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set of matrices: $\mathbb{R}^{m\times n}$

$$m \text{ rows} \left\{ \underbrace{\begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}}_{n \text{ columns.}} \right.$$

$$A \times B = \begin{pmatrix} a_1^T \\ | \\ a_m^T \end{pmatrix} \begin{pmatrix} | & & | \\ b_1 & \cdots & b_p \\ | & & | \end{pmatrix}$$

$A \in \mathbb{R}^{m\times n}$; $B \in \mathbb{R}^{n \times p}$ $\qquad (A\times B)_{ij} = a_i^T \cdot b_j$

$$A \times B = \begin{pmatrix} a_1^T b_1 & \cdots & a_1^T b_p \\ a_2^T b_2 & & | \\ | & & \\ a_m^T b_1 & & a_m^T b_p \end{pmatrix}$$

$$(A \times B)_{ij} = a_i^T b_j \quad \text{(inner product.}$$

$$= \sum_{k=1}^{n} a_{ik} b_{kj}$$

$AB \neq BA \quad ; \; A \in \mathbb{R}^{m \times n}$

$\qquad\qquad\qquad\quad B = \mathbb{R}^{n \times p}$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; ; \; B = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 2a & 4b \\ 2c & 4d \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 4c & 4d \end{pmatrix}$$

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$$(A^T)_{ij} \Rightarrow A_{ji}$$

$$(AB)^T = B^T A^T$$

Proof: $C = AB$, $D = B^T A^T$

$$(C^T)_{ij} = C_{ji} = \sum_{k=1}^{n'} a_{jk} b_{ki}$$

$$D_{ij} = \sum_{k=1}^{n} (B^T)_{ik} (A^T)_{kj} =$$

$$= \sum_{k=1}^{n} b_{ki} a_{jk}$$

$$\Rightarrow (C^T)_{ij} = D_{ij} \Rightarrow C^T = D$$

$$\Rightarrow (AB)^T = B^T A^T$$

A is symmetric iff $A^T = A$.

AA^T is symmetric.

Proof:

$$(AA^T)^T = (A^T)^T A^T = AA^T$$

Permutation matrix:

Every row and every column has exactly one "1" and the remaining elements are "0"

Exple:

$$B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} ; \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xleftrightarrow{\text{Swipe } 2^{nd} \text{ row with } 3^{rd} \text{ row}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Swipe 2^{nd} row with 1^{st} row

$$B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} ; \quad A \times B = \begin{pmatrix} b & c & a \\ e & f & d \\ h & i & g \end{pmatrix}$$

$\Longrightarrow$ rearranges the columns.

$$B \times A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$= \begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix} \Rightarrow \text{rearranges the rows.}$$

Inverse of a matrix:

$A \in \mathbb{R}^{n \times n}$

A is invertible iff there exists $B \in \mathbb{R}^{n \times n}$

Such that $AB = BA = I$

We denote $B = A^{-1}$

Exple:

$n=1$; $a \neq 0 \Rightarrow a^{-1} = \frac{1}{a}$

$n=2$: $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $ad - bc \neq 0$
$\det(A) \neq 0$

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

Proof: $(A.B)(B^{-1}A^{-1}) = A \underbrace{BB^{-1}}_{I} A^{-1}$

$= AA^{-1} = I$

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$.

$$(A^T)^{-1} = (A^{-1})^T$$

Proof: $A^T \times (A^{-1})^T = (A^{-1}A)^T = I$

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