MATH 532 DISCUSSION WED 02/09

$p(n) = \#$ partitions of n 1, 2, 3, 5, 7, 11, 15, 22, ...

$$P(x) = \sum_{n\geq 0} p(n) x^n = \prod_{i\geq 1} \left(\frac{1}{1-x^i}\right)$$

choose $(x^i)^k$ where i occurs k times in λ.

$$= \prod_{i\geq 1} (1 + x^i + x^{2i} + x^{3i} + \cdots)$$

$\lambda = (9^5, 5^{12}, 3^2, 1^3) \vdash 5\cdot 9 + 12\cdot 5 + 2\cdot 3 + 3\cdot 1 = 114$

$[x^{114}]$

$$\frac{1}{1-x^9} = (1 + x^9 + x^{18} + \cdots + x^{45} + \cdots)$$

$(x^9)^5$

$$\frac{1}{1-x^8} = (1 + x^8 + x^{16} + \cdots)$$

$(x^8)^0$

$(x^5)^{12}$

$$\frac{1}{1-x^5} = (1 + x^5 + x^{10} + x^{15} + \cdots + x^{60} + \cdots)$$

1. $q(n) = \#$ partitions of n into distinct parts

$$Q(x) = \sum_{n\geq 0} q(n) x^n$$

Ex. $(40, 36, 20, 9, 6, 2, 1)$

$$= \prod_{i\geq 1} (1 + x^i)$$

don't take i take i

2. $p_{odd}(n) = \#$ partitions of n into odd parts

$$P_{odd}(x) = \sum_{n\geq 0} p_{odd}(n) x^n$$

Ex. $(9^5, 5^{12}, 3^2, 1^3)$

$$= \prod_{\substack{i\geq 1 \\ i \text{ odd}}} \left(\frac{1}{1-x^i}\right)$$

as many as you like.

only get odd parts

Prop. $q(n) = p_{odd}(n)$.

$$Q(x) = \prod_{i \geq 1} (1 + x^i) \qquad P_{odd}(x) = \prod_{\substack{i \geq 1 \\ i \text{ odd}}} \left(\frac{1}{1 - x^i} \right)$$

$$1 + x^i = \frac{(1 - x^{2i})}{1 - x^i} \qquad \prod_{i \geq 1} \left(\frac{1}{1 - x^{2i-1}} \right) = \frac{\prod_{i \geq 1} 1 - x^{2i}}{\prod_{i \geq 1} 1 - x^i}$$

$$(1+q)(1-q) = (1-q^2)$$

$$Q(x) = \prod_{i \geq 1} (1 + x^i) = \prod_{i \geq 1} \left(\frac{1 - x^{2i}}{1 - x^i} \right) = \prod_{i \geq 1} \left(\frac{1}{1 - x^{2i-1}} \right) = P_{odd}(x).$$

$(9^5, 5^{12}, 3^2, 1^3)$ $\rightarrow$ write exponents in binary

P_{odd}

$9(1+4) + 5(4+8) + 3(2) + 1(1+2)$

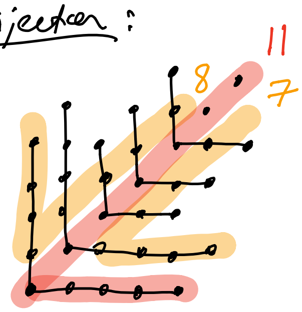
$9, 36, 20, 40, 6, 1, 2$

$(40, 36, 20, 9, 6, 2, 1)$

Q

<u>HAOYANG's Bijection:</u>

$\dfrac{9955511}{P_{odd}}$



$\dfrac{(11, 8, 7, 6, 3)}{Q}$