

# Brief Papers

## A Multiregion Fuzzy Logic Controller for Nonlinear Process Control

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**Abstract**—Although a fuzzy logic controller is generally nonlinear, a PI-type fuzzy controller that uses only control error and change in control error is not able to detect the process nonlinearity and make a control move accordingly. In this paper, a multiregion fuzzy logic controller is proposed for nonlinear process control. Based on prior knowledge, the process to be controlled is divided into fuzzy regions such as *high-gain*, *low-gain*, *large-time-constant*, and *small-time-constant*. Then a fuzzy controller is designed based on the regional information. Using an auxiliary process variable to detect the process operating regions, the resulting multiregion fuzzy logic controller can give satisfactory performance in all regions. Rule combination and controller tuning are discussed. Application of the controller to pH control is demonstrated.

**Index Terms**—Fuzzy logic control, nonlinear process control, fuzzy PI controllers, pH control.

### I. INTRODUCTION

**F**UZZY logic control (FLC) has been widely applied to industries in recent years. Although many of the applications are relatively small in scale—such as in washing machines, elevators, and automobiles—there is a considerable amount of interest in applying fuzzy logic systems to process control [4]–[6], [11]. Early process control applications include a cement kiln control [8], where FLC is particularly useful because of the difficulty in derivation of a mathematical model for the process. Recent applications of FLC spread over various areas of automatic control, which are surveyed in [11] and [5]. In the field of process control, much research has been conducted in using fuzzy logic similar to conventional PID controllers [13], [15]. In [6], fuzzy logic has been applied in a gain scheduler which adapts PID controller parameters based on process nonlinearity. Theoretical studies including stability of FLC are of particular interest [3], [10].

A typical fuzzy logic controller is composed of three basic parts: input signal fuzzification, a fuzzy engine that handles rule inference, and defuzzification that generates continuous signals for actuators such as control valves. Fig. 1 depicts such a fuzzy logic controller. The fuzzification block transforms the continuous input signal into linguistic fuzzy variables such as *Small*, *Medium*, and *Large*. The fuzzy engine carries out rule inference where human experience can easily be injected through linguistic rules. The defuzzification block converts the inferred control action back to a continuous signal

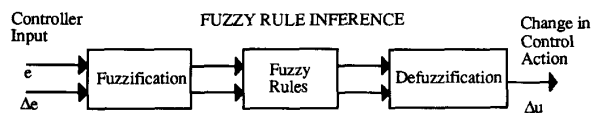


Fig. 1. A schematic diagram for a typical fuzzy logic controller.

that interpolates between simultaneously fired rules. Owing to defuzzification, fuzzy logic is sometimes referred to as continuous logic or interpolative reasoning [14]. The resulting relation is actually a nonlinear functional relationship rather than a logic relationship.

Two distinct features of fuzzy logic control are 1) that human experience can easily be integrated, and 2) that fuzzy logic provides a nonlinear relationship induced by membership functions, rules, and defuzzification. These features make fuzzy logic promising for process control where conventional control technologies do a poor job and human operator experience exists. Although there are many published works in the field of fuzzy logic control, most of them only use control error ( $e$ ) and change of the control error ( $\Delta e$ ) as controller inputs. Using only these inputs, the fuzzy controller is not able to differentiate in which region the process operates. Therefore, such a fuzzy controller cannot make a control action based on the knowledge of the process nonlinearity associated with different regions. In this paper, a multiregion fuzzy logic controller that uses auxiliary process variables as controller inputs is proposed. The auxiliary variable is used to indicate in which region the process is operating. Such a fuzzy controller can compensate for process nonlinearity so that the control performance can be made more uniform throughout different regions of the process non-linearity.

The organization of the paper is divided into five sections. In Section I, we have provided a brief introduction to fuzzy logic control and application. Section II presents a controller framework that uses control error, change of error, and an auxiliary process variable to make appropriate control moves corresponding to process nonlinearity. Also discussed are issues of tuning the controller and rule combination. Section III demonstrates application of the multiregion fuzzy controller to a pH control in a simulated continuously stirred tank reactor (CSTR) which has dramatic gain nonlinearity. Section IV discusses several practical issues related to the fuzzy controller and fuzzy rule reduction. Conclusions are given in the Section V.

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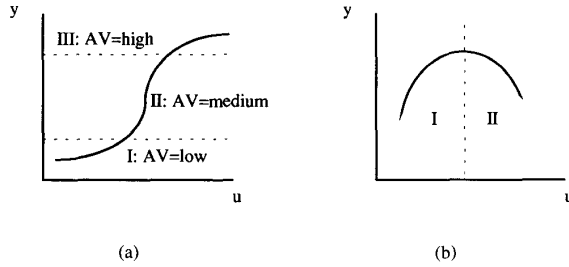


Fig. 2. Two typical cases of processes with nonlinear gains: (a) sigmoidal; and (b) process gain changing signs.

