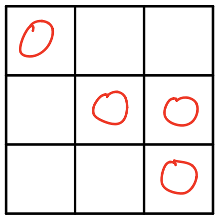
MATH 532 Discussion Monday 28/09



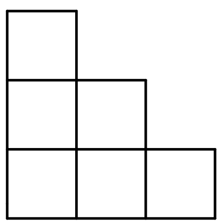
$[n] \times [n] \leftarrow n$ non-attacking rooks $\Longleftrightarrow w \in S_n$
↳ no two same col
no two same row

$B \subseteq$ "BOARD"

Rook numbers for B

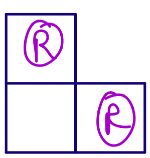
r_k = # ways to place k nonattacking rooks onto cells of B

Ex. Consider $\delta_n = (n-1, n-2, \ldots, 2, 1) \vdash \binom{n}{2}$
"staircase"

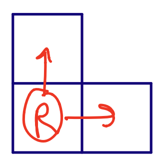
$\delta_4 =$ 

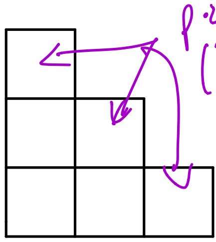
$(3,2,1)$

Let δ_n be the board B.
Compute r_k on B.

 $\subseteq [2] \times [2]$

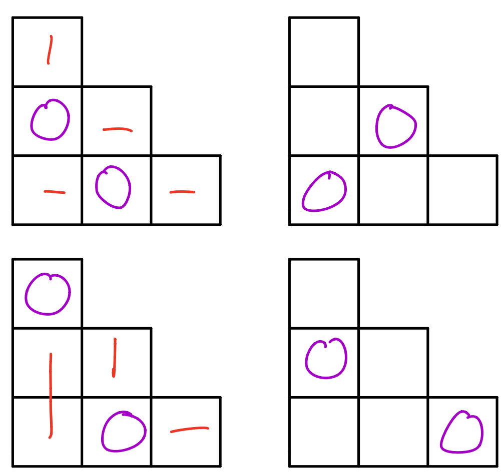
$r_0 = 1$
$r_1 = 3$
$r_2 = 1$

 no sleud!



p: 2 ⊢ 2
$\binom{3}{2} = 3$

$r_0 = 1$
$r_1 = 6$
$r_2 = 7$
$r_3 = 1$



IN General:

$\sigma_0 = 1$

$\sigma_1 = \binom{B}{1} = |\delta_n| = \binom{n}{2}$

$\sigma_{n-2} = ??$

$\sigma_{n-1} = 1$

δ_5

$\sigma_0 = 1$

$\sigma_1 = 10$

$\sigma_2 = 25$

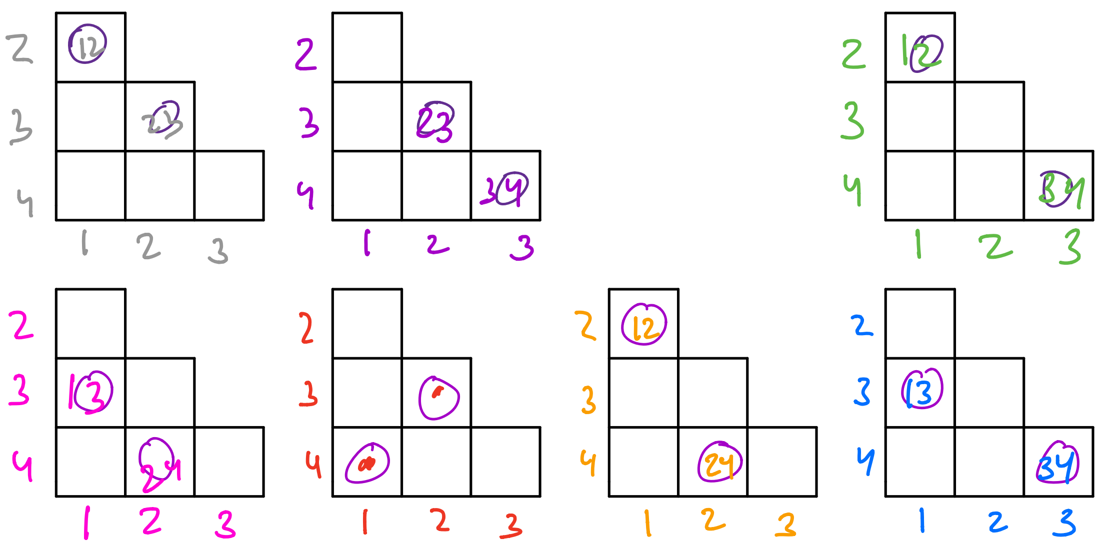
$\sigma_3 = 15$

$\sigma_4 = 1$

$S(n, n-k)$

Stirling #s of 2nd Kind.

set partitions of $[n]$ into k nonempty blocks.



Set partitions of $[n]$ into $n-2$ blocks $(n=4)$

1 − 234

2 − 134

3 − 124

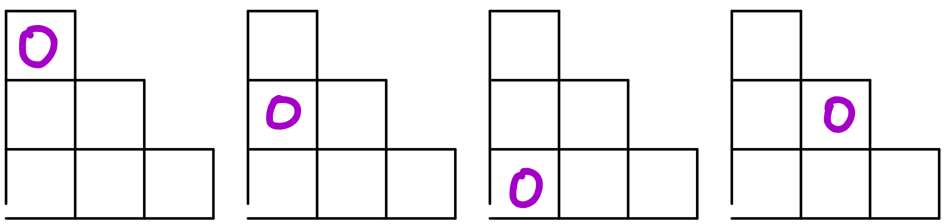
4 − 123

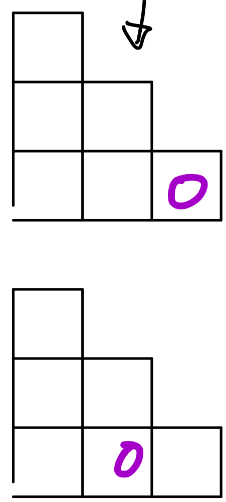
12 − 34

13 − 24

14 − 23

$$\binom{\binom{4}{2}}{1}$$





[4] into 4-1 blocks :

12 - 3 - 4	23 - 1 - 4
13 - 2 - 4	24 - 1 - 3
14 - 2 - 3	34 - 1 - 2

$$\binom{4}{2}$$