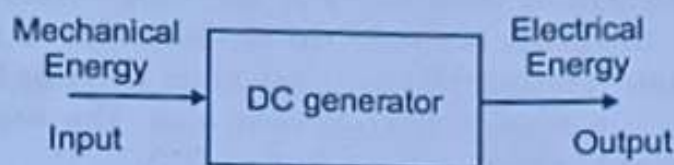


## 4.1 DC GENERATOR - INTRODUCTION

An electrical generator is a rotating machine which converts mechanical energy into electrical energy. It is shown in figure 4.1. This energy conversion is based on the principle of electromagnetic induction. According to Faraday's laws of electromagnetic induction, whenever a conductor is moved in a magnetic field, dynamically induced e.m.f is produced in the conductor.



*Figure 4.1*

When an external load is connected to the conductor, the induced e.m.f. causes a current to flow in the load. Thus the mechanical energy, which is given in the form of motion to the conductor is converted into electrical energy. If a single conductor is used, the e.m.f. produced is small. Large number of conductors are used to obtain greater e.m.f. and the rotating conductor assembly is called an armature.

### 4.1.1 Constructional Details

Figure 4.2 shows a DC generator with its major parts as given below.

1. Magnetic frame or Yoke
2. Poles, interpoles, windings, pole shoes
3. Armature
4. Commutator
5. Brushes, bearings and shaft

#### **Magnetic frame**

The magnetic frame or yoke serves two purposes.

1. It acts as a protecting cover for the whole machine and provides mechanical support for the poles.
2. It carries the magnetic flux produced by the poles. The flux per pole divides at the yoke so that the yoke carries only half the flux produced by each pole.

### 4.1.2 Principle of Operation

Let us consider a single turn coil ABCD (figure 4.7) rotated on a shaft within a uniform magnetic field of flux density. It is rotated in an anticlockwise direction.

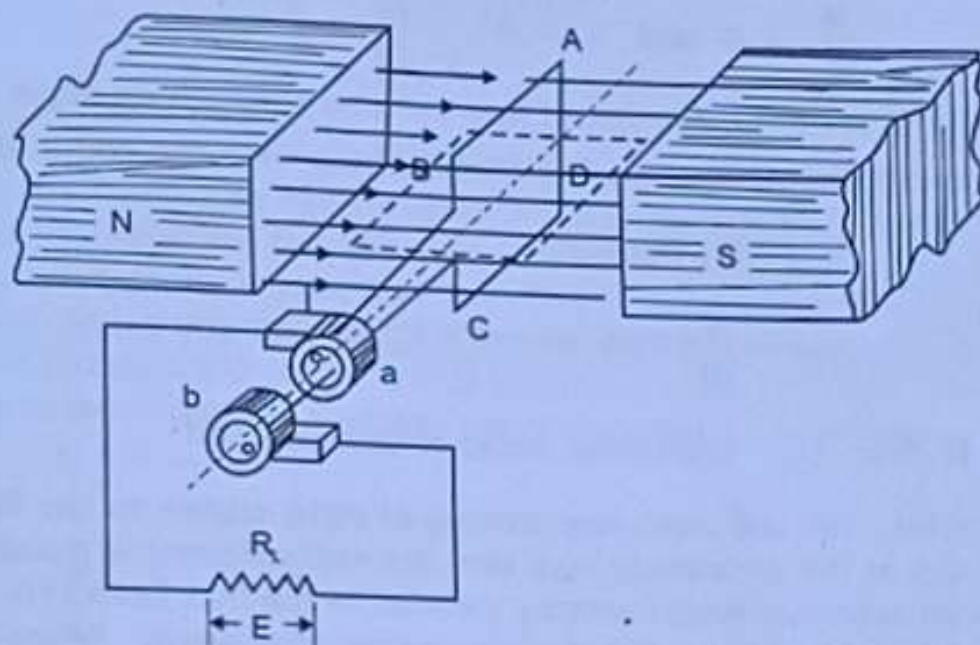


Figure 4.7

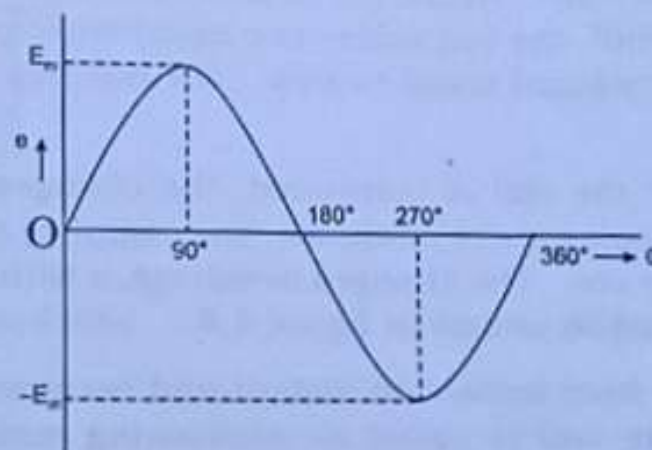


Figure 4.8

Let ' $\ell$ ' be the length and ' $b$ ' be the breadth of the coil in meters. When the coil sides AB and CD are moving parallel to the magnetic field, the flux lines are not being cut and no emf is induced in the coil. At this position, we assume the angle of rotation ' $\theta$ ' as zero.



This vertical position of the coil is the starting position. According to Faraday's law II, the emf induced is proportional to the rate of change of flux linkages.

$$e = -N \frac{d\phi}{dt} \quad \dots (4.1)$$

where "N" is the number of turns, " $\phi$ " is the flux and "t" is the time.

$$\text{As } N = 1, e = -\frac{d\phi}{dt} \text{ volts}$$

Initially, when the coil is moving parallel to the flux lines, no flux line is cut and hence

$$\frac{d\phi}{dt} = 0 \text{ and } e = 0 \quad \dots (4.2)$$

After time "t" secs, the coil would have rotated through an angle  $\omega t$  radians in the anti-clockwise direction. The flux then linking with the coil is  $B \ell b \cos \omega t$ .

$$\phi = B \ell b \cos \omega t$$

$$\therefore e = -\frac{d}{dt}(B \ell b \cos \omega t) = E_m \sin \omega t \quad \dots (4.3)$$

where  $E_m = B \ell b \omega$ ,  $E_m$  - maximum value of induced emf

When  $\theta = 90^\circ$ , the coil sides are moving at right angles to the flux lines. The flux lines are cut at the maximum rate and the emf induced is maximum. When  $\theta = 180^\circ$ , the coil sides are again moving parallel to the flux lines (AB and CD have exchanged positions) and the emf induced, is zero once again. When  $\theta = 270^\circ$ , the coil sides again move at right angles to the flux lines but with their position reversed when compared with  $\theta = 90^\circ$ . Hence the induced emf is maximum in the opposite direction. When  $\theta = 360^\circ$ , the coil sides once again move parallel to the magnetic field making the induced emf equal to zero. The coil has now come back to the starting point.

If the rotation of the coil is continued, the changes in the emf are again repeated. For the two pole generator shown, one complete cycle of changes occurs in one revolution of the coil. The changes in voltage, e with respect to the angle or even time can be plotted as shown in figure 4.8.

The emf changes from instant to instant and becomes alternatively positive and negative. Such an emf is called an alternating emf. If the coil sides are connected to two slip rings 'a' and 'b' and an external resistance R is connected across them, a current flows through the resistor, which is again alternating.

The induced emf in the coil, can be increased by

- 1) increasing the flux density (B) and
- 2) by increasing the angular velocity ( $\omega$ ).

### 4.1.3 E.M.F. Induced in a DC Generator

Let  $\phi$  be the flux per pole in webers.

Let  $P$  be the number of poles.

Let  $Z$  be the total number of conductors in the armature. All the  $Z$  conductors are not connected in series. They are divided into groups and let  $A$  be the number of parallel paths into which these conductors are grouped.

Each parallel path will have  $Z/A$  conductors in series.

Let  $N$  be the speed of rotation in revolutions per minute (rpm).

Consider one conductor on the periphery of the armature. As this conductor makes one complete revolution, it cuts  $P\phi$  webers. As the speed is  $N$  rpm, the time taken for one revolution is  $60/N$  Secs.

Since the emf induced in the conductor = rate of change of flux cut,

$$e \propto \frac{d\phi}{dt} = \frac{P\phi}{60/N}$$

$$e = \frac{NP\phi}{60} \text{ volts} \quad \dots (4.4)$$

Since there are  $Z/A$  conductors in series in each parallel path the emf induced

$$E_s = \frac{NP\phi}{60} \frac{Z}{A} = \frac{\phi ZN}{60} \frac{P}{A} \text{ Volts} \quad \dots (4.5)$$

The armature conductors are generally connected in two different ways, viz, lap winding and wave winding. For lap wound armatures, the number of parallel paths is equal to the number of poles (i.e.,  $A = P$ ). In wave wound machines,  $A = 2$ , always.



**EXAMPLE 1**

Calculate the emf generated by a 6 pole DC generator having 480 conductors and driven at a speed of 1200 rpm. The flux per pole is 0.012 wb. Assume the generator to be (a) lap wound, (b) wave wound.

**Solution:**

$$E_g = \frac{\phi Z N}{60} \times \frac{P}{A} \text{ volts}$$

a) For a lap wound machine,  $A = P = 6$

$$E_g = \frac{0.012 \times 480 \times 1200 \times 6}{60 \times 6} = 115.2 \text{ volts}$$

$$E_g = 115.2 \text{ V}$$

b) For a wave wound machine,  $A = 2$

$$E_g = \frac{0.012 \times 480 \times 1200 \times 6}{60 \times 2} = 345.6 \text{ volts}$$

$$E_g = 345.6 \text{ V}$$

**EXAMPLE 2**

A wave connected armature winding has 19 slots with 54 conductors per slot. If the flux per pole is 0.025 wb and number of poles is 8, find the speed at which the generator should be run to give 513V. Also find the speed if the armature is lap connected.

**Solution:**

**Given data:**

$$P = 8, \quad \phi = 0.025 \text{ wb}, \quad Z = 19 \times 54 = 1026, \quad A = 2 \text{ (for wave)}$$

$$E_g = 513 \text{ volts}; \quad E_g = \phi \frac{Z N P}{60 A}$$

$$\text{Substituting, } N = \frac{60 \times 513 \times 2}{0.025 \times 19 \times 54 \times 8} = 300 \text{ rpm}$$

$$N = 300 \text{ rpm}$$

For lap wound,  $A = P = 8$

$$\therefore N = \frac{60 \times 513 \times 8}{0.025 \times 1026 \times 8} = 1200 \text{ rpm}$$

$$N = 1200 \text{ rpm}$$

**EXAMPLE 3**

The armature of a 4-pole, 600 rpm, lap wound generator has 100 slots. If each coil has 4 turns, calculate the flux per pole required to generate an emf of 300V.

