Robust H-infinity Control for the pH Neutralization Process Based on Fuzzy Models

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***Abstract*— pH process has many nonlinear and time varying characteristics so modelling and control of this process is a challenging problem. To deal with the difficulties of the process, Takagi-Sugeno fuzzy approach for describing nonlinear model with several local linear models is proposed in this paper. The parallel distributed compensation method is considered for fuzzy control of overall nonlinear system and a LMI approach has been used for designing H-infinity control of each linear system. Proposed model and controller have been simulated and results show a satisfying performance.**

Keywords*— pH process; fuzzy modeling; PDC approach; robust control

I. INTRODUCTION

The pH process is prevalent in the chemical processes and in many industry branches such as biotechnological, pharmaceutical industries, food production, fermentation and etc. It is necessary to sustain the measure of pH in constant default level in industrial processes. There is different classification for pH process. As we know acids are divided in to two groups, weak or strong, so one categorization is based on weakness or strength of acid that has been used and another one is upon output flow. Based on output flow pH process is divided in to two classes that are called batch and continuous. We have two configuration for continuous process, CSTR-pH process and in-line pH process. There are numerous papers about this issue.

It is available to use various methods for modelling the pH process. Wiener and Seborg models are rather more common than other approaches. Electrical charge equivalence and material balance are two other famous approaches for modelling the system. The Wiener model of system gives linear dynamic equation together with nonlinear static equation..

The control of this process frequently faces many problems that are derived nonlinearity and time varying characteristics of process so it is very arduous to get satisfactory appropriate performance and robust control with using ordinary controllers. In fact, The difficulty of pH control stems from the nonlinearity of the process described by titration curve that illustrates the steady-state pH values versus the corresponding flow rate of the manipulated acid or base. Moreover the process nonlinearities are time varying due to changes in the buffering capacity. This means that the process may have several titration curve. To control the system it is good idea to use linearized model of system under some of conditions. With medial disturbances and designing an efficient controller we can expect that linear approximation has small deviation. But modelling the system with only one local linear model has a low performance and for the plants that have different operating points is not applicable. Conventional PID schemes, adaptive control and gain scheduling methods, genetic control algorithms have been used by authors in papers. There are numerous work on modelling and control that we point to some of them. Kalafatis, Wang and Cluett used a PI control based on Wiener model approach [1]. Henson and Sebrog proposed an adaptive nonlinear control strategy based on a modified input output linearization approach [2]. Darab, Hodera, Crisan and Naseu developed control of industrial pH process using internal model control strategy [3]. Dumont applied the Languerre method for modelling and control of pH process [4]. Katarina Kavsek-Biasizo, Igor Skrjanc and Drago Matko [5] and Kyn-Hyung Cho, Yeong-Koo Yeo, Jin-Sung Kim and Seung-Tae Koh [6] present a predictive control for pH process based on T-S fuzzy model of process.

we prposed Takagi-Sugeno fuzzy solution to linearize pH process. The T-S fuzzy approach for linearization and identification is based on input-output data and parameter estimation. Due to nonlinear and uncertain characteristics of pH process, robust control based on H-infinity control is then used to control the pH value. The so-called parallel distributed compensation (PDC) is an approach for designing a controller based on T-S fuzzy model of system. It determines the Lyapunov stability conditions of overall nonlinear fuzzy system with a criterion and the common P matrix of criterion is obtained by using linear matrix inequality (LMI) technique.

II. SYSYEM MODELLING

A. System Description

We have considered the pH process involving the mixing of reacting streams (a strong base and a weak acid) for this paper. The pH process is depicted by Fig. 1. This is a simplified model of pH process in industry. This process has three inputs and 2 outputs. F_a is the acid flow rate, C_a is concentration of acid solution, F_b is base flow rate, C_b is concentration of alkaline solution and V is the volume of mixing tank reactor

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that supposed to be constant. F_a , F_b and buffer flow rate are inputs of process and pH stream that expected to have constant pH measure. Two out of three inputs, acid and buffer streams, are considered to have constant flow as well as volume of tank is supposed to be constant so we can control the pH value with Base flow that is controlled and regulated by a valve. We use an agitator in container to mixed stream of inputs uniformly. By definition, the pH value specifies the activity of hydrogen ions in the solution and can be determined by

$$pH = -\log_{10}[\mathrm{H}^+] \tag{1}$$

Where $[\mathrm{H}^+]$ denotes hydrogen ion concentration.

