

Partially Ordered SETS

Equivalence Relations on S

- reflexive for all $x \in S$ $x = x$
- symmetric for all $x, y \in S$ if $x = y$, then $y = x$
- transitive $\forall x, y, z \in S$, if $x = y$ & $y = z$ then $x = z$

A partial order on a set S is a relation \leq (\downarrow prec)
 \downarrow precedes

- reflexive $\forall x \in S$ $x \leq x$
- antisymmetry if $x \leq y$ & $y \leq x$ then $x = y$
- transitive $\forall x, y, z \in S$ if $x \leq y$ & $y \leq z$ then $x \leq z$.

NOTATION $P = (S, \leq)$ \leq_P \downarrow usual

EXAMPLES • $P = ([n], \leq)$ $i \leq_P j$ if $i \leq j$
 $\{1 < 2 < 3 < \dots < n\}$ TOTAL / LINEAR ORDER

• B_n = Boolean poset on $2^{[n]}$

Objects are subsets of $[n]$
 order is containment.

$n=3$: $\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

$\{2\} \leq \{1,2\}$

$\{2\}$ & $\{1,3\}$ are incomparable

$(2^{[n]}, S \leq T \text{ if } \underline{\#S} \leq \underline{\#T}) \cong [n+1]$

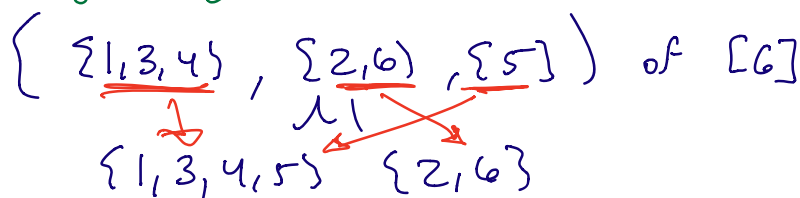
• $D_n = ([n], i \leq_n j \text{ iff } j \text{ is evenly divisible by } i)$

$n=12$ what #s are ≤ 12 ? 1, 2, 3, 4, 6, 12
 what #s are ≤ 8 ? 1, 2, 4, 8

incomparable: i incmp j iff $\gcd(i, j) \neq i$ or j

1 is the unique minimal element. $\hat{0}$

• Π_n partial order on set partitions of $[n]$
 given by refinement.



$\{B_i\} \leq \{C_i\}$ if $\forall i B_i \subseteq C_j$ some j .

minimal element? $\{1\} \{2\} \dots \{n\}$

maximal element? $\{1, 2, \dots, n\}$.

• $B_n(12)$ all subposets of Π_n
 ordered by inclusion.

List All Posets on 4 Objects

HASSE DIAGRAM
 b
 $|$ if $a \leq b$ and
 a $\nexists c$ s.t. $a \leq c \leq b$
 $a \neq b$