*Given data:*

Numbers of poles = 4; Speed = 600 rpm; Numbers of slots = 100;  $E_g = 300V$

Each turn has two active conductors and 100 coils are required to fill 100 slots. Therefore number of conductors  $Z = 100 \times 4 \times 2 = 800$ ; for lap wound generator  $A = P = 4$ .

*To find:*

Flux per pole ( $\phi$ ).

**Solution:**

$$\text{Generated emf } E_g = \frac{P\phi ZN}{60A}$$

$$\therefore \text{Flux / pole } \phi = \frac{E_g \times 60A}{PZN} = \frac{300 \times 60 \times 4}{4 \times 800 \times 600}$$

$$\boxed{\phi = 37.5 \text{ mwb}}$$

**EXAMPLE 4**

A 6-pole, lap wound armature rotated at 350 rpm is required to generate 300V. The useful flux per pole is 0.05wb. If the armature has 120 slots, calculate the number of conductors per slot.

*Given data:*

Numbers of poles,  $P = 6$ , Speed,  $N = 350 \text{ rpm}$ , Generated emf,  $E_g = 300V$ ,

Flux per pole,  $\phi = 0.05\text{wb}$ , Numbers of slots = 120, For lap wound generator,  $A = P = 6$

*To find:*

Numbers of conductors / slot:

**Solution:**

$$\text{Generated emf } E_g = \frac{P\phi ZN}{60A}$$

$$\text{Numbers of conductors } Z = \frac{E_g \times 60A}{P\phi N} = \frac{300 \times 60 \times 4}{4 \times 0.05 \times 350} = 1029$$

$$\therefore \text{Numbers of conductors / slot} = \frac{1029}{120} = 8.575$$

$$\boxed{\text{Conductors / slot} = 9}$$



**EXAMPLE 5**

The armature of a 4 pole DC generator has 85 slots and the commutator has 245 segments. It is wound to give lap winding having one turn per coil. If the flux per pole is 35mwb, calculate the generated emf at a speed of 1200rpm.

Given data:

Numbers of slots = 85, Numbers of commutator segments = 245,  $P = 4$

Flux / pole = 35 mwb, For lap wound  $A = P = 4$ ,  $N = 1200$  rpm

To find:

Generated emf  $E_g$ .

**Solution:**

The number of coils is equal to the number of commutator segments. Each turn has 2 active conductors.

$\therefore$  Numbers of conductors  $Z = 245 \times 2 = 490$

$$E_g = \frac{P\phi ZN}{60A} = \frac{4 \times 35 \times 10^{-3} \times 490 \times 1200}{60 \times 4}$$

$$E_g = 343 \text{ V}$$

**EXAMPLE 6**

A 4-pole, wave wound generator has 40 slots and 10 conductors are placed per slot. Find, the generated emf when the generator is driven at 1200 rpm and  $\phi = 0.02$  wb.

(AU/Mech-Dec 2005)

Given data:

Number of poles  $P = 4$

Total number of conductors  $Z = \text{Numbers slots} \times \text{conductors per slot}$   
 $Z = 40 \times 10 = 400$

Flux per pole  $\phi = 0.02$  wb, Speed  $N = 1200$  rpm.

To find:

Generated emf ( $E_g$ )

**Solution:**

Generated emf  $E_g = \frac{P\phi ZN}{60A}$  ; for a wave wound machine  $A = 2$

$$E_g = \frac{4 \times 0.02 \times 400 \times 1200}{60 \times 2}$$

$$E_g = 320 \text{ V}$$

**EXAMPLE 7**

A 4-pole machine has 60 slots and 8 conductors per slot. The total flux per pole is 20mwb. For relative speed of 1500rpm, between field flux and armature winding, calculate the generated armature voltage if the machine is a DC machine with lap winding.

(AU/EEE - Dec 2003)

*Given data:*Number of poles  $P = 4$ Number of conductors  $Z = \text{Number of slots} \times \text{conductor per slot} = 60 \times 8 = 480$ Speed  $N = 1500 \text{ rpm}$ Flux per pole  $\phi = 20\text{mwb}$ For lap winding  $A = P = 4$ *To find:*Generated armature voltage ( $E_g$ )**Solution:**

$$E_g = \frac{P\phi ZN}{60A} = \frac{4 \times 20 \times 10^{-3} \times 480 \times 1500}{60 \times 4}$$

$$E_g = 240 \text{ V}$$

**EXAMPLE 8**

A 8 pole, DC generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 wb. The machine is running at 250 rpm. Calculate the induced armature voltage.

(GATE - 2004)

*Given data:*Number poles  $P = 8$ Total number of conductors  $Z = 2 \times 32 \times 6 = 384$ Flux per pole  $\phi = 0.06 \text{ wb}$ For wave wound  $A = 2$ **Solution:**

$$\text{Induced armature voltage } E_g = \frac{P\phi ZN}{60A} = \frac{8 \times 0.06 \times 384 \times 250}{60 \times 2}$$

$$E_g = 384 \text{ V}$$



#### 4.1.4 Types of DC Generators

DC generators can be classified according to their methods of field excitation. There are two types of d.c. generators on the basis of excitation.

1. Separately excited d.c. generators
2. Self excited d.c. generators.

##### Separately excited DC generators

If the field winding is excited by a separate d.c. supply, then the generator is called separately excited d.c. generator. Figure 4.10 shows the diagram of a separately excited generator.

The field winding has large number of turns of thin wire.

From this diagram,

Armature current  $I_a$  = Load current  $I_L$

$R_a$  = Resistance of the armature winding

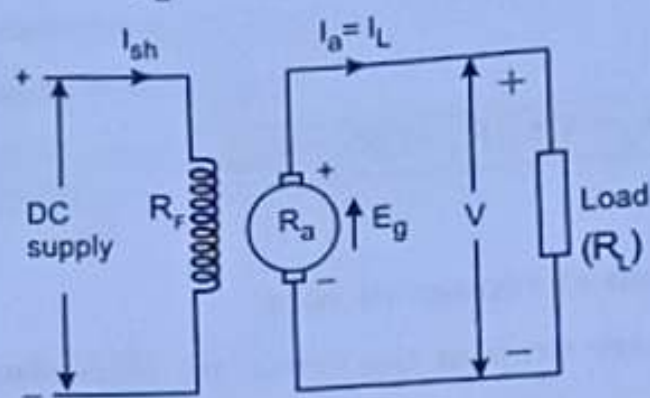


Figure 4.10

Terminal voltage  $V = E_g - I_a R_a - V_{\text{brush}}$

$V_{\text{brush}}$  = voltage drop at the contacts of the brush.

Generally  $V_{\text{brush}}$  is neglected because of very low value.

Generated emf  $E_g = V + I_a R_a + V_{\text{brush}}$

Electric power developed =  $E_g I_a$

Power delivered to load =  $V I_a$

##### Self-excited DC Generators

If in a d.c. generator field winding is supplied from the armature of the generator itself, then it is called a self-excited d.c. generator. Residual flux is present in the poles. When the armature is rotated, a small emf is produced in the armature winding because of residual flux. This emf produces a small field current in the field winding. Then flux per pole increases. The increased flux increases the induced emf, which further increases the field current. Because of this cumulative process, generator produces its rated voltage. The self-excited generators can be classified depending upon how the field winding is connected to the armature.

There are three types,

1. Series generator,
2. Shunt generator,
3. Compound generator.

### i) Series Generator

The field winding is connected in series with the armature. This type of D.C. generator is called D.C. series generator.

Here, the armature current flows through the field winding as well as the load. The d.c. series generator connection diagram is shown in figure 4.11.

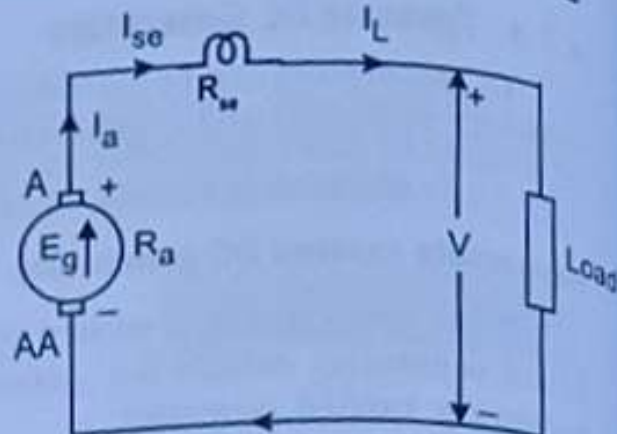


Figure 4.11

The field winding has less number of turns of thick wire. It has low resistance.

It is denoted by  $R_{se}$ . Here, armature, field and load are all in series. So they carry the same current.

$$\therefore I_a = I_{se} = I_L$$

Generated emf

$$E_g = V + I_a R_a + I_a R_{se} + V_{brush}$$

where

$V$  = terminal voltage in volts

$I_a R_a$  = voltage drop in the armature resistance.

$I_a R_{se}$  = voltage drop in the series field winding resistance

$V_{brush}$  = brush drop

$\therefore$  Terminal voltage  $V = E_g - I_a R_a - I_a R_{se} - V_{brush}$

Power developed in the armature =  $E_g I_a$

Power delivered to load =  $V I_a$  or  $V I_L$

### ii) Shunt Generator

In a d.c. shunt generator, field winding is connected across the armature. The load is also connected across the armature.

The shunt field winding has more number of turns of thin wire. It has high resistance.

Therefore, a small amount of current flows through the field winding and large amount of current flows through the armature. Figure 4.12 shows connections diagram of a D.C. shunt generator.

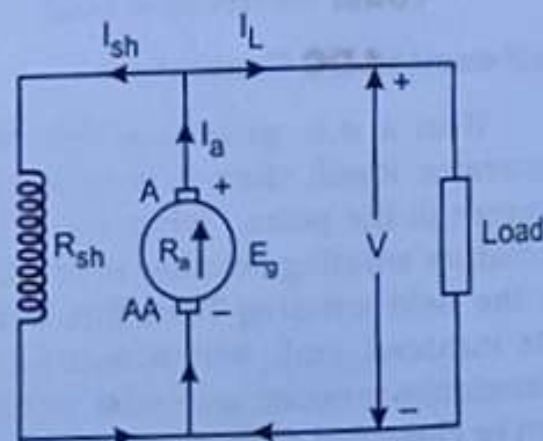


Figure 4.12



Terminal voltage  $V = E_g - I_a R_a$

Shunt field current  $I_{sh} = \frac{V}{R_{sh}}$

Armature current  $I_a = I_L + I_{sh}$

Power developed by armature  $= E_g I_a$

Power delivered to load  $= V I_L$

### iii) Compound Generator

The compound generator consists of both shunt field and series field windings. One winding is in series and other winding is in parallel with the armature. Depending upon the shunt field and series field connections, compound generator can be classified as

1. Long shunt compound generator
2. Short shunt compound generator

#### Long shunt compound generator

Figure 4.13 shows connection diagram of a long shunt compound generator. Here, shunt field winding is connected across both series field and armature windings.

