Grammar of PL:

Rule 0:
Capital letters A, B, C,... are sentences of PL. We will call them *atomic sentences*.

Rule 1:
If φ is a sentence of PL, then ¬φ is a sentence of PL.

Rule 2:
If φ is a sentence of PL and ψ is a sentence of PL, then:

+ (φ ∨ ψ) is a sentence of PL;
+ (φ ⊃ ψ) is a sentence of PL;
+ (φ ∧ ψ) is a sentence of PL; and
+ (φ ≡ ψ) is a sentence of PL.

Rule 3:
Nothing else is a sentence of PL

Warm Up

Identify whether the following arguments are valid.

Argument 1:

1. Either coffee is expensive or clothing is expensive.
2. Clothing is expensive.
3. So, coffee is expensive.

Not valid.

Argument 2:

1. Either coffee is expensive or clothing is expensive.
2. Clothing is not expensive.
3. So coffee is expensive.

Valid.

Identify whether the following strings of characters are sentences of PL. If it is a PL-sentence, prove it.

1. ¬(A ∧ (¬B)) Not a sentence of PL.
2. (¬¬A ∨ ¬¬C) Yes, it's a PL-sentence.

(i) A and C are PL-sentence. (Rule 0)

(ii) ¬A is a PL-sentence. (Rule 1, i)

(iii) ¬¬A is a PL-sentence. (Rule 1, ii)

(iv) ¬C is a PL-sentence. (Rule 1, i)

(v) ¬¬C is a PL-sentence. (Rule 1, iv)

(vi) (¬¬A ∨ ¬¬C) is a PL-sentence. (Rule 2, iii, v).

Convention

It's ok to omit outer parentheses when writing a PL-sentence.

Translations

Translate the following sentences of English into sentences of PL. Be sure to preserve all the logical constants in the English language sentence.

Translation Key:

D = Sam is a donkey

H = Kevin is a horse

O = Jingyi is an ostrich.

Straightforward translations

1. Sam is a donkey and Kevin is a horse.
   (D ∧ H)
2. Either Sam is a donkey or Kevin is a horse.
   (D ∨ H)
3. It's not the case that Jingyi is an ostrich.
   ¬O

A bit harder translations

4. Either Sam is a donkey, or Kevin is a horse and Jingyi is an ostrich.
   (D ∨ (H ∧ O))
5. Either Sam is a donkey or Kevin is a horse; and Jingyi is an ostrich.
   ((D ∨ H) ∧ O)

Conditionals

6. If Sam is a donkey, then Kevin is a horse.
   (D ⊃ H)

7. Kevin is a horse, if Sam is a donkey.
   (D ⊃ H)
8. Sam is a donkey only if Kevin is a horse.
   (D ⊃ H)
9. Only if Kevin is a horse, is Sam a donkey.
   (D ⊃ H)
10. Sam is a donkey if and only if Kevin is a horse.
    (D ≡ H)

Nonstandard

11. Jingyi is an ostrich but Kevin is a donkey.
    (O ∧ D)

Long and complex

12. Either Jingyi is an ostrich, or Sam is a donkey only if Kevin is a horse.
    (O ∨ (D ⊃ H))

13. If Kevin isn't a horse, then Sam is a donkey, if Jingyi isn't an ostrich.
    (¬O ⊃ (¬H ⊃ D))