Generalized Predictive Control of pH neutralization Process Based on Fuzzy Inverse Model

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***Abstract*— Control of pH neutralization process has always been one of challenging problem in process control. The method presented here to control this process is the fuzzy identification of systems using Wiener model, and then multiplying the measured signal by the inverse of the nonlinear part of model. Therefore, we can design a linear controller for this new augmented system. This strategy is implemented in a generalized predictive control. One of the advantages of this control structure is consideration of explicit constraint in control of systems which also included in proposed fuzzy predictive control. At the end, the proposed method is tested on the model of a pH neutralization process.**

Keywords— pH process, Model Predictive Control, Fuzzy Systems, Fuzzy Inverse Model

I. INTRODUCTION

Model Predictive controller (MPC) is the most popular controller for implementation in the process industries [1]. Among the methods of predictive control, generalized predictive control (GPC) is the most common method that gives possibility of tuning of control and prediction horizons and adding of control signal rate to the cost function and considering explicit constraints. However, this controller has been presented for linear processes. Nevertheless, many of the processes are highly nonlinear so that linear system identification may not result good performance for them. Composition of MPC as a solution for systems with high dead time, with Fuzzy systems as a tool for dealing with uncertainty as well as the use of human knowledge, has been studied in various researches [2, 3].

Despite fuzzy systems have the ability to describe uncertain phenomenon, fuzzy is an explicit theory with exact relations. Fuzzy theory was first introduced in 1965 by Zadeh [4]. In 1973, Zadeh by concept of linguistic variables and if-then rules for formulating of human knowledge, proposed a solution for the analysis of complex systems and established fuzzy control foundation [5]. In later years, other application of fuzzy was developed such as controllers, signal processing, expert systems, system identification and so on.

Fuzzy modeling or fuzzy identification was introduced first time by Takagi and Sugeno [6]. Unlike the classical model, which has a poor performance in dealing with uncertainty, fuzzy inference system employing the if-then rules can model the uncertainties. In 1993, Zhang proposed an Adaptive Neural Fuzzy Inference System (ANFIS) for purpose of fuzzy identification based on human knowledge (in the form of if-then fuzzy rules) and based on input-output system data and used the combinational training procedure to minimize the cost function [7]. In recent years, different applications of fuzzy logic control was seen in industries [8], such as tank level control [9], temperature control [10], warm water Process [11], Oxygenation process in the furnace steel [12], Biological process [13], Wastewater treatment [14], Heat exchange [2] and so on.

Different structures and strategies for model predictive controllers and fuzzy systems are proposed in recent works. Dynamic Matrix Control (DMC) and Generalized Predictive Control (GPC) are linear predictive controllers mostly uses in inverse model strategy in structure of adaptive control and multiple control. Authors in [18] obtained step responses for different operating points from the nonlinear fuzzy model and then developed a modified linear DMC algorithm. A fuzzy predictive control using multiple models strategy presented [19]. Also a multiple model predictive control (MMPC) strategy based on Takagi–Sugeno (T-S) fuzzy models are developed in [20]. A direct adaptive fuzzy predictive control is proposed by [21] based on Takagi-Sugeno fuzzy models.

In inverse model control strategy, nonlinear part of system is eliminated by inverse of it and then resultant linear system is controlled by a linear controller. Wiener and Hammerstein structures have ability to model very large group of systems. Hammerstein model is appropriate for systems that have nonlinearity in their actuators [22]. Design of a linear GPC by use of Takagi-Sugeno adaptive fuzzy controller with inverse Hammerstein model for elimination of nonlinear static part of system is proposed in [23]. Also [24] use fuzzy Hammerstein models as part of a model predictive control strategy. In another work a fuzzy-neural model by applying Hammerstein model is used in a predictive controller structure to control of lyophilization plant [25].