This is obvious from (1) that hydrogen ions for pH value of 5 is ten times less than pH value of 4.

Buffering is a property of weak solutions that cause this solution resist changes in pH value. For a acid-base reacting system the nonlinearity of process is described by titration curve that is an “S” shape curve and illustrates the relation between input stream and steady-state pH value. Titration is a convential method that specifies the acidity or basicity of solution. This nonlinearlity makes it difficult to model this process. Fig. 2, shows the titration curve of proposed pH process.

B. Wiener and pH Process Modelling

All processes have nonlinearity features in a range between low and high level. There are many approaches for gaining nonlinear model of a processes with high level nonlinearity. One common approach is Wiener model. pH neutralization processes are a group of Wiener-type systems. This model consists of a linear dynamic model that describes the mixing dynamics in the tank in series with nonlinear static model that represents the titratin curve and is shown in Fig. 3.

In this process we have considered acetic acid and sodium hydroxide as acid and base repectively. The ionization reactions are given by

$$CH_3COOH \leftrightarrow CH_3COO^- + H^+ \tag{2}$$

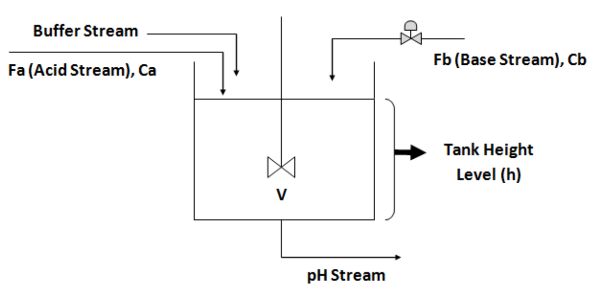

Fig. 1. pH process discription

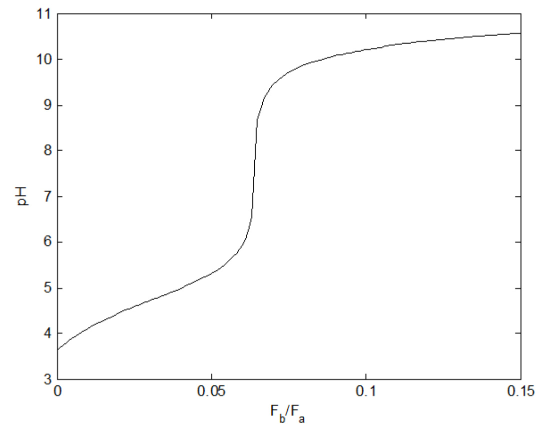

Fig. 2. Titration curve of pH process

$$NaOH \leftrightarrow Na^+ + OH^- \tag{3}$$

$$H_2O \leftrightarrow H^+ + OH^- \tag{4}$$

We define the x_a and x_b as total ion concentration of the acid and base respectively by

$$x_a = [\mathrm{CH_3COO^-}] + [\mathrm{CH_3COOH}], \quad x_b = [\mathrm{Na^+}] \tag{5}$$

The linear model of mixing dynamics in the tank can be described by (6) and (7).

$$V\frac{dx_a}{dt} = F_aC_a - (\mathrm{F}_a + \mathrm{F}_b)\,\mathrm{x}_a \tag{6}$$

$$V\frac{dx_b}{dt} = F_aC_a - (\mathrm{F}_a + \mathrm{F}_b)\,\mathrm{x}_b \tag{7}$$

$$\frac{K_a x_a}{K_a + [\mathrm{H}^+]} + \frac{K_w}{[\mathrm{H}^+]} = [\mathrm{H}^+] + \mathrm{x}_b \tag{8}$$

Equation (8) shows the nonlinear static model that represents the titration curve.