## II. A FUZZY CONTROLLER WITH AN AUXILIARY PROCESS VARIABLE

Most industrial processes demonstrate considerable nonlinearity with respect to different regions of operation. Depending on process characteristics, the process regions can often be categorized as *high-gain*, *low-gain*, *large-time-constant*, and *long-time-delay*, etc. If the process gain ( $G_p$ ), the time constant ( $T_c$ ) and the time delay ( $T_d$ ) are used to characterize a nonlinear process, they depend on different process regions. For example, Fig. 2 illustrates two situations of gain nonlinearity. The sigmoidal shape of the nonlinear gain in Fig. 2(a) has two low-gain regions and one high-gain region. If an auxiliary variable ( $AV$ ) is used to indicate different regions of the nonlinear process, the process nonlinearity can be represented as the following fuzzy relationships:

- IF  $AV$  is in Region I, THEN  $G_p = low$ ;  
 IF  $AV$  is in Region II, THEN  $G_p = high$ ;  
 IF  $AV$  is in Region III, THEN  $G_p = low$ .

Fig. 2(b) gives another example where the process changes sign.

It is difficult to design a fuzzy controller for the two situations by using only the control error and change in the control error (as shown in Fig. 1). If one is to design such a fuzzy controller that works for the case as shown in Fig. 2(a), for example, one has to assure that the controller is stable in the high-gain region while sacrificing performance in the low-gain regions. In other words, a fuzzy controller that does not differentiate between the high- and low-gain regions cannot provide satisfactory performance in all regions. The case illustrated in Fig. 2(b) is less often encountered, but it provides further difficulty in controller design. If one were to design a uniform controller for all regions, one would have negative feedback in one region and positive feedback in another. Therefore, the controller designed as such cannot guarantee stability.

To design a fuzzy logic controller that gives satisfactory performance for different regions of gain nonlinearity, a multi-region fuzzy controller that uses an auxiliary process variable is proposed to allow designing of different control strategies in different regions. The fuzzy control framework is depicted in Fig. 3. In addition to using control error and change in the control error as inputs, an auxiliary variable is used as another input to determine in which region the process is

operating. The functional relationship represented by such a fuzzy controller can be described as follows:

$$\Delta u = FLC(\Delta e, e, AV) \quad (1)$$

where  $FLC(\cdot)$  stands for the nonlinear relationship of the fuzzy controller. The auxiliary variable ( $AV$ ) can be the process input ( $u_k$ ) or the process output ( $y_k$ ), depending on how the operation regions should be defined. For example, it is convenient to use  $y_k$  as the  $AV$  in the case of sigmoidal nonlinear gain as shown in Fig. 2(a), but one has to use  $u_k$  as the  $AV$  in the case of Fig. 2(b).

Given a particular region of  $AV$ , the fuzzy controller can be designed based on the process knowledge associated with that region. The remaining relation is merely between  $\Delta e$ ,  $e$ , and  $\Delta u$ , and therefore it can be designed similar to a PI-type of fuzzy controller [15]. For example, if the process is in a region where the process gain is *low*, an aggressive PI-type fuzzy controller should be designed. If the process gain is *high*, the control strategies should be less aggressive. During the transition from one region to another, the control action is smoothed naturally by the interpolative capability of the fuzzy logic. Detail design consideration is given in the following subsections.

### A. Membership Function Definition

The fuzzy membership functions associated with each controller input can be defined based on prior knowledge about the process. To illustrate how to define the membership function, the sigmoidal nonlinear gain in Fig. 2(a) is used as an example. The auxiliary variable for the case of Fig. 2(a) should have three regions: *High*, *Medium*, and *Low*. For the control error ( $e$ ), change of control error ( $\Delta e$ ), and control action ( $\Delta u$ ), it is convenient to use scaled variables as

$$e^* = \frac{e}{S_e} \quad (2)$$

$$\Delta e^* = \frac{\Delta e}{S_{\Delta e}} \quad (3)$$

$$\Delta u^* = \frac{\Delta u}{S_{\Delta u}} \quad (4)$$

where  $S_e$ ,  $S_{\Delta e}$ , and  $S_{\Delta u}$  are scaling factors for  $e$ ,  $\Delta e$ , and  $\Delta u$ , respectively. A set of fuzzy membership functions for  $AV$ ,  $e^*$ ,  $\Delta e^*$ , and  $\Delta u^*$  are given in Fig. 4. The membership partitions for  $e^*$ ,  $\Delta e^*$ , and  $\Delta u^*$  are usually symmetric from  $-1$  to  $1$ , however, the partition of  $AV$  should be according to process knowledge. The number of membership functions for each variable can vary, depending the resolution required for that variable. Generally speaking, more membership functions offer more degrees of freedom to the functional relationship of the controller, however, it requires more effort to implement. It is shown by Ying *et al.* [13] that a conventional PID controller can be reproduced using fuzzy logic with two membership functions for each variable. Therefore, the effect of using more than two membership functions is merely adding nonlinearity. It should be noted that the change in the control action ( $\Delta u^*$ ) usually requires higher resolution than other variables because the controller must be able to make aggressive and less aggressive control moves based on process nonlinearity.

