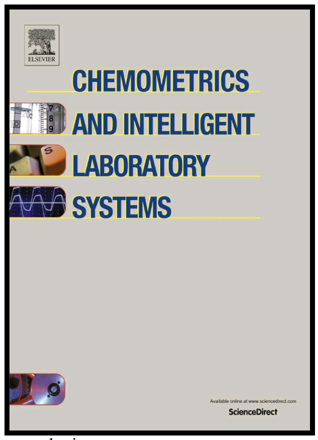

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An implication of Fuzzy ANOVA: Metal uptake and transport by corn grown on a contaminated soil

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Abstract

Three organic fertilizers of manure sheep, sewage sludge and municipal waste were mixed with soil at the rates of 0%, 1.25 % and 2.5 % (w/w), respectively. In a greenhouse experiment, corn seed were grown on pots of 3 kg treated contaminated soils and irrigated. Sixty days after sowing, the aerial plant parts were harvested and analyzed for Cadmium (Cd) and Lead (Pb) contents. Since all of data in this research are fuzzy, we need an extended version of analysis of variance (ANOVA) to investigate on these fuzzy observations. In this paper, as a method to compare several populations, the fuzzy analysis of variance (FANOVA) has been used where the collected data considered fuzzy rather than crisp numbers and therefore all calculations are based on FANOVA method. Although, ANOVA based on vague data can lead to a fuzzy decision, but measuring the vagueness of this fuzzy decision is one of advantages of proposed method from the applied point of view.

Keywords

Fuzzy numbers; Fuzzy statistics; Fuzzy analysis of variance; Organic fertilizers; Cadmium and Lead

1. Introduction and background

Soil organic matters have always been considered in the research of nutrient absorption by plants. These materials can impact on nutrient concentration and absorption by plants directly or indirectly. This is due to the effect of organic matters on cation exchange capacity and metallic cation replacement and exchange in the form of stable complexes with organic ligand. Therefore, knowledge about the mechanism and extent of the effect of organic substances in the absorption and micronutrients concentrations can be used and is required for effective use of these materials as soil amendments. Nevertheless, land disposal of organic matters may either directly or indirectly alter the heavy metal status of the soil by dissociation kinetics or affecting metal solubility (Del Castilho et al., 1993, Karaca, 2004). Soil pollution is becoming harmful to environment and human health. Heavy metals are among the most important elements of soil pollution and the environment which has attracted much attention among the researchers. Some of heavy metals are include Cadmium (Cd), Zinc (Zn), Copper (Cu), Lead (Pb), Nickel (Ni), Chromium (Cr) and Arsenic (Ar). Most heavy metals infiltrate and accumulate in the top of soil. Accumulation of heavy metals in soil is incremental and therefore in the long term results are increased the levels of contamination, so that it may reach limits that can constitute a real threat to food safety for human civilization (Okoronkwo et al., 2005).

A review of the investigation results shows that the heavy metals concentrations depend upon and varies with the metal type, soil condition and type of plant variety, but normally the levels of concentration in aerial parts of a plant or crop is significantly higher than other parts and significantly lower in seeds (Giordanoand Mays., 1977; Kabatta and Pendias., 2001). Sappin-Didier et al. (2005) worked on Cd absorption capacity of transgenic tobaccos (Nicotiniana tabacum). Moreover, some plants species are characterized by a lower or higher tendency to absorb trace metal (Mench, 1998). For example, Menon et al. (2005) studied on Cd influence in Norway spruce (Picea abies), poplar (Populus tremula), willow (Salix viminalis) and birch (Betula pendula) trees and a variety of herbaceous under storey plants for three years. Root growth and evapotranspiration were decreased in metal polluted treatments, independent of the type of subsoil. Göthberg et al. (2004) recorded Cadmium content in aerial and bellow spinach (Ipomoea aquatic) parts. Cao et al. (2009) analyzed the potential ecological risk of Cd, Pb and Ar in agricultural black soil on both roots and shoots growth in soybean, and they showed that soil contamination from Pb contamination for almost all sampling sites had moderate ecological risk; Cd in some samples had high potential ecological risk; while soil contamination from Ar had low ecological risk. McBride (1995) reported that the mobility of heavy metal was most nearly associated with soil pH and metal-organic complexation. McGrath et al. (1988) indicated the addition of organic fertilizer decreases solution Cd and Ni concentrations, but increases the Zn extractability

Environmental analyses are usually plagued by uncertainties in data, fuzziness in assumptions and imprecision in available modelling tools. These limits affect policy decisions and so must be communicated and quantified during the process of decision making. Therefore, the researcher may be confronted with uncertainties which are available as linguistics expressions and could be modelled by fuzzy numbers in some applied cases. For the last decades, investigation and research have been developed such that a coalition of fuzzy sets theory and statistics has been established with the bellow purposes (Coppi et al., 2006):

(i) to introduce new data analysis problems in which the major target involves either using fuzzy terms or using fuzzy relationships;

(ii) to find out well-formalized models for elements combining fuzziness and randomness;

(iii) to develop statistical methods to handle fuzzy observations or imprecise data; and

(iv) to incorporate fuzzy sets to help solving conventional and classical statistical problems with crisp or precise data.

Recently studies shows that the observations that are including vagueness can be analyzed by using the fuzzy set theory (FST). In other words, the vagueness which is included in data can be defined exactly by using a membership function of the FST, and therefore, the vague data can be figured by the membership functions. Such vague data is processed directly by the aid of the membership function in the statistical processes. The calculation process becomes more complicated with respected to the classical statistical process (Konishi et al., 2006), because it is necessary to perform the calculation precisely using the membership functions. In many environmental and applied sciences such as social sciences, geology, economics and agriculture, there are several real-life populations where imprecise values can be assigned to their experimental outcomes. In this way, the FST is a suitable model to handle and formulize these populations in real cases, which is the reason of our need to the FST in the analysis of variance (ANOVA). In recent years, some papers on different areas of ANOVA based on the FST have been published. These areas are as follows: investigating on the behavior of one-way fuzzy analysis of variance (FANOVA) and comparing it with regression model (De Garibay., 1987), exact one-way ANOVA testing under normal fuzzy random variables (Montenegro et al., 2004b), bootstrap method for approximating the asymptotic one-sample tests by fuzzy random variables (Montenegro et al., 2004a), developing a one-way ANOVA approach for the functional data on a given Hilbert space (Cuevas et al., 2004), processing ANOVA method using the moment correction for vague data (Konishi et al., 2006), bootstrap asymptotic multi sample testing of means assuming fuzzy random variables (Gil et al., 2006; González-Rodríguez et al., 2011), considering the cuts of fuzzy random variables for one-way ANOVA problem on the basis of fuzzy data based on optimization approach (Wu, 2007), and extending one-way ANOVA for fuzzy observations based on extension principle approach (Nourbakhsh et al., 2011).

