

A ambient set
 $Y \subseteq S$ collection of
 properties.

$$f_{\bar{=}}(\emptyset) = \sum_Y (-1)^{\#Y} f_{\geq}(Y)$$

\uparrow exactly properties in \emptyset \uparrow at least properties of Y

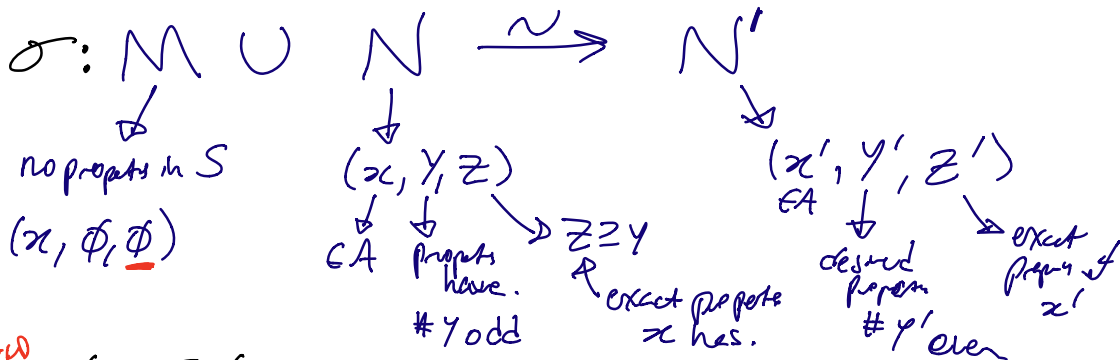
Ex. $f_{\bar{=}}(T) = \#\{\omega \in S_n \mid \text{fixed pts of } \omega \text{ are exactly } T\}$
 $\omega(i) = i \text{ iff } i \in T$

here. $f_{\geq}(T) = \#\{\omega \in S_n \mid \omega(i) = i \text{ if } i \in T\}$
 \uparrow more allowed
 easy to count directly.

$f_{\geq}(Y) = (n - \#Y)!$ If $\#Y = \#X$, same count.

$\Rightarrow f_{\bar{=}}(\emptyset) = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)!$
 derangements \uparrow choose Y of size k \uparrow $f_{\geq}(Y)$.

$$f_{\bar{=}}(\emptyset) + \sum_{\#Y \text{ odd}} f_{\geq}(Y) = \sum_{\#Y \text{ even}} f_{\geq}(Y)$$



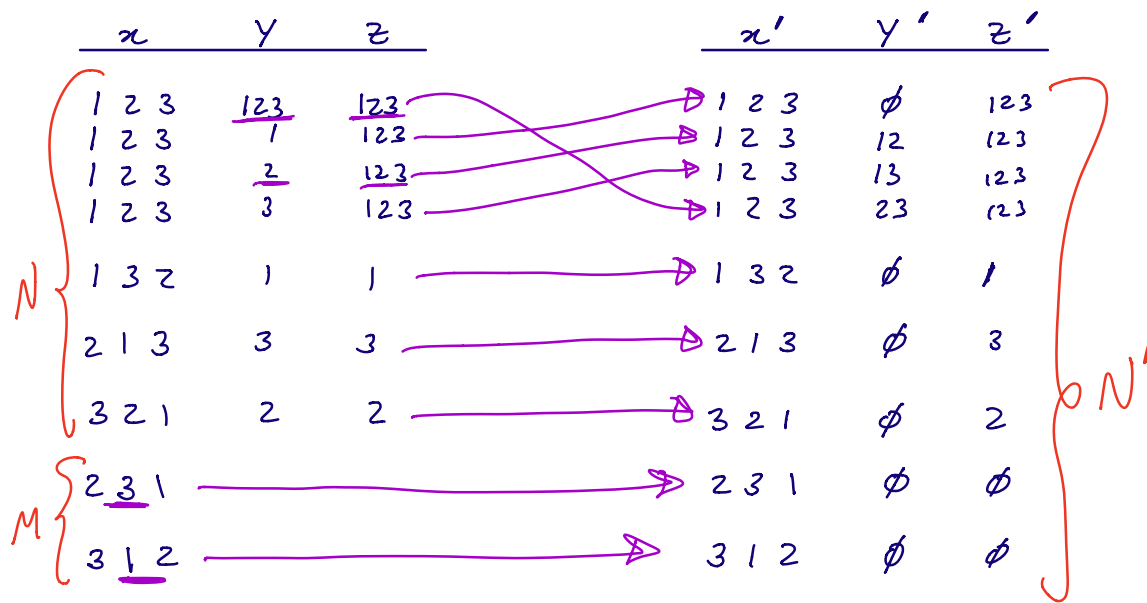
Fixed pts $\sigma(x) = (x, \emptyset, \emptyset)$ if $x \in M$.

$$\sigma(x, Y, Z) = \begin{cases} (x, Y - i, Z) & \text{if } \min Y = \min Z = i \\ (x, Y \cup i, Z) & \text{if } i = \min Z < \min Y \end{cases}$$

S_3

write the sets M, N, N' lying to the map.

$$M = \{231, 312\}$$



$$N \cup N' \xrightarrow{\tau} N \cup N' \text{ involution.}$$

\uparrow neg \uparrow pos

τ involution on $N \cup N'$ satisfying:

- If $\tau(x) = y$ and $y \neq x$ then exactly one of x, y belongs to N (exactly one to N')
- If $\tau(x) = x$, then $x \in N^+$ (positive half)

$$wt(x) = \begin{cases} +1 & x \in N' \text{ & pos} \\ -1 & x \in N \text{ & neg} \end{cases}$$

$$\#f_X(\tau) = \sum_{x \in X} wt(x)$$