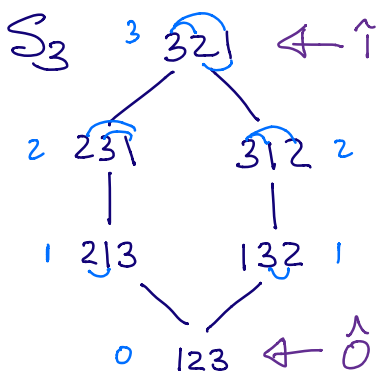


Weak order on S_n

$u < w$ if get w by swapping adjacent letters $u_i u_{i+1}$ where $u_i < u_{i+1}$



• Is there always $\hat{0}$? $\hat{1}$?

• What is the rank function?

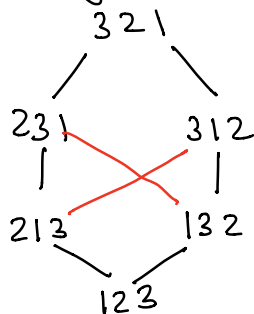
• Anything else? $\hookrightarrow \text{rk}(u) = \text{inv}(w)$
 $= \# \{(i, j) \mid u_i > u_j\}$

$$F(q) = \sum_{w \in S_n} q^{\text{rk}(w)}$$

$$= 1 + 2q + 2q^2 + q^3$$

$$= (1+q)(1+q+q^2) = [n]_q!$$

Strong order on S_n



$u < w$ if (A) $u_i < u_{i+1}$ swap u_i & u_{i+1}

(B) i left of $i+1$ swap i & $i+1$
 $\hat{0}$ same b/c still sort

$$213 < 231$$

$$213 < 312$$

\uparrow ? same

position = value of identity $u_i = i$

An atom of a ranked poset w/ $\hat{0}$ is an element with only $\hat{0}$ below it.

Is $\text{rk}(u) = \text{inv}(u)$? $u < w$ $\text{inv}(w) \neq \text{inv}(u) + 1$