In this paper, the mean absorption of Cadmium and Lead in aerial and bellow corn parts to conclude whether it is dependent on the added levels of organic fertilizers have been analyzed. So, this research aimed to create a link between environmental practice and theoretical analysis of variance.

The rest of this paper is organized as follows: After introducing fuzzy numbers, arithmetic operations on triangular fuzzy numbers (TFN) have been summarized into Section 2. A new approach based of fuzzy measurements to analysis of variance as an alternative of ANOVA method has been detailed into Section 3. Section 4 includes an experimental study on Cd absorption based on a real-world agricultural data (generated in a Lab. at Tehran University by Ivani (2007)) presented. The obtained results and future research directions for the proposed fuzzy ANOVA method have been discussed in Section 5, Conclusions.

2. Fuzzy sets and arithmetic operations on fuzzy numbers

2.1. Fuzzy set theory

The fuzzy set theory (FST) models the situations in which the uncertainty is due to the non-precise (fuzzy) environment. A method to handle practical problems, particularly those connected with vagueness and uncertainty about input values and theoretical relationships is using the proposed FST theory proposed by Zadeh (1965). A usual crisp set has a clearly defined boundary, such as any real number which is either a member of a crisp set or it is not. But a fuzzy set is a set without a crisp boundary and it can contain elements with degree of membership as a number between 0 and 1. Using the FST is inevitable with the situations such as uncertain, imprecise or the cases that include linguistic expressions. Fuzzy logic is a branch of mathematics that allows a computer to model the real world in the same way that people do. It provides a simple way to reason with vague, ambiguous, and imprecise input or knowledge (Engin et al., 2008; Kaya, 2009; Kaya and Kahraman, 2011, 2001a; Turanoğlu et al., 2012).

For example, suppose one define the optimum range of Zn absorption in a plant as the interval [15,150] $mg.kg^{-1}$.D.M. Using traditional set theory, it is possible to define the equilibrium absorption amount as a crisp value set containing the element 25 $mg.kg^{-1}$.D.M. In this case, a crisp set considered and any given absorption amount is either in or not in the equilibrium range. But in contrast, fuzzy sets allow for partial membership and an absorption amount 25 $mg.kg^{-1}.D.M$ might regarded as having partial membership of equilibrium set and partial membership of a below-equilibrium set. This flexibility allows users to deal with uncertainty and imprecision which is in the nature of many real world problems (Shepherd and Shi, 2006). See Parchami et al. (2011) for a similar implication of testing fuzzy hypotheses based on a real-world data. In the following we will discuss and review some of the mathematics of the FST.

2.2. Fuzzy data: why and where?

Real observations for the quantities of continuous variables are not precise numbers and they are more or less imprecise (Viertl, 2011). The best description of such observations is using an especial numbers which called imprecise (fuzzy) numbers. In real world, the fuzziness of an observed continuous variable often happens in one of three following cases:

(*i*) The first case is because of technical conditions of measurements where the continuous variable cannot measured exactly and so in this case, the results of experiments cannot recorded clearly with precise numbers and only in linguistic terms to justify the required tolerance of the errors in measurements. In other words, the experimenter has not a capable instrument to measure the exact value of the nature. For example, the data readings on analogue measurement equipment or the given data by color intensity pictures can considered as fuzzy numbers instead of precise numbers (Filzmoser and Viertl, 2004).

(*ii*) The second case is due to the fact the continues variable will be given by linguistic forms, such as linguistic report of an expert or the report of a farmer about his products, which are not numeric (Nourbakhsh et al., 2011).

(*iii*) In the third case, the true value of continues quantity has not exactly a precise amount, and therefore the experimenter cannot be able to exactly record and present its value by precise numbers. For instance, the water level of a river cannot be measured in an exact way because of the fluctuation (Kaya and Kahraman, 2011; Wu, 2007). Another typical example for third case is the lifetime of a battery which cannot, in general, be described exactly by a real number since the time at which the lifetime ends is not a crisp number but is more or less imprecise.

2.3. Arithmetic operations on fuzzy numbers

Assume that two fuzzy numbers defined as $\tilde{A}=(a,b,c)$ and called triangular fuzzy number (TFN), and defined as $\tilde{A}=(a,b,c,d)$ and called trapezoidal fuzzy number (TrFN). The membership functions of them are as follows:

$$T_{a,b,c}(x)=\begin{cases}\left(\dfrac{x-a}{b-a}\right) & for \quad a\le x\le b\\ \left(\dfrac{c-x}{c-b}\right) & for \quad b\le x\le c\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

$$T_{a,b,c,d}(x)=\begin{cases}\left(\dfrac{x-a}{b-a}\right), & \text{for} \quad a\le x\le b\\ 1, & \text{for} \quad b\le x\le c\\ \left(\dfrac{d-x}{d-c}\right), & \text{for} \quad c\le x\le d\\ 0 \quad , & Otherwise\end{cases}$$

and they are denoted as $T(a,b,c)$ and $T(a,b,c,d)$. The real number a called the core value and the positive real numbers b and c called left and right spreads of TFN, respectively. $F_T(R)$ and $F_T(R^+)=\{T_{a,b,c}\,|\,a,b,c\in R^+\}$, respectively, denote the set of all TFNs and the set of all positive TFNs, where R^+ is the set of all positive real numbers. Also, symmetric triangular fuzzy number (STFN) symbolically denoted by $T(a,b)$ in this paper where left and right spreads considered equal to b .