Wiener model is used in systems that have nonlinearity in their sensors [26] e.g. pH neutralization system [27, 15]. A model predictive control based on a Wiener model with a piecewise linear model is presented in [28]. Another Wiener-model-based nonlinear predictive control combines the advantages of linear-model-based predictive control and gain scheduling [29]. Some of proposed fuzzy predictive controllers employed for pH system [18, 27].

PH control process was modeled first by Seborg for a batch-type process [15, 16] based on the Wiener model. In this model, base flow is considered as control input. Acid flow is supposed to be a constant input; in some cases, it is used as a disturbance. For control of Tank level, a PID controller with buffer control input is used. According to this model structure, if we could somehow remove this nonlinear part, also called titration curve, we can say that we are dealing with a linear system.

In this paper, we design a fuzzy predictive controller for nonlinear systems with Wiener modeling by combination of model predictive control and fuzzy systems and then multiplying the measured signal by the inverse of the nonlinear part of model. Therefore, we can design a linear controller for this new augmented system. Inverse of nonlinear static is modeled by a fuzzy system and parameters of inverse model have been updated online. Proposed method tested on pH neutralization process as a plant with high nonlinearity and advantages of proposed method in compare with linear GPC have been shown.

For using of fuzzy inverse model control, identification of nonlinear static system with the fuzzy inverse model is required. An efficient solution to obtain inverse model of system is identification of it using output-input data pairs. Considerations for selecting appropriate data are discussed to identify the inverse model of the system with better accuracy. The results indicate the efficiency of the proposed method compared with the local generalized predictive control.

The paper organized as follows. In section 2 Generalized Predictive control was brought up. In section 3 first we explain inverse model idea and then the structure of fuzzy system was introduced. A procedure for tuning of model parameters was introduced. In last part of section 3 some considerations of fuzzy inverse model identification are discussed. Finally simulation results in section 4 shows the advantage of proposed method.

II. Generalized Predictive Control

Generalized predictive control (GPC) is one of the most common methods of predictive control. Here the design of GPC is carried out based on [17]. It is supposed that a model of plant is expressed in terms of the following CARIMA structure:

$$A(z^{-1})y(t) = B(z^{-1})z^{-d}u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \qquad (1)$$

where $u(t)$ is the control input, $y(t)$ the measured variable and $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ are the polynomials in the backward shift operator z^{-1} as follows:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \qquad (2)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b} \qquad (3)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{n_c} z^{-n_c} \qquad (4)$$

The function of the Δ operator $\Delta = 1 - z^{-1}$ is to guarantee integral action in the controller which eliminate offset. d is The pure delay of system. At each sampling time, the control signal is obtained by minimization of a quadratic cost function

$$J(t) = \sum_{k=N_1}^{N_2} \left[\hat{y}(t+k) - r(t+k)\right]^2 + \sum_{k=1}^{N_u-1} \lambda_k \left[\Delta u(t+k)\right]^2 \qquad (5)$$

where $\hat{y}(t)$ and $r(t)$ are predicted model output and reference trajectory respectively. Parameter λ_k is the control signal weighting coefficient. The lower and upper bounds of prediction horizon are denoted by N_1 and N_2 respectively and Δ is difference operator.

By minimizing the cost function (5) and perform some simplification, control signal is obtained as [17]:

$$\mathbf{u} = -\mathbf{H}^{-1}\mathbf{b} = (\mathbf{G}^T\mathbf{G} + \lambda\mathbf{I})^{-1}\mathbf{G}^T(\mathbf{w} - \mathbf{f}) \qquad (6)$$

where the elements of the matrix $\mathbf{G}$ are the open-loop step response coefficients of the plant and the vector f is the vector of free response predictions. These parameters are calculated in form of (7)

$$\mathbf{G} = \begin{bmatrix} g_0 & 0 & 0 & 0 \\ g_1 & g_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{N-1} & g_{N-1} & \cdots & g_0 \end{bmatrix} \quad ; \quad G_j = E_j B \qquad (7)$$

where E_j is obtained by solution of Diophantine equation (8)

$$1 = E_j(z^{-1})\widetilde{A}(z^{-1}) + z^{-j}F_j(z^{-1}) \qquad (8)$$

and f is

$$F(z^{-1}) = \begin{bmatrix} F_{d+1}(z^{-1}) \\ F_{d+2}(z^{-1}) \\ \vdots \\ F_{d+N}(z^{-1}) \end{bmatrix} \qquad (9)$$