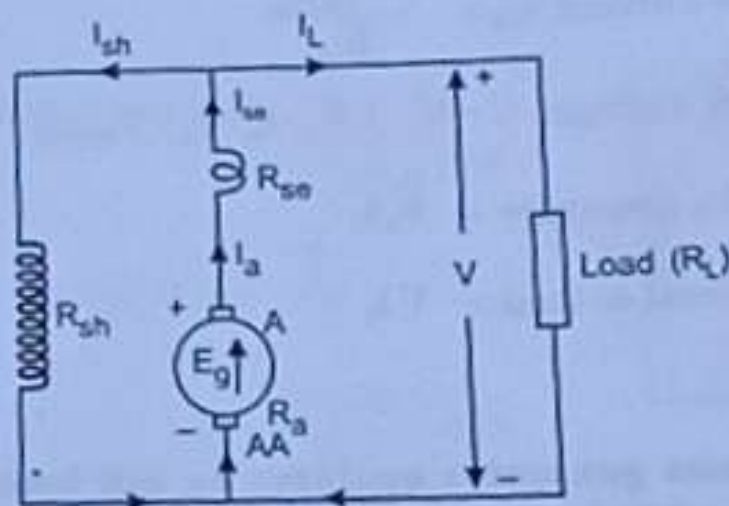


Figure 4.13

From this figure 4.13, series field current  $I_{sc} = I_a = I_L + I_{sh}$

Shunt field current  $I_{sh} = \frac{V}{R_{sh}}$

Generated emf  $E_g = V + I_a (R_a + R_{se}) + V_{brush}$

Terminal voltage  $V = E_g - I_a (R_a + R_{se}) - V_{brush}$

Power developed in armature  $= E_g I_a$

Power delivered to load  $= V I_L$

## Short Shunt Compound Generator

Figure 4.14 shows short shunt compound generator. Here, shunt field winding is connected in parallel with the armature and this combination is connected in series with series field winding.

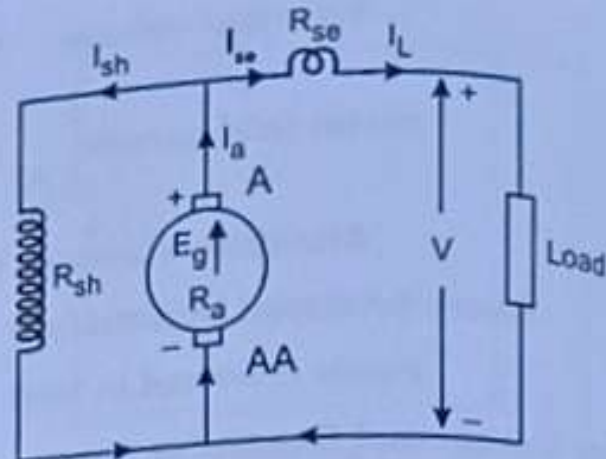


Figure 4.14

From this figure 4.14,

$$\text{Series field current} = I_{se} = I_L$$

$$\text{Load current} = I_L$$

$$\text{Armature current } I_a = I_{sh} + I_{se}$$

$$\text{Generated emf } E_g = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$\text{Voltage across shunt field winding} = I_{sh} R_{sh}$$

$$I_{sh} R_{sh} = E_g - I_a R_a - V_{brush}$$

$$= V + I_a R_a + I_{se} R_{se} + V_{brush} - I_a R_a - V_{brush}$$

$$= V + I_{se} R_{se}$$

$$\therefore \text{Shunt field current } I_{sh} = \frac{V + I_{se} R_{se}}{R_{sh}}$$

$$\text{Terminal voltage } V = E_g - I_a R_a - I_{se} R_{se} - V_{brush}$$

$$\text{Power developed in armature} = E_g I_a$$

$$\text{Power delivered to load} = V I_L$$

### EXAMPLE 9

A 50 kW, 250-volt shunt generator operates on full load at 1500rpm. The armature has 6 poles and is lap wound with 200 turns. Find the induced emf and the flux per pole at full load. Given that the armature and field resistances are 0.01 and 125 Ohms respectively. Neglect armature reaction.

**Solution:**

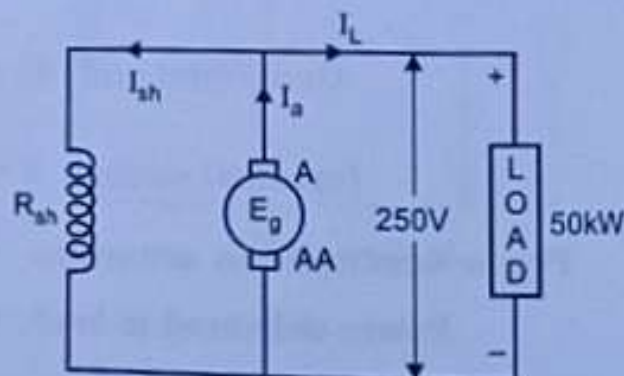
For a load power of 50kW at a terminal voltage of 250-volts, load current,

$$I_L = \frac{50 \times 10^3}{250} = 200 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2 \text{ A}$$

For a shunt generator,

$$I_a = I_L + I_{sh} = 202 \text{ A}$$





$$\text{Induced emf, } E_g = V + I_L + I_{sh} \text{ (neglecting armature reaction and other drops)}$$

$$= 250 + 202 \times 0.01 = 252.02V$$

$$E_g = 252.02V$$

$$Z = 200 \times 2 \text{ (since 1 turn = 2 conductors)}$$

$$N = 1500 \text{ rpm}$$

$$P = A = 6 \text{ (lap wound)}$$

$$\text{Therefore } \phi = \frac{252.02 \times 60 \times 6}{400 \times 1500 \times 6} = 0.025205 \text{ Wb}$$

$$\phi = 0.025205 \text{ wb}$$

### EXAMPLE 10

A 4-pole lap connected shunt generator has  $R_{sh} = 100\Omega$  and  $R_a = 0.1\Omega$  and supplies sixty lamps each rated 40W, 200V. Calculate the armature current, induced emf and current in each parallel path of the armature. Allow a brush drop of 1V per brush.

**Solution:**

$$\text{Total load supplied } P_o = 60 \times 40$$

$$= 2400 \text{ watts}$$

$$\text{Load current} = \frac{P_o}{V}$$

$$I_L = \frac{2400}{200} = 12A$$

$$\text{Field current } I_{sh} = \frac{V}{R_{sh}} = \frac{200}{100} = 2A$$

$$\text{Armature current } I_a = I_L + I_{sh} = 12 + 2 = 14A$$

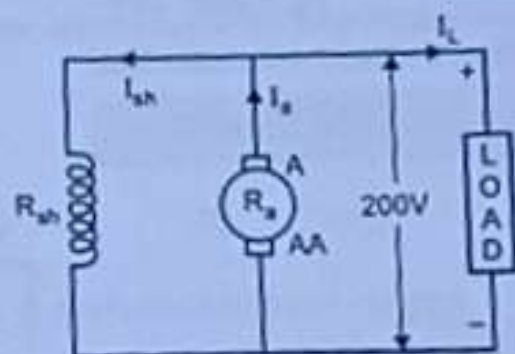
$$\text{Numbers of parallel paths} = \text{Numbers of poles} = 4 \text{ (lap)}$$

$$\text{Therefore current per path} = 14/4 = 3.5A$$

$$\text{Induced e.m.f.} = V + I_a R_a + \text{brush drop}$$

$$= 200 + 14 \times 0.1 + 2 \times 1$$

$$E_g = 203.4 \text{ volts}$$



**EXAMPLE 11**

A compound generator delivers a load current of 50A at 500V. The resistances are  $R_a = 0.05\Omega$ ,  $R_{se} = 0.03\Omega$  and  $R_{sh} = 250\Omega$ . Find the induced emf if contact drop is 1.0 V per brush. Neglect armature reaction. Assume (a) long shunt, b) short shunt connection.

**(MKU/Nov-2000)****Solution:****Long shunt connection**

$$V = 500V$$

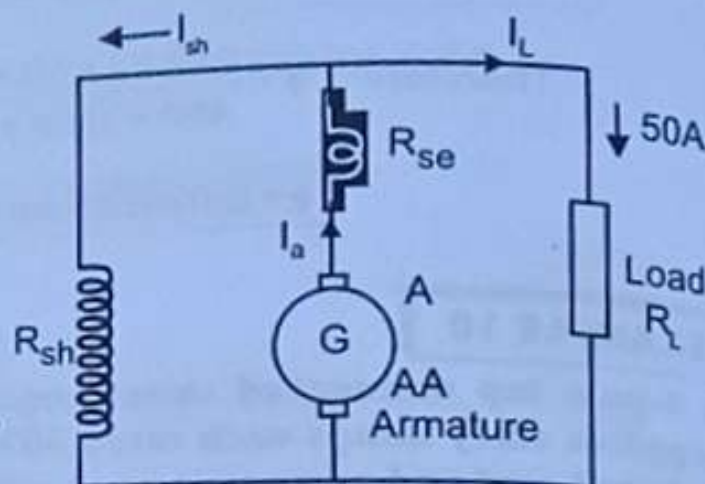
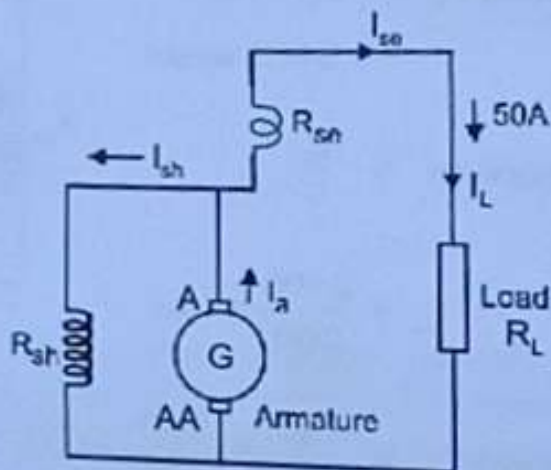
$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A$$

$$I_a = I_{sh} + I_L = 2 + 50 = 52A$$

$$E_g = V + I_a(R_{se} + R_a) + \text{Brushdrop}$$

$$= 500 + 52(0.03 + 0.05) + 2 \times 1$$

$$E_g = 506.16V$$

**Short shunt connection**

Now the shunt field current is obtained by dividing  $(V + I_L R_{se})$  by the shunt field resistance.

$$I_{sh} = \frac{500 + 50 \times 0.03}{250} = 2.006 \text{ Amps}$$

$$I_a = I_L + I_{sh} = 50 + 2.006 = 52.006 A$$

$$E_g = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$= 500 + 52.006 \times 0.05 + 50 \times 0.03 + 2$$

$$E_g = 506.1003V$$



**EXAMPLE 12**

A separately excited generator has induced emf of 250V and a full load terminal voltage of 240V. If the value of  $R_a = 0.01\Omega$ , find the full load current and output of the generator. Neglect armature reaction and brush drop.

