Homework 1 **Hongyu Yan (516030910595)**
CS 259 Numerical Methods for Data Science ACM Class, Zhiyuan College, SJTU
Prof. **David Bindel** Due Date: Month Day, Year
TA. **Yurong You, Xinran Zhu** Submit Date: June 19, 2018

Problem 1

The plot is shown in figure 1:

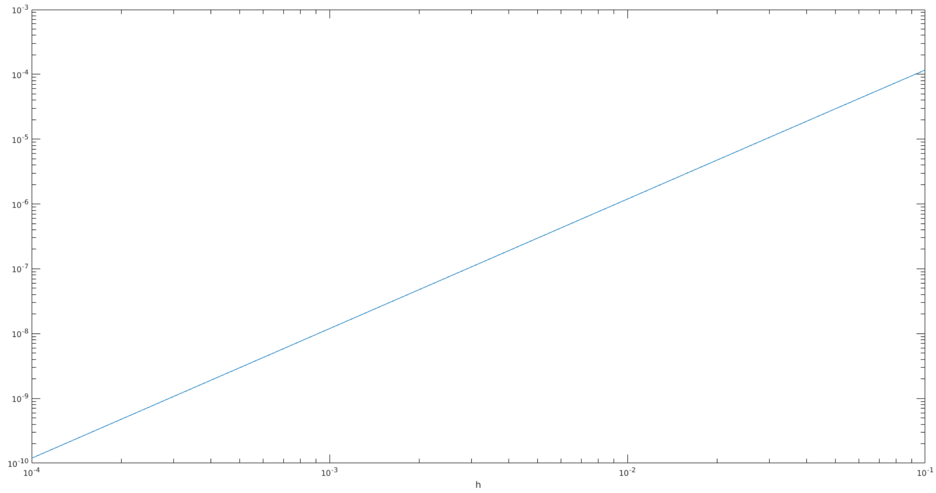

Figure 1: the log-log plot of the norm versus h

The result of fitting the curve with MATLAB Curve Fitting Tool is as below.

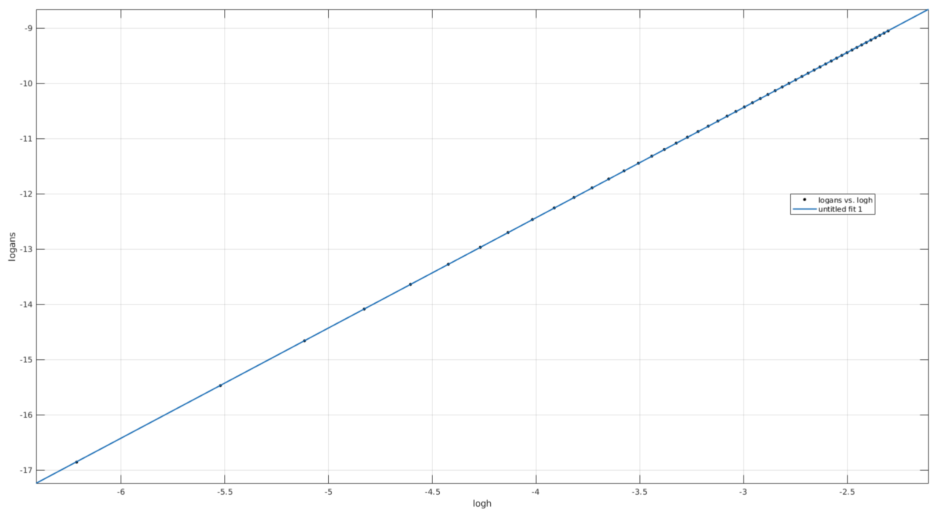

Figure 2: the fitting curve of log(h) versus log(norm)

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Linear model Poly1:
     f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):
       p1 =        1.994  (1.993, 1.995)
       p2 =       -4.458  (-4.461, -4.456)

Goodness of fit:
  SSE: 0.0002814
  R-square: 1
  Adjusted R-square: 1
  RMSE: 0.002421
```

Proof

$$
\begin{aligned}
&(A+hE)^{-1}-(A^{-1}-hA^{-1}EA^{-1})\\
=&(A+hE)^{-1}[I-(A+hE)(I-hA^{-1}E)]A^{-1}\\
=&(A+hE)^{-1}[I-A(I+hA^{-1}E)(I-hA^{-1}E)]A^{-1}\\
=&(A+hE)^{-1}[I-I-Ah^2(A^{-1}E)^2A^{-1}]\\
=&h^2(A+hE)^{-1}A(A^{-1}E)^2A^{-1}
\end{aligned}
$$

Since h is small, we have $A+hE\approx A$. Then the result approximates $||h^2(A^{-1}E)^2A^{-1}|| = ||h^2|| * ||(A^{-1}E)^2A^{-1}||$, so $\frac{log(result)}{log(h)} = 2$, which means the asymptotic slope on the log-log plot is 2.

Problem 2

The KKT conditions

The lagrangian is

$$L(x,\lambda,\mu) = ||x-u||^2 + \lambda\sum_i x_i + \sum_i \mu_i(-x_i)$$

So the KKT conditions are:

$$
\begin{aligned}
2x-2u+\lambda e-\mu &= 0 && \\
\sum_i x_i - 1 &= 0, && \text{equality constraints}\\
-x_i &\le 0, && \text{inequality constraints}\\
\mu_i &\ge 0, && \text{non-negativity of multipliers}\\
-x_i\mu_i &= 0, && \text{complementary slackness}
\end{aligned}
$$

e is $[1,1,\cdots,1]^T$, μ is $[\mu_1,\mu_2,\cdots,\mu_n]^T$

Proof

From the KKT conditions we have:

$$
\begin{aligned}
2x - 2u + \lambda e - \mu = 0 \\
x = u - \frac{\lambda e - \mu}{2} \\
x_i = u_i - \frac{\lambda - \mu_i}{2}
\end{aligned}
$$

So in some conditions, solution satisfies $x_i = max(u_i + C, 0)$.
In such condition, $C = \frac{\lambda}{2}$, according to $-x_i \mu_i = 0$, we have:

$$
\begin{aligned}
x_i = 0, u_i - \frac{\lambda}{2} < 0, & \qquad when\ \mu_i > 0 \\
x_i = u_i - \frac{\lambda}{2}, & \qquad when\ \mu_i = 0
\end{aligned}
$$