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Influence of sampling on the tuning of PID controller parameters

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Abstract: The paper deals with an analysis of automatic control system with continuous and discrete PID controllers. A method of tuning the parameters of the continuous controller is presented, which is optimal according to the ITAE criterion. The behavior of control systems with discrete controllers whose parameters were tuned using the mentioned method are described. The impact of changes in the sampling period of controlled signal on the control quality is shown. Changes of the values of optimal parameters of discrete PID controllers in relation to changes of the sampling rate of controlled signal are characterized.

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1. INTRODUCTION

The general rules of the sampling period selection depend on the parameters of the control object model. The parameters are often: T_{max} – the dominant time constant, L – the transport delay time constant, T – inertia time constant (Kalman et al. 1958), (Astrom et al. 1984). There are also known rules determining sampling period with respect to the control quality indicators such as: t_s – settling time and t_r – rise time, (Isermann 1981), (Astrom et al. 1984). These rules allow one to estimate the signal sampling period with respect to the identified controller parameters (T_i – integration time, T_d – differentiation time constant) and are presented in: (Astrom et al. 1984), (Fertik 1975). These rules do not specify precisely what value of sampling period Δt ought to be used. They allow one to only roughly estimate the value of interval Δt.

It was assumed that the continuous control system is the reference system. It makes it easier to analyze the impact of sampling period of control signal on the control quality of the discrete system and the to choose of the optimal settings of discrete controllers. In the continuous system, the controller constantly monitors the controlled signal (process value) and the reference signal (setpoint value). On the basis of these signals it generates a control signal.

The settings of PI and PID controller are often selected using methods that are designed for continuous controllers (Ziegler et al. 1942), (Astrom et al. 1984), (Astrom et al. 1995). Badly selected continuous controller parameters can cause poor quality of control. The quality of control can deteriorate even more if the selected settings are used with a controller which responds to the input signals periodically – a discrete controller. To avoid this, the controller parameters are selected using an optimization method taking into account the sampling period. Such a method was proposed in (Wcislik et al. 2011) and it is briefly described in the next section.

2. OPTIMAL SETTINGS OF CONTINUOUS PID CONTROLER

The control system with negative feedback shown in figure 1 was analyzed as a basic control system.

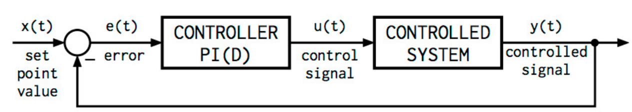

Fig. 1. The diagram of the basic automatic control system with continuous controller.

It was analyzed in (Wcislik et al. 2011). It was assumed that PID controller has the form:

$$G_{c(s)} = K_c \left[1 + 1/(sT_i) + (sT_d)/(s(T_d/N)+1)\right] \tag{1}$$

where: K_c – proportional gain, T_i – integral time, T_d – derivative time, N – dimensionless coefficient.

The value of the dimensionless coefficient N is determined by bibliography analysis. Usually the value of the coefficient is in the range of 2 to 30 (O'Dwyer 2006). It was assumed that $N = 20$ (Wcislik et al. 2011).

The dynamics of the controlled system is approximated by a first-order inertial model with transport delay.

$$G_{(s)} = K\, e^{-sL}/(1 + sT) \tag{2}$$

where: K – static gain, T – inertia time constant, L – transport delay time constant.

The model description can map the dynamics of a wide range of industrial processes with satisfactory accuracy. It also makes it possible to model the steady state. The presence of a transport delay allows an approximation of potentially

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unstable processes. The ITAE was selected as an optimality criterion:

$$ITAE = \int_0^t t \left| e_{(t)} \right| dt \tag{3}$$

The procedure for the selection of the optimal settings of PI and PID controllers consists of a few steps. First, the proportional and derivative parts of the PID controller are disconnected. For the PI controller, only the proportional part is disconnected. Next, the gain of the integral part is increased to get the closed loop system to border stability. At this stage, the controller ultimate gain K_i and the sustained oscillation angular frequency ω_{osc} of the controlled variable y are assessed. On the basis of these parameters the time constant T of an approximating model is identified.