**Solution:**

$$V = 240 \text{ V}$$

$$E_g = 250 \text{ V}$$

$$E_g = V + I_a R_a$$

$$I_a = \frac{E_g - V}{R_a}$$

$$I_a = \frac{250 - 240}{0.01} = 1000 \text{ A} = I_L$$

$$\text{Output power} = VI_L = 240 \times 1000 = 240 \text{ kW}$$

$$P = 240 \text{ kW}$$

**EXAMPLE 13**

A 30 kW, 300V dc shunt generator has armature and field resistances of  $0.05\Omega$  and  $100\Omega$  respectively. Calculate the total power developed by the armature when it delivers full load output.

(AU/Mech - Dec 2012)

**Given data:**

$$P = 30 \text{ kW}; \quad V = 300 \text{ V}; \quad R_a = 0.05\Omega; \quad R_{sh} = 100\Omega$$

**To find:**

Total power developed by the armature when it delivers full load output.

**Solution:**

Figure shows the dc shunt generator on load.

$$\begin{aligned} \text{Load current, } I_L &= \frac{P}{V} = \frac{30 \times 10^3}{300} \\ &= 100 \text{ A} \end{aligned}$$

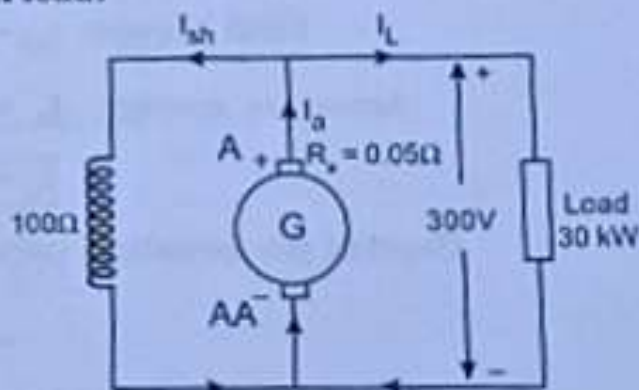
$$\begin{aligned} \text{Field current, } I_{sh} &= \frac{V}{R_{sh}} = \frac{300}{100} \\ &= 3 \text{ A} \end{aligned}$$

$$\therefore \text{Armature current } I_a = I_L + I_{sh}$$

$$I_a = 100 + 3 = 103 \text{ A}$$

$$\text{Generated emf } E_g = V + I_a R_a = 300 + 103 \times 0.05 = 305.15 \text{ V}$$

$$\text{Power developed by armature} = E_g I_a = 305.15 \times 103 = 31.43 \text{ kW.}$$



**EXAMPLE 14**

A 4-pole shunt generator, with a lap wound armature has field resistance of  $50\Omega$  and armature circuit resistance of  $0.1\Omega$ . The generator is supplying sixty  $100\text{V}$ ,  $40\text{W}$  lamps. Find the total armature current in each armature conductor and generated emf. The brush contact drop is  $1\text{V}$  per brush.

(AU/CSE-Dec 2003, 2007)

Given data:

Poles  $P = 4$

Armature resistance  $R_a = 0.1\Omega$

Brush drop  $V_{\text{brush}} = 2 \times 1 = 2\text{V}$

Field resistance  $R_{sh} = 50\Omega$

Total power supplied  $P_0 = 60 \times 40 = 2400\text{W}$

Terminal voltage  $V = 100\text{V}$

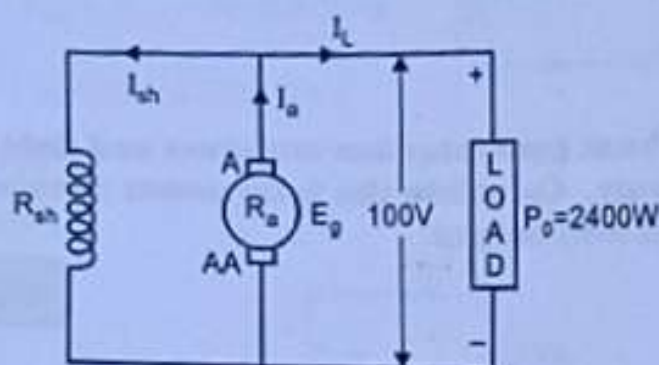
To find:

Armature current  $I_a$

Generated emf  $E_g$

**Solution:**

Figure shows circuit diagram of dc shunt generator.



Total power supplied  $P_0 = 60 \times 40 = 2400\text{W}$

Load current  $I_L = \frac{P_0}{V} = \frac{2400}{100} = 24\text{A}$

Field current  $I_{sh} = \frac{V}{R_{sh}} = \frac{100}{50} = 2\text{A}$

Armature current  $I_a = I_L + I_{sh} = 24 + 2 = 26\text{A}$

$I_a = 26\text{A}$

Current per armature parallel path

$= \frac{I_a}{A} = \frac{I_a}{P} = \frac{26}{4} = 6.5\text{A}$  [For lap wound  $A = P$ ]

Generated emf  $E_g = V + I_a R_a + V_{\text{brush}}$   
 $= 100 + 26 \times 0.1 + 1 \times 2$

$E_g = 104.6\text{V}$



**EXAMPLE 15**

An 8-pole DC shunt generator with 778 wave connected armature conductors and running at 500 rpm supplies a load of  $12.5\Omega$  resistance at a terminal voltage of 250V. The armature resistance is  $0.24\Omega$  and field resistance is  $250\Omega$ . Find the armature current, the induced emf and the flux per pole.

(MKU/EEE-Nov2002)

Given data:

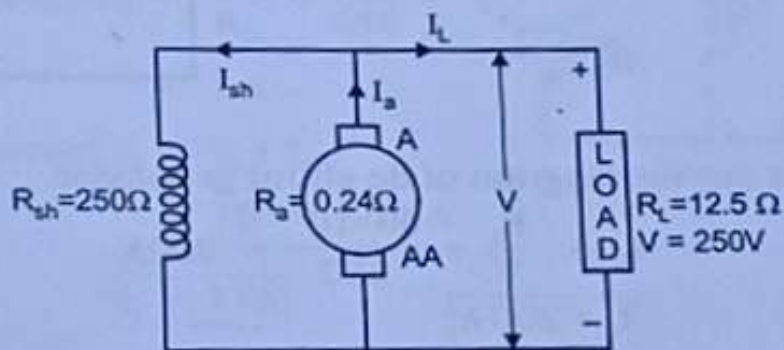
Numbers of poles	$P = 8$
Numbers of armature conductors	$Z = 778$
Speed	$N = 500 \text{ rpm}$
Load resistance	$R_L = 12.5\Omega$
Terminal voltage	$V = 250\text{V}$
Armature resistance	$R_a = 0.24\Omega$
Shunt field resistance	$R_{sh} = 250\Omega$
Wave connected machine	$A = 2$

To find:

Armature current ( $I_a$ ), Induced emf ( $E_g$ ), Flux per pole ( $\phi$ ).

**Solution:**

Figure shows circuit diagram of dc shunt generator.



i) Armature current ( $I_a$ )

$$\text{Load current } I_L = \frac{V}{R_L} = \frac{250}{12.5} = 20\text{A}$$

$$\text{Shunt field current } I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1\text{A}$$

$$\therefore I_a = I_L + I_{sh} = 20 + 1 = 21\text{A}$$

$$\boxed{I_a = 21\text{A}}$$

ii) Induced emf  $E_g$

$$E_g = V + I_a R_a = 250 + 21 \times 0.24 = 255.04 \text{ V}$$

$$E_g = 255.04 \text{ V}$$

iii) Flux per pole ( $\phi$ )

$$E_g = \frac{P \phi Z N}{60 A}$$

$$\phi = \frac{E_g 60 A}{P Z N} = \frac{255.04 \times 60 \times 2}{8 \times 778 \times 500} = 9.83$$

$$\phi = 9.83 \text{ mwb}$$

### EXAMPLE 16

A 50kW, 250V, dc shunt generator has a field circuit resistance of  $60\Omega$  and an armature resistance of  $0.02\Omega$ . Calculate (i) load current, field current and armature current. (ii) the generated armature voltage when delivering rated current at rated speed and voltage.

(MKU/ EEE-Apr2003)

Given data:

Terminal voltage  $V = 250\text{V}$

Output power  $P_o = 50 \text{ kW}$

Armature resistance  $R_a = 0.02\Omega$

Shunt field resistance  $R_{sh} = 60 \Omega$

$R_{sh} = 60\Omega$

To find:

i)  $I_L$ ,  $I_{sh}$ ,  $I_a$

ii)  $E_g$

**Solution:**

Figure shows circuit diagram of dc shunt generator.

i) Load current  $I_L = \frac{P_o}{V} = \frac{50 \times 10^3}{250} = 200\text{A}$

$$I_L = 200\text{A}$$

Field current  $I_{sh} = \frac{V}{R_{sh}} = \frac{250}{60} = 4.166\text{A}$

$$I_{sh} = 4.166\text{A}$$

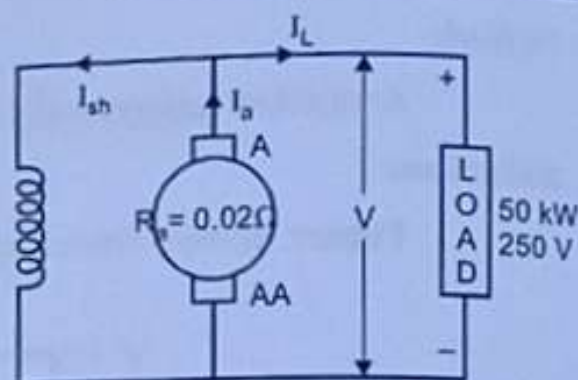
Armature current  $I_a = I_L + I_{sh} = 200 + 4.166$

$$I_a = 204.166\text{A}$$

ii) Generated emf  $E_g$

$$E_g = V + I_a R_a = 250 + 204.166 \times 0.02$$

$$E_g = 254.08\text{V}$$





#### **4.1.5 Applications of DC Generators**

DC supply has for almost all applications been replaced by alternating current. AC has the chief advantage that the voltage level can be easily stepped up or down. However, DC is in use for some special applications and where the DC equipment is still in operation.