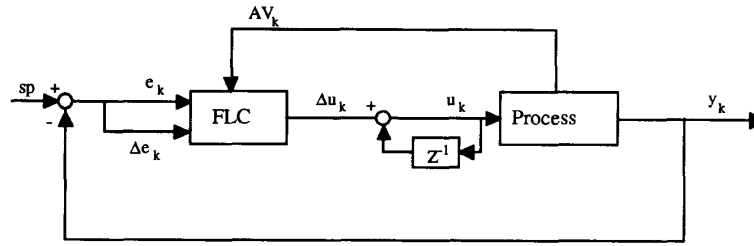


Fig. 3. A fuzzy controller structure that uses an auxiliary process variable to detect where the process is operating and compensate for gain nonlinearity.

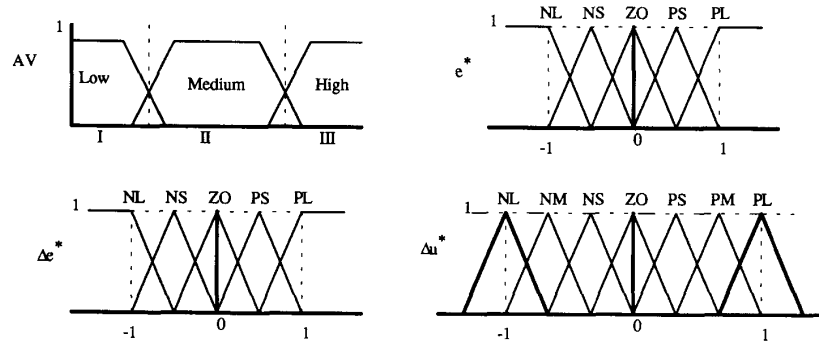


Fig. 4. Membership function definition for the fuzzy controller. NL = Negative Large, NM = Negative Medium, nS = Negative Small, ZO = Zero, PS = Positive Small, PM = Positive Medium, PL = Positive Large.

TABLE I  
FUZZY RULES FOR A 3-REGION FUZZY CONTROLLER

AV = Low						AV = Medium						AV = High					
e	Δe					e	Δe					e	Δe				
	NL	NS	ZO	PS	PL		NL	NS	ZO	PS	PL		NL	NS	ZO	PS	PL
NL	PL	PL	PL	PM	ZO	NL	PM	PM	PM	PS	ZO	NL	PL	PL	PM	PS	ZO
NS	PL	PL	PM	ZO	NM	NS	PM	PM	PS	ZO	NS	NS	PL	PL	PM	ZP	NS
ZO	PM	PM	ZO	NM	NL	ZO	PM	PS	ZO	NS	NM	ZO	PL	PM	ZO	NM	NM
PS	PS	ZO	NM	NL	NL	PS	PS	ZO	NS	NM	NM	PS	PM	ZO	NM	NL	NL
PL	ZO	NS	NM	NL	NL	PL	ZO	NS	NM	NM	NM	PL	ZO	NM	NL	NL	NL

This consideration will be discussed in rule definition in the next subsection.

### B. Rule Definition

A general fuzzy inference rule for the multiregion fuzzy controller can be described as follows:

{If  $AV$  is  $A_i$  and  $e$  is  $B_i$  and  $\Delta e$  is  $C_i$  then make  $\Delta u D_i$ } (5)

where  $A_i, B_i, C_i$ , and  $D_i$  are adjectives for  $AV, e, \Delta e$ , and  $\Delta u$ , respectively. These adjectives could be descriptors such as *Negative Small*, *Positive Large*, or *Zero*. Within each region of the process, designated as the auxiliary variable ( $AV$ ), the controller can be designed as a PI-type of fuzzy controller [15]. A fundamental requirement for these rules is that they have to perform negative feedback control for the sake of stability. An exemplar set of rules is listed in Table I. In real process applications, the fuzzy rules need to be further adjusted from prior control experience of human operators. In general, when the process is operating in the low-gain region, aggressive control action is demanded. On the other hand,

when the process is in a high-gain region, mild to low control action should be used. The aggressiveness of control action can be seen by whether it activates a *Large* control action or not. To make more aggressive actions, *Positive Large* and *Negative Large* control moves are activated. However, these control moves should never be activated to make a mild control action.

