Interactive Lecture - One: Solutions

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Interactive web applications available at
https://apps.eeng.dcu.ie/ESOA/index.html

Question One:
Figure (??) depicts, at a specific moment in time, the electric field of a plane wave propagating with frequency $f = 100MHz$ in a lossless medium. Given the data in the picture compute ϵ_r for the material. Note that for this simulation ϵ_r was chosen to be an integer and you can assume that $\mu_r = 1$. What is the phase velocity of this wave and what is the impedance of the material? If instead you were told that $f = 200MHz$ what would you compute ϵ_r to be?
The form of the wave is $E = \cos(\omega t - \beta z)$. The dots shown approximately depict a local maximum and minimum on the wave. The spacing between the two dots on the graph is one and a half wavelengths and so

$$1.83 - 0.705 \simeq 1.5\lambda$$

and so

$$\lambda \simeq 0.75 \text{ and so } \beta \simeq \frac{2\pi}{0.75} = \frac{8\pi}{3}$$

We have therefore

$$\omega\sqrt{\mu\epsilon} \simeq \frac{8\pi}{3}$$

and so

$$\begin{aligned} \sqrt{\epsilon_r} &\simeq \frac{8\pi}{3}\frac{c_0}{\omega} \\ &= \frac{8\pi}{3}\frac{3\times 10^8}{2\pi(10^8)} \\ &= 4 \end{aligned}$$

yielding $\epsilon_r = 16$.

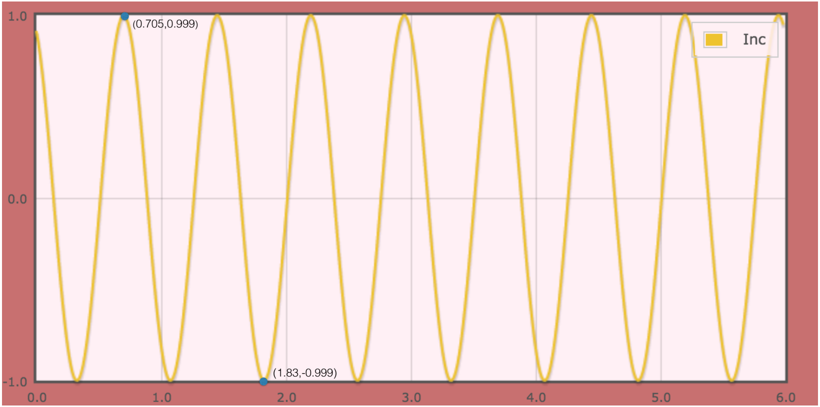

Figure: Figure for Question One

Question Two: Given that

$$\alpha = \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right)^{\frac{1}{2}}$$

$$\beta = \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]\right)^{\frac{1}{2}}$$

Show that for good dielectrics (satisfying $\left(\frac{\sigma}{\omega\epsilon}\right)^2 \ll 1$)

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\beta \simeq \omega\sqrt{\mu\epsilon}$$

We have for α

$$
\begin{aligned}
\alpha &= \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right)^{\frac{1}{2}} \\
&= \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[1+\frac{1}{2}\left(\frac{\sigma}{\omega\epsilon}\right)^2-1\right]\right)^{\frac{1}{2}} \\
&= \omega\sqrt{\mu\epsilon}\left(\frac{1}{4}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right)^{\frac{1}{2}} \\
&= \omega\sqrt{\mu\epsilon}\frac{1}{2}\frac{\sigma}{\omega\epsilon} \\
&= \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}
\end{aligned}
$$

Similar analysis for β.

Question Three: Consider the scenario in figure (??). It depicts, at a specific moment in time, the electric field of a plane wave propagating with frequency $f = 1GHz$ in a good dielectric with some loss. Given the data in the figure estimate σ and ϵ_r. Note that σ was chosen to be $n \times 10^{-3}$ for n an integer while ϵ_r was also chosen to be an integer. NB The points chosen are adjacent local maxima.

