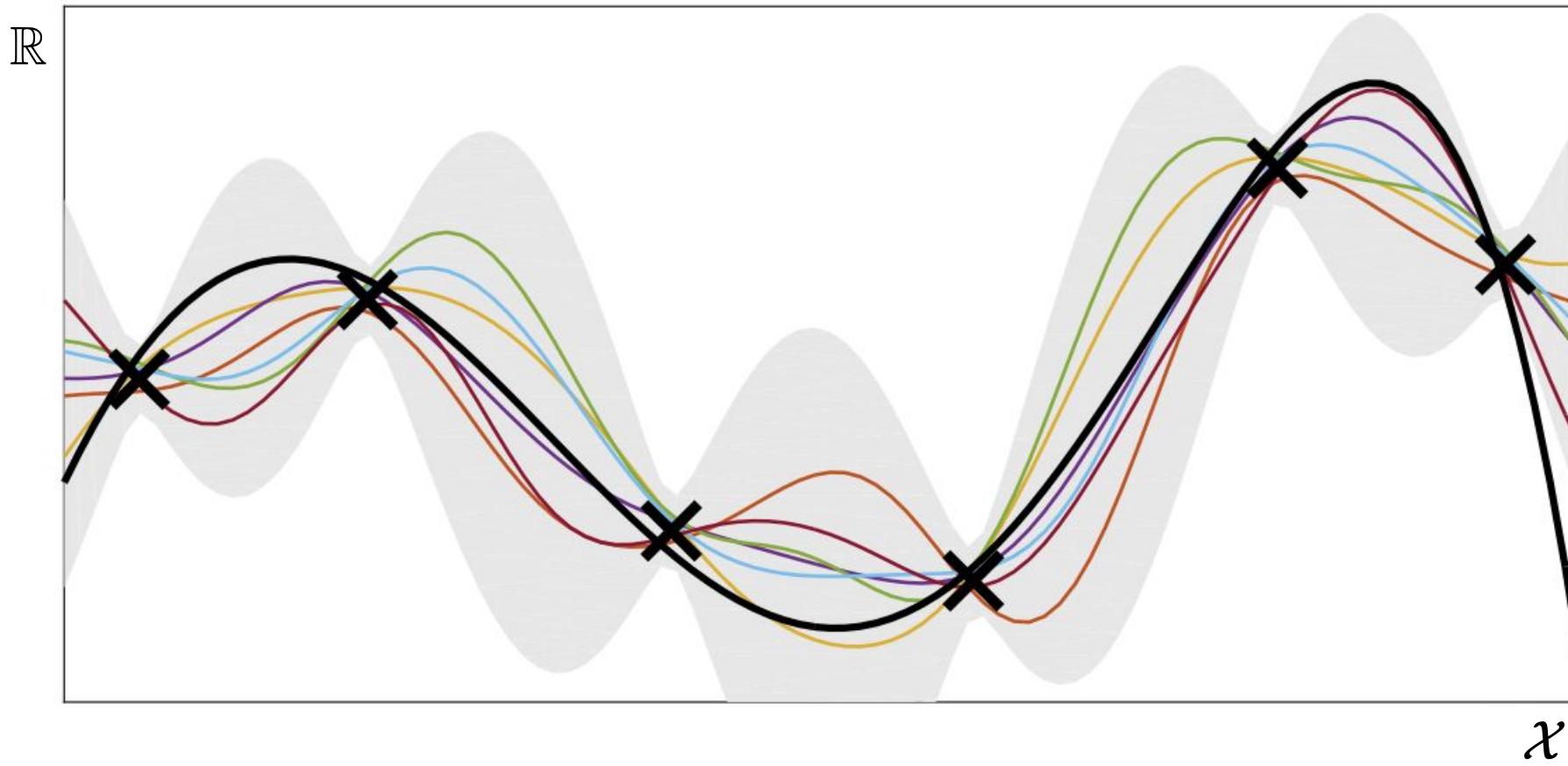
Example (mat sci): inputs are material compositions, outputs are energies



Gaussian Processes – Intuition

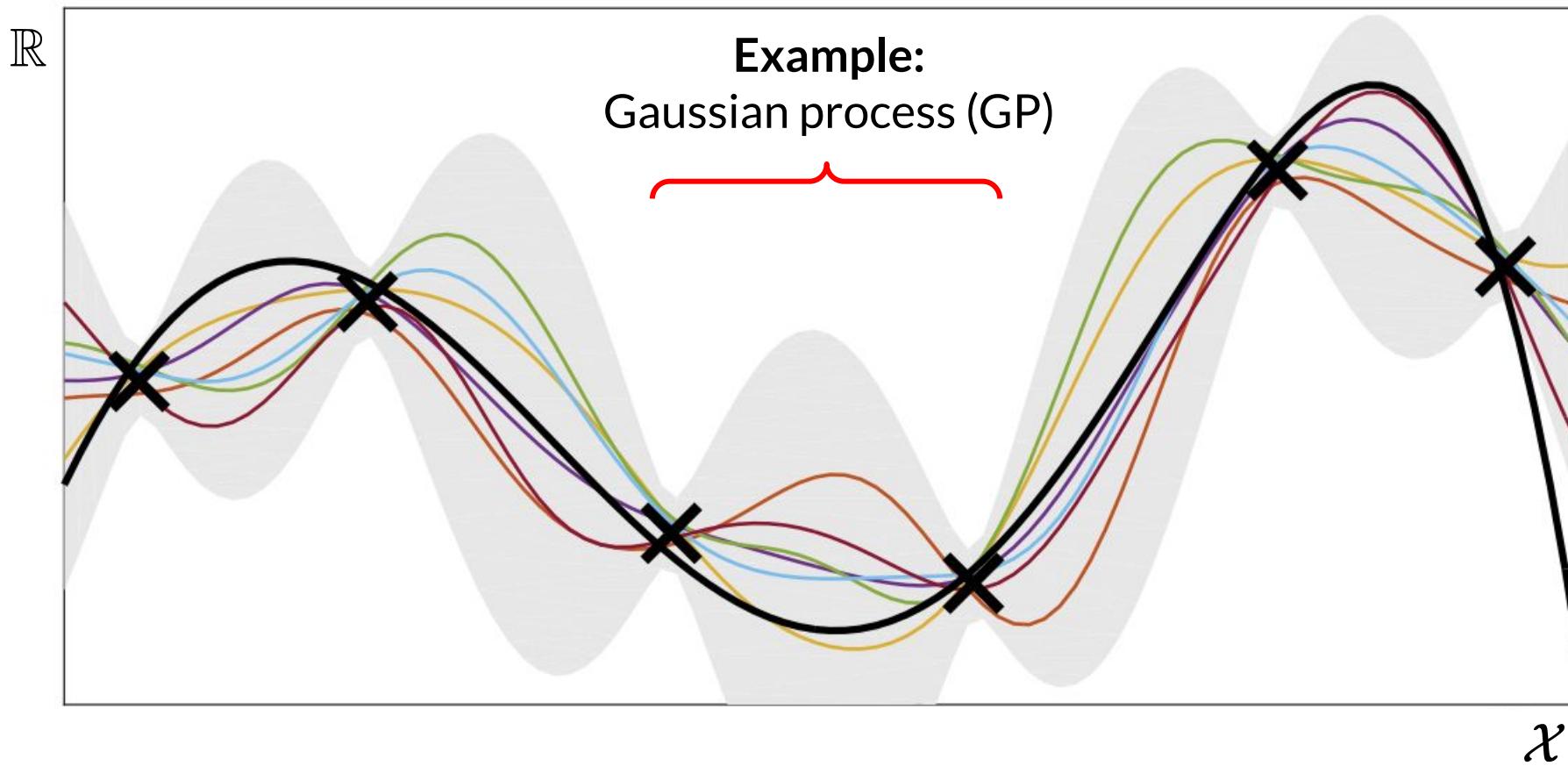


Gaussian Processes – Intuition

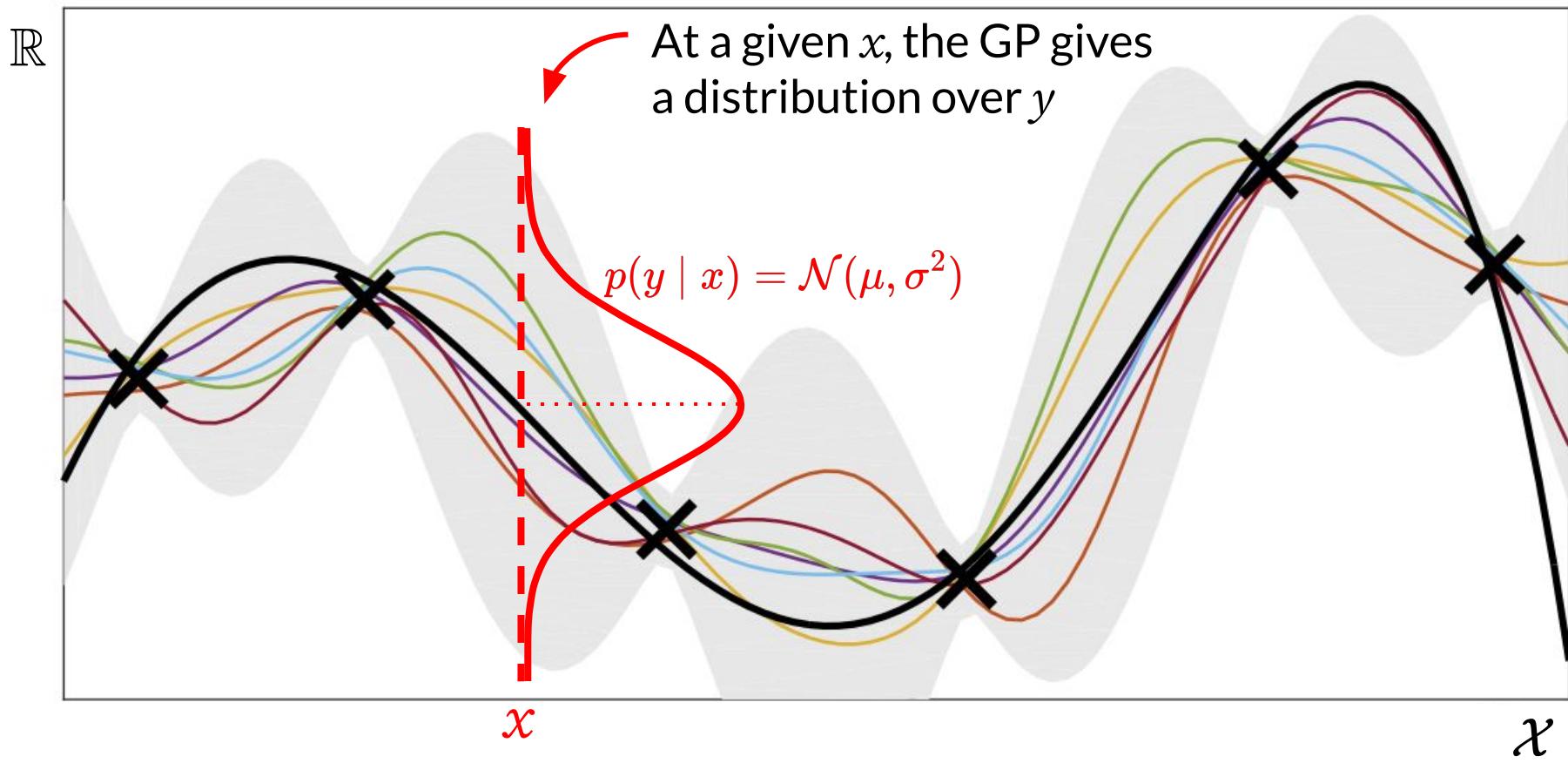


Given this dataset, can fit *predictive uncertainty* model.

Gaussian Processes – Intuition

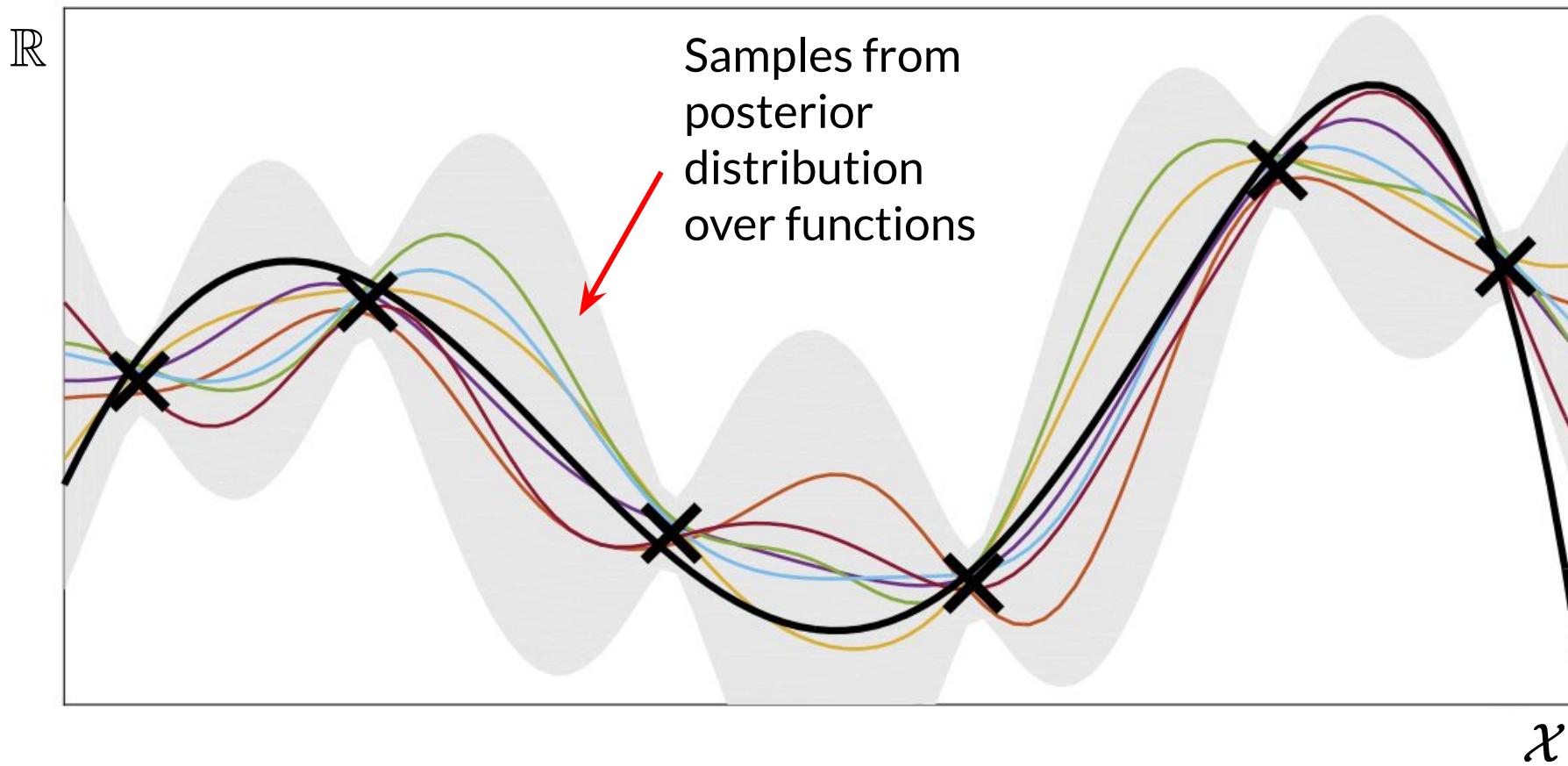


Gaussian Processes – Intuition



A popular choice for this model is a **Gaussian Process**.

Gaussian Processes – Intuition



Gaussian Processes – Background on Multivariate Normal

So, as mentioned a GP models the distribution over *functions* using a:
Multivariate normal (MVN) distribution.

Most of us are familiar, but for a quick review of important details:

Gaussian Processes – Background on Multivariate Normal

Multivariate normal (MVN) distribution.

The PDF, for $x \in \mathbb{R}^d$ is:

Gaussian Processes – Background on Multivariate Normal

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$$\mathcal{N}(x; \mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x - \mu)^\top \Sigma^{-1}(x - \mu)\right)$$

Gaussian Processes – Background on Multivariate Normal

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With parameters:

$\mu \in \mathbb{R}^d$ — Mean vector.

$\Sigma \in \mathbb{R}^{d \times d}$ — Covariance matrix.

Gaussian Processes – Background on Multivariate Normal

Where a **covariance matrix** parameter Σ is the $n \times n$ square matrix with entries $\Sigma_{ij} = \text{Cov}[X_i, X_j]$, written as:

$$\Sigma = \begin{bmatrix} \text{Cov}[X_1, X_1] & \cdots & \text{Cov}[X_1, X_n] \\ \vdots & \ddots & \vdots \\ \text{Cov}[X_n, X_1] & \cdots & \text{Cov}[X_n, X_n] \end{bmatrix}$$

Gaussian Processes – Background on Multivariate Normal

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And the **covariance** of two random variables X and Y is defined to be

$$\text{Cov}[X, Y] = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$$

⇒ Describes whether two variables tend to move in the same direction (positive covariance) or opposite directions (negative covariance).

Gaussian Processes – Background on Multivariate Normal

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Gaussian Processes – Background on Multivariate Normal

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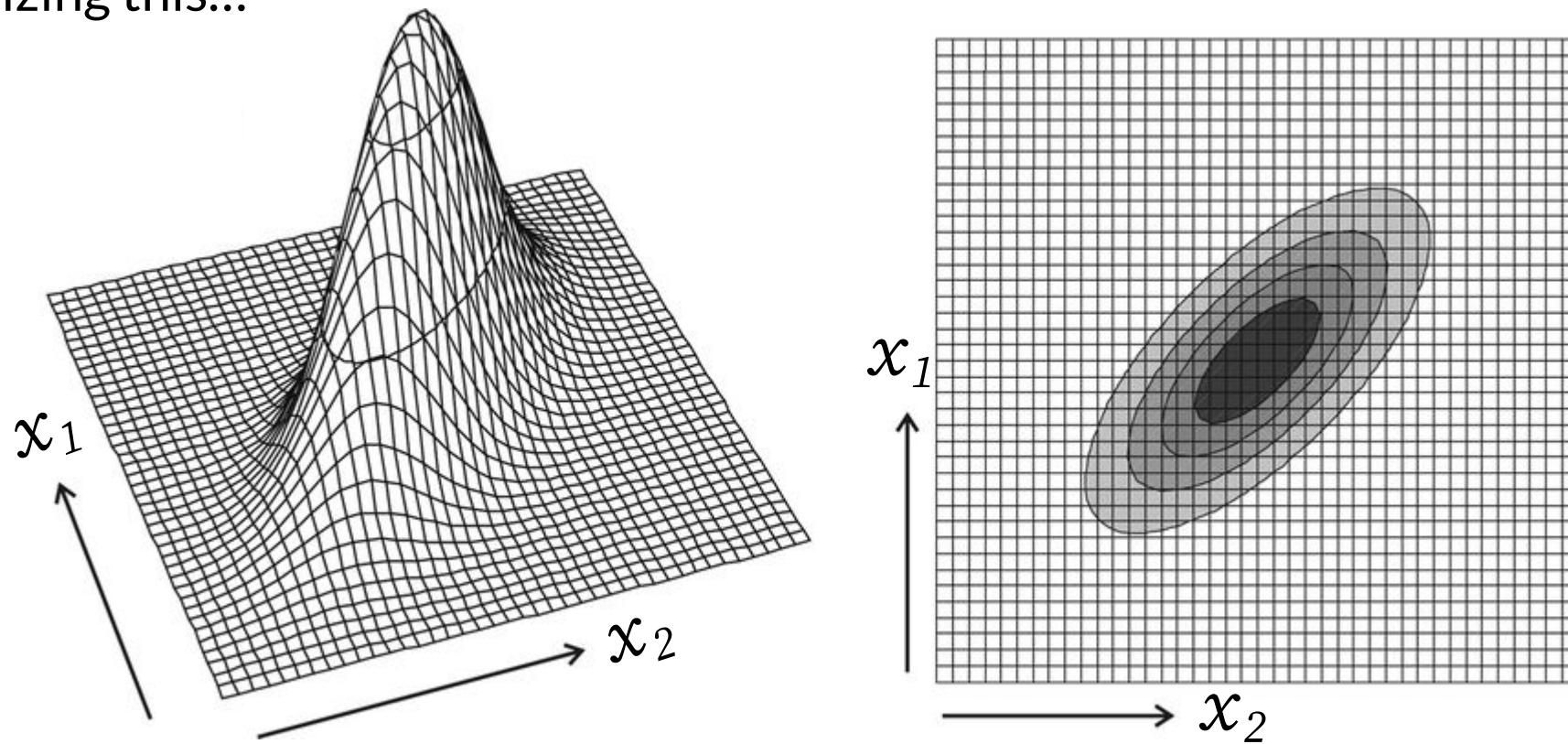
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Gaussian Processes – Background on Multivariate Normal

Visualizing this...

Gaussian Processes – Background on Multivariate Normal

Visualizing this...

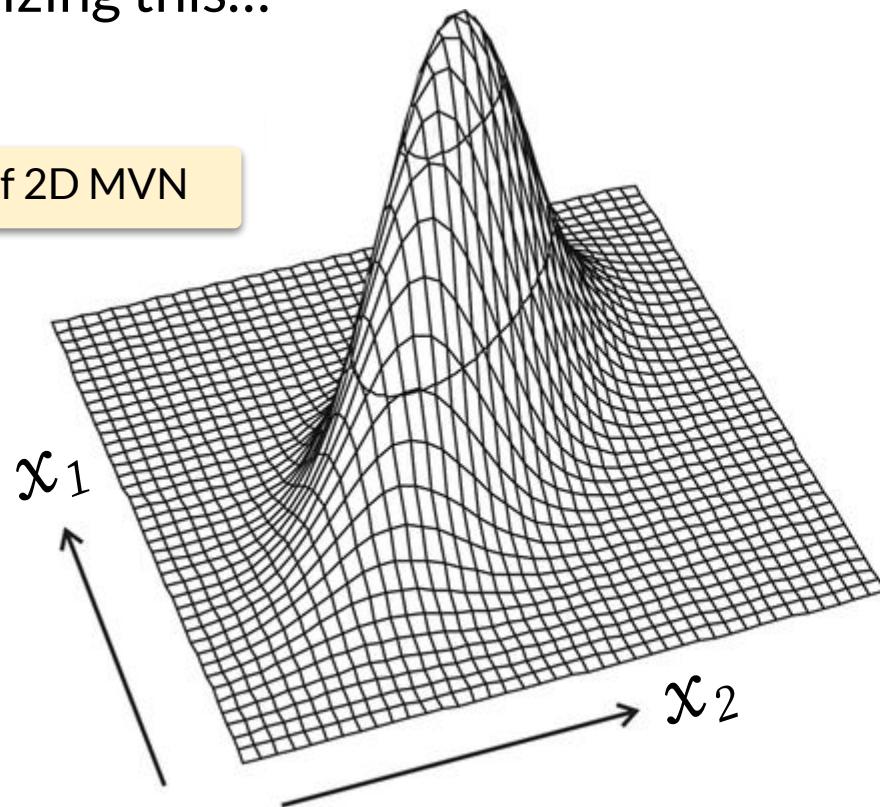


Source: Keith Sircombe, "Mountains in the shadows of time"

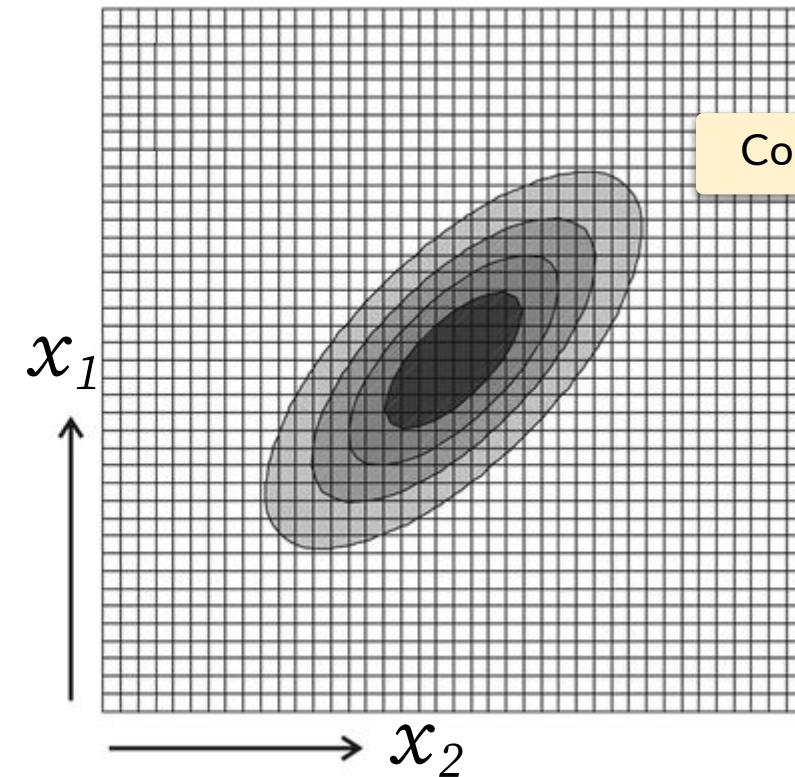
Gaussian Processes – Background on Multivariate Normal

Visualizing this...

PDF of 2D MVN



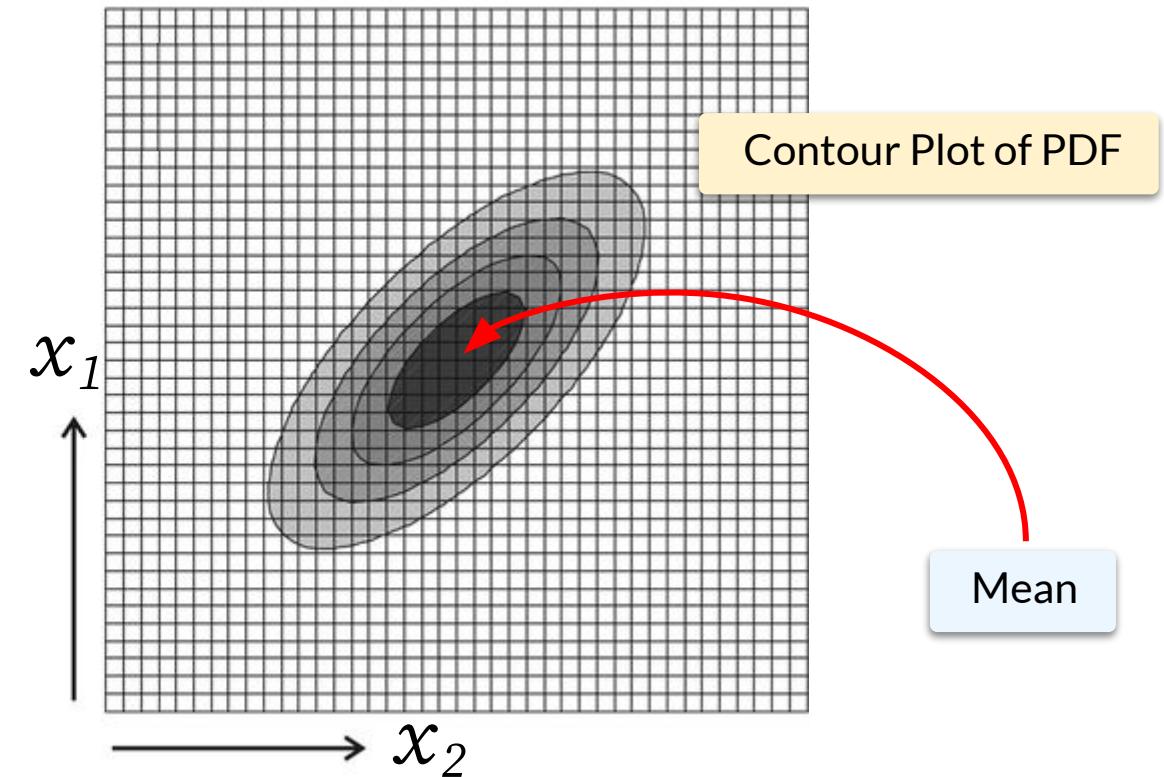
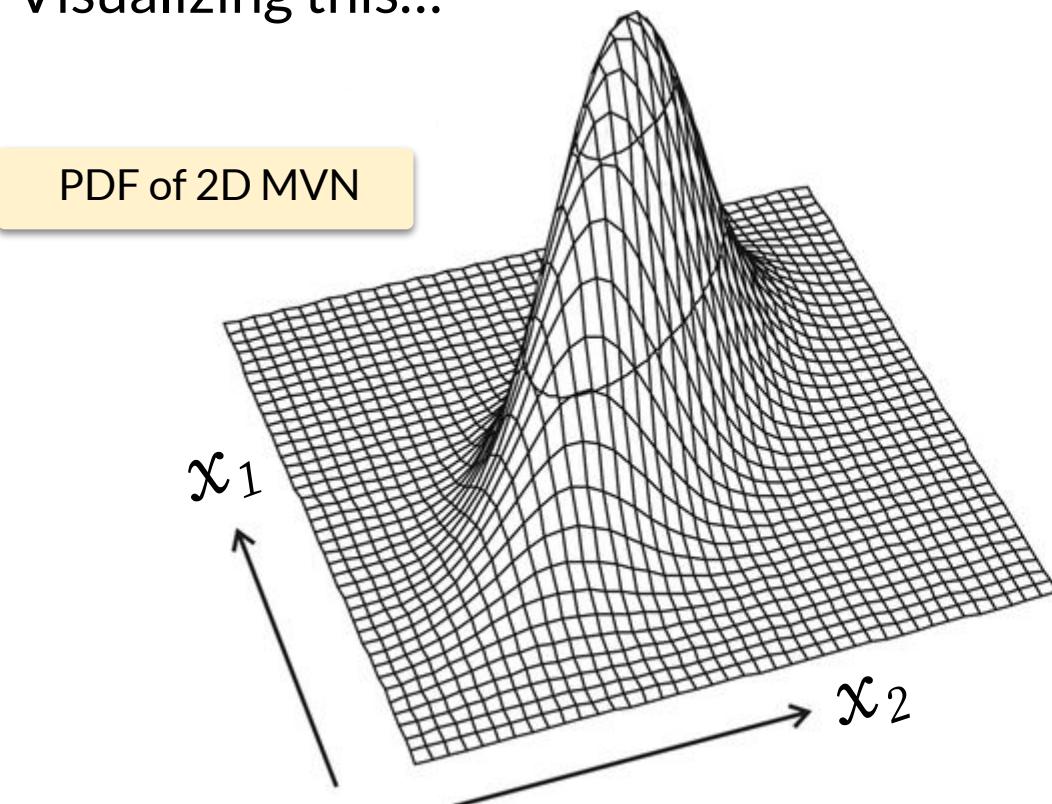
Contour Plot of PDF



Source: Keith Sircombe, "Mountains in the shadows of time"

Gaussian Processes – Background on Multivariate Normal

Visualizing this...

