Fractional Order Control of pH Neutralization Process Based on Fuzzy Inverse Model

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***Abstract*—pH neutralization process is a nonlinear process with time varying characteristics. Usually, the model based control of this process is a difficult task. Regarding to the appropriate modelling of the pH process by a Wiener model, in this paper an inverse model based control scheme is presented in which a fractional order controller is used as a linear controller. To apply this control scheme, firstly the inverse model of the nonlinear static part of the pH process is modelled benefiting from a fuzzy modelling method. Then by using the inverse model of the nonlinear static part, an augmented system is obtained whose behavior is similar to a linear system. Therefore, simple controllers such as PID type controllers can be used to control the augmented plant. In this paper, a fractional order PID (FOPID) controller is applied to the augmented system as a linear controller. The simulations presented in this paper show the efficiency of this type of controller in comparison with the classical PID controller in a similar structure.**

Keywords—Fractional Order PID Controller; pH Process; Fuzzy Inverse Model; Wiener Model

1. Introduction

In recent years, fractional calculus has attracted the interests of the researchers in different fields. In the field of control engineering, recently many researchers have been done focusing on designing different types of fractional order controllers [1-4]. Most widely used form of fractional order controllers include PI^{λ}, PD^{μ} , and $PI^{\lambda}D^{\mu}$ controllers proposed by Podlubny [5]. The freedom in tuning λ and μ parameters provide us with more precise adjustment of the control system properties [4, 6-8]. It has been shown that these controllers are more efficient than their corresponding integer order controllers in some applications [6-8]. Since many industrial processes are nonlinear in their essence, linear controllers may not result in good performance for all set-points.

Combination the fractional order controller with fuzzy system, as a tool for dealing with uncertainty as well as the use of human knowledge can employ adjectives of both methods [9]. Also, this combination is a useful tool to control of nonlinear systems [10]. In this paper, the combination of a fractional order PID (FOPID) controller and a fuzzy system is employed in control of pH neutralization process.

pH neutralization process is used in the wide range of industries, e.g. biotechnology, fermentation, and chemical processing. In this control system, maintaining the pH value of a liquid in a special level is a general purpose. In some cases, fractional order PI (FOPI) and FOPID controllers have been used to control pH process only in a particular set-point [11, 12]. But due to the nonlinear and time varying characteristic of the pH process, usually the performance objectives are hardly achieved by the use of linear controllers. Moreover, the high sensitivity of this process to even small disturbances when the system works near its equilibrium point is another difficulty in the control procedure [13].

In literature, different models have been presented for describing the dynamic behavior of the pH process [14-16]. In [14, 15], pH process has been modelled by Seborg for a batch type process based on the Wiener model. Wiener model has been used for systems possessing nonlinearity in their sensors such as pH process [17, 18]. Each Wiener model consist a linear dynamic and a nonlinear static part [19] which by removing its effect it would be possible to benefit from linear controllers such as PID type controllers. As demonstrated by several researchers, the nonlinear static part of the pH process is reflected in the titration curve of the process stream [20, 21]. One of the useful tools for modelling the titration curve of the pH process is fuzzy logic system [22-24]. The control of the pH neutralization process based on its inverse model has been also investigated in literature [17, 24, 25]. Despite fuzzy systems have the ability to describe uncertain phenomenon, it is an explicit theory with exact relations. Unlike the classical model which has a poor performance in dealing with uncertainty, fuzzy inference system by employing the if-then rules can model the uncertainties [26]. Jang proposed an adaptive network-based fuzzy inference system (ANFIS) for purpose of fuzzy identification based on human knowledge (in the form of if-then fuzzy rules) and based on input-output system data and used the combinational training procedure to minimize the cost function [27].

In this paper, firstly the inverse model of the nonlinear static part of the pH neutralization process is identified by using a fuzzy modelling method. Then by multiplying the process output by the obtained static inverse model, an augmented system is achieved whose behavior is similar to a linear system. Therefore, it is possible to obtain a First Order Plus Dead Time (FOPDT) model based on the step response of the augmented system. By using the obtained FOPDT model a FOPID controller is designed and applied in the inverse model based control structure. Finally, by presenting simulation results the efficiency of applying FOPID controller is shown in comparison with the classical PID controller in the similar structure.

