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## Adaptive fuzzy sliding mode control in PH neutralization process

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### Abstract

In chemical industry process, PH neutralization process consists of a complex and multi-variable nonlinear coupling system. This paper combines the fuzzy control and sliding mode control, and fully using the system state information to simplify the fuzzy control rules. This paper gives the adaptive fuzzy sliding mode control algorithm as well as the stability-state analysis. The quantification scale factor changing adaptively adjusts the universe of fuzzy control system, makes system control signals soften and reduces the chattering of sliding mode control. Designed control system has strong robustness and good adaptive ability. Simulation results show that even a major change in operating point or suffer greater interference, the systems still have good anti-disturbance and strong robustness.

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### 1. Introduction

Sliding mode control (SMC) has strong robustness, suitable for nonlinear system control. It depends on the mathematic model of system. Fuzzy control do not depend on the mathematic model, but more input dimension and control rules make the control method more complex. The combination of fuzzy control and slide control can reduce the fuzzy controller input number and fully utilize the information of system, this method is ease to create fuzzy rule table.

At present, most study of PH neutralization process are aimed at single-input and single-output (SISO) systems<sup>[1-4]</sup>. The PH neutralization reaction of multi-input and multi-output (MIMO) systems is

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more complicated. This paper uses fuzzy sliding mode control (FSMC) in multivariable PH neutralization reaction. The creation of fuzzy control table is based on the principle of Lyapunov, make the closed-loop system asymptotically stable. The proportion quantify scale factors are obtained automatically by different values of sliding surface variable, So as to achieve variable universe adaptive fuzzy control purposes, Based on the study on multivariable pH neutralization reaction, the proposed method has good tracking performance, robustness and good dynamic performance for nonlinear uncertain MIMO systems.

## 2. PH model

The neutralization process essentially involves a strong acid, a buffer and a strong base. The model described as follows four equations:

$$\dot{W}_a = \frac{1}{Ah} [(W_{a1} - W_a)q_1 + (W_{a2} - W_a)q_2 + (W_{a3} - W_a)q_3] \quad (1)$$

$$\dot{W}_b = \frac{1}{Ah} [(W_{b1} - W_b)q_1 + (W_{b2} - W_b)q_2 + (W_{b3} - W_b)q_3] \quad (2)$$

$$W_a + 10^{PH-14} + W_b \frac{1 + 2 \times 10^{PH-pk_2}}{1 + 10^{pk_1-PH} + 10^{PH-pk_2}} - 10^{-PH} = 0 \quad (3)$$

$$\dot{h} = \frac{1}{A} (q_1 + q_2 + q_3 - C_v h^{0.5}) \quad (4)$$

Where  $W_a = [H^+] - [OH^-] - [HCO_3^-] - 2[CO_3^{2-}]$ ,  $W_b = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}]$ . Define the system state as follows:  $x = [W_a \quad W_b \quad h]$ ,  $u_1 = q_1$ ,  $u_2 = q_3$ ,  $y_1 = PH$ ,  $y_2 = h$ . Then PH neutralization process can defined as follow:

$$\dot{x} = f(x) + g(x)u \quad (5)$$

## 3. FSMC controller design

### 3.1. Multivariable FSMC controller design

The sliding surface of sliding mode control defined as:

$$S(\tilde{x}, t) = \left(\frac{d}{dt} + c\right)^{n-1} \tilde{x} = \sum_{i=1}^{n-1} c_i \tilde{x}_i + \tilde{x}_n \quad (6)$$

Where  $c_i > 0$ ,  $\tilde{x} = x_i - x_{di}$ , ( $i = 1, 2, \dots, n$ ),  $\tilde{x}$  is state error and  $x_{di}$  is set value. Upon reaching sliding surface, the control law of system is equivalent control. System is less sensitive for model uncertainty, bounded disturbance and model parameter's change. It makes the system reduced susceptibility state vector  $x_i$  tracking  $x_{di}$ . Control law is always meeting the conditions as follow:

$$\frac{1}{2} \frac{d}{dt} S^2 \leq -\eta |S| \leq 0 \quad (7)$$

Where,  $\eta$  is positive constant. The controller design output  $u$  called equivalent control  $u_{eq}$ . Because the system contains uncertainty and disturbance, so the control law must add a discrete condition: Switch control quantity, it meet the sliding surface sliding condition and stability.

$$u = u_{eq} + u_s = u_{eq} + k \text{sgn}(s) \quad (8)$$

Where,  $k$  is a constant and  $\text{sgn}(s)$  is a switch function. The controller design is based on formula (7). We adopted  $V(t) = (1/2)S^2$  as Lyapunov function of fuzzy control system. We define  $S$  and  $\dot{S}$  are two inputs of the fuzzy controller and  $u$  is the controller output. Based on the analysis shows that the input and output following rules: If  $S > 0$ , decreasing  $u$  causes  $S\dot{S} < 0$  and if  $S < 0$ , increasing  $u$  makes  $S\dot{S} < 0$ . And it meet the controller stability. In this paper, we using seven fuzzy sets assigned to each control variable {NB, NM

, NS, Z, PS, PM, PB}, and respectively represent {negative big, negative medium, negative small, zero, positive small, positive medium, positive big}. The rule table shows at table 1.