The vector $\mathbf{w}$ is derived from the reference signal w as (10)

$$\mathbf{w} = \begin{bmatrix} w(t+d+1) & w(t+d+2) & \cdots & w(t+d+N) \end{bmatrix}^T \tag{10}$$

More details on GPC controller could be found in [17].

III. Generalized Predictive Control Based on Fuzzy Inverse Model

A. Inverse Model

In this paper we decide to design a linear GPC with considering a Wiener model for the system and multiplying the process by the inverse of the nonlinear static part. Schematic of Wiener model is shown in Fig. 1. $G(q)$ and $h(.)$ is the transfer function of linear dynamic and nonlinear static mapping respectively. $u(t)$ is system input and $\hat{y}(t)$ is the output of wiener model and also $v(t)$ is supposed to be an internal signal that is output of linear model and input of nonlinear static mapping. According to Fig. 1, we have

$$v(t) = G(q)u(t) \tag{11}$$

$$\hat{y}(t) = h(v(t)) \tag{12}$$

By passing of system output y(t) through the inverse of the static nonlinear block, we have

$$z(t) = \hat{h}^{-1}(y(t)) \tag{13}$$

where $z(t)$ is output of inverse model. we have

$$\hat{h}^{-1}(\hat{y}(t)) = \hat{h}^{-1}(h(v(t))) \tag{14}$$

If the system identification is accurate enough

$$\lim_{t\to\infty}\left(\hat{h}^{-1}(y(t)) - h^{-1}(\hat{y}(t))\right) = 0 \tag{15}$$

and then

$$z(t) \approx v(t) \tag{16}$$

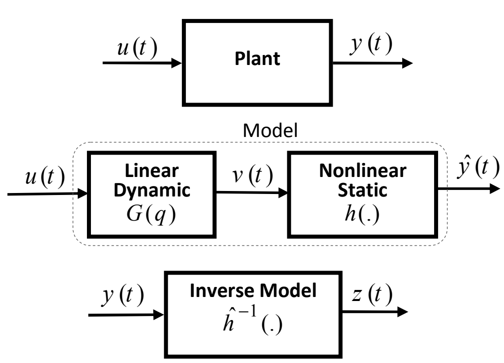

Fig. 1. Introducing the input and output of plant, Wiener Model and inverse model.

So $z(t) \approx G(q)u(t)$ can be supposed as linear system, which is controlled by a linear GPC. Schematic of the proposed method is shown in Fig. 2.

B. Adaptive Neural Fuzzy Inference System (ANFIS)

In Fig. 2, general schematic of the proposed method is shown. To construct the nonlinear static inverse model, we can use the structure of a Takagi-Sugeno fuzzy system [10]. Input to the inverse model is the output of the plant and the output of the inverse model is the output of the linear part of the model

$$w_i = A_i(x) \tag{20}$$

constructed through identification of the pH neutralization process by a Weiner model.

The fuzzy system has singleton fuzzifier, product inference engine and center average defuzzifier. Relationship of this structure is shown in Fig. 3.

$$f(x) = \sum_{i=1}^{M} \bar{w}_i f_i(x) \tag{17}$$

where

$$f_i(x) = m_i x + n_i \tag{18}$$

$$\bar{w}_i = \frac{w_i}{\sum_{j=1}^{M} w_j} \tag{19}$$

where x is input of fuzzy model, f is output of model, f_i is output of ith rule, w_i firing strength for the ith rule and A_i is the membership function of ith input.