The following equations have been proved in (Dubois and Prade, 1980) for any $T(a,b,c)$, $T(a',b',c')\in F_T(R)$ by Zadeh's extension principle:

$$T(a,b,c)\oplus T(a',b',c')=T\left(a+a',b+b',c+c'\right), \tag{2}$$

$$T(a,b,c)-T(a',b',c')=T\left(a-a',b+c',c+b'\right). \tag{3}$$

Also, for $T(a,b,c)$, $T(a',b',c')\in F_T(R^+)$, operations $\otimes$ and $\oslash$ are given by approximation as:

$$T(a,b,c)\otimes T(a',b',c')\cong T(aa',ab'+a'b,ac'+a'c), \tag{4}$$

and

$$T(a,b,c)\oslash T(a',b',c')\cong T\left(\frac{a}{a'},\frac{ac'+a'b}{a'^2},\frac{ab'+a'c}{a'^2}\right), \qquad a'\neq 0. \tag{5}$$

Meanwhile, the scalar multiplication of $T(a,b,c) \in F_T(R)$ and $k \in \{0\} \cup R^+$ is defined by:

$$T(a,b,c) \odot k = k \odot T(a,b,c) = T(ka,kb,kc). \tag{6}$$

3. FANOVA based on triangular fuzzy numbers

In this section, we review a new extended version of fuzzy ANOVA which we call it FANOVA. For more details see (Wu, 2007). Let r denotes the number of levels of the factor under study, any one of these levels is denoted by the index i, $i = 1,...,r$. The number of cases for the i th factor level is denoted by n_i, and the total number of cases in the study is denoted by n_t, i.e. $n_t = \sum_{i=1}^{r} n_i$. The index j will be used to identify the given case or trial for a particular factor level. Therefore, Y_{ij} denotes the j th observation on the response (dependent) variable for the i th factor level. For example Y_{ij} can be the productivity of the j th employee in the i th plant. The number of cases or trials for the i th factor level is denoted by n_i, and so $j = 1,...,n_i$. Similar to the classical ANOVA model, FANOVA model can state by (Wu, 2007):

$$Y_{ij} = \mu_i + \varepsilon_{ij}, \quad \text{for } i = 1,...,r \text{ and } j = 1,...,n_i, \tag{7}$$

in which, Y_{ij} 's are the values of the response variables in the j th trial for the i th factor level, μ_i 's are the factor level means, and ε_{ij} 's are independent random variables having normal distribution $N(0,\sigma^2)$. Therefore, one can expect that Y_{ij} 's are independent random variables having the normal distribution $N(\mu_i,\sigma^2)$ for any $i = 1,...,r$. Let the total sum of squares (*SST*), the treatment sum of squares (*SSTR*), and the error sum of squares (*SSE*) be respectively defined by $SST = \sum_{i=1}^{r}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_{..})^2 = \sum_{i=1}^{r}\sum_{j=1}^{n_i} Y_{ij}^{\ 2} - \frac{Y_{..}^{\ 2}}{n_t}$, $SSTR = \sum_{i=1}^{r} n_i(\overline{Y}_{i.} - \overline{Y}_{..})^2 = \sum_{i=1}^{r}\frac{Y_{i.}^{\ 2}}{n_i} - \frac{Y_{..}^{\ 2}}{n_t}$ and $SSE = \sum_{i=1}^{r}\sum_{j=1}^{n_i}(Y_{ij} - \overline{Y}_{i.})^2 = \sum_{i=1}^{r}\sum_{j=1}^{n_i} Y_{ij}^{\ 2} - \sum_{i=1}^{r}\frac{Y_{i.}^{\ 2}}{n_i}$, where $\overline{Y}_{..} = \frac{Y_{..}}{n_t} = \frac{1}{n_t}\sum_{i=1}^{r}\sum_{j=1}^{n_i} Y_{ij}$ and $\overline{Y}_{i.} = \frac{Y_{i.}}{n_i} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$, for $i = 1,...,r$. Thus, the treatment mean square and error mean square are obtained by $MSTR = \frac{SSTR}{r-1}$ and $MSE = \frac{SSE}{n_t - r}$, respectively. And finally, FANOVA test statistic is (Wu 2007):

$$F = \frac{MSTR}{MSE}.$$

The FANOVA is able to handle the fuzzy-valued data on the basis of Zadeh's extension principle. Considering the above discussion, it is assumed that we are concerned with a classical ANOVA, where the entire theoretical elements of the model such as random variables, statistical hypothesis and parameters of populations are crisp, but only the observed values of the classical random variables can be considered as fuzzy numbers. In such cases, observations and recorded data can be considered as TFNs $\tilde{y}_{ij} = T\,(y_{ij}, a_{ij}, b_{ij})$, where $\tilde{y}_{ij}$ is interpreted as "approximately y_{ij}", for $i = 1,2,...,r$ and $j = 1,2,...,n_i$. As a result, the observed statistics in FANOVA can be obtained as follows by using extension principle:

$$sst = \left[\overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} \tilde{y}_{ij}^{2}\right] \text{Ä} \left[\frac{1}{n_t} \odot \tilde{y}_{..}^{2}\right] = \left[\overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} \left(\tilde{y}_{ij} \otimes \tilde{y}_{ij}\right)\right] \text{Ä} \left[\frac{1}{n_t} \odot \left(\tilde{y}_{..} \otimes \tilde{y}_{..}\right)\right], \tag{11}$$

$$ssr = \left\{\overset{r}{\underset{i=1}{\oplus}} \left[\frac{1}{n_i} \odot \tilde{y}_{i.}^{2}\right]\right\} \text{Ä} \left[\frac{\tilde{y}_{..}^{2}}{n_t}\right] = \left\{\overset{r}{\underset{i=1}{\oplus}} \left[\frac{1}{n_i} \odot \left(\tilde{y}_{i.} \otimes \tilde{y}_{i.}\right)\right]\right\} \text{Ä} \left[\frac{1}{n_t} \odot \left(\tilde{y}_{..} \otimes \tilde{y}_{..}\right)\right] \tag{12}$$

and

$$sse = \left[\overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} \tilde{y}_{ij}^{2}\right] \text{Ä} \left[\overset{r}{\underset{i=1}{\oplus}} \left(\frac{1}{n_t} \odot \tilde{y}_{i.}^{2}\right)\right] = \left[\overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} \left(\tilde{y}_{ij} \otimes \tilde{y}_{ij}\right)\right] \text{Ä} \left[\overset{r}{\underset{i=1}{\oplus}} \left(\frac{1}{n_t} \odot \left(\tilde{y}_{i.} \otimes \tilde{y}_{i.}\right)\right)\right], \tag{13}$$

in which

$$\begin{aligned}\tilde{y}_{i.} &= \overset{n_i}{\underset{j=1}{\oplus}} \tilde{y}_{ij} = \tilde{y}_{i1} \oplus \tilde{y}_{i2} \oplus \cdots \oplus \tilde{y}_{in_i} = \overset{n_i}{\underset{j=1}{\oplus}} T\left(y_{ij}, a_{ij}, b_{ij}\right) \\ &= T\left(\sum_{j=1}^{n_i} y_{ij}, \sum_{j=1}^{n_i} a_{ij}, \sum_{j=1}^{n_i} b_{ij}\right) = T\left(y_{i.}, a_{i.}, b_{i.}\right),\end{aligned}$$

