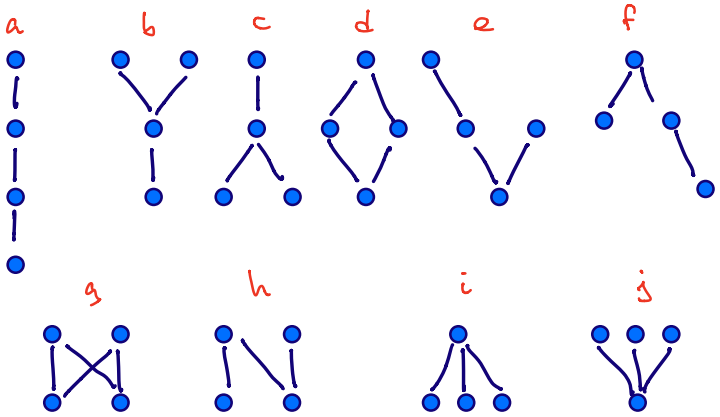
19/10/2020

MATH 532 Discussion

Connected Posets on 4 elements



Defn An upper bound for $s,t \in P$ is an $u \in P$ s.t. $s \le u$ & $t \le u$.

A least upper bound for $s,t \in P$ is an upper bound u s.t. if u' is also an upper bound, then $u \le u'$.

Defn A lattice is a poset in which every pair of elts has a lub & glb.

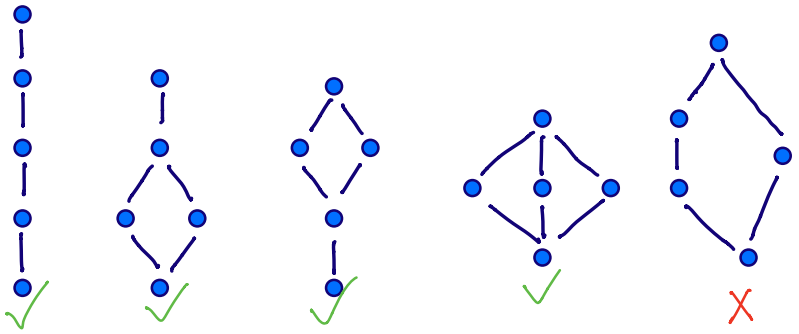
Q: Which of these posets is a lattice?

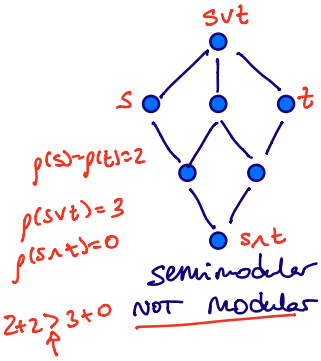
a & d

NOTATION: lub (least upper bound): $s \vee t$ "join"
glb (greatest lower bound): $s \wedge t$ "meet"

OBSERVE: A (finite) lattice always has $\hat{0}$ & $\hat{1}$.

LATTICES ON 5 ELEMENTS





Prop. L finite lattice. TFAE

(i) L is graded and the rk fn ρ satisfies $\rho(s)+\rho(t) \ge \rho(s \vee t)+\rho(s \wedge t)$

(ii) If both s & t cover $s \wedge t$, then $s \vee t$ covers s & t.

Defn A lattice satisfying (i) or (ii) is called semi-modular

A modular lattice is graded with $\rho(s)+\rho(t)=\rho(s \vee t)+\rho(s \wedge t)$.

Prop L modular iff $s \vee (t \wedge u) = (s \vee t) \wedge u$