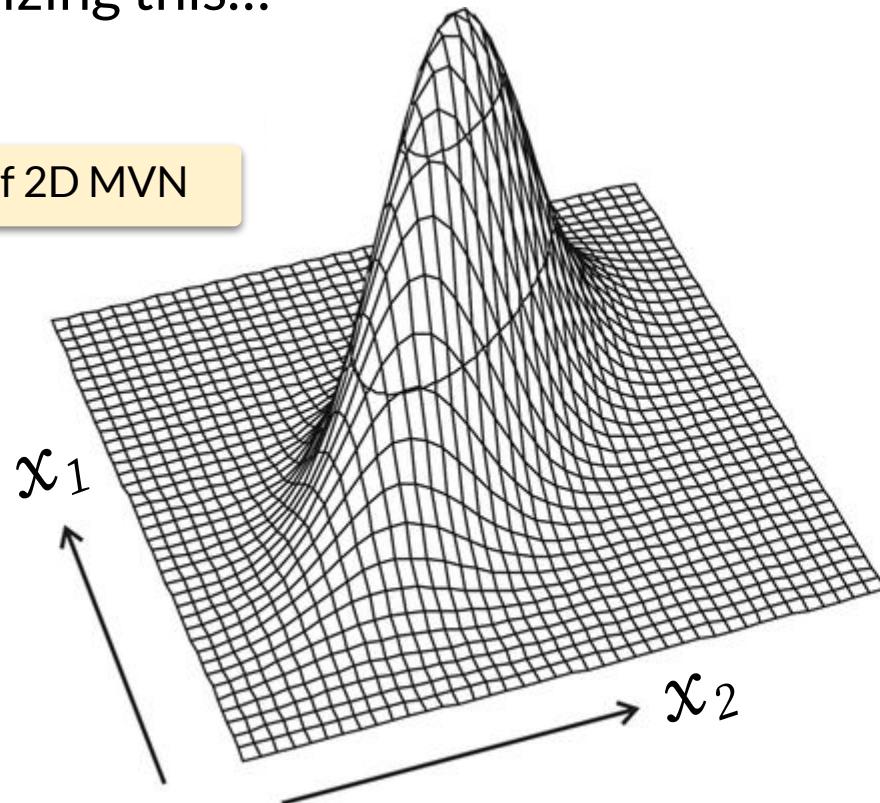


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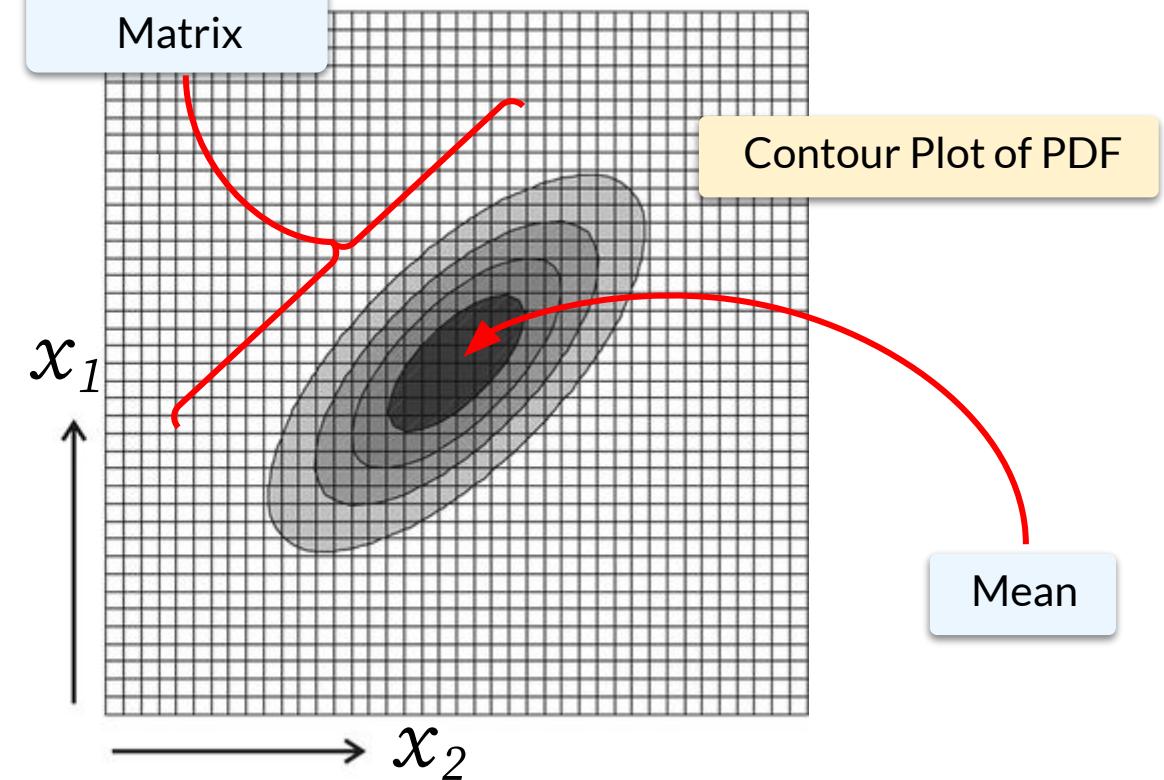
Gaussian Processes – Background on Multivariate Normal

Visualizing this...

PDF of 2D MVN



Covariance Matrix



Contour Plot of PDF

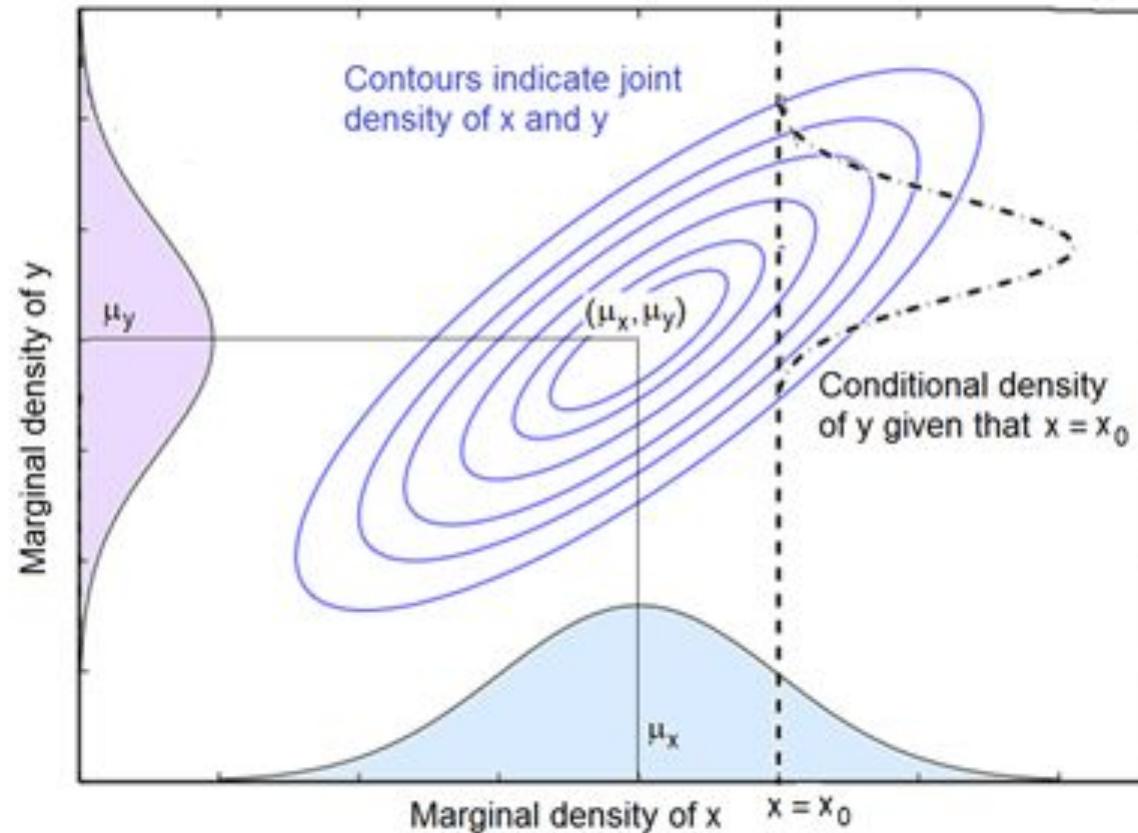
Mean

Source: Keith Sircombe, "Mountains in the shadows of time"

Gaussian Processes – Background on Multivariate Normal

Visualizing this...

Showing
marginal and
conditional
distributions.

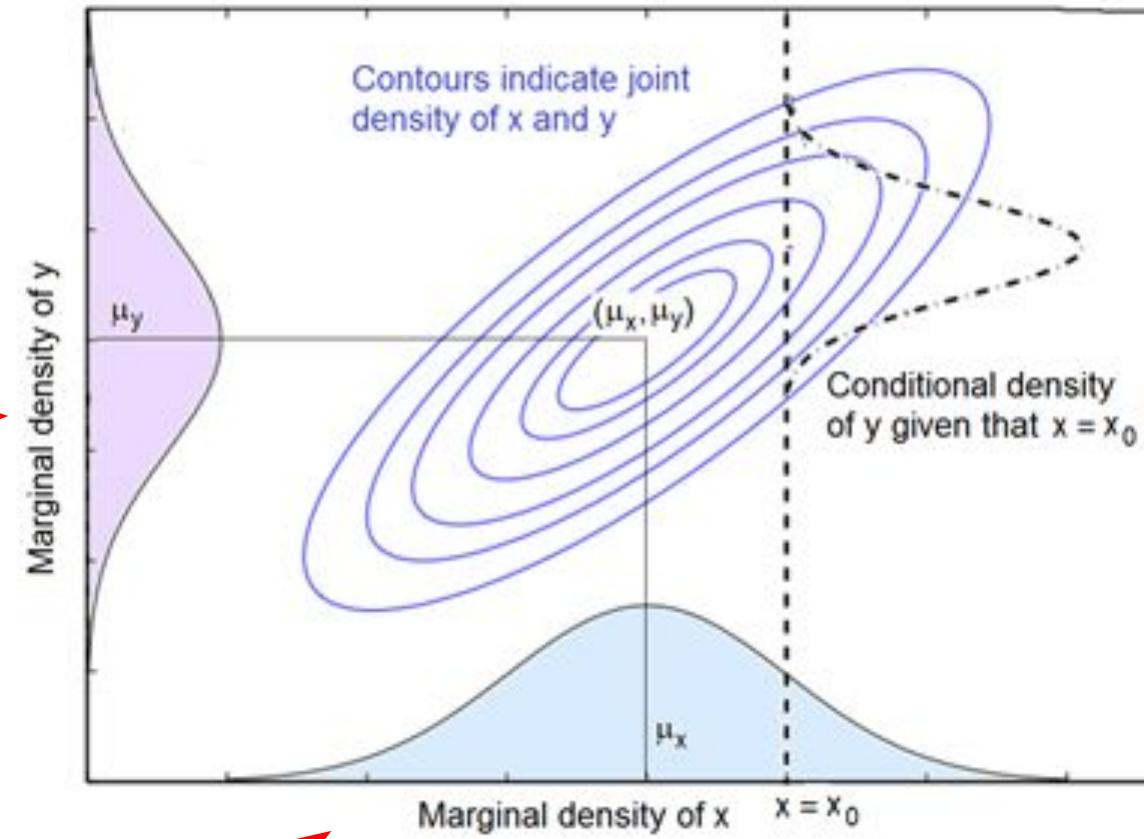


Gaussian Processes – Background on Multivariate Normal

Visualizing this...

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conditional
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Marginal distributions
are Gaussian!

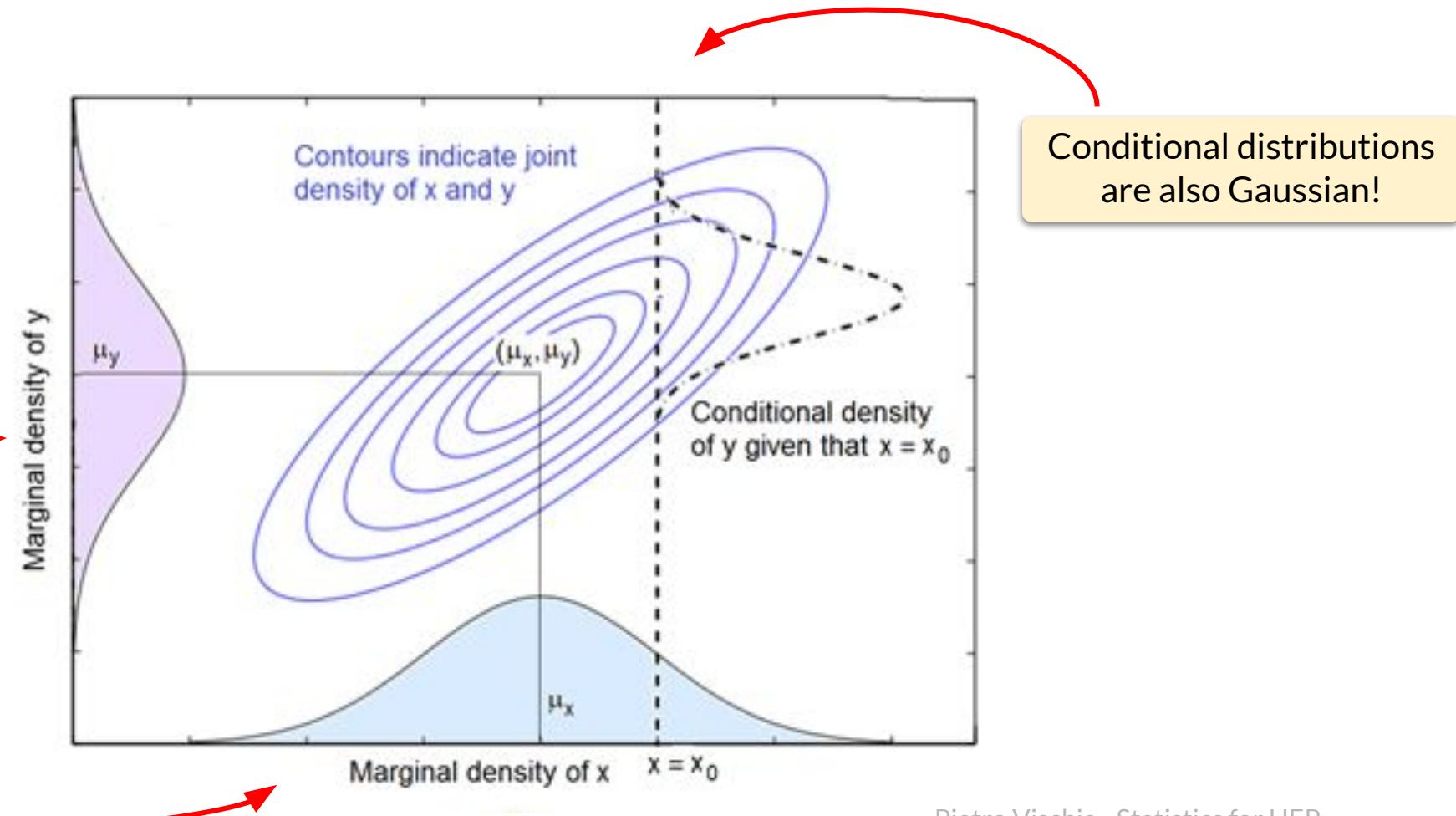


Gaussian Processes – Background on Multivariate Normal

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Gaussian Processes – More Formally

Gaussian Processes – More Formally

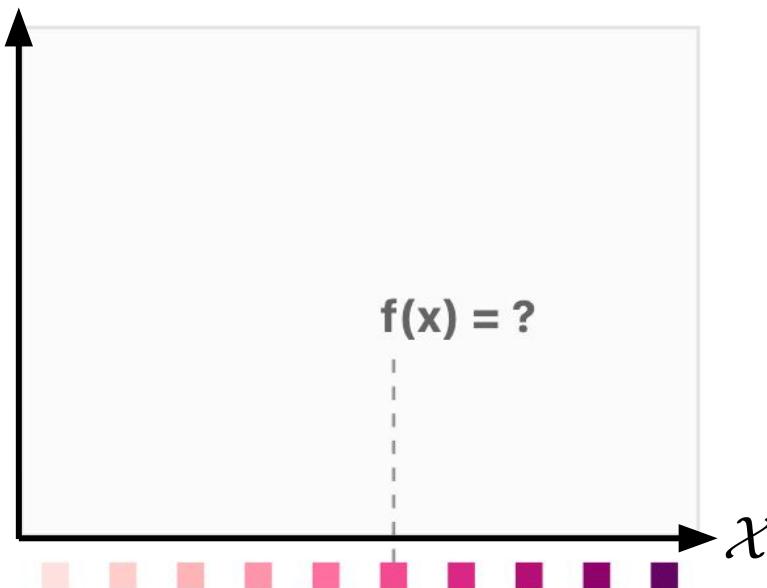
A Gaussian Process (GP) is a *stochastic process* – a collection of random variables – with the property:

“Every finite collection of those RVs has a multivariate normal distribution”

What does this mean?

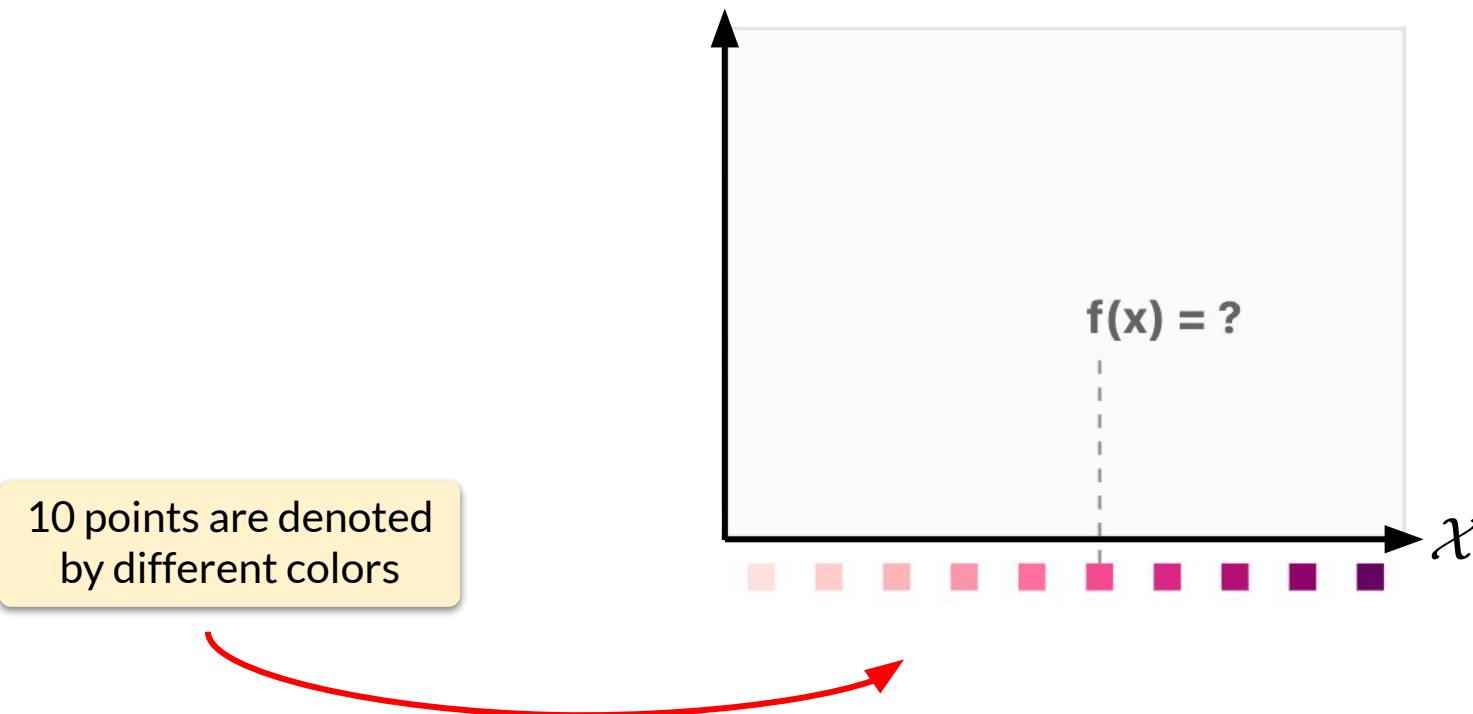
Gaussian Processes – More Formally

Suppose we pick a set of $d=10$ points on the x -axis, and want to define a **function** (or set of functions) over these points:



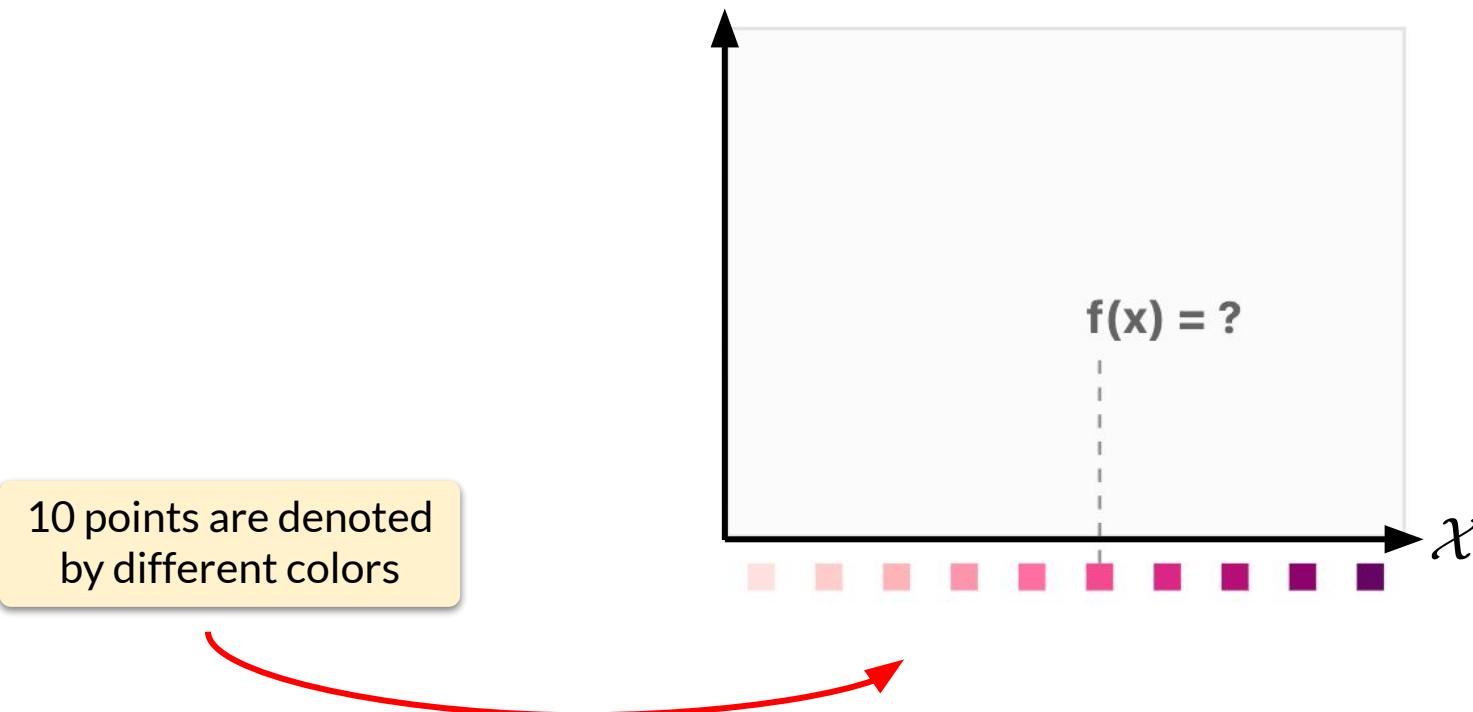
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Gaussian Processes – More Formally

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We can define a *distribution of functions*, defined on these 10 points, in the following way...

Gaussian Processes – More Formally

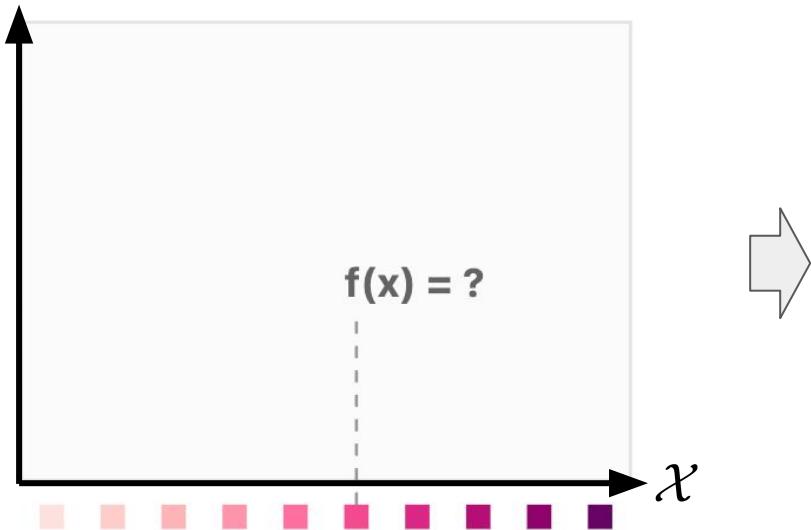
We define a MVN of d dimensions, with a mean and covariance matrix parameter.

Mean parameter $\mu \in \mathbb{R}^d$ – assume we set this to $\mathbf{0}$ for now.

Covariance matrix $\Sigma \in \mathbb{R}^{d \times d}$ – define using a *kernel function* based on distance between input points (next slide).