With a 3-region fuzzy controller using 5 adjectives, the number of possible rules is 75. If the controller uses 7 adjectives, a total of 147 rules are resulted. Because more rules require more computing time and memory, it is recommended to reduce the number of rules without affecting the controller performance. Since the control logic for each region is similar to one another, it is possible to reduce the number of rules by combining similar rules for different regions. For example, no matter what  $AV$  is, the *Zero* control action ( $\Delta u$ ) is fired under the same condition of  $e$  and  $\Delta e$ . By combining these rules in Table I, one can easily eliminate 10 of the rules without changing functionality. Moreover, if the control rules for low and high  $AV$  are the same, elimination of 25 rules can be

accomplished. Thus, rule reduction can be done by combining similar rules in the multiregion fuzzy controller.

### C. Tuning of Fuzzy Controllers

Tuning of the multiregion fuzzy controller includes 1) tuning of scaling factors, 2) tuning of fuzzy membership functions, 3) tuning of fuzzy rules, and 4) tuning of  $AV$  membership functions for smooth regional transitions. The scaling factors should be tuned with first priority because they are global tuning parameters that affect the overall control performance. A membership function, which has an effect on one subset of rules, can be tuned secondly. Individual fuzzy rules should be tuned last because they affect only the local nonlinearity of the controller.

Within each region, the fuzzy controller can be tuned by using techniques similar to those used for a fuzzy PI controller [13]. For a fuzzy controller shown in Fig. 1, with two membership functions for control error, two membership functions for change in control error, and four control rules, Ying *et al.* [13] have shown that the resulting controller has the following relation:

$$\Delta u^* = \frac{e^* + \Delta e^*}{2(2 - \max(|e^*|, |\Delta e^*|))}; \quad |e^*| \leq 1, |\Delta e^*| \leq 1. \quad (6)$$

Furthermore, if a linear defuzzification is used, the resulting fuzzy control relation becomes

$$\Delta u^* = 0.5(e^* + \Delta e^*) \quad (7)$$

or

$$\Delta u = 0.5 \frac{S_{\Delta u}}{S_{\Delta e}} \left( \Delta e + \frac{S_{\Delta e}}{S_e} e \right) \quad (8)$$

in terms of unscaled variables. Given a PI controller that has the following form:

$$u(t) = K_p \left[ e(t) + \frac{1}{T_i} \int e(t) dt \right] \quad (9)$$

or in a discrete-time form

$$\Delta u = K_p \left( \Delta e + \frac{\Delta t}{T_i} e \right) \quad (10)$$

one can easily see that  $K_p$  and  $T_i$  relate to the scaling factors through the following equations:

$$K_p = 0.5 \frac{S_{\Delta u}}{S_{\Delta e}} \quad (11)$$

$$T_i = (S_e / S_{\Delta e}) \cdot \Delta t \quad (12)$$

where  $K_p$  is the proportional gain,  $T_i$  is the integral time constant, and  $\Delta t$  is the controller sampling time constant. Therefore, if one wants to strengthen proportional action, one can either increase  $S_{\Delta u}$  or decrease  $S_{\Delta e}$ . Since  $S_{\Delta u}$  is constrained by the physical response speed of the actuator, the ultimate tuning parameter for proportional control is actually  $S_{\Delta e}$ . To strengthen the integral control action, one should either decrease  $S_e$  or increase  $S_{\Delta e}$  because the small integral time constant represents strong integral control action. Since  $S_{\Delta e}$  is the ultimate tuning parameter for proportional action,

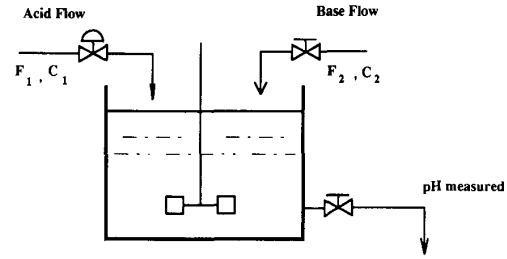


Fig. 5. A continuously stirred tank reactor for pH titration control.

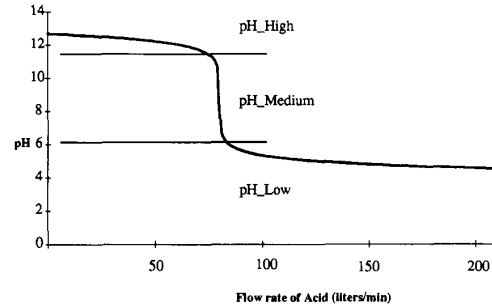


Fig. 6. The steady-state nonlinear relation of pH versus acid flow rate.

the ultimate tuning parameter for integral action should be  $S_e$ . Note that the controller sampling time ( $\Delta t$ ) can also affect the integral time, but it should not be used as a tuning parameter, in general.