This paper is organized as follows. In Section 2, the pH process is introduced. In Section 3, the inverse nonlinear static part of the pH process is identified by ANFIS. In the end of this section, a FOPDT model is presented for describing the linear dynamic behavior of the pH process augmented with its inverse static model. Section 4 devotes to presenting some preliminaries about fractional order calculus and FOPID controllers. The method of designing FOPID controller has been described in Section 5. In Section 6, firstly we show that a single FOPID controller could not result in a good performance in all set-points. Secondly, the results of applying FOPID controller in the inverse model based control structure have been compared with the results of applying their integer order counterparts in the presence of external disturbance and measurement noise. Also, it is shown that a more practicable control signals can be achieved by applying a set-point weighted FOPID controller. Finally, the paper is concluded in Section 7.

2. pH Neutralization Process

The inputs of the pH process are base, acid, and buffer flow. In this paper, water is added to reactor and the acid input is supposed to be constant. The outputs of the process are the pH value and the height of the tank content. The control objective in this process is control of the pH value by adjusting the rate of the base stream. Since in this process the interactions effects are negligible, it is possible to control the height of the tank content in a separated loop by a PID controller [13, 28]. The control structure of pH process is shown in Fig. 1. The high sensitivity of this process to the measurement noise and disturbance signals is a challenging problem in designing an effective control system. Different types of disturbances may occur in this process which can be categorized as change of acid concentration, change of acid flow rate, change of base concentration, and change of level set-point which change both the flow of the water and volume of liquid in the reactor.

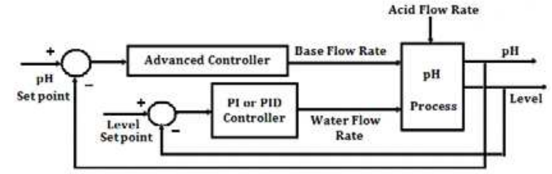

Fig. 1. Control structure of the pH process [28]

3. Fuzzy Inverse Model

To apply fuzzy inverse model control strategy it is required to identify the nonlinear static part of the system with a fuzzy inverse model at first.

The Wiener model of system is defined as

$$y(t) = h(G(q)u(t)). \tag{1}$$

In this structure, $G(q)$ and $h(.)$ are a transfer function of the linear dynamic and a mapping of the nonlinear static, respectively. Also, q is the forward-shift operator, $u(t)$ is the input and $y(t)$ is the output of the Wiener model. By passing $y(t)$ as the system output through the estimation of the inverse of the static nonlinear block, we have

$$v(t) = \hat{h}^{-1}(y(t)), \tag{2}$$

where, $v(t)$ is the output of the nonlinear static inverse model. If the estimation $\hat{h}(.)$ of $h(.)$ has an enough accuracy, then the augmented system from input $u(t)$ to output $v(t)$ has a linear behavior and we can use linear theory approach to control this system. To model the inverse of the static nonlinear part, we can use ANFIS model structure [27]. The schematic of ANFIS structure used in this paper is shown in Fig. 2.

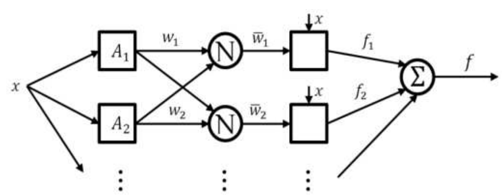
Fig. 2. The schematic of ANFIS structure [27]

According to Fig. 2, the output is a function of input as follows.

$$f(x) = \sum_{i=1}^{M} \bar{w}_i f_i(x) = \frac{\sum_{i=1}^{M} A_i(x)\left(m_i x + n_i\right)}{\sum_{j=1}^{M} A_j(x)}. \tag{3}$$

