# An Integrated Performance and Robust Stability Based Multi-Model Control of Nonlinear Systems: A pH neutralization reactor

M. Ahmadi
Advanced Control Systems Lab
Electrical Engineering Department
Sharif University of Technology
Tehran, Iran
Email: mahdiahmadi@ee.sharif.edu

M. Haeri
Advanced Control Systems Lab
Electrical Engineering Department
Sharif University of Technology
Tehran, Iran
Email: haeri@sharif.ir

*Abstract*—**This paper presents a new multi-model control scheme which integrates the stability and performance requirements to determine the nominal local models. Gap metric and stability margin concepts are applied to measure the distance between the local models and to grid the operation range of nonlinear system, respectively. The minimization of the integral of time absolute error is included as performance criterion. Furthermore, the multi-model decomposition and local controllers design are integrated in an optimization problem which provides avoidance of the redundancy issue. A fixed PI structure is considered as the local controllers to simplify the global multi-model controller structure. Therefore, the advantages of the presented method are: to avoid the dependency on experience, to eliminate the redundancy problem, to guarantee the stability and performance, and to have a simple structure. A pH neutralization reactor is simulated to evaluate the efficiency of the proposed method. Results demonstrate that this approach is systematic and works better than the traditional multi-model controller in terms of set-point tracking and disturbance rejection.**

***Keywords-multi-model; gap metric; stability margin; performance; pH neutralization reactor.***

## I. INTRODUCTION

Multi-model methods are powerful techniques developed to deal with nonlinear systems [1]. These approaches have become one of ideal candidates to handle strong nonlinear systems either operate in a wide range of operation space or have large set-point changes [1-3]. This idea has attracted much attention in the past years [2-8]. The key point is to represent a complicated nonlinear system by a combination of local linear subsystems so that well-known control techniques such as PID, MPC, LQR, robust control, and others can be utilized to design a global controller. Though, the questions of how many and which local models are required to span the global operation space are the most important issues in multi-model methods, they remain unanswered yet [7].

The methods to select the nominal local models are broadly classified into three families: experience [3, 4, 9], stability [8, 10-12], and integrated stability and performance based methods [5, 7]. In the first family, either a prescribed distance level [3, 4, 13] or a priori knowledge about the behavior of nonlinear system [9] is employed to grid the operation space and select the nominal local models. This family of methods suffers not only from dependency on experience, but also the redundancy problem which influences directly the complexity of the global controller in multi-model form.

In the second family of methods, robust stability of local models is considered as a criterion to grid the operation range. To this end, gap metric turns out to be a useful tool to measure the distance between the local models. The gap, $\upsilon$-gap, and H-gap have been utilized in [8], [10], and [14], respectively. Though, local robust stability of the nonlinear system is guaranteed and dependency on experience is largely reduced by the obtained global controller, it is not so easy to design a global controller that satisfies both stability and performance requirements concurrently. In order to overcome this weakness, the third family of methods have introduced in recent years.

Robust stability and closed-loop performance have been integrated to design the global multi-model controller in the third family of methods. In [7], the authors have introduced a method, which is an improved version of the one in [8], to select the nominal local models based on robust stability and performance. The $H_\infty$ controllers are designed for each local model so that the performance requirements are satisfied. Then, the local models are classified into different sub-regions based on their stability margins. The time spent to design the $H_\infty$ controllers will be a major concern if the number of local models is large. To fix the problem, a threshold value, the minimum of a tuning parameter and maximum stability margin, is introduced to cluster the local models bank. This strategy has used again in [5] where the local controllers are MPC. Though, the method presented in [5, 7] is effective, the dependency on a priori knowledge to choose the tuning parameter still exists while the performance requirements cannot be included in $H_\infty$ design procedure as well. These issues are the main consideration of our work.

In this paper, we introduce a new method which is included

978-1-5090-5963-8/17/$31.00 ©2017 IEEE

in the third category. The proposed method incorporates the stability criterion and closed-loop performance to choose the nominal local models and design the local controllers. The minimization of the integral of time absolute error (ITAE) is considered as performance criterion. In other words, the selection of nominal local models and design of the local controllers are integrated in an optimization problem which is non-convex and will be solved using Heuristic Kalman Algorithm (HKA) [15, 16]. Also an updating threshold is introduced which not only helps to avoid the redundancy problem, but also removes the dependency on a priori knowledge. In comparison with previous works, our method provides a simple and systematic approach to design multi-model controller which eliminates redundancy and dependency on experience, and guarantees the robust stability and performance. To evaluate the effectiveness of the method, a $pH$ neutralization reactor is considered. This system exhibits a highly nonlinear behavior and its control is challenging in the chemical process industry and cannot be effectively controlled with a conventional PI controller.

