

$L(w) = (3, 0, 4, 2, 3, 0, 0, 0, 0)$
 $w = 4, 1, 7, 5, 8, 2, 3, 6 \in S_9$



$D(w) = \{(i, j) \mid w_j > i\}$

Death rays at (i, w_i)

Rothe diagram
 Inversion diagram

$inv(w) = \{(i, j) \mid w_i > w_j\}$

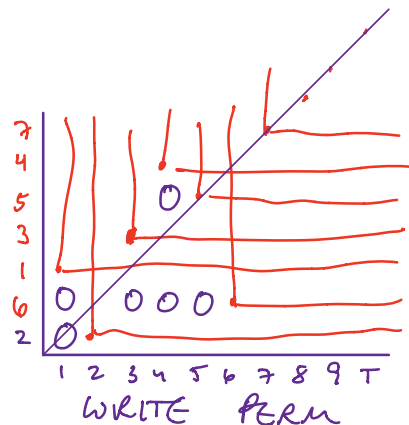
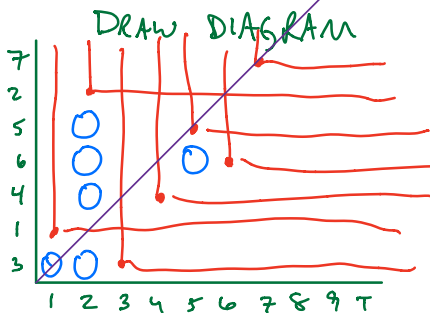
$L(w)_i = \# \text{ bubbles in row } i \text{ from bottom}$

Claim $inv(w) = \# \text{ bubbles.}$

Observations

- Sherron: not symmetric along $y=x$
- Rebel: death rays take leftmost non-bubble, no dead cell
- Bo: bubble stacks come at alternating spots
- Huang: same # above & below line in this ex.
- Xiao: # bubbles 1, 2, 3, 6 pattern? groups matter?
- Sam: ϕ must be a bubble!
- Taorui: pattern like this
- Saipad: death ray bubble patterns.
- John: 12 rows & 12 bubbles.

$w = 3, 1, 4, 6, 5, 2, 7$

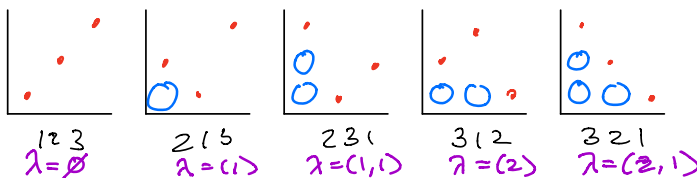


$u = 2, 6, 1, 3, 5, 4, 7$

Proposition $D(w) = D(w^{-1})$ Transpose & flip across $y=x$ $w_1^{-1} \dots w_j^{-1} \dots w_k^{-1}$

Defn w is 132-avoiding iff $\nexists i < j < k$ s.t. $w_i < w_k < w_j$

Theorem w is 132-avoiding iff $\exists \lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k$ s.t. $\forall i \geq 0$, its row of $D(w)$ is first λ_i cells



Corollary

$\sum_{w \in S_{132}(n)} q^{inv(w)} = \sum_{\lambda \vdash n} q^{|\lambda|}$