The wave is of the form $E = e^{-\alpha z} \cos(\omega t - \beta z)$. The points chosen are adjacent local maxima which means that

$$\begin{aligned} \lambda &\simeq 0.116 - 0.056 = 0.06 \\ \Longrightarrow \beta &\simeq \frac{2\pi}{0.06} = 104.72 \end{aligned}$$

and so

$$\begin{aligned} \omega\sqrt{\mu\epsilon} &\simeq 104.72 \\ \Longrightarrow \sqrt{\epsilon_r} &\simeq 104.72\frac{c_0}{\omega} \\ &\simeq 104.72\frac{3 \times 10^8}{2\pi \times 10^9} \\ &= 104.72\frac{3}{20\pi} = 5 \end{aligned}$$

which means that $\epsilon_r = 25$ (given the information in the question that an integer value was chosen).

The points chosen are local maxima which means that $\cos(\omega t - \beta z) = 1$ in both cases. Therefore

$$\frac{0.995}{0.997} \simeq e^{-\alpha(0.116-0.056)}$$

and so

$$\begin{aligned} e^{-0.06\alpha} &\simeq 0.99799 \\ \Longrightarrow \alpha &\simeq \frac{-\ln 0.99799}{0.06} \\ &= 0.0335 \end{aligned}$$

Therefore

$$\begin{aligned} \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} &\simeq 0.0335 \\ \Longrightarrow \sigma &= \frac{0.067}{\eta} \\ &= \frac{\sqrt{\epsilon_r}0.067}{\eta_0} \\ &= \frac{5\,(0.067)}{(377)} \\ &= 0.0009 \end{aligned}$$

The actual figure used in the simulation was $\sigma = 0.001$ or 1×10^{-3}

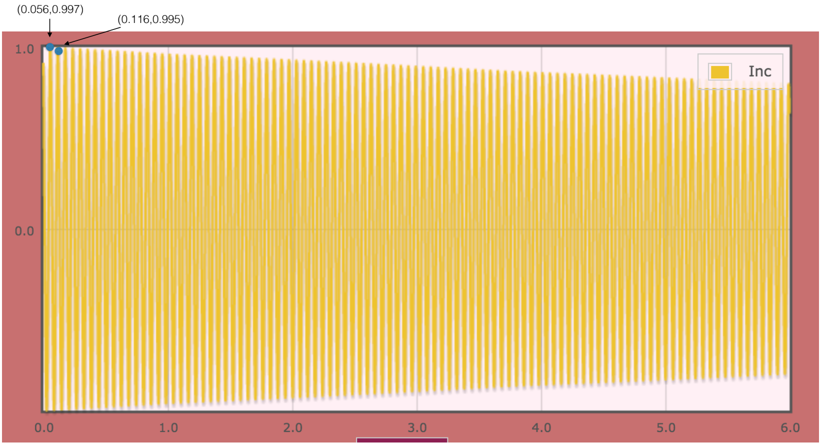

Figure: Figure for Question Three

Question Four: A plane wave propagates through free space. At a particular point at time $t = 0$ the electric field E has amplitude 1. At time $t = \frac{1}{8\times10^9}$ seconds the magnetic field H has amplitude $\frac{1}{377\sqrt{2}}$. What are the possible values for the frequency f?

At time $t = 0$ we have

$$E = \cos\left(\omega\left(0\right) - \beta z\right) = 1$$

Without loss of generality let the point in question be $z = 0$. At time $t = \frac{1}{8\times10^9}$ we have

$$\begin{aligned} H &= \frac{1}{\eta_0}\cos\left(\frac{\omega}{8\times10^9} - \beta(0)\right) \\ &= \frac{1}{377}\frac{1}{\sqrt{2}} \end{aligned}$$

This means that

$$\cos\left(\frac{\omega}{8\times10^9} - \beta(0)\right) = \frac{1}{\sqrt{2}}$$

which means that

$$\frac{2\pi f}{8\times10^9} = \frac{\pi}{4} + 2n\pi = \pi\left(2n + \frac{1}{4}\right)$$

Measuring f in MHz we have

$$f = 4\times(10^3)\left(2n + \frac{1}{4}\right)$$