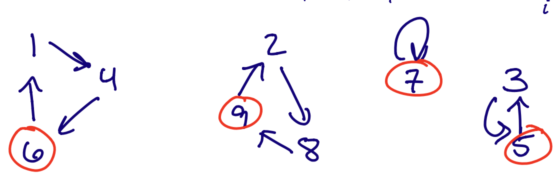
MATH 532 Discussion Monday 31/08

Office hours:
Tuesdays 1:30 – 2:30
& by appointment

Cycle type $(c_1\ c_2\ c_3 \dots c_n)$ $c_i = \#\ i\text{-cycles}$

$(1, 1, 2, 0, \dots 0)$ $\sum_i i \cdot c_i = n$



$(5\ 3)(6\ 1\ 4)(7)(9\ 2\ 8)$

$(2\ 3\ 1\ 3) \to \{\{2,3,1,3\}\} = (3,3,2,1)$

$3+3+2+1=9$

integer partitions of n $\lambda = (\lambda_1 \geq \dots \geq \lambda_k)$ $\lambda_i > 0$.

$\sum \lambda_i = n$

λ_i is a part of λ.

Valid cycle types for $\omega \in S_n$ are partitions of n.

List all partitions of $n = 4, 5, 6$

$p(n) =$ 1	2	3	5	7	11
(1)	(2)	(3)	(4)	(5)	(6)
	(1,1)	(2,1)	(31)	(41)	(51)
		(1,1,1)	(22)	(32)	(42)
			(211)	(311)	(411) ✓
			(1111)	(221)	(33)
				(2111)	(321) ✓
				(11111)	(3111)
					(222) ✓
					(2211)
					(21111)
					(111111)

$p(n) = 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, \dots$

$p(n) = \#$ partitions of n
$= \#\{\lambda \vdash n\}$

$p_k(n) = \#\{(\lambda_1 \geq \dots \geq \lambda_k > 0) \mid \sum_i \lambda_i = n\}$
$= \#$ partitions of n into k parts.

$P_3(6) = 3$

Find a recurrence relation for $P_k(n)$.

$P_1(n) = 1 \quad \forall n.$

$\underline{\text{end in } 1} \quad \uplus \quad \underline{\text{don't end in } 1}$

$(\lambda_1, \dots, \lambda_{k-1}, \cancel{1})$ $\quad$ $(\lambda_1 - 1, \dots, \lambda_k - 1)$ $\lambda_k \geq 2$

$$P_k(n) = P_{k-1}(n-1) + P_k(n-k)$$

$P_{\leq k}(n) = \#$ partitions of n into at most k parts.
$= P_1(n) + P_2(n) + P_3(n) + \dots + P_k(n)$

$P(n) = P_{\leq n}(n).$