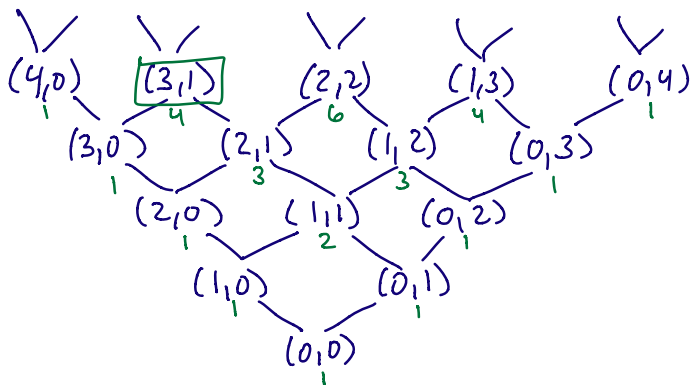


$$P = \mathbb{N} \times \mathbb{N} \quad \{(a,b) \mid a,b \in \mathbb{N}\}$$

$$(a,b) \leq (a',b') \text{ if } \begin{matrix} a \leq a' \\ b \leq b' \end{matrix}$$



• cover relations?
add 1 to x or y comp.

• rank function?
 $\rho(a,b) = a+b$

• how many elts of rk i ?
 $i=4$

Count SATURATED CHAINS from $\hat{0}$ to z . $\frac{i+1}{i=4}$.

↑
skip no steps

↑
linear order (go up!)

↖ antichain

Recall an order ideal for a subset $A \subseteq P$ is

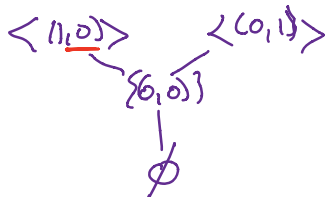
$$\{s \in P \mid s \leq t \text{ for some } t \in A\}$$

A principle order ideal is where $A = \{t\}$.

The lattice of order ideals of P is $\mathcal{J}(P)$,

the set of order ideals ordered by inclusion is sets

• order ideals \leftrightarrow antichains \rightarrow set of pairwise comp elts.



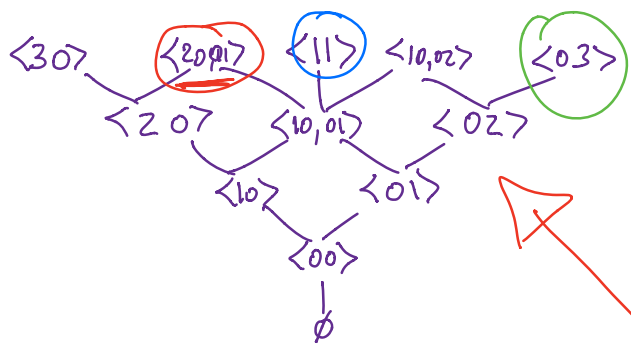
• Draw ≥ 2 more levels

• cover relations?

• rank function?

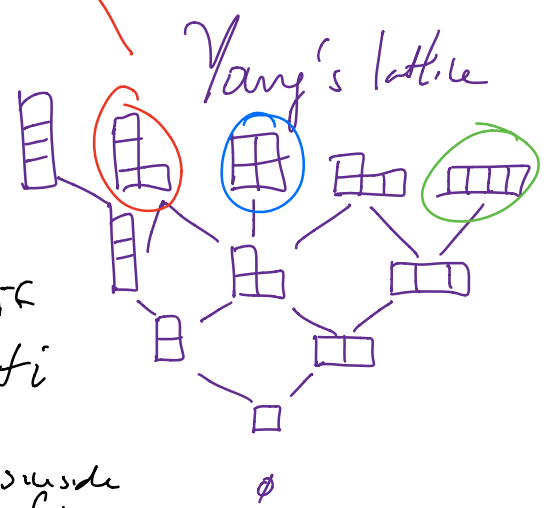
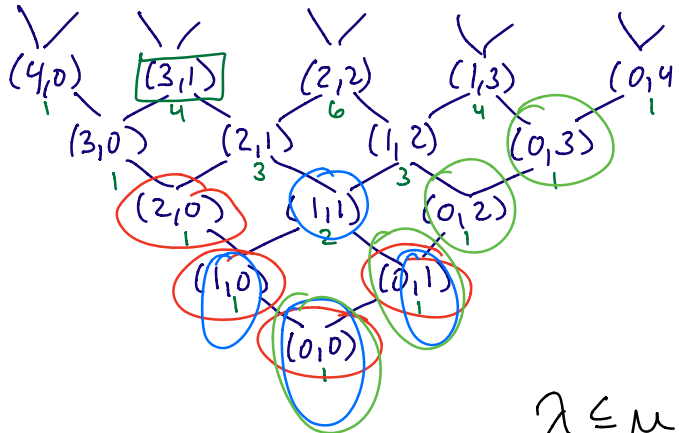
• how many elts of each rank?

• how many saturated chains?



If $s \leq t$ in P ,
is $I_s \leq I_t$ in $J(P)$?

No... $\langle 0,1 \rangle \not\leq \langle 1,1 \rangle$



$\lambda \leq \mu$ if
 $\lambda_i \leq \mu_i \forall i$

diagm of λ fits inside
diagm of μ