PROBLEMS: ANALYSIS I

(1) Let $x \in (0,1)$ and define the sequence a_n via $a_0 = 1+x$ and $a_n = a_{n-1}(1+x^{2^n})$. Does a_n converge as $n \to \infty$ and if so what is its limit?

(2) Define the sequence $x_{n+1} = \frac{1}{2}(x_n + \frac{K}{x_n})$, for arbitrary positive x_0 and K. Prove that this sequence converges and find its limit.

(3) Prove that $a_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln(n+1)$ is convergent.

(4) Define $f : \mathbb{R} \to \mathbb{R}$ via $f(x) = x$ for $x \leq e$ and recursively $f(x) = xf(\ln(x))$ whenever $x > e$. Does the sequence

$$s_n = \sum_{k=1}^{n} \frac{1}{f(k)}$$

converge?

(5) Define the sequence a_n as $a_0 = 1$ and $a_{n+1} = a_n + e^{-a_n}$. Does the sequence $a_n - \ln(n)$ have a limit as $n \to \infty$?

(6) Find (and solve, or ask me about) an interesting (and hard) problem from the Putnam (or similar) which fits the current topic. As sources you can use the Putnam Archive of Kiran Kedlaya, or IMO, or "AoPS", or any of the sources listed in the syllabus.