and

$$\begin{aligned}\tilde{y}_{..} &= \overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} \tilde{y}_{ij} = \tilde{y}_{11} \oplus \tilde{y}_{12} \oplus \cdots \oplus \tilde{y}_{rn_r} = \overset{r}{\underset{i=1}{\oplus}}\overset{n_i}{\underset{j=1}{\oplus}} T\left(y_{ij}, a_{ij}, b_{ij}\right) \\ &= T\left(y_{11}, a_{11}, b_{11}\right) \oplus T\left(y_{12}, a_{12}, b_{12}\right) \oplus \cdots \oplus T\left(y_{rn_r}, a_{rn_r}, b_{rn_r}\right) \\ &= T\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}, \sum_{i=1}^{r}\sum_{j=1}^{n_i} a_{ij}, \sum_{i=1}^{r}\sum_{j=1}^{n_i} b_{ij}\right) = T\left(y_{..}, a_{..}, b_{..}\right).\end{aligned}$$

By the way,

$$msr = \frac{1}{r-1} \odot ssr, \quad mse = \frac{1}{n_t - r} \odot sse, \tag{14}$$

and

$$\tilde{f} = msr \oslash mse = \frac{n_t - r}{r-1} \odot \left(ssr \oslash sse \right). \tag{15}$$

In FANOVA model, based on positive TFNs $\tilde{y}_{ij} = T(y_{ij}, a_{ij}, b_{ij})$, $i = 1, 2, ..., r$, $j = 1, 2, ..., n_i$, Nourbakhsh et al. (2011) calculated the observed value of fisher statistics by

$$\begin{aligned} \tilde{f} &= msr \oslash mse \\ &= \frac{n_t - r}{r-1} \odot \left(ssr \oslash sse \right) \\ &\cong T\left(f_C, f_L, f_U \right), \end{aligned} \tag{16}$$

in which

$$f_C = \frac{n_t - r}{r-1} \left[\frac{\sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{n_t}}{\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i}} \right],$$

$$f_L = \frac{2(n_t - r)}{r-1} \left[\frac{\left[\left(\sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{n_t} \right) \left((\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij} b_{ij} + \sum_{i=1}^{r} \frac{y_{i.} a_{i.}}{n_i}) \right) \right]}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right)^2} + \frac{\left[\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right) \left(\sum_{i=1}^{r} \frac{y_{i.} a_{i.}}{n_i} + \frac{y_{..} b_{..}}{n_t} \right) \right]}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right)^2} \right]$$

and

$$f_U = \frac{2(n_t - r)}{r-1} \left[\frac{\left[\left(\sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{n_t} \right) \left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij} a_{ij} + \sum_{i=1}^{r} \frac{y_{i.} b_{i.}}{n_i} \right) \right]}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right)^2} + \frac{\left[\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right) \left(\sum_{i=1}^{r} \frac{y_{i.} b_{i.}}{n_i} + \frac{y_{..} a_{..}}{n_t} \right) \right]}{\left(\sum_{i=1}^{r}\sum_{j=1}^{n_i} y_{ij}^2 - \sum_{i=1}^{r} \frac{y_{i.}^2}{n_i} \right)^2} \right].$$

To test whether factor level means μ_i 's are equal or not, FANOVA has been considered to decide whether to reject (or accept) the null hypothesis “$H_0 : \mu_1 = \mu_2 = ... = \mu_r$

", against the alternative hypothesis "H_1: not all μ_i's are equal", based on fuzzy data. Rather than comparing f and $I_{\{F_{1-\alpha,r-1,n_t-r}\}}$, FANOVA decision rule constructed on the basis of comparing the real numbers $F_1 = \int_0^{F_{1-\alpha,r-1,n_t-r}} f(f)df$ and $F_2 = \int_{F_{1-\alpha,r-1,n_t-r}}^{\infty} f(f)df$, where $F_{1-\alpha;r-1,n_t-r}$ is the αth quantile of the fisher distribution with $r-1$ and $n_t - r$ degrees of freedom (see Figure 1). Therefore, at the given significance level α, we accept the null hypothesis H_0 with degree of acceptance $D_{H_0} = \frac{F_1}{F_1+F_2}$ if $F_1 > F_2$ and otherwise we reject H_0 with the degree of rejection $D_{H_1} = \frac{F_2}{F_1+F_2}$ (Nourbakhsh et al., 2011).

From a practical point of view, especially when we work in a fuzzy environment, it is helpful to introduce a criterion for the certainty degree of our decision. Although, in FANOVA the decision is fuzzy, but the fuzziness (vagueness) amount of the fuzzy decision is measurable. We measure and define *the fuzziness of the decision* in FANOVA based on TFNs by

$$FD = 1-2\left|D_{H_0} - 0.5\right| = 1-2\left|D_{H_1} - 0.5\right|. \tag{17}$$

Therefore, the measure of *FD* is zero for crisp data and in this case we do not have vagueness in our decision, which coincides to crisp decisions (accept/reject H_0 without vagueness) in classical ANOVA.

4. Materials and methods

4.1. Soil and organic fertilizers description

Polluted soil was conducted on agriculture soil (47% clay, 29.7% silt and 23.3% sand) collected from outside a Zinc concentration processing factory in Zanjan province, Iran. The soil samples were taken at a depth of 0-25 cm, air dried and sieved to <2 mm for analysis. Three kinds of organic fertilizers such as animal manure, municipal waste compost, and sewage sludge passed through a 5 mm sieve. The properties of the organic fertilizers were estimated according to Baran et al. (1995). Physical and chemical properties of soil and organic fertilizers are presented in Table 1 and Table 2, respectively.

4.2. Greenhouse culture

Three organic wastes were added at three rates of 0%, 1.25% and 2.5% to air-dried contaminated soil (w/w), mixed well to get a homogenized soil mixture with organic fertilizer. Then, they were filled in plastic pots of 3 kg and five fenugreek seeds were sown in each pot and all pots were irrigated with distilled water, during growing season. Sixty days after sowing, the above plant parts were harvested, dried, powdered by electric grinder and samples kept in electric oven at 550^{0C} and then acid digested and plant extracts were prepared (Waling, et al. 1989) for Pb and Cd concentrations in plant.