Where K_a and K_w are equilibrium constants of acid and water respectively and are given by

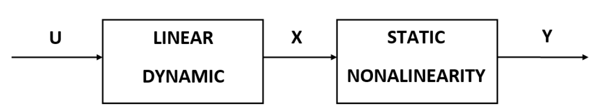

Fig. 3. The Wiener model

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$$K_a = \frac{[CH_3COO^-][\mathrm{H}^+]}{CH_3COOH} \tag{9}$$

$$K_w = [\mathrm{H}^+][OH^-] \tag{10}$$

Which [X] denotes the concentration of ions in solution. Table 1 shows simulation parametes for wiener model in this work.

III. Fuzzy Modelling

A. T-S modelling

It is necessary to have a simple and general mathematical tool for description of system model. Identification with using input-output data and derivation from given nonlinear system equations are two methods for constructing fuzzy models. In modelling of plants such as pH process that their representation in analytical models are too difficult, the first method is suitable. As it mentioned identification with applying input-output data based on Takagi-Sugeno approach has been used in this work.

There are two steps for fuzzy modelling, structure identification and parameter estimation. Suppose the rules of fuzzy system are as follows

Rule i: (i =1, 2,…,R)

If z_1 is A_1^i and z_2 is A_2^i and …and z_n is A_n^i Then

$$y = f(\mathrm{z}_1, \mathrm{z}_2, ..., \mathrm{z}_n) \tag{11}$$

Where R is the number of fuzzy rules, $A_1^i, A_2^i, \ldots, A_n^i$ are fuzzy sets, $z_1, z_2, ..., z_n$ are input variables of the fuzzy model and y is the output that usually is in linear form of

$$y = \alpha_1^i z_1 + \alpha_2^i z_2 + ... + \alpha_n^i z_n + r_i \tag{12}$$

And an estimated model output can be represent by:

$$y = \frac{\sum_{i=1}^{R} \mathrm{w}_i(\mathrm{z})\,\mathrm{y}_i}{\sum_{i=1}^{R} \mathrm{w}_i(\mathrm{z})} \tag{13}$$

TABLE I. SIMULATION PROCESS VARIABLES

variable	***value***
Base consentration	0.05 M
Acid concentration	0.0032 M
Volume	11001 ml
Asetic acid flow	10 ml/s
Asetic acid disociation constant	1.727e-005

Where

$$\mathrm{w}_i(\mathrm{z}) = \prod_{i=1}^{n} \mu_{A_i}(\mathrm{z}_i) \tag{14}$$

B. pH process fuzzy modelling

The described fuzzy model, gives a static nonlinear input-output relation of system. Dynamic systems are usually modelled by feeding back delayed input and output signals.

To describe nonlinear system with local linear models, IF-THEN Takagi-Sugeno fuzzy rules are as follow

Rule i: (i=1, 2,…,R)

If $y(\mathrm{t}-1)$ is A_1^i and $y(\mathrm{t}-2)$ is A_2^i and …and $y(\mathrm{t}-\mathrm{n})$ is A_n^i Then

$$y(\mathrm{t}) = \sum_{p=1}^{n} a_p^i y(\mathrm{t}-\mathrm{p}) + = \sum_{q=1}^{m} b_q^i\, \mathrm{u}(\mathrm{t}-q) \tag{15}$$

Where n is order of output and m is the order of input.

For gathering input-output data of process, It has been supposed that system just has one input and it is base flow rate. With using amplitude modulated pseudo random binary signal as system input, the changes of output, pH value, has been investigated. Sample time is 2 seconds. Fig. 4, shows changes of pH value versus base flow changes.

The membership functions used for this work are Gaussian and a fuzzy clustering method is used for determining the position and number of this functions. Table 2 and Fig. 5 show membership functions for pH process.

In this work, second order linear model has been chosen for each rule so we have four parameters for estimating in each rule. The recursive least square method is used for estimating the parameters of conclusion parts.

