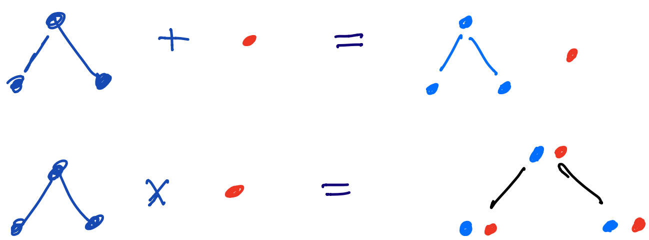
MATH 532 Discussion 23/10/2020

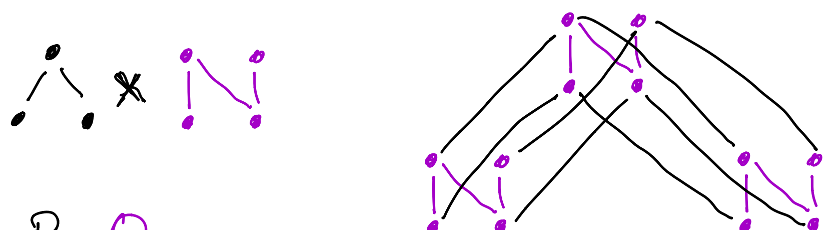
$P, Q$ posets

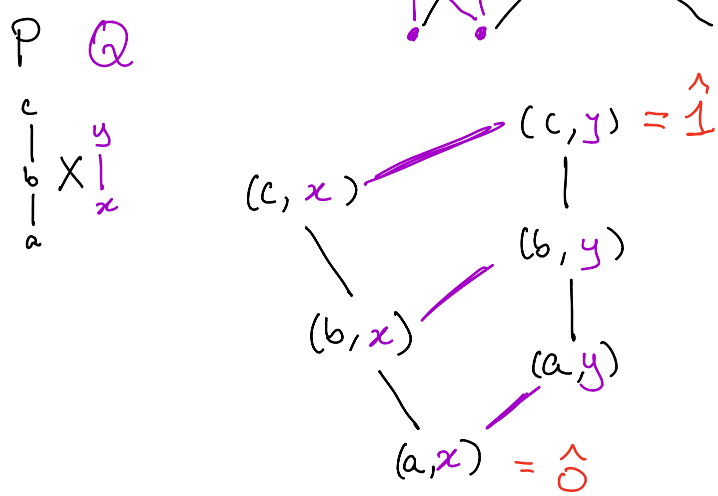
$P+Q$ ($P \cup Q$) $\quad s \leq t$ if $s,t \in P$ & $s \leq t$; $s,t \in Q$ & $s \leq t$

$P \times Q$ ($(p,q) \in P \times Q$) $\quad (p,q) \leq (p',q')$ if $p \leq p'$ and $q \leq q'$

EXAMPLES







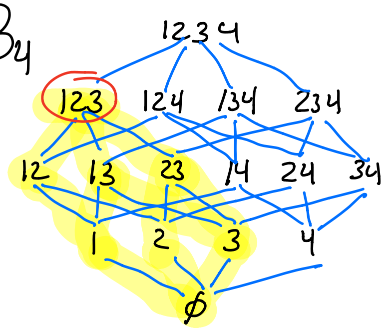

$(b,x)$ $\quad (a,y)$

$b >_P a \quad x <_Q y$

Defn An order ideal of $A$ in $P$ $(A \subseteq P)$ is

$$I_A = \{ s \mid s \leq t \text{ for some } t \in A \}$$

$B_4$



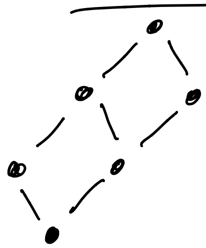

$\mathcal{I}_{\underline{123}} \cong B_3 \cong \mathcal{I}_{134}$

234

$\mathcal{I}_{12,234} = \left\{ \begin{matrix} 12 & & 23 \quad 24 \quad 34 \\ 1 \quad 2 & & 3 \quad\quad 4 \\ \phi & & \end{matrix} \right\} \geq \mathcal{I}_{12}$

**Defn** Given a poset $P$, define the lattice of order ideals $J(P)$ with inclusion as relation.

$P$

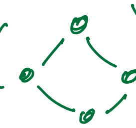


$J(P)$

lattice

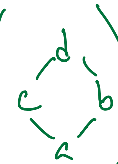
WARM UP :

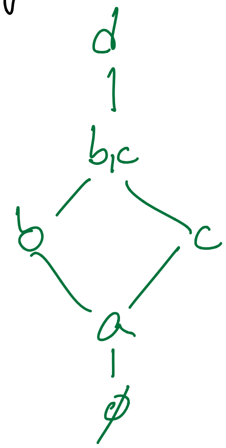
$\to J($  $)$

$\to J($  $)$

**Defn** An antichain is a subset $A \subseteq P$ s.t. $x, y$ incomparable $\forall\, x, y \in A$ distinct.

$J\left( \begin{matrix} c \\ | \\ b \\ | \\ a \end{matrix} \right) = \begin{matrix} \mathcal{I}_c = \{a,b,c\} \\ | \\ \mathcal{I}_b = \{a,b\} \\ | \\ \mathcal{I}_a = \{a\} \\ | \\ \mathcal{I}_\phi = \phi \end{matrix}$

$J($  $) =$




**Defn** A lattice $\mathcal{L}$ is distributive if

$$s \vee (t \wedge u) = (s \vee t) \wedge (s \vee u)$$
$$s \wedge (t \vee u) = (s \wedge u) \vee (s \wedge t)$$

$B_n$ is dist.
$B_n(q)$ is not.

**Theorem** $\mathcal{L}$ finite distributive lattice. Then $\exists!$ poset $P$ s.t. $\mathcal{L} \cong J(P)$.