After being tuned in each region, the fuzzy controller tuning should be coordinated over all regions. For example, if one uses the scaling factor  $S_{\Delta u}$  to tune the low-gain region, one can tune the high-gain region by adjusting the position of the inner membership functions of  $\Delta u$  which give mild control actions. To achieve smooth transition between regions, the membership functions for  $AV$  can be tuned based on process knowledge. In summary, the following procedure can be used to tune the multiregion fuzzy controller.

- 1) Tune the scaling factors ( $S_e$ ,  $S_{\Delta e}$ , and  $S_{\Delta u}$ ) for the low-gain region.
- 2) Tune the position of the inner membership functions for  $\Delta u$  for the high-gain region. This basically results in mild control actions.
- 3) Tune the membership functions of the auxiliary variable ( $AV$ ) over all regions to achieve smooth transition of control.
- 4) Fine-tune the membership functions and the rules to achieve desired control performance.

### III. CONTROL OF A pH CSTR

A continuously stirred tank reactor (CSTR) for pH titration is shown in Fig. 5. The CSTR has two input streams—an acid and a base. In this simulation, the acid flow rate ( $F_1$ ) is used to control the pH value of the solution, and the base concentration ( $C_2$ ) is used as a disturbance. The mathematical equations of the CSTR can be described as follows by assuming the tank

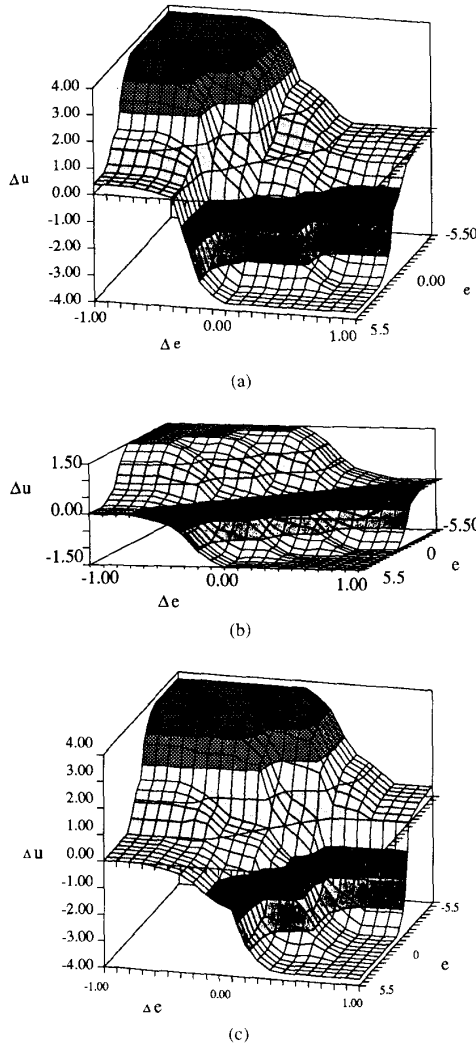


Fig. 7. Control actions versus error and change in the error different pH values (a) pH = 13 (High); (b) pH = 9 (Medium); and (c) pH = 4 (Low).

level is controlled perfectly [7]:

$$V \frac{d\xi}{dt} = F_1 C_1 - (F_1 + F_2) \xi \quad (13)$$

$$V \frac{d\zeta}{dt} = F_2 C_2 - (F_1 + F_2) \zeta \quad (14)$$

$$[H^+]^3 + (K_a + \zeta)[H^+]^2 + (K_a(\zeta - \xi) - K_w)[H^+] - K_w K_a = 0 \quad (15)$$

$$pH = -\log_{10} [H^+] \quad (16)$$

where

$$\xi \cong [HAC] + [AC^-] \quad (17)$$

$$\zeta \cong [Na^+]. \quad (18)$$

The physical meaning of each variable and its associated initial values is listed in Table II.

TABLE II  
THE PHYSICAL PARAMETERS USED IN THE SIMULATED  
pH CONTINUOUSLY STIRRED TANK REACTOR

Variable	Meaning	Initial Setting
$V$	Volume of Tank	1000 l
$F_1$	Flow Rate of Acid	81 l/min
$F_2$	Flow Rate of Base	512 l/min
$C_1$	Concentration of Acid in $F_1$	0.32 moles/l
$C_2$	Concentration of Acid in $F_2$	0.05005 moles/l
$K_a$	Acid Equilibrium Constant	$1.8 \times 10^{-5}$
$K_w$	Water Equilibrium Constant	$1.0 \times 10^{-14}$

The pH CSTR is used here for testing the multiregion fuzzy controller. Fig. 6 depicts the steady-state nonlinear relationship of pH with respect to the acid flow rate ( $F_1$ ). Roughly, three regions of nonlinear gains can be identified: pH-High, pH-Medium, and pH-Low. In the regions of pH-High and pH-Low, the process gain is extremely small, while in the pH-Medium region the process gain is extremely high. In the high-gain region, a small change of the acid flow rate would result in a large change in pH value, while in the low-gain regions, considerable changes have to be made in the acid flow rate to make an appreciable change in the pH value. This example presents a difficult task to a conventional PID controller. One has to use low controller gains to maintain stability in the high-gain region, but dynamic response in the low-gain region will be very sluggish.