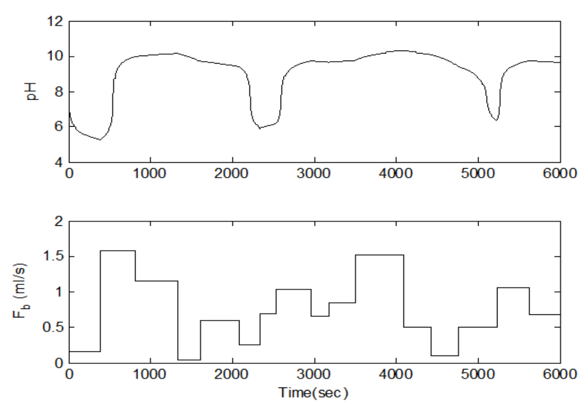

Fig. 4. Dynamic model response of pH process

TABLE II. MEMBERSHOP FUNCTIONS SPECIFICATINS

Membership function	*Center*	*Standard deviation*
A_1	7.6145	1.1
A_2	10.0152	1.1

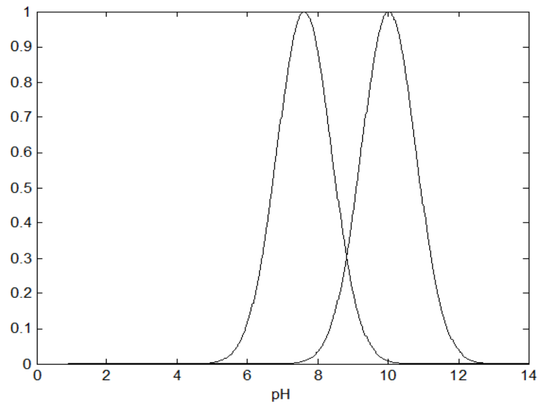

Fig. 5. Membership functions over pH values

We have determined two membership functions for dividing output space so we have four subspaces and consequently four rules. The estimated parameters for each rule are presented in Table 3.

Fig. 6 shows the proposed fuzzy model output in comparison with Wiener model output. We have used half of all data set for estimating the parameters. The estimation error has been shown too.

IV. CONTROL SCHEME

A. Parallel Distributed Compensation

There are many model-based approaches for robust control of linear systems. As modelling of pH process is not straightforward, we represent local linear input-output relation of nonlinear system by T-S fuzzy model.

TABLE III. ESTIMATED PARAMETERS FOR LINEAR MODELS

Rule	**parameters**			
	a_1	a_2	b_1	b_2
1	0.9904	0.006	0.0109	0.0292
2	0.3539	0.299	0.0638	0.0429
3	0.3595	11.0030	-0.1521	-0.1551
4	0.6367	0.3623	0.0083	0.0042

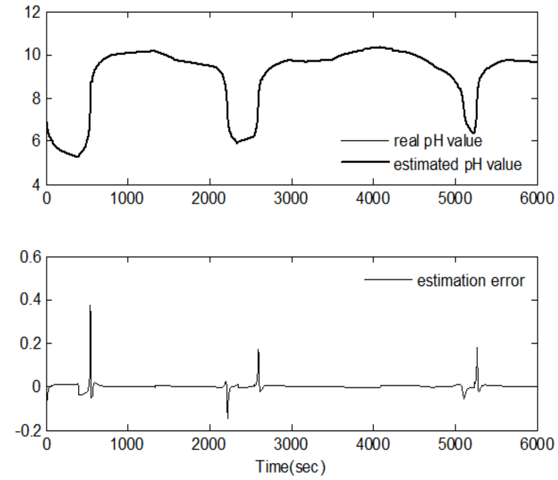

Fig. 6. Validation of fuzzy model for simulated data

The parallel distributed compensation approach (PDC) offers a procedure to design a fuzzy controller from a given T-S fuzzy model. The stability of control system is analyzed in [7]. In the PDC design each control rule is designed from the corresponding rule of a T-S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in premise parts. A Discrete fuzzy system can be shown as

Model Rule i: (i=1, 2, …, R)

If $z_1(\mathrm{k})$ is A_1^i and $z_2(\mathrm{k})$ is A_2^i and …and $z_n(\mathrm{k})$ is A_n^i Then

$$x(\mathrm{k}+1) = \mathrm{A}_i\, x(\mathrm{k}) + \mathrm{B}_i\, u(\mathrm{k}) + g(\mathrm{k}) \tag{16}$$

