Solutions to Recurrence Relation Problems

Question: 1. Find the value of $T(2)$ for the recurrence relation $T(n) = 3T(n-1) + 12n$, given that $T(0) = 5$.

Answer:

1. Given: $T(0) = 5$

2. Calculate $T(1)$:

   $T(1) = 3T(0) + 12 \times 1 = 3 \times 5 + 12 = 15 + 12 = 27$

3. Calculate $T(2)$:

   $T(2) = 3T(1) + 12 \times 2 = 3 \times 27 + 24 = 81 + 24 = 105$

Answer: $T(2) = 105$

Question: 2. Given a recurrence relation, solve it using the substitution method:

a. $T(n) = T(n-1) + c$

b. $T(n) = 2T(n/2) + n$

c. $T(n) = 2T(n/2) + c$

d. $T(n) = T(n/2) + c$

Answer:

(a) $T(n) = T(n-1) + c$ expands linearly. Solution: $T(n) = O(n)$.

(b) $T(n) = 2T(n/2) + n$ follows the Master Theorem Case 2. Solution: $T(n) = O(n \log n)$.

(c) $T(n) = 2T(n/2) + c$ follows the Master Theorem Case 1. Solution: $T(n) = O(n)$.

(d) $T(n) = T(n/2) + c$ has a logarithmic expansion. Solution: $T(n) = O(\log n)$.

Question: 3. Given a recurrence relation, solve it using the recursive tree approach:

a. $T(n) = 2T(n-1) + 1$

b. $T(n) = 2T(n/2) + n$

Answer:

(a) $T(n) = 2T(n-1) + 1$ has a depth of $O(n)$ with work doubling at each level. Solution: $T(n) = O(2^n)$.

(b) $T(n) = 2T(n/2) + n$ follows the Master Theorem. Solution: $T(n) = O(n \log n)$.