The rest of this paper is organized as follows. In Section II, the relevant backgrounds of multiple-model methods and gap metric theorems are discussed. The integrated nominal local models selection and local controllers design are established in Section III. In Section IV, simulation results and analysis are provided and discussed. Finally, Section V concludes the paper.

## II. PRELIMINAIES

Let consider nonlinear systems described by

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= h(x,u), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^r$ is the manipulated variable vector, and $y \in \mathbb{R}^p$ is the output vector. Also $f: \mathbb{R}^{n\times r} \to \mathbb{R}^n$ and $h: \mathbb{R}^{n\times r} \to \mathbb{R}^p$ are nonlinear differentiable vector-valued functions.

It is assumed that nonlinear system (1) has an equilibrium manifold, denoted by $\Psi$, defined as $\{(x_e,u_e,y_e) | f(x_e,u_e) = 0 \text{ and } y_e = h(x_e,u_e)\}$ where $(x_e,u_e,y_e)$ is an equilibrium point. By linearizing nonlinear system (1) at the $i^{th}$ equilibrium point, the corresponding local model ($P_i$) is determined as

$$\begin{aligned} \delta\dot{x}_i &= A_i\delta x_i + B_i\delta u_i, \\ \delta y_i &= C_i\delta x_i + D_i\delta u_i, \end{aligned} \tag{2}$$

where $A_i = \partial f/\partial x|_{(x_{ei},u_{ei})}$, $B_i = \partial f/\partial u|_{(x_{ei},u_{ei})}$, $C_i = \partial h/\partial x|_{(x_{ei},u_{ei})}$, $D_i = \partial h/\partial u|_{(x_{ei},u_{ei})}$, $\delta x_i = x - x_{ei}$, $\delta u_i = u - u_{ei}$, and $\delta y_i = y - y_{ei}$. Each of local models has a validity region $\psi_i \subset \Psi$ which indicates they cannot represent the behavior of original nonlinear system individually. The multi-model method combines the local models in the overall model as follows

$$\begin{aligned} \dot{x}^{li} &= \textstyle\sum_{i=1}^{N_s} w_i(t)(A_i x_i + B_i u + \alpha_i), \\ y^{li} &= \textstyle\sum_{i=1}^{N_s} w_i(t)(C_i x_i + D_i u + \beta_i), \end{aligned} \tag{3}$$

where $\alpha_i = -A_i x_{ei} - B_i u_{ei}$ and $\beta_i = h(x_{ei},u_{ei}) - C_i x_{ei} - D_i u_{ei}$. Also, $w_i(t)$ denotes the time-dependent weighting function corresponds to $P_i$ at time $t$ that has to satisfy the following convex combination constraint.

$$\textstyle\sum_{i=1}^{N_s} w_i(t) = 1, \text{ where } 0 \le w_i(t) \le 1 \tag{4}$$

To determine the number ($N_s$) and location ($(x_{ei},u_{ei}), i = 1{:}N_s$) of required nominal local models, the gap metric can be applied which measures the distance between local models. Let consider $P(s)$ be a rational transfer matrix and described by normalized right coprime factorization as follow.

$$P(s) = \mathcal{N}(s)\mathcal{M}(s)^{-1}, \text{ where } \widetilde{\mathcal{M}}(s)\mathcal{M}(s) + \widetilde{\mathcal{N}}(s)\mathcal{N}(s) = I, \tag{5}$$

where $(\tilde{\cdot})$ denotes complex conjugate, i.e., $\widetilde{\mathcal{M}}(s) = \mathcal{M}^T(-s)$ and $\widetilde{\mathcal{N}}(s) = \mathcal{N}^T(-s)$. For finite-dimensional linear systems with the same number of inputs and outputs, the gap metric can be computed as follows [17].