Here, $z(\mathrm{k}) = [\mathrm{x(k)}, \mathrm{x(k-1)}]^T$ is fuzzy model input, $x(\mathrm{t}) \in \mathbb{R}^n$ is the state vector, $u(\mathrm{t}) \in \mathbb{R}^m$ is the input vector, $y(\mathrm{k}) \in \mathbb{R}^m$ is the output vector and $g(\mathrm{k})$ is the unknown disturbance. By observable state space realization of models that have obtained in pervious section, four state space models of pH are achieved.

For model (16), fuzzy controller via PDC can be represent as:

If $z_1(\mathrm{k})$ is A_1^i and $z_2(\mathrm{k})$ is A_2^i and …and $z_n(\mathrm{k})$ is A_n^i Then

$$u(\mathrm{k}) = -K_i\, x(\mathrm{k}) \tag{17}$$

The fuzzy control rules have a state feedback law designed by H-infinity technique. The overall fuzzy controller have a nonlinear law and can be represent as:

$$u(\mathrm{k}) = -\frac{\sum_{i=1}^{R} w_i(\mathrm{z(k)})\,\mathrm{K}_i\, x(\mathrm{k})}{\sum_{i=1}^{R} w_i(\mathrm{z(k)})} \tag{18}$$

Fig. 7 shows the structure of fuzzy model and controller with PDC approach. If the subsystems be stable, the overall nonlinear system is not stable in general. Substituting (18) into (16), we obtain the closed loop system as:

$$\mathrm{x(k+1)} = -\sum_{i=1}^{R}\sum_{i=1}^{R} h_i(\mathrm{z(k)})h_j(\mathrm{z(k)})\{\mathrm{A}_i - \mathrm{B}_i\mathrm{K}_j\}x(\mathrm{k}) + g(\mathrm{k}) \tag{19}$$

Where

$$h_i(\mathrm{z(k)}) = -\frac{w_i(\mathrm{z(k)})}{\sum_{i=1}^{R} w_i(\mathrm{z(k)})} \tag{20}$$

The fuzzy control (17) uses the local linear structure but the gains k_i should be determined using global conditions.

There is a method based on linear matrix inequality that gives the common P systematically and find the feedback gains that will be discussed in next section.

B. Robust Control Design

In fuzzy control systems, the robustness is an important issues especially for the systems that have to deal with uncertainties and external disturbances. Uncertain dynamics have bad influence in performance of system control. The object of robust fuzzy control is to find the feedback gain for each rule such that stabilize overall system and minimize the norm of uncertain block.

The H-infinity performance defined as:

$$\sum_{k=0}^{t_f} x^T(\mathrm{k})\mathrm{Q}\mathrm{x(k)} \le \eta^2 \sum_{k=0}^{t_f} g^T(\mathrm{k})\,\mathrm{g(k)} \tag{21}$$

Where t_f is terminal time of control, η determine the effect of $g(\mathrm{k})$ on $x(\mathrm{k})$ *and* Q is a positive definite matrix. By this definition, we should design each controller such that the effect of uncertainties on states be minimized.

A stability criterion is given by Theorem 1 [8] to guarantee the stability and control performance of system (18).

Theorem 1: The closed loop fuzzy system (19) is stable if there exist symmetric positive definite matrix P and a positive constant γ and the feedback gains K_i shown in (18) are chosen to satisfy the following conditions:

$$\begin{aligned}&[(\mathrm{A}_i - \mathrm{B}_i\mathrm{K}_i)^T\ \mathrm{P}(\mathrm{A}_i - \mathrm{B}_i\mathrm{K}_i)] + \gamma^{-1}P^2 + Q < 0\\ &for\quad i = 1,2,..,R\end{aligned} \tag{22}$$

$$\begin{aligned}&[\mathrm{H}_{il}^T\mathrm{P} + \mathrm{P}\mathrm{H}_{il}] + \gamma^{-1}P^2 + Q < 0\\ &for\quad i \le l \le r\end{aligned} \tag{23}$$

