Lecture Notes 8 for 9/24

Inference rules

1. MP
2. MT
3. Add
4. Simp
5. HS
6. Dil
7. DS
8. Conj

Warm up proofs.

Proof 1

1. ¬A ⊃ ¬B
2. A ⊃ C
3. Z ⊃ W
4. ¬C ∧ ¬W
5. Derive: ¬B ∨ W

Proof 2

1. P ⊃ (Q ⊃ (R ∨ S))
2. P ∧ Q
3. S ⊃ T
4. ¬T ∨ ¬W
5. ¬¬W
6. Derive: R

Replacement Rules

1. DN
2. Dup
3. Comm
4. Assoc
5. Contrap
6. BE
7. Dist
8. DeM
9. CE
10. Exp

Justify Each Step. Derive ¬(A ∨ H)

1	¬A ≡ ¬(B ≡ C)	Pr
2	¬(D ∨ C) ∨ ¬B	Pr
3	¬B ⊃ ¬(E ⊃ F)	Pr
4	¬(E ∨ H)	Pr
5	¬E ∧ ¬H	DeM 4
6	¬H	Simp 5
7	¬E	Simp 5
8	¬E ∨ F	Add 7
9	E ⊃ F	CE
10	(E ⊃ F) ⊃ B	Contrap 3
11	B	MP 9, 10
12	¬¬B	DN 11
13	¬(D ∨ C)	DS 2, 12
14	¬D ∧ ¬C	DeM 13
15	¬C	Simp 14
16	¬¬B ∧ ¬C	Conj 12, 15
17	¬(¬B ∨ C)	DeM 16
18	¬(B ⊃ C)	CE 17
19	¬(B ⊃ C) ∨ ¬(C ⊃ B)	Add 18
20	¬((B ⊃ C) ∧ (C ⊃ B))	DeM 19
21	¬(B ≡ C)	BE 20
22	(¬A ⊃ ¬(B ≡ C)) ∧ (¬(B ≡ C) ⊃ ¬A)	BE 1
23	¬(B ≡ C) ⊃ ¬A	Simp 22
24	¬A	MP 23, 21
25	¬A ∧ ¬H	Conj 6, 24
26	¬(A ∨ H)	DeM 25

Derivations:

1. (A ∨ B) ⊃ ¬C
2. D ⊃ (C ∨ C)
3. (F ∧ (E ∧ D))
4. Derive: ¬(A ∨ B)

1	(A ∨ B) ⊃ ¬C	Pr
2	D ⊃ (C ∨ C)	Pr
3	(F ∧ (E ∧ D))	Pr
4	(F ∧ E) ∧ D	Assoc 3
5	D	Simp 4
6	C ∨ C	MP 5, 2
7	C	Dup 6
8	¬¬C	DN 7
9	¬(A ∨ B)	MT 1, 8

Another one:

1. ¬(S ∧ T) ⊃ W
2. W ⊃ ¬(A ∨ B)
3. A
4. Derive: S

1	¬(S ∧ T) ⊃ W	Pr
2	W ⊃ ¬(A ∨ B)	Pr
3	A	Pr
4	A ∨ B	Add 3
5	¬(S ∧ T) ⊃ ¬(A ∨ B)	HS 1, 2
6	(A ∨ B) ⊃ (S ∧ T)	Contrap 5
7	S ∧ T	MP 4, 6
8	S	Simp 7

Another one:

1. ¬B
2. Derive: ¬(A ∧ B)

1	¬B	Pr
2	¬A ∨ ¬B	Add 1
3	¬(A ∧ B)	DeM 2

Another one:

1. ¬A ⊃ ¬B
2. B
3. Derive A

1	¬A ⊃ ¬B	Pr
2	B	Pr
3	B ⊃ A	Contrap 1
5	A	MP 2,3