$$\delta(P_1,P_2) = \max\left\{ \inf_{Q\in H_\infty} \left\| \begin{bmatrix}\mathcal{M}_1\\ \mathcal{N}_1\end{bmatrix} - \begin{bmatrix}\mathcal{M}_2\\ \mathcal{N}_2\end{bmatrix} Q \right\|_\infty, \inf_{Q\in H_\infty} \left\| \begin{bmatrix}\mathcal{M}_2\\ \mathcal{N}_2\end{bmatrix} - \begin{bmatrix}\mathcal{M}_1\\ \mathcal{N}_1\end{bmatrix} Q \right\|_\infty \right\}. \tag{6}$$

The gap metric has properties such as: 1) $0 \le \delta(P_1,P_2) \le 1$ and 2) a small gap between two linear systems implies not only they behave similarly in the closed-loop sense, but also there exist at least a feedback controller which can stabilize both of them (more details can be found in [17]). An useful connection between gap metric and stability is given by Proposition 1.

**Proposition 1 [17].** Assume a feedback system with pair $(P,K)$ is stable. Let $\mathcal{P}$ be defined as $\mathcal{P} \triangleq \{P_\Delta | \, \delta(P,P_\Delta) < \delta_P\}$. Then the feedback system with pair $(P_\Delta,K)$ is stable for all $P_\Delta \in \mathcal{P}$ if and only if

$$\delta_P = \delta(P,P_\Delta) \le b_{P,K}, \tag{7}$$

where $b_{P,K}$ is the stability margin and is defined for rational transfer matrix $P(s)$ and stabilizing controller $K$ as follows

$$b_{P,K} = \left\| \begin{bmatrix} I \\ K(s) \end{bmatrix} (I + P(s)K(s))^{-1} [I \quad P(s)] \right\|_\infty^{-1} \tag{8}$$

Moreover, it has been proven in [8] that there exists at least a controller $K$ that robustly stabilizes $P$ if and only if

$$\delta_P = \delta(P,P_\Delta) \le b_{\mathrm{opt}}(P), \tag{9}$$

where $b_{\mathrm{opt}}$ is the maximum stability margin and defined as follows

$$b_{\mathrm{opt}}(P) = \left\{ \inf_{K \text{ stablizing}} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I + PK)^{-1} [I \quad P] \right\|_\infty \right\}^{-1}. \tag{10}$$

It is indicated in [17] that $b_{\mathrm{opt}}(P)$ is an inherent property of $P(s)$ and can be computed as follows

$$b_{\mathrm{opt}}(P) = \sqrt{1 - \|[\widetilde{N} \quad \widetilde{M}]\|_H^2} < 1, \tag{11}$$

where $\widetilde{N}$ and $\widetilde{M}$ are normalized left coprime factorization of $P$, i.e. $P(s) = \widetilde{M}^{-1}\widetilde{N}$ and $\|\cdot\|_H$ is Hankel norm. Note that, (7) and (8) are key inequalities that have been utilized in [5, 7, 8, 11] to grid the operation range of nonlinear systems.

## III. ROBUST MULTI-MODEL CONTROL BASED ON INTEGRATED STABILITY AND PERFORMANCE

### A. Fixed Structure Controller

Let us assume that the $i^{th}$ equilibrium point is a nominal one and the corresponding local model can be obtained as

$$P_i = C_i(sI - A_i)^{-1}B_i + D_i \tag{12}$$

where $(A_i, B_i, C_i, D_i)$ are the linear model's matrix defined in (2) and $I \in \mathbb{R}^{n\times n}$ is an identity matrix. To reduce the complexity of the global multi-model controller, the local controllers assume PI structure which is widely used in industrial applications. The $i^{th}$ local controller is as

$$K_i(s,q) = k_{Pi} + \frac{k_{Ii}}{s}, \tag{13}$$

where $k_{Pi} \in \mathbb{R}^{r\times p}$, $k_{Ii} \in \mathbb{R}^{r\times p}$, $i \in \{1,2,\dots,N_s\}$ and $q = [\mathrm{vec}(k_{Pi}) \quad \mathrm{vec}(k_{Ii})]^T$ is the vector of design parameters. It is assumed that $q$ belongs to a known set $\mathcal{D}$ which is defined as

$$q \in \mathcal{D} = \left\{ q \in \mathbb{R}^{n_q} | \underline{q} \le q \le \overline{q} \right\}, \tag{14}$$

where $\underline{q}$ and $\overline{q}$ are the design and search domain boundaries.

