EE-588: Homework # 5

Due on Wednesday, November 20, 2019

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Contents

Problem 1

$$y_i = \phi(a_i^T x + v_i)$$
$$a \leq \phi'(u) \leq b$$
$$v_i \sim \mathcal{N}(0, \sigma^2)$$

Define $w_i = a_i^T x + v_i = \phi^{-1}(y_i)$.
Given that y_i are sored in non-decreasing order, i.e., $y_1 \leq y_2 \cdots \leq y_m$ The derivative $\phi'(u)$ is given by:

$$\phi'(y_{i+1}) = \frac{y_{i+1} - y_i}{w_{i+1} - w_i}$$

Since,

$$\frac{1}{\beta} \leq \frac{1}{\phi'(u)} \leq \frac{1}{\alpha}$$

we get,

$$\frac{1}{\beta}(y_{i+1} - y_i) \leq w_{i+1} - w_i \leq \frac{1}{\alpha}(y_{i+1} - y_i)$$

The likelihood is given by:

$$v_i = w_i - a_i^T x$$
$$L(w, x) = \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{\frac{-(w_i - a_i^T x^2)}{2\sigma^2}}$$
$$\log L(w, x) = -\frac{1}{2\sigma^2} \sum_{i=1}^{m} (w_i - a_i^T x_i)^2 - \frac{m}{2} \log 2\pi\sigma^2$$

$-\frac{m}{2} \log 2\pi\sigma^2$ is a constant, so we maximize the following function:

$$\text{maximize } -\frac{1}{2\sigma^2} \sum_{i=1}^{m} (w_i - a_i^T x_i)^2$$
$$\text{subject to } w_i = a_i^T x + v_i$$

Problem 2

Given:

$$0 \leq x(1) \leq x(2) \cdots x(N)$$
$$y(t) = \sum_{\tau=1}^{k} h(\tau)x(t-\tau) + v(t) \quad t = 2, \ldots, N+1 v(t) \qquad \sim \mathcal{N}(0,1)$$

Following problem 1, the log likelihood is given by:

$$\log L(y,x) = -\frac{1}{2}\sum_{t=2}^{N+1} v(t)^2$$
$$= -\frac{1}{2}\sum_{t=2}^{N+1}(y(t) - \sum_{\tau=1}^{k} h(\tau)x(t-\tau))^2$$

which can be expressed as COP:

maximize $-\dfrac{1}{2}\displaystyle\sum_{t=2}^{N+1} v(t)^2$

subject to $x(N) \geq x(N-1) \geq x(N-2) \cdots x(1) \geq 0$

Problem 3

Given the inequaliies,

$$f_1(x^{(j)}) > \max\{f_2(x^{(j)}), f_3(x^{(j)})\}$$
$$f_2(y^{(j)}) > \max\{f_1(y^{(j)}), f_3(y^{(j)})\}$$
$$f_3(y^{(j)}) > \max\{f_1(z^{(j)}), f_2(z^{(j)})\}$$

are symmetric, we can relax the strict inequality:

$$f_1(x^{(j)}) \geq \max\{f_2(x^{(j)}), f_3(x^{(j)})\} + 1$$
$$f_2(y^{(j)}) \geq \max\{f_1(y^{(j)}), f_3(y^{(j)})\} + 1$$
$$f_3(y^{(j)}) \geq \max\{f_1(z^{(j)}), f_2(z^{(j)})\} + 1$$

' the form of the above affine functions is as follows:

$$f_i(z) = a_i^T z - b_i \quad i = [1, 2, 3]$$

The above inequalities are shift invariant for both a_i and b_i, i.e. they would not change if any constants were to be added to a_i and/or b_i across the three coefficients. WLOG, we can hence fix, $a_1 + a_2 + a_3 = 0$ and $b_1 + b_2 + b_3 = 0$ to solve the convex optimzation problem under these two equality constraints.

