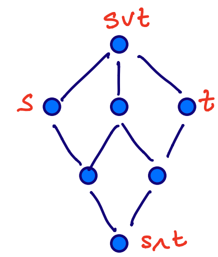
MATH 532 Discussion 21/10/2020

Finite Poset



- meet semilattice ($s \wedge t$ exists) $\Rightarrow \hat{0}$
- join semilattice ($s \vee t$ exists) $\Rightarrow \hat{1}$
- lattice : meets & joins
  - semimodular : graded & $\rho(s)+\rho(t) \geq \rho(s\wedge t)+\rho(s\vee t)$
    s, t covering $s \wedge t$ & both covers $\Rightarrow$ s, t covered by $s \vee t$ ← also covers.
  - modular : graded & $\rho(s)+\rho(t) = \rho(s\wedge t)+\rho(s\vee t)$
  - atomic : an atom is any $s \in \mathcal{L}$ s.t. $\hat{0} \lessdot s$
    ↳ every $u \in \mathcal{L}$ can be written $u = s_1 \vee s_2 \vee \cdots \vee s_k$ s_i's atoms
  - a lattice is complemented if $\forall s \in \mathcal{L}$ $\exists t \in \mathcal{L}$ s.t. $s \vee t = \hat{1}$ & $s \wedge t = \hat{0}$.

geometric lattice is a finite, semimodular atomic lattice.

Classify the following:

- $([n], \leq)$ modular lattice (not atomic, not complemented)
- $B_n = (S \subseteq [n], \subseteq)$ modular, complemented, atomic, geometric
- $D_n = ([n], i \leq j \text{ if } j \equiv 0 \bmod i)$
- Π_n = (set partitions of $[n]$, refinement) atomic geometric not modular.
- $B_n(q) = (\text{subspaces of } \mathbb{F}_q^n, \subseteq)$

BONUS weak order on S_n