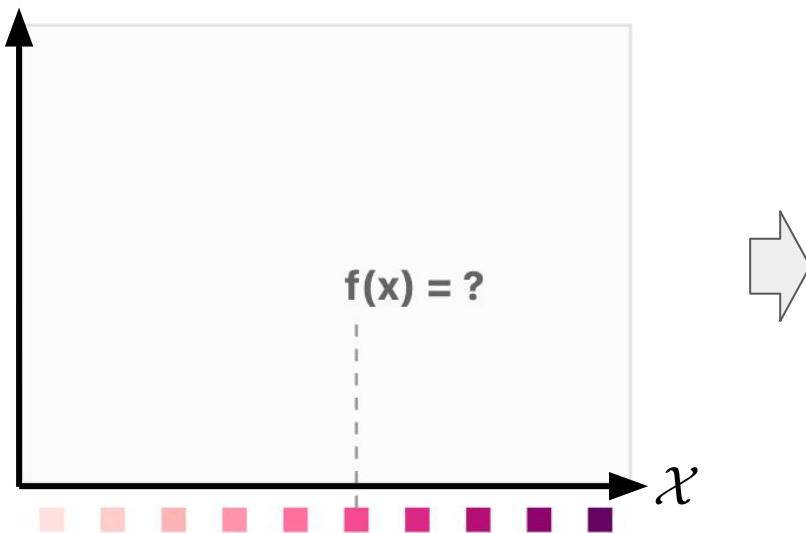
Gaussian Processes – More Formally

For these 10 points:

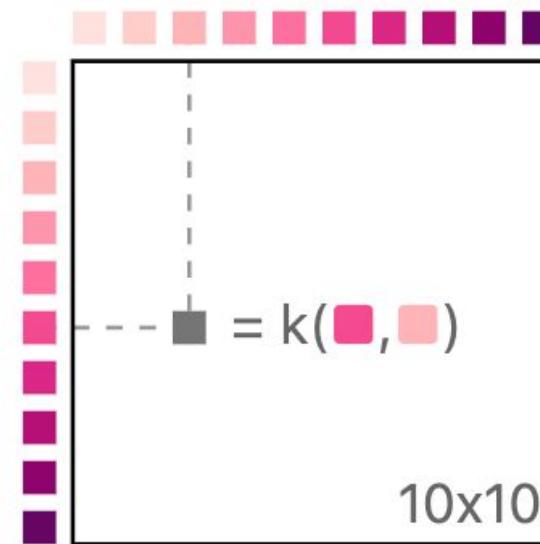


Gaussian Processes – More Formally

For these 10 points:



Covariance matrix:



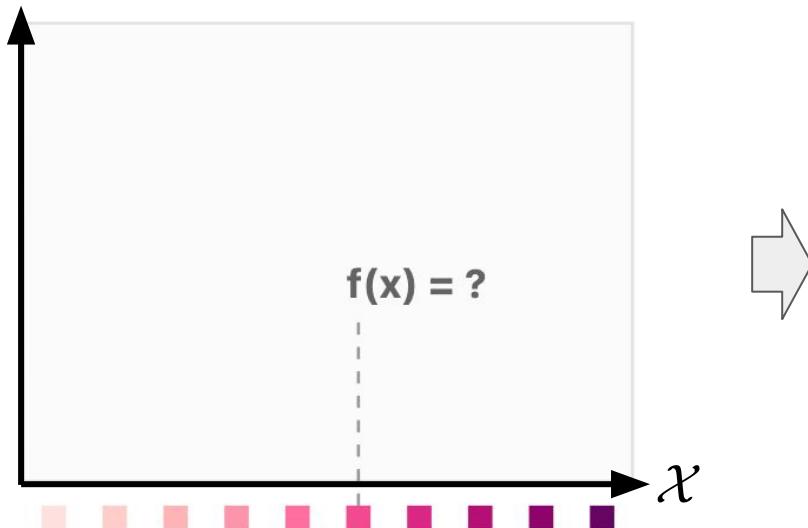
Define covariance between any pair of inputs using kernel function $k(\cdot, \cdot)$.

One example: *RBF kernel*,

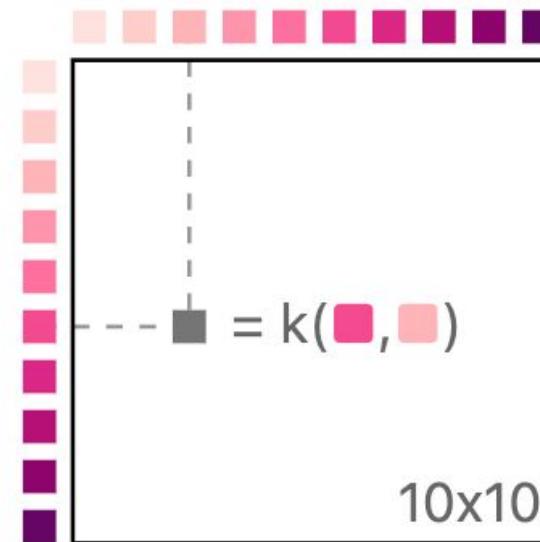
$$k(x_1, x_2) = \sigma^2 \left(-\frac{\|x_1 - x_2\|^2}{2\ell^2} \right)$$

Gaussian Processes – More Formally

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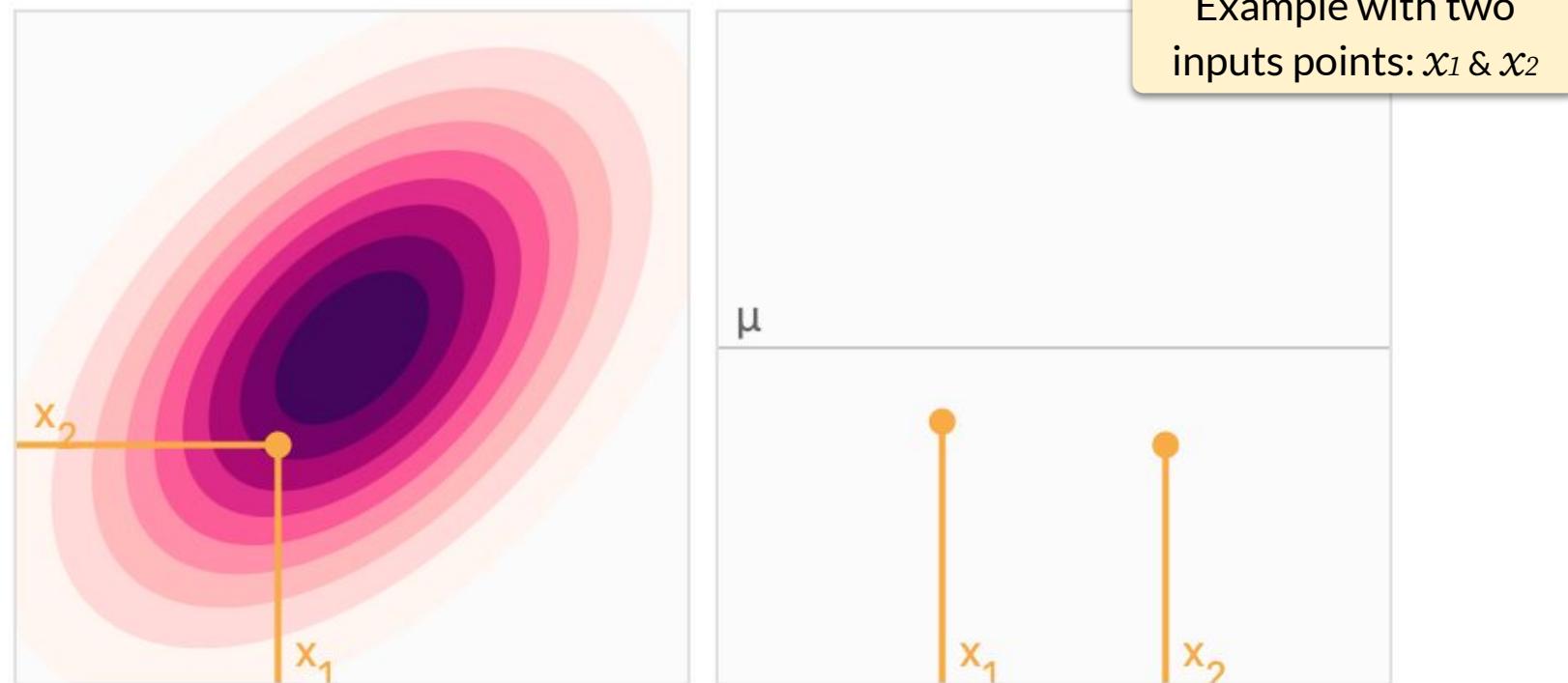
Highest when $x_1 = x_2$, lowest when x_2 very different from x_2 .

Gaussian Processes – More Formally

This yields a MVN, where samples give function values for the d input points:

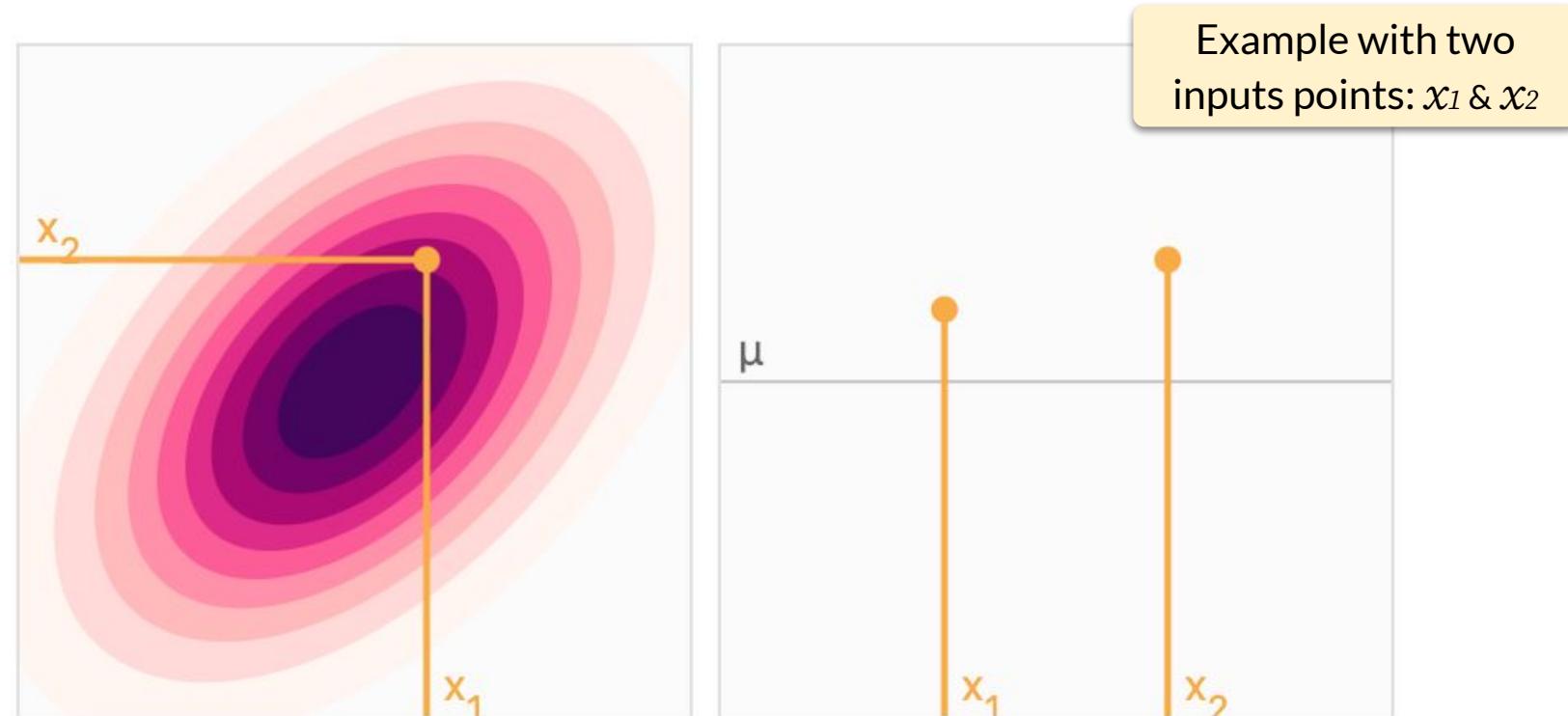
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Gaussian Processes – More Formally

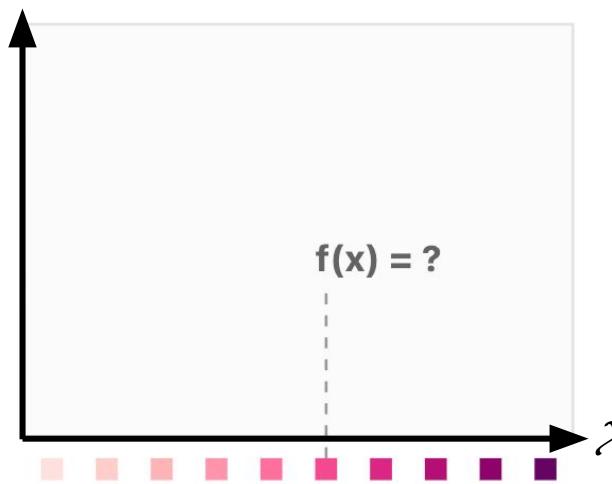
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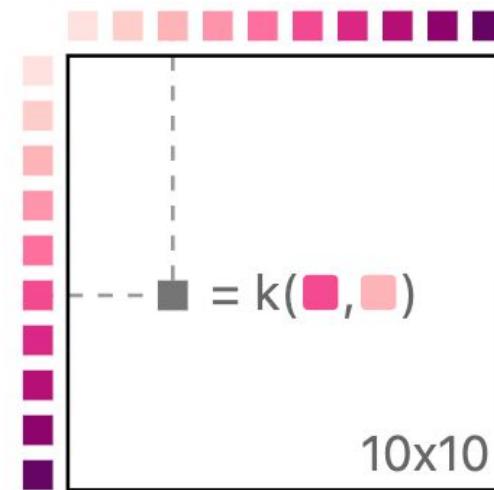
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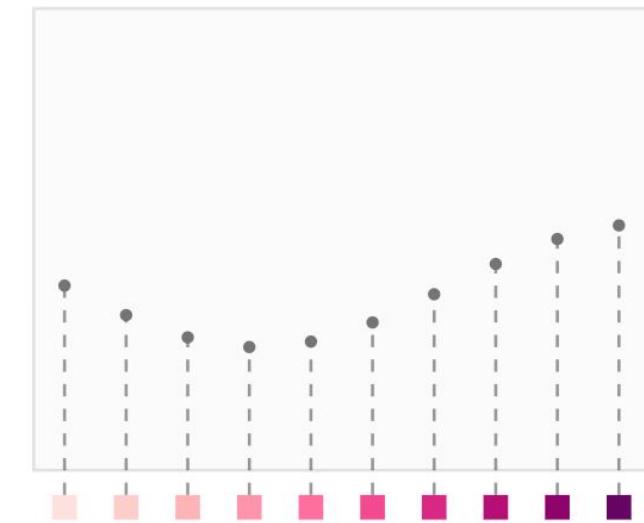
For these 10 points:



Covariance matrix:



Sampled function values



Gaussian Processes – More Formally

All together:

Gaussian Processes – More Formally

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We define a **Gaussian Process (GP)** as a distribution over functions, parameterized by a *mean function* $m(\cdot)$ and *covariance kernel function* $k(\cdot, \cdot)$, written

$$f \sim \text{GP}(m(\cdot), k(\cdot, \cdot))$$

Gaussian Processes – More Formally

All together:

We define a **Gaussian Process (GP)** as a distribution over functions, parameterized by a *mean function* $m(\cdot)$ and *covariance kernel function* $k(\cdot, \cdot)$, written

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Where, for any finite collection of d input points, we can:

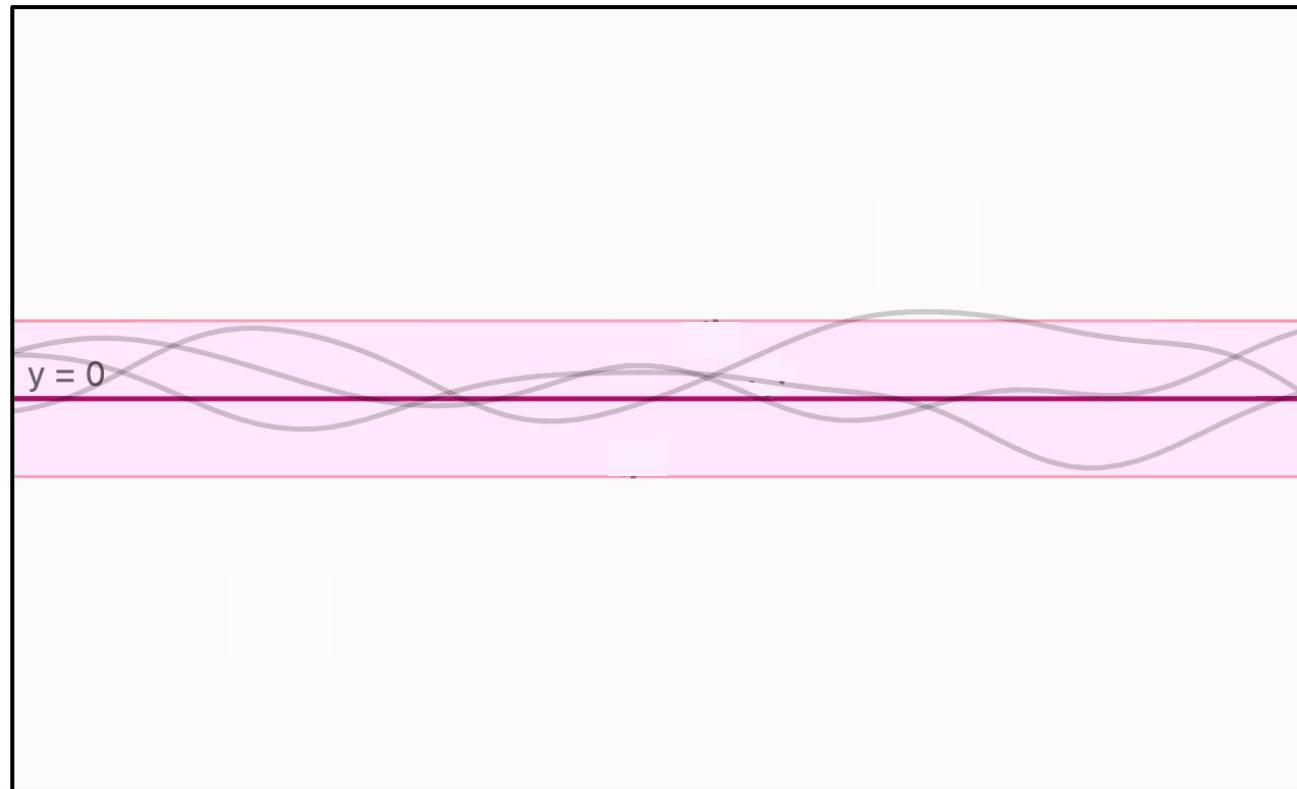
- Define a MVN on these d points, using the mean/kernel functions to define a mean parameter and covariance matrix.
- And this MVN yields a distribution over function values.

Gaussian Processes – Sausage Plot

What does this distribution look like?

Gaussian Processes – Sausage Plot

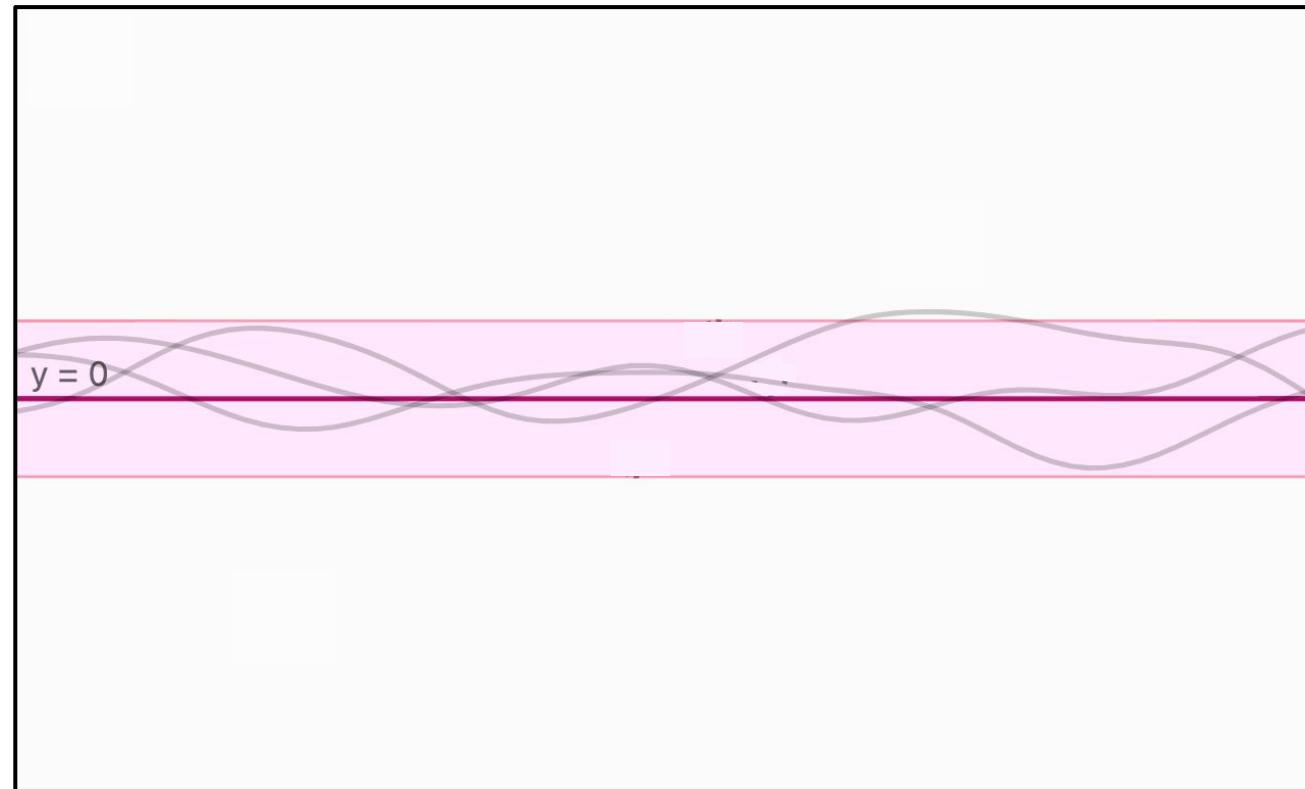
What does this distribution look like? → often drawn like this:



Gaussian Processes – Sausage Plot

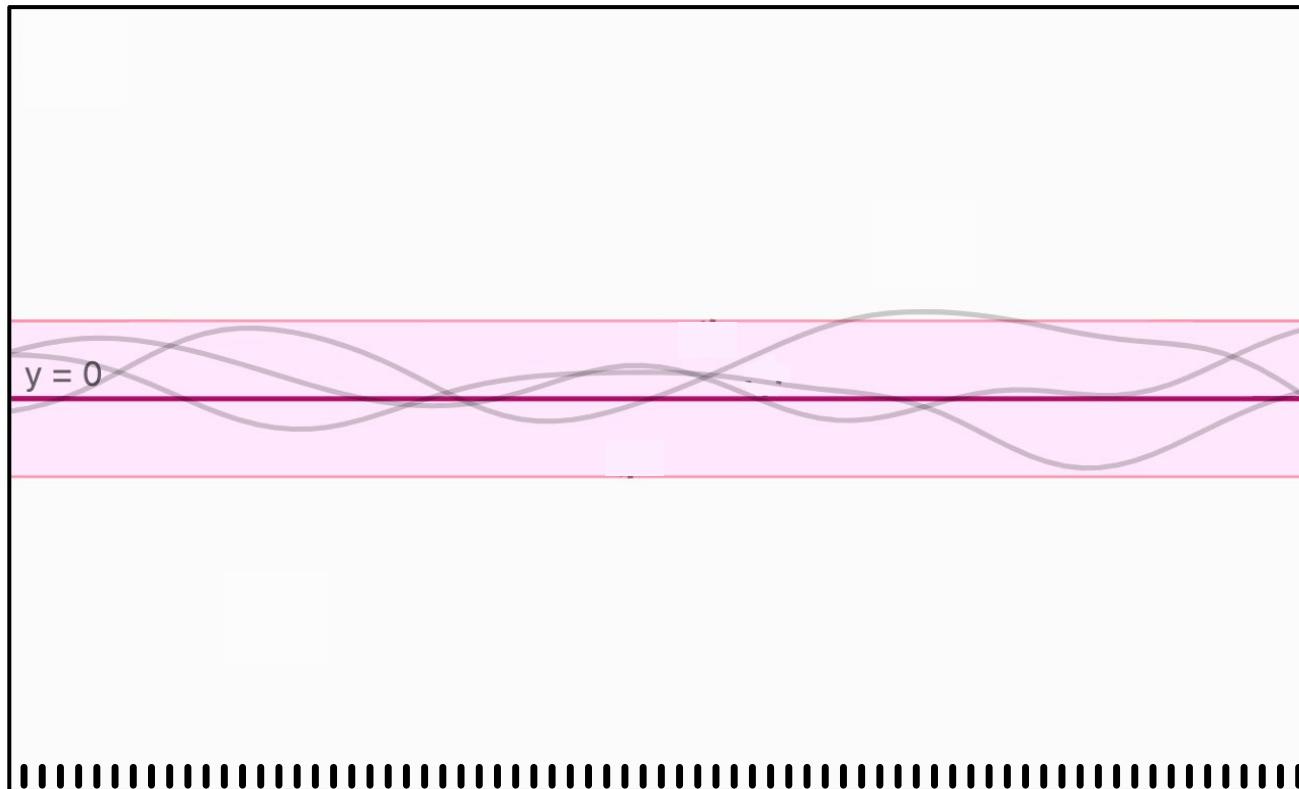
Called a *Sausage Plot* because it looks like a sausage

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Gaussian Processes – Sausage Plot

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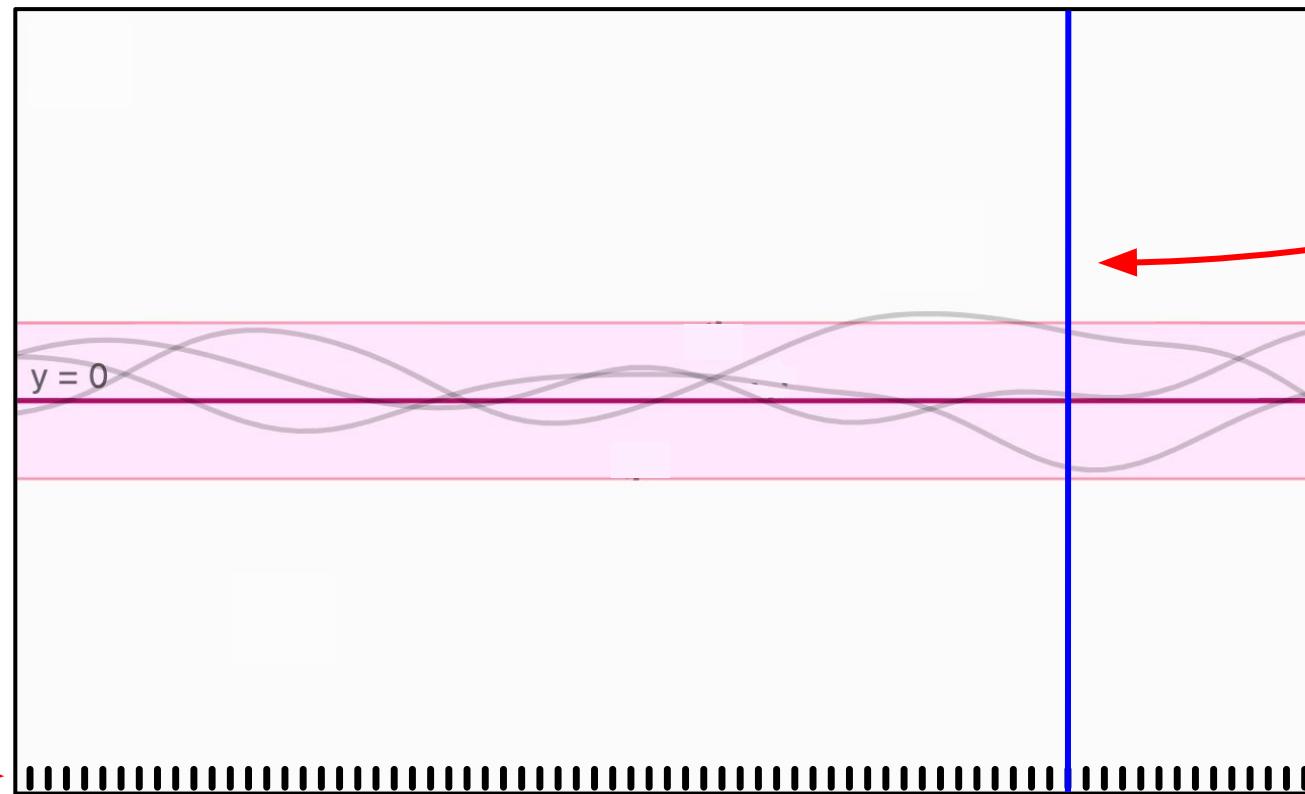


