MATH 532 Discussion MONDAY 24/08

A (strong) composition of n is a sequence of positive integers that sum to n.

$$\underline{a} = (a_1, a_2, \dots, a_k) \qquad \sum_{i=1}^{k} a_i = n \qquad a_i \in \mathbb{P}.$$

Ex. Compositions of 4:

$8 = 2^3$

(4)
(3,1) (1,3)
(2,2)
(2,1,1) (1,2,1) (1,1,2)
(1,1,1,1)

1 (1,1,1,1)
3 (1,2,1) (2,1,1) (1,1,2)
3 (1,3) (3,1) (2,2)
1 (4)

$$\sum_{k=1}^{4} (1 + x + \cdots + x^4)^k \qquad [x^4]$$

append 1 to end

(1,1,1,1,1)
(1,2,1,1) (2,1,1,1) (1,1,2,1)
(1,3,1) (3,1,1) (2,2,1)
(4,1)

All have $a_{-1} = 1$

ASIDE

Online Encyclopedia of Integer Sequences

OEIS

$$\#\text{comp of } n = 2^{n-1}$$

Comp(n)

↓ Bijective map.

Comp(n-1) ⊔ Comp(n-1)

append 1 add 1 to last

add 1 to a_{-1} = last part of a

(1,1,1,1)
(1,2,1) (2,1,1) (1,1,2)
(1,3) (3,1) (2,2)
(4)

$2^{n-1} = \#\text{Comp}(n) = \#\text{subsets of } [n-1]$

TASK: Give a bijection between

comp of n w/ k parts $\longleftrightarrow$ $(k-1)$-element subsets of $[n-1]$

$n=4$

$(1,1,1,1)$ $\qquad \{1,2,3\}$

$(1,2,1)$ $(2,1,1)$ $(1,1,2) \longleftrightarrow \{1,2\}\ \{1,3\}\ \{2,3\}$

$(1,3)$ $(3,1)$ $(2,2) \longleftrightarrow \{1\}\ \{2\}\ \{3\}$

(4) $\qquad \emptyset$

$(a_1, a_2, \dots, a_k) \xrightarrow{\phi} \{a_1, a_1+a_2, \dots, a_1+\dots+a_{k-1}\}$

$\{i_1, \dots, i_{k-1}\} \xrightarrow{\phi^{-1}} (i_1, i_2-i_1, i_3-i_2, \dots, n-i_{k-1})$

$n=15$

$\{2,3,6,10,13\} \subseteq [14]$ $\qquad$ $(3,4,1,6,1)$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$(2,1,5,2,3,2)$ $\qquad$ $\{3,7,8,14,15\} \subseteq [14]$

1. Describe $\phi(\text{rev}(a))$ in terms of $\phi(a)$
2. Describe $\phi^{-1}([n] \setminus S)$ in terms of $\phi^{-1}(S)$