4.3. The methods of measuring observation

In soil science laboratory at agriculture department of Tehran University (Ivani, 2007), soil pH and electrical conductivity (EC) was measured in water with 1:5 ratio of soil by Richards (1954). Organic fertilizer was estimated based on Walkley and Black (1934). Available heavy metals in soils and organic fertilizers were extracted with DTPA solution based on Lindsay and Norvell (1978). All solutions with Lead (Pb) and Cadmium (Cd) were analyses by atomic absorption spectrophotometer (AAS) shimatzu 670 with flame or graphite furnace. The precise data for investigation on the effect of three organic fertilizers on Cd and Pb concentration in above and below parts of corn plant is presented in Tables 3-5 which are gadered in soil science laboratory at agriculture department of Tehran University (Ivani, 2007). But, Pb content has been analyzed in root of plant because Atomic Absorb Machine cannot read the Pb concentration in aerial parts of plant.

4.4. Results and discussion

Regarding to Tables 3-5, the factor levels are zero, 1.25% and 2.50% for each three kinds of organic fertilizers (municipal waste compost, manure sheep and sewage sludge). For example, the first three lines of Table 3 are the absorbed Cadmium amounts in aerial parts of corn plant from the polluted soil by municipal waste compost that can be taken as the factor levels as zero, 1.25% and 2.50% in ANOVA. Now, we want to test whether the factor level means μ_i 's are equal or not for each three kinds of organic fertilizers at the significance level 0.05, where $i = 1, 2, 3$. In other words, we want to decide to accept only one of the following hypotheses based on the observed data in Tables 3-5:

$$H_0 : \mu_1 = \mu_2 = \mu_3,$$

$$H_1 : \text{not all } \mu_i\text{'s are equal, } i = 1, 2, 3.$$

Considering Model (7), for example in testing the effect of different municipal waste compost levels on the Cadmium absorption in aerial parts of corn plant, variable Y_{ij} denotes the Cadmium concentration volume of the j th pot in the i th level of municipal waste compost where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$. Although the validity of normality tests is very low for small observed sample size, but the normal distribution assumption for random variable Y_{ij} comes from the essence of random variable which is rooted from nature. For instance, we can accept that the growth rate of plants in a specific time period, the uptake amount of heavy metals through the roots of the plants in a specific greenhouse experiment, or the weight of seeds picked from a particular plant type, are all normal random variables with suitable means and variances.

There are some unavoidable elements in experiment which could be cause the vagueness in the recorded data by Atomic Absorption Spectrophotometer (AAS). These are:

(a) disability of AAS in exactly measuring Cd and Pb for each plant.

(b) the possibility of instruments errors in recording data of Tables 3-5 by a long process in 60 days. The stages of this process are harvesting, drying, grinding, powdering samples, keeping in electric oven and digesting by HCl_2N, respectively.

(c) the possibility of laboratory errors in the process construction of DTPA solution.

(d) the possibility of human errors in measuring, reading and keeping records.

(e) the limit of the digital laboratory scales of precision.

Therefore, one can conclude that a preferred way to record the observations is to use fuzzy numbers. From now on, in this experiment, we decide to rewrite the data by symmetric triangular fuzzy numbers, to cover the unavoidable elements described earlier. The core values of STFNs set equal to the precise recorded data in Table 3, and the vagueness of these STFNs can be considered as a coefficient of the precise recorded data in Table 3. In other words, to cover the mentioned unavoidable elements justified in (a)-(e) above, we convert the precise observations y_{ij}'s in Table 3 to the STFNs $\tilde{y}_{ij} = T(y_{ij}, k\, y_{ij})$, where $i = 1, 2, 3$, $j = 1, 2, 3, 4$ and $k = 0, 0.0001, 0.001, 0.01, 0.1$ (see Figure 3 for $k = 0.1$). It is obvious that the classical ANOVA method cannot analyze these fuzzy values, and we need FANOVA that is the extended version of ANOVA presented in Section 3, to processing these fuzzy measurements. Table 6 contains the results of fifteen FANOVA tests based on the fuzzified data of Table 3 for various spreads of STFNs. For example, the membership function of the observed test statistic in FANOVA model for investigation on the effect of three levels (0%, 1.25% and 2.50%) municipal waste compost on Cd concentration in aerial parts of corn plant has been calculated as $\tilde{f} = T(117,216)$ for $k = 0.001$ by (16) depicted Figure 2. Therefore, after calculating $F_1 = 24.8$ and $F_2 = 191$ one can decide to accept the hypothesis "equality of the mean amounts of Cd absorption for three different levels of municipal waste compost" with certainty $D_{H_1} = 0.885$ at significance level of 0.05. In other words, with degree of certainty 0.885 one can claim that "Cd absorption in aerial parts of corn plant" has not depended on "the added levels of municipal waste compost in soil", when the fuzziness of the recorded data in Table 3 is considered equal to 0.001. Meanwhile, although we faced a vague decision in FANOVA based on imprecise data, but this level of uncertainty can be exactly measured by (17) in the proposed decision making method. It must be mentioned that all calculations of Table 6 are done by a computer program in *R* software (Rizzo, 2008) which is available upon request on the basis of the presented approach in Section 3.

As presented in Table 6, the non-equality hypothesis for the mean amounts of Cd absorption in three different levels of municipal waste compost (MWC) has been accepted with degree of acceptance $D_{H_1} = 0.505$ where k = 0.1. Similarly, the hypothesis H_1 has been accepted for a such case when manure sheep (MS) or sewage sludge (SS) is added to the soil at level 1.25%. Therefore the difference between means is significant at level 0.05.

Using TFNs arithmetic, one can compute the observed fuzzy mean amount of Cd absorption for each level (0%, 1.25% and 2.50%) of three kinds of organic fertilizers. For example, the observed fuzzy mean of Cadmium concentration in aerial parts of corn plant at level 1.25% municipal waste compost, manure sheep and sewage sludge, are

$$mean_{1.25\%MWC} = T(0.2225, 0.02225),$$

$$mean_{1.25\%MS} = T(0.3425, 0.03425),$$

and

$$mean_{1.25\%SS} = T(0.5825, 0.05825),$$

respectively, where k = 0.1. Also, it must be cleared for instance, the observed mean amount of Cd absorption in aerial parts of corn symbolically denoted by $mean_{1.25\%MWC}$ when 1.25% municipal waste compost has been added to soil. For these cases, the fuzzy observations of Cd concentration in aerial parts have been drown in Figure 3 and also their observed mean values have been drown in Figure 3 by bold lines. Also, one can compare these observed mean values as follows:

$$mean_{1.25\%MWC} < mean_{1.25\%MS} < mean_{1.25\%SS} \text{, for } k = 0, 0.0001, 0.001, 0.01, 0.1,$$

since the supports of observed mean values have not subscription in these cases. For other cases, the comparison is possible but by using any ordering or ranking function which we avoid to present them for shortening length of paper.