This plot depicts a MVN
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Gaussian Processes – Sausage Plot

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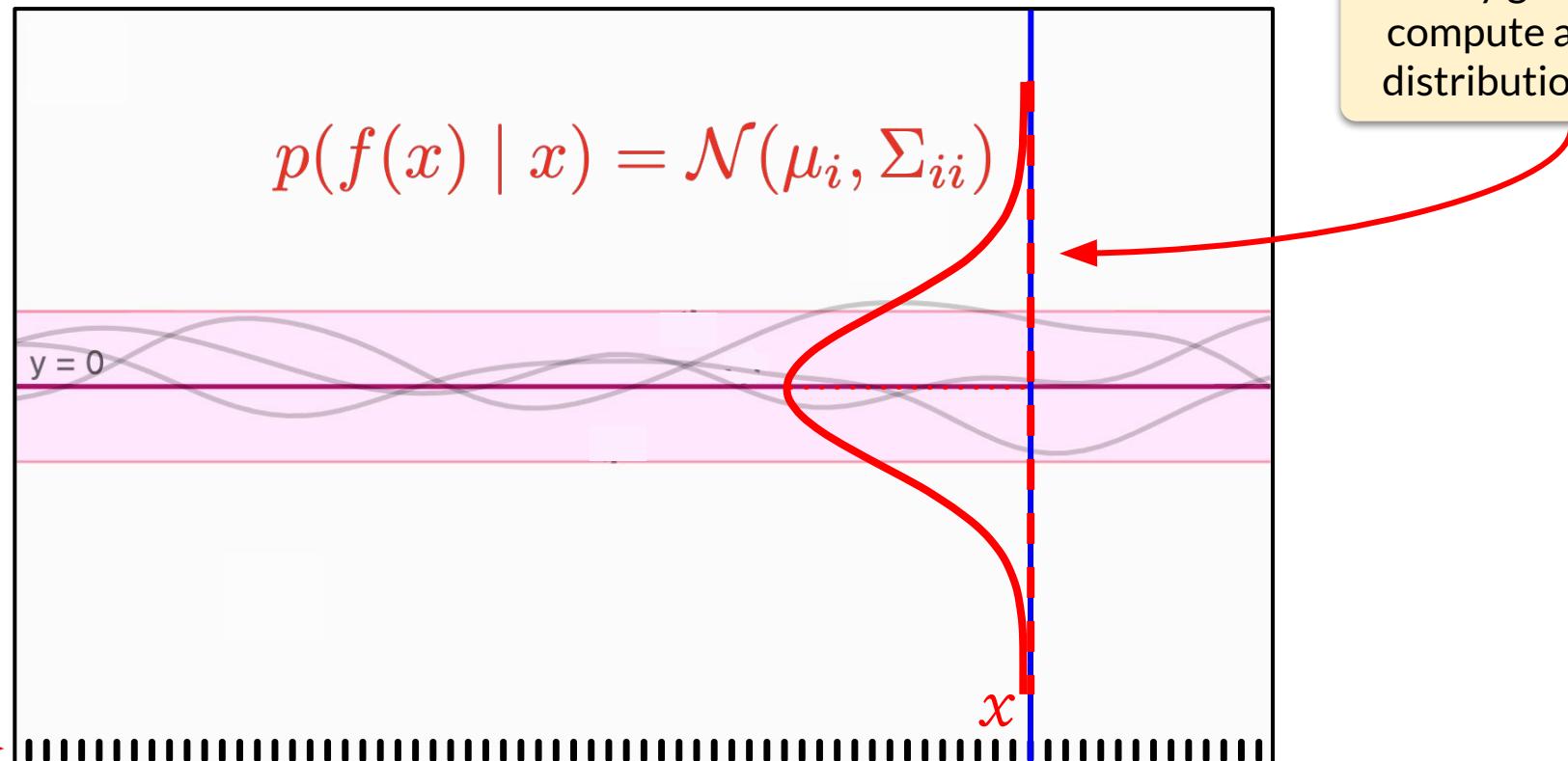


For any grid point, can compute a marginal distribution of MVN

Gaussian Processes – Sausage Plot

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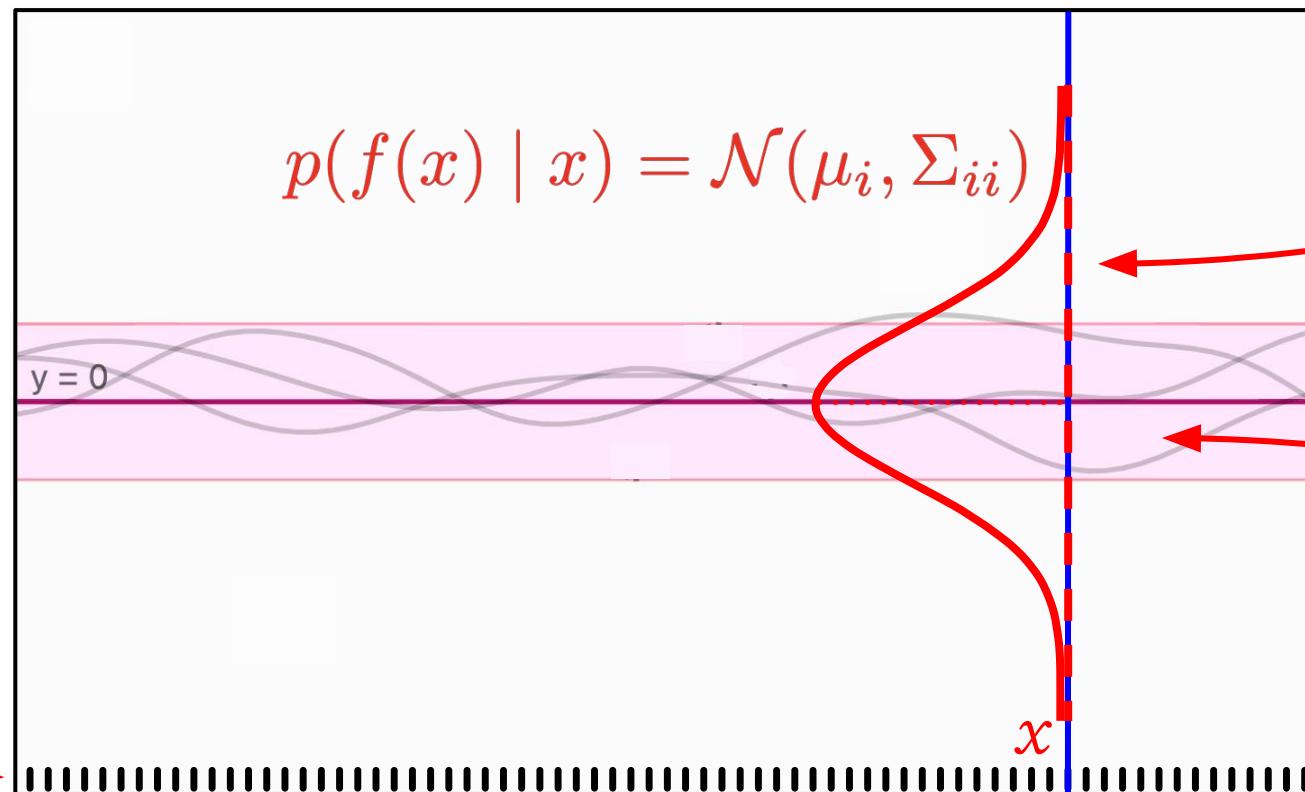
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For any grid point, can compute a marginal distribution of MVN

Samples from the MVN are shown in grey (appear as functions!)

Gaussian Processes – Posterior

So this lets us define a distribution over functions on any collection of inputs points.

Next, how do we use GPs for predictive UQ models?

Gaussian Processes – Posterior

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- *I.e.*, we have a dataset of (x, y) pairs.
- And we want to learn a predictive model, and then make predictions (along with uncertainty) on new test inputs.

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⇒ We will do Bayesian inference using this model!

Gaussian Processes – Posterior

In particular, we will:

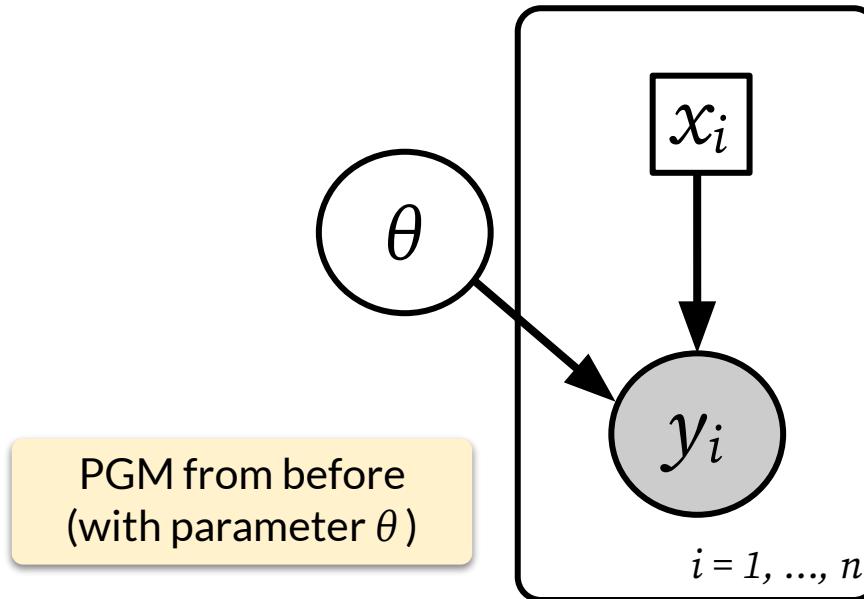
- Use a GP to define a prior over functions.
- Observe a set of (x, y) pairs.
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Gaussian Processes – Posterior

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Prior on Parameter

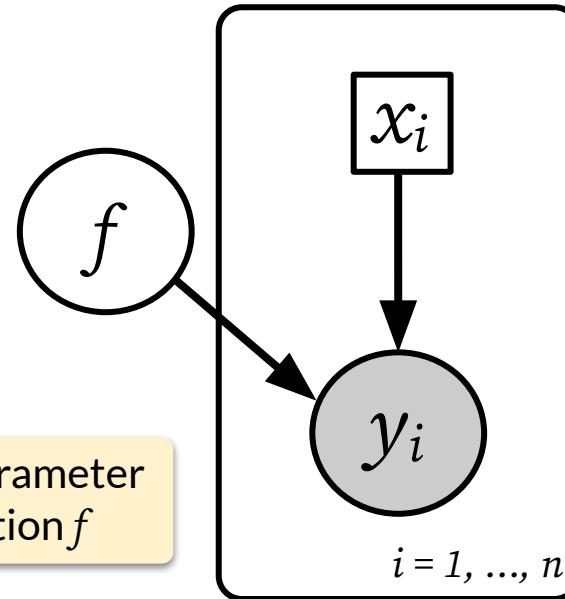


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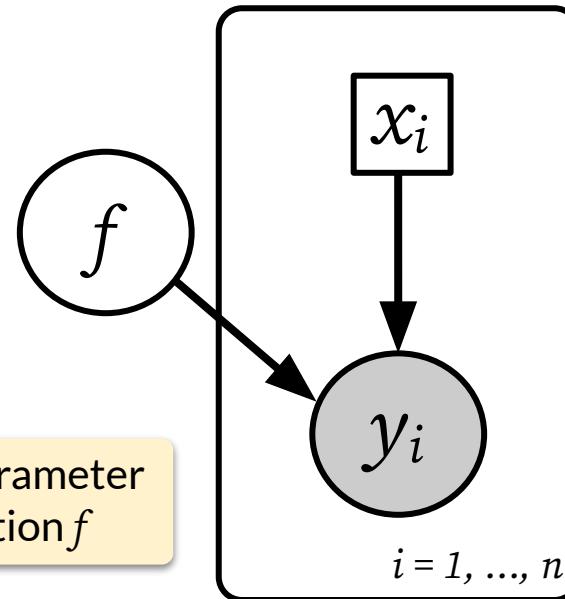
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Gaussian Processes – Posterior

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In GPs, the parameter
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We want GP posterior:
 $p(f | \{y_i\})$

Gaussian Processes – Posterior

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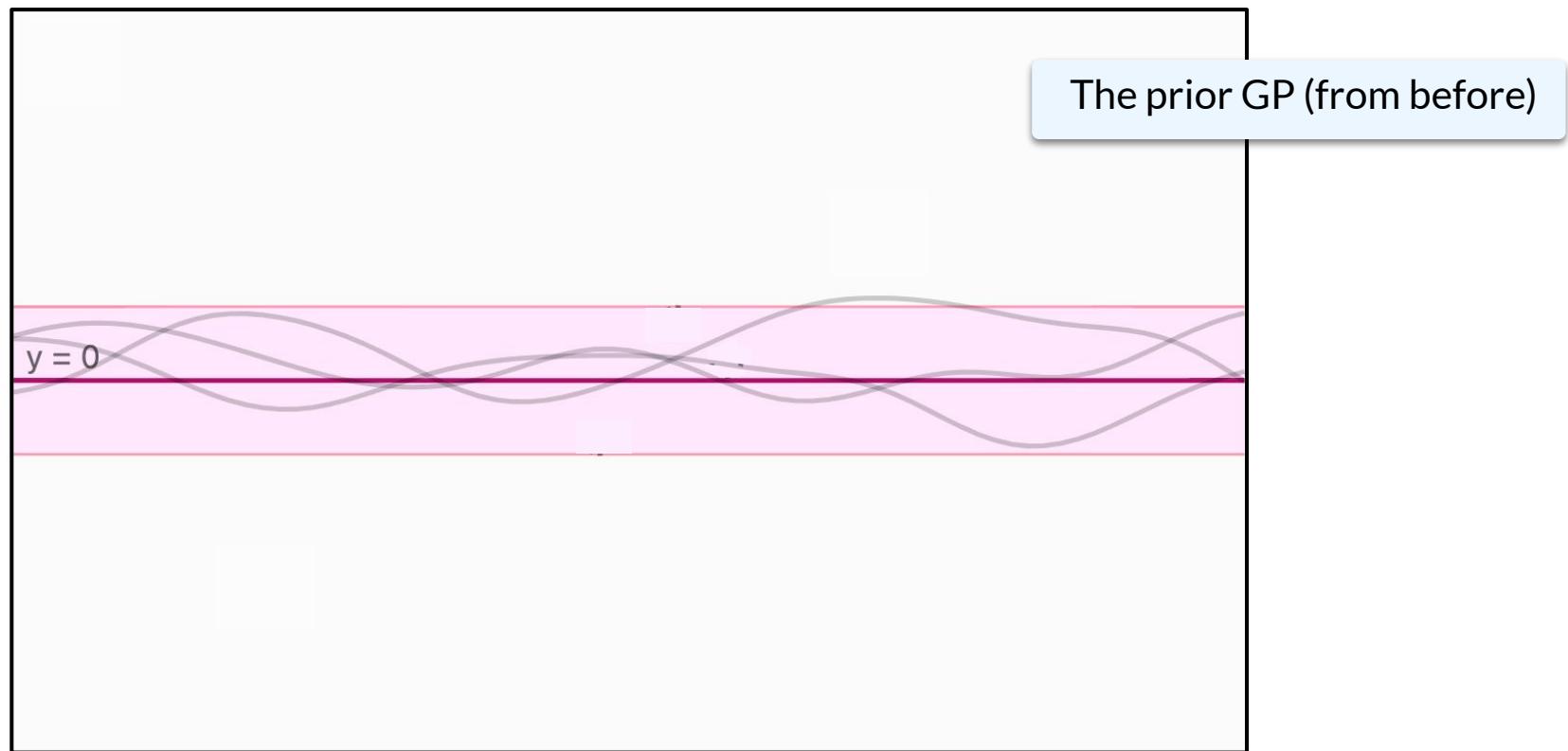
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- \Rightarrow Posterior is (*essentially*) also a multivariate normal distribution.

Posterior is technically a **posterior Gaussian process**, so it's another GP with a mean function and covariance kernel parameters

\Rightarrow which can be computed in closed form.

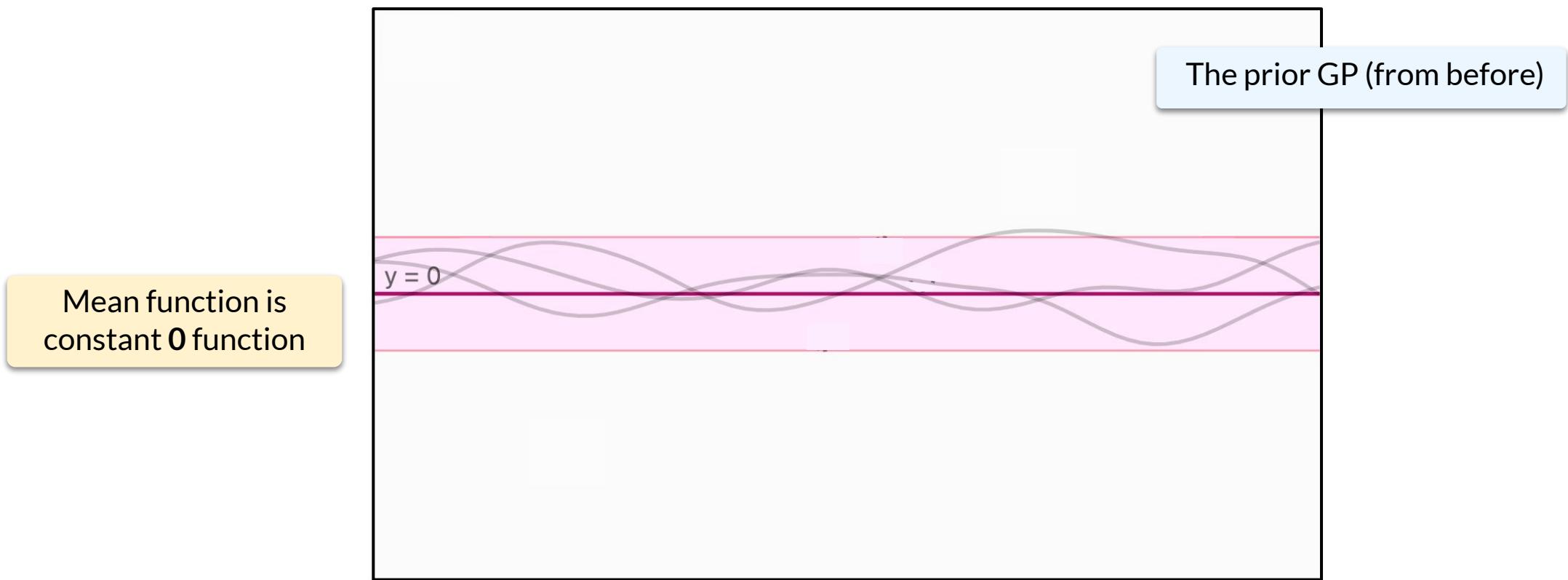
Gaussian Processes – Posterior

Visualizing this:



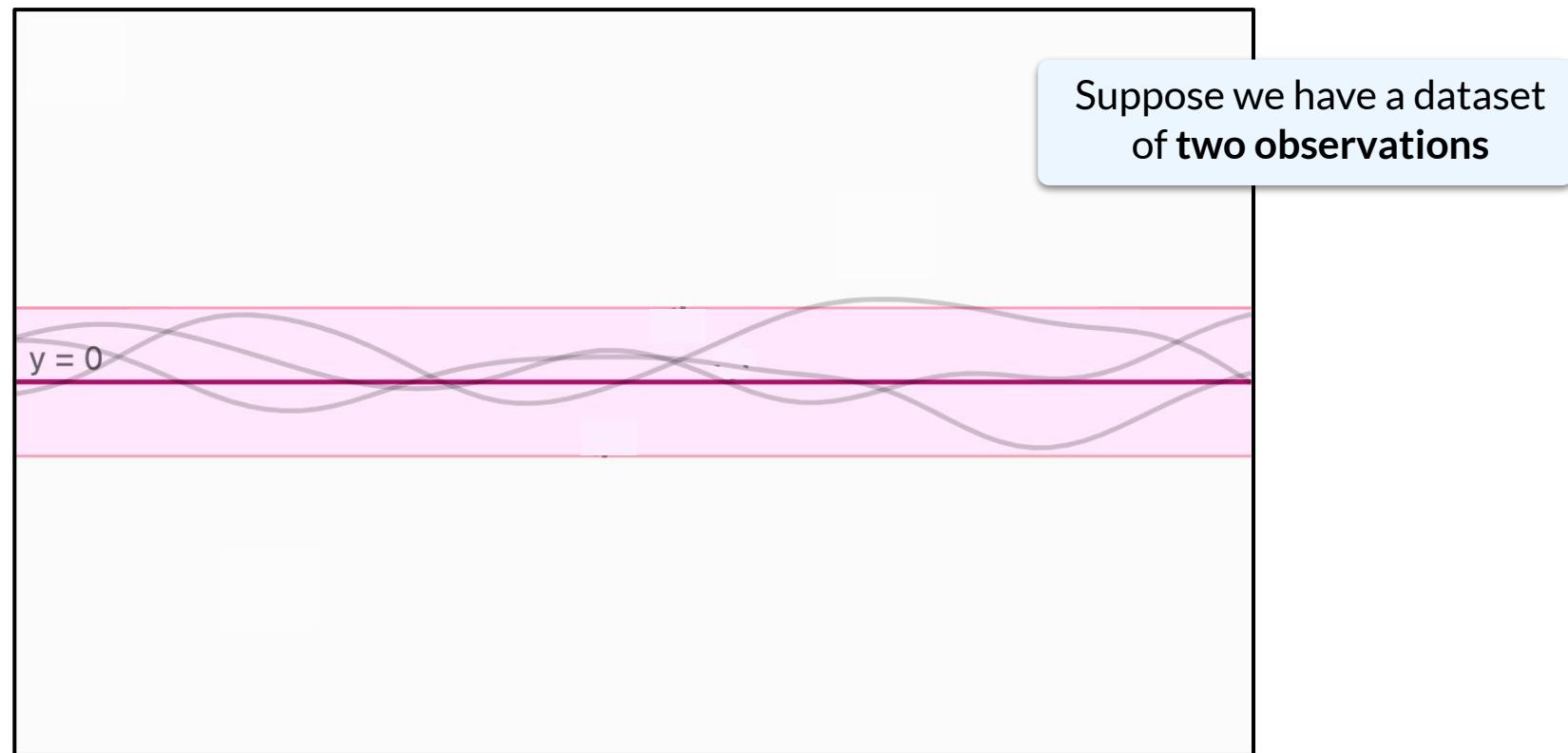
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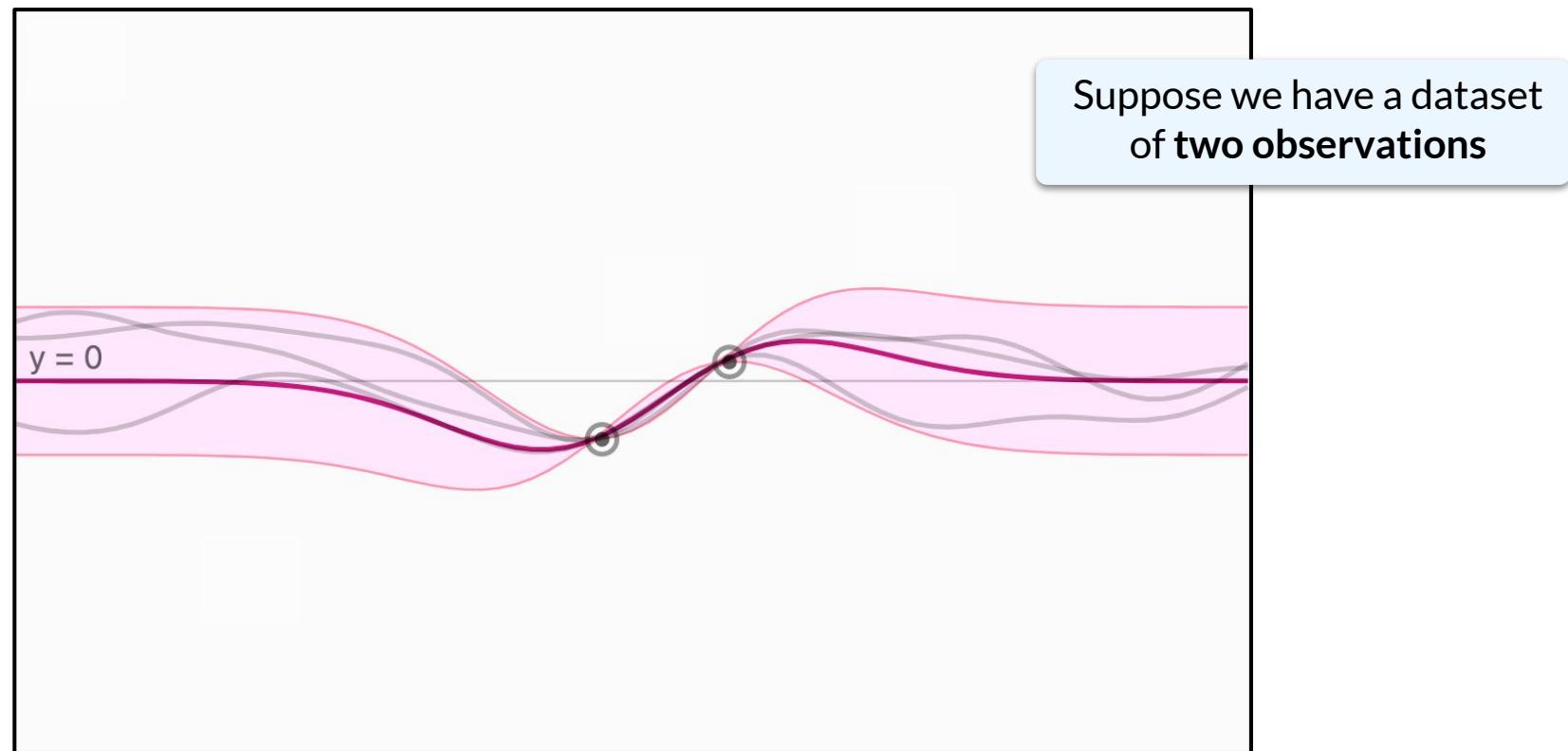
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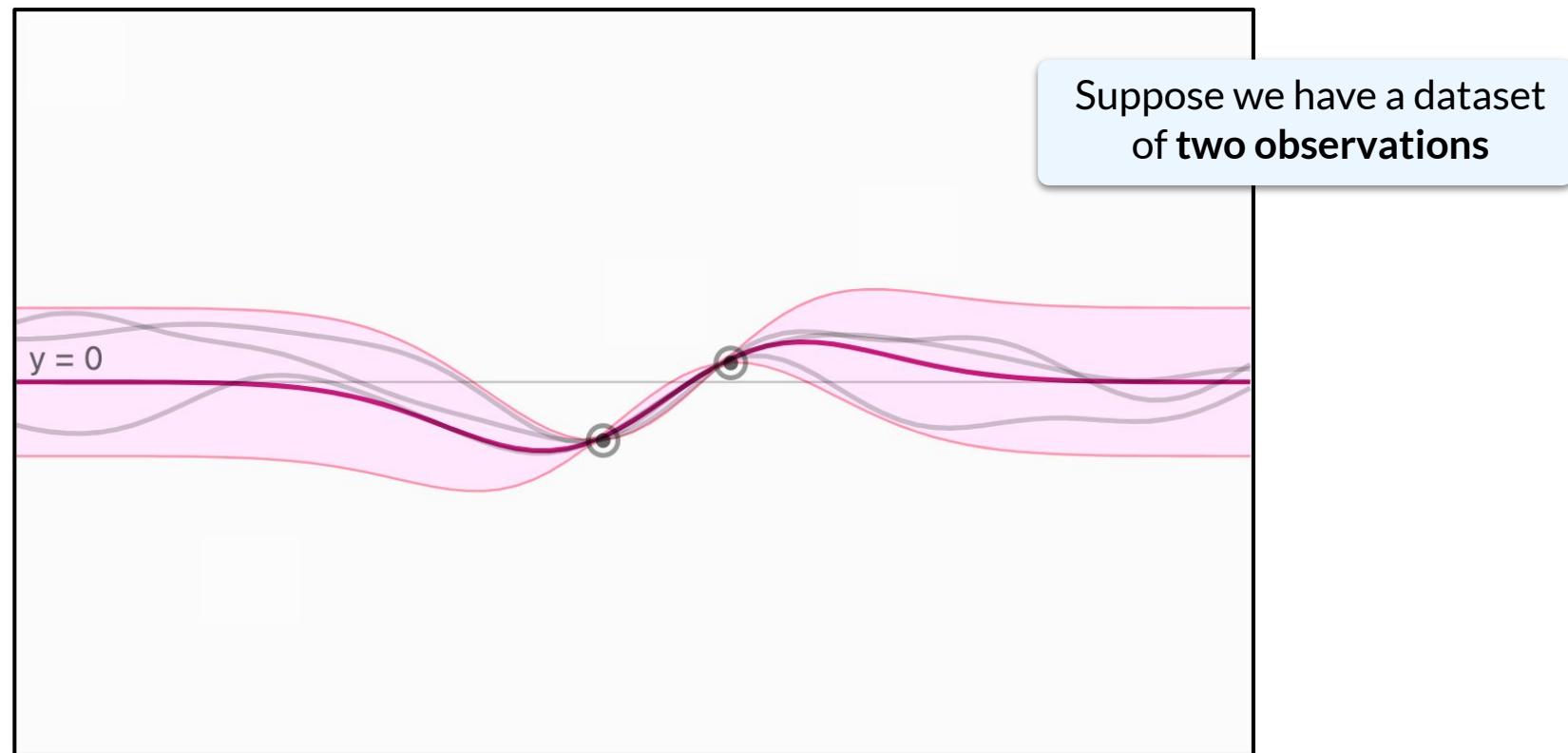
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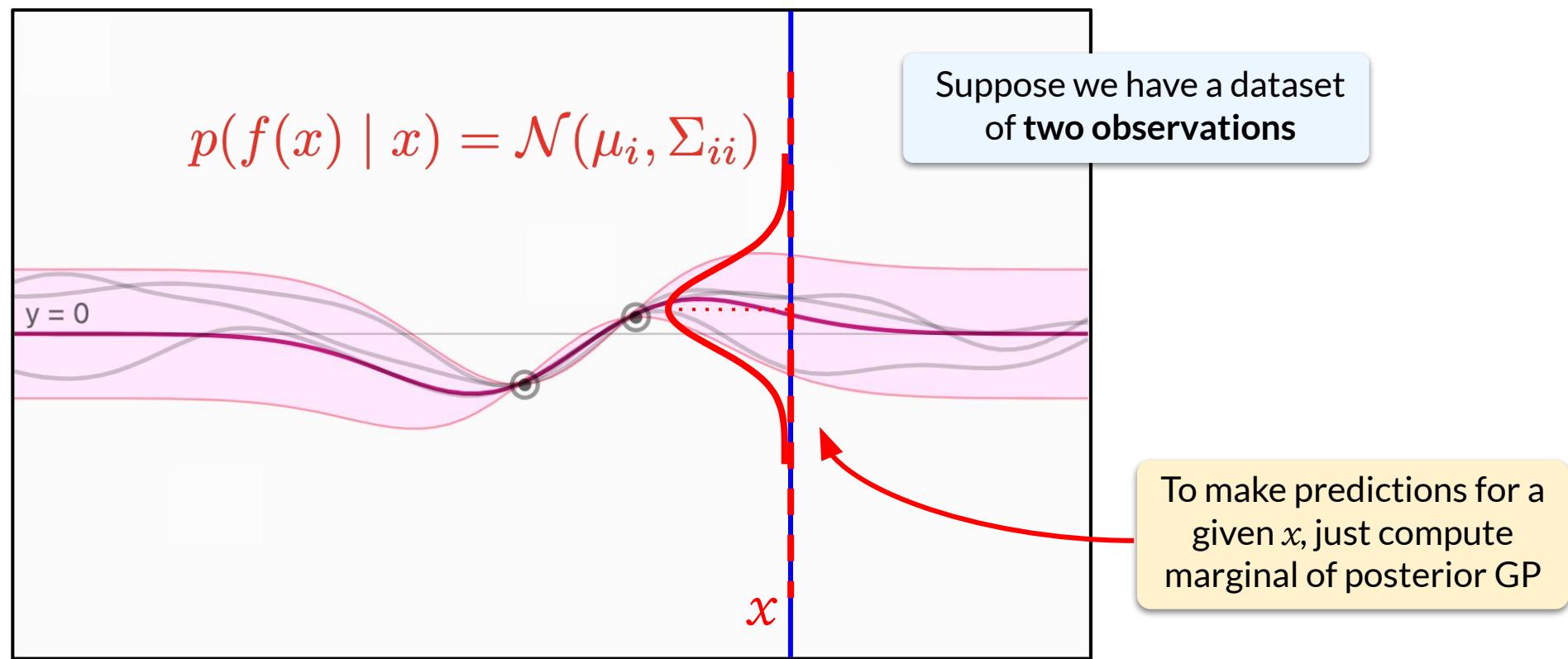
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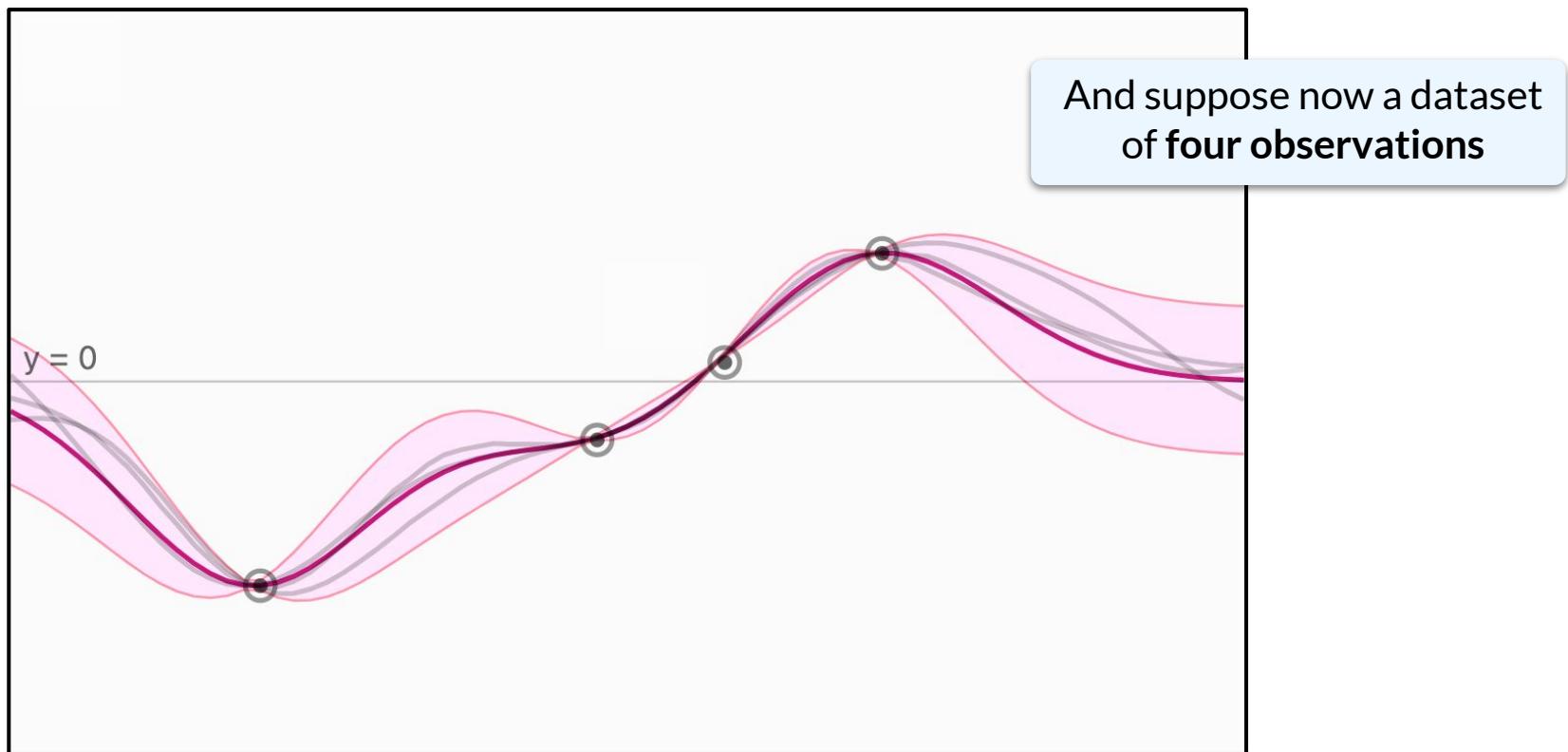
Gaussian Processes – Posterior

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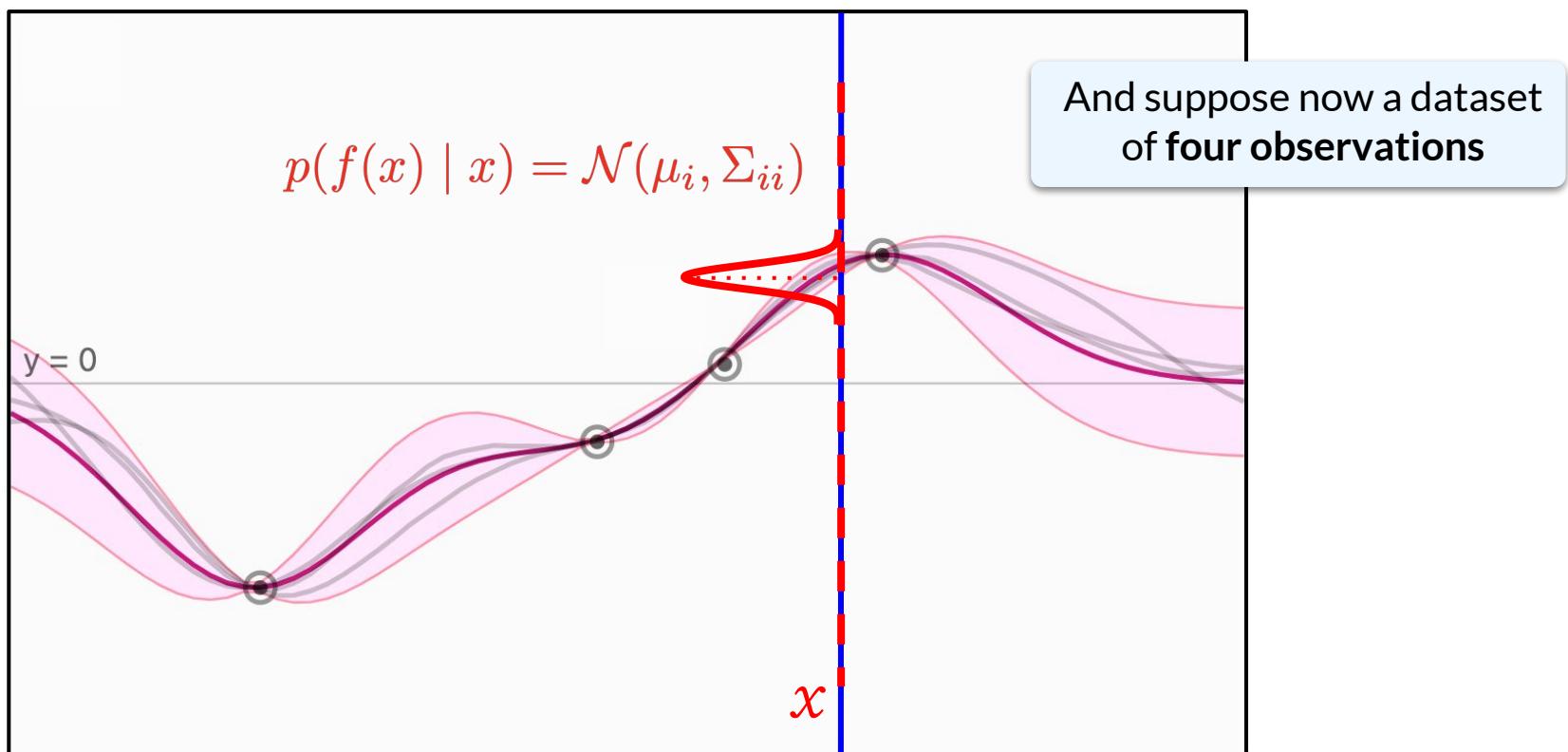
Gaussian Processes – Posterior

Visualizing this:



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Visualizing this:



Gaussian Processes – Summary

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- ⇒ This yields a predictive UQ model!

Deep Uncertainty Models

We can also define predictive UQ models using neural networks.

Deep Uncertainty Models

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I will go through a few examples of these.

Deep Uncertainty Models

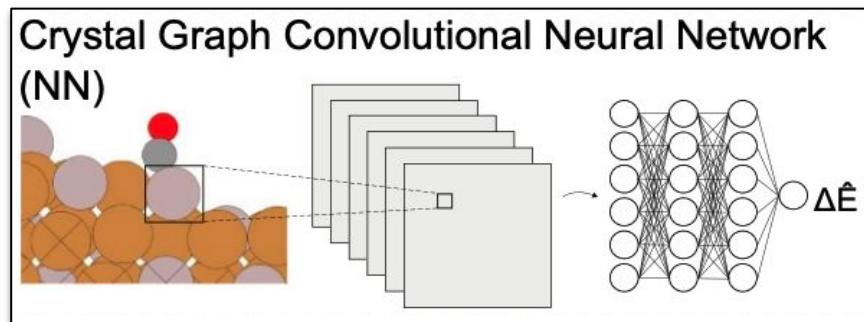
As a running example, we'll look at methods for incorporating predictive uncertainty into the following neural network:

Deep Uncertainty Models – Example: Computational Catalyst Design

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Xie, Tian, and Jeffrey C. Grossman. "Crystal graph convolutional neural networks for an accurate and interpretable prediction of material properties." *Physical review letters*, 2018.

CGCNN Model (*a graph convolutional neural network model*)

- *Inputs:* three-dimensional atomic structure (a graph).
- *Outputs:* DFT-calculated site adsorption energies ΔE .
(\Rightarrow Regression).

"Methods for comparing uncertainty quantifications for material property predictions", *Tran, *Neiswanger, et al., MLST. 2020

"Computational catalyst discovery: Active classification through myopic multiscale sampling", *Tran, *Neiswanger, et al., J. Chem. Phys. 2021

Deep Uncertainty Models – Example: Computational Catalyst Design

Suppose we want to *incorporate uncertainty into this neural network.*

(I.e., use neural network for both point predictions and uncertainties).

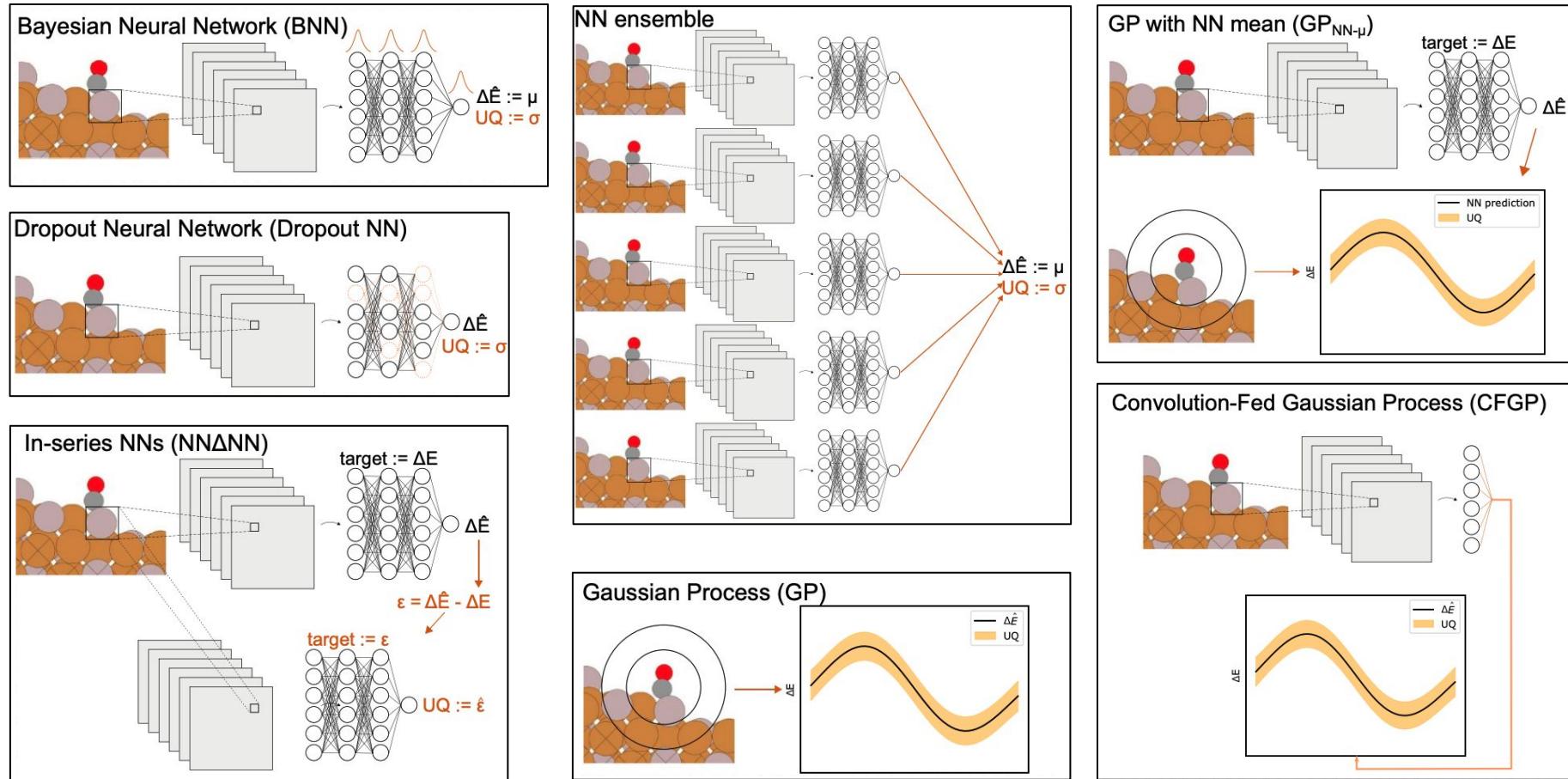
Deep Uncertainty Models – Example: Computational Catalyst Design

Suppose we want to *incorporate uncertainty into this neural network.*

(I.e., use neural network for both point predictions and uncertainties).

⇒ there are many ways this can be done, and it's an active area of research!