Input membership function must have two properties: be able to identify nonlinearity and be differentiable. G-bell membership function and Gaussian membership function have these properties. Derivative of Gaussian membership function is relatively simple; which is a privilege for this membership function. Therefore, Gaussian membership function is used for the fuzzy system. Gaussian membership function has the form of (21)

$$\mathrm{A}_i(x) = \exp\left(-\frac{(x-c_i)^2}{2\sigma_i^2}\right) \tag{21}$$

where c_i and σ_i is center and variance of membership function of the ith input, respectively.

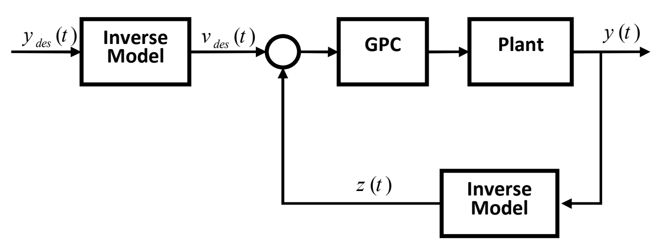

Fig. 2. Schematic of inverse model control.

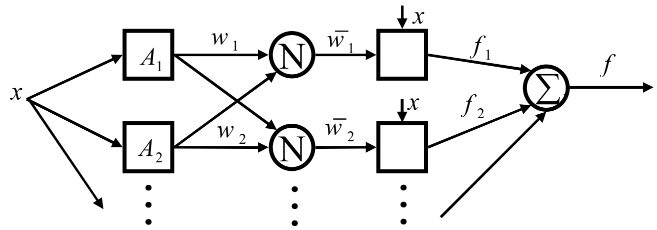

Fig. 3 ANFIS structure

C. Tuning of Model Parameters

Training of fuzzy system parameters, such as parameters of input membership function A_i and parameters of output linear functions, are carried out by online hybrid optimization method. Also for an initial value for these parameters, offline estimation is carried out. Nonlinear parameters are estimated by gradient descent and RLS are used for estimation of linear parameters. Nonlinear parameters of fuzzy system such as c_i and σ_i is updated by back propagation learning method and using gradient descent optimization algorithm, based on the cost function of identification. Cost function is defined as (22)

$$J=\sum_{i=1}^{N} e_i^2 \tag{22}$$

where N is the number of training set and e_i is ith training error and is defined as (23)

$$e=\hat{v}-v=f(x)-T \tag{23}$$

where T is target values that we try to fit output model on it.

In (24) relationship of output of fuzzy system and these linear parameters are shown. As mentioned, tuning of linear parameters such as m_i and n_i is carried out by RLS.

$$v(t)=f(\mathrm{x})=\sum_{i=1}^{M}\bar{w}_i(m_i x+n_i) \tag{24}$$

In formulation of RLS method, repressors vector is $\mathrm{X}=[\bar{\mathrm{w}}_1\mathrm{x} \quad \bar{\mathrm{w}}_1 \quad \bar{\mathrm{w}}_2\mathrm{x} \quad \bar{\mathrm{w}}_2 \quad \ldots \quad \ldots]$ and parameter vector is $\theta=[\mathrm{m}_1 \quad \mathrm{n}_1 \quad \mathrm{m}_2 \quad \mathrm{n}_2 \quad \ldots \quad \ldots]$.

D. Consideration of Fuzzy Inverse Model Identification

Here we have discussion about offline estimation as an initial value for online estimation. Offline identification of inverse system is carried out by output–input coupled data. A very important consideration is the distribution of identification input-output data. If the input data is uniformly distributed in the range of the input signal, the output data distribution is not uniform in all this range. This has a considerable impact on accuracy of identification. For instance, if the inputs are chosen in the range of 1 to 10 uniformly, the output data distribution of the pH neutralization process is not uniform in the range of 4 to 14, as illustrated in Fig. 4.a.

