MATH 532 Discussion 14/10/2020

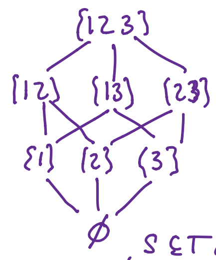
Hasse Diagram has node for every elt of P,
edge $\begin{matrix} t \\ | \\ s \end{matrix}$ iff
- $s \leq t$, $s \neq t$
- $\forall r \in P$ s.t. $s \leq r \leq t$, either $r = s$ or $r = t$.

"cover relations"

Ex. $([n], \leq)$

$F_P(x) = \sum_{i=0}^{n-1} x^i = (x)_n$

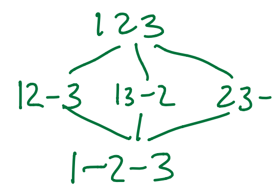
$B_n = (S \subseteq [n], \subseteq)$



$S \lessdot T$ if $S \subseteq T$ & $\underline{\# T \setminus S = 1}$

$F_P(x) = \sum_{i=0}^{n} \binom{n}{k} x^k = (1+x)^n$

$\Pi_n = (B \vdash [n], \text{refinement})$



$B \lessdot C$ if merge 2 blocks

$rk(B) = n - \#$ blocks

$F_P(x) = \sum_{i=0}^{n-1} S(n, n-i)\, x^i$

- Say P has $\hat{0}$ if $\exists$ $\hat{0} \in P$ s.t. $\hat{0} \leq t$ $\forall t \in P$.
  $\hat{1}$ $\hat{1} \in P$ $t \leq \hat{1}$ $\forall t \in P$.
- Chain in P is a seq $(t_1, \dots, t_k)$ s.t. $t_i \leq t_{i+1}$
  - maximal if it's not properly contained in another chain.
  - saturated if $\nexists u \in P$ s.t. $t_i < u < t_{i+1}$ some i
- RANK of a finite poset P is $\ell(P) =$ max length saturated chain in P.

P is graded if all max'l chains have same length.

$\underline{\ell(P) = 5}$

If P is graded, then for $t \in P$

$\ell(t) = rk(t) =$ max'l length chain $s_1 \lessdot \cdots \lessdot s_k \lessdot \underline{t}$

$$F_P(x) = \sum_{i=0}^{n} f_i x^i$$

where $f_i = \#$ elts of P of rk i