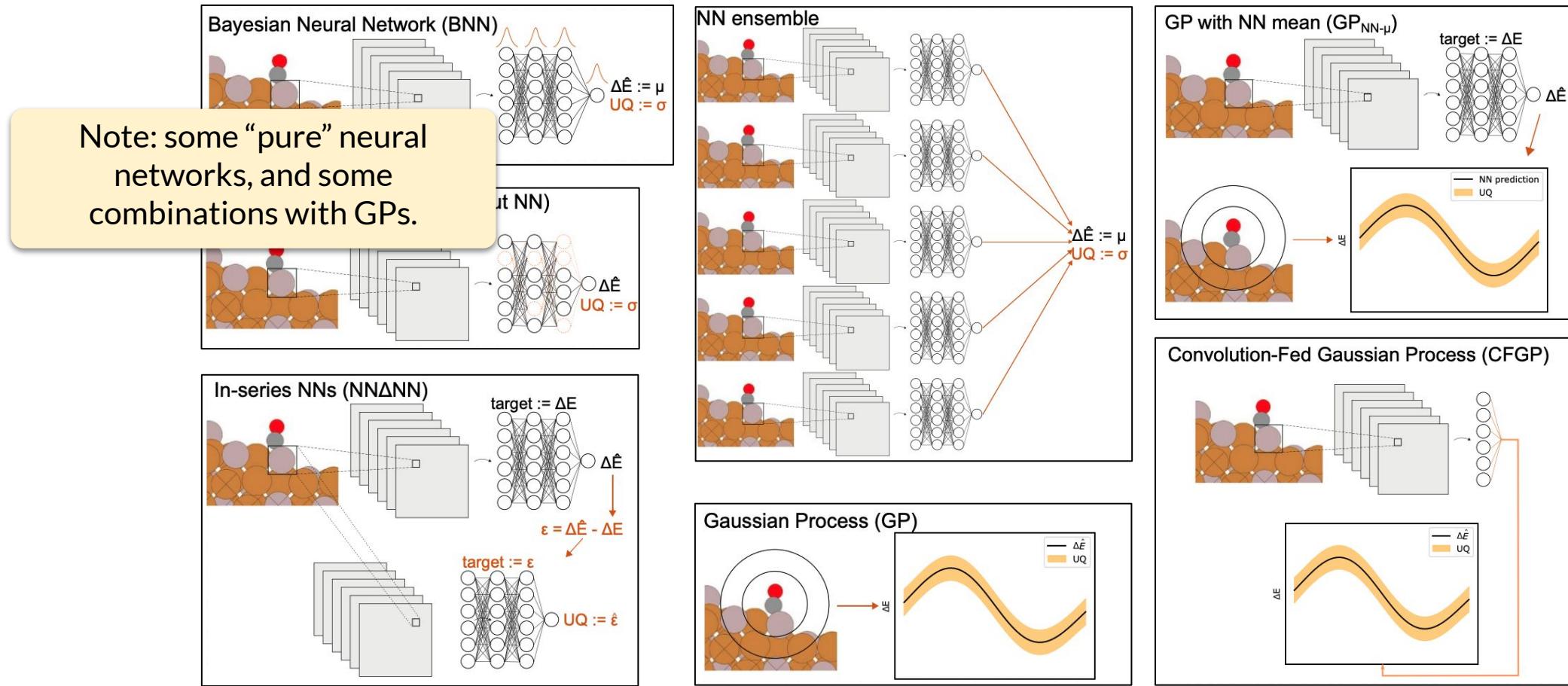
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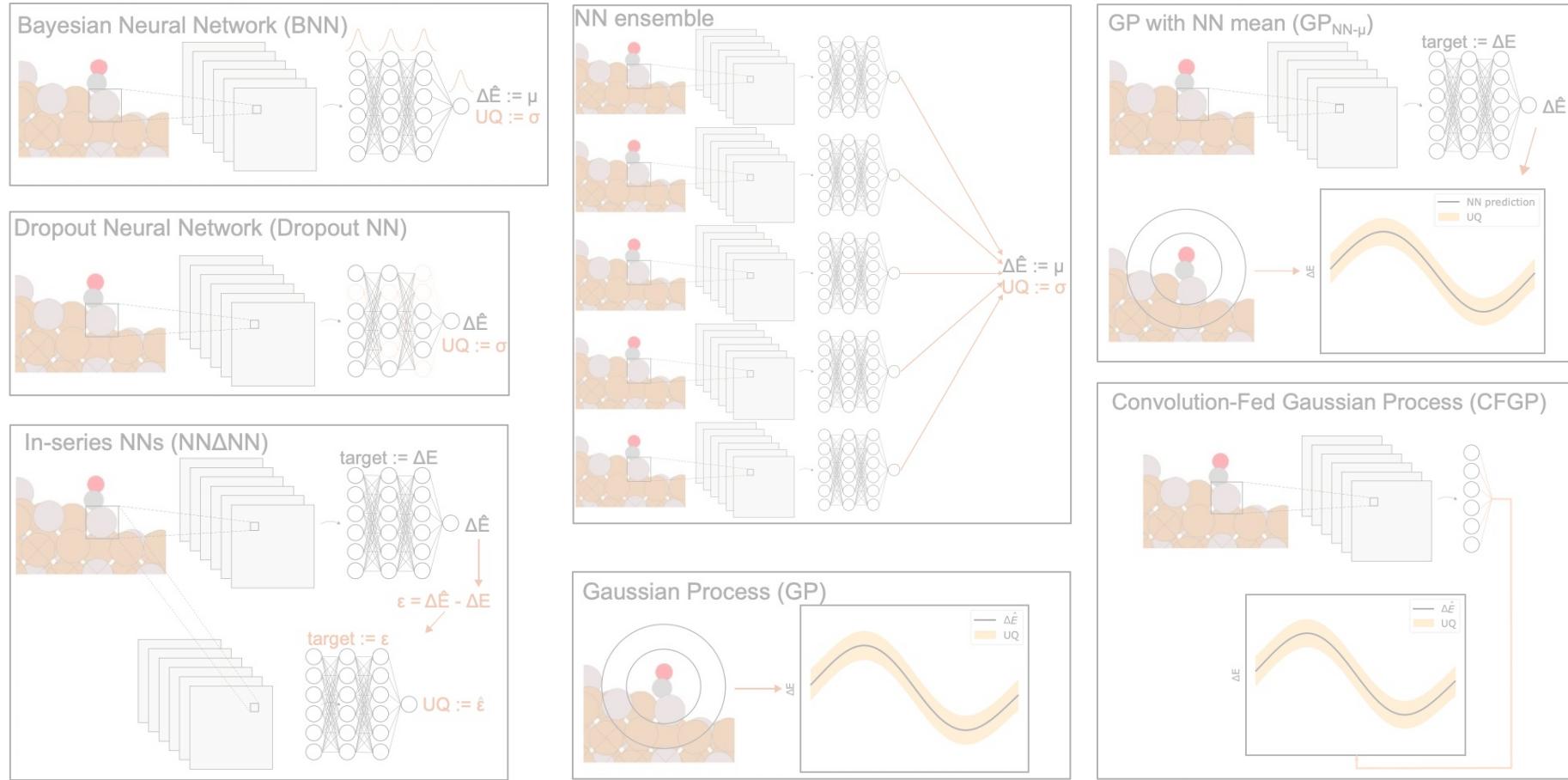
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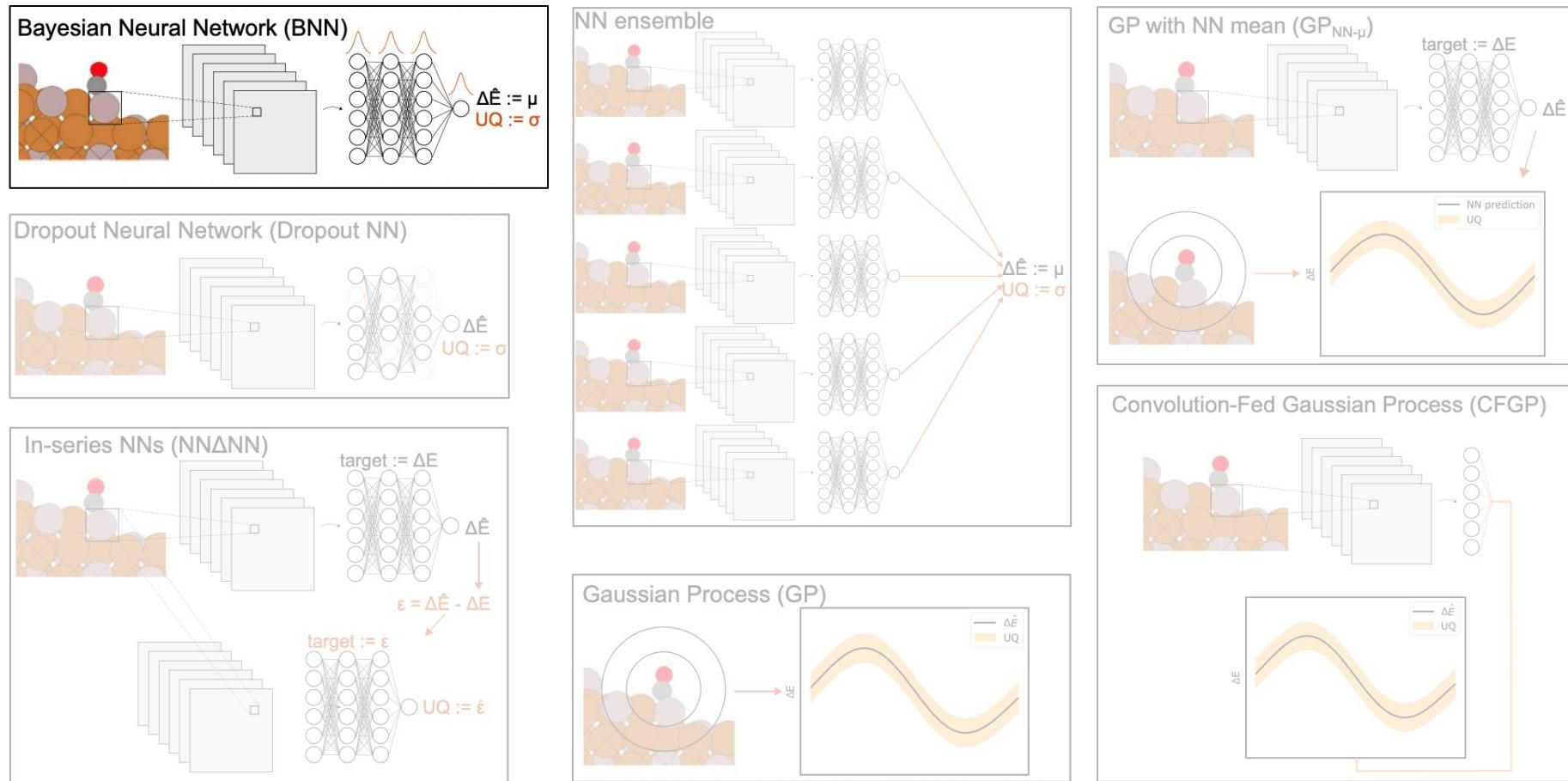
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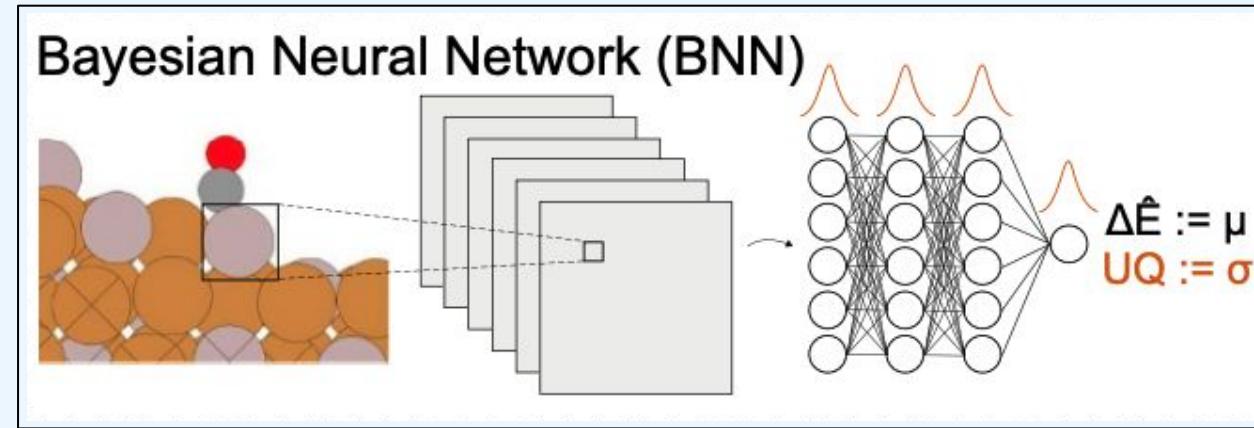
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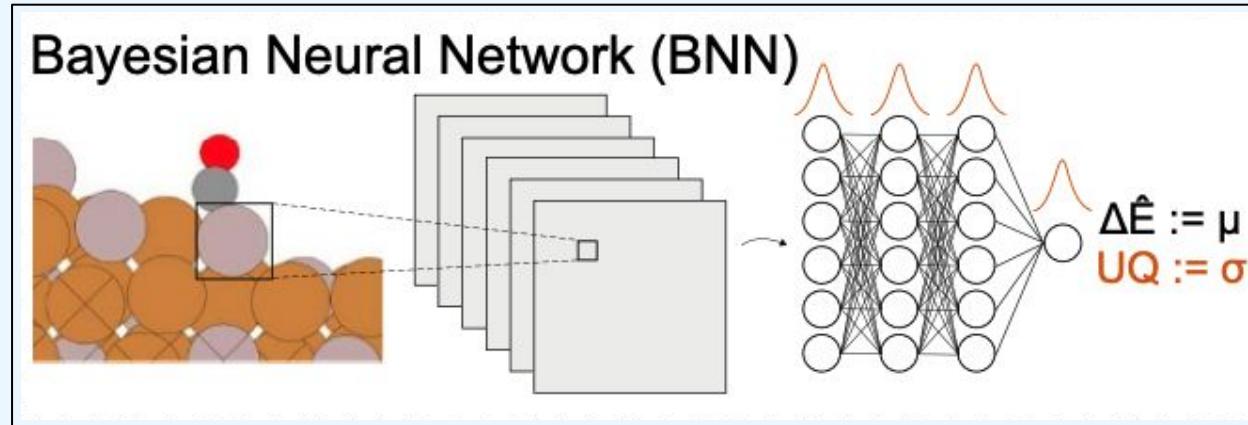
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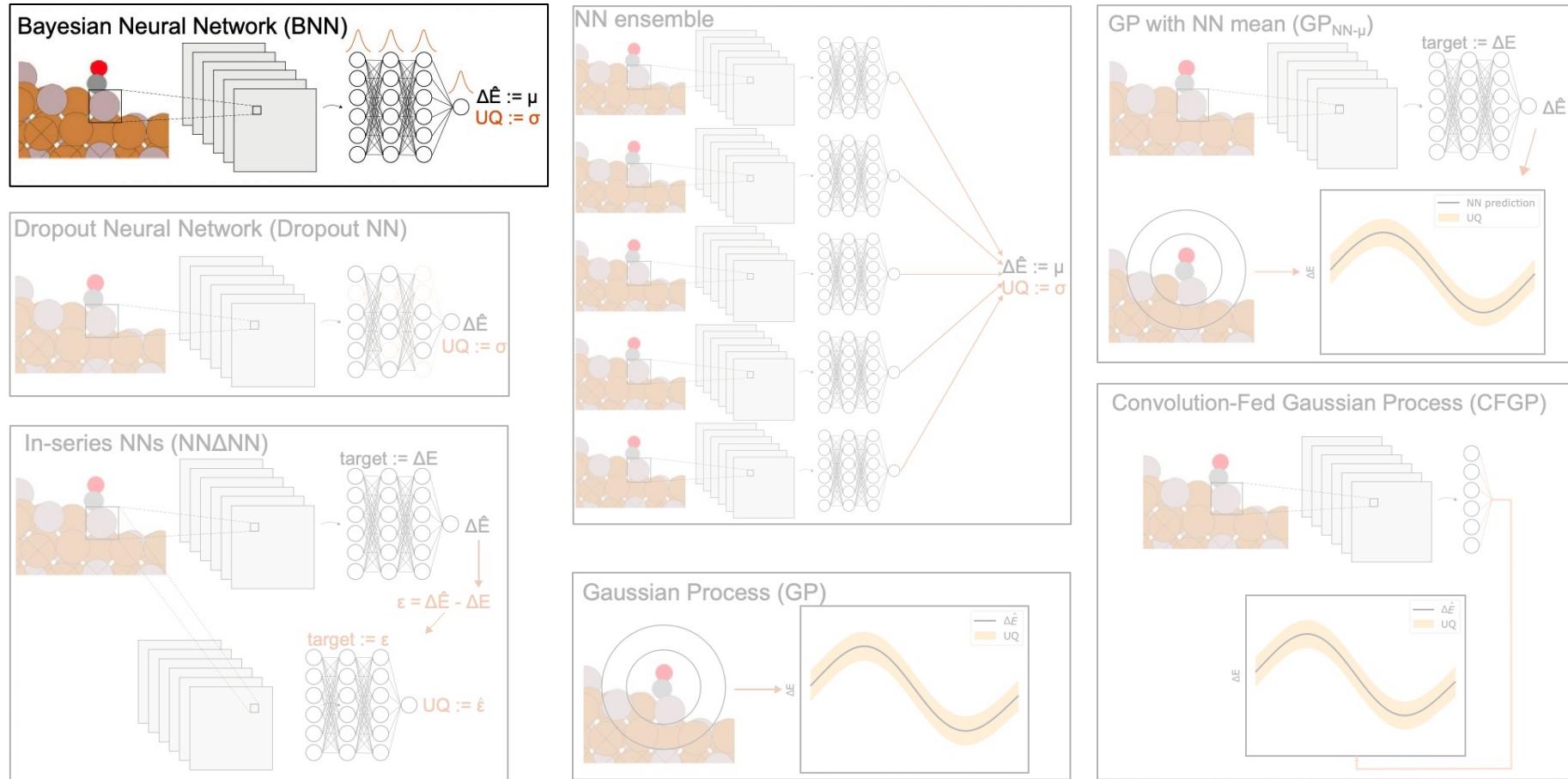


- Define a prior on neural network weights.
- In training, compute posterior over neural network weights (rather than a point estimate).
- Often using some type of MCMC (e.g., LMC) or variational inference.

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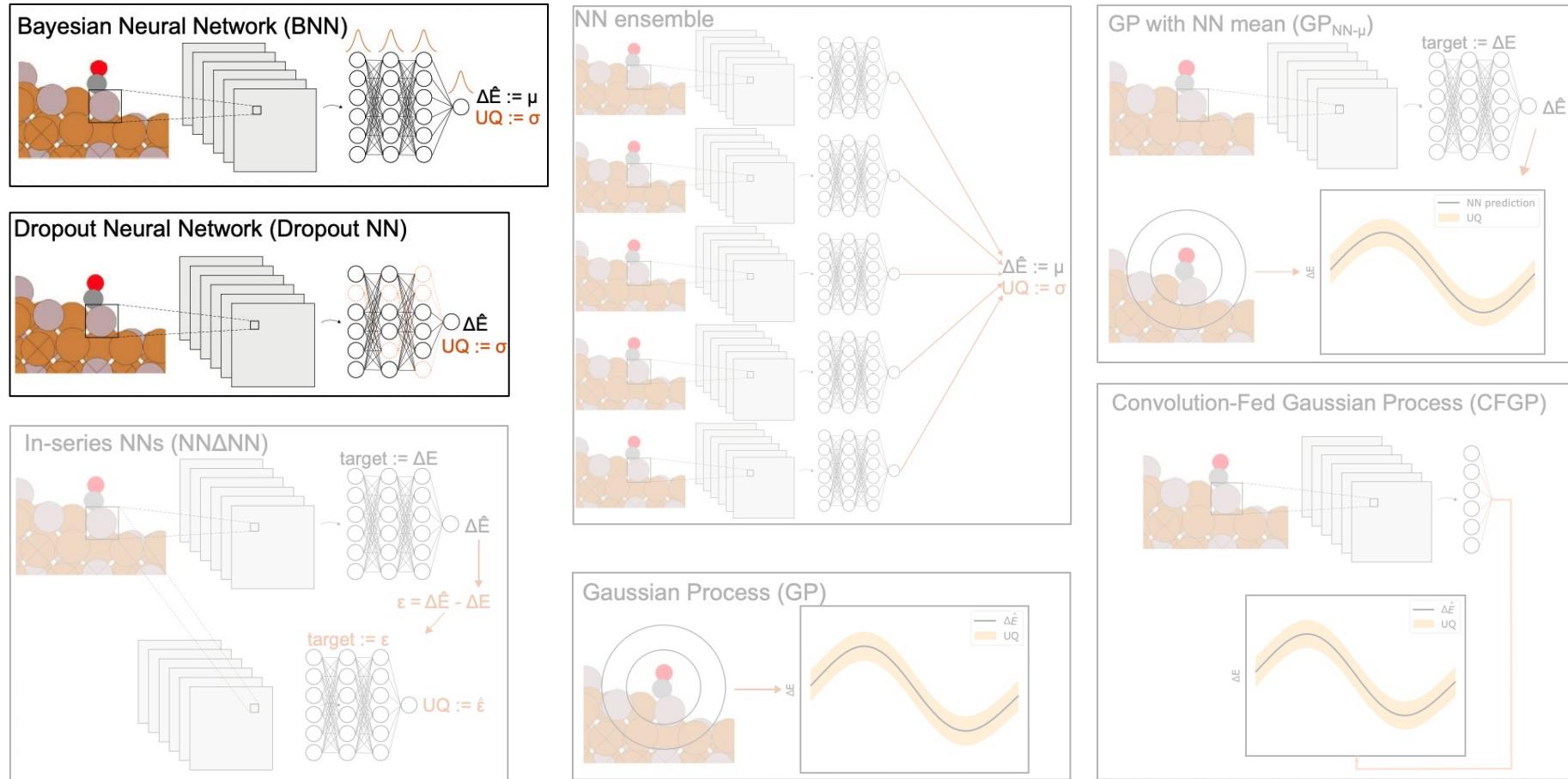
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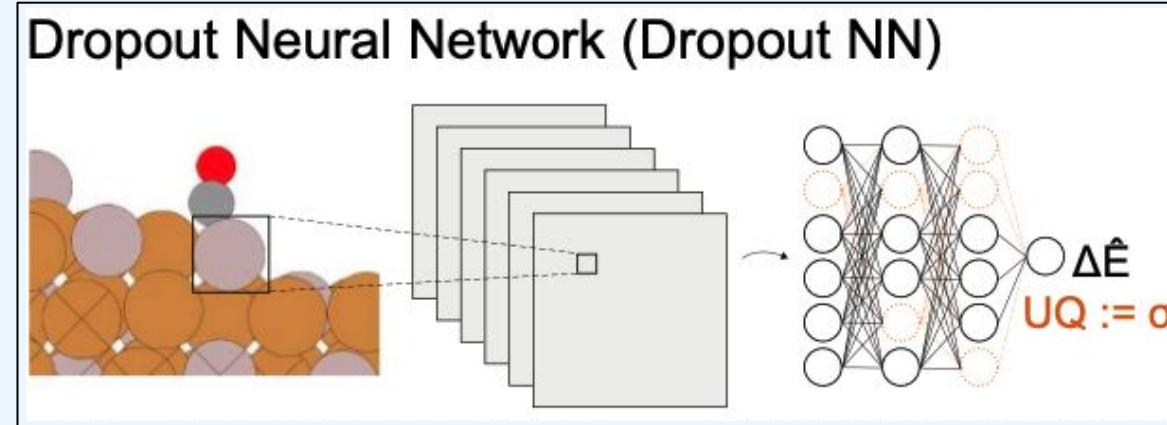
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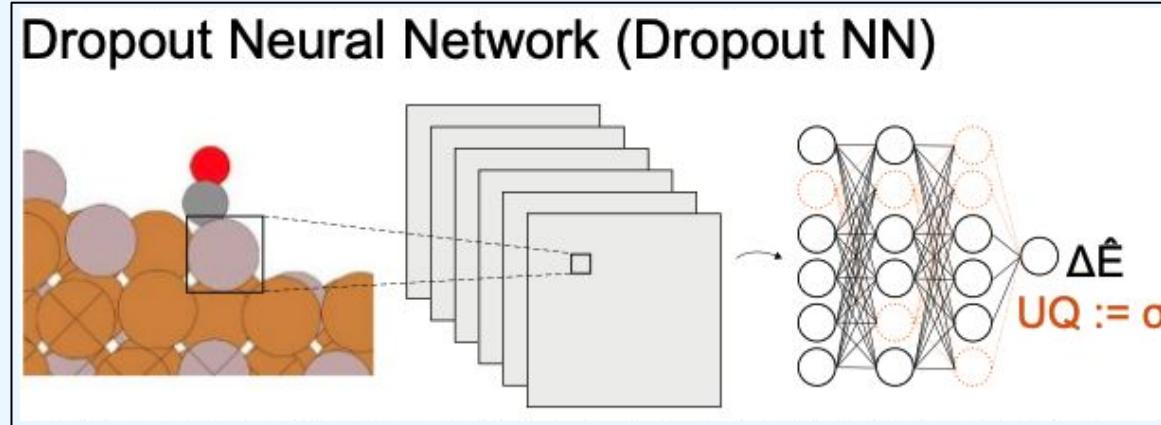
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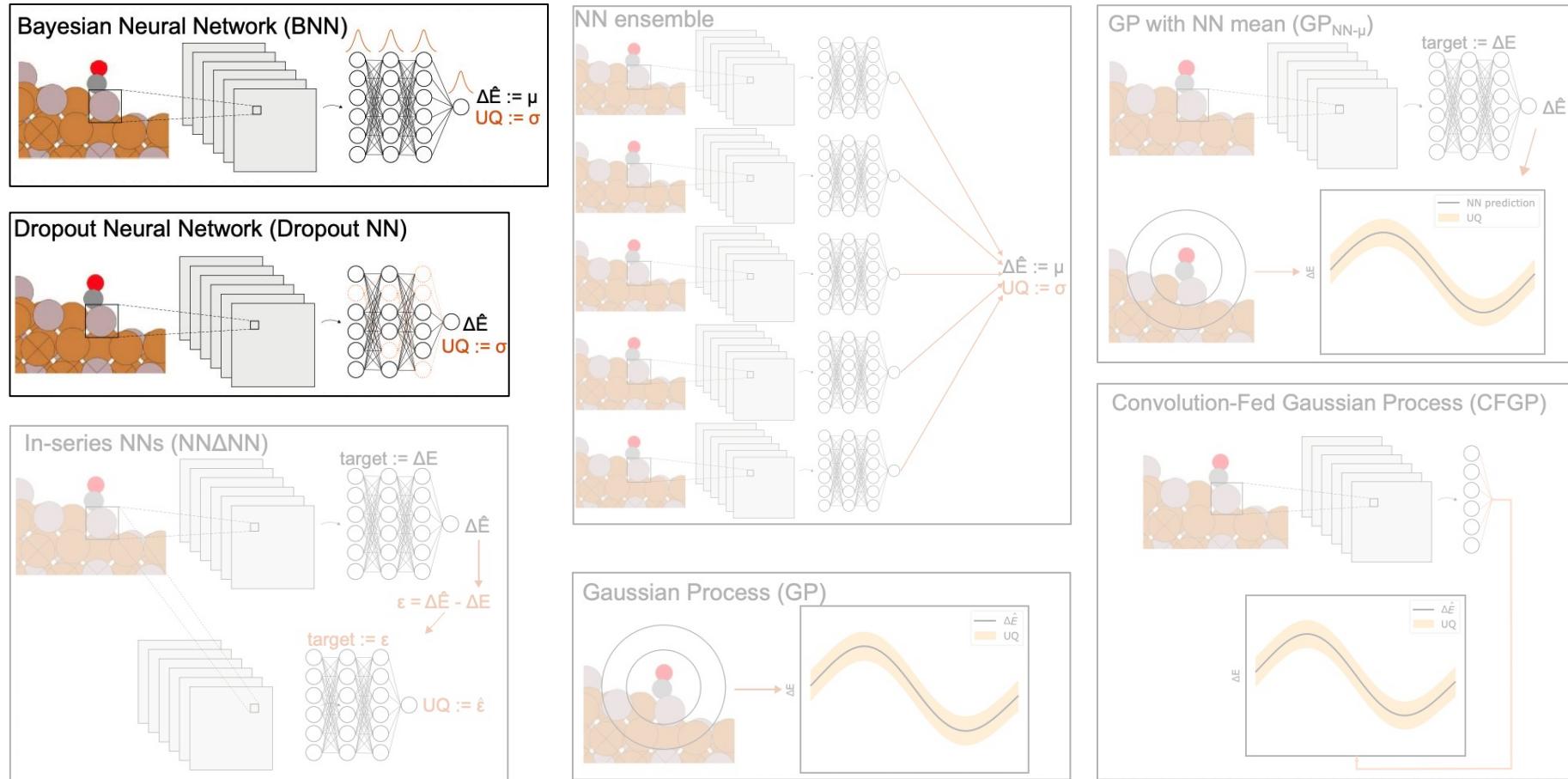


- During train/inference-time, perform *Monte Carlo Dropout*.
- (*I.e.*, set a random fraction of the neuron's outputs to zero during forward pass of the model).
- ⇒ A set of forward passes yields a set of samples from an output distribution.

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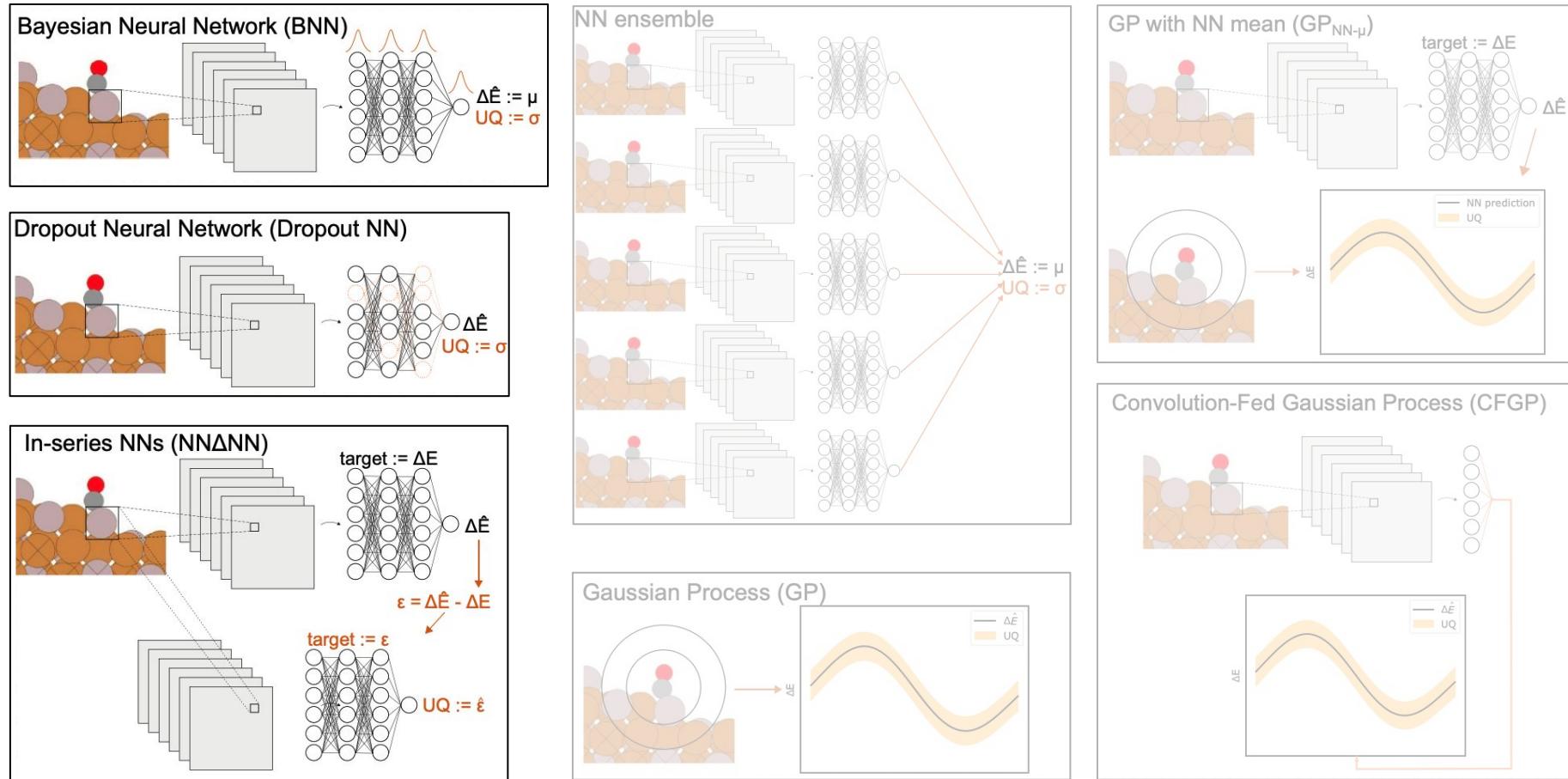
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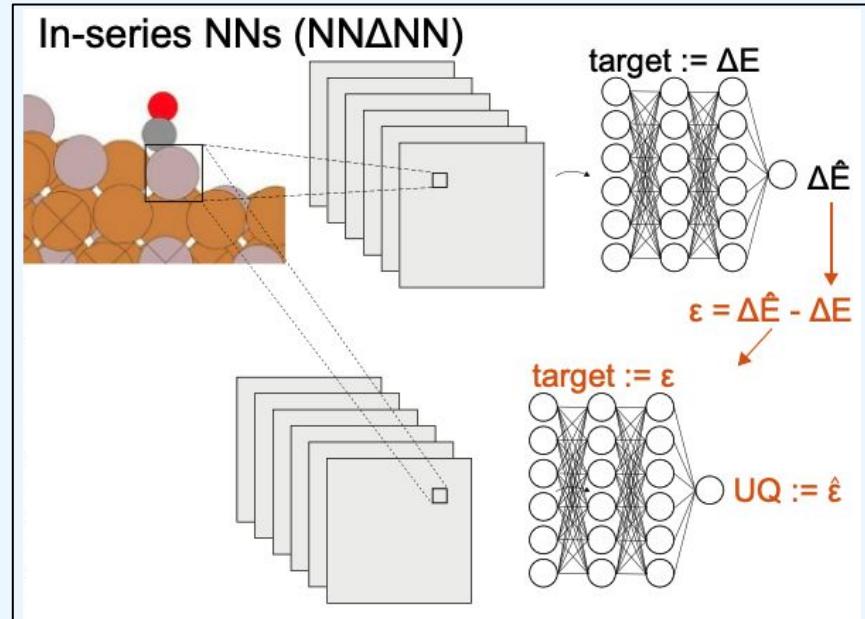
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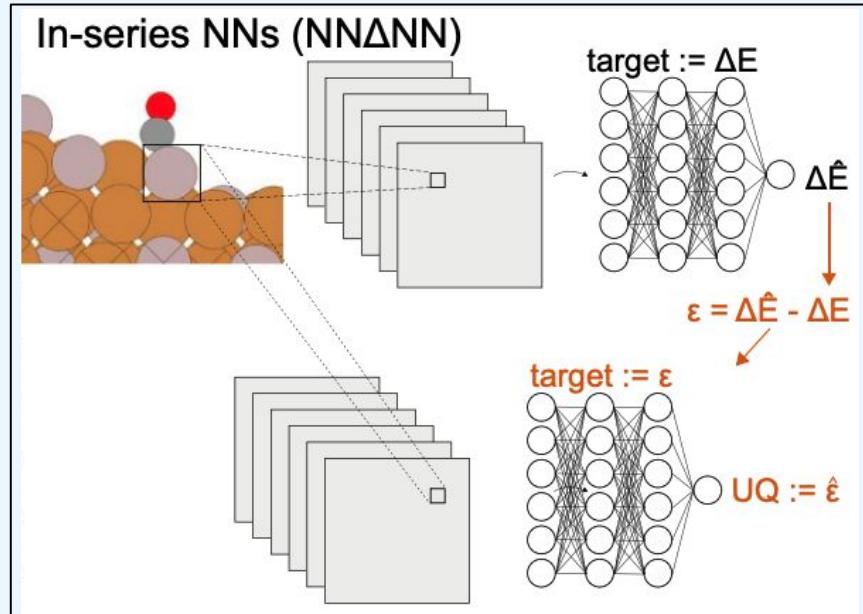
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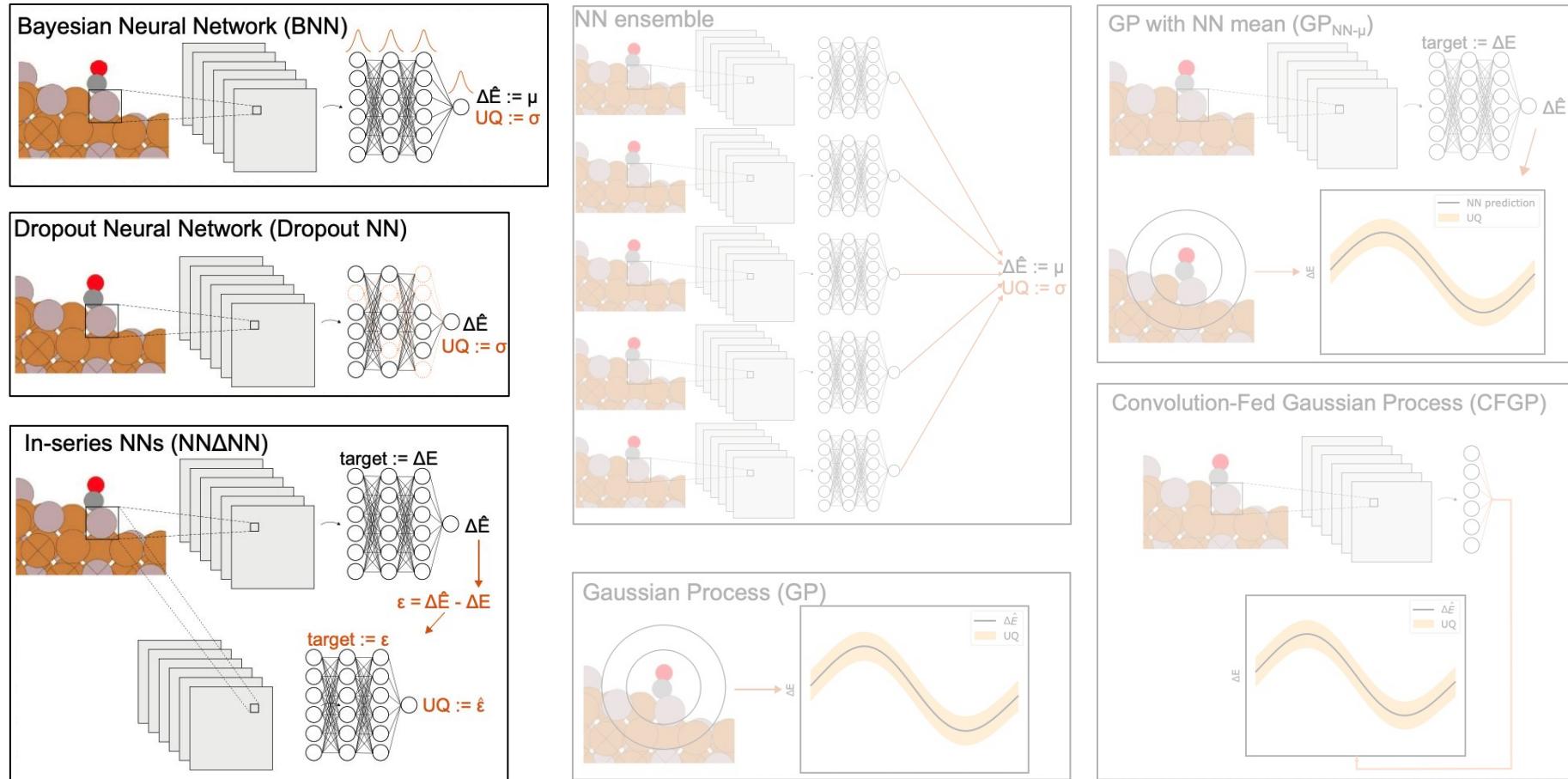


- First train a neural network for point prediction.
- Then compute residuals on a held-out set.
- Then use another neural network to try and predict residuals.
- ⇒ Second network predicts error, which can be used as UQ model.