To achieve good control of pH values over all regions, a 3-region FLC is designed for the pH CSTR. The pH value is used as the auxiliary variable. The three regions can generally be defined using human knowledge of the process. Here it is defined based on the steady-state relation as shown in Fig. 6. After designing the fuzzy rules for each process gain region, the control response surfaces (control action versus control error and change in control error) are depicted in Fig. 7 for three different regions. One can see that even under the same condition of control error and change in control error, the control action is different in low-gain regions and the high-gain region. In the low-gain regions, the control action is more aggressive. However, in the high-gain region, the control action is made less aggressive. This feature cannot be designed in the regular PI-type of fuzzy controllers. Another observation is that the control surfaces are fairly nonlinear, which cannot be achieved using conventional PID controllers. It should be noted that for transient regions between the three regions, the control surface is actually an interpolation of the three control surfaces implied by the

To examine the control performance over all regions, the setpoint of pH is changed from 5.5 to 7.0, 10.0, and 11.5, and then from 11.5 to 10.0, 7.0, and 5.5. Fig. 8 depicts the dynamic response with setpoint changes. One can see that the same step change in acid flow rate does not result in the same time response in pH. Further, even in the same region, positive and negative step changes result in a different time response. When the step change is from a low-gain region to a high-gain region, a larger overshoot is expected. However, when the step change is from a high-gain region to a low-gain region, little overshoot occurs.

For comparison, a PI-type of fuzzy controller [15] is designed to control the pH CSTR. To assure the stability in all

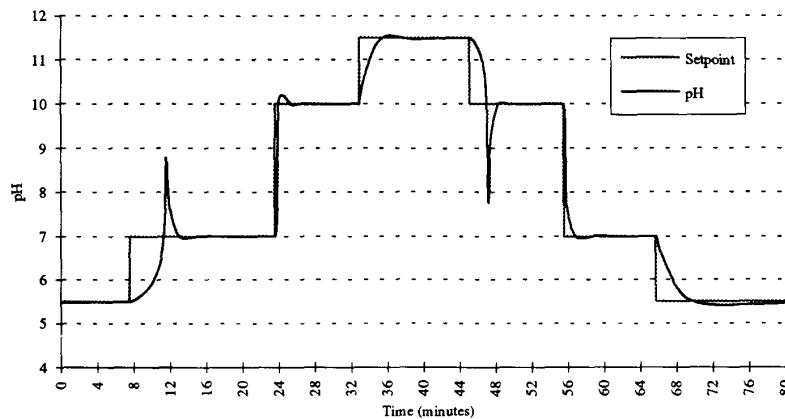


Fig. 8. Step response of the pH CSTR controlled by 3-region fuzzy logic controller.

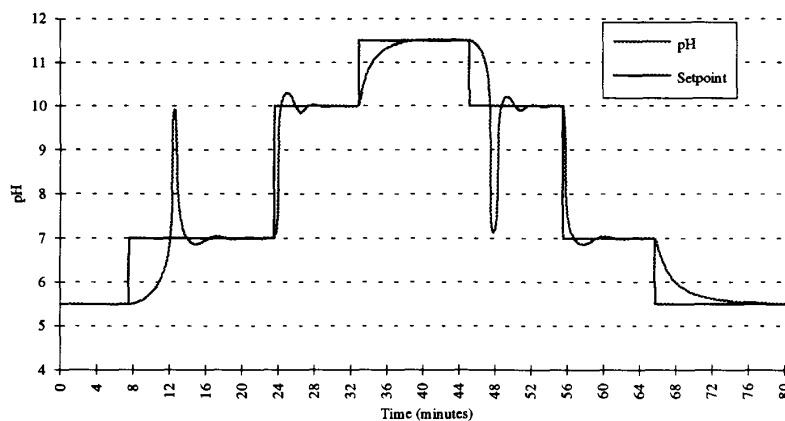


Fig. 9. Step response of the pH CSTR controller by a PI type of fuzzy controller.

three regions, the controller gain has to be carefully adjusted so that the CSTR is stable in the high-gain region. To control performance of this controller is shown in Fig. 9, where similar step changes are applied to the controller. By comparing the one-region FLC to the 3-region FLC, it is seen that the 3-region FLC performs better in that it has less overshoot in moving from the low-gain region to high-gain region. In addition, the overall settling time for the 3-region controller is less than that for the 1-region controller. This result is expected because the 3-region FLC offers more flexibility than the 1-region PI-type of FLC in handling the nonlinear gain of the pH CSTR.