*B. Local Controller Designing Formulation*

For performance requirement, ITAE is used which is defined in (15). It should be noted that, other performance measures like IAE, ISE and etc., can be utilized as well. The minimization of ITAE is considered as an objective for designing a fixed structure PI controller.

$$\mathrm{ITAE}(q) = \int_0^\infty t|e(t)|\, dt \tag{15}$$

The fixed controller in (13) is substituted in (8) and the stability margin defined in (16) is taken as a constraint.

$$g(q) = \left\| \begin{bmatrix} I \\ K_i(s,q) \end{bmatrix} (I + P_i(s)K_i(s,q))^{-1} [I \quad P_i(s)] \right\|_\infty^{-1} \tag{16}$$

Finally, the control design problem is formulated as an optimization problem given below

$$\begin{aligned} &\min_q \mathrm{ITAE}(q)\,, \\ &\text{sub. to } g(q) \ge \gamma, \quad q \in \mathcal{D} \end{aligned} \tag{17}$$

where $\gamma \le 1$ is the required stability margin that has a key role in classifying of local models bank. $\gamma$ is taken as a large value at first and then is updated along the multi-model controller design procedure. The constrained optimization problem defined in (17) can be converted to an unconstraint problem by using penalty function concept.

$$J(q) = \mathrm{ITAE}(q) + \rho\big(\max(\gamma - g(q), 0)\big), \tag{18}$$

where $\rho$ penalizes the violation of constraint and was set to 1000 in all simulations of this paper. Then, the optimal parameters of the $i^{th}$ local controller can be found by solving the optimization problem defined in (19).

$$q_{\mathrm{opt}} = \underset{q\in\mathcal{D}}{\operatorname{argmin}}\, J(q) \tag{19}$$

*C. Multi-Model Controller Design Algorithm*

From (17), it can be deduced that the robust stability and performance requirement are connected together for the design of local controllers. To build the overall multi-model controller, it is required to select the nominal linear models among local models bank. To this end, a straightforward algorithm is proposed which provides an optimization method to select the nominal models. Each of nominal models $(P^*)$ is found in a sub-region where consists some consecutive local models $(P_i)$ so that $\delta(P^*, P_i) \le \gamma$. Therefore, $\gamma$ influences directly the number of sub-regions and local controllers correspondingly. To avoid the dependency on experience, it is not defined as a fix threshold. $\gamma$ is set to a large value ($\gamma = 0.8$) and then updated until the design conditions satisfied. Therefore, the proposed method removes the redundancy and then the obtained global multi-model controller has a simpler structure. The algorithm of nominal local models selection and global multi-model controller design has been presented in detail as follow.

**S1.** Grid the nonlinear system's operation space using the gap metric and dichotomy gridding algorithm [3] and determine the linearization points in the equilibrium manifold. Then linearize the considered nonlinear system at these points and obtain $N$ linearized models ($P_i$ ($i = 1,\dots,N$)).

**S2.** Compute the gap between all pairs of $N$ linearized models according to (6), and build an $N \times N$ matrix gap $= [\delta_{i,j}] = [\delta(P_i, P_j)]_{N\times N}$. Then calculate the maximum stability margin vector for $N$ local models based on (11) and make an $N \times 1$ vector $B_{\mathrm{opt}} = [b_{\mathrm{opt}}(P_i)]_{N\times 1}$.

**S3.** Set $i = 1$.

**S4.** Set $\gamma = 0.8$.

**S5.** Set $j = i + 1$.

**S6.** Find the best local model $(P^*)$ between the $i^{th}$ to $j^{th}$ local models based on the following criterion.

$$P^* \triangleq \left\{ P_k | \min_{i\le k\le j}\big(\max_{i\le m\le j}(\delta(P_m, P_k))\big) \right\}.$$

**S7.** Find the biggest gap between $P^*$ and the other linearized models as $\delta^* = \max_{i\le m\le j}(\delta(P^*, P_m))$.

**S8.** If $\delta^* < \gamma$ then set $j = j + 1$ and go back to **S6** (include another linearized model into the current sub-region). Otherwise, go to **S9**.

**S9.** Set $j = j - 1$. The $j - i + 1$ local models are classified in one sub-region. If $\gamma > b_{\mathrm{opt}}(P^*)$, decrease $\gamma$ and go back to **S5**. Otherwise ($\gamma < b_{\mathrm{opt}}(P^*)$), for the nominal local model $P^*$, solve the optimization problem in (19). If the optimization problem is feasible, get the designed controller $K$. Otherwise, decrease $\gamma$ and go back to **S5**.