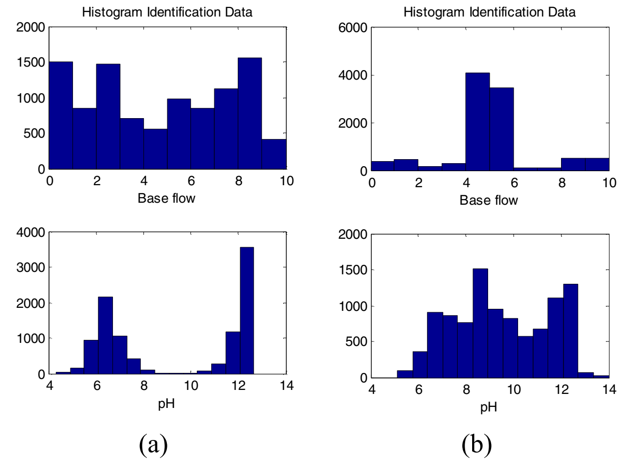

Fig. 4. Distribution of input-output data. a) Data collection with uniform distribution input data. b) Data collection with uniform distribution output data.

One solution is to distribute the input signal according to the inverse of an approximation of the derivative of the function. This approximation could be obtained by focus on titration curve or nonlinear static characteristic as shown in Fig. 5. As a result, the input data distribution is considered as shown in Fig. 4. b. 70% of data is selected as training data and 30% of it is selected as test data. Optimal order of the linear dynamic part of the model is obtained by testing of transfer function with different order and then computing the cost function of every of them.

IV. Results

A. Inverse Model

As mentioned in (11), linear part of system is formulated as the following:

$$v(t)=G(q)u(t) \tag{25}$$

where the estimated linear system is obtained as (26). The steady state gain of this model is unit.

$$G(q)=\frac{0.02319q^2+0.02695q-0.04586}{q^3-1.1711q^2-0.3802q+0.5558} \tag{26}$$

As mentioned, for offline estimation of the nonlinear static part of system as an initial value for online identification, the input training data U is applied to model linear dynamics (25) and data pairs (v, y) are obtained. As mentioned previously, the static nonlinearity is estimated by a fuzzy system. Number of input and output membership functions and rules are determined by cost function diagram.

Using 9 membership functions presents the best result. Therefore, a fuzzy system is constructed using 9 membership functions, 9 rules and 9 outputs.

Diagram of inverse model output $\hat{v}$ in term of its input pH is compared with titration curve in Fig. 5. As can be seen due to the unity of steady state gain of linear dynamic model, the two curves are approximately identical with good accuracy.

Fig. 6 shows that fuzzy inverse model is identified accurately. Where red curve is output of identified fuzzy inverse model that is applied to nonlinear static part of system and blue curve is identity function.

B. GENERALIZED PREDICTIVE CONTROL BASED ON FUZZY INVERSE MODEL

Results of designing GPC for linear dynamic system and applying to the set of pH process and inverse fuzzy model is presented in this section. We expected that with tracking $v_{des}(t)$ by $z(t)$, output (pH) can follow $y_{des}(t)$. The result of applying the fuzzy predictive controller is shown in Fig. 8. In comparison with linear GPC (Fig. 7), especially in critical operating points of pH, i.e. 8, 9 and 10 which have high nonlinearity, it can be seen that proposed fuzzy predictive controller have better performance (Fig. 8).

Fig. 9 shows response of proposed fuzzy GPC in existence of disturbance. It is considered that the acid concentration changed in t = 1600 sec.

V. CONCLUSION

If we can identify the nonlinear system with Wiener model, then the inverse part of model can be used to augment the process model into a linear model and then it can be controlled by a linear controller. Moreover, in existence of disturbance and uncertainty, by use of online identification, we can solve this problem.

