MATH 532 Discussion

A ambient set

$Y \subseteq S$ collection of properties.

$$f_=(\emptyset) = \sum_Y (-1)^{\#Y} f_\geq(Y)$$

exactly properties in $\emptyset$

at least properties of Y

Ex. $f_=(T) = \#\{\omega \in S_n \mid$ fixed pts of ω are exactly $T\}$
$\omega(i) = i$ iff $i \in T$

here.

$f_\geq(T) = \#\{\omega \in S_n \mid \omega(i) = i$ if $i \in T\}$

more allowed

easy to count directly.

$f_\geq(Y) = (n-\#Y)!$ If $\#\cancel{Y} = \#X$, same count.

$$\Rightarrow f_=(\emptyset) = \sum_{k=0}^{n} (-1)^k \binom{n}{k}(n-k)!$$

derangements

choose Y of size k

$f_\geq(Y)$.

$$f_=(\emptyset) + \sum_{\#Y \text{ odd}} f_\geq(Y) = \sum_{\#Y \text{ even}} f_\geq(Y)$$

$$\sigma : M \cup N \xrightarrow{\sim} N'$$

no props in S

$(x, \emptyset, \underline{\emptyset})$

(x, Y, Z)

$\in A$ props have. $\#Y$ odd

$Z \supseteq Y$ exact props x has.

(x', Y', Z')

$\in A$

desired props $\#Y'$ even

exact props of x'

FIXED PTS

$\sigma(x) = (x, \emptyset, \emptyset)$ if $x \in M$.

$\in N'$

$$\sigma(x, Y, Z) = \begin{cases} (x, Y - i, Z) & \text{if } \min Y = \min Z = i \\ (x, Y \cup i, Z) & \text{if } i = \min Z < \min Y \end{cases}$$

$\underline{\underline{S_3}}$

write the sets M, N, N' line up the map.

$M = \{231, 312\}$

	x	y	z	x'	y'	z'	
N	123	$\underline{123}$	$\underline{123}$	123	$\emptyset$	123	N'
	123	1	123	123	12	123	
	123	$\underline{2}$	123	123	13	123	
	123	3	123	123	23	123	
	132	1	1	132	$\emptyset$	1	
	213	3	3	213	$\emptyset$	3	
	321	2	2	321	$\emptyset$	2	
M	$2\underline{3}1$			231	$\emptyset$	$\emptyset$	
	$3\underline{1}2$			312	$\emptyset$	$\emptyset$	

$N \uplus N'$ $\circlearrowleft$ τ involution.

N ← neg, N' ← pos

τ involution on $N \cup N'$ satisfying:

- If $\tau(x) = y$ and $y \neq x$ then exactly one of x, y belongs to N (exactly one to N')
- If $\tau(x) = x$, then $x \in N^+$ (positive half)

$$wt(x) = \begin{cases} +1 & x \in N' \leftarrow \text{pos} \\ -1 & x \in N \leftarrow \text{neg} \end{cases}$$

$$\#\mathrm{Fix}(\tau) = \sum_{x \in X} wt(x)$$