In (3), x and f are respectively the input and output of fuzzy model. f_i and w_i are the output and firing strength of the i-th rule and also, A_i is the membership function of the i-th input. The above fuzzy system has singleton fuzzifier, product inference engine and center average defuzzifier. In this paper, we use Gaussian membership function defined as

$$w_i = A_i(x) = \exp\left(-\frac{(x-c_i)^2}{2\sigma_i^2}\right), \tag{4}$$

where, c_i and σ_i are respectively the center and variance of i -th membership function. Parameters c_i and σ_i are updated by back propagation learning method using gradient descent optimization algorithm. The cost function in this optimization is considered as sum of the squared error (SSE). As can be seen in (3), parameters m_i and n_i will appear in a linear structure equation and can be estimated using recursive least square (RLS) method,

$$f(x)=[\bar{w}_1 x \quad \bar{w}_1 \quad \cdots \quad \bar{w}_M x \quad \bar{w}_M]^T [m_1 \quad n_1 \quad \cdots \quad m_M \quad n_M]$$
$$=X^T\theta \ , \tag{5}$$

where, X and θ are regressor and parameter vectors.

An efficient solution to obtain an inverse model of a system is an offline identification based on its output-input data pairs. One of the important considerations of inverse model identification is distribution of input-output identification data. In Fig. 3 (a-1), the input data has a uniform distribution in the range of 1 to 10, but the output data of pH neutralization does not have a uniform distribution in the range of 4 to 14 (See Fig. 3 (a-2)). To obtaining a uniform distribution in $y(t)$, based on knowledge about pH process and titration curve, if we choose an input data with distribution as Fig. 3 (b-1), output has a uniform distribution as Fig. 3 (b-2).

Number of the input and output membership functions and rules are determined by cost function diagram which is obtained as 9. Therefore, the fuzzy system is constructed using 9 membership functions, 9 rules, and 9 outputs. In Fig. 4, the mapping of the obtained fuzzy inverse model is compared with pH titration curve. Due to the unity gain of the linear dynamic model, these two curves are fitted with good accuracy.

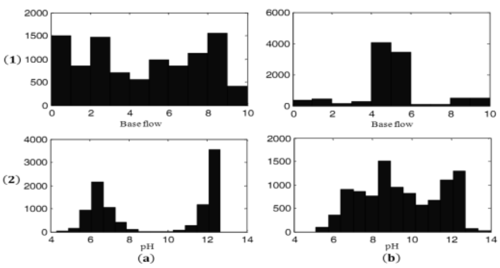

Fig. 3. Distribution of input-output data pair. a) Data collection with uniform distributed input data. b) Data collection with uniform distributed output data.

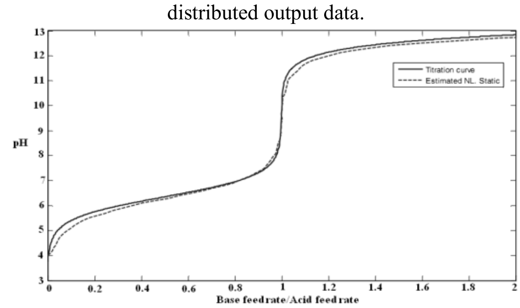

Fig. 4. Titration curve and its estimation

Since linear dynamic part of a Wiener model can be estimated by a FOPDT model [19], by augmenting the obtained inverse static model and the plant, the FOPDT model (6) is estimated based on step response parameter estimation method.

$$G(s)=\frac{1}{33.675s+1}e^{-6s}. \tag{6}$$

This model describes the dynamic behavior of the augmented system. The delay term in the linear model (6) is appeared because of the existence of a dead time between the actuator and the plant in a real system.

4. Fractional Order PID Controller

Fractional calculus is a generalization of integration and differentiation to non-integer order fundamental operator ${}_aD_t^{\alpha}$ where a and t are respectively the lower and the upper terminals of the operation. A fractional order integral of order γ is defined as follows [29].

$${}_0I_t^{\gamma}x(t)=\frac{1}{\Gamma(\gamma)}\int_0^t (t-\tau)^{\gamma-1}x(\tau)d\tau \ , \ \gamma\in\mathbb{R}^+ \ . \tag{7}$$

In (7), $\Gamma(\gamma)$ is the Euler's Gamma function.