In the actual industrial process, general fuzzy controller of control rules are experience summary, the adaptive ability is bad and can't reach the expected effect of control. The problem is how to make use of the input and output variables rule and get optimal control effect. Hongxing Li<sup>[5]</sup> proposed a theory of "variable universe". The input and output can range domain according to control conditions and certain standards. It can improve the accuracy of the controller. Therefore, this paper used this method in FSMC and the control performance is better. Variable universe fuzzy control is based on the fuzzy control and the input of the fuzzy controller added an expansion factor  $\alpha(S)$ . Contraction-expansion factor used as follows:  $\alpha(S) = 1 - 0.7e^{-0.1s^2}$ ,  $\lambda \in (0, 1), k \geq 1$ . After using the contraction-expansion factor  $[-\alpha_1(S)\tilde{x}_{i\max}, \alpha_1(S)\tilde{x}_{i\min}]$ , the universe  $\alpha(x)$  became  $[-\tilde{x}_{i\max}, \tilde{x}_{i\min}] \cdot \alpha(x)$ . In puts  $S$  and  $\dot{S}$  are effect before fuzzy inference.  $S/\alpha_1(S)$  and  $\dot{S}/\alpha_2(\dot{S})$  as new inputs by fuzzy inference. This method is equal to contraction-expansion  $\alpha(x)$  action on universe in control and also can be equal to adjust the quantization factor  $k_1, k_2$  of fuzzy controller. According to the PH model, we choose contraction-expansion as follows:  $k_1 = (1 - 0.7e^{-0.1s^2}) * 2$ ,  $k_2 = (1 - 0.7e^{-5\dot{s}^2}) * 3$ . This method in the application process, don't modify the original fuzzy control rules and improve the control efficiency.

### 3.2. The stability analysis of variable universe FSMC controller

Fuzzy basis function defined as follows:

$$p_j(x) = \prod_{i=1}^n A_{ij} \left( \frac{x_i(t)}{\alpha(x_i(t))} \right) \quad (9)$$

Then the output of the fuzzy controller is:

$$u_f = \mu(y) \sum_{j=1}^M p_j(x) y_j = \sum_{j=1}^M p_j(x) (\mu(y) \cdot y_j) = \sum_{j=1}^M p_j(x) \theta_j = \mu \eta \quad (10)$$

Fuzzy control variables are defined in a bounded domain of the fuzzy variables such as  $\text{Sup}|u^* - u_f| < \varepsilon$ . When the system state variables are in the bounded area, Fuzzy control output  $u_f$  to approximate sliding mode control law  $u^*$ . As we all know, there is a best approximation of fuzzy control parameters:  $\theta^* = [\theta_1^* \ \theta_2^* \ \dots \ \theta_m^*]^T$ . So the optimal output of the fuzzy controller was:

$$u_f^* = \sum_{j=1}^M p_j(x) \theta_j^* \quad (11)$$

$$u_f - u_f^* = \sum_{j=1}^M p_j(x) (\theta_j - \theta_j^*) = \sum_{j=1}^M p_j(x) \tilde{\theta}_j \quad (12)$$

Where,  $\tilde{\theta} = \theta_j - \theta_j^*$ . From (5), we have

$$u_{eq} = \frac{1}{g} [-f(x) - d(x) - \sum_{i=1}^{n-1} c_i \tilde{x}^{(i)} + \dot{x}_d^{(n)}] \quad (13)$$

From (6), we have

$$\dot{S} = c_1 \tilde{x}_1 + c_2 \tilde{x}_2 + \dots + \tilde{x}_n = \sum_{i=1}^{n-1} c_i \tilde{x}^{(i)} + \dot{\tilde{x}}_n = \sum_{i=1}^{n-1} c_i \dot{\tilde{x}}_i + \dot{x}^{(n)} - \dot{x}_d^{(n)} \quad (14)$$

And also, from (5), (13) and (14), we have  $\dot{S} = g(x)(u_{eq} - u_f)$

$$\text{So} \quad \dot{S} = g(x)(u - u_s - u_f) \quad (15)$$

$$\text{Then (15) can be reduce to } S\dot{S} = S \cdot g(x)(u - u_s - u_f) \quad (16)$$

For nonlinear system, we choose lyapunov function as follows:

$$V = \frac{1}{2g(x)} S^2 + \frac{1}{2} \tilde{\theta}^T \tilde{\theta} \quad (17)$$

From (17), we have

$$\dot{V} = \frac{1}{g} S(\tilde{x}) g(x) (u - u_s - u_f) + \sum_{j=1}^M \dot{\tilde{\theta}}_j \tilde{\theta}_j \quad (18)$$