**S10.** Set $i = j$ and go back to **S4**. Repeat the above steps until all of the $N$ local models are classified.

**S11.** Let us suppose that $N_s$ sub-regions are obtained. Therefore, $N_s$ PI controllers are designed which minimize the ITAE and guarantee the robust stability at the same time. Finally, build the global multi-model controller by proper combining (hard/soft switching) of these local controllers as given below

$$u(t) = \sum_{i=1}^{N_s} w_i(t) u_i(t), \tag{20}$$

where $u_i$ is the manipulated variable corresponds to the $i^{th}$ local controller and $w_i(t)$ is weighting function that determines the $i^{\text{th}}$ local controller belongs to which sub-region at time $t$. $w_i(t)$ is formulated as follow

$$w_i(t) = \begin{cases} 1 & \theta(t) \in \psi_i\,, i \in \{1,\dots,N_s\} \\ 0 & \text{otherwise} \end{cases}, \tag{21}$$

where $\theta$ is a scheduling vector of variables which could be one or more variables that characterize(s) the operating behavior of the system. The states, inputs, outputs, and disturbances can be selected as the scheduling variables. Typically, one or more control inputs $u$, and/or one or more outputs $y$ act as the scheduling variables.

**Remark.** Note that (19) is a non-convex optimization problem. Among the non-convex optimization methods, HKA requires only three "user-defined" tuning parameters and therefore is applied to solve (19).

## IV. Simulation Results

The pH neutralization process is a benchmark model in the multi-model literature and has been studied in many previous researches due to its highly nonlinear behavior. The mathematical model of the pH neutralization process is given as

$$\begin{aligned}
\dot{x}_1 &= \frac{q_A}{V}\left(x_{1,i} - x_1\right) - \frac{q_A}{V} x_1 u, \\
\dot{x}_2 &= -\frac{q_A}{V} x_2 + \frac{q_A}{V}\left(x_{2,i} - x_2\right) u, \\
\dot{x}_3 &= -\frac{q_A}{V} x_3 + \frac{q_A}{V}\left(x_{3,i} - x_3\right) u, \\
h(x,y) &= \xi + x_2 + x_3 - x_1 - \frac{K_W}{\xi} - \frac{x_3}{1+K_x\xi/K_W},
\end{aligned} \tag{22}$$

where $u = q_B/q_A$ is the input, $y = pH$ is the output of the system and $\xi = 10^{-y}$. The model parameters are listed in Table I. The equilibrium manifold can be determined by setting derivatives in (22) to zero. The following equation indicates the steady-state curve.

$$\xi + \frac{x_{2i}u}{1+u} - \frac{x_{1i}}{1+u} - \frac{K_W}{\xi} + \frac{x_{3i}u}{1+u}\left(\frac{K_x\xi/K_W}{1+K_x\xi/K_W}\right) = 0 \tag{23}$$

$pH$ is chosen as the scheduling variable. The operating points can be obtained by gridding the range of $pH$ and solving the nonlinear steady-state equation in (23). For gridding the range of $pH$, dichotomy gridding algorithm is implemented and 92 static operating points are obtained where the gap between each two consecutive local models is smaller than 0.1. Fig. 1 presents the gap between all pair of these 92 local models. The biggest gap between these 92 local models is 0.975 which implies that pH neutralization process has a highly nonlinear behavior. The 35th local model is the best nominal local model among the linear models bank when $P^* \triangleq \left\{P_k \middle| \min_{1\le k\le 92}\left(\max_{1\le m\le 92}\left(\delta(P_m, P_k)\right)\right)\right\}$ is implemented. By considering $\max_{1\le i\le 92} \delta(P_{35}, P_i) = 0.79$ and $b_{opt}(P_{35}) = 0.71$, it can be deduced that stabilizing of pH neutralization reactor will not be possible if only a linear controller is applied.

TABLE I. Model Parameters for pH Neutralization Reactor System.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $x_{1,i}$ | 0.0012 mol HCl/L | $K_x$ | $10^{-7}$ mol/L |
| $x_{2,i}$ | 0.002 mol NaOH/L | $K_w$ | $10^{-14}$ mol$^2$/L$^2$ |
| $x_{3,i}$ | 0.0025 mol NaHCO$_3$/l | $q_A$ | 1 L/min (16.67 ml |
| | | $V$ | 2500 mL |

To design the multi-model controller, the presented method is implemented. To this end, the search space is set as $\mathcal{D} = \{q \in \mathbb{R}^2 | (0,0)^T \le q \le (2,1)^T\}$ in all regions. Three "user-defined" parameters of HKA are set as $N = 50$, $N_\xi = 7$, and $\alpha = 0.3$. The results are tabulated in Table II.

