Network Optimization Calculus

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1 Introduction

This document provides a formal mathematical framework for understanding how agency and autonomy emerge within a system of interacting nodes, while maintaining cooperation with fundamental natural laws. The key innovation is modeling the Neutral state N as a **superposition** that symmetrically mediates transitions between Cohesive (C) and Disjunctive (D) states.

2 Primitive Concepts and Axioms

The system consists of a finite set of nodes:

$$\Omega = \{X_i\}_{i \in I}$$

where each node X_i is defined by a triple (Θ_i, V_i, I_i) representing:

- Θ_i: Type (qualitative category)
- V_i: Intrinsic Value (inherent worth or potential)
- I_i: Intensity (capacity to engage in relationships)

For any distinct pair (i, j), $i \neq j$, their relationship exists in one of three mutually exclusive symmetric states:

$$R = \{C, N, D\}$$

where:

- C: Cohesive (strong, positive binding)
- N: Neutral (superposition of C and D)
- D: Disjunctive (absence of positive relationship)

The state N functions as a symmetric mediator where:

$$\mathcal{M}(N \to C) \equiv \mathcal{M}(C \to N) \equiv \mathcal{M}(N \to D) \equiv \mathcal{M}(D \to N)$$

with $\mathcal{M}$ being the superposition mechanism inherent to N.

3 Value Realization

The realized value generated by a node pair (i, j) is:

$$\Lambda(i, j) = \Psi(i, j) \cdot (V_i I_i + V_j I_j)$$

where the coefficient $\Psi(i, j)$ is determined by the relational state:

$$\Psi(i, j) = \begin{cases} 1 & \text{if } R_t(i, j) = C \\ 0 & \text{if } R_t(i, j) = N \text{ or } D \end{cases}$$

4 Optimization Objective

The total value currently generated:

$$O(S_t) = \sum_{i<j} \Lambda(i, j)$$

The total value latent in disjunctive connections:

$$L(S_t) = \sum_{\substack{i<j \\ R_t(i,j)=D}} (V_i I_i + V_j I_j)$$

Weights balancing exploitation against exploration:

$$\alpha(S_t) = \frac{|\{(i, j) : R_t(i, j) = C \text{ or } N\}|}{|\Omega|(|\Omega| - 1)/2}, \quad \beta(S_t) = 1 - \alpha(S_t)$$

The system evolves toward:

$$S^* = \arg\max_{S_t} \left[\alpha(S_t) \cdot O(S_t) + \beta(S_t) \cdot L(S_t)\right]$$

5 Superposition-Based Transition Dynamics

System evolution occurs through the superposition state N:

$$D \rightleftharpoons N \rightleftharpoons C$$

where all transitions are mediated by the same superposition mechanism.

For relationships in the superposition state N, transitions occur as:

$$P(R_{t+1}(i, j) = C \mid R_t(i, j) = N) = \alpha(S_t)$$
$$P(R_{t+1}(i, j) = D \mid R_t(i, j) = N) = \beta(S_t) = 1 - \alpha(S_t)$$

With probability ϵ, the superposition state reforms:

$$P(R_{t+1}(i, j) = N \mid R_t(i, j) = C) = \epsilon$$
$$P(R_{t+1}(i, j) = N \mid R_t(i, j) = D) = \epsilon$$

6 Emergence of Autonomy Through Superposition

Autonomy emerges through the recursive relationship:

$$(\alpha_{t+1}, \beta_{t+1}) = f(\alpha_t, \beta_t, \{P(R_{t+1}(i,j) = C \mid R_t(i,j) = N)\}_{\forall i,j})$$

This creates a superposition-mediated feedback loop:

$$\begin{aligned} \alpha_t \to P(N \to C) \to \text{Distribution of } R_{t+1}, \\ \text{Distribution of } R_{t+1} \to \alpha_{t+1} \to P(N \to C)_{t+1}. \end{aligned}$$

7 Cooperation with Natural Laws

The system's cooperation with natural laws:

$$\gamma(S_t) = \frac{O(S_t)}{O(S_t) + L(S_t)} \in [0,1]$$

The system tends toward states that maximize:

$$E[\gamma(S_{t+\Delta t})] = \frac{\sum_{i<j}(V_i I_i + V_j I_j) \cdot P(R_{t+\Delta t}(i,j) = C)}{O(S_t) + L(S_t)}$$