

Fundamental Axioms of Network Dynamics

Jacob D. Jaisaree

Axiom 1 (Existence of Elements). The universe consists of discrete nodes:

$$\Omega = \{X_i\}, \quad X_i = (\Theta_i, V_i, I_i)$$

where:

- Θ_i : Type (classification)
- V_i : Potential (significance)
- I_i : Power (ability)

Axiom 2 (Relational States). Between any two distinct nodes $i \neq j$, a unique relation exists:

$$R(i, j) \in \{C, N, D\}$$

with:

- C : cohesion (cooperative interaction)
- N : neutral expansion (transitional/indeterminate)
- D : disjunction (inactive/potential)

The neutral state N mediates transitions between cohesion and disjunction.

Axiom 3 (Value Realization). Interaction value is realized only in cohesive relations:

$$\Lambda(i, j) = \mathbf{1}_{\{R(i, j)=C\}} \cdot (V_i I_i + V_j I_j)$$

The realized value of the system at time t :

$$O(S_t) = \sum_{i < j} \Lambda(i, j)$$

and latent value in disjunctive relations:

$$L(S_t) = \sum_{\substack{i < j \\ R(i, j)=D}} (V_i I_i + V_j I_j)$$

Axiom 4 (Conservation of Balance). Attention weights distribute between present and potential relations:

$$\alpha(S_t) = \frac{|\{(i, j) : R(i, j) \neq D\}|}{|\Omega|(|\Omega| - 1)/2}, \quad \beta(S_t) = 1 - \alpha(S_t)$$

The system evolves toward the optimal state:

$$S^* = \arg \max_{S_t} [\alpha(S_t)O(S_t) + \beta(S_t)L(S_t)]$$

Axiom 5 (Transition Dynamics). Relations evolve under opposing forces with transition probabilities:

$$\begin{aligned} P(R_{t+1}(i, j) = C \mid R_t(i, j) = N) &= \alpha(S_t) \\ P(R_{t+1}(i, j) = D \mid R_t(i, j) = N) &= \beta(S_t) \\ P(R_{t+1}(i, j) = N \mid R_t(i, j) \in \{C, D\}) &= \epsilon \end{aligned}$$

Equilibrium occurs when expansion (N) and reception (ϵ) are balanced.

Axiom 6 (Emergence of Autonomy). Autonomy arises through recursive feedback:

$$(\alpha_{t+1}, \beta_{t+1}) = f(\alpha_t, \beta_t, P(N \rightarrow C), \epsilon)$$

following the causal loop:

$$\alpha_t \rightarrow P(N \rightarrow C) \rightarrow R_{t+1} \rightarrow \alpha_{t+1}$$

Axiom 7 (Efficiency Principle). System efficiency is defined as:

$$\gamma(S_t) = \frac{O(S_t)}{O(S_t) + L(S_t)} \in [0, 1]$$

Expected future efficiency:

$$\mathbb{E}[\gamma(S_{t+\Delta t})] = \frac{\sum_{i < j} (V_i I_i + V_j I_j) \cdot P(R_{t+\Delta t}(i, j) = C)}{O(S_t) + L(S_t)}$$

Optimal dynamics occur when realized value, latent value, expansion, and reception reach equilibrium.