Background definitions

Logical consequence
A conclusion is a logical consequence of some premises if the conclusion follows in virtue of the logical form of the premises and the conclusion.

Logical form
The logical form of a sentence is what you get when you abstract from the content of the sentence and keep just the sentence's logical constants.

Validity
An argument is valid when its conclusion is a logical consequence of its premises.

Warm up

Identify whether the following arguments are valid. If they aren't, propose a way to fix them so that they become valid.

Argument 1: *valid*

1. Sam is tall.
C. So, Sam is tall.

Argument 2: *valid*

C. Either Sam is tall or Sam is not tall.

Argument 3: *invalid*

1. Brian says Sam is tall.
C. So, Sam is tall.

A possible fix:

1. Brian says Sam is tall.
2. If Brian says Sam is tall, Sam is tall.
C. So, Sam is tall.

Argument 4: *valid*

1. If there are 50 people in the room, then God exists.
2. There are 50 people in this room.
3. So, God exists.

Fallacy of equivocation.

Example 1:

You see that Sam is near a financial institution that stores money. So you reason as follows.

1. If Sam is near a financial institution that stores money, then he is near a bank.
2. If Sam is near a bank, then he is near a river.
3. Sam is near a financial institution that stores money.
4. So, Sam is near a river.

What's gone wrong?

The argument equivocates between two meanings of the word *bank*.

The problem

Sentences don't always have their logical form on their sleeves.

That's a problem. It makes it hard to tell what's a logical consequence of what.

So: let's build a logical language whose sentences are never ambiguous in this way.

Another problem

We want to construct a set of sentences of the following form:

$(A \land B)$; $\neg\neg\neg\neg A$; $((C \supset D) \supset E)$ and so on.

Let's call this set PL.

But there's way too many such sentences to simply list them.

So: How do we articulate what sentences are members of PL and what sentences aren't members of PL?

Definition for what sentences are members of PL

The solution is to make use of a recursive definition.

Rule 0:

Capital letters A, B, C,... are sentences of PL. We will call them *atomic sentences*.

Rule 1:

If φ is a sentence of PL, then $\neg\varphi$ is a sentence of PL.

Rule 2:

If φ is a sentence of PL and ψ is a sentence of PL, then:

+ $(\varphi \vee \psi)$ is a sentence of PL;
+ $(\varphi \supset \psi)$ is a sentence of PL;
+ $(\varphi \wedge \psi)$ is a sentence of PL; and
+ $(\varphi \equiv \psi)$ is a sentence of PL.

Rule 3:

Nothing else is a sentence of PL

Exercises

Prove that the following are sentences of PL.

1. $(A \vee B)$

Example proof:

(i) A is a PL-sentence. (rule 0)

(ii) B is a PL-sentence. (rule 0)

(iii) $(A \vee B)$ is a PL-sentence. (i, ii, rule 2)

2. $(A \wedge B) \supset (C \equiv D)$
3. $\neg\neg\neg(A \vee \neg B)$
4. $(C \vee (D \wedge E))$

The *major operator* of a sentence of PL is the operator that you apply last when you construct the sentence using PL's construction rules.

In the following sentences, identify the major operator of the sentence.

5. $(A \vee B)$
6. $(A \wedge B) \supset (C \equiv D)$
7. $\neg\neg\neg(A \vee \neg B)$
8. $(C \vee (D \wedge E))$

Induction

Suppose you encounter a long line of men.

And suppose you know:

1. The first man in the line has a hat.
2. Given any arbitrary man in the line, if they have a hat, then the man after them also has a hat.

Then you can infer that every man in the line has a hat.

We can use a similar style of argument to prove things about sentences as PL.

If you can:

1. Show that every atomic sentence has a property F;
2. And also show that:
   a. for any two sentences of PL φ and ψ:
   b. if φ has F and ψ has F, then
      i. $\neg \varphi$ has F
      ii. $(\varphi \vee \psi)$ has F,
      iii. $(\varphi \wedge \psi)$ has F,
      iv. $(\varphi \supset \psi)$ has F,
      v. and $(\varphi \equiv \psi)$ has F;

Then you have just shown that every sentence of PL has property F.

Challenge:

Prove that every sentence of PL has the same number of parentheses on its left as on its right.