

DC3 Algorithm Sketch

- 1 Construct the suffix array of a sample of the suffixes. In the sample, we include the suffixes starting at positions $i \bmod 3 \neq 0$. We recursively find the suffix array of a string of two-thirds length of the original string.
- 2 Construct the suffix array of the remaining suffixes using the result of the first step.
- 3 Merge the two suffix arrays into one using comparison-based merging.

DC3 Step 1: Construct the sample.

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Let S_i denote a suffix starting at index i in T . Let $C = B_1 \cup B_2$ be the set of sample start indices and S_C is the set of sample suffixes.

$$C = \{1, 4, 7, 10, 2, 5, 8, 11\}$$

$$S_C = \{S_1, S_4, S_7 \dots S_8, S_{11}\}.$$

DC3 Step 2: Construct R to sort sample suffixes.

Recall that $T = yabbadabbado$

2. Construct a new string R to sort the sample suffixes.

Let t_i be the i -th element of T . For $k = 1, 2$, we can construct the strings

$$R_k = [t_k t_{k+1} t_{k+2}][t_{k+3} t_{k+4} t_{k+5}] \dots [t_{\max B_k} t_{\max B_k + 1} t_{\max B_k + 2}].$$

What do R_1 and R_2 look like?

$$R_1 = [abb][ada][bba][do0] \text{ and } R_2 = [bba][dab][bad][o00].$$

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$$R = [abb][ada][bba][do0][bba][dab][bad][o00]$$

The nonempty suffixes of R correspond to S_C of sample suffixes. By sorting the suffixes of R , we get the order of the sample suffixes S_C .

DC3 Step 3: Sort the characters of R

Recall that $T = \text{yabbadabbado}$.

3. Sort the suffixes of R . First, radix sort the *characters* of R (the triples $[t_i t_{i+1} t_{i+2}]$) and rename them with their ranks to obtain a new string R' .

$$R = [\text{abb}][\text{ada}][\text{bba}][\text{do0}][\text{bba}][\text{dab}][\text{bad}][\text{o00}]$$

Rank	Index in R	Character
1	0	abb
2	1	ada
3	6	bad
4	2	bba
4	4	bba
5	5	dab
6	3	do0
7	7	o00

Index in R	R' (Rank)
0	1
1	2
2	4
3	6
4	4
5	5
6	3
7	7

DC3 Step 4: Sort the suffixes of R' (if needed).

Recall that $T = \text{yabbadabbado}$, and $R' = [1, 2, 4, 6, 4, 5, 3, 7]$.

4. If any of the characters of R are the same, recursively sort the suffixes of R' .

Rank	Index in R'	Suffix
1	8	\$
2	0	12464537\$
3	1	2464537\$
4	6	37\$
5	4	4537\$
6	2	464537\$
7	5	537\$
8	3	64537\$
9	7	7\$

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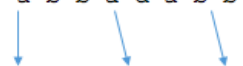
But how does this relate to the suffixes of the original string T ?

DC3 Step 4: Sort the suffixes of R' (if needed).

We can write the correspondence between start indices of the suffixes R' to the start indices of T .

$T = y \ a \ b \ b \ a \ d \ a \ b \ b \ a \ d \ o$

$R = [a \ b \ b] [a \ d \ a] [b \ b \ a] [d \ o \ 0] \dots$



Start Index of Suffix in R'	Start Index of Suffix in T
0	1
1	4
2	7
3	10
4	2
5	5
6	8
7	11

DC3 Step 5: Use sorted order of R' to sort the sample suffixes of T

5. Combining the results in the last two tables, we see that we can assign a rank to each suffix in S_C .

Suffix Start Index in T	Rank of Suffix
0	x
1	1
2	4
3	x
4	2
5	6
6	x
7	5
8	3
9	x
10	7
11	8
12	x
13	0
14	0

DC3 Step 6: Sort non-sample suffixes

6. The non-sample suffixes are the suffixes with start indices in B_0 . We represent each of these suffixes S_i by a tuple, $(t_i, \text{rank}(S_{i+1}))$.

Start Index of Suffix in T	Tuple Representation
0	(y, 1)
3	(b, 2)
6	(a, 5)
9	(a, 7)
12	(0, 0)

We can compare these suffixes as follows

$$S_i \leq S_j \iff (t_i, \text{rank}(S_{i+1})) \leq (t_j, \text{rank}(S_{j+1})).$$

Radix-sorting the tuples gives us an ordering of the non-sample suffixes.

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Radix-sorting the tuples gives us an ordering of the non-sample suffixes.

What is the sorted order of these suffixes? $S_{12}, S_6, S_9, S_3, S_0$.

DC3 Step 7: (Almost done!) Merge

7. We can merge the two sorted sets of suffixes using standard comparison-based merging.

To compare a suffix $S_i \in B_1$ with $S_j \in B_0$,

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To compare a suffix $S_i \in B_2$ with $S_j \in B_0$,

$$S_i \leq S_j \iff (t_i, t_{i+1}, \text{rank}(S_{i+2})) \leq (t_j, t_{j+1}, \text{rank}(S_{j+2})).$$

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$$T(n) = T\left(\frac{2n}{3}\right) + O(n).$$