$$T = \sqrt{(K\,K_i)^2 - \omega_{osc}^2} \,/\, \omega_{osc} \tag{4}$$

Then the coefficient $\theta = L/T$ is calculated

$$\theta = arcctg(\omega_{osc} T) / \omega_{osc} T \tag{5}$$

The optimal settings of the PI controller were defined in (Wcislik et al. 2011):

$$\begin{aligned} K_c &= \left[10^{\left(0.49/\sqrt{\theta}\right)-0.67}\right] / K \\ T_i &= T \cdot \left[0.0058\theta^2 + 0.31\theta + 0.91\right] \end{aligned} \tag{6}$$

and of the PID controller:

$$\begin{aligned} K_c &= \left[10^{\left(0.81/\sqrt[3]{\theta}\right)-0.79}\right] / K \\ T_i &= T \cdot \left[0.4\theta + 0.97\right] \\ T_d &= T \cdot \left[0.48\sqrt{\theta} - 0.16\right] \end{aligned} \tag{7}$$

The equations (6) and (7) were obtained using the approximation of the set of PI and PID optimal settings. The least squares method was used in this purpose. The obtained formulas provide acceptable accuracy for θ = <0.2, 2>.

Examples of step responses of the continuous systems with PID controllers were shown in figure 2. The controllers parameters were selected using the Ziegler-Nichols method (Ziegler et al. 1942) and the proposed method (Wcislik et al. 2011). The controlled system has a transfer function described by (2) with $\theta = L/T = 0.2$.

The use of the proposed method causes a slight increase of the rise time as well as a decrease of the settling time and the overshoot value. The control quality is significantly better than for the Ziegler-Nichols method.

The transients for the proposed method in figure 2 have some pulse disturbances. They arise from an interaction between the derivative part of the PID controller and the transport delay of the controlled system.

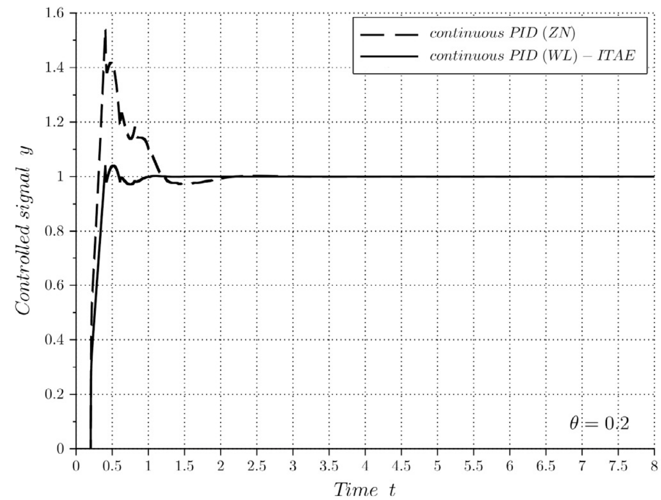

Fig. 2. The transients of the continuous control system with the PID controller.

3. OPTIMAL SETTINGS OF DISCRETE TIME PID CONTROLER

The block diagram of an automatic control system with discretized control signal with sapling period Δt is shown in figure 3.

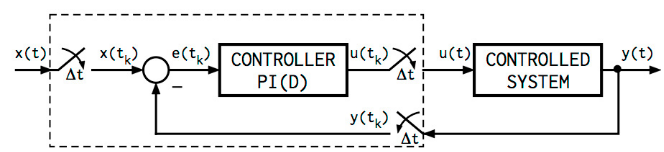

Fig. 3. Discretized automatic control system.

Simulation of the system presented in figure 3 requires the separation of the elements that are solved with different periods. The summation node as well as the controller are solved with period Δt. For the continuous part of the system, the model of the controlled system is solved with the step δt. The step δt may be either fixed or variable. It depends on the chosen method of solving differential equations describing the controlled system.

In SIMULINK environment, the above discrete control system requires two ZOH (zero-order hold) extrapolators, which must be placed before and after the continuous model of the controller (fig. 4).