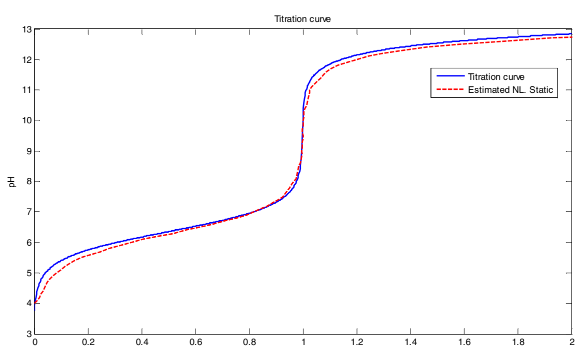

Fig. 5. Nonlinear static part of system (titration curve) and it's estimation

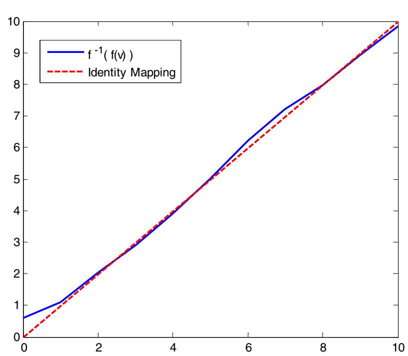

Fig. 6. Output of identified fuzzy inverse model that is applied to nonlinear static part of system

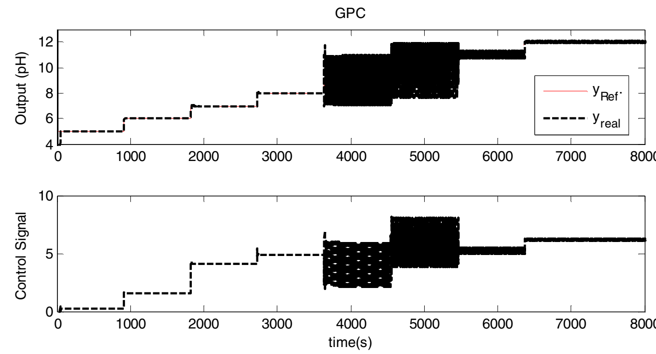

Fig. 7. response of system with linear GPC

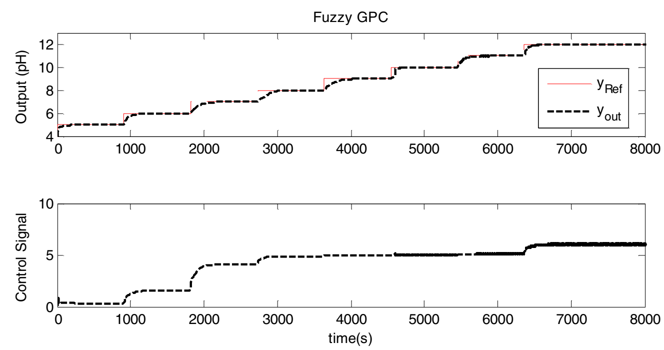

Fig. 8 response of system with fuzzy GPC

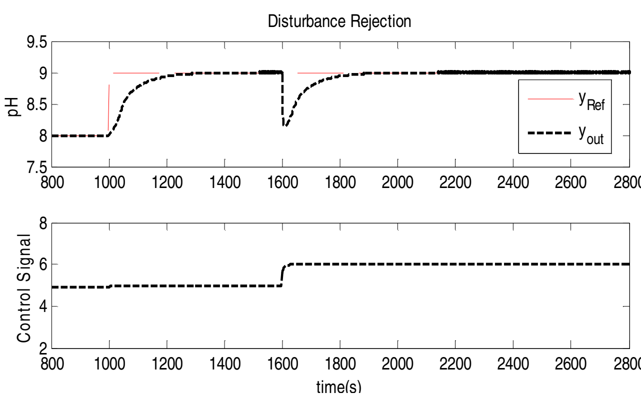

Fig. 9 response of system with fuzzy GPC in existence of disturbance

VI. REFERENCES

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