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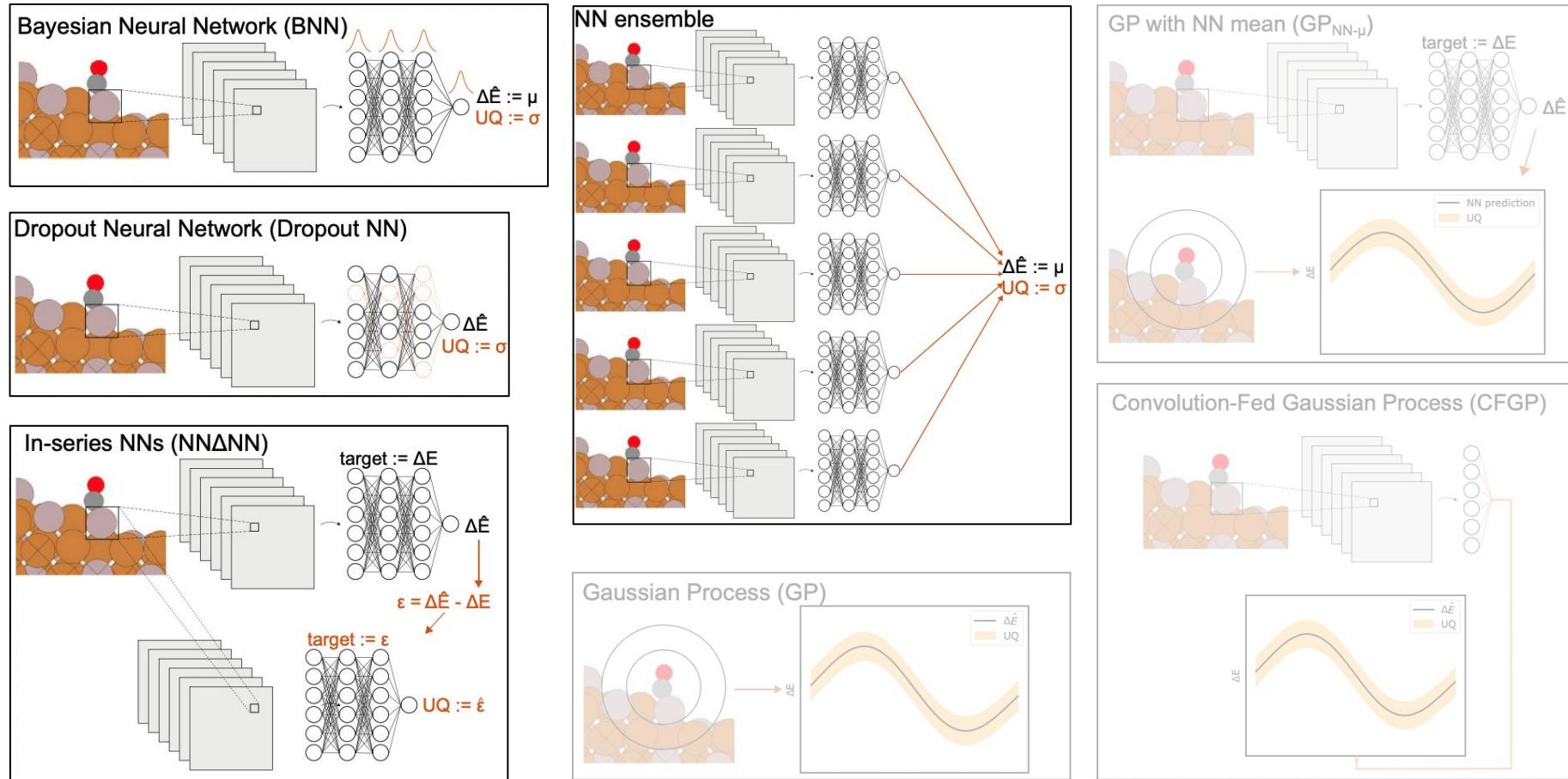
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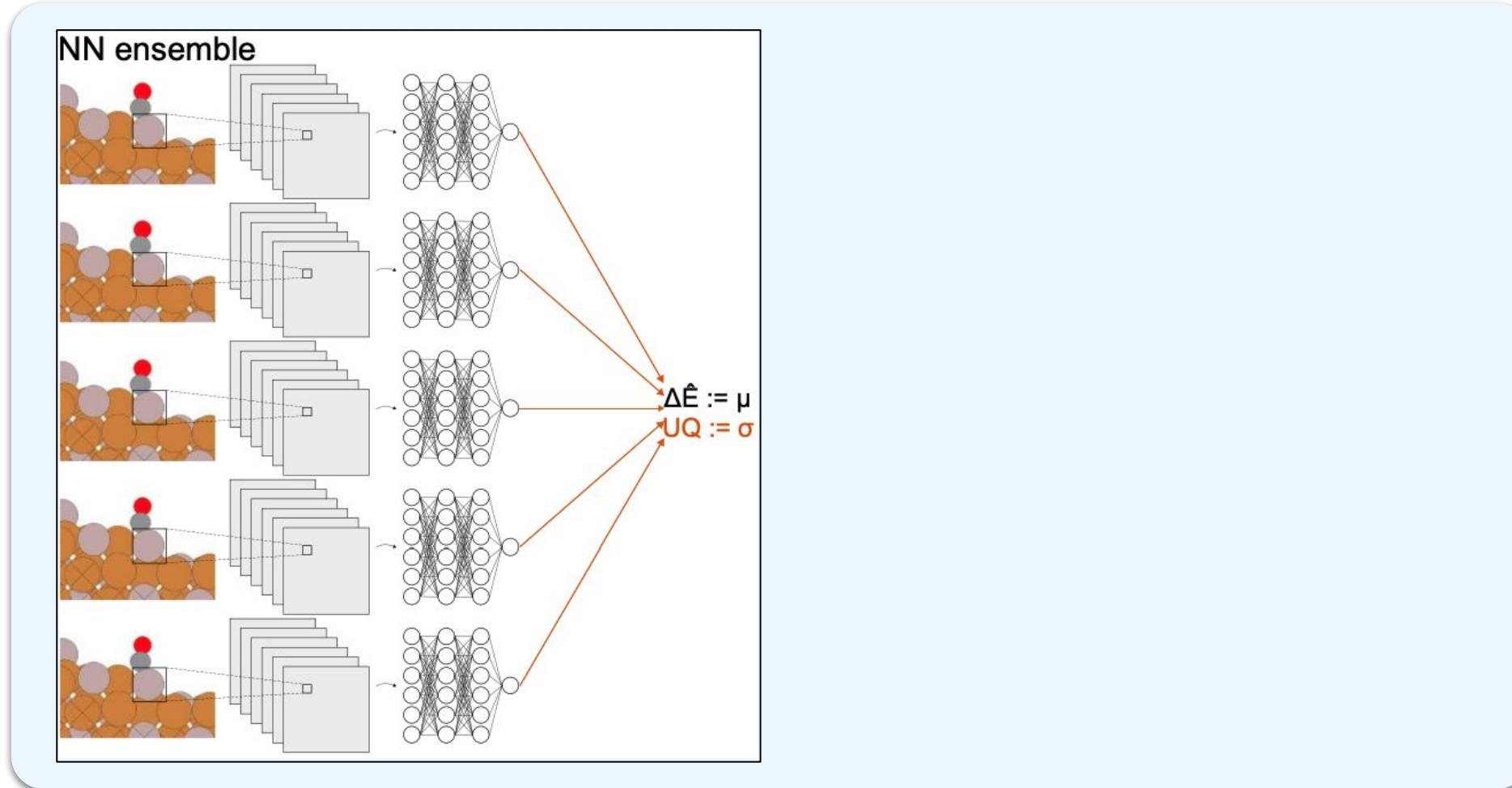
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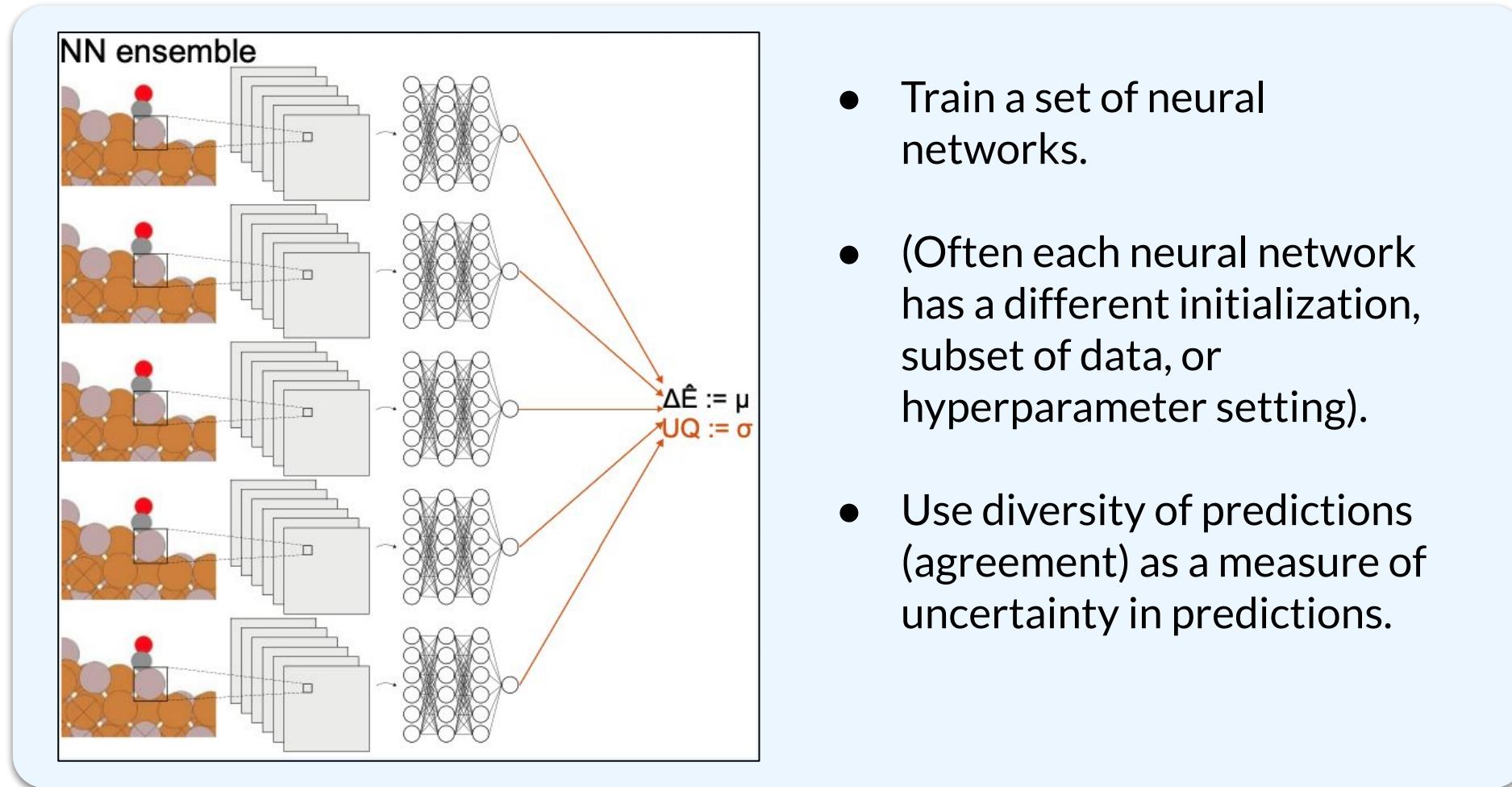
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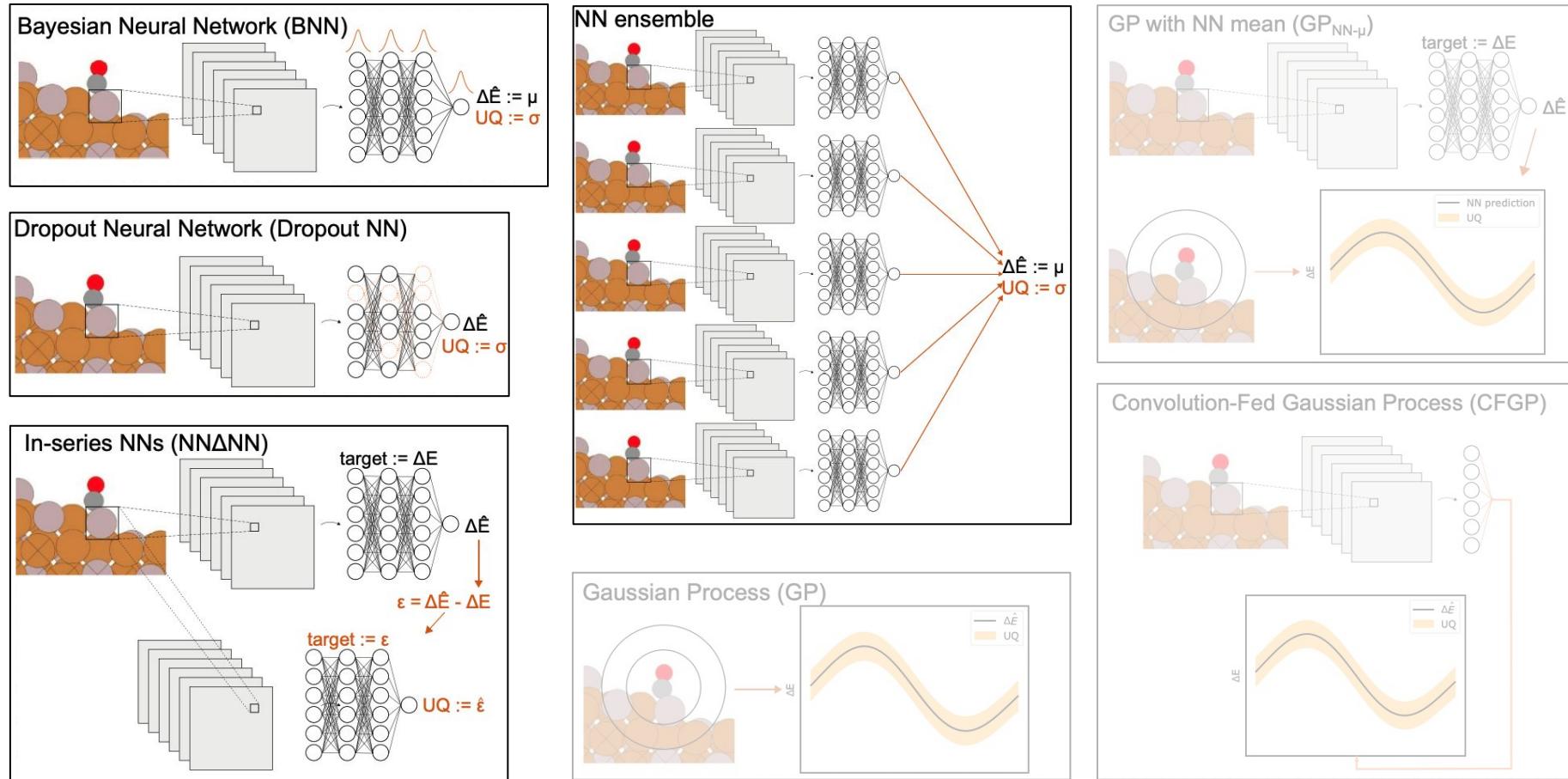
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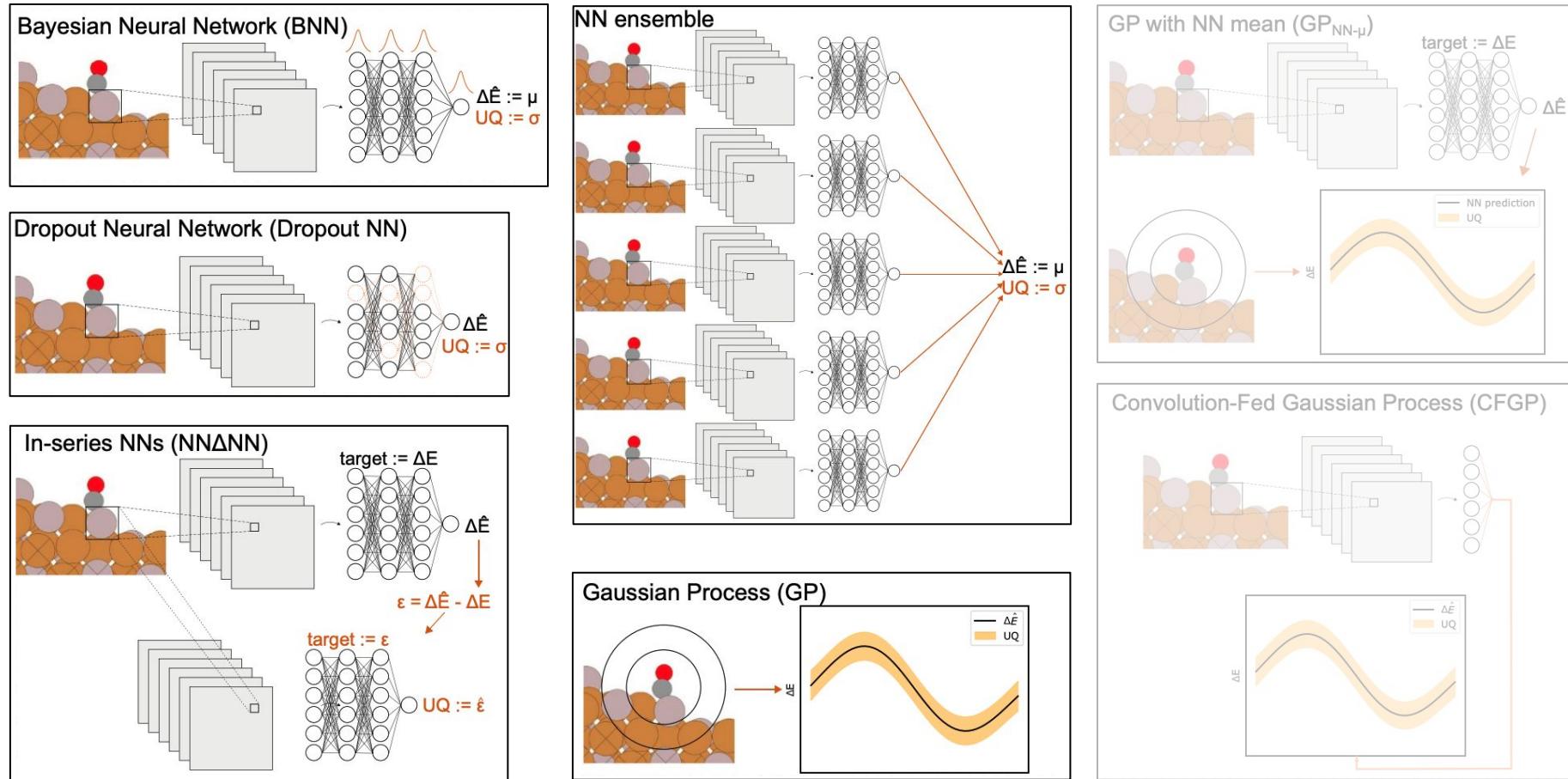
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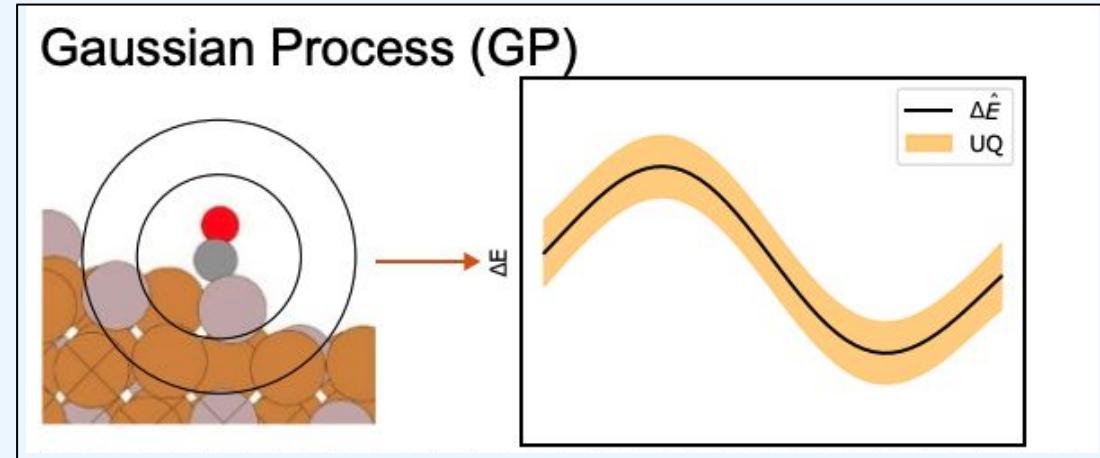
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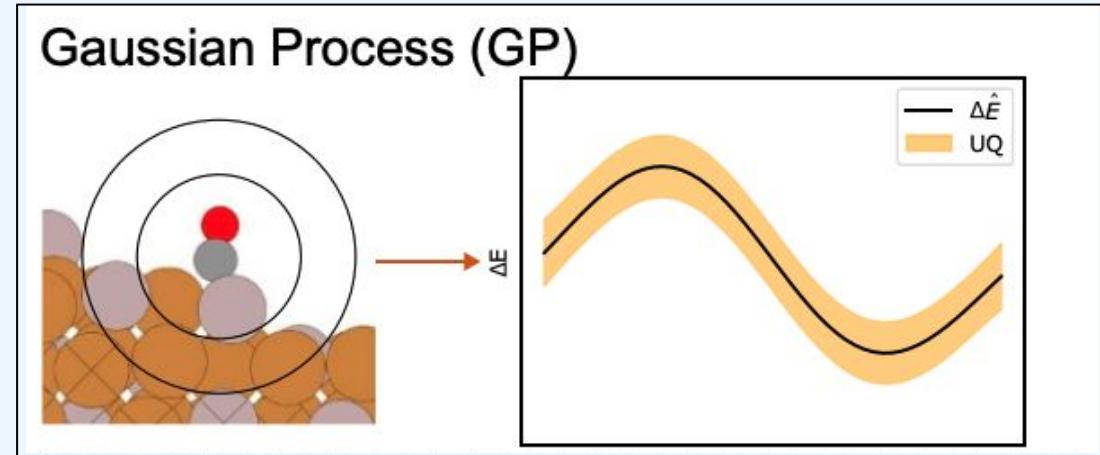
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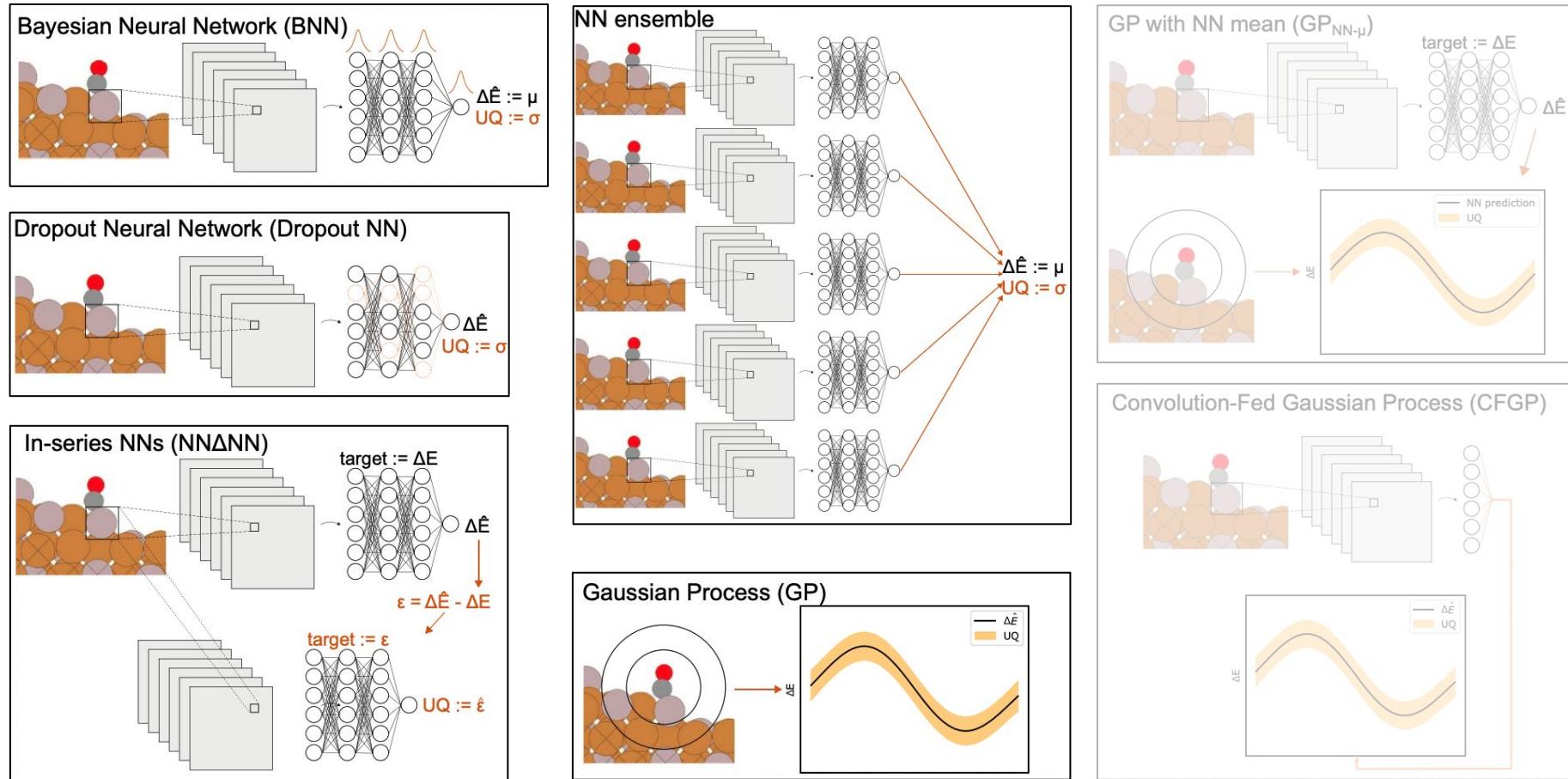


- Define some appropriate mean and kernel function on graph inputs.
- This yields a valid GP, which can make predictions on new inputs (new graphs)
- *Difficulty:* hard to hand-design good kernel function, & doesn't use NN.