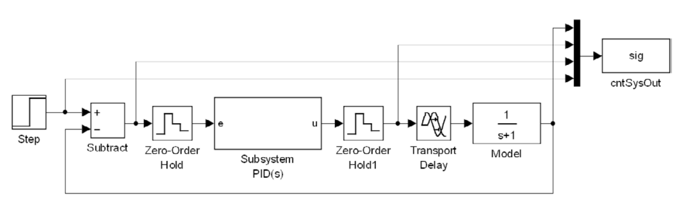

Fig. 4. Diagram of the discrete control system in SIMULINK.

Simulation results show that the insertion of the ZOH extrapolators into the continuous system (fig. 4) causes system instability, even if the original continuous system was stable and the transient of the controlled signal was optimal.

For the coefficient N = 20 the instability occurs even for small values of the period Δt. Optimal settings of the controller (1) which are used with the system of figure 4 have to be selected taking into account the sampling period Δt. The problem may be avoided by using the discrete form of the PID controller.

The discrete controller form discussed above was used in control system shown in figure 5.

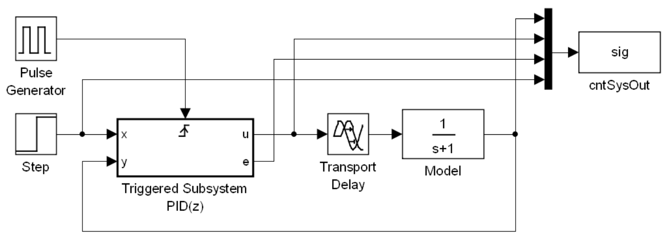

Fig. 5. Diagram of control system with a discrete controller in SIMULINK

In block diagram in figure 5 the controller equations were obtained using the Euler-Forward method (Buso et al. 2006), which gives the PID controller equation applied to (1). It makes the system in figure 5 correspond to the system in figure 4. That form of equation of the PID controller is often implemented in control devices (Liptak 2006):

$$u_k = K_c\left[e_k + \Delta t\, S_k / T_i + T_d\left(e_k - e_{k-1}\right)/\Delta t\right] \qquad (8)$$

where: $S_k = S_{k-1} + e_k$.

Equation (8) is called the position algorithm. Its SIMULINK diagram is shown in figure 6.

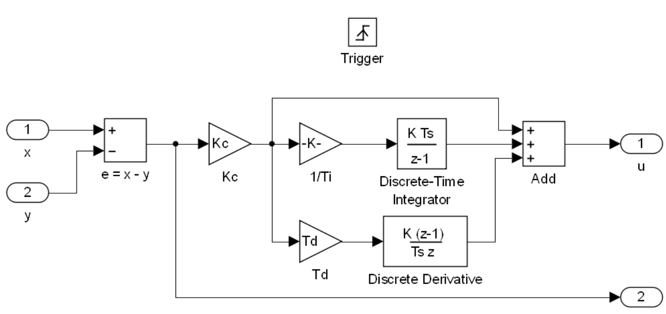

Fig. 6. Position PID algorithm – diagram in SIMULINK

The block diagram from the figure 6 is placed within the "Triggered Subsystem PID(z)" block in diagram in figure 5.

The transients of the output (controlled) signal for the control systems with the continuous (1) controller and with the discrete (8) controller are shown in figure 7 for the signal sampling period: $\Delta t = 0.001$. According to the time scaling of the control system, the sampling period Δt is related to the inertia time constant T.

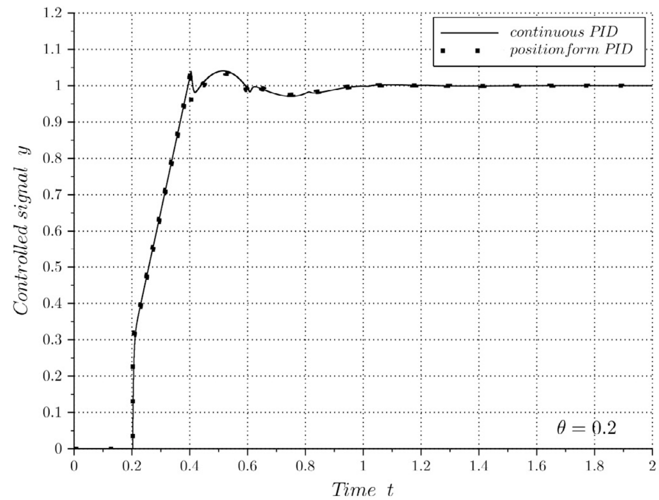

Fig. 7. Transients of the continuous and discrete control system.

