

Lecture Notes 7 for 9/19

The basic idea

In this class we are investigating what it means for a conclusion to logically follow from some premises.

One way of thinking about what that means has to do with interpretations.

A conclusion logically follows from some premises, when there is no interpretation where the premises are true but the conclusion is false.

Today we will be exploring a second way of thinking about things. On this second way, a conclusion is a logical consequence of some premises, when there is a derivation from the premises to the conclusion which uses nothing but rock solid inference rules.

Inference rules

To start, I introduce 8 inference rules.

Modus Ponens (MP)

1. $p \supset q$
2. p
3. $\therefore q$

Modus Tollens (MT)

1. $p \supset q$
2. $\neg q$
3. $\therefore \neg p$

Hypothetical Syllogism (HS)

1. $p \supset q$
2. $q \supset r$
3. $\therefore p \supset r$

Simplification (Simp)

1. $p \wedge q$
2. $\therefore p$
1. $p \wedge q$
2. $\therefore q$

Conjunction (Conj)

1. p
2. q
3. $\therefore p \wedge q$

Disjunctive Syllogism (D.S.)

1. $p \vee q$
2. $\neg p$
3. $\therefore q$
1. $p \vee q$
2. $\neg q$
3. $\therefore p$

Addition (Add.)

1. p
2. $\therefore p \vee q$
1. q
2. $\therefore p \vee q$

Dilemma (Dil.)

1. $p \supset q$
2. $r \supset s$
3. $p \vee r$
4. $\therefore q \vee s$

Rule Citation

Problem 1:

1. $(\neg A \vee \neg B) \vee C$
2. $\neg C$
3. So: $\neg A \vee \neg B$

Problem 2:

4. $\neg(A \vee B) \supset (\neg A \vee \neg B)$
5. $\neg(\neg A \vee \neg B)$
6. So: $\neg\neg(A \vee B)$

Problem 3:

7. $(\neg B \vee \neg C) \supset \neg D$
8. $\neg A \supset (\neg B \vee \neg C)$
9. $\neg A \supset \neg D$

Sample derivation

The task: derive M from the following premises.

1	$(W \wedge S)$	Premise
2	$((W \wedge S) \wedge (R \vee F)) \supset (P \vee M)$	Premise
3	$(P \supset B)$	Premise
4	$\neg B \wedge L$	Premise
5	$\neg R \wedge F$	Premise
6	F	Simp 5
7	$R \vee F$	Add 6
8	$(W \wedge S) \wedge (R \vee F)$	Conj 1, 7
9	$(P \vee M)$	MP 2, 8
10	$\neg B$	Simp 4
11	$\neg P$	MT 3, 10
12	M	DS 9, 11

Sample derivation

The task: derive $\neg P \vee (\neg F \wedge \neg H)$

1	$(A \vee \neg B) \supset \neg(C \vee \neg D)$	Premise
2	$(F \wedge \neg H) \vee A$	Premise
3	$\neg(F \wedge \neg H) \wedge (\neg O \wedge \neg P)$	Premise
4	$(\neg Z \wedge X) \supset O$	Premise
5	$\neg(F \wedge \neg H)$	Simp 3
6	A	DS 2, 5
7	$\neg O \wedge \neg P$	Simp 3
8	$\neg O$	Simp 7
9	$\neg(\neg Z \wedge X)$	MT 4, 8
10	$A \vee \neg B$	Add 6

11	$\neg P$	Simp 7
12	$\neg(C \vee \neg D)$	MP 10,
13	$\neg P \vee (\neg F \wedge \neg H)$	

Proofs

Derive $\neg F$ from:

1. $(C \wedge D) \supset \neg F$ Pr
2. $(A \supset C) \wedge (B \supset D)$ Pr
3. $(A \wedge B)$ Pr
4. A Simp 3
5. $A \supset C$ Simp 2
6. C MP 4, 5
7. B Simp 3
8. $B \supset D$ Simp 2
9. D MP 7, 8
10. $C \wedge D$ Conj 6, 9
11. $\neg F$ MP 1, 10

Derive C from:

1. $(A \supset B) \supset (B \supset C)$ Pr
2. $(A \wedge B) \supset (A \supset B)$ Pr
3. $(\neg F \supset A) \wedge (\neg D \supset B)$ Pr
4. $(F \supset T) \wedge (D \supset S)$ Pr
5. $\neg T \wedge \neg S$ Pr
6. $\neg T$ Simp 5
7. $F \supset T$ Simp 4
8. $\neg F$ MT 6, 7
9. $\neg S$ Simp 5
10. $D \supset S$ Simp 4
11. $\neg D$ MT 9, 10
12. $\neg F \supset A$ Simp 3
13. A MP 8, 12
14. $\neg D \supset B$ Simp 3
15. B MP 11, 14
16. $A \wedge B$ Conj 13, 15
17. $A \supset B$ MP 2, 16
18. $B \supset C$ MP 1, 17
19. C MP 15, 18