

Card shuffling.  $GRS$   
 $\swarrow \searrow \searrow$   
 Gilbert Rads Shannon  
 N-card deck.

Riffle Shuffle

Cut deck

$$\text{Prob}(\text{cut } k \text{ cards off top}) = \frac{\binom{N}{k}}{2^N}$$

Drop cards

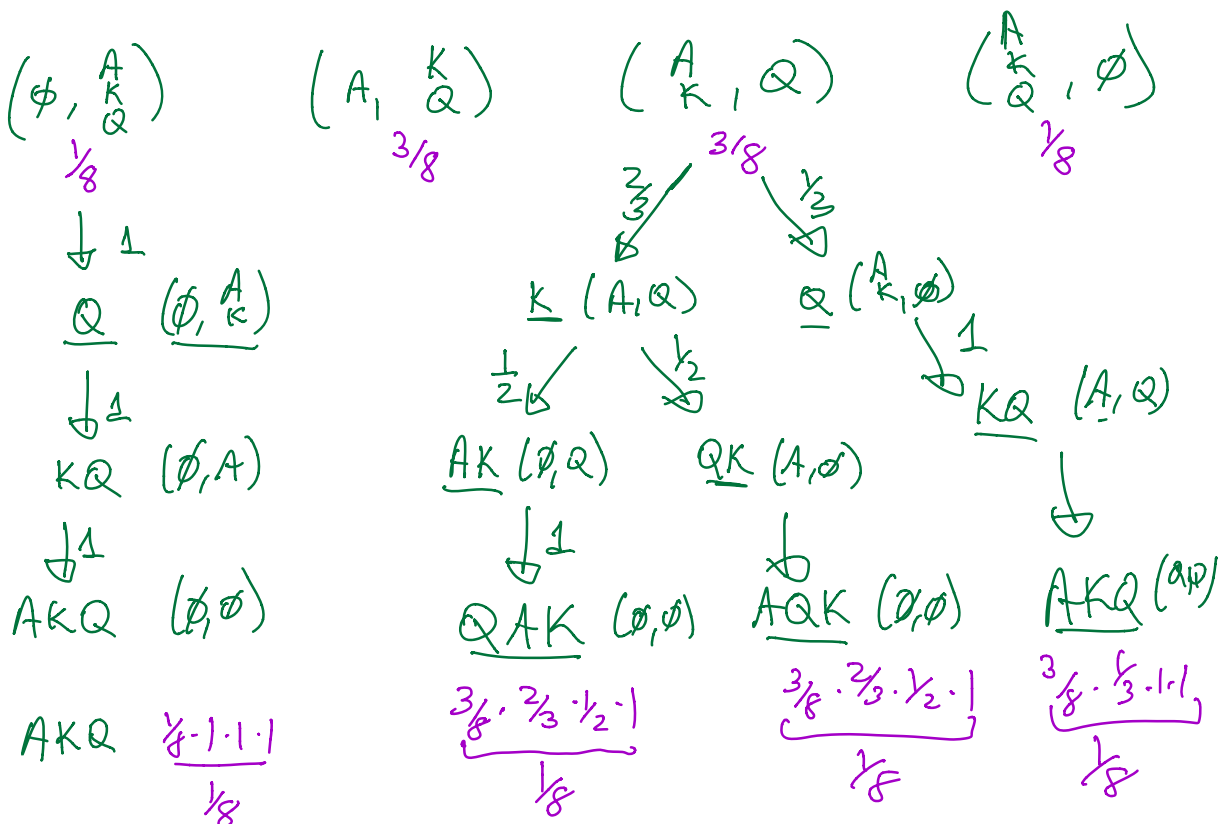
$$\text{Prob}(L \text{ had}) = \frac{\#L}{\#L + \#R}$$

Silly Example

$N=3$

$AKQ$

$\begin{matrix} A \\ K \\ Q \end{matrix}$



$Q_2(\sigma) = \text{prob}(\sigma \text{ results from a single shuffle}).$

	AKQ	AQK	KAQ	KQA	QAK	QKA
$Q_2(\sigma)$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	0
$U(\sigma)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$n=3 \quad TV = \frac{1}{3}$$

Total Variation  $\leq 1$

$$\|Q_2 - U\|_{TV} = \frac{1}{2} \sum_{\sigma \in S_n} |Q_2(\sigma) - U(\sigma)|$$

$Q_a$  an  $a$ -shuffle

$$\text{Cut} : \frac{1}{a^n} \binom{n}{k_1 k_2 \dots k_a}$$

$$\text{Drop} : \frac{\#H_i}{\sum_{j=1}^a \#H_j}$$

Theorem (Bayer-Discreet 1992)

$$Q_a * Q_b = Q_{ab}$$

Ex. 3-shuffles w/2 heads = 1 shuffle w/8 heads.

$$Q_2 * Q_2 * Q_2 = Q_8$$

$$Q_a(\sigma) = \binom{n+a-r}{n} \frac{1}{2^a}$$

$r = \# \text{ risky sequence in } \sigma$