To examine how the 3-region fuzzy logic controller performs in rejecting disturbance changes, the base concentration ( $C_2$ ) is used as the source of disturbance. Fig. 10 shows the 3-region fuzzy controller performance in the presence of disturbances in base concentration. Both positive and negative disturbance changes are applied while the pH setpoint is fixed at 7. It is seen that the fuzzy controller performs well in rejecting disturbance changes in both directions.

#### IV. DISCUSSIONS

Having demonstrated the concept of multiregion fuzzy control for nonlinear systems, several issues in applying fuzzy

logic to process control are discussed in this section. The first question which arises from applying fuzzy logic to process control is: what benefit can be achieved if the existing controllers are replaced by fuzzy controllers? Another question is: is fuzzy logic control mature enough to be applied to process control in terms of tuning and stability analysis? [3], [10].

Since many industrial processes are controlled by PID controllers, it is logical to compare fuzzy logic to PID controllers. A comparison of FLC with classical controllers is given by Tang *et al.* [12]. In fact, a PI-type of fuzzy logic controller is simply a nonlinear version of conventional PI controllers. Ying *et al.* [13] have shown that a simple fuzzy controller with two membership functions for each variable is equivalent to a nonlinear PID controller. Ying *et al.* have further demonstrated with simulation that fuzzy controllers perform better than conventional PID controllers. Below are a few comments on the differences between FLC's and conventional controllers.

- 1) Fuzzy logic controllers have constraints in the change of control moves with definition of the scaling factors, which is a good feature for designing controllers for industrial processes. The constraints are related to the physical response speed of the actuators. PID controllers,

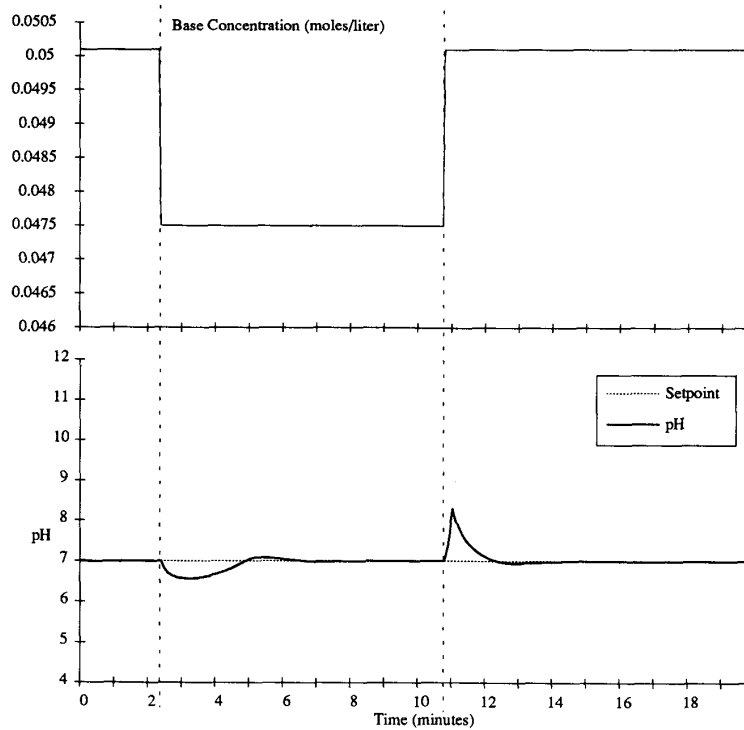


Fig. 10. The 3-region pH control performance for positive and negative disturbances in the base concentration.

however, do not have constraints on control moves unless specifically designed. Therefore, one should expect superior control response from fuzzy controllers when conventional PID controllers run against hard constraints of actuators.

- 2) Fuzzy controllers are very flexible in implementing a nonlinear relationship, while standard PID controllers are usually linear. As shown by Ying *et al.* [13], a fuzzy controller with more than two membership functions is essentially more flexible than a PID controller.
- 3) Conventional control technologies such as PID and PID with automatic tuning capability [1], [9] perform well in the control of many industrial processes. When PID controllers perform equally well as fuzzy controllers, PID controllers are preferred because of easier implementation and tuning. Fuzzy logic controllers should seek applications where conventional control technologies perform poorly but human operators can do an excellent job.
- 4) The multiregion fuzzy controller is similar in concept to gain schedulers [2]. The difference is that the multiregion fuzzy controller is based on fuzzy logic rules, while gain schedulers are usually based on PID controllers.
- 5) The concept of fuzzy control is intuitive and simple, but the tuning can be much more complex than PID tuning. The reason is that there are many parameters to adjust in a fuzzy controller, such as scaling factors, rules, and membership functions, while PID controllers have only three parameters to tune. A convenient tuning method

for fuzzy logic controllers is desirable.