From Table II, the entire range of operation space $\Psi$ has been divided into 4 sub-regions. In the first sub-region, the $11^{th}$ local model is marked as nominal model $P_1^*$ where the biggest gap between $P_1^*$ and the other local models in the first sub-region is 0.59. Also, the gained stability margin for $K_1$ is 0.6 which is larger than $\max_{1\le i\le 24} \delta(P_1^*, P_i)$. Therefore, all pairs $(P_\Delta, K_1)$ are stable under Proposition 1 which $P_\Delta = \{P_i | 1 \le i \le 24\}$. The other columns of Table II are interpreted as the same. Note that, the robust stability condition is preserved in the other sub-regions i.e. $b_{P_i^*,K_i} > \delta_{\max}(P_i^*)$ for $i = 2, 3, 4$ as well. Therefore, the robust stability requirement is satisfied in the entire range of operation space.

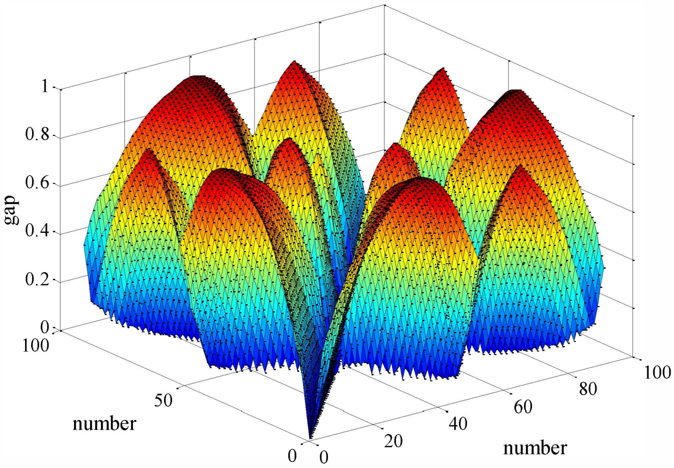


Figure 1. The gap between 92 local models.

TABLE II. Linear Models Bank and Local Controllers Under the Proposed Method.

| Sub-region | 1st | 2nd | 3rd | 4th |
|---|---|---|---|---|
| **linearized models set** | $1-24$ | $24-49$ | $49-85$ | $85-92$ |
| **Operation point ($pH, u$)** | 11th (3.59,0.198) | 42th (5.99,0.28) | 75th (9.78,0.648) | 89th (10.45,0.866) |
| $\boldsymbol{\delta_{max}}$ | 0.59 | 0.54 | 0.54 | 0.25 |
| **Scheduling variable range ($pH$)** | $2.92-4.21$ | $4.21-6.66$ | $6.66-10.26$ | $10.26-10.6$ |
| **Controller** | $K_1 = 1.33 + \frac{0.013}{s}$ | $K_2 = 1.33 + \frac{0.02}{s}$ | $K_3 = 1.33 + \frac{0.0176}{s}$ | $K_4 = 3 + \frac{0.041}{s}$ |
| **Stability margin** | $b_{P_1^*,K_1} = 0.6$ | $b_{P_2^*,K_2} = 0.6$ | $b_{P_3^*,K_3} = 0.6$ | $b_{P_4^*,K_4} = 0.31$ |

The closed-loop response of pH neutralization process using the proposed global multi-model controller is shown in Fig. 2. $pH_{\mathrm{MM1}}$ and $u_{\mathrm{MM1}}$ denote the output and manipulated variable, respectively. The designed multi-model controller in [9] is picked up as well. In [9], the entire range of operation space has been divided into 5 sub-regions and local models have been selected based on a priori knowledge about the nonlinear system. Moreover, 5 PI controllers have been designed based on IMC tuning rules. The simulation results are presented in Fig. 2 where $pH_{\mathrm{MM2}}$ and $u_{\mathrm{MM2}}$ indicate the output and control signal under [9]. To have a fair comparison, the reference input is the same in both designs.