Settings for both controllers were calculated from equations (7). The controlled system was defined by the formula (2) with coefficient $\theta = L/T = 0.2$. Transients of the controlled signal obtained from discrete and continuous control systems are very close (fig. 7). The similar behavior of the continuous (1) and the discrete (8) controllers is observed for a relatively high value of the coefficient N. As was previously mentioned, the value of N is equal to 20. This value of the coefficient N lowers the filtering influence in the derivative component of the continuous controller (1).

The step responses of the control systems with the PI and the PID controllers are shown in figures 8 and 9. Simulations were made for various values of Δt. In each simulation the controller parameters were set up to the optimal values calculated for the continuous controller (1). It can be seen that despite the selecting optimal parameters, the increase of the Δt causes loss of the control quality for both types of controllers. The PID controller is more sensitive to the change of Δt (fig. 9).

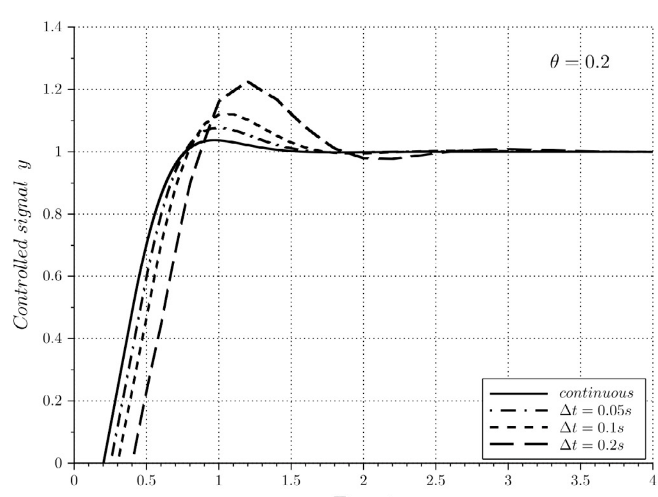

Fig. 8. Transients of the controlled variable in the control system with PI controller for different sampling periods.

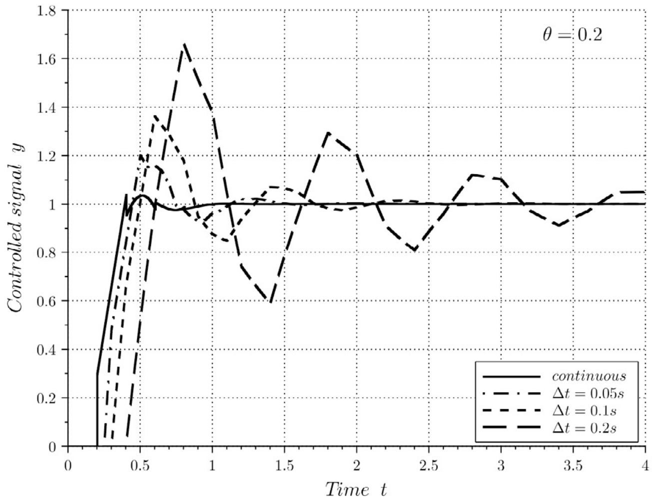

Fig. 9. Transients of the controlled variable in the control system with PID controller for different sampling periods.