Shunt generators are used for supplying nearly constant loads. They are used for battery charging, for supplying the fields of synchronous machines and separately excited DC machines.

Since the output voltage of a series generator increases with load, series generators are ideal for use as boosters for adding a voltage to the transmission line and to compensate for the line drop. The series generator is connected in series with the line and operated in the straight line portion (unsaturated) of the characteristic.

Compound generators maintain better voltage regulation and hence find use where constancy of voltage is required eg., for a self contained generator unit.

## 4.2 DC MOTORS - INTRODUCTION

While a DC generator converts mechanical energy in the form of rotation of the conductor (armature) into electrical energy, a motor does the opposite. The input to a DC motor is electrical and the output is mechanical rotation or torque. It is shown in figure 4.15. The fundamental principles and construction of the DC motors are identical with DC generators which have the same type of excitation. A DC machine that runs as a motor will also operate as a generator.

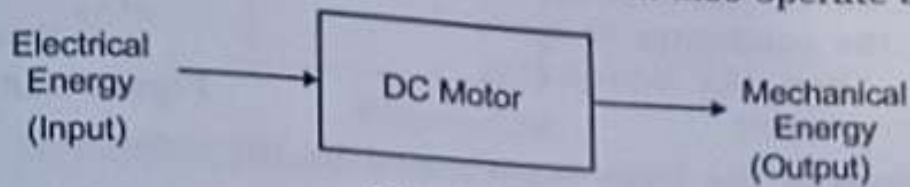


Figure 4.15

### 4.2.1 Principle of Operation

The basic principle of operation of d.c. motor is, "whenever a current carrying conductor is placed in a magnetic field, the conductor experiences a force tending to move it."

The magnetic field between two poles N and S is shown in figure 4.16.

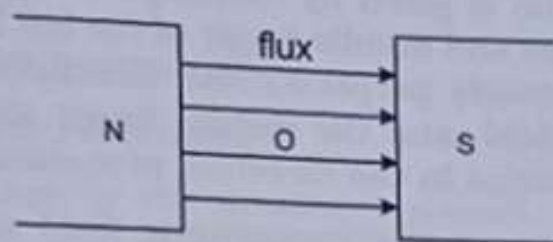


Figure 4.16

A current carrying conductor is shown in figure 4.17 along with the direction of the flux loops around it.



Figure 4.17

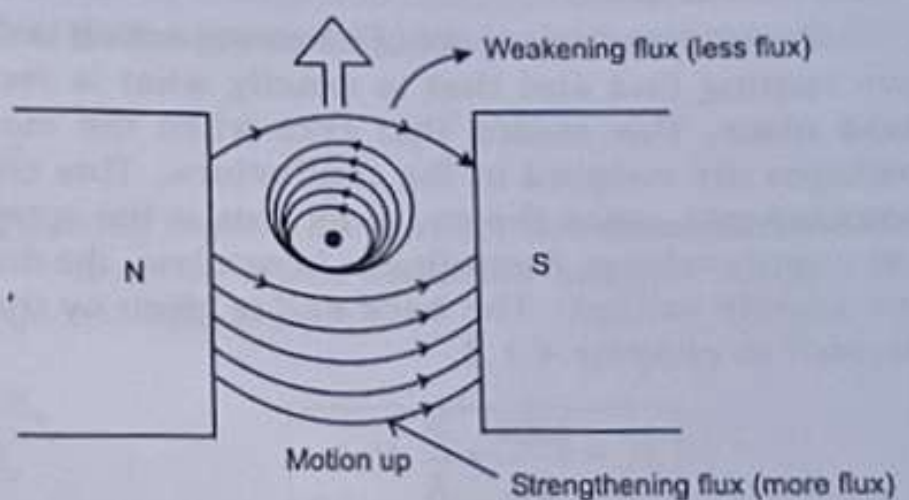


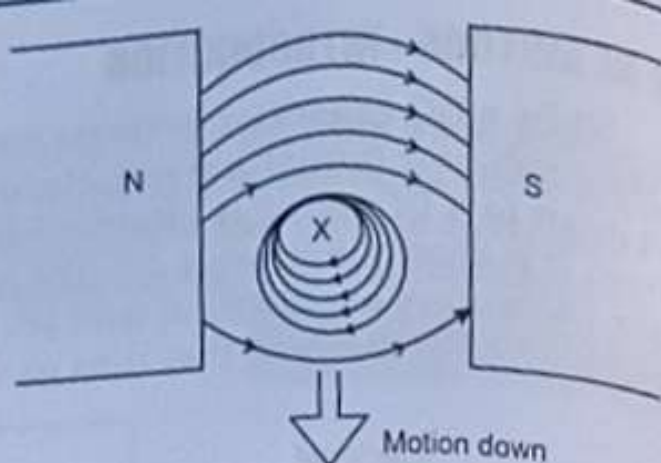
Figure 4.18

If a current carrying conductor is placed between two magnetic poles as shown in figure 4.18, both the fields will be distorted. Above the conductor, the field is weakened (less flux) and below the conductor, the field is strengthened. Therefore the conductor tends to move upwards.



The force exerted upwards depends upon the intensity of the main field flux and the magnitude of the current.

Then the direction of the current through the conductor is reversed as shown in figure 4.19. Here, the field below the conductor is less (weakened) and field above the conductor is more (strengthened). Then the conductor tends to move downwards.



**Figure 4.19**

The magnitude of the force experienced by the conductor in a motor is given by

$$F = BIl \text{ Newtons,}$$

where

$B$  = Magnetic field density in  $\text{wb/m}^2$ .

$I$  = current in amperes

$l$  = length of the conductor in metres.

The direction of motion is given by Fleming's Left Hand rule, which states that if the thumb, fore finger and middle finger of the left hand are held such that the fingers show three mutually perpendicular directions and if the fore finger indicates direction of the field, and the middle finger indicates the direction of current, then the thumb points in the direction of motion of the conductor.

In a d.c. motor, a strong electromagnetic field and a large number of conductors housed in an armature and carrying current, make the armature rotate.

#### 4.2.2 Back EMF

An interesting aspect of motoring action is detailed below. The conductors are cutting flux and that is exactly what is required for generator action to take place. This means that even when the machine is working as a motor, voltages are induced in the conductors. This emf is called as the back emf or counter emf, since the cause for this is the rotation, which, in turn, is due to the supply voltage. According to Lenz's law, the direction of the back emf opposes the supply voltage. The back emf is given by the equation for induced emf as derived in chapter 4.1.3,

$$E_b = \frac{\phi ZN}{60} \times \frac{P}{A} \text{ Volts} \quad \dots (4.6)$$

where the symbols  $f, P, A, Z$  and  $N$  have the same meaning as in chapter 4.1.3. Figure 4.20 shows the equivalent circuit of a motor. Here, the armature circuit is equivalent to a source of emf  $E_b$ , in series with a resistance of  $R_a$  and then a DC supply is applied across these two. The voltage equation of this DC motor is

$$V = E_b + I_a R_a \text{ Volts} \quad \dots (4.7)$$

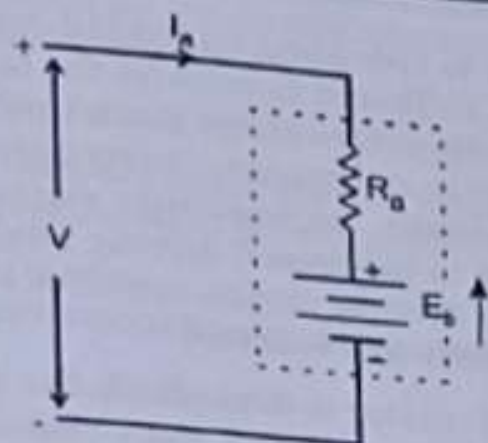


Figure 4.20

From this equation, armature current

$$I_a = \frac{V - E_b}{R_a} \text{ Amps} \quad \dots (4.8)$$

where

$V$  - applied voltage

$E_b$  - back emf

$I_a$  - armature current

$R_a$  - armature resistance

$V - E_b$  - net voltage in the armature circuit.

From equations (4.6) and (4.7), the induced emf in the armature of a motor  $E_b$  depends upon (i) armature speed, (ii) armature current.  $I_a$  depends upon the back emf  $E_b$  for a constant input voltage  $V$ , and armature resistance  $R_a$ .

If the motor speed is high, back emf  $E_b$  is large and therefore armature current is small. If the motor speed is low, then back emf  $E_b$  will be less and armature current is more.

### 4.2.3 Importance of Back EMF

The DC motor is a self regulating machine because, the development of back emf makes the DC motor to draw as much armature current which is just sufficient to develop the required load torque.

Armature current  $I_a = \frac{V - E_b}{R_a}$

1. When the DC motor is operating on no load condition, small torque is required to overcome the friction and windage losses. Therefore the back emf is nearly equal to input voltage and armature current is small i.e.,  $I_a$  is very low.

$$\therefore E_b \cong V$$



- When the DC motor is operating on loaded condition, driving torque of the DC motor is not sufficient to counter the increased retarding torque due to load. Hence, armature slows down (motor speed decreases) and motor back emf  $E_b$  also decreases. Corresponding armature current  $I_a$  increases. The increase in armature current results in increased driving torque. Due to increased driving torque, the motor continues to slow down till the driving torque matches the load torque and then steady state conditions are reached.
- When the load on DC motor is decreased, the driving torque developed is momentarily in excess of the load requirement so that, motor speed is accelerated (motor speed increases). As the motor speed increases, the back emf  $E_b$  also increases, causing armature current to decrease. The decrease in armature current causes decrease in driving torque and steady state conditions are obtained when the driving torque is equal to the load torque.

From the above three important points, the back emf  $E_b$  of a DC motor regulates the armature current and it makes a motor as self regulating.

