Interactive Lecture - Three

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Interactive web applications available at
https://apps.eeng.dcu.ie/ESOA/index.html

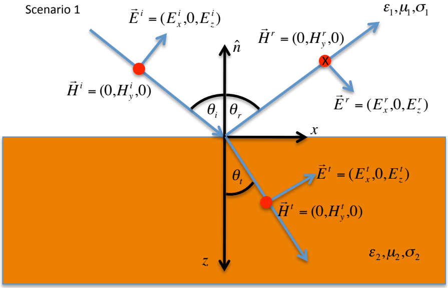
Reflection Coefficient

$$\Gamma_{\parallel} = \frac{-\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$\theta_t = \sin^{-1}\left(\frac{\gamma_1}{\gamma_2}\sin\theta_i\right)$$

Transmission Coefficient

$$T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$



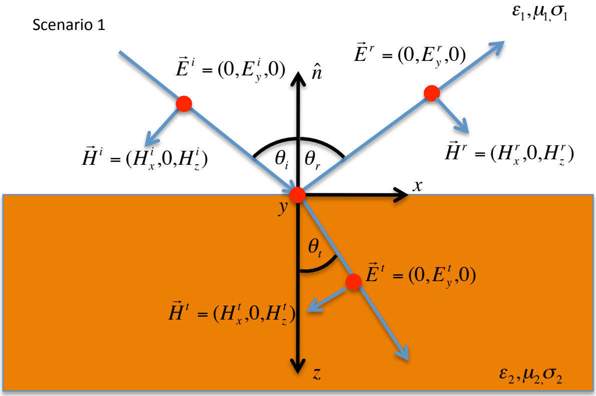
Reflection Coefficient

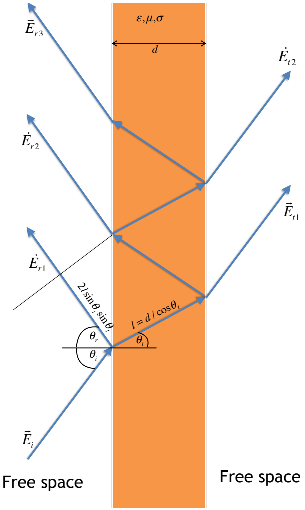
$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Transmission Coefficient

$$T_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Scenario 1





The reflected field can be expressed as:

$$\Gamma = \Gamma_{01} + T_{01}T_{10}\sum_{n=1}^{\infty}\Gamma_{10}^{2n-1}\left(P_d\right)^{2n}\left(P_a\right)^{n}$$

where

$$\begin{aligned} P_d &= e^{-\gamma_1 \ell} \\ P_a &= e^{2\jmath\beta_0\ell\sin\theta_i\sin\theta_t} \end{aligned}$$

This can be simplified to:

$$\Gamma = \frac{\Gamma_{01}\left(1 - P_d^2 P_a\right)}{1 - \Gamma_{01}^2 P_d^2 P_a}$$

Question One: A perpendicularly polarised wave travelling in free space impinges on a medium with $\epsilon_r = 5$ at an angle of 45°. Compute the reflection and transmission coefficient. Does the reflection coefficient change if the wave is instead travelling from the medium $\epsilon_r = 5$ to free-space?

Question Two: Repeat the above exercise for a parallel polarised wave.

Question Three: At what angle will the reflection coefficient go to zero? Investigate this for both polarisations and scenarios. Where possible derive an expression for the angle that produces zero reflection (Brewster angle). Assume in all cases that $\mu = \mu_0$ for all materials

Question Four: At what angle will the reflection coefficient go to one? Investigate this for both polarisations and scenarios. Where appropriate derive an expression for the angle that produces unity reflection (critical angle).

Question Five: Investigate the behaviour of the reflection coefficients as the incident wave approaches grazing, ie. $\theta_i \to 90^\circ$.

Question Six: Derive the expressions for Γ and T for both polarisations. This can be done by enforcing continuity of the tangential components of both $\vec{E}$ and $\vec{H}$ along the boundary ($z = 0$).

Question Seven: Demonstrate that the expression for the reflection coefficient from a slab for oblique incidence reduces to that for normal incidence when $\theta_i = 0$.