The Caputo definition of the fractional order derivative of order γ, where γ is a positive non-integer number is given by

$${}_0^C D_t^{\gamma}x(t)={}_0I_t^{r+1-\gamma}\left\{\frac{d^{r+1}}{dt^{r+1}}x(t)\right\}, \ r<\gamma<r+1 \ , \ r\in N\cup\{0\} \ . \tag{8}$$

Also, if the initial conditions are considered as zero, the Laplace transform of the Caputo fractional derivative is given by

$$L\left\{{}_0^C D_t^{\gamma}x(t)\right\}=s^{\gamma}L\left\{x(t)\right\} \ , \ r<\gamma<r+1 \ , \tag{9}$$

where $L\{.\}$ denotes the Laplace transform operator [29]. Generally, a fractional order transfer function $G(s)$ is represented as

$$G(s)=\frac{Q(s)}{P(s)}=\frac{b_{n-1}s^{\beta_{n-1}}+b_{n-2}s^{\beta_{n-2}}+\cdots+b_1s^{\beta_1}+b_0}{s^{\alpha_n}+a_{n-1}s^{\alpha_{n-1}}+\cdots+a_1s^{\alpha_1}+a_0}, \tag{10}$$

where $P(s)$ and $Q(s)$ have no common zeros, $\alpha_n>\alpha_{n-1}>\cdots>\alpha_1\geq 0$ and $\beta_{n-1}>\cdots>\beta_1\geq 0$ are arbitrary real numbers. For $\beta_{n-1}\leq\alpha_n$ transfer function $G(s)$ in (10) would be a proper transfer function [29].

The most common fractional order controller is the FOPID controller which is described as

$$C(s)=k_p+\frac{k_i}{s^{\lambda}}+k_d s^{\mu} \ , \ 0<\lambda<2 \ , \ 0<\mu<2 \ . \tag{11}$$

In (11), the parameters λ and μ are respectively the fractional order of the integrator and the differentiator part of the controller. Also, k_p, k_i, and k_d are respectively the proportional, integral, and derivative gains of the FOPID controller. The approximation method presented in [30] is used for implementation of the designed FOPID controller in this paper. Also, the order and the frequency range of the approximation filter are considered as $N=5$ and $\omega=[0.001,1000](rad/\sec)$ respectively.

5. Linear Controller Design

The general schematic of an inverse model based control structure is shown in Fig. 5. In literature, different types of linear controllers have been used to control pH process by an inverse model based control system [22, 24, 25]. Among these controllers, PID controllers have been mostly used due to their simplicity and effectiveness [25]. In this paper for achieving more desired performance specifications, FOPID controller is used as the linear controller.

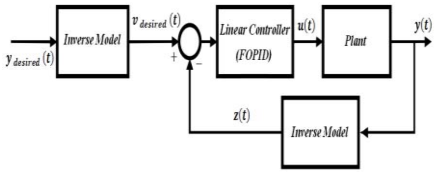

Fig. 5. Schematic of an inverse model based control system [25]

Recalling from Section 2, in the pH neutralization process the height of the tank content can be controlled in a separated loop by a PID controller [13, 28]. The intended PID controller is considered as

$$C_{level}(s) = 50 + \frac{4}{s} + 0.1s\,. \tag{12}$$

5.1 FOPID Controller Design

In this subsection, the FOPID controller (11) is designed based on the method presented in [8]. In this method, the controller is obtained such that for a given cross over frequency ω_c in which

$$|L(j\omega_c)| = 0(dB)\,, \tag{13}$$

the relations

$$Arg(L(j\omega_c)) = -\pi + \phi_m\,, \tag{14}$$

and

$$\left(\frac{d(Arg(L(j\omega_c)))}{d\omega}\right)_{\omega=\omega_c} = 0\,, \tag{15}$$

hold. In these equations, $L(s) = G(s)C(s)$ where $C(s)$ is the controller and $G(s)$ is the transfer function of the plant. In (14), ϕ_m is the required phase margin. Also, for achieving good reference tracking and disturbance rejection in the frequency bandwidth, the inequality