#### 4.2.4 Voltage Equation of DC Motor

From the figure 4.21,

- $V$  - input voltage,
- $E_b$  - back emf,
- $R_a$  - armature resistance
- $I_a$  - armature current,
- $I_{sh}$  - shunt field current,
- $R_{sh}$  - shunt field resistance

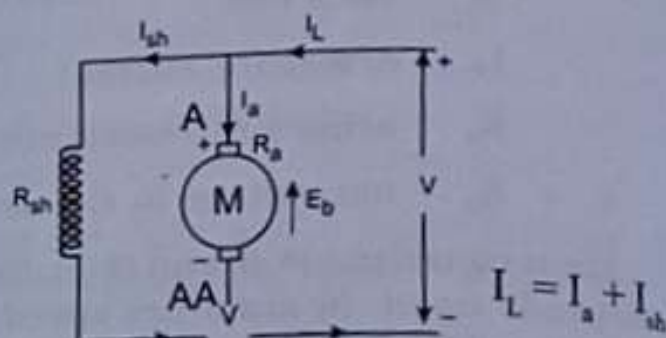


Figure 4.21

Here, the current flowing in the armature is given by,

$$I_a = \frac{V - E_b}{R_a}$$

$$\text{or } \boxed{V = E_b + I_a R_a} \quad \dots (4.9)$$

This equation is known as voltage equation of a DC motor.

#### 4.2.5 Power Relationship of DC motor

The voltage equation is  $V = E_b + I_a R_a$

Multiplying each term of the voltage equation by  $I_a$ , we get

$$VI_a = E_b I_a + I_a^2 R_a \quad \dots (4.10)$$

This equation is known as power equation of a DC motor

$VI_a$  - electric power supplied to armature (input to the armature)

$E_b I_a$  - power developed by the motor armature (output of the armature ie mechanical output)

$I_a^2 R_a$  - power loss in the armature (armature copper loss)

The armature copper loss is a small portion (about 5%) of power input to the armature. The remaining portion  $E_b I_a$  is converted into mechanical power within the armature.

$P_m$  is nothing but mechanical power developed by the motor. It is given by

$$\begin{aligned} P_m &= E_b I_a \\ &= VI_a - I_a^2 R_a \end{aligned}$$

Here,  $V$  and  $R_a$  are fixed values. So the power developed by the motor depends upon the motor armature current  $I_a$ . Differentiating both sides with respect to armature current  $I_a$  we have  $\frac{dP_m}{dI_a} = V - 2I_a R_a$

For maximum mechanical power,  $\frac{dP_m}{dI_a}$  is zero.

$$\text{or } V - 2I_a R_a = 0$$

$$\therefore I_a R_a = \frac{V}{2}$$

$$\therefore V = E_b + I_a R_a$$

$$\text{becomes } V = E_b + \frac{V}{2}$$

or

$$\boxed{E_b = \frac{V}{2}}$$

.. (4.11)

$\therefore$  The power developed in armature is maximum when the back emf is equal to half of the input voltage.

### Disadvantages

This is not attained in practice, because

- Under this condition  $\left(E_b = \frac{V}{2}\right)$ , the motor armature current  $I_a$  is very large. This armature current is more than that of rated current of the motor.
- Half of the input power is wasted in the armature. So taking other losses (iron and mechanical) into account, the efficiency will be less than 50%.



**EXAMPLE 22**

A DC motor connected to a 460V supply has an armature resistance of  $0.15\Omega$ . calculate (a) the value of back emf when the armature current is 120A. (b) the value of armature current when the back emf is 447V.

Given data :

$$V = 460V;$$

$$R_a = 0.15\Omega;$$

$$I_a = 120A$$

To find :

$$E_b = ?$$

Formula used :

$$V = E_b + I_a R_a$$

**Solution:**

a) The value of back emf when the armature current is 120A.

$$E_b = V - I_a R_a = 460 - (120 \times 0.15)$$

$$E_b = 442V$$

b) The value of armature current when the back emf is 447 V.

$$I_a R_a = V - E_b$$

$$I_a = \frac{V - E_b}{R_a} = \frac{460 - 447}{0.15} = \frac{13}{0.15}$$

$$\therefore \text{Armature current} = 86.67A$$

**EXAMPLE 23**

A DC motor connected to a 460V supply takes an armature current of 120A on full load. If the armature circuit has a resistance of  $0.25\Omega$ , calculate the value of the back emf at this load.

Given data :

$$V = 460V;$$

$$I_a = 120A;$$

$$R_a = 0.25\Omega$$

To find :

$$E_b = ?$$

Formula used :

$$V = E_b + I_a R_a$$

**Solution:**

$$E_b = V - I_a R_a$$

$$= 460 - (120 \times 0.25) = 460 - 30$$

$$\therefore \text{Back emf} = 430V$$

**EXAMPLE 24**

A four pole DC motor takes an armature current of 150A at 440V. If its armature circuit has a resistance of  $0.15\Omega$ , what will be the back emf at this load?

Given data :

$$P = 4$$

$$R_a = 0.15\Omega$$

$$I_a = 150A$$

$$V = 440V$$

To find :

$$E_b = ?$$

Formula used :

$$V = E_b + I_a R_a$$

Solution:

$$E_b = V - I_a R_a = 440 - (150 \times 0.15) = 440 - 22.5$$

$$\therefore \text{Back emf} = 417.5V$$

**EXAMPLE 25**

A 250 V dc shunt motor takes 41A at full load. Resistances of motor armature and shunt field windings are  $0.1\Omega$  and  $250\Omega$  respectively. Find the back emf on full load.

(AU/CSE - May 2006)

Given data:

$$V = 250V$$

$$I_L = 41A$$

$$R_a = 0.1\Omega$$

$$R_{sh} = 250\Omega$$

To find:

Back emf on full load ( $E_b$ )

Solution:

$$\text{Shunt field current } I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1A$$

$$\begin{aligned} \text{Armature current } I_a &= I_L - I_{sh} \\ &= 41 - 1 = 40A \end{aligned}$$

$$\begin{aligned} \text{Back emf } E_b &= V - I_a R_a \\ &= 250 - 40 \times 0.1 = 246V \end{aligned}$$

$$E_b = 246V$$



### 4.2.6 Types of DC Motors

The classification of DC motors is similar to that of the DC generators. The classification is based on the connections of field winding in relation to the armature. The types of DC motors are

- i) Separately excited DC motor
- ii) Self excited DC motor
  - Series motor
  - Shunt motor
  - Compound motor
    - a) Long shunt compound motor
    - b) Short shunt compound motor

#### Separately excited DC motor

Figure 4.22 shows connection diagram of a separately excited DC motor.

Here, the field winding and armature are separated. The field winding is excited by a separate DC source. That is why it is called separately excited DC motor.

From this diagram,

Armature current  $I_a$  = line current  $I_L$

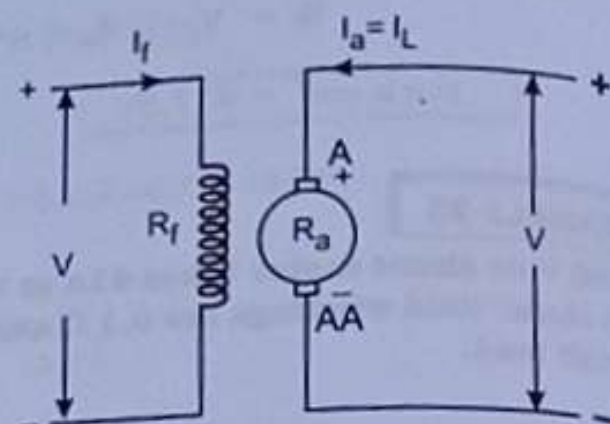


Figure 4.22

Input voltage  $V = E_b + I_a R_a + V_{\text{brush}}$

Back emf  $E_b = V - I_a R_a - V_{\text{brush}}$

$V_{\text{brush}}$  is very small and therefore it is neglected.

#### DC series motor

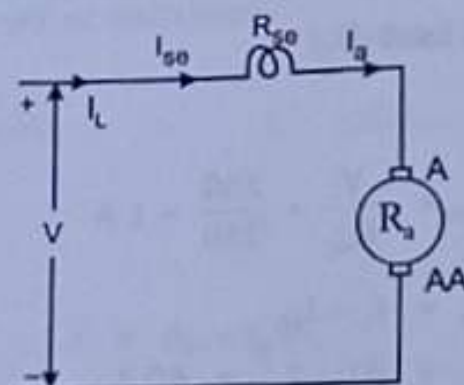


Figure 4.23

DC series motor means, the field winding is connected in series with armature. Figure 4.23 shows connection diagram of a DC series motor.

The field winding should have less number of turns of thick wire.  $R_{sc}$  is the resistance of the series field winding. Normally  $R_{sc}$  value is very small.

In a DC series motor,

$I_L$  = line current drawn from the supply

Armature current  $I_a$  = series field current  $I_{sc} = I_L$

$$I_a = I_{sc} = I_L$$

The voltage equation is given by

$$V = E_b + I_a R_a + I_{sc} R_{sc} + V_{brush}$$

where,  $I_a = I_{sc}$

$$\therefore V = E_b + I_a (R_a + R_{sc}) + V_{brush}$$

$V_{brush}$  = Voltage drop in the brush. Normally it is neglected.

and hence  $V = E_b + I_a (R_a + R_{sc})$

In a DC series motor, full armature current flows through the series field winding. Therefore, flux produced is directly proportional to the armature current i.e.,

$$\phi \propto I_{sc} \propto I_a$$

### DC shunt motor

In a DC shunt motor, the field winding is connected across the armature. Figure 4.24 shows the connection diagram of a DC shunt motor.

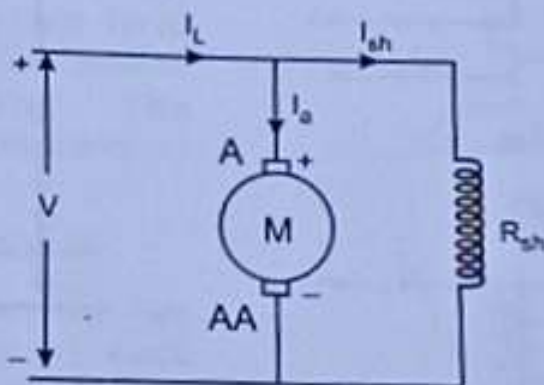


Figure 4.24

Here, the shunt field winding has more number of turns and less cross-sectional area.  $R_{sh}$  is the shunt field winding resistance.  $R_a$  is the armature resistance. The value of  $R_a$  is very small and  $R_{sh}$  is quite large. The input voltage  $V$  is equal to the voltage across the armature and field winding.

