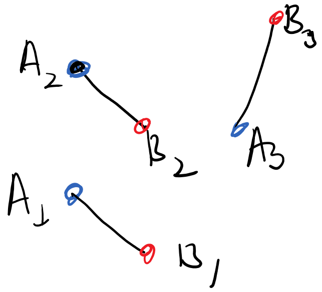
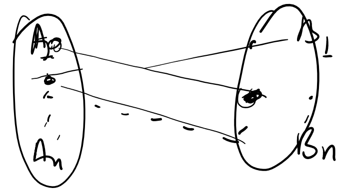
Monday, February 8, 2021 6:29 PM



Matching $\to \underbrace{(A_1,B_1),\ (A_2,B_2),\ (A_3,B_3)}_{M}$

$$f(M) = \|A_1B_1\| + \|A_2B_2\| + \|A_3B_3\|$$

n blue points n red points



$A_1 \to n$ choices
$A_2 \to n-1$ choices remaining w.
$\vdots$
$A_n \to 1$ choice.

$n!$

$$f(M_1),\ f(M_2),\ \dots,\ f(M_{n!}) \in \mathbb{R}_{>0}$$

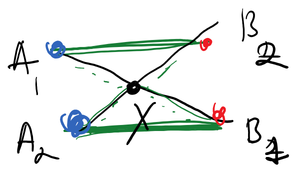
take minimal M_i, s.t. $f(M_i) \le f(M_j)$ for j –

will have no crossings.

to prove that

argue by contradiction:

suppose not true $\Rightarrow$ M_i has at least one crossing.



$$f(M_i) = \|\cdot\| + \cdot \|\cdot\| + \|A_2 B_1\| + \|A_1 B_2\|$$

$M_i' \to \dots\dots (A_1, B_2),\ (A_2, B_1)$

$$f(M_i') = \|\cdot\| + \dots \|\cdot\| + \|A_1B_2\| + \|A_2B_1\|$$

$$A_1B_1 + A_2B_2 = \underbrace{A_1X + XB_2}_{\ge \|A_1B_2\|} + \underbrace{A_2X + XB_1}_{\ge \|A_2B_1\|}$$

contradiction with the minimality of $f(M_i)$.

Graphs continued.

Ramsey theory : $R(4,3) \le 9$, $K_9 \to$ complete graph on 9 vertices

(edges $\to \binom{9}{2} = 36$

Ramsey $R(4,3) = 9$, K_9

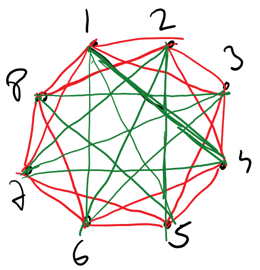
edges $\to \binom{9}{2} = 36$

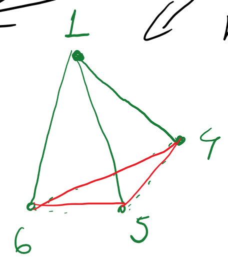
each edge $\to$ red or green

$\Rightarrow$ there is

↙ neither of those

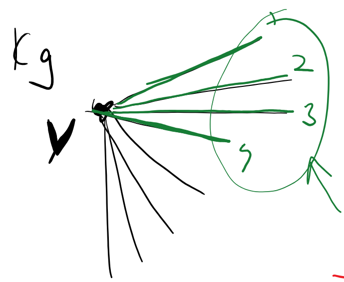
K_8





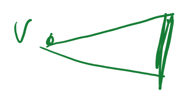
(*) Assume we have a coloring no red K_4, no green $\triangle$

K_9

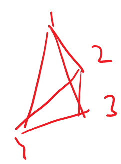


suppose ≥ 3 green edges.

no green •—•, else

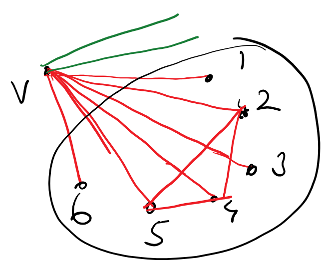


$\Rightarrow$



$\to$ no

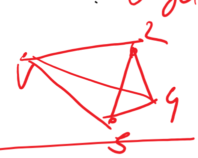
If there is a vertex with ≤ 2 green edges $\Rightarrow \geq 6$ red edges.



last time $(R(3,3))$

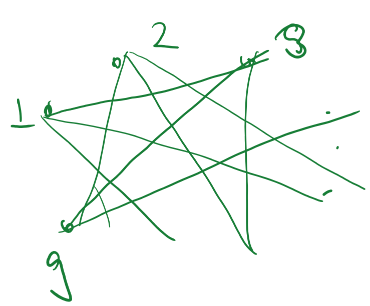
$\to$ either red $\triangle$ or green $\triangle$ no, by assumption (*)

get



$\to$ contradiction with (*)

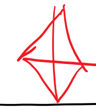
$\Rightarrow$ Every vertex has exactly 3 green edges.