According to Fig. 2, $pH_{\mathrm{MM1}}$ tracks the reference input without steady-state error. The response comes with an overshoot smaller than 10%. Also, it can be seen that $pH_{\mathrm{MM1}}$ settles faster than $pH_{\mathrm{MM2}}$. Moreover, the control input $u_{\mathrm{MM1}}$ is smooth while $u_{\mathrm{MM2}}$ has aggressive action. To have a detailed insight, the Mean Square Error (MSE) values are computed which are $\mathrm{MSE}(pH_{\mathrm{MM1}}) = 0.14$ and $\mathrm{MSE}(pH_{\mathrm{MM2}}) = 0.33$. Therefore, the proposed method exhibits outstanding results with preserving the local robust stability while the multi-model controller in [9] cannot mathematically guarantee the stability.

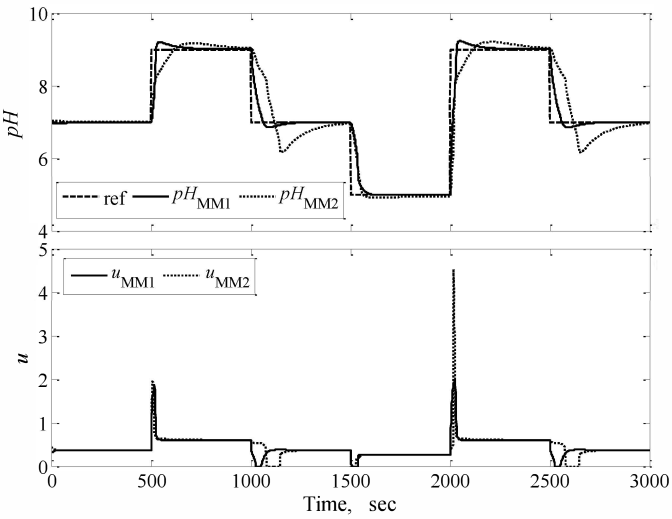


Figure 2. The closed-loop response of pH neutralization process using proposed method (MM1) and [9] (MM2).

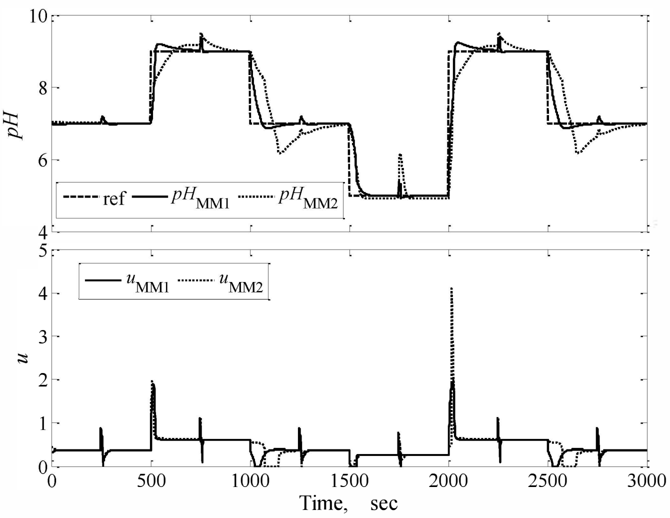


Figure 3. Disturbance rejection response of the proposed method and [9].

Figure 3 shows the disturbance rejection simulation results under the proposed method and the presented multi-model controller in [9]. To test the disturbance rejection ability, pulse disturbances have been added to the input at seconds $t = 250$, $750$, $1250$, $1750$, and $2250$ with duration and amplitude as 10 seconds and 0.5, respectively. According to Fig. 3, $pH_{\mathrm{MM1}}$ gets quickly back to the reference input while $pH_{\mathrm{MM2}}$ settles slowly. Furthermore, the sudden changes can be seen in $pH_{\mathrm{MM2}}$ at second 1750. Also, the manipulated variable $u_{\mathrm{MM1}}$ is still smooth whenever the disturbance has occurred. The MSE values are $\mathrm{MSE}(pH_{\mathrm{MM1}}) = 0.15$ and $\mathrm{MSE}(pH_{\mathrm{MM2}}) = 0.34$. Therefore, the results imply that the proposed method performs better both in set-point tracking and in disturbance rejection.