The Nelder-Mead method was used to find optimal (3) settings of PI and PID controllers. Optimal parameters of the PI controller are shown in figure 10. The results show the influence of the sampling period of the control signal on the control quality and optimal parameters of the controller. It can be seen that the increase of the sampling period Δt causes the decrease of the optimal value of the proportional gain K_c. Simultaneously, the optimal value of the integral time T_i increases. The system behaves this way for small values of θ. Along with the increase of the coefficient θ, the optimal values of K_c decrease and are almost independent of the sampling time. The optimal values of T_i are more sensitive to the sampling period. It means that for relatively quick systems (systems with small θ) the use of the optimal settings of the continuous controller for the discrete controller requires one to change both of its parameters: K_c and T_i. For the slower systems (systems with larger θ) only the T_i value ought to be changed.

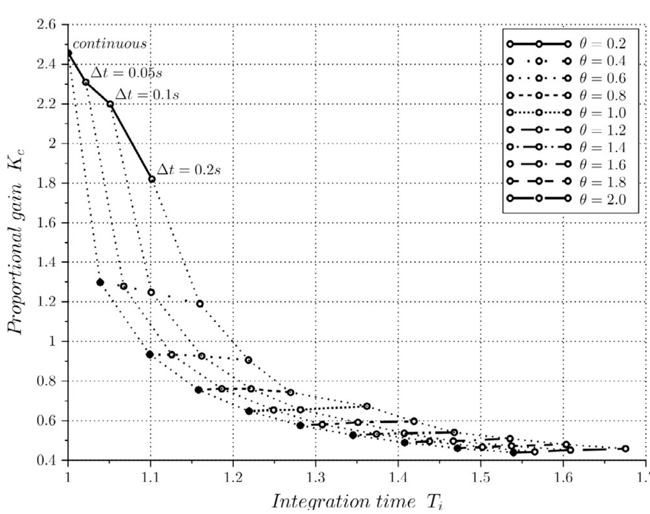

Fig. 10. Optimal parameters of the PI controllers for different values of: the θ coefficient and the sampling period Δt .

The ITAE index optimal values as a function of the sampling period Δt of control signal are shown in figure 11. These values correspond to the settings shown in figure 10.

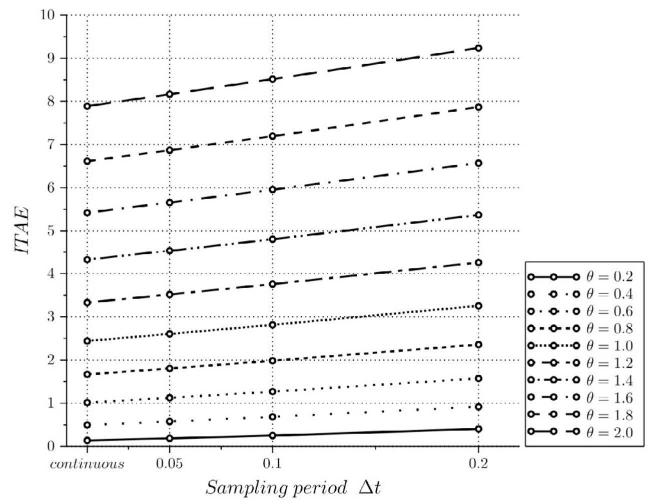

Fig. 11. ITAE optimal values as a function of Δt, for a system with PI controller.

Continuous control systems are characterized by the lowest values of the ITAE index (fig. 11). The phenomenon occurs for all values of θ. It means that the continuous system gives the best control quality. The use of a discrete controller only degrades the quality of control. The controller responds to the system signals in a periodic manner with the period Δt, therefore a part of information about the control system state between samples is lost. The discrete controller generates the control signal based on the deficient data. It has to lead to a loss of control quality.

The step responses of the system with PI controller are shown in figure 12. The controller settings were chosen according to figure 10. The controlled system has $\theta = 0.2$.

