Homework 8

Compute the singular-value decompositions of the following matrices. Do not use a computer!

Don't use eigenvalues either:

$$A = \begin{pmatrix} 0 & 0 & 3 & 5 \\ 0 & 0 & 3 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1$$

Pseudoinverses

2 By hand, compute the pseudoinverses of the matrices

$$A = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_{1 \times n}$$

$$B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}_{n \times 1}$$

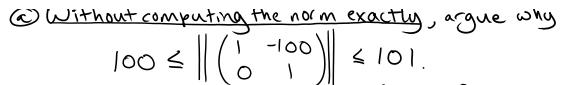
$$C = \begin{pmatrix} 1 & 2 & 3 & 0 \\ -4 & 2 & 0 & 2 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 4 \end{pmatrix}$$

3 Is $(AB)^{\dagger} = B^{\dagger}A^{\dagger}$ always true for pseudoinverses? No! Give a counterexample.

SVD and condition number

(9) Without computing the norm exactly, argue why



- (b) What is the inverse of $A = (0^{-100})$? What is the condition number of A? What is the condition number of A^{-1} ?
- © Find vectors $\overrightarrow{b} \in \mathbb{R}^2$ and $\overrightarrow{f} \in \mathbb{R}^2$ such that $\|f\|$ is "small" compared to $\|b\|$, and yet $\|A^{-1}(b+f) A^{-1}b\|$

is "large" compared to MA'bM. (Use your own judgment for what should rount as small or large.)
(Hint: Compute the SVD and experiment a bit,)
using Matlab.

Compute $b = H_{SX}$ for x = (1, 1, 1) and x = (0, 6, -3.6). A small change Δb produces a large change Δx .

© Compute numerically the largest and smallest singular values of the 7×7 Hilbert matrix H_7 , a matrix whose (i,j) entry is $\frac{1}{i+j-1}$.

(Hint: Google the Matlab "hilb" command.)

- @ If Htx = b with ||b||=1, how large can ||x|| be?
 If b has roundoff error less than 10-16 in norm,
 how large an error can this cause in x
- © Let A be an $m \times n$ real matrix, with singular-value decomposition $A = \sum_i \sigma_i \vec{v}_i \vec{u}_i^T$

Assume that the singular values are sorted 0,70,700,00.

A= 10 viviui.
Assume that the singular values are sorted, 077027...70.

a) Give the SVD for the matrix AAT. What is its condition number? Why is R(AAT)=R(A)?

Why is R(AAT)=R(A)? Why is N(ATA)=N(A)?

b) Give the SYD for the (m+n) × (m+n) matrix

 $\mathcal{B} = \begin{pmatrix} \mathcal{O} & \mathcal{A} \\ \mathcal{A}^{\mathsf{T}} & \mathcal{O} \end{pmatrix}.$

What is its condition number?

In terms of At, what is the pseudoinverse Bt?

(Hint: What is B applied to (0, vi) & IRM+1 ?

How about B applied to (vi, 0) & IRM+1 ?

Least-squares regression

Find the projection of b onto the column space of A:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}.$$

Split b into p+q, with p in the column space and q perpendicular to that space. Which of the four subspaces contains q?

R(A), $R(A^{T})$, N(A) or $N(A^{T})$

b) Using your answer from part@, find the x = R2 that minimizes || Ax-6||.

For $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^m$, prove that \mathbf{x}_2 is a least squares solution for $\mathbf{A}\mathbf{x} = \mathbf{b}$ if and only if \mathbf{x}_2 is part of a solution to the larger system

$$\begin{pmatrix} \mathbf{I}_{m \times m} & \mathbf{A} \\ \mathbf{A}^T & \mathbf{0}_{n \times n} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix}.$$