Lecture Notes 5 for 9/10

Sentential Operators

Let's define a *sentential operator* as follows:

+ A sentential operator is a fragment of a sentence with some blanks, such that when you input declarative sentences into those blanks, you get a new declarative sentence.

Some examples of sentential operators include:

+ It is not the case that __________
+ Antonio believes that __________
+ Either ______ or _________

Truth-functionality

Some sentential operators are *truth-functional*. Others aren't.

Consider for instance:

+ *it is not the case that* ______

If you know whether the inputted sentence is true, then you know whether the new sentence generated is true, too.

A sentential operator is truth functional just in case the truth value of any new sentence generated by plugging in sentences into the operator's blank(s) is a function fo the truth value(s) of the imputed sentences.

But other sentential operators aren't truth-functional. For example.

+ Antonio believes that __________

There are some true sentences such that if you plug them into *Antonio believes* _______ you get a new truth.

But there are other truths that Antonio doesn't believe. In which case if you plug them in you get a falsehood.

The semantics of logical operators in propositional logic

One way of understanding PL is as the study of truth-functional operators.

A big stipulation we will be making is that each of the operators in our language is truth-functional.

Because of this, it is possible to represent the meaning of each of our operators in the form of a truth-table.

1. Each operator can be understood as a *function* from truth value input(s) to a truth value output.
2. One way of spelling out this function is via a truth-table.

The definitions.

Let p and q be variables ranging over sentences of PL.

$\neg$p is true iff p is false..

p	$\neg$p
T	F
F	T

(p $\vee$ q) is true iff p is true or q is true.

p	q	(p $\vee$ q)
T	T	T
T	F	T
F	T	T
F	F	F

$(p \wedge q)$ is true iff p is true and q is true.

p	q	$(p \wedge q)$
T	T	T
T	F	F
F	T	F
F	F	F

$(p \equiv q)$ is true iff p and q are both true or p and q are both false.

p	q	$(p \equiv q)$
T	T	T
T	F	F
F	T	F
F	F	T

$(p \supset q)$ is true iff p is false or q is true.

$(p \supset q)$ is true iff it's not the case that p is true but q is false.

p	q	$(p \supset q)$
T	T	T
T	F	F
F	T	T
F	F	T

Some problems

Let A, B, and C be true and X, Y, and Z be false.

Compute the truth value of each of the following sentences.

1. $\neg A \vee \neg B$

False

2. $(A \wedge X) \vee (B \wedge Y)$

False

3. $\neg B \supset \neg(X \vee Y)$

True

4. $(A \equiv B) \equiv (X \equiv Y)$

True

An interpretation of PL

An i*nterpretation* of PL is a function from atomic sentences of PL to either true or false.

PL Valuation

For any interpretation of PL I, a valuation for PL V_I is a function from sentences of PL to true or false such that:

For any atomic sentence α and PL-sentences φ and ψ:

1. $V_I(\alpha) = I(\alpha)$
2. $V_I(\varphi)$ = False iff $V_I(\neg \varphi)$ is True;
3. $V_I(\varphi \vee \psi)$ = True iff $V_I(\varphi)$ is true or $V(\psi)$ is true;
4. $V_I(\varphi \wedge \psi)$ = True iff $V_I(\varphi)$ is true and $V_I(\psi)$ is true;
5. $V_I(\varphi \equiv \psi)$ = True iff $V_I(\varphi)$ and $V_I(\psi)$ are both true or both false;
6. $V(\varphi \supset \psi)$ = True iff $V_I(\varphi)$ is false or $V_I(\psi)$ is true.

Tautology

A sentence of PL is a tautology iff on any interpretation of PL, the value of the sentence is true.

Or: a sentence of PL is a tautology iff for any interpretation I of PL, $V_I(\varphi)$ = True.

Contradiction

A sentence of PL is a contradiction iff on any interpretation of PL, the value of the sentence is false.

Or: a sentence of PL, φ, is a contradiction iff for any interpretation I of PL, $V_I(\varphi)$ = True.

Contingency

A sentence of PL is a contingency iff there are interpretations on which the sentence is true and interpretation on which the sentence is false.

Or: a sentence of PL, φ, is a contingency iff there is an interpretation I1 on which $V_{I1}(\varphi)$ = True and there is also an interpretation I2 on which $V_{I2}(\varphi)$ = False.

Some more problems

Compute truth tables for these sentences of PL. Identify whether each sentence is a tautology, a contradiction, or a contingency.

1. ¬(A ≡ ¬B) ≡ ¬(A ≡ B)

A	B	¬	(A	≡	¬	B)	**≡**	¬	(A	≡	B)
T	T	T	T	F	F	T	**F**	F	T	T	T
T	F	F	T	T	T	F	**F**	T	T	F	F
F	T	F	F	T	F	T	**F**	T	F	F	T
F	F	T	F	F	T	F	**F**	F	F	T	F

Contradiction

2. A ⊃ (B ⊃ A)

A	B	A	⊃	(B	⊃	A)
T	T	T	**T**	T	T	T
T	F	T	**T**	F	T	T
F	T	F	**T**	T	F	F
F	F	F	**T**	F	T	F

Tautology

3. (¬A ∧ B) ∨ A

A	B	C	(¬	A	∧	B)	∨	C
T	T	T	F	T	F	T	**T**	T
T	T	F	F	T	F	T	**F**	F
T	F	T	F	T	F	F	**T**	T
T	F	F	F	T	F	F	**F**	F
F	T	T	T	F	T	T	**T**	T
F	T	F	T	F	T	T	**T**	F
F	F	T	T	F	F	F	**T**	T
F	F	F	T	F	F	F	**F**	F