$I_L$  is the line current drawn from the supply. The line current is divided into two paths, one through the field winding and second through the armature i.e.,

$$I_L = I_a + I_{sh}$$



where,  $I_a$  = armature current ;  
 $I_{sh}$  = shunt field current

$$I_{sh} = \frac{V}{R_{sh}}$$

Voltage equation of a DC shunt motor is given by

$$V = E_b + I_a R_a + V_{brush}$$

In shunt motor, flux produced by field winding is proportional to the field current  $I_{sh}$ .

i.e.,  $\phi \propto I_{sh}$

Here, the input voltage is constant and so the flux is also constant. Therefore, DC shunt motor is also called a constant flux motor or constant speed motor.

### DC compound motor

The DC compound motor consists of both series and shunt field windings.

#### a) Long shunt compound motor

In this motor, the shunt field winding is connected across both armature and series field winding. Figure 4.25 shows connection diagram of a long shunt compound motor.

From this diagram,

$$I_L = I_{se} + I_{sh}$$

$$I_{se} = I_a$$

$$\therefore I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

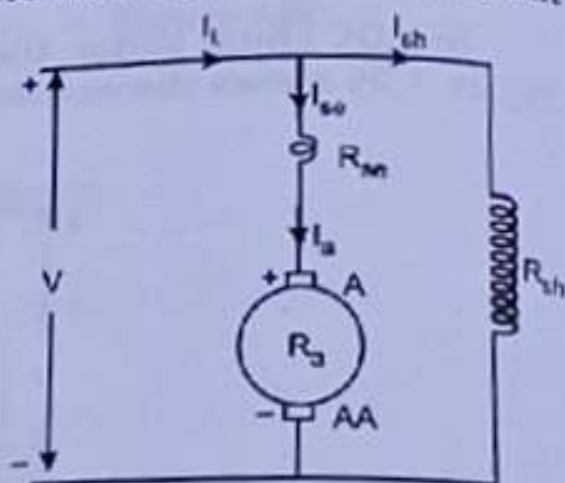


Figure 4.25

The voltage equation of this motor is given by

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

where  $I_a = I_{se}$ ,  $\therefore V = E_b + I_a (R_a + R_{se}) + V_{brush}$

#### b) Short shunt compound motor

In this type of motor, the shunt field winding is across the armature and series field winding is connected in series with this combination. Figure 4.26 shows connection diagram of a short shunt compound motor.

$$I_L = I_{sc}, \quad I_L = I_a + I_{sh}$$

$$\therefore I_L = I_{sc} = I_a + I_{sh}$$

The voltage across the shunt field winding can be found out from the voltage equation.

$$V = E_b + I_a R_a + I_{sc} R_{sc} + V_{brush}$$

$$I_{sc} = I_L$$

$$V = E_b + I_a R_a + I_L R_{sc} + V_{brush}$$

Voltage drop across the shunt field winding is  $= V - I_L R_{sc}$

$$V_{sh} = E_b + I_a R_a + V_{brush}$$

$$\therefore I_{sh} = \frac{V - I_L R_{sc}}{R_{sh}}$$

The compound motors again can be classified into two types

i) Cumulative compound motor

ii) Differential compound motor

### Cumulative compound motor

In this type of motor, the two field winding fluxes aid each other i.e., flux due to the series field winding strengthens the flux due to the shunt field winding. The winding connection diagram is shown in figure 4.27.

### Differential compound motor

In this type of motor, the two field winding fluxes oppose each other i.e., flux due to series field winding weakens the flux due to shunt field winding. Figure 4.28 shows winding connection diagram of this motor.

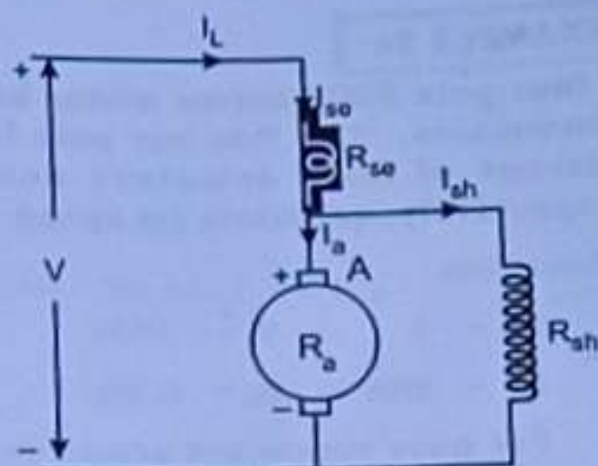


Figure 4.26

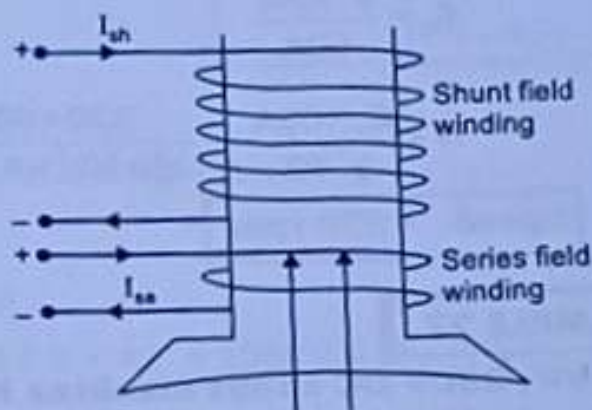


Figure 4.27

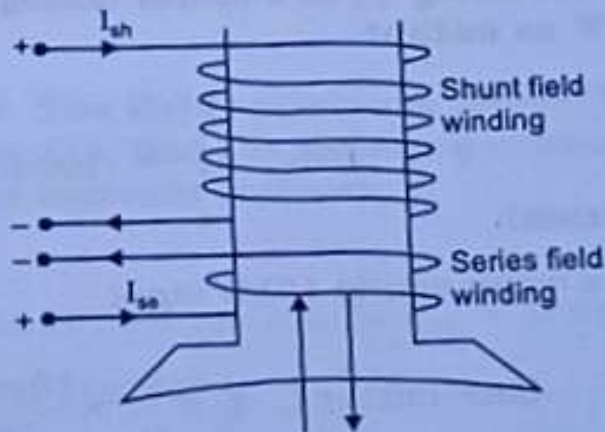


Figure 4.28



**EXAMPLE 32**

A 4 pole, 500 V dc shunt motor has 700 wave connected conductors on its armature. The full load armature current is 60 A and flux per pole is 30 mwb. Calculate the full load speed if the motor armature resistance is  $0.2\Omega$  and the brush drop is 1 volt per brush.

(AU/ECE - June 2007)

Given data:

Number of poles  $P = 4$

Number of conductors  $Z = 700$

Flux per pole  $\phi = 30 \text{ mwb}$

Brush drop = 1 V per brush  
 $= 2 \times 1 = 2 \text{ V}$

Supply voltage  $V = 500 \text{ V}$

Full load armature current  $I_a = 60 \text{ A}$

Armature resistance  $R_a = 0.2 \Omega$

For wave connection  $A = 2$

To find:

Full load speed (N)

Solution:

$$\begin{aligned} \text{Back emf } E_b &= V - I_a R_a - \text{brush drop} \\ &= 500 - 60 \times 0.2 - 2 = 486 \text{ V} \end{aligned}$$

$$E_b = \frac{P\phi ZN}{60A}$$

$$N = \frac{E_b 60A}{P\phi Z} = \frac{486 \times 60 \times 2}{4 \times 30 \times 10^{-3} \times 700}$$

$$N = 694.28 \text{ rpm}$$

#### 4.2.7 Torque Equation

Torque is nothing but turning or twisting force about an axis.

Torque is measured by the product of force and the radius at which the force acts. Consider a wheel of radius 'r' metres, acted on by a circumferential force 'F' Newton as shown in figure 4.29. Let the force 'F' cause the wheel to rotate at 'N' rpm. The angular velocity of the wheel is

$$\omega = \frac{2\pi N}{60} \text{ rad / sec}$$

$$\text{Torque, } T = F \times r \text{ N-m}$$

$$\text{Work done per revolution} = F \times \text{distance moved}$$

$$= F \times 2\pi r \text{ joules}$$

$$\text{Power developed, } P = \frac{\text{work done}}{\text{time}}$$

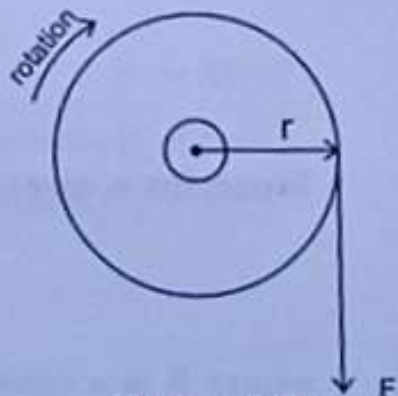


Figure 4.29

$$= \frac{F \times 2\pi r}{\text{time for 1 rev}} = \frac{F \times 2\pi r}{\frac{60}{N}}$$

$$\left( \text{rpm} = N; \text{ rps} = \frac{N}{60}; \text{ time for one 1 rev} = \frac{60}{N} \right)$$

$$P = (F \times r) \frac{2\pi N}{60}$$

$$P = T\omega \text{ watts}$$

where  $T$  = torque in N-m,  $\omega$  = angular speed in rad / sec

The torque developed by a DC motor is obtained by looking at the electrical power supplied to it and mechanical power produced by it. It is also called as armature torque. The gross mechanical power developed in the armature is  $E_b I_a$ . Then

$$\text{Power in armature} = \text{Armature torque} \times \omega$$

$$E_b I_a = T_a \times \frac{2\pi N}{60}; E_b = \frac{\phi PNZ}{60A}$$

$$\frac{\phi PNZ}{60A} I_a = T_a \times \frac{2\pi N}{60}; T_a = \frac{\phi I_a}{2\pi} \frac{PZ}{A}$$

$$T_a = 0.159 \phi I_a \frac{PZ}{A} \text{ N-m}$$

..(4.12)

The above equation is torque equation of a DC motor.

$T$  is proportional to  $\phi I_a$ . Hence the torque of a given DC motor is proportional to the product of the armature current and the flux.

#### 4.2.8 Speed and Torque Equation

For a DC motor, the speed equation is obtained as follows.

We know

$$E_b = V - I_a R_a = \frac{\phi ZN}{60} \cdot \frac{P}{A}$$

$$\text{or } V - I_a R_a = \frac{\phi ZN}{60} \times \frac{P}{A}$$

$$\therefore N = \frac{V - I_a R_a}{\phi Z} \times \frac{60A}{P}$$

Since for a given machine,  $Z$ ,  $A$  and  $P$  are constants

$$N = \frac{K(V - I_a R_a)}{\phi}$$

...(4.13)

where  $K$  is a constant.



