1. Definition of a sentence of PL

Rule 0: All capital English letters on their own are sentences of PL.

Rule 1: If φ is a PL-sentence, then $\neg \varphi$ is a PL-sentence.

Rule 2: If φ and ψ are PL-sentences, then:

+ $(\varphi \wedge \psi)$ is a PL-sentence
+ $(\varphi \vee \psi)$ is a PL-sentence
+ $(\varphi \supset \psi)$ is a PL-sentence
+ $(\varphi \equiv \psi)$ is a PL-sentence

Rule 3: Nothing else is a PL-sentence.

2. Truth table definitions of the logical operators

p	q	¬ p	(p ∧ q)	(p ∨ q)	(p ⊃ q)	(p ≡ q)
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

3. Inference Rules

MP p ⊃ q p ∴ q	MT p ⊃ q ¬q ∴ ¬p	HS p ⊃ q q ⊃ r ∴ p ⊃ r	Simp p ∧ q ∴ p	Simp p ∧ q ∴ q	Conj p q ∴ p ∧ q
Dil p ⊃ q r ⊃ s p ∨ r ∴ q ∨ s	DS p ∨ q ¬q ∴ p	DS p ∨ q ¬p ∴ q	Add p ∴ p ∨ q	Add q ∴ q ∨ p	

4. Replacement Rules

Dup p :: (p ∨ p) P :: (p · p)	Comm (p ∨ q) :: (q ∨ p)	Assoc ((p ∨ q) ∨ r) :: (p ∨ (q ∨ r)) ((p ∧ q) ∧ r) :: (p ∧ (q ∧ r))
CE (p ⊃ q) :: ¬p ∨ q	DeM ¬(p ∨ q) :: (¬p ∧ ¬q) ¬(p ∧ q) :: (¬p ∨ ¬q)	BE (p ≡ q) :: ((p ⊃ q) ∧ (q ⊃ p))
DN p :: ¬¬p	Contrap (p ⊃ q) :: (¬q ⊃ ¬p)	Dist (p · (q ∨ r)) :: ((p ∧ q) ∨ (p ∧ r)) (p ∨ (q ∧ r)) :: ((p ∨ q) ∧ (p ∨ r))
		Exp ((p ∧ q) ⊃ r) :: (p ⊃ (q ⊃ r))

5. Condition Proof and Indirect Proof

Conditional Proof

```
n       → p          Assp for CP
          .
          .
          .
m         q
m+1       p ⊃ q      CP n-m
```

Indirect Proof

```
n       → p          Assp for IP
          .
          .
          .
m         q ∧ ¬q
m+1       ¬p         IP n-m
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