Interactive Lecture - One

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Interactive web applications available at
https://apps.eeng.dcu.ie/ESOA/index.html

Example - Plane wave in lossy medium

Consider a wave travelling in $\hat{z}$ direction with $\vec{E}$ in x-direction and no variation in y or x. Therefore Helmholtz equation becomes

$$\frac{d^2}{dz^2}E_x(z) = \gamma^2 E_x(z)$$

where $\gamma = \alpha + \jmath\beta$ and so

$$\begin{aligned} E_x(z) &= E_0 e^{-\gamma z} \\ \mathcal{E}_x(z,t) &= \Re\left(E_0 e^{-\gamma z} e^{\jmath\omega t}\right) \\ &= |E_0|\, e^{-\alpha z}\cos\left(\omega t - \beta z + \phi\right) \text{ where } E_0 = |E_0|\, e^{\jmath\phi} \end{aligned}$$

Quantity	Symbol	Expression
Attenuation constant	α	$\omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right\}^{\frac{1}{2}}$
Phase constant	β	$\omega\sqrt{\mu\epsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]\right\}^{\frac{1}{2}}$
Wave impedance	η	$\sqrt{\frac{\jmath\omega\mu}{\sigma+\jmath\omega\epsilon}}$
Wavelength	λ	$\frac{2\pi}{\beta}$
Velocity	v	$\frac{\omega}{\beta}$
Skin depth	δ	$\frac{1}{\alpha}$

Question One:
Figure (1) depicts, at a specific moment in time, the electric field of a plane wave propagating with frequency $f = 100MHz$ in a lossless medium. Given the data in the picture compute ϵ_r for the material. Note that for this simulation ϵ_r was chosen to be an integer and you can assume that $\mu_r = 1$. What is the phase velocity of this wave and what is the impedance of the material? If instead you were told that $f = 200MHz$ what would you compute ϵ_r to be?

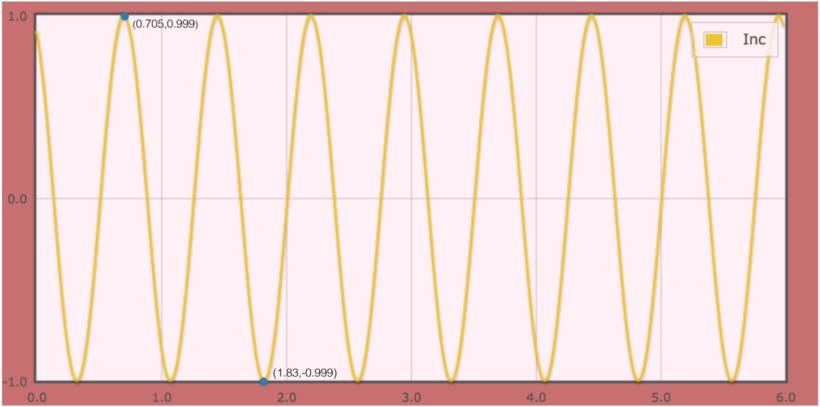

Figure: Figure for Question One

Question Two: Given that

$$\alpha = \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]\right)^{\frac{1}{2}}$$

$$\beta = \omega\sqrt{\mu\epsilon}\left(\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]\right)^{\frac{1}{2}}$$

Show that for good dielectrics (satisfying $\left(\frac{\sigma}{\omega\epsilon}\right)^2 \ll 1$)

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$$

$$\beta \simeq \omega\sqrt{\mu\epsilon}$$

Question Three: Consider the scenario in figure (2). It depicts, at a specific moment in time, the electric field of a plane wave propagating with frequency $f = 1GHz$ in a good dielectric with some loss. Given the data in the figure estimate σ and ϵ_r. Note that σ was chosen to be $n \times 10^{-3}$ for n an integer while ϵ_r was also chosen to be an integer. NB The points chosen are adjacent local maxima.

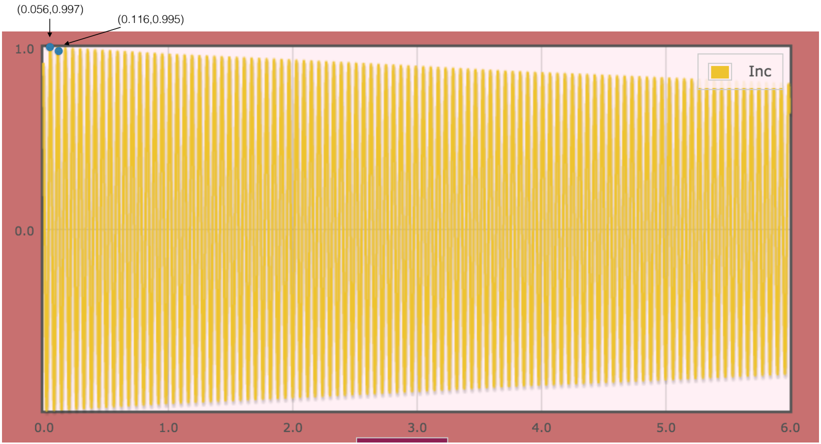

Figure: Figure for Question Three

Question Four: A plane wave propagates through free space. At a particular point at time $t = 0$ the electric field E has amplitude 1. At time $t = \frac{1}{8 \times 10^9}$ seconds the magnetic field H has amplitude $\frac{1}{377\sqrt{2}}$. What are the possible values for the frequency f?