

PROBLEMS: CONTINUOUS PROBABILITY, INTEGRALS

- (1) Two points are chosen uniformly at random on a stick of length 1m and then the stick is cut at these points into 3 smaller sticks. What is the probability that you can form a triangle out of these sticks?
- (2) What is the probability that 4 points selected uniformly at random on a circle all lie in a semicircle?
- (3) A needle of length ℓ is dropped on a floor tiled with square tiles of size $s \times s$. What is the probability that the needle lies entirely in one tile?
(Your answer will depend on ℓ and r .)
- (4) Find the elementary function equal to the following (indefinite) integral:

$$\int \frac{1 + e^{2t}}{1 + e^t} dt$$

- (5) Find the integral

$$\int \frac{x^4}{24 + 24x + 12x^2 + 4x^3 + x^4} dx.$$

- (6) Find (and solve, or ask me about) an interesting (and hard) problem from the Putnam (or similar) which fits the current topic. As sources you can use the Putnam Archive of Kiran Kedlaya, or IMO, or “AoPS”, or any of the sources listed in the syllabus.