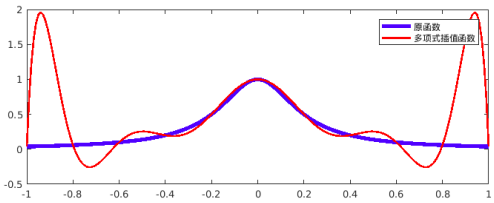
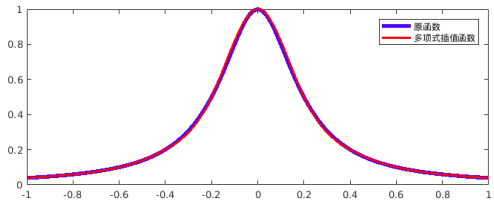
1 计算实习题 P50 第 2 题

1.1 解题方案

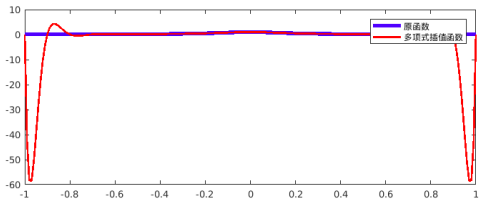
代码如下，将主程序中 n 的值修改为 10 和 20 来计算不同的情况

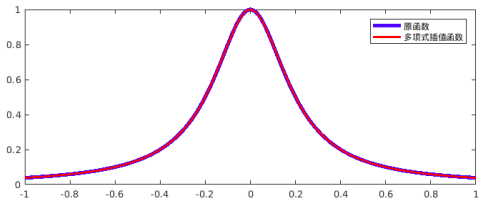
1.2 图片





n=10





n=20

1.3 代码

1.3.1 主程序

```
n = 10;

% 生成插值点
x = [-1:2.0/n:1];
y = 1./(1+25.*x.*x);
x0 = [-1:0.01:1];

% 多项式插值
y0 = poly_interpolation(x, y, x0);

subplot(2,1,1);
plot(x0, 1./(1+25.*x0.*x0), 'b', 'LineWidth', 4);
hold on;
plot(x0, y0, 'r', 'LineWidth', 2);
legend(' 原函数',' 多项式插值函数');

% 三次样条插值
f0 = (-50.0 * -1) / ((1.0 + 25)^2);
fn = (-50.0 * 1) / ((1.0 + 25)^2);
y0 = spline_interpolation(x, y, f0, fn, x0, 1);
```

```
subplot(2,1,2);
plot(x0, 1./(1+25.*x0.*x0), 'b', 'LineWidth', 4);
hold on;
plot(x0, y0, 'r', 'LineWidth', 2);
legend(' 原函数',' 多项式插值函数');
```

1.3.2 多项式插值

```
function [y0] = poly_interpolation(x, y, x0)
% lagrange interpolation

n = length(x);
m = length(x0);
y0 = zeros(m,1);

for i = 1:m
  s=0.0;
  z = x0(i);
  for k = 1:n
    p=1.0;
    for j=1:n
      if j~=k
        p=p*(z-x(j))/(x(k)-x(j));
      end
    end
    s=p*y(k)+s;
  end
  y0(i)=s;
end
```

1.3.3 三次样条插值

```
function y0 = spline_interpolation(x, y, f0, fn, x0, kind)

n = length(x); m = length(x0);
y0 = zeros(m,1);

% 中间变量定义
miu = zeros(n);
lambda = zeros(n);
d = zeros(n);
h = zeros(n);

% 中间变量计算
for i = 1:n-1
  h(i) = x(i+1) - x(i);
end
for i = 2:n-1
```

```
  miu(i) = h(i-1)/(h(i-1)+h(i));
  lambda(i) = h(i)/(h(i-1)+h(i));
  d(i) = 6*( (y(i+1)-y(i))/(x(i+1)-x(i))-(y(i)-y(i-1))/(x(i)-x(i-1)) ) / (h(i-1)+h(i));
end
% 三种不同的边界条件
if kind == 1
  lambda(1) = 1;
  miu(n) = 1;
  d(1) = 6/h(1)*((y(2)-y(1))/(x(2)-x(1))-f0);
  d(n) = 6/h(n-1)*(fn-(y(n-1)-y(n))/(x(n-1)-x(n)));
end
if kind == 2
  lambda(1) = 0;
  miu(1) = 0;
  d(1) = 2*f0;
  d(n) = 2*fn;
end
if kind == 3
  lambda(1) = 0;
  miu(1) = 0;
  lambda(n) = h(1) / (h(1) + h(n-1));
  miu(n) = 1-lambda(n);
  d(1) = 0;
  d(n) = 6*((y(2)-y(1))/(x(2)-x(1))-(y(n)-y(n-1))/(x(n)-x(n-1)))/(h(1)+h(n-1));
end

% 列出方程
A = zeros(n,n);
for i = 1:n
  A(i,i) = 2;
  if (i>1)
    A(i,i-1) = miu(i);
  end
  if (i<n)
    A(i,i+1) = lambda(i);
  end
end

if kind == 3
  A(2,n) = miu(2);
  A(n,2) = lambda(n);
end

% 解方程
M = A\d;
if kind == 3
```

```
  M(1) = M(n);
end

% 计算在给定点的插值函数值
for i = 1:m
  z = x0(i);
  for j = 1:(n-1)
    if z<=x(j+1)
      y0(i) = M(j)*((x(j+1)-z)^3)/6/h(j)+...
              M(j+1)*((z-x(j))^3)/6/h(j)+...
              (y(j)-M(j)*(h(j)^2)/6)*(x(j+1)-z)/h(j)+...
          (y(j+1)-(M(j+1)*h(j)^2)/6)*(z-x(j))/h(j);
      break;
    end
  end
end
```

2 补充练习

2.1 代码

2.1.1 主程序

```
n = 20;
x = [-1:2.0/n:1];
y = 1./(1+25.*x.*x);
x0 = [-1:0.01:1];

% 第一种边界条件
f0 = (-50.0 * -1) / ((1.0 + 25)^2);
fn = (-50.0 * 1) / ((1.0 + 25)^2);
y0 = spline_interpolation(x, y, f0, fn, x0, 1);

subplot(3,1,1);
plot(x0, 1./(1+25.*x0.*x0), 'b', 'LineWidth', 4);
hold on;
plot(x0, y0, 'r', 'LineWidth', 2);
legend(' 原函数',' 多项式插值函数');
title(' 第一种边界条件插值');

% 第二种边界条件
f0 = (3750-50.0) / (1+25)^3;
fn = (3750-50.0) / (1+25)^3;
y0 = spline_interpolation(x, y, f0, fn, x0, 2);

subplot(3,1,2);
plot(x0, 1./(1+25.*x0.*x0), 'b', 'LineWidth', 4);
```

```
hold on;
plot(x0, y0, 'r', 'LineWidth', 2);
legend(' 原函数',' 多项式插值函数');
title(' 第二种边界条件插值');

% 第三种边界条件
f0 = -1;
fn = 1;
y0 = spline_interpolation(x, y, f0, fn, x0, 3);

subplot(3,1,3);
plot(x0, 1./(1+25.*x0.*x0), 'b', 'LineWidth', 4);
hold on;
plot(x0, y0, 'r', 'LineWidth', 2);
legend(' 原函数',' 多项式插值函数');
title(' 第三种边界条件插值');
```

2.1.2 三次样条插值

同上一题的代码。写完第一题的代码后，在其基础上改得第二题代码，但是没有保留原始版本，只好共用了一份代码。

2.2 插值图像