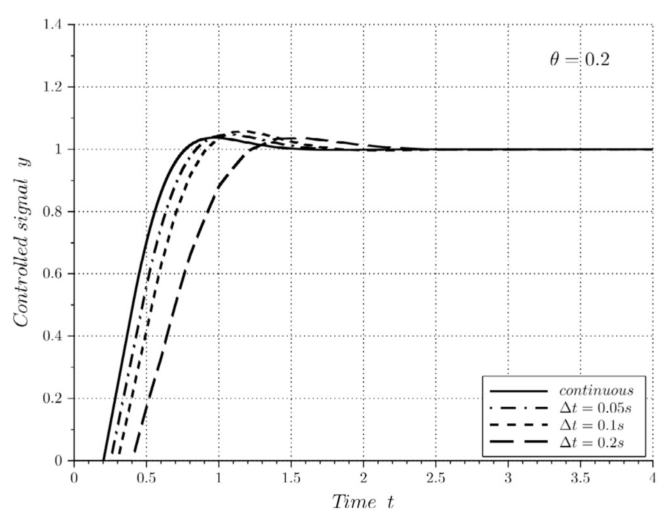

Fig. 12. Transients of the controlled signals of control system with PI controller for optimal settings related to the period Δt.

Plots in figures 8 and 12 show that the overshoot was reduced, especially for the system with a large value of sampling period Δt. The rise time was elongated. It causes the slowing down of the transient of the controlled variable. However, the settling time has been improved.

Analogically identified, optimal settings for PID controller are shown in figures 13, 14 and 15.

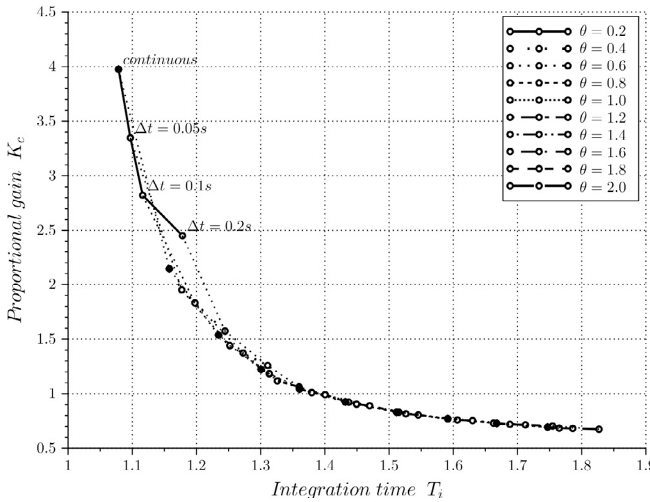

Fig. 13. Optimal values: K_c, T_i of the PID controllers for different values of: the θ coefficient and the sampling period Δt.

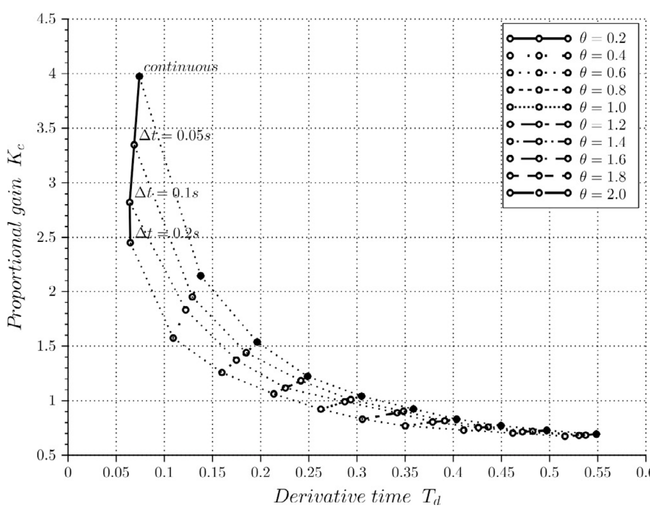

Fig. 14. Optimal values: K_c, T_d of the PID controllers for different values of: the θ coefficient and sampling period Δt.

As it is shown in figure 13 for the small value of θ the optimal value of the proportional gain K_c decreases with the increase of Δt. Simultaneously, the optimal value of integral time T_i increases. The optimal value of derivative time T_d decreases along with an increase in Δt - figure 14. It should be noted that for small values of θ the change of T_d is relatively small (fig. 14). Along with the increase of θ the range of optimal values of K_c decreases, while the ranges of changes of optimal values of T_i and T_d increase.