∴ Speed equation becomes  $N \propto \frac{V - I_a R_a}{\phi}$

or

$$N \propto \frac{E_b}{\phi}$$

Hence the speed of the motor is directly proportional to back emf  $E_b$  and inversely proportional to flux  $\phi$ . By varying the flux and voltage, the motor speed can be changed.

The torque equation of a DC motor is given by

$$T \propto \phi I_a$$

Here, the flux  $\phi$  is directly proportional to the current flowing through the field winding i.e.,

$$\phi \propto I_f$$

For DC shunt motor, the shunt field current  $I_{sh}$  is constant as long as input voltage is constant. Therefore flux is also constant.

Hence  $T \propto \phi I_a$  becomes  $T \propto I_a$

For DC shunt motor, torque is directly proportional to the armature current. For DC series motor, the series field current is equal to the armature current  $I_a$ . Here, the flux  $\phi$  is proportional to the armature current  $I_a$ .

$$\phi \propto I_a$$

Hence  $T \propto \phi I_a$  becomes  $\therefore T \propto I_a^2$

For DC series motor, the torque is directly proportional to the square of the armature current. The speed and torque equations are mainly used for analyzing the various characteristics of DC motors.

**EXAMPLE 34**

A 4 pole DC motor takes an armature current of 50A. The armature has 480 lap connected conductors. The flux per pole is 20mwb. Calculate the gross torque developed by the motor.

(AU/ECE - May 2004)

Given data:

Number of poles  $P = 4$ ,  
 Number of conductors  $Z = 480$ ,  
 For lap connection  $A = P$

Armature current  $I_a = 50\text{A}$ ,  
 Flux per pole  $\phi = 20\text{ mwb}$ ,

To find:

Gross torque ( $T_a$ )

**Solution:**

$$\text{Gross torque } T_a = 0.159 \phi \frac{I_a P Z}{A}$$

$$T_a = 0.159 \times 20 \times 10^{-3} \times \frac{50 \times 4 \times 480}{4}$$

$$T_a = 76.32\text{N-m}$$

**EXAMPLE 35**

A 250 V, 4 pole wave wound dc series motor has 782 conductors on its armature. It has armature and series field resistance of  $0.75 \Omega$ . The motor takes a current of 40A. Determine its speed and gross torque developed if it has a flux per pole of 25 mwb.

(AU/EEE - May 2006)

Given data:

$$V = 250\text{ V}$$

$$Z = 782$$

$$I_a = I_L = 40\text{ A}$$

$$\text{For wave wound } A = 2$$

$$P = 4$$

$$R_a + R_{se} = 0.75 \Omega$$

$$\phi = 25\text{ mwb}$$

To find:

1. Speed (N)

2. Gross torque ( $T_a$ )



**Solution:**

1. Speed (N)

$$\begin{aligned}\text{Back emf } E_b &= V - I_a (R_a + R_{se}) \\ &= 250 - 40 \times 0.75 = 220 \text{ V}\end{aligned}$$

$$E_b = \frac{P\phi ZN}{60A}$$

$$N = \frac{E_b 60A}{P\phi Z} = \frac{220 \times 60 \times 2}{4 \times 25 \times 10^{-3} \times 782}$$

$$N = 337.6 \text{ rpm}$$

2. Gross torque ( $T_a$ )

$$T_a = 0.159 \frac{\phi I_a PZ}{A} = \frac{0.159 \times 25 \times 10^{-3} \times 40 \times 4 \times 782}{2}$$

$$T_a = 248.7 \text{ N-m}$$

### EXAMPLE 36

A 4-pole dc motor has a wave-wound armature with 594 conductors. The armature current is 40A and flux per pole is 7.5mwb. Calculate the torque developed by the motor.

(AU/Mech - Dec 2007)

*Given data:*

Number of poles  $P = 4$

For wave wound  $A = 2$

Number of conductors  $Z = 594$

Flux per pole  $\phi = 7.5 \text{ mwb}$

Armature current  $I_a = 40 \text{ A}$

*To find:*

Torque developed by the motor

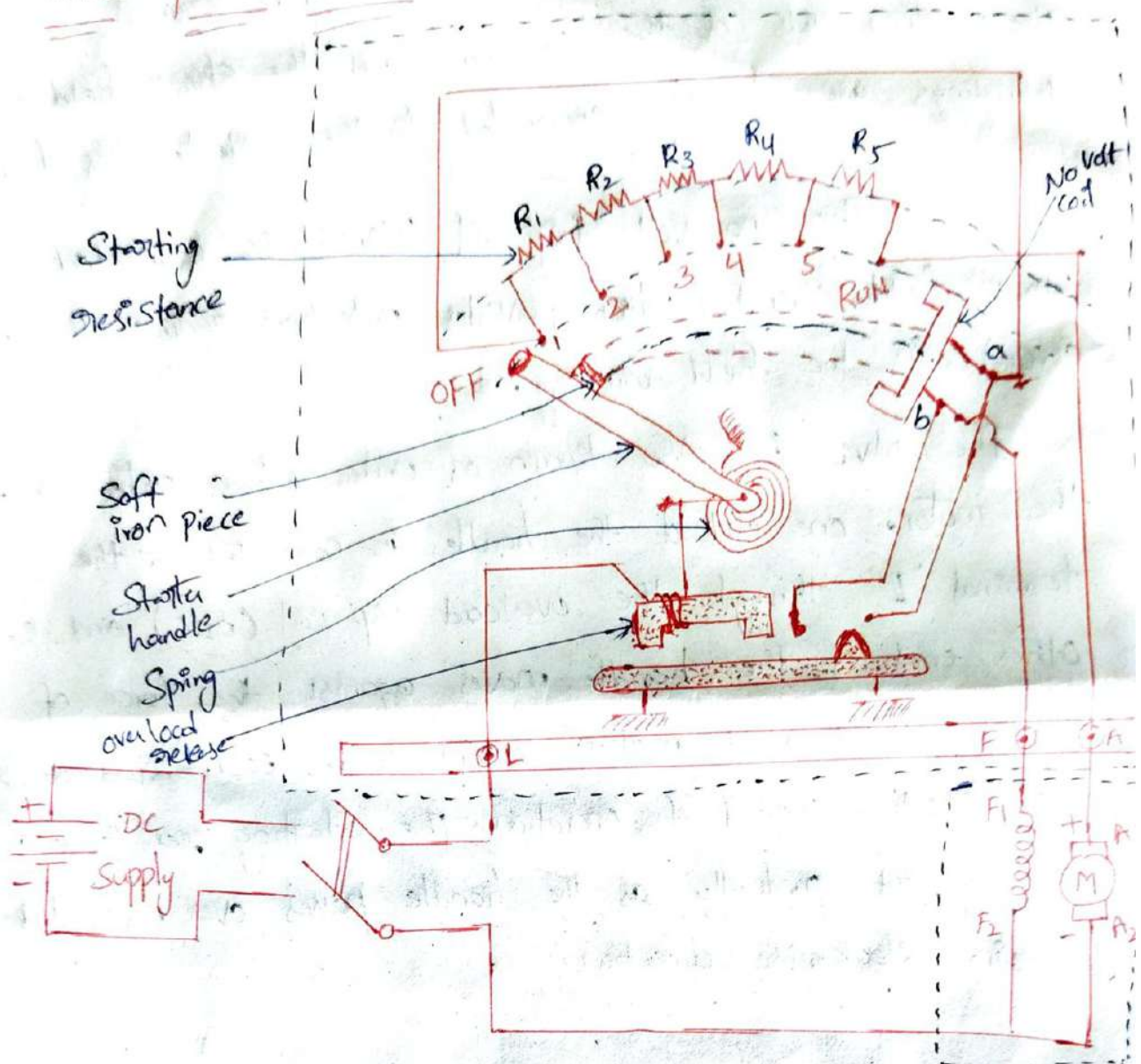
**Solution:**

$$T_a = 0.159 \phi \frac{I_a PZ}{A}$$

$$= 0.159 \times 7.5 \times 10^{-3} \times \frac{40 \times 4 \times 594}{2}$$

$$T_a = 56.66 \text{ N-m}$$

## Three point Starter



The circuit diagram of the three point starter is shown in above figure.

It is called three point starter because it has three terminals L, F & A. It consists of a graded starting resistance to limit the starting current and is connected in series with the armature of the motor. The tapping points of the starting resistance are taken out to a no. of studs.



The three terminals L, 2 & A of the Starter are connected to the positive terminal and the armature terminal respec.

The other ends of the armature and the Shunt field windings are directly connected to the -ve terminals of Supply.

The no volt trip coil (NVC) is connected in shunt field circuit, which provides protection against the open circuit in the field winding.

The NVC is also known as under voltage protection of the motor. one end of the handle is connected to the terminal 'L' through the overload trip coil (OLC) and the other end of the handle moves against the force of control spring and makes contact with each stud during the starting period of operation. The starting resistance is cutting out gradually as the handle passes over each stud in clockwise direction.

### Working principle of three point Starter :-

1. Initially, the DC Supply is Switched on with handle in the off position.
2. The handle is now moved to the 1st stud. When it comes in contact with the 1st stud, the whole starting resistance is inserted in series with the armature winding and the shunt field winding is directly connected across the DC Supply.



3. As the handle is gradually moved over to the final Stud, the starting resistance is cut out from the armature circuit in steps. After reaching to the final Stud the handle is held magnetically by the NVC which is energized by the Shunt field Current.

4. If the Supply Voltage is interrupted or if an open ckt is occurred in the field ckt, the NVC is de energized and the handle goes back to the off position under the pull of the Control Spring.

If the NVC were not used, then in case of failure of Supply, the handle would remain in contact with the final Stud. When the supply is restored, the motor will be directly connected across the full Supply Voltage resulting in an excessive armature current and may damage the motor.

5. If the motor is overloaded or if a Short ckt is occurred, it will draw a large Current from the Supply.

The excessive Current will increase the mmf of the OLC and pull the plunger P, which Short circuits the NVC.

Hence NVC is de energized and the handle is pulled to the OFF position by Control Spring.

Therefore, the motor is automatically isolated from the Supply.



# Drawbacks of Three-Point Starter

The three-point starter suffers from a serious drawback for motors with large variation of speed by the adjustment of the field rheostat. As in the 3-point starter, the NVC is connected in series with the shunt field circuit, thus it carries the shunt field current.

While exercising the speed control through the field rheostat, the shunt field current may reduce to such an extent that the NVC may not be able to hold the handle in the ON position during the normal operation of the motor. This may disconnect the motor from the line, which is not desirable.