Network Optimization Calculus: A Systems-Theoretic Framework

Jacob D. Jaisaree

1 Introduction

This framework models how cooperation, potential, and disconnection shape the evolution of networked systems. It formalizes how stability and adaptability interact, and how autonomy emerges from their balance.

2 Primitive Concepts and Axioms

The system consists of a finite set of nodes:

$$\Omega = \{X_i\}_{i \in I},$$

where each node $X_i = (\Theta_i, V_i, I_i)$ has:

- Θ_i: Type (qualitative category),
- V_i: Intrinsic value (inherent potential),
- I_i: Intensity (capacity to engage in relationships).

Any pair (i, j), $i \neq j$, holds a relationship in one of three states:

$$R(i, j) \in \{C, N, D\},$$

where

- C: Cohesive (cooperation),
- N: Neutral (potential),
- D: Disjunctive (inactive).

The neutral state mediates transitions between cohesion and disjunction.

3 Value Realization

The realized value of a pair is

$$\Lambda(i, j) = \mathbf{1}_{\{R(i,j)=C\}} \cdot (V_i I_i + V_j I_j),$$

where $\mathbf{1}$ is the indicator function.

4 Optimization Objective

The system's realized value at time t is

$$O(S_t) = \sum_{i<j} \Lambda(i,j).$$

The latent value stored in disjunctive pairs is

$$L(S_t) = \sum_{\substack{i<j \\ R(i,j)=D}} (V_i I_i + V_j I_j).$$

Weights balance realized vs. latent value:

$$\alpha(S_t) = \frac{|\{(i,j) : R(i,j) \neq D\}|}{|\Omega|(|\Omega|-1)/2}, \quad \beta(S_t) = 1 - \alpha(S_t).$$

The system evolves toward the state

$$S^* = \arg\max_{S_t} \Big[\alpha(S_t)O(S_t) + \beta(S_t)L(S_t)\Big].$$

5 Transition Dynamics

For neutral relationships:

$$P(R_{t+1}(i,j) = C \mid R_t(i,j) = N) = \alpha(S_t),$$

$$P(R_{t+1}(i,j) = D \mid R_t(i,j) = N) = \beta(S_t).$$

With reopening probability ϵ:

$$P(R_{t+1}(i,j) = N \mid R_t(i,j) \in \{C, D\}) = \epsilon.$$

6 Emergence of Autonomy

Autonomy arises from recursive feedback between realized and latent value:

$$(\alpha_{t+1}, \beta_{t+1}) = f(\alpha_t, \beta_t, P(N \to C)).$$

This loop captures the trade-off between stability and adaptability:

$$\alpha_t \;\to\; P(N \to C) \;\to\; \text{Distribution of } R_{t+1} \;\to\; \alpha_{t+1}.$$

7 Cooperation with Systemic Constraints

The system's efficiency relative to its potential is

$$\gamma(S_t) = \frac{O(S_t)}{O(S_t) + L(S_t)} \in [0, 1].$$

Future cooperation is guided by

$$E[\gamma(S_{t+\Delta t})] = \frac{\sum_{i<j} (V_i I_i + V_j I_j)\, P(R_{t+\Delta t}(i, j) = C)}{O(S_t) + L(S_t)}.$$

Optimal cooperation occurs when realized value and adaptive potential are in balance.