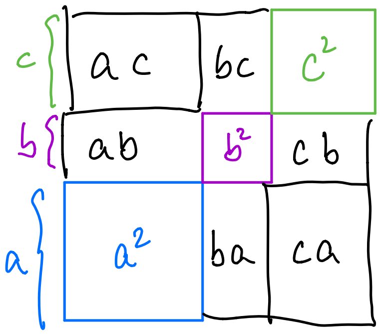
MATH 532 Discussion Friday 21/08



$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

Find a sequence of numbers $a_0 = 1, a_1, a_2, \dots$ ($a_1 = \frac{1}{2}$)

$$\underline{\sum_{k=0}^{n} a_k a_{n-k} = 1} \qquad A(x) = \sum_{n \geq 0} a_n x^n$$

$$(A(x))^2 = \left(\sum_{n\geq 0} a_n x^n\right)^2 = \sum_{n \geq 0}\left(\sum_{k=0}^{n} a_k a_{n-k}\right) x^n$$

$$= \sum_{n \geq 0} x^n = \frac{1}{1-x}$$

$$(A(x))^2 = \frac{1}{1-x}$$

$$A(x) = (1-x)^{-1/2} = \sum_{k \geq 0} \binom{-1/2}{k}(-x)^k = \sum_{k\geq 0} \underbrace{(-1)^k \binom{-1/2}{k}}_{a_k} x^k$$

$$\binom{-1/2}{k} = \frac{(-1/2)\overset{-3/2}{(-1/2 - 1)} \overset{-5/2}{\cdots} \overset{\frac{-2k+1}{2}}{(-1/2 - k + 1)}}{k!}$$

↳ simplify this!

Binomial Thm

$$(1+x)^n = \sum_{k \geq 0}^{\infty} \binom{n}{k} x^k$$

$$a_k = \frac{1}{4^k}\binom{2k}{k} \leftarrow \text{best?}$$

$$a_k = \frac{1}{2}\prod_{m=1}^{k}\left(\frac{1}{m} - k\right) \leftarrow \text{less good}$$

$$a_k = \frac{1\cdot 3 \cdot 5 \cdots (2k-1)}{2^k \cdot k!} \leftarrow$$

$$a_k = \frac{1}{4^k}\binom{2k}{k} = \frac{(2k)!}{2^k\, 2^k k!\, k!}$$

$$\boxed{2\cdot 4 \cdot 6 \cdots 2k}$$