With $H_{il} = \dfrac{(\mathrm{A}_i - \mathrm{B}_i\mathrm{K}_l) + (\mathrm{A}_l - \mathrm{B}_l\mathrm{K}_i)}{2}$, $P = P^T > 0$

With a LMI approach the fuzzy controller for each rule can be computed. A controller with integral action is used for set point tracking. Fig. 8 shows the results of proposed controller for simulated system. The Wiener model developed before is used as pH process and the fuzzy model is used in design of robust controller. As it is obvious from this figure, the set point is tracked by controller successfully.

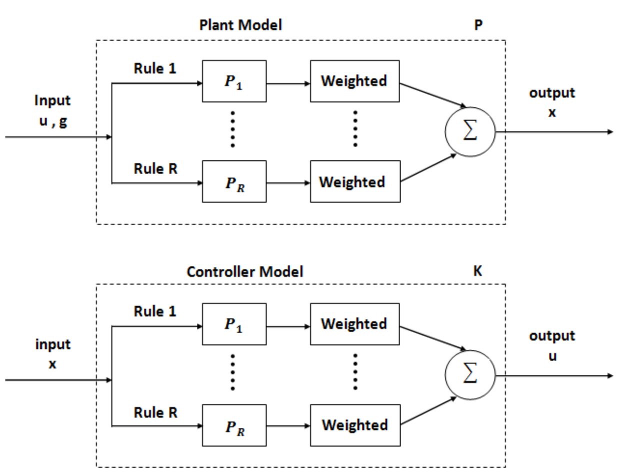

Fig. 7. Structure of fuzzy model and control with PDC approach

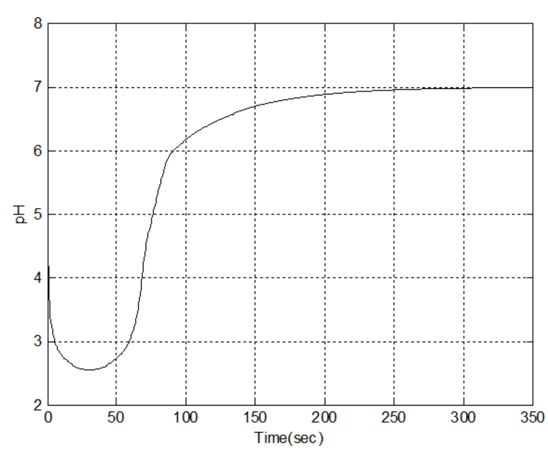

Fig. 8. Robust controll based on fuzzy model

The changes of acid flow rate is used as external disturbance. Fig. 9 shows the controller performance in presence of changes of acid flow rate. The controller can track the set point also in presence of disturbance and robust performance of system is satisfied with this controller.

V. Conclusion

A pH process modelling and control strategy has been proposed in this paper. The process is modelled and linearized by T-S fuzzy approach. We derived four rules for pH system so we have four local linear input-output relation. It is available to makes more than four rules but it will be complicated to satisfy the conditions of stability for them and as it is obvious with having four rules we have gotten satisfactory performance. For control of process a robust control is developed. Reference tracking and disturbance attenuation are tested for proposed control.

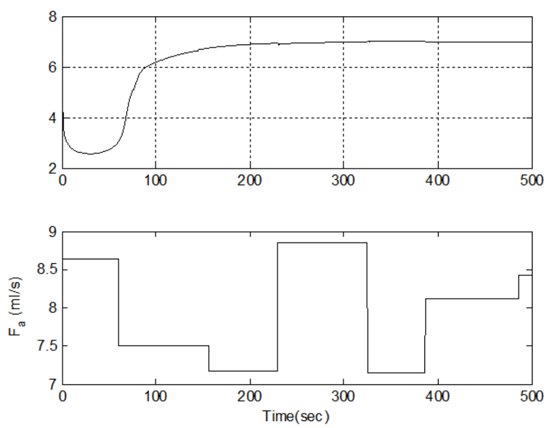

Fig. 9. Robust control in presence of disturbance

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