$$|S(j\omega)| = \left|\frac{1}{1+L(j\omega)}\right| \le B(dB) \quad \forall \omega \le \omega_s\,, \tag{16}$$

should be held. In (16), the parameter B is determined by the designer. Finally, by holding the following inequality

$$|T(j\omega)| = \left|\frac{L(j\omega)}{1+L(j\omega)}\right| \le A(dB) \quad \forall \omega \ge \omega_t\,, \tag{17}$$

it would be possible to have an appropriate noise reduction for high frequencies. In (17), the value of parameter A is chosen according to the intended control objective on noise reduction.

6. Simulation Results

In this section, firstly we show that a single FOPID controller could not yield to the desired performance in all set-points. Secondly, the results of applying FOPID controller are shown in comparison with those of applying a classical PID controller in the inverse model based control of the pH neutralization process. At the end of this section, the result by applying the FOPID controller based on the set-point weighted control structure is presented.

6.1 Single FOPID Controller Design

In this subsection, firstly we estimate a FOPDT model for three set-points of pH neutralization process. Based on the pH titration curve, three set-points are selected as follows.

$$pH = 6.297 : P_1(s) = \frac{0.3486}{31s+1}e^{-6s}, \tag{18}$$

$$pH = 8.207 : P_2(s) = \frac{6.417}{33s+1}e^{-6s}, \tag{19}$$

and

$$pH = 8.909 : P_3(s) = \frac{29.96}{33s+1}e^{-6s}. \tag{20}$$

For $P_1(s)$ in (18) and by considering

$$\omega_{c_1} = 0.01(\frac{rad}{\sec}), \phi_{m_1} = 80^o,\ \omega_{t_1} = 1(\frac{rad}{\sec})\,,$$

$$\omega_{s_1} = 0.001(\frac{rad}{\sec}), A_1 = -20(dB), B_1 = -20(dB)\,, \tag{21}$$

based on the method described in Subsection 5.1, the FOPID controller is obtained as

$$C_{FOPID\_P_1}(s) = 0.9029 + \frac{0.0205}{s^{1.0827}} + 0.3086\, s^{0.3894}\,. \tag{22}$$

Similarly, for $P_2(s)$ in (19) and by considering

$$\omega_{c_2} = 0.03(\frac{rad}{\sec}), \phi_{m_2} = 80^o,\ \omega_{t_2} = 5(\frac{rad}{\sec})\,,$$

$$\omega_{s_2} = 0.001(\frac{rad}{\sec}), A_2 = -20(dB), B_2 = -20(dB)\,, \tag{23}$$

the FOPID controller is achieved as

$$C_{FOPID\_P_2}(s) = 0.1569 + \frac{0.0042}{s^{1.057}} + 0.1295\, s^{0.5307}\,. \tag{24}$$

Finally, for $P_3(s)$ in (20) and by considering

$$\omega_{c_3} = 0.08(\frac{rad}{\sec}), \phi_{m_3} = 80^o,\ \omega_{t_3} = 10(\frac{rad}{\sec})\,,$$

$$\omega_{s_3} = 0.003(\frac{rad}{\sec}), A_3 = -20(dB), B_3 = -20(dB)\,, \tag{25}$$

the FOPID controller is resulted as

$$C_{FOPID\_P_3}(s) = 0.0672 + \frac{0.0039}{s^{0.9965}} + 0.2575\, s^{0.6456}\,. \tag{26}$$

The results of applying three FOPID controllers (22), (24), and (26) are shown in Fig. 6 (a), Fig. 6 (b), and Fig. 6 (c) respectively. As can be seen in Fig. 6, FOPID controllers (22) and (24) become unstable (See Fig. 6 (a) and Fig. 6 (b)). Also, applying FOPID controller (26) does not result in good performance for all set-points (See Fig. 6 (c)). Thus, according to the simulation results it is inferred that a single FOPID controller is unable to control pH process with good performance in all set-points.

