CSCI 670: Theoretical Thinking 1

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1 Finding the Paper

First, I had to decide what type of paper I wanted. Initially, I wondered whether I could find something in the domain of agent-agent or human-agent negotiation: my area of interest. On Google, I tried combinations of words and phrases related with topics from my interests and our class–e.g. combinations of "human-agent," "agent-agent", "negotiation," "game-theory," "sub-modular," etc. Additionally, I wanted the paper to be relatively recent, so as to get perspective on the current trajectory of research. Ultimately, I found *Distributed greedy algorithm for multi-agent task assignment problem with submodular utility functions* by Guannan *et al.* [3].

This, actually, was the second paper I found. Initially, I wanted to pursue *Evaluating Practical Automated Negotiation Based on Spatial Evolutionary Game Theory* by Chen *et al.*[1]. While, this aligned a little better with my interests, I landed on the other paper since this was published in 2014 (seven years ago), and did not seem to match our class as well as my ultimate selection. The Guannan *et al.* paper's relevance to the class and topic of multi-agent systems ultimately drew me to further investigate it, despite it not aligning as well with my research interests. Particularly, this paper covers sub-modular functions and greedy algorithms.

2 Problem Formulation and Result

This problem is one of task assignment among agents. The following problem formulation is from Guannan *et al.* [3]. Let $\mathcal{A}$ be a set of agents; $\mathcal{T}$ be a set of tasks; $\mathcal{T}_{a\in\mathcal{A}}$ be the set of admissible tasks for agent a; Π an assignment profile (e.g. pair (a_i, τ_j) implies agent a_i selected task τ_j); the authors further define $T_a(\Pi)$ to be the set of all tasks τ that agent a selected; $A_\tau(\Pi)$ is the set of agents that have selected task τ; and each task has a utility function $U_\tau : 2^{\mathcal{A}} \to \mathbb{R}$, which the authors define as non-decreasing, sub-modular, and normal (the global utility function $U(\Pi)$ is simply $\Sigma_{\tau\in\mathcal{T}} U_\tau(A_\tau(\Pi))$) [3].

Guannan *et al.* purport to show a method by which they can maximize the global utility function U with some agent-task assignment, subject to some constraints [3]. Specifically, that an agent may select at most one task, and that agents must select tasks from their admissible task set $\mathcal{T}_a$ [3]. The authors consider local communication amongst agents–i.e. two agents can communicate given they share some task τ in their admissible task set (i.e., as they put it, if $\mathcal{T}_a \cap \mathcal{T}_{a'} \neq \emptyset$) [3]. The authors note several applications related to this assumption–e.g. "satellite-location, weapon-target assignment, and sensor coverage" [3].

Per the authors, maximizing a sub-modular function is NP-Hard, so they create greedy solutions that attain some fraction of the optimal answer [3]. Guannan *et al.* use a definition of curvature for a sub-modular function from Conforti *et al.*, which is as follows

$$\kappa_\tau = \max_{a\in\mathcal{A}:U_\tau(\{a\})>0} 1 - \frac{U_\tau(\mathcal{A}) - U_\tau(A/\{a\})}{U_\tau(\{a\})} \qquad [2].$$

So, they define κ as

$$\kappa = \max_{\tau\in\mathcal{T}} \kappa_\tau \qquad [3].$$

As previously mentioned, the authors propose two greedy algorithms: global, and local versions.

In the global version, a centralized algorithm greedily chooses an agent-task pairing π at time-step k that maximizes $U(\Pi^k \cup \{\pi\}) - U(\Pi^k)$, where Π^k is the current agent-task assignment [3]. They claim their algorithm attains at least $\frac{1}{1+\kappa}$ of optimal; i.e. $\frac{U(\Pi^G)}{U(\Pi^*)} \geq \frac{1}{1+\kappa}$, where Π^G is the assignment by their global greedy algorithm, Π^* is the optimal assignment, and κ is the maximum curvature of any of the task utility functions U_τ [3]. To back this claim, they cite Thm. 2.3 of Conforti *et al.* [2], which states:

Theorem 1 *If X is a matroid and Z is a nondecreasing submodular set function with $Z(\emptyset) = 0$, and total curvature α, then*

$$Z^G \geq \frac{1}{1+\alpha} Z^*$$

In the local algorithm of Guannan *et al.*, the algorithm runs locally: i.e. from the perspective of each agent [3]. In this algorithm, at each iteration, agents may be active of inactive; if active, they can attempt to select tasks, if not, they still receive information [3]. Their algorithm works as follows: if agent a is active at time k, then it will some candidate task τ that maximizes $U_\tau(A_\tau^k \cup \{a\}) - U(A_\tau^k)$ [3]. Then, agent a will negotiate with other agents who have the same candidate task at time k; ultimately one agent will select that task, and all others will select nothing at this time step [3]. Next, the winning agent will implement its selected task, communicate this selection, and terminate, while all other agents will enter another iteration of the algorithm [3]. This algorithm performs the same of the global greedy one; i.e. $\frac{U(\Pi^G)}{U(\Pi^*)} \geq \frac{1}{1+\kappa}$, where Π^D, where Π^D is the assignment by this distributed greedy algorithm [3].

3 Significance

The authors state their contribution in this paper to be the creation of a distributed algorithm for agent task selection, as "existing solutions ... require a global coordinator with access to global information" [3]. One example they give that lends to the importance of this contribution is in the context of distributed space systems, in which satellites coordinate in a task selection task, but have limited communication [3]. Here, a distributed algorithm would be useful. This idea can be extended to myriad domains with limited communication.

4 Open Question

Guannan *et al.* state two open questions related to their paper. First, recall the definition of curvature they use to calculate κ_τ. The authors designate investigating how alternative definitions of curvature might affect the bounds of their algorithms as their future work [3]. Secondly, they mention they intend to look at different communication restraints: i.e., currently agents can only communicate if they share tasks in their admissible task set, so they would look at different rules [3]. I would be curious to see if there could be some type of hybrid approach to this problem using evolutionary computation.

References

[1] Siqi Chen, Jianye Hao, Gerhard Weiss, Karl Tuyls, and Ho-fung Leung. Evaluating practical automated negotiation based on spatial evolutionary game theory. In Carsten Lutz and Michael Thielscher, editors, *KI 2014: Advances in Artificial Intelligence*, pages 147–158, Cham, 2014. Springer International Publishing.

[2] Michele Conforti and Gérard Cornuéjols. Submodular set functions, matroids and the greedy algorithm: tight worst-case bounds and some generalizations of the rado-edmonds theorem. *Discrete applied mathematics*, 7(3):251–274, 1984.

[3] Guannan Qu, Dave Brown, and Na Li. Distributed greedy algorithm for multi-agent task assignment problem with submodular utility functions. *Automatica*, 105:206–215, 2019.