$\to$ no such graph exists.

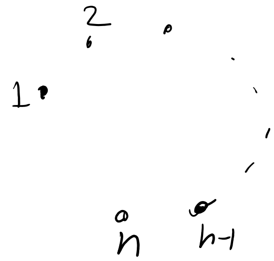
$$\sum_{v \in V} \deg(v) = 2|E| \quad \text{(handshake lemma)}$$

$9 \cdot 3 = 2|E| \to$ impossible $\to$ contradiction sign.

→ exhausted all cases → contradiction with (K) ⇒  or Δ.

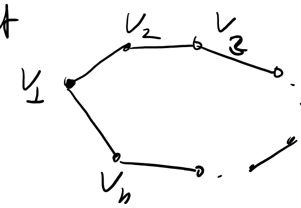
$R(3,3,3) \le 18$
r g b

Problem:

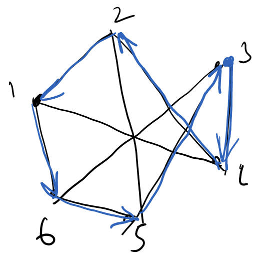


$\deg(v) \ge n/2$ for every vertex $v \in V$

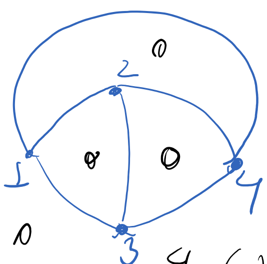
→ show that



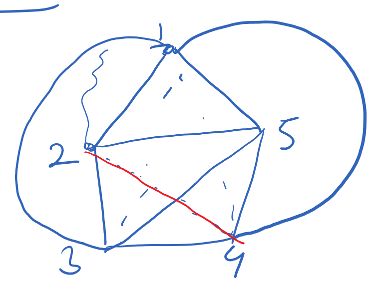
(Hamiltonian circuit)



Planar graphs:



← planar.

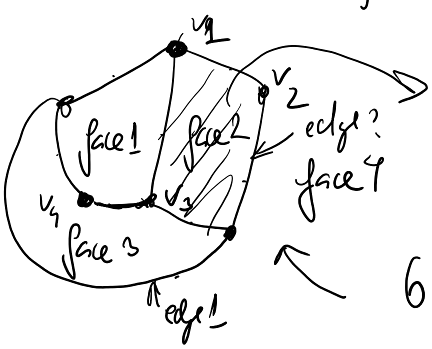


← K_5 not planar.

$4-6+4=2$

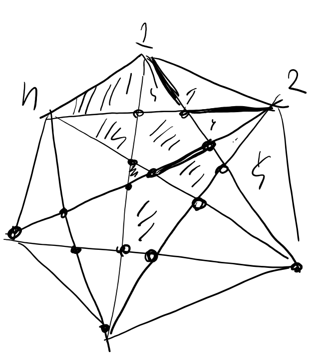
Euler's formula:

$V - E + F = 2$ ← proof by induction.



$\deg(\text{face } 2) = 4$

$6-8+4=2$



→ number of regions ↔

$V - E + F = 2$

segments from HW1.

$$n + \binom{n}{4} - \left(\binom{n}{4} + 2\binom{n}{2} - n\right) + F = 2$$

$$\Rightarrow F = \binom{n}{4} + \ldots$$

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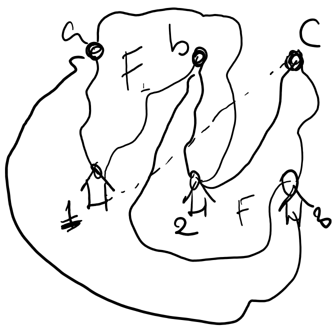


Proof by contradiction:

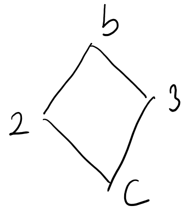
Suppose $(1,2,3) \leftrightarrow (a,b,c)$, can draw without intersection.

$\Rightarrow V - E + F = 2 \Rightarrow F = 5$

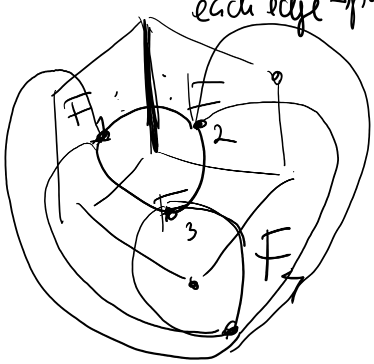
$V = 6$, $E = 9$

each edge - part of 2 faces

degree of a face $\rightarrow$ # edges around it



$\deg F \geq 4$



$\left(\sum \deg(F_i)\right) = 2|E|$

$F_i \rightarrow$ face

$|V|$

$4 \cdot 5$

$\Rightarrow 2|E| \geq 4 \cdot 5$

$|E| \geq 10$

9 contradiction.

no

$\Rightarrow$ not planar