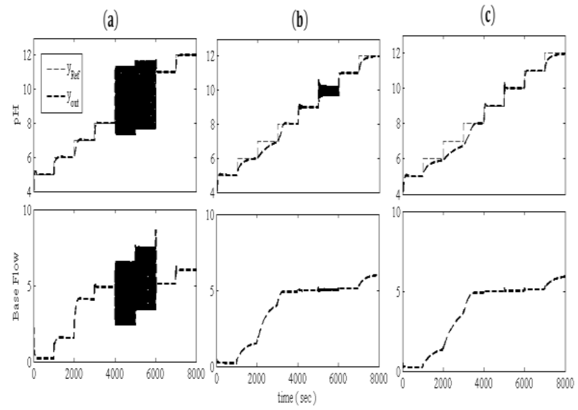

Fig. 6. Output by applying three FOPID controllers (22, 24, and 26)

6.2 FOPID and Fuzzy Inverse Model

The identified FOPDT model $G(s)$ in (6) has been considered as the linear model describing the dynamical behavior of the pH system augmented with its static inverse model. Therefore, for $G(s)$ in (6) and by considering

$$\omega_c = 0.0263(\frac{rad}{\sec}), \phi_m = 70^o, \omega_t = 0.3(\frac{rad}{\sec}),$$

$$\omega_s = 0.002(\frac{rad}{\sec}), A = -20(dB), B = -20(dB), \tag{27}$$

based on the method described in Subsection 5.1, a FOPID controller is obtained as follows.

$$C_{FOPID}(s) = 0.6975 + \frac{0.0252}{s^{1.0733}} + 0.6787\, s^{0.4042}. \tag{28}$$

Also, by considering the three conditions brought in (13), (14), and (15), a classical PID controller can be designed as

$$C_{PID}(s) = 0.6794 + \frac{0.0377}{s} + 10.7965 s. \tag{29}$$

Besides the PID controller (29), the performance of two other PID controllers designed based on Chien-Hrones-Roswick (CHR) and Amigo method are verified in this paper [31, 32]. The PID controllers obtained based on Amigo and CHR design methods are given by

$$C_{AMIGO}(s) = 2.7256 + \frac{0.145}{s} + 7.7620 s, \tag{30}$$

$$C_{CHR}(s) = 3.3675 + \frac{0.5612}{s} + 10.1025 s. \tag{31}$$

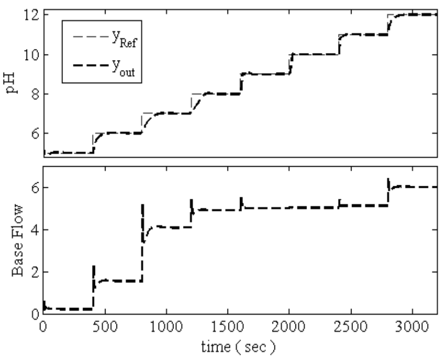

Fig. 7. Output by applying the FOPID controller (28)

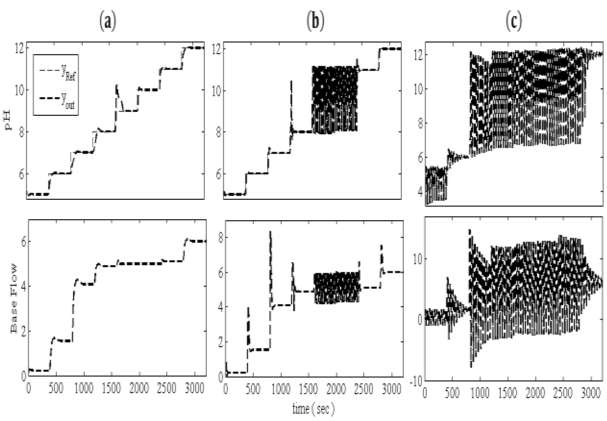

Fig. 8. Output by applying three PID controllers (29, 30, and 31)

In Fig. 7, the performance of the FOPID controller (28) has been investigated in the inverse model based control of the pH process. Also, the performances of the three classical PID controllers (29), (30), and (31) are shown in Fig. 8 (a), Fig. 8 (b), and Fig. 8 (c) respectively. This investigation has been done for 8 different set-points. As these figures reveal, the FOPID controller (28) has an appropriate set-point tracking for every 8 set-points while the PID controller (29) can't track the fifth set-point $(pH = 9)$ effectively and the PID controllers (30) and (31) become unstable.

