Courier Problems

Theoretical Thinking 2

Jared Coleman

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Abstract

This report is an introduction to the practical applications, variations, and existing work on a new class of problems involving cooperative mobile agents: Courier Problems.

1 Introduction

The problem of delivering a message or package from one location to another is at least as old as human civilization. Before the telegraph, telephone, and eventually the internet made wireless communication cheap and practical, nearly all important long-distance message delivery relied on couriers to hand-deliver messages to their destinations. Modern technology hasn't eliminated all need for courier-like delivery, though. In fact, the face-to-face communication model still applies to many practical situations, such as when the message being delivered is a physical object (as opposed to information which can be encoded and replicated) or wireless communication is not possible (i.e. in remote areas like deep wilderness or exoplanets, in military situations where communication networks are being jammed, when couriers are weak autonomous robots without communication hardware, etc.).

Thus, courier problems are a class of problems which involve one or many mobile agents transporting one or more messages from one location to another. There are many potential variants to the problem - only a few of which have been given considerable attention in the literature. Data delivery with energy-constrained robots on graphs has been studied in some depth, though it is a very active area of research [1, 5]. We recently proposed the Pony Express Communication Problem [7], which poses the question: How can a team of mobile agents with different speeds best cooperate to deliver a message from one location to another? We studied the problem on the line [7] and in the plane [6]. This report will focus mainly on this problem, its variants, and potential extensions.

Courier problems are related to problems in search theory as well, such as the cow-path problem [2] and its many variants (with multiple agents [8, 3], using randomized algorithms [11], and in different topologies such as the bounded line [4], the ring [12, 10], the disk [13], the plane [9], and graphs [3]).

2 The Pony Express Communication Problem

We initially studied Pony Express Communication Problem on the line: Given a line segment with endpoints 0 and 1, a team of agents with different speeds initially placed arbitrarily along the segment, and a message placed at 0 which must be delivered to 1, how can the agents cooperate (via face-to-face handover) to deliver the message to its destination in as little time as possible [7]. In general, we are interested in finding offline and online algorithms for these kinds of problems. In the offline setting, agents know the initial position and speed of all other agents in the system. In the online setting, however, they know only their own position and speed. For this simplest version of the problem, we gave optimal offline and online algorithms. Then, we considered the anycast and broadcast variants:

> **Anycast**: Given a line segment with endpoints -1 and 1 and the message initially placed at 0, minimize the time to deliver the message to *either* endpoint.

> **Broadcast**: Given a line segment with endpoints -1 and 1 and the message initially placed at 0, minimize the time to deliver the message to *both* endpoints.

For the anycast variant, we gave an optimal offline algorithm and a $3/2$-competitive online algorithm (we call an algorithm c-competitive when it guarantees delivery in at most c times the optimal delivery time of a full-knowledge optimal offline algorithm). For the broadcast variant, we proposed an offline FPTAS (Fully Polynomial Time Approximation Scheme) and a $9/5$-competitive online algorithm. Further, we showed that the proposed online algorithms are optimally competitive. In other words, no better online algorithm exists.

Then, we extended these results to the plane in [6]. We provided an optimal offline algorithm for 2-robot systems by connecting the problem with an ancient theorem by the Greek philosopher Apollonius. For systems of n robots, we provided a $\sqrt{2}$-approximation algorithm by discretizing the continuous space and leveraging Carvalho et al.'s optimal graph algorithm. We also show that, interestingly, an exceedingly simple online algorithm where every robot attempts to deliver the message entirely independently is 2-competitive. Finally, we showed two lower bounds using different techniques, but neither appears to be tight.

3 Conclusion

The Pony Express Communication Problem is just one of many potential courier problems. Other problems may consider multiple messages, destination boundaries rather than points, agents with more complex dynamics (acceleration, limited range, terrain, etc.) and/or communication abilities (short range communication, full communication with dead-zones, etc.), and other environments (obstacles, air, etc.). Systems with faulty, self-interested, or competing teams of agents are also important areas of future work, especially for military/defense applications. The Pony Express problem itself remains to be studied for many topologies, such as the ring, the disk, and in graphs. Similarly, the energy-constrained courier problem studied in [1] also remains to be studied in non-graph environments.

Courier problems are fundamental problems to the study of algorithms for systems of cooperative, autonomous mobile agents. They have practical applications everywhere and many open problems remain to be solved.

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