It means that for relatively quick systems (systems with small θ) the use of the optimal settings of the continuous controller for the discrete controller requires a significant change of K_c, small changes of T_i and almost no change of T_d. For the slower systems (the systems with larger θ) both: T_i and T_d must be changed, while K_c may remain almost unchanged. In relation to the PI controller the control system with use of PID controller is more sensitive to proportional gain K_c (fig. 10 and 13).

The ITAE index optimal values as a function of the sampling period of the controlled signal for a system with PID controller are shown in figure 15.

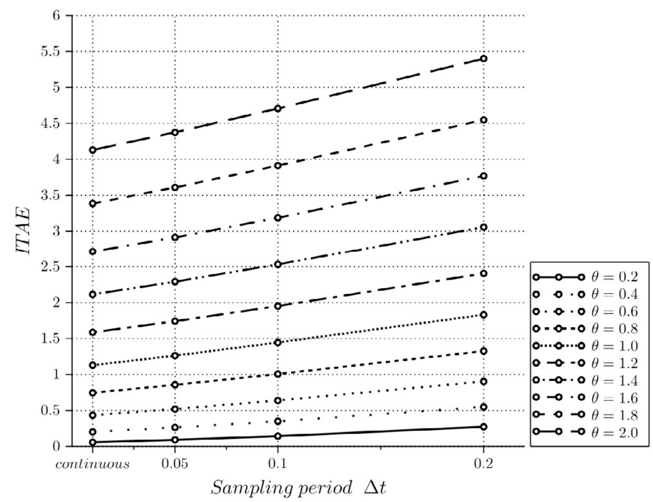

Fig. 15. ITAE optimal values as a function of Δt, for a system with PID controller.

Just as for the system with PI controller the best control quality is observed for continuous control system with PID controller. The use of the discrete controller only degrades control quality. The use of the PID controller improves control quality, especially for the systems with large values of the θ - figures 11 and 15. The optimal step responses of the system with PID controller are shown in figure 16.

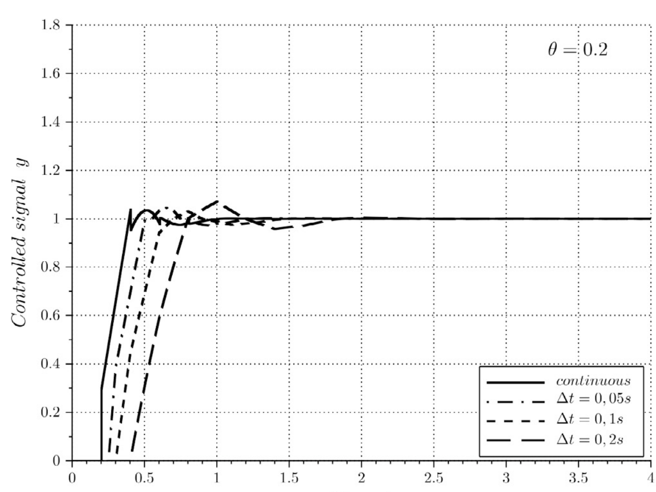

Fig. 16. Step responses of the control system with PID controller for optimal settings related to the period Δt.

The controller settings corresponding to the values are shown in figure 13 and 14. Comparing the plots in figures 9 and 16 it can be seen that the control quality has been significantly improved especially for the systems with large value of the sampling period. The overshoot has been reduced and the settling time became shorter.

6. CONCLUSIONS

The settings of the PI and the PID controllers chosen using the authors' method (Wcislik et al. 2011) allows one to achieve optimal (3) control quality of the controlled system. These settings used with discrete controllers do not provide optimal control quality. The control quality decreases with the increase of the sampling period Δt of the control signal. To maintain the required quality, value correction of the controller settings is needed.

Changes of control quality and the changes of controller settings they require depend on the chosen form of the discrete controller. It was noted that the discrete implementation of continuous controller whose equation was derived using the Euler-Forward method is very sensitive to changes of the control signal sampling rate, especially for the higher values of the N coefficient in (1). Even a small value of Δt may destabilize a system which is stable and optimally tuned with the continuous controller.

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