Interactive Lecture - Four: Two Ray Model, UTD and fading

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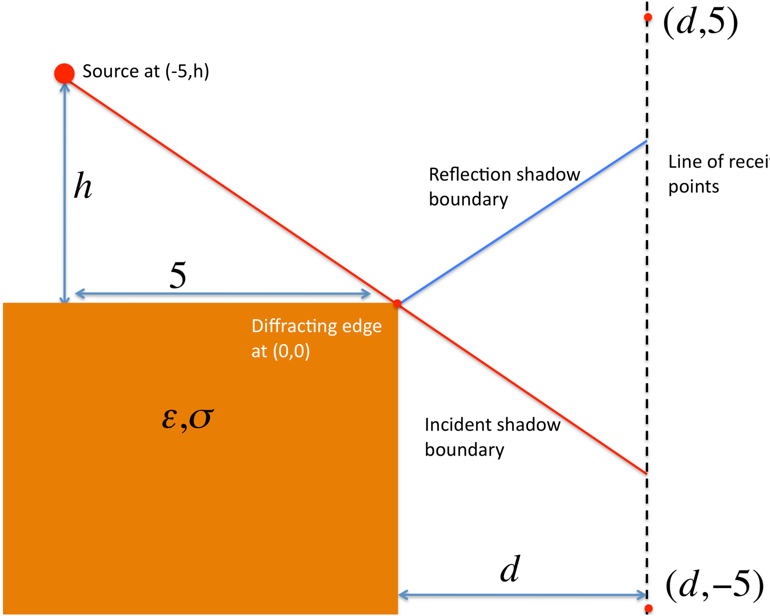
Interactive web applications available at
https://apps.eeng.dcu.ie/ESOA/index.html


(d,5)
Source at (-5,h)
h
Reflection shadow boundary
Line of receiver points
5
Diffracting edge at (0,0)
ε,σ
Incident shadow boundary
d
(d,–5)

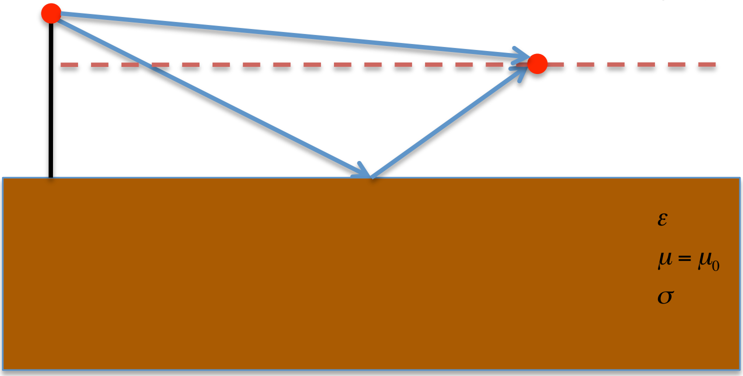
Question One: For the case where $h = 2$ and $d = 3$ find where the incident and reflected shadow boundaries intersect the line of receivers. Use the interactive demo to explore what happens the fields in the vicinity of these points.

Consider receivers along dotted line. Create log plot of power and compare to free space and $1/R^4$ loss.

$\varepsilon = \varepsilon_0$

$\mu = \mu_0$

$\sigma = 0$



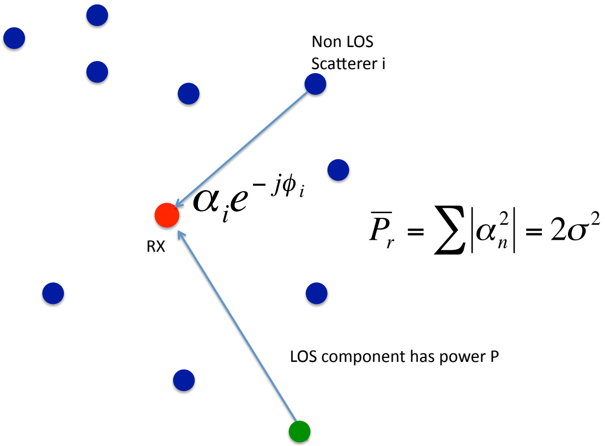
ε

$\mu = \mu_0$

σ

Question Two: For the case of a metallic conductor show that the power decays as $1/R^4$ for large values of R.

Question Three: Show that this results hold even for non-perfectly conducting terrain.


Non LOS
Scatterer i
$\alpha_i e^{-j\phi_i}$
RX
$\overline{P}_r = \sum |\alpha_n^2| = 2\sigma^2$
LOS component has power P

Rayleigh fading

$$
\begin{aligned}
r(t) &= \Re\left\{ \left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-\jmath\phi_n(t)} \right] e^{\jmath 2\pi f_c t} \right\} \\
&= r_I(t)\cos 2\pi f_c t - r_Q(t)\sin 2\pi f_c t \\
z(t) &= \sqrt{r_I^2(t) + r_Q^2(t)} \\
\text{As } N \to \infty : P_Z(z) &= \frac{z}{\sigma^2} \exp\left[\frac{-z^2}{2\sigma^2}\right] \\
P_{Z^2}(x) &= \frac{1}{2\sigma^2} \exp\left[\frac{-x}{2\sigma^2}\right] \\
\bar{P}_r &= \sum_n E\left[\alpha_n^2\right] = 2\sigma^2
\end{aligned}
$$

Rician fading

$$
\begin{aligned}
P_Z(z) &= \frac{z}{\sigma^2}\exp\left[\frac{-\left(z^2+s^2\right)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right) \\
\bar{P}_r &= s^2 + 2\sigma^2 \\
K &= \frac{s^2}{2\sigma^2} \\
P_Z(z) &= \frac{2z\left(K+1\right)}{\bar{P}_r}\exp\left[-K-\frac{\left(K+1\right)z^2}{\bar{P}_r}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{\bar{P}_r}}\right)
\end{aligned}
$$

Question Four: Consider a channel with Rayleigh fading. What is the probability that the received power is more than 3dB below the average received power?

Question Five: Consider a channel with Rician fading. The NLOS average power is $-20dBm$ while the LOS component has power $-10dBm$. Estimate the probability that the received signal amplitude is less than 0.01