Similarly, Table 7 and Table 8 are contain the results of several FANOVA tests based on the fuzzified data of Table 4 and Table 5 for various spreads of STFNs, respectively.

By the results of Table 5, one can claim that increasing the vagueness of the data, fast causes the increase of the uncertainty and therefore causes more doubt in the decision. One can draw this nonlinear relation between the fuzziness of inputted data and the fuzziness of the outputted decision from the proposed FANOVA model by using (17). For instance, the mentioned relation for investigation on Pb concentration in root of corn plant at level 2.5% sewage sludge has been drown in Figure 4.

Conclusions and further research

The need for the fuzzy analysis of variance (FANOVA) based on vague data emerged from the attempt of providing a rigorous mathematical framework for precisely dealing with uncertain phenomena expressed by fuzzy numbers. As a practical problem, one may face fuzzy/vague observation rather than crisp data. For instance, an experimenter can use a fuzzy number to show the observed information for the absorbed Cadmium in a plant. In analyzing such vague numbers, we use an extended version of one-way ANOVA method which can measure the fuzziness of decision and this can be important form the applied point of view. By comparing the obtained fuzzy test statistics and the significance level 0.05, the degree of acceptance or rejection of the null hypothesis can be computed in the proposed FANOVA method. The presented degree of acceptance of null hypothesis in this research belongs to [0,1], but in the ordinary testing hypotheses it belongs to {0,1}, and this is one of the benefits of using fuzzy approaches instead of the conventional and classical methods in routine environmental practices containing uncertainties from nature. In the applied section of this paper, three organic fertilizers of manure sheep, sewage sludge and municipal waste were mixed with soil at the rates of 0%, 1.25 % and 2.5 % (w/w), respectively. In greenhouse experiment, corn seed were grown on pots of 3 kg treated contaminated soils and irrigated. Sixty days after sowing, the aerial plant parts were harvested and analyzed for Cadmium and Lead contents. Since all of data in this research are fuzzy, we use from FANOVA to investigate/comparison between the considered three populations on the basis of fuzzy observations. The results of investigation on the effect of levels of Cadmium and Lead concentration in aerial and root parts of corn plant are presented in Tables 6-8 for above mentioned manures, by considering $3\times5\times3$ FANOVA tests. The results showed that the proposed FANOVA test is a rational substitution for classical ANOVA when the observed data are fuzzy. Also, the relation between "fuzziness of inputted data" and "fuzziness of the outputted decision" is exactly computable in the proposed FANOVA model. In this research, increasing of the vagueness of the observed data nonlinearly causes the increase of the uncertainty and doubt in the obtained decision.

Theoretical and applied study on ANOVA model based on fuzzy hypotheses and fuzzy observations are two potential subjects for further research. The study of the applicability of the proposed FANOVA model in agricultural, industrial, social, chemical or economical sciences, are possible topics for further applied research. Moreover, the investigate of the FANOVA from a Bayesian perspective is another potential subject for more study.

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Figures

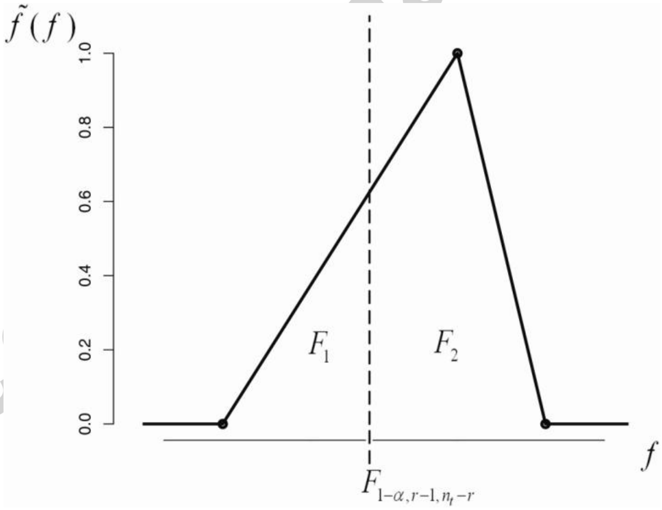

Figure 1. The membership function of the observed FANOVA test statistic and the indicator function of the α th quantile of Fisher distribution with $r-1$ and $n_t - r$ degrees of freedom

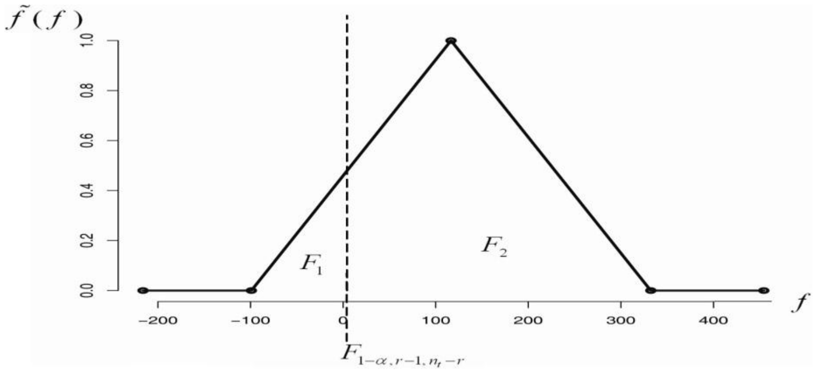

Figure 2. The membership function of the observed FANOVA test statistic and the indicator function of the α th quantile of Fisher distribution in FANOVA model for investigation on the effect of three levels (0%, 1.25% and 2.50%) municipal waste compost on Cadmium concentration in aerial parts of corn plant where $k = 0.001$