## V. CONCLUSION

An integrated multi-model controller design is studied in this paper. To avoid the redundancy issue, not only the decomposition of operation space is incorporated with together the design of local controllers, but also the stability and performance criteria are integrated in local controller designing step. Furthermore, to eliminate the dependency on the experience and a priori knowledge about system, an updating threshold is introduced. Also, the local controllers have a fixed PI structure that reduces the complexity of the global multi-model controller. The design of multi-model controller is formulated as an optimization problem which is non-convex. The HKA method is applied to get the optimal parameters of local controllers. In conclusion, the designed multi-model controller guarantees the robust stability and performance and no dependency on experience while avoids the redundancy problem.

A pH neutralization process is studied that has big gap between the local models that implies this process is a strong nonlinear system. Also, the comparisons have been made between the proposed approach and a traditional multi-model controller. The closed-loop simulations demonstrate that the proposed method performs significantly better both in set-point tracking and disturbance rejection.

## REFERENCES


[1] C. Song, B. Wu, J. Zhao, and P. Li, "An integrated state space partition and optimal control method of multi-model for nonlinear systems based on hybrid systems," J. Process Control, vol. 25, pp. 59-69, 2015.

[2] X. Tao, D. Li, Y. Wang, N. Li, and S. Li, "Gap metric based multiple model predictive control with polyhedral stability region," Ind. Eng. Chem. Res., vol. 54, pp. 11319-11329, 2015.

[3] J. Du, C. Song, Y. Yao, and P. Li, "Multilinear model decomposition of MIMO nonlinear systems and its implication for multilinear model-based control," J. Process Control, vol. 23, pp. 271-281, 2013.

[4] J. Du, C. Song, and P. Li, "Application of gap metric to model bank determination in multilinear model approach," J. Process Control, vol. 19, pp. 231-240, 2009.

[5] J. Du and T.A. Johansen, "Integrated multilinear model predictive control of nonlinear systems based on gap metric," Ind. Eng. Chem. Res., vol. 54, pp. 6002-6011, 2015.

[6] J. Du and T.A. Johansen, "A gap metric based weighting method for multimodel predictive control of MIMO nonlinear systems," J. Process Control, vol. 24, pp. 1346-1357, 2014.

[7] J. Du and T.A. Johansen, "Integrated multimodel control of nonlinear systems based on Gap metric and stability margin," Ind. Eng. Chem. Res., vol. 53, pp. 10206-10215, 2014.

[8] J. Du, C. Song, and P. Li, "Multimodel control of nonlinear systems: An integrated design procedure based on gap metric and $H_\infty$ loop shaping," Ind. Eng. Chem. Res., vol. 51, pp. 3722-3731, 2012.

[9] O. Galán, J.A. Romagnoli, and A. Palazoglu, "Real-time implementation of multi-linear model-based control strategies- an application to a bench-scale pH neutralization reactor," J. Process Control, vol. 14, pp. 571-579, 2004.

[10] A.A. Jalali and H. Golmohammad, "An optimal multiple-model strategy to design a controller for nonlinear processes: A boiler-turbine unit," Comput. Chem. Eng., vol. 46, pp. 48-58, 2012.

[11] A.A. Haj Salah, T. Garna, J. Ragot, and H. Messaoud, "Transition and control of nonlinear systems by combining the loop shaping design procedure and the gap metric theory," Trans. Inst. Meas. Control, 2015.

[12] A.A. Haj Salah, T. Garna, J. Ragot, and H. Messaoud, "Synthesis of a robust controller with reduced dimension by the Loop Shaping Design Procedure and decomposition based on Laguerre functions," Trans. Inst. Meas. Control, 2015.

[13] J. Du, C. Song, and P. Li, "Multilinear model control of Hammerstein-like systems based on an included angle dividing method and the MLD-MPC strategy," Ind. Eng. Chem. Res., vol. 48, pp. 3934-3943, 2009.

[14] S. Hosseini, A. Fatehi, T.A. Johansen, and A.K. Sedigh, "Multiple model bank selection based on nonlinearity measure and H-gap metric," J. Process Control, vol. 22, pp. 1732-1742, 2012.

[15] R. Toscano and P. Lyonnet, "Heuristic Kalman algorithm for solving optimization problems," IEEE Trans. Syst. Man Cybern. Part B Cybern., vol. 39, pp. 1231-1244, 2009.

[16] R. Toscano and P. Lyonnet, "Robust PID controller tuning based on the heuristic Kalman algorithm," Automatica, vol. 45, pp. 2099-2106, 2009.

[17] K. Zhou and J.C. Doyle, Essentials of Robust Control. Prentice Hall, 1998.