Interactive Lecture - Four: Two Ray Model, UTD and fading

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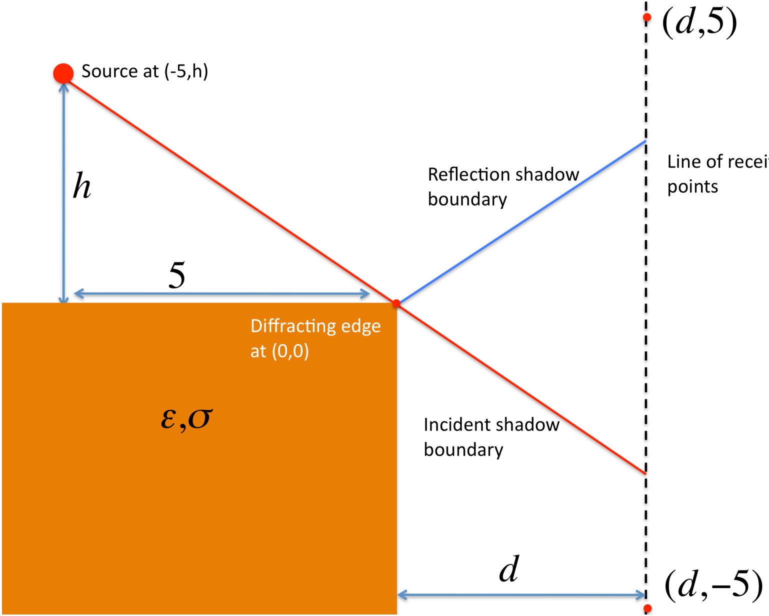
Interactive web applications available at
https://apps.eeng.dcu.ie/ESOA/index.html



$(d,5)$
Source at (-5,h)
h
Reflection shadow boundary
Line of receiver points
5
Diffracting edge at (0,0)
ε,σ
Incident shadow boundary
d
$(d,-5)$

Question One: For the case where $h = 2$ and $d = 3$ find where the incident and reflected shadow boundaries intersect the line of receivers. Use the interactive demo to explore what happens the fields in the vicinity of these points.

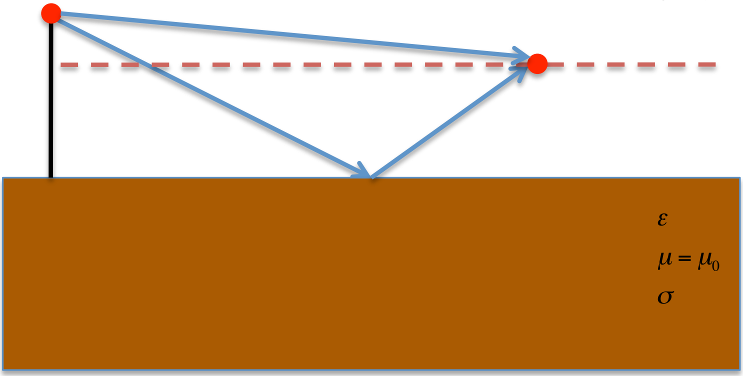
It is simple to show that if the diffracting edge is at $(0, 0)$ then the incident shadow boundary intersects the line $x = d$ at $(d, -\frac{dh}{5})$ while the reflection shadow boundary intersects at $(d, \frac{dh}{5})$. For the values given $\frac{dh}{5} = 1.2$. The geometric optics field displays discontinuities in the vicinity of these points which are compensated for by the diffracted field, producing a continuous total field.

Consider receivers along dotted line.
Create log plot of power and compare
to free space and $1/R^4$ loss.

$\varepsilon = \varepsilon_0$

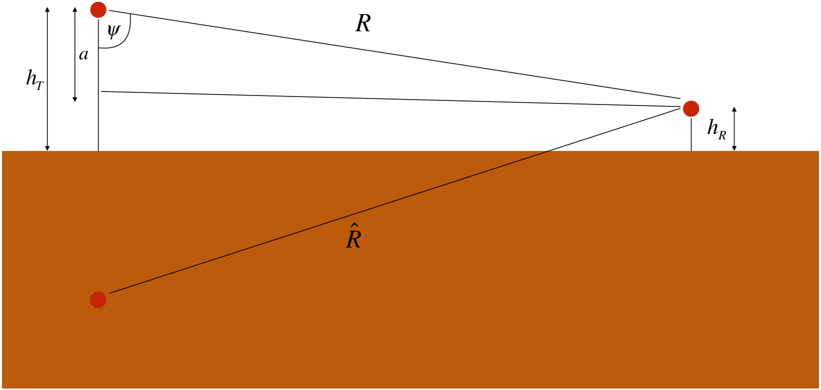
$\mu = \mu_0$

$\sigma = 0$



Question Two: For the case of a metallic conductor show that the power decays as $1/R^4$ for large values of R.

Let $a = h_T - h_R$. R is the distance along the line of sight path while $\hat{R}$ is the distance along the reflected path. We have according to the cosine rule



$$\hat{R}^2 = R^2 + (2h_T)^2 - 4h_T R \cos\psi$$

where

$$\cos\psi = \frac{a}{R}$$

and so

$$\begin{aligned}
\hat{R} &= R\left(1 + \frac{4h_T^2}{R^2} - \frac{4h_T a}{R^2}\right)^{\frac{1}{2}} \\
&\simeq R\left(1 + \frac{2h_T^2}{R^2} - \frac{2h_T a}{R^2}\right) \\
&= R + \frac{2h_T}{R}\left(h_T - a\right) \\
&= R + \frac{2h_T h_R}{R}
\end{aligned}$$

The electric field at the receiver is proportional to

$$E = \frac{1}{R} e^{-j\beta_0 R} - \frac{1}{\hat{R}} e^{-j\beta_0 \hat{R}}$$

where we have assumed a perfect conducting terrain with $\Gamma = -1$.