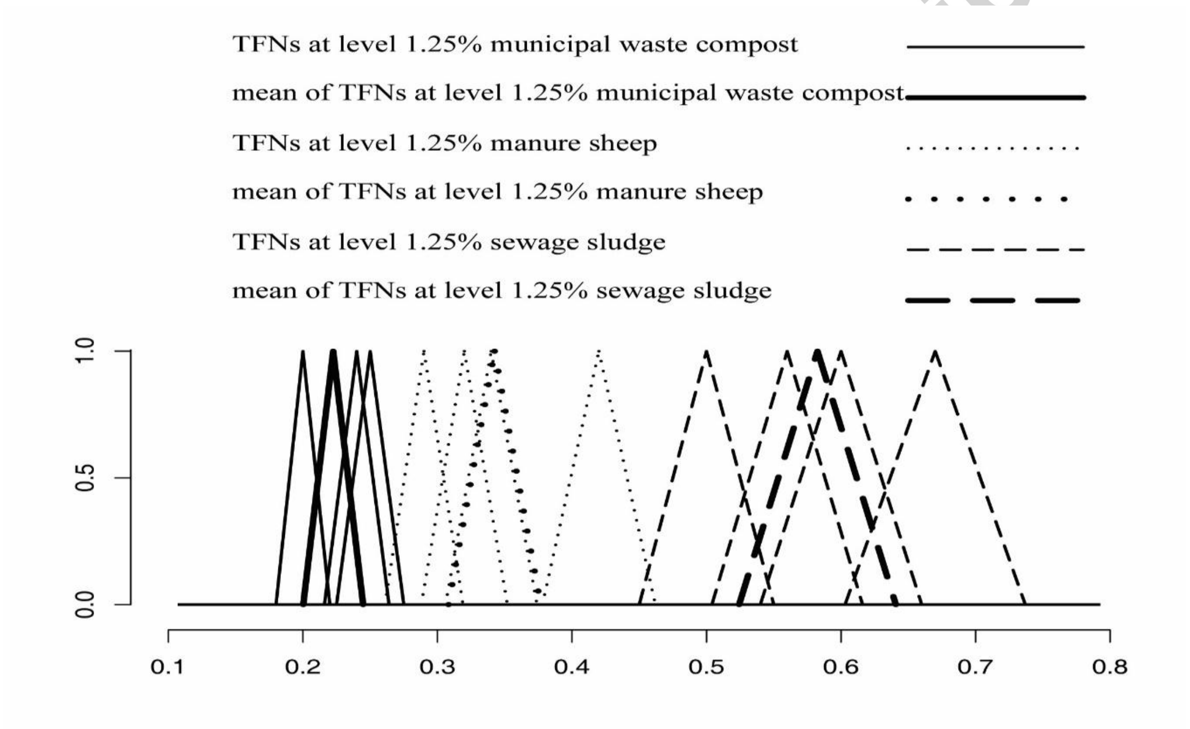

Figure 3. Fuzzy observations of Cadmium concentration in aerial parts of corn plant at level 1.25% municipal waste compost, manure sheep and sewage sludge, with $k = 0.1$

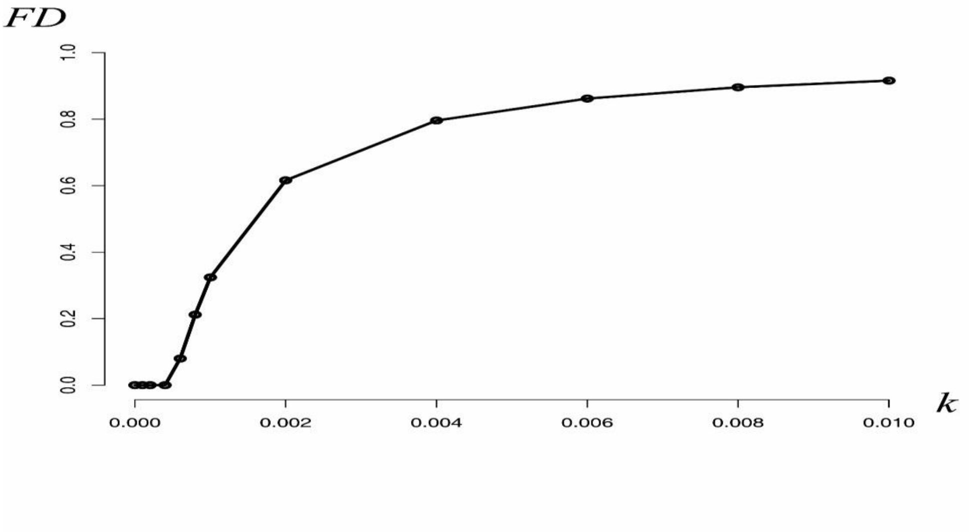

Figure 4. The relation between the fuzziness of inputted data and the fuzziness of the exported decision from the proposed FANOVA model, for investigation on lead concentration in root of corn plant at level 2.5% sewage sludge

Tables

Table 1. Some Chemical and physical properties of soil

Texture	F.C	CEC	pH	Ec	$Caco_3$	OC	Pb	Cd
Silty clay	22.3 %	21.32 $Cmol.kg^{-1}$	7.6	0.6 $dS.m^{-1}$	16 %	1.1 %	20.16 $mg.kg^{-1}$	2.98 $mg.kg^{-1}$

Table 2. Some Chemical properties of organic fertilizers

organic fertilizers	pH	EC	CEC	OC	Pb	Cd
municipal waste compost	7.7	7.35 $dS.m^{-1}$	32.28 $Cmol.kg^{-1}$	14.61%	60.4 $mg.kg^{-1}$	1.2 $mg.kg^{-1}$
manureSheep	8.08	22.9 $dS.m^{-1}$	61.5 $Cmol.kg^{-1}$	69%	0 $mg.kg^{-1}$	0 $mg.kg^{-1}$
sewage sludge	7.6	2.41 $dS.m^{-}$	52.28 $Cmol.kg^{-}$	32.14%	183.15 $mg.kg^{-1}$	2.1 $mg.kg^{-1}$

Table 3. The effect of three levels (0%, 1.25% and 2.50%) municipal waste compost, manure sheep and sewage sludge on Cadmium concentration in aerial parts of corn plant

added manure to soil	percent by weight	Cadmium concentration in aerial parts ($mg.kg^{-1}$)			
municipal waste compost	0%	0.43	0.4	0.4	0.41
	1.25%	0.2	0.25	0.24	0.2
	2.50%	0.3	0.3	0.3	0.29
manureSheep	0%	0.43	0.4	0.4	0.41
	1.25%	0.29	0.34	0.42	0.32
	2.50%	0.55	0.88	0.88	1.21
sewage sludge	0%	0.43	0.4	0.4	0.41
	1.25%	0.56	0.67	0.6	0.5
	2.50%	1.13	1.1	1.53	1.24

