

$\text{Des}(\omega) = \{i \mid \omega_i > \omega_{i+1}\}$  descent set  $\subseteq [n-1]$

Given a set  $S \subseteq [n-1]$ , define

$$\alpha(S) = \# \{ \omega \in S_n \mid \text{Des}(\omega) \subseteq S \}.$$

$$\beta(S) = \# \{ \omega \in S_n \mid \text{Des}(\omega) = S \}.$$

Exercise 1. Compute  $\alpha(\{3, 8\})$  (Assume  $n \geq 9$ )

$$\binom{9}{3} \cdot \binom{6}{5} \binom{n}{3} \binom{n-3}{5} \underbrace{\omega_1 < \omega_2 < \omega_3}_{\text{?}} \underbrace{\omega_4 < \omega_5 < \omega_6 < \omega_7 < \omega_8}_{\text{?}} \underbrace{\omega_9 < \dots}_{\text{?}}$$

2. Give a formula for  $\alpha(S)$ .

$$S = \{s_1, \dots, s_k\}$$

$$\alpha(S) = \binom{n}{s_1, s_2 - s_1, s_3 - s_2, \dots, s_k - s_{k-1}, \underline{n - s_k}}$$

$$\underline{\binom{n}{k} = \binom{n}{k, n-k}}$$

3. Write a formula relating  $\alpha(S)$  &  $\beta(S)$ .

$$\alpha(S) = \sum_{T \subseteq S} \beta(T)$$

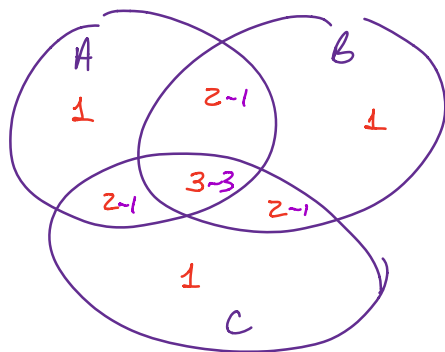
4. Compute  $\beta(\{3, 8\})$ .

$$\alpha(3, 8) - \alpha(3) - \alpha(8) + \alpha(\emptyset)$$

5. Invert your formula for 3.

$$\beta(S) = \sum_{T \subseteq S} (-1)^{\#(S-T)} \alpha(T)$$

$A, B, C$   
sets.



$$|A \cup B \cup C| =$$

$$|A| + |B| + |C|$$

$$- |A \cap B| - |A \cap C| - |B \cap C|$$

$$+ |A \cap B \cap C|$$