Network Optimization Calculus

Jacob D. Jaisaree

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1 Introduction

This document provides a formal mathematical framework for understanding how agency and autonomy emerge within a system of interacting nodes, while maintaining cooperation with fundamental natural laws. The key innovation is modeling the Neutral state N as a **superposition** that symmetrically mediates transitions between Cohesive (C) and Disjunctive (D) states.

2 Primitive Concepts and Axioms

The system consists of a finite set of nodes:

$$\Omega = \{X_i\}_{i \in I}$$

where each node X_i is defined by a triple (Θ_i, V_i, I_i) representing:

- Θ_i: Type (qualitative category)
- V_i: Intrinsic Value (inherent worth or potential)
- I_i: Intensity (capacity to engage in relationships)

For any distinct pair (i, j), $i \neq j$, their relationship exists in one of three mutually exclusive symmetric states:

$$R = \{C, N, D\}$$

where:

- C: Cohesive (strong, positive binding)
- N: Neutral (superposition of C and D)
- D: Disjunctive (absence of positive relationship)

The state N functions as a symmetric mediator where:

$$\mathcal{M}(N \to C) \equiv \mathcal{M}(C \to N) \equiv \mathcal{M}(N \to D) \equiv \mathcal{M}(D \to N)$$

with $\mathcal{M}$ being the superposition mechanism inherent to N.

3 Value Realization

The realized value generated by a node pair (i, j) is:

$$\Lambda(i, j) = \Psi(i, j) \cdot (V_i I_i + V_j I_j)$$

where the coefficient $\Psi(i, j)$ is determined by the relational state:

$$\Psi(i, j) = \begin{cases} 1 & \text{if } R_t(i, j) = C \\ 0 & \text{if } R_t(i, j) = N \text{ or } D \end{cases}$$

4 Optimization Objective

The total value currently generated:

$$O(S_t) = \sum_{i<j} \Lambda(i,j)$$

The total value latent in disjunctive connections:

$$L(S_t) = \sum_{\substack{i<j \\ R_t(i,j)=D}} (V_i I_i + V_j I_j)$$

Weights balancing exploitation against exploration:

$$\alpha(S_t) = \frac{|\{(i,j) : R_t(i,j) = C \text{ or } N\}|}{|\Omega|(|\Omega|-1)/2}, \quad \beta(S_t) = 1 - \alpha(S_t)$$

The system evolves toward:

$$S^* = \arg\max_{S_t} [\alpha(S_t) \cdot O(S_t) + \beta(S_t) \cdot L(S_t)]$$

5 Superposition-Based Transition Dynamics

System evolution occurs through the superposition state N:

$$D \rightleftharpoons N \rightleftharpoons C$$

where all transitions are mediated by the same superposition mechanism.

For relationships in the superposition state N, transitions occur as:

$$P(R_{t+1}(i,j) = C \mid R_t(i,j) = N) = \alpha(S_t)$$
$$P(R_{t+1}(i,j) = D \mid R_t(i,j) = N) = \beta(S_t) = 1 - \alpha(S_t)$$

With probability ϵ, the superposition state reforms:

$$P(R_{t+1}(i,j) = N \mid R_t(i,j) = C) = \epsilon$$
$$P(R_{t+1}(i,j) = N \mid R_t(i,j) = D) = \epsilon$$

6 Emergence of Autonomy Through Superposition

Autonomy emerges through the recursive relationship:

$$(\alpha_{t+1}, \beta_{t+1}) = f(\alpha_t, \beta_t, \{P(R_{t+1}(i,j) = C \mid R_t(i,j) = N)\}_{\forall i,j})$$

This creates a superposition-mediated feedback loop:

$$\alpha_t \to P(N \to C) \to \text{Distribution of } R_{t+1},$$
$$\text{Distribution of } R_{t+1} \to \alpha_{t+1} \to P(N \to C)_{t+1}.$$

7 Cooperation with Natural Laws

The system's cooperation with natural laws:

$$\gamma(S_t) = \frac{O(S_t)}{O(S_t) + L(S_t)} \in [0, 1]$$

The system tends toward states that maximize:

$$E[\gamma(S_{t+\Delta t})] = \frac{\sum_{i<j}(V_i I_i + V_j I_j) \cdot P(R_{t+\Delta t}(i,j) = C)}{O(S_t) + L(S_t)}$$

8 Interpretation: Superposition as Fundamental Mechanism

This framework reveals three key principles enhanced by the superposition perspective:

1. **Symmetric Agency in Superposition:** The Neutral state N represents genuine choice points where the same superposition mechanism mediates transitions in both directions, creating true symmetry in relationship dynamics.

2. **Autonomy from Network Superposition:** System-wide behavior emerges from local interactions mediated by superposition states, creating global patterns through distributed superposition collapses and formations.

3. **Cooperation with Natural Laws through Superposition:** The system evolves toward states that balance exploration and exploitation while respecting the fundamental superposition nature of relational transitions.

The mathematical structure demonstrates how objective laws can coexist with genuine choice and emergent autonomy through the fundamental mechanism of superposition.