Lecture Notes 6 for 9/12

Warm Up

Let's calculate the truth table for (¬A ⊃ B) ⊃ C.

Kinds of Sentence

Tautology: true on every interpretation.

E.g. (A ∨ ¬A)

Contradiction: false on every interpretation.

E.g. (A ∧ ¬A)

Contingency: there is an interpretation where the sentence is true and an interpretation where the sentence is false.

E.g. (A)

What does it mean for a sentence to be true on an interpretation?

An *interpretation* is an assignment of truth values to atomic sentences.

The problem is that if all we have is an interpretation then we don't yet know the value of sentences of PL that aren't atomic sentences.

So we need to say that the *value of a sentence φ, given an interpretation I* is given by the function $V_I(\varphi)$ which maps every sentence of PL to a truth value.

The function V_I is defined as follows.

1. If α is an atomic sentence, then $V_I(\varphi) = I(\varphi)$.

So if I interprets A as true and B as false, then $V_I(A)$ = True and $V_I(B)$ = false.

Then we give the following rules for negation.

2. If $V_I(\varphi)$ = true, then $V_I(\neg\ \varphi)$ = false.
3. If $V_I(\varphi)$ = false, then $V_I(\neg\ \varphi)$ = true.

Now we can say not just what the value of A is, but also what the value of ¬A is and what the value of ¬¬A is.

And we give the following rules for disjunction.

4. If $V_I(\varphi)$ = True or $V_I(\psi)$ = True, then $V_I(\varphi \vee \psi)$ = True.
5. Otherwise, $V_I(\varphi \vee \psi)$ = False.

And we give the following rules for conjunction.

6. If $V_I(\varphi)$ = True and $V_I(\psi)$ = True, then $V_I(\varphi \vee \psi)$ = True.
7. Otherwise, $V_I(\varphi \vee \psi)$ = False

And the following rules for the biconditional.

8. If $V_I(\varphi) = V_I(\psi)$, then $V_I(\varphi \equiv \psi)$ = True.
9. Otherwise, $V_I(\varphi \equiv \psi)$ = False.

And the following rules for the horseshoe.

10. If $V_I(\varphi)$ = True and $V_I(\psi)$ = False, then $V_I(\varphi \supset \psi)$ = False.
11. Otherwise, $V_I(\varphi \supset \psi)$ = True.

Equivalence

Two sentences are *equivalent* if they have the same truth value on every interpretation.

+ E.g (¬A ⊃ A) and A

Consistency and inconsistency

Let's now talk about some properties that a bunch of sentences of PL can have.

A set of sentences is *consistent* if there is an interpretation on which each of the sentences is true.

+ E.g. {A, B}.

A set of sentences is *inconsistent* if there is no interpretation on which each of the sentences is true.

+ E.g. {A, ¬B, A ⊃ B}.

True or false:

1. A set of sentences that features a contradiction is always inconsistent. **True**
2. A set of sentences that features a tautology is always consistent. **False**

3. If a set of sentences is consistent, then the sentences are all true. **False**
4. If the antecedent of a conditional is a contradiction, then the conditional sentence is a contradiction, too. **False**

Semantic validity

Recall that this class is principally interested in the logical consequence relation.

The logical consequence relation is a relation that obtains between a set of premises and a conclusion, where somehow the conclusion follows from the premises, in virtue of the logical form of the premises and the conclusion.

We're now in a position to make this precise:

Logical Consequence:

\+ A conclusion is a logical consequence of some premises, when there is no interpretation on which the premises are true but the conclusion is false.

And recall that:

Valid argument:

\+ An argument is valid if its conclusion is a logical consequence of its premises.

Some examples.

1. $A \supset B, A, \therefore B$
2. $A \vee B, \neg A, \therefore B$
3. $A, \therefore A \vee B$
4. $A \therefore \neg\neg A$

Some more examples.

5. $A, \therefore B$
6. $A \supset B, B, \therefore A$

An interpretation is a **counterexample** to an argument's validity, if it is an interpretation on which the premises of the argument are true, but the conclusion of the argument is false.

Tricky examples.

7. $(A \wedge \neg A) \therefore C$
8. $A, \therefore (B \vee \neg B)$

True or false:

1. An argument that features a contradiction as a premise is never valid. **False**
2. An argument that features a tautology as a conclusion is always valid. **True**
3. If the premises of an argument are inconsistent, then the argument is valid. **True**
4. If one of the premises of an argument is also the argument's conclusion, then the argument is valid. **True**