## V. CONCLUSION

The multiregion fuzzy logic controller proposed in this paper uses an auxiliary variable to indicate different nonlinear regions of processes and yield better control performance than a 1-region fuzzy controller. It is demonstrated through simulation that the multiregion fuzzy controller performed well in controlling a pH CSTR which is highly nonlinear process. The fuzzy controller is flexible in implementing a nonlinear control strategy. The final fuzzy controller tuning deserves further study.

## APPENDIX NOMENCLATURE

- $\Delta e$  change of control error
- $\Delta e^*$  scaled change of control error
- $\Delta t$  sampling time constant
- $\Delta u$  change of controller output
- $\Delta u^*$  scaled change of controller output
- AV auxiliary variable
- $C_1$  concentration of acid in  $F_1$
- $C_2$  concentration of acid in  $F_2$
- $e$  control error (the difference between setpoint and process output)
- $e^*$  scaled control error

- $F_1$  flow rate of acid  
 $F_2$  flow rate of base  
 $K_a$  acid equilibrium constant  
 $K_p$  proportional gain of PI controllers  
 $K_w$  water equilibrium constant  
 $S_{\Delta e}$  scaling factor for change of control error  
 $S_{\Delta u}$  scaling factor for change of controller output  
 $S_e$  scaling factor for control error  
 $T_i$  integral time constant of PI controllers  
 $V$  volume of tank  
 $\xi$  as defined in equation (13)  
 $\zeta$  as defined in equation (14)

## REFERENCES

- [1] K. J. Åström and T. Hägglund, *Automatic Tuning of PID Controllers*. Res. Triangle Park, NC: Instrument Soc. of Amer., 1988.
- [2] K. J. Åström and B. Wittenmark, *Adaptive Control*. Reading, MA: Addison Wesley, 1989.
- [3] C. J. Harris and C. G. Moore, "Phase plane analysis tools for a class of fuzzy control," in *Proc. 1st IEEE Int. Conf. Fuzzy Syst.*, Mar. 8–12, 1992, pp. 511–518.
- [4] C. L. Karr and E. J. Gentry, "Fuzzy control of pH using genetic algorithms," *IEEE Trans. Fuzzy Syst.*, vol. 1, pp. 46–53, 1993.
- [5] C. C. Lee, "Fuzzy logic in control systems: Fuzzy logic controllers—Parts I & II," *IEEE Trans. Syst., Man, Cybern.*, vol. 20, pp. 404–435, 1990.
- [6] C. Ling and T. Edgar, "A new fuzzy gain scheduling algorithm for process control," in *Proc. ACC'92*, Chicago, IL, 1992, pp. 2284–2290.
- [7] T. J. McAvoy, "Time optimal and Ziegler–Nichols control," *Ind. Eng. Chem. Process Des. Develop.*, vol. 11, no. 1, pp. 71–78, 1972.
- [8] J. J. Ostergaard, "Fuzzy logic control of a heat exchange process," in *Fuzzy Automata and Decision Processes*, M. M. Gupta, G. N. Saridis, and B. R. Gaines, Eds. Amsterdam: North Holland, 1977, pp. 285–320.
- [9] F. G. Shinskey, *Process Control Systems: Application, Design, and Adjustment*, 3rd ed. New York: McGraw-Hill, 1988.
- [10] S. Singh, "Stability analysis of discrete fuzzy control systems," in *Proc. 1st IEEE Int. Conf. Fuzzy Syst.*, Mar. 8–12, 1992, pp. 527–536.
- [11] M. Sugeno, "An introductory survey of fuzzy control," *Inform. Sci.*, vol. 36, pp. 59–83, 1985.
- [12] K. L. Tang and R. J. Mulholland, "Comparing fuzzy logic with classical controller design," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-17, pp. 1085–1087, 1987.
- [13] H. Ying *et al.*, "Fuzzy control theory: A nonlinear case," *Automatica*, vol. 26, no. 3, pp. 513–520, 1990.
- [14] L. A. Zadeh, "Interpolative reasoning in fuzzy logic and neural network theory," in *Proc. 1st IEEE Int. Conf. Fuzzy Syst.*, p. 1, Mar. 8–12, 1992.
- [15] L. Zheng, "A practical guide to tune of proportional and integral (PI) like fuzzy controller," in *Proc. 1st IEEE ICFS*, Mar. 8–12, 1992, pp. 663–640.