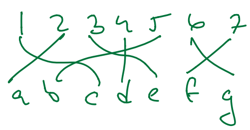
Defn A permutation of a finite set S of size n is a bijection $w : [n] \longrightarrow S$.

Ex. Let $S = \{a,b,c,d,e,f,g\}$. Then $n = 7$.

$w = (g,a,b,c,d,e,f)$ one line notation. compact, good for lots of things.

$w = (w(1), w(2), \dots, w(7))$



wiring diagram — great for inversions (sorting, braids)

$(acdf)(bg)(e)$ cycle notation — algebraic

Defn A permutation statistic is a map $\text{stat} : \text{Perm}(S) \to \mathbb{Z}$

Ex. A statistic on sets could be $f : 2^S \to \mathbb{Z}$

$f(x) = \# x$

2^S = collection of subsets of S

$\# 2^S = 2^{\# S}$

$$\sum_{x \in 2^S} 1 = 2^{\# S}$$

$x \subseteq S$

replace 1 with a statistic

$$\sum_{x \in 2^S} q^{f(x)} = \sum_{k=0}^{n} \binom{n}{k} q^k$$

Setting $q = 1$, gets back $2^{\# S}$

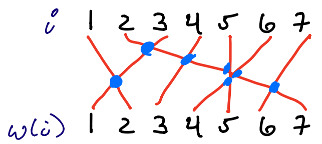
inversion number

$\text{inv}(w) = \#$ pairs $(i<j)$ s.t. $w(i) > w(j)$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$
$(2,7,1,3,5,4,6) : [7] \to [7]$

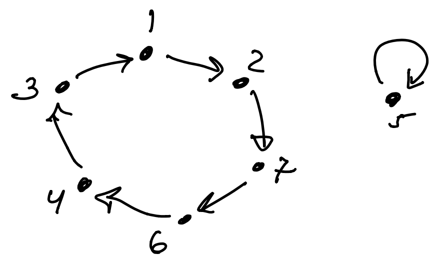
$1+5+1 = 7$

WIRING DIAGRAM



CYCLE NOTATION

$(1,2,7,6,4,3)(5)$



$i < j \Rightarrow i$ left of j

$w(i) > w(j) \Rightarrow w(j)$ left of $w(i)$

<u>least # of inversions?</u> $1\ 2\ 3 \cdots n$ has <u>0</u>.

<u>most # of inversions?</u> $n\ n{-}1 \cdots 3\ 2\ 1$ $\qquad \frac{n(n-1)}{2} = \binom{n}{2}$

<u>Theorem</u> inv(w) is the # of steps required to sort w using adjacent transpositions.

2 <u>7 1</u> 3 5 4 6
2 1 7 3 <u>5 4</u> 6
2 1 <u>7 3</u> 4 5 6
<u>2 1</u> 3 7 4 5 6
1 2 3 <u>7 4</u> 5 6
1 2 3 4 <u>7 5</u> 6
1 2 3 4 5 <u>7 6</u>
1 2 3 4 5 6 7