In Fig. 9, the efficiency of the FOPID controller (28) is shown in reducing the noise effect. For this purpose, a noise is injected to the output signal as the measurement noise. In this simulation, the noise power is considered equal 0.005.

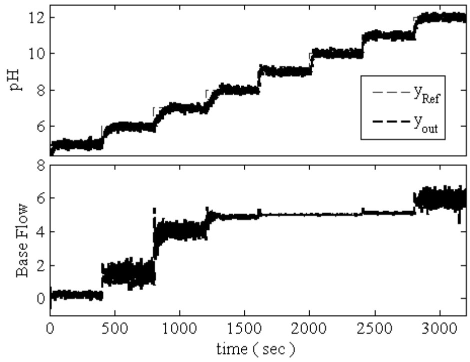

Fig. 9. Noise reduction by the FOPID controller (28)

To verify the performance of the FOPID controller (28) in disturbance rejection, increase of acid concentration at time instant $t = 2700$ sec is applied as the disturbance signal in Fig. 10. This figure shows that the disturbance rejection is done more efficiently by the FOPID controller (28) in comparison with what is done by the PID controller (29).

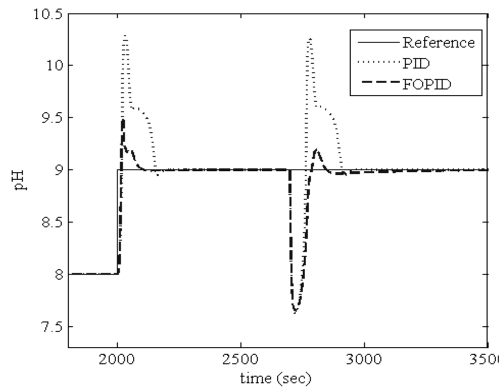

Fig. 10. Disturbance rejection of two controllers (28, 29)

Up to now, the efficiency of the FOPID controller (28) has been shown in reference tracking, noise reduction and disturbance rejection of the pH process compensated by an inverse model based control structure. But as Fig. 7 show, the FOPID controller (28) may not be applicable due to the rapid change of its control signal in a short timeframe. The structure of set-point weighted FOPID controller has been presented in [33]. Different methods for adjusting the set-point weight (a) are given in [33, 34]. This parameter impacts only on tracking the reference signal. In this paper, regarding to the purpose of achieving an appropriate response for every 8 set-points, the value of the set-point weight has been chosen as $a = 0.6$. The result of applying set-point weighted

FOPID controller is shown in Fig. 11. This figure shows that the control signal by applying the set-point weighted FOPID controller become smoother in comparison with the control signal obtained by applying FOPID controller.

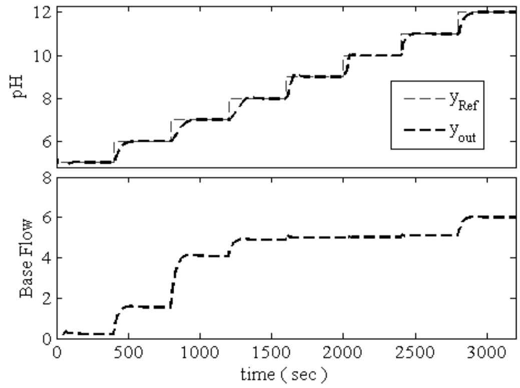

Fig. 11. Output by applying the set-point weighted FOPID controller

7. Conclusions

In this paper, firstly it was shown that the performance of a single FOPID controller would not be appropriate for all set-points. Secondly, by benefiting from a FOPID controller in an inverse model based control system the pH value of the pH process was controlled effectively. Generally speaking, in this paper by the combination of an inverse fuzzy model and a FOPID controller in an inverse model based control system a simple and effective control strategy was presented for the pH value control of the pH neutralization process.

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