Assuming $\hat{R} \simeq R$ for amplitude variations and the above approximation for phase approximations we can write this as:

$$
\begin{aligned}
E &= \frac{1}{R} e^{-j\beta_0 R} - \frac{1}{\hat{R}} e^{-\jmath\beta_0 \hat{R}} \\
&= \frac{1}{R} e^{-j\beta_0 R} \left(1 - e^{-\jmath\beta_0 \frac{2h_T h_R}{R}} \right)
\end{aligned}
$$

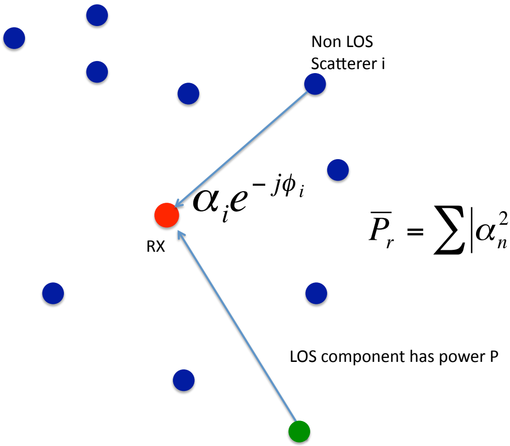
For R large we have

$$
\begin{aligned}
E &= \frac{1}{R} e^{-j\beta_0 R} \left(1 - 1 + \jmath\beta_0 \frac{2h_T h_R}{R} \right) \\
E &= e^{-j\beta_0 R} \jmath\beta_0 \frac{2h_T h_R}{R^2}
\end{aligned}
$$

and so we see that $E \propto \frac{1}{R^2}$ and so power is proportional to $\frac{1}{R^4}$. For large R the phase term $\beta_0 \frac{2h_T h_R}{R}$ approaches zero. The change to $\frac{1}{R^4}$ begins roughly once this phase term is less than π. Hence we require

$$
\begin{aligned}
\beta_0 \frac{2h_T h_R}{R} &< \pi \\
\implies R &> \frac{4h_T h_R}{\lambda}
\end{aligned}
$$

Question Three: Show that this results hold even for non-perfectly conducting terrain. Even if the ground is imperfectly conducting the reflection coefficients tends to -1 for grazing angles, which holds for large R and so the above analysis holds.



$$\overline{P}_r = \sum \left|\alpha_n^2\right| = 2\sigma^2$$

Rayleigh fading

$$
\begin{aligned}
r(t) &= \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-\jmath\phi_n(t)}\right] e^{\jmath 2\pi f_c t}\right\} \\
&= r_I(t)\cos 2\pi f_c t - r_Q(t)\sin 2\pi f_c t \\
z(t) &= \sqrt{r_I^2(t) + r_Q^2(t)} \\
\text{As } N \to \infty : P_Z(z) &= \frac{z}{\sigma^2}\exp\left[\frac{-z^2}{2\sigma^2}\right] \\
P_{Z^2}(x) &= \frac{1}{2\sigma^2}\exp\left[\frac{-x}{2\sigma^2}\right] \\
\bar{P}_r &= \sum_n E\left[\alpha_n^2\right] = 2\sigma^2
\end{aligned}
$$

Rician fading

$$
\begin{aligned}
P_Z(z) &= \frac{z}{\sigma^2}\exp\left[\frac{-\left(z^2+s^2\right)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right)\\
\bar{P}_r &= s^2+2\sigma^2\\
K &= \frac{s^2}{2\sigma^2}\\
P_Z(z) &= \frac{2z\left(K+1\right)}{\bar{P}_r}\exp\left[-K-\frac{\left(K+1\right)z^2}{\bar{P}_r}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{\bar{P}_r}}\right)
\end{aligned}
$$

Question Four: Consider a channel with Rayleigh fading. What is the probability that the received power is more than 3dB below the average received power?

We require that

$$10\log_{10}\frac{r^2}{2\sigma^2} \quad < \quad -3$$
$$\implies r^2 < 10^{-0.3}2\sigma^2$$

The cdf is

$$P(r^2 < x) \quad = \quad \int_0^x \frac{1}{2\sigma^2}e^{-\frac{z}{2\sigma^2}}\,dz$$
$$= \quad 1 - e^{-\frac{x}{2\sigma^2}}$$

We get

$$P\left(r^2 < 10^{-0.3}2\sigma^2\right) = 1 - e^{-10^{-0.3}} \simeq .3942$$

Question Five: Consider a channel with Rician fading. The NLOS average power is $-20dBm$ while the LOS component has power $-10dBm$. Estimate the probability that the received signal amplitude is less than 0.01

Using the simulator with the above settings we can plot the cumulative distribution function for the Ricean distribution (Choose cumulative distribution function under "toggle between outputs" option). On the x axis we see signal amplitude values while the y axis gives the probability that the signal amplitude is less than or equal to these values. For 0.01 the probability is roughly 0.468.