Hi all, I thought I'd show you guys an example of how to prove something about every sentence of PL using induction.

Hopefully this will be helpful for those of you who want to tackle the challenge problem.

**A sample proof**

I want to prove (1).

1. For every sentence x of PL, the number of parentheses in x is twice the number of two place operators in x.

I prove this by induction.

First, the base case.

Observe that every atomic sentence of PL has 0 parentheses and 0 two place operators. Hence every atomic sentence of PL is such that it has twice the number of parentheses as two place operators. 2*0 = 0.

Next, the inductive step.

Suppose that φ and ψ are such that they both have twice the number parentheses as two place operators.

I want to show that, if φ and ψ are like that, then the same holds for:

+ ¬ φ
+ (φ ∨ ψ)
+ (φ ∧ ψ)
+ (φ ⊃ ψ)
+ (φ ≡ ψ)

If I can do that, then I will have shown that every sentence of PL has twice as many parentheses as two place operators.

Let's start with ¬ φ.

By inductive hypothesis, we know that φ has twice the number of parentheses as two place operators.

But now observe that if we simply add a negation this doesn't change either the number of two-place operators or the number of parentheses.

Hence if φ has twice the number of parentheses as two place operators, then so too does ¬ φ.

Next, let's consider (φ ∧ ψ), (φ ∨ ψ), (φ ⊃ ψ), and (φ ≡ ψ) as a group.

By hypothesis both φ and ψ have twice the number of parentheses as two place operators.

It follows that φ has some number *n* two place operators and 2*n* parentheses; and ψ has some number *m* two place operators and *2m* parentheses.

Now observe that in each case under consideration – φ ∧ ψ), (φ ∨ ψ), (φ ⊃ ψ), and (φ ≡ ψ) – a *single* new two place operator is added and *two* new parentheses are added.

So the total number of two place operators in each sentence is equal to: *The number of two place operators in φ* plus *the number of two place operators in ψ* plus *1*. Or n + m + 1, for short.

Meanwhile, the total number of parentheses in each sentence is equal to this: The number of parentheses in φ plus the number of parentheses in ψ plus 2. Or 2n + 2m + 2, for short.

But now observe that 2n +2m +2 is twice n + m + 1.

Hence (φ ∧ ψ), (φ ∨ ψ), (φ ⊃ ψ), and (φ ≡ ψ) each have twice the number of parentheses as two place operators.

Hence the inductive step: if two sentence of PL φ and ψ are such that they have twice the number of parentheses as two place operators, then this holds for ¬ φ, (φ ∨ ψ), (φ ∧ ψ), (φ ⊃ ψ), and (φ ≡ ψ), also.

Hence my conclusion: every sentence of PL has twice as many parentheses as two place operators.