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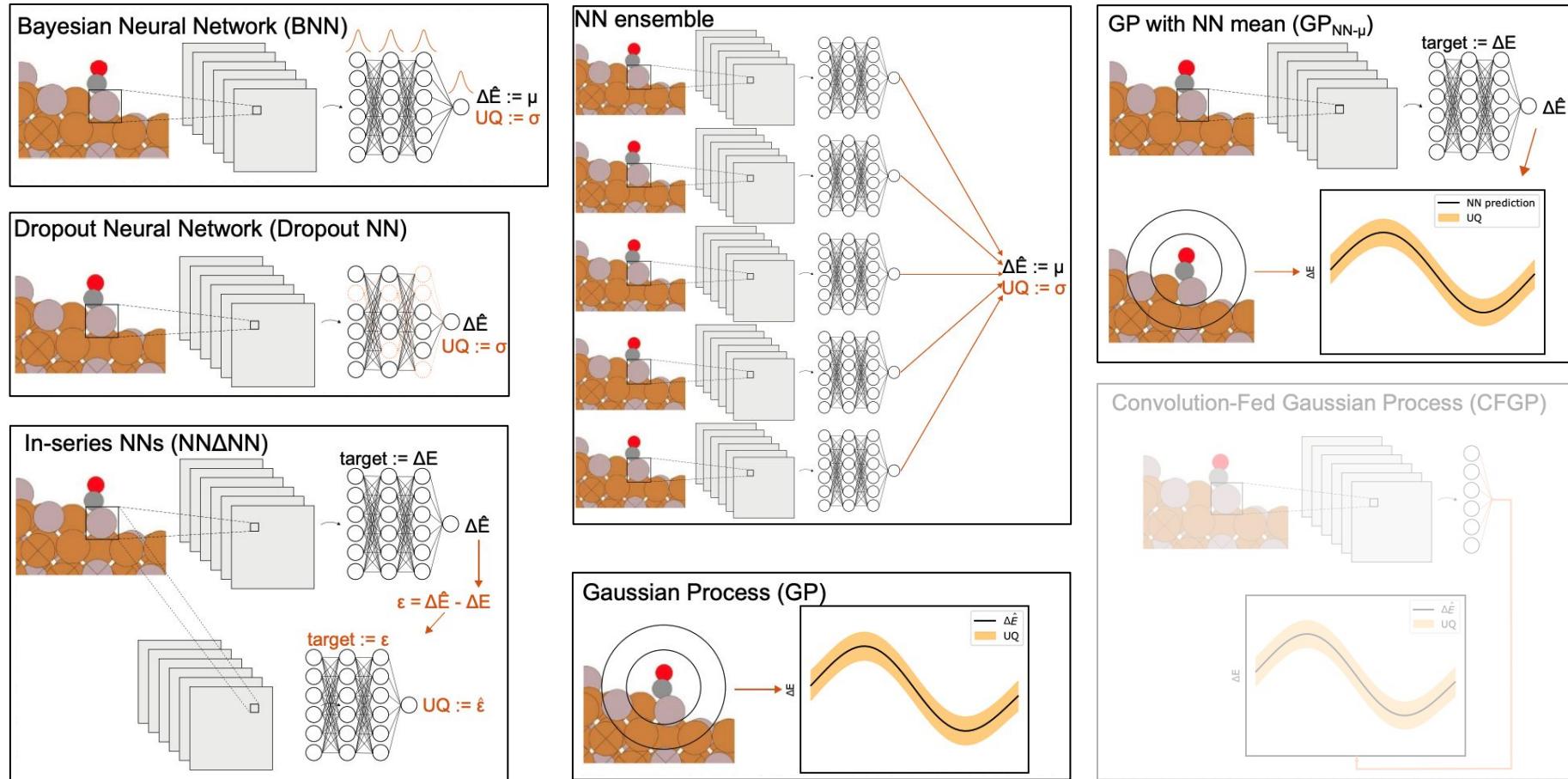
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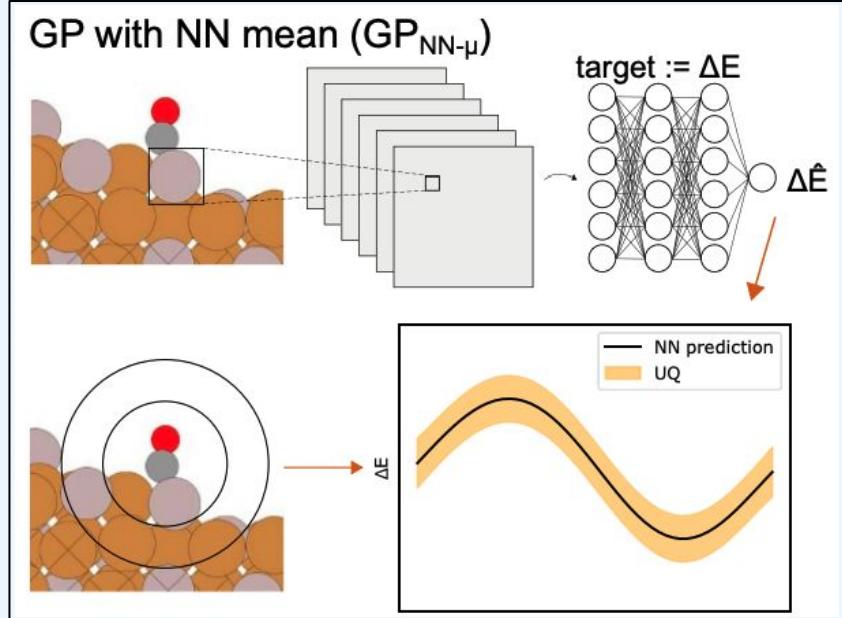
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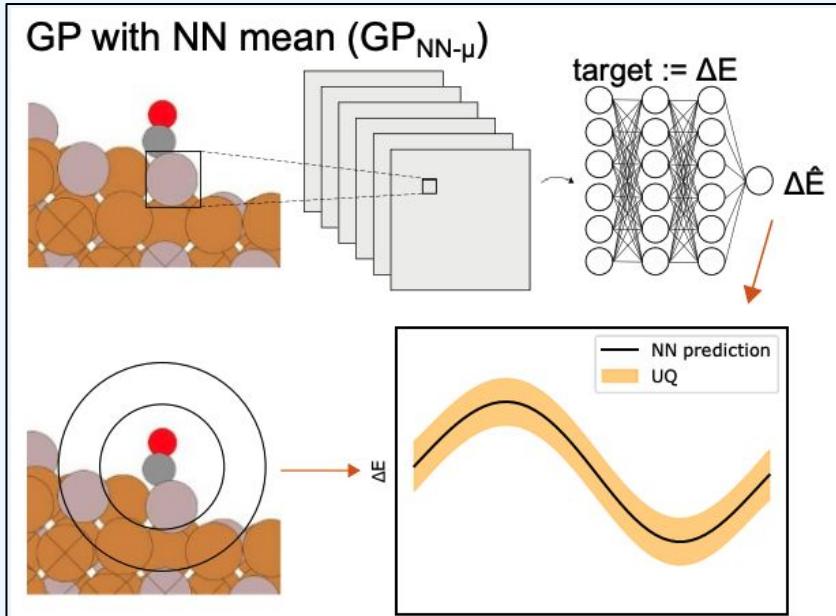
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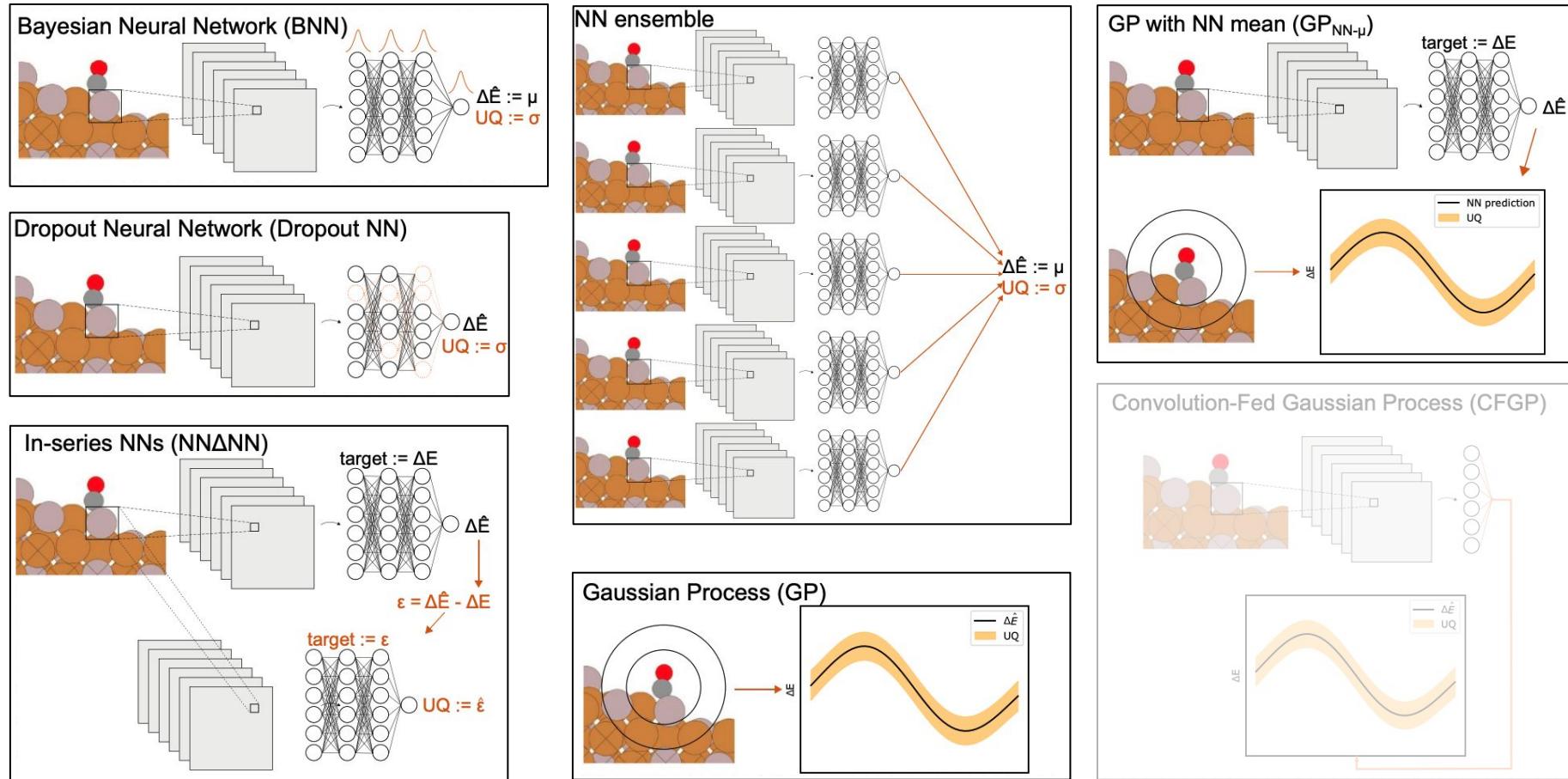


- In GPs, the prior mean fn is usually set to be **0**.
- But it could be anything...
- Here, we can set the prior mean function to be the output of a neural network model!
- Downside: still need to define a kernel function on graph inputs.

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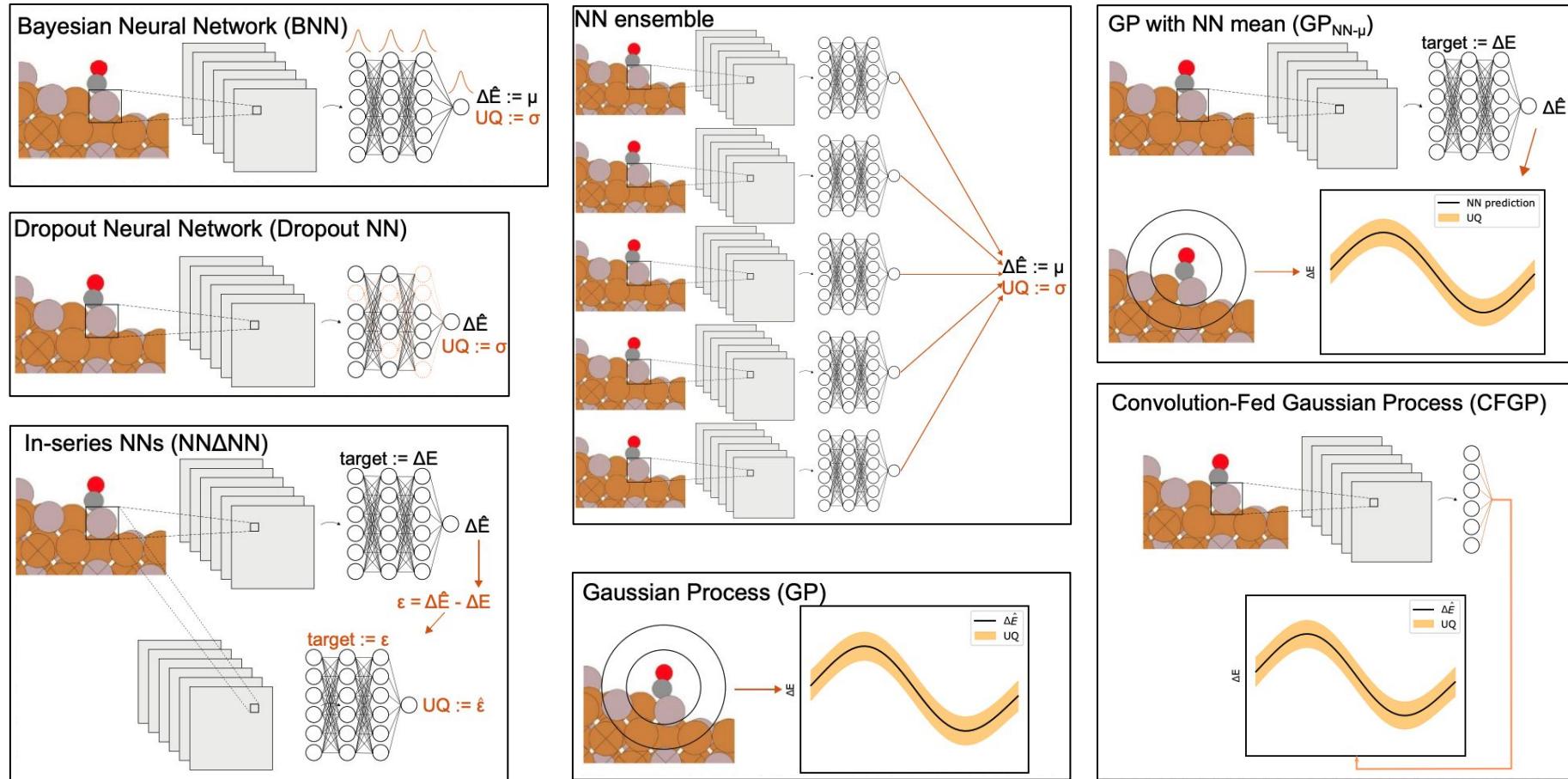
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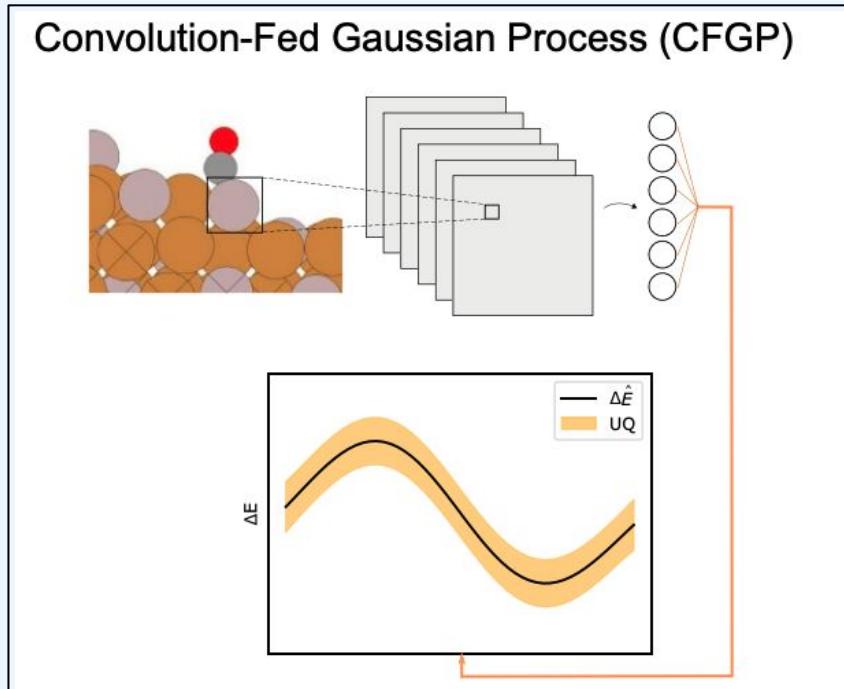
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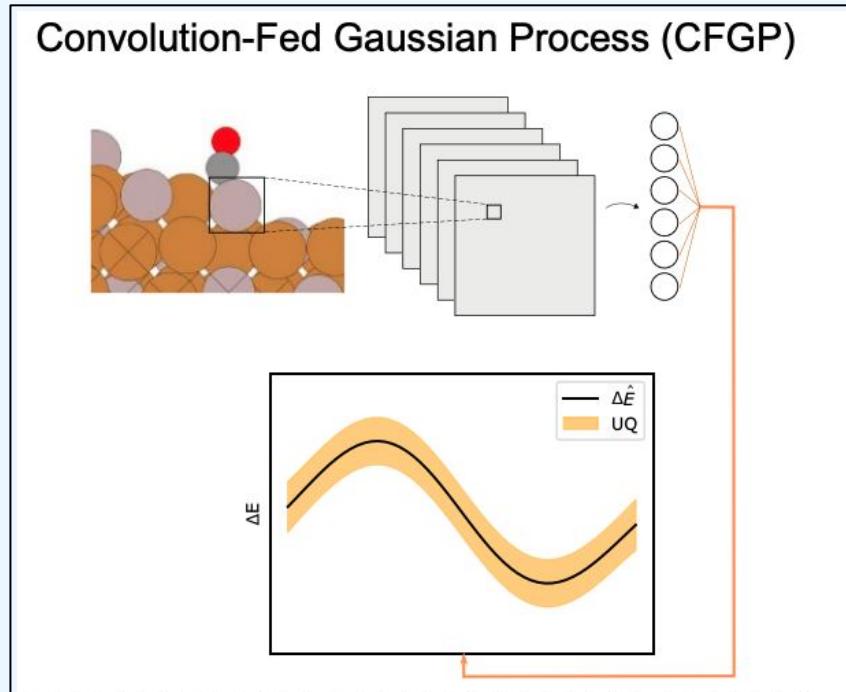
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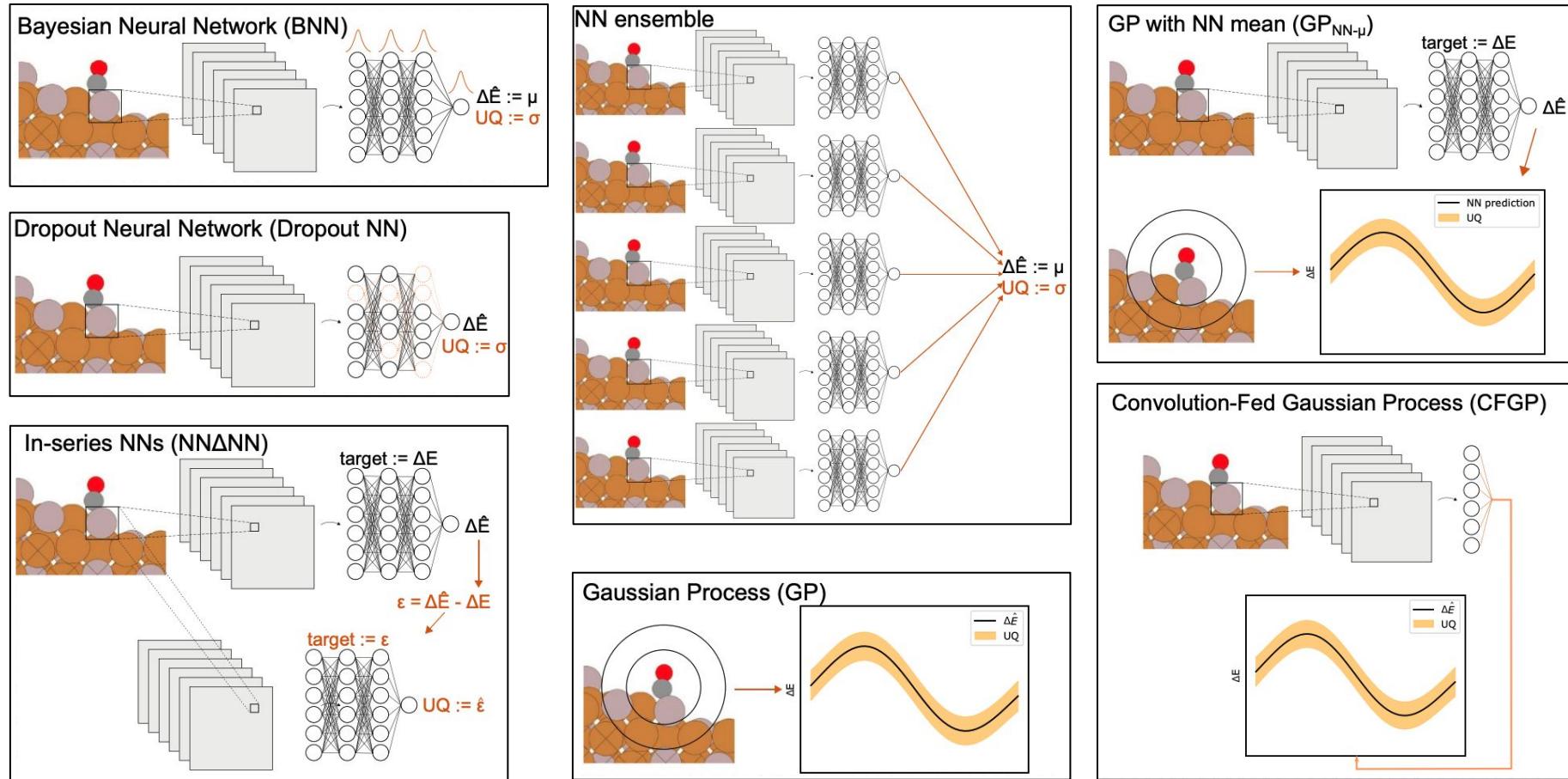


- Here, we use the neural network to learn good representations.
- For any input, first make a prediction w/ neural network.
- Use output at an intermediate layer as new transformed input (into a continuous space).
- Then define a Gaussian process on this new representation of inputs.

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Next Time

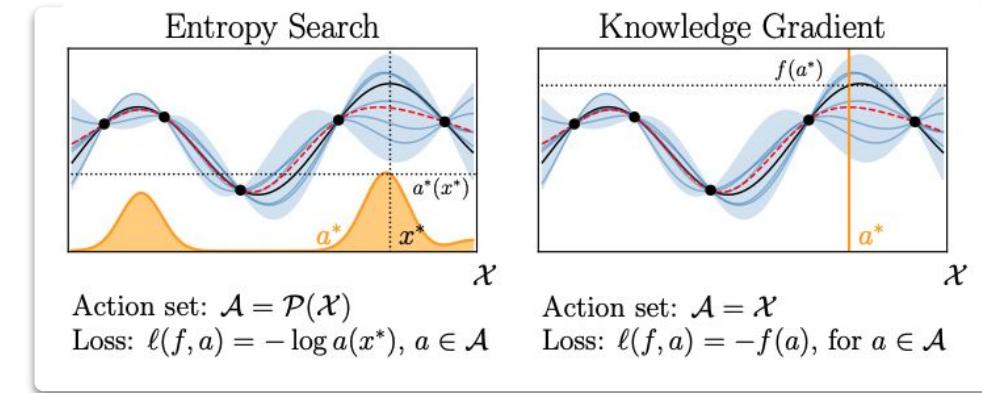
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Lecture: Using predictive UQ models for active learning and sequential decision making, including Bayesian optimization and optimal experimental design.

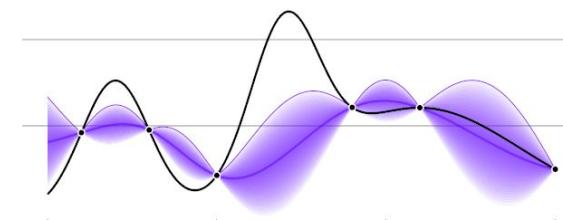
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- Decision making under uncertainty.
- Active learning.
- Bayesian optimization.
- Optimal experimental design.
- Extensions: Bayesian algorithm execution.



"Generalizing Bayesian Optimization with Decision-theoretic Entropies", 2022

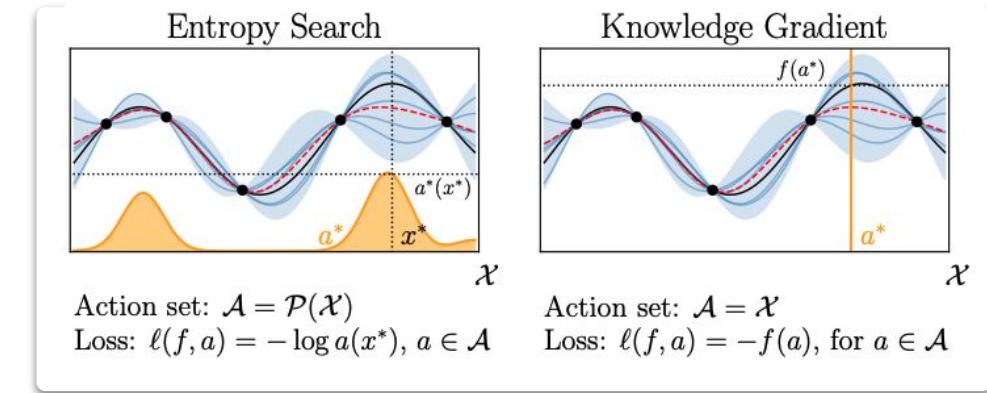


Bayesian
Optimization,
Wikipedia

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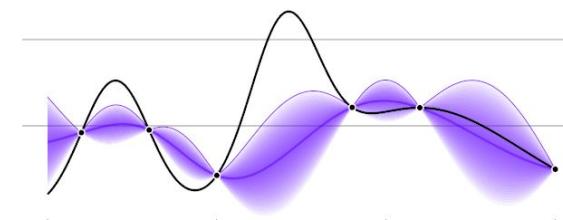
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"Generalizing Bayesian Optimization with Decision-theoretic Entropies", 2022

After:

- Paper presentations from students.



Bayesian
Optimization,
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