Table 4. The effect of three levels (0%, 1.25% and 2.50%) municipal waste compost, manure sheep and sewage sludge on Cadmium concentration in root of corn plant

added manure to soil	percent by weight	Cadmium concentration in root($mg.kg^{-1}$)			
municipal waste compost	0%	3.88	4.43	3.88	3.33
	1.25%	6.4	6.4	6.64	6.17
	2.50%	0.77	0.77	0.7	0.85
manureSheep	0%	3.88	4.43	3.88	3.33
	1.25%	1.82	2.16	2.16	2.50
	2.50%	4.37	4.37	5.1	3.65
sewage sludge	0%	3.88	4.43	3.88	3.33
	1.25%	2.67	2.67	2.64	2.7
	2.50%	5.86	5.86	6.1	5.63

Table 5. The effect of three levels (0%, 1.25% and 2.50%) municipal waste compost, manure sheep and sewage sludge on Lead concentration in root of corn plant

added manure to soil	percent by weight	lead concentration in root($mg.kg^{-1}$)			
municipal waste compost	0%	6.05	6.68	6.36	6.36
	1.25%	7.7	7.7	8.05	7.34
	2.50%	2.6	2.6	2.32	2.86
manureSheep	0%	6.05	6.68	6.36	6.36
	1.25%	1.18	1.18	1.22	1.14
	2.50%	1.52	1.52	1.51	1.53
sewage sludge	0%	6.05	6.68	6.36	6.36
	1.25%	4.83	4.83	4.4	5.26
	2.50%	3.73	3.73	3.81	3.65

Table 6. Investigation on the effect of levels on Cadmium concentration in aerial parts of corn plant by FANOVA for several manures on the basis of several sets of fuzzy data

vagueness of data (k)	0 (precise data)	0.0001	0.001	0.01	0.1
municipal waste compost	$f=117$ $D_{H_1}=1.000$	$\tilde{f}=T(117,21.6)$ $D_{H_1}=1.000$	$\tilde{f}=T(117,216)$ $D_{H_1}=0.885$	$\tilde{f}=T(117,2157)$ $D_{H_1}=0.551$	$\tilde{f}=T(117,21567)$ $D_{H_1}=0.505$
Manuresheep	$f=13.6$ $D_{H_1}=1.000$	$\tilde{f}=T(13.6,0.134)$ $D_{H_1}=1.000$	$\tilde{f}=T(13.6,1.34)$ $D_{H_1}=1.000$	$\tilde{f}=T(13.6,13.4)$ $D_{H_1}=0.952$	$\tilde{f}=T(13.6,134.5)$ $D_{H_1}=0.567$
sewage sludge	$f=54$ $D_{H_1}=1.000$	$\tilde{f}=T(54,1.48)$ $D_{H_1}=1.000$	$\tilde{f}=T(54,14.8)$ $D_{H_1}=1.000$	$\tilde{f}=T(54,148)$ $D_{H_1}=0.780$	$\tilde{f}=T(54,1475)$ $D_{H_1}=0.533$

Table 7. Investigation on the effect of levels on Cadmium concentration in root parts of corn plant by FANOVA for several manures on the basis of several sets of fuzzy data

vagueness of data (k)	0 (precise data)	0.0001	0.001	0.01	0.1
municipal waste compost	$f=394$ $D_{H_1}=1.000$	$\tilde{f}=T(394,49.7)$ $D_{H_1}=1.000$	$\tilde{f}=T(394,497)$ $D_{H_1}=0.977$	$\tilde{f}=T(394,4969)$ $D_{H_1}=0.575$	$\tilde{f}=T(394,49688)$ $D_{H_1}=0.508$
Manuresheep	$f=25.7$ $D_{H_1}=1.000$	$\tilde{f}=T(25.7,0.99)$ $D_{H_1}=1.000$	$\tilde{f}=T(25.7,9.95)$ $D_{H_1}=1.000$	$\tilde{f}=T(25.7,99.5)$ $D_{H_1}=0.692$	$\tilde{f}=T(25.7,995.5)$ $D_{H_1}=0.521$
sewage sludge	$f=130$ $D_{H_1}=1.000$	$\tilde{f}=T(130,17)$ $D_{H_1}=1.000$	$\tilde{f}=T(130,170)$ $D_{H_1}=0.967$	$\tilde{f}=T(130,1702)$ $D_{H_1}=0.571$	$\tilde{f}=T(130,17015)$ $D_{H_1}=0.507$

Table 8. Investigation on the effect of levels on lead concentration in root of corn plant by FANOVA for several manures on the basis of several sets of fuzzy data

vagueness of data (k)	0 (precise data)	0.0001	0.001	0.01	0.1
municipal waste compost	$f=423$ $D_{H_1}=1.000$	$\tilde{f}=T(423,122)$ $D_{H_1}=1.000$	$\tilde{f}=T(423,1220)$ $D_{H_1}=0.784$	$\tilde{f}=T(423,12199)$ $D_{H_1}=0.534$	$\tilde{f}=T(423,121986)$ $D_{H_1}=0.503$
manuresheep	$f=1499$ $D_{H_1}=1.000$	$\tilde{f}=T(1499,526)$ $D_{H_1}=1.000$	$\tilde{f}=T(1499,5264)$ $D_{H_1}=0.744$	$\tilde{f}=T(1499,52639)$ $D_{H_1}=0.528$	$\tilde{f}=T(1499,526393)$ $D_{H_1}=0.503$
sewage sludge	$f=108$ $D_{H_1}=1.000$	$\tilde{f}=T(108,24.1)$ $D_{H_1}=1.000$	$\tilde{f}=T(108,241)$ $D_{H_1}=0.838$	$\tilde{f}=T(108,2414)$ $D_{H_1}=0.542$	$\tilde{f}=T(108,24141)$ $D_{H_1}=0.504$

Highlights

- In a greenhouse experiment, corn seed were grown on pots of 3 kg treated contaminated soils and irrigated.
- Sixty days after sowing, the aerial plant parts were harvested and analyzed for Cd and Pb contents.
- All followed data in this research are fuzzy and therefore we need an extended version of analysis of variance (ANOVA) to investigate on these fuzzy observations.
- As the method to compare several populations, the fuzzy analysis of variance, FANOVA, has been used where the collected data considered fuzzy rather than crisp numbers.