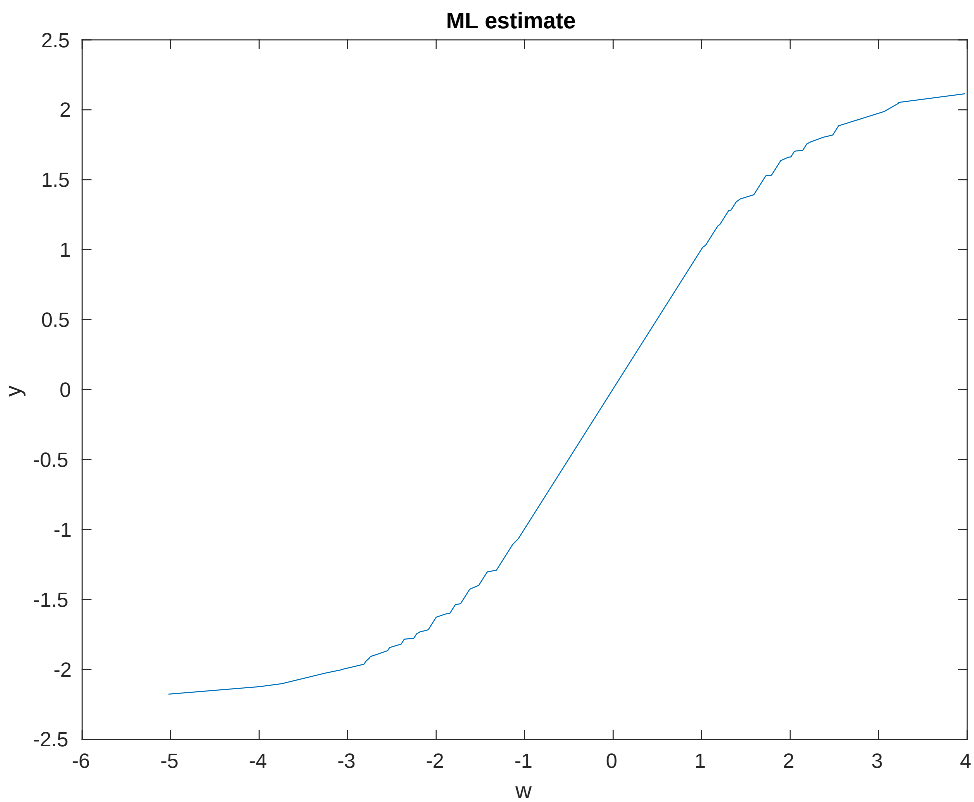

Figure 1: Problem 1

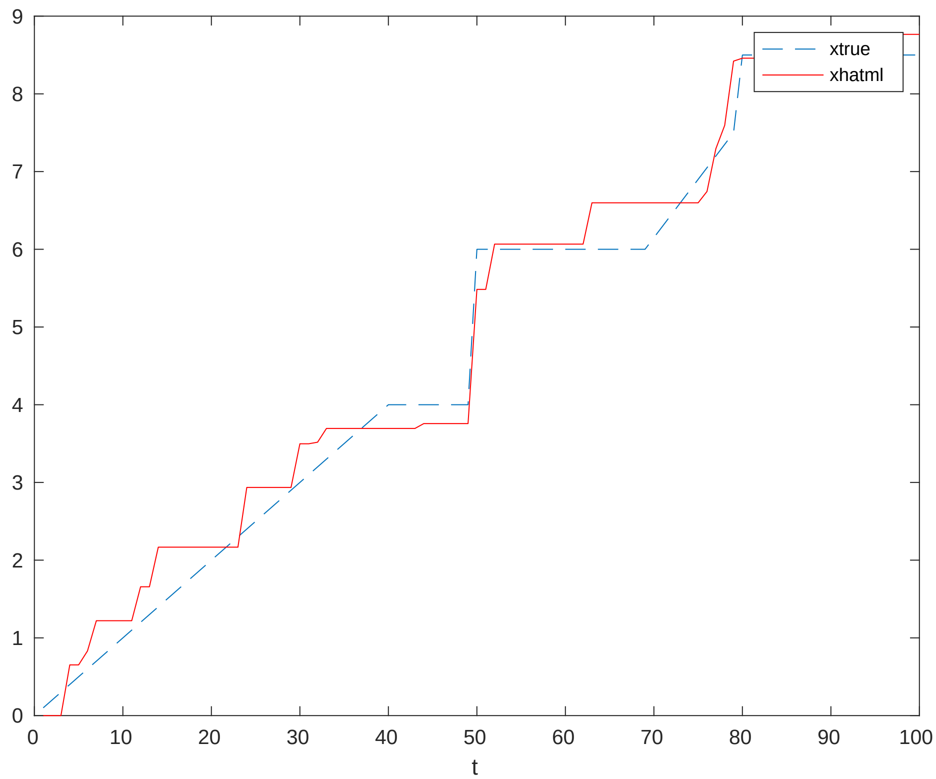

Figure 2: Problem 2

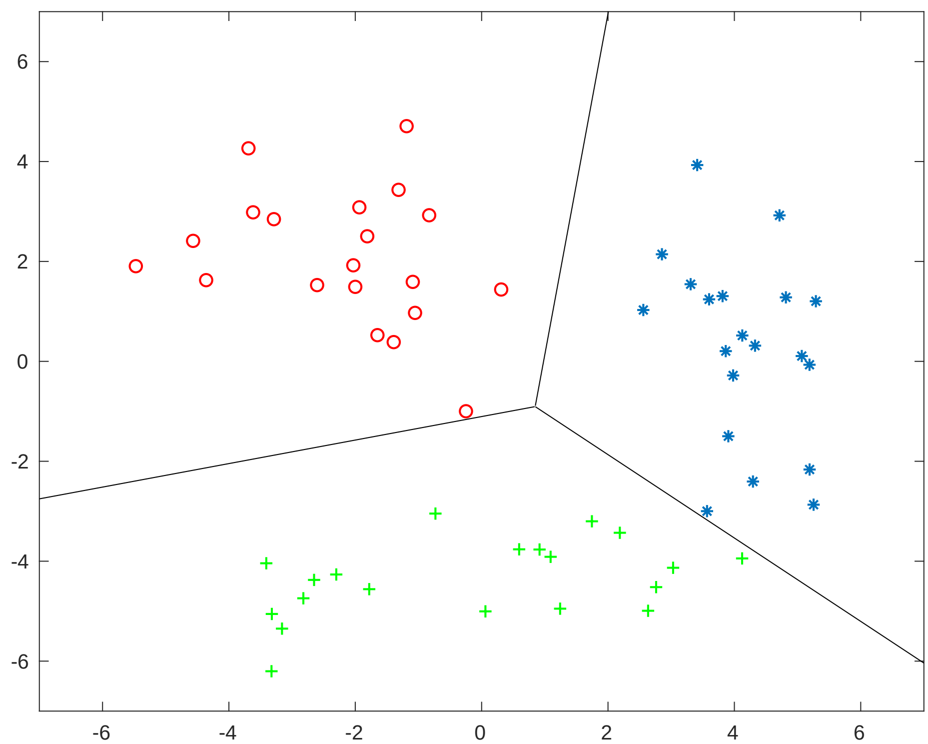

Figure 3: Problem 3