We choose  $u_s = k \operatorname{sgn}(S)$  and  $k > 0$ , so from (18) we have

$$\dot{V} \leq S(\tilde{x}) \varepsilon - S(\tilde{x}) \sum_{j=1}^M p_j(x) \tilde{\theta}_j - S(\tilde{x}) k \operatorname{sgn}(S(\tilde{x})) + \sum_{j=1}^M \dot{\tilde{\theta}}_j \tilde{\theta}_j \quad (19)$$

$$\dot{V} \leq |S(\tilde{x})| \varepsilon - |S(\tilde{x})| k + \sum_{j=1}^M (\dot{\tilde{\theta}}_j - |S(\tilde{x})| p_j(x)) \tilde{\theta}_j \quad (20)$$

When using variable universe function as follows:

$$\dot{\tilde{\theta}}_j = \tilde{\theta} = |S(\tilde{x})| p_j(x) \quad (21)$$

From (21), we have  $\dot{V} \leq |s(\tilde{x})| \varepsilon - |s(\tilde{x})| k = |s(\tilde{x})| (\varepsilon - k) \leq 0$ . So the controller is stable.

Table.1.Fuzzy rule table of MIMO system

| $\begin{smallmatrix} \dot{S}_j \\ S_j \end{smallmatrix}$ | NB | NM | NS | ZE | PS | PM | PB |
|--|----|----|----|----|----|----|----|
| PB   | ZE | NS | NM | NB | NB | NB | NB |
| PM   | PS | ZE | NS | NM | NB | NB | NB |
| PS   | PM | PS | ZE | NS | NM | NB | NB |
| ZE   | PB | PM | PS | ZE | NS | NM | NB |
| NS   | PB | PB | PM | PS | ZE | NS | NM |
| NM   | PB | PB | PB | PM | PS | ZE | NS |
| NB   | PB | PB | PB | PB | PM | PS | ZE |

#### 4. Simulation results

In this paper, we studied on PH Neutralization process. Three liquid streams: an acid flow ( $q_1$ ), a bluffer flow( $q_2$ ), and a base flow ( $q_3$ ). It is desired to control the pH of the effluent stream and the level height  $h$  at a specified set point in the presence of disturbances in buffer flow rate. The parameters<sup>[6]</sup> of the model and operating conditions are summarized in Table 2. PH sensor delays in the real process, it is important to note a delay of 30 seconds was used.

When the disturbance of buffer flow changes as a square wave alternating between 0.55 and 0 at time points  $t=900s$  and  $1800s$ , each for 300s duration. The simulation results shows in Fig.1 (a) and (b). According to these figures, the controller has a good result with flow disturbance of buffer flow.

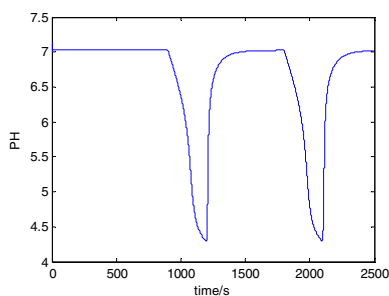
#### 5. Conclusion

Variable universe FSMC as a nonlinear robust intelligent control method can be applied to nonlinear systems. Simulation results show that this method has good tracking performance and robustness. Despite the fact that the variable universe FSMC performs similarly to a conventional SMC controller, it should be noted that the SMC controller uses all the information obtained from the process model and depends greatly on the canonical representation of the process model, whereas the variable universe FSMC

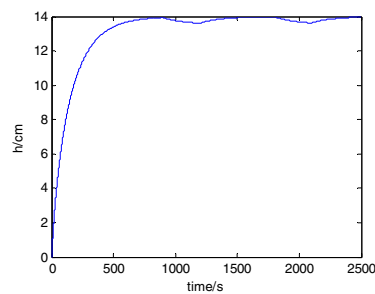
method relies simply on the input/output behavior (data) associated with the process. Simulation results show that despite its ease of use, the designed controller quite well.

Table2. The parameters of the pH neutralization reactor

| Variable | Value                                 | Variable | Value                                 |
|----------|---------------------------------------|----------|---------------------------------------|
| $Pk_1$   | 6.35                                  | $W_{b3}$ | $0.00005\text{mol}\cdot\text{L}^{-1}$ |
| $Pk_2$   | 10.25                                 | $q_1$    | $16.6\text{ml}\cdot\text{s}^{-1}$     |
| $W_{a1}$ | $0.003\text{mol}\cdot\text{L}^{-1}$   | $q_2$    | $0.55\text{ml}\cdot\text{s}^{-1}$     |
| $W_{a2}$ | $-0.03\text{mol}\cdot\text{L}^{-1}$   | $q_3$    | $15.6\text{ml}\cdot\text{s}^{-1}$     |
| $W_{a3}$ | -                                     | $A$      | $207\text{cm}^2$                      |
| $W_{b1}$ | $0.0\text{mol}\cdot\text{L}^{-1}$     | $\theta$ | 30s                                   |
| $W_{b2}$ | $0.00005\text{mol}\cdot\text{L}^{-1}$ | $h$      | 14.0cm                                |



(a) PH



(b)h

Fig.1 Closed-loop response of the system to buffer flow rate changes

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