2.8 SINGLE PHASE AC CIRCUITS

In chapter 1, we have seen what happens when a DC source is applied to a circuit. The current that flow in such circuits obey Ohm's law and Kirchoff's laws.

Circuit Parameters

In AC circuits apart from resistor, we have two more parameters. They are inductor and capacitor. These are deal with in the subsequent sections.

2.8.1 Resistor

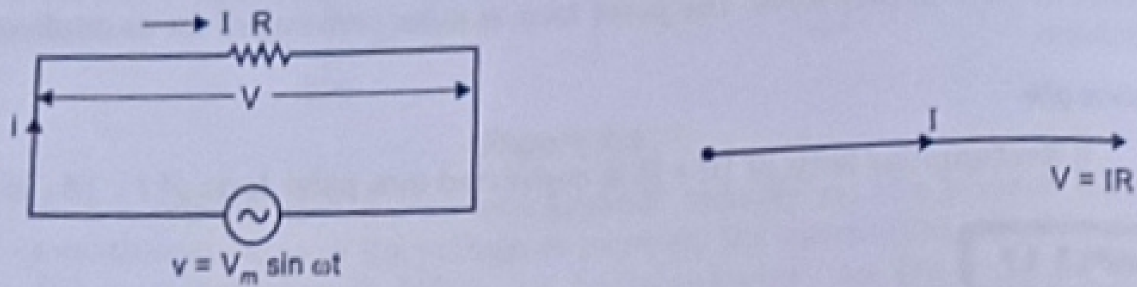


Figure 2.6

We have already learnt about this parameter. When a sinusoidal voltage

$v = V_m \sin\omega t$ ...(11)

is applied to it, the instantaneous value of current flowing through the resistance R is given by

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} \quad ...(12)$$

The value of current will be maximum when $\sin \omega t = 1$ or ($\omega t = 90°$)

$$\therefore\ I_m = \frac{V_m}{R} \quad ...(13)$$

The above equation substituting in equation (12) we get,

$$i = I_m \sin \omega t \quad ...(14)$$

Comparing (11) and (14), we find that alternating voltage and current are in phase with each other as shown in figure 2.7 (a).

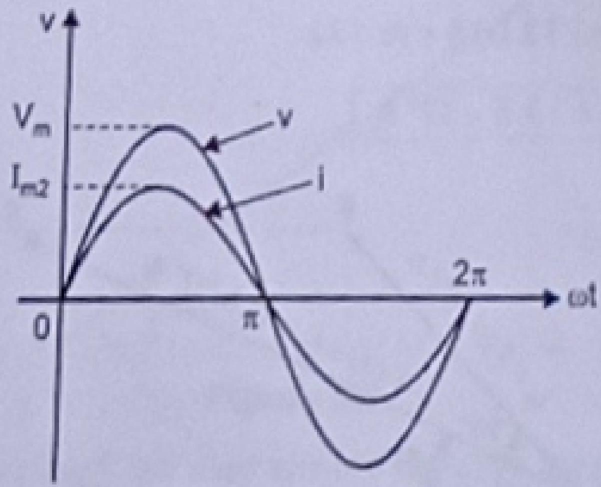


Figure 2.7 (a)

Figure 2.7 (b) shows instantaneous voltage, current and power waveforms.

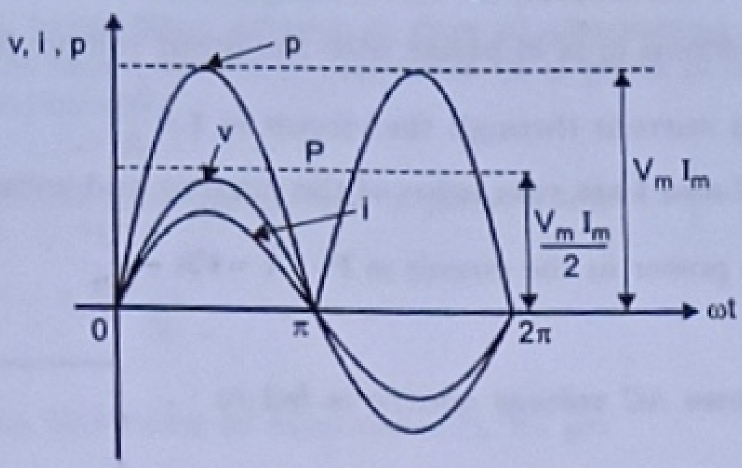


Figure 2.7 (b)

The power dissipated in the resistor is obtained by finding the average of the instantaneous power.

$$p = v \cdot i = V_m \sin \omega t \cdot I_m \sin\omega t$$

$$\text{Average power,} P = \frac{1}{2\pi}\int_0^{2\pi} p \cdot d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} (V_m \sin \omega t \cdot I_m \sin\omega t) \cdot d(\omega t)$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \sin^2 \omega t \cdot d(\omega t) = \frac{V_m I_m}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)$$

$$= \frac{V_m I_m}{2\pi}\left(\omega t - \frac{\sin 2\omega t}{2}\right)_0^{\pi} = \frac{V_m I_m}{2\pi}(\pi - 0 - 0)$$

$$= \frac{V_m \cdot I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = VI$$

$$P = VI \qquad \text{... (15)}$$

$$P = I^2R = \frac{V^2}{R}$$

where

V = Rms value of input voltage

I = Rms value of the current

From figure 2.7(b), we can observe that no part of the power cycle at any time becomes negative. In order words the power in a purely resistive circuit never becomes zero.

From the above discussion, we can conclude that

1. Input current (i) is in phase with the input voltage (v)
2. The rms current through the circuit is $I = \frac{V}{R}$

   where V and I are rms value of the current and voltage.
3. Average power in the circuit is $P = VI = I^2R = \frac{V^2}{R}$

2.8.2 Inductor

Figure 2.8 shows AC voltage source is fed to the pure inductor.

AC input voltage is fed to the circuit. The input voltage is given by

$$v = V_m \sin \omega t \qquad .....(16)$$

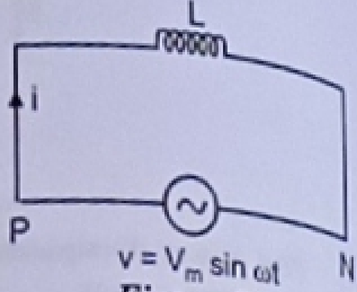


Figure 2.8

Whenever an alternative voltage is applied to a purely inductive coil, varying flux is produced. Because of this change in flux, a voltage is induced in the inductor proportional to the rate of change of current.

$$\text{i.e., emf induced} \propto \frac{di}{dt}$$

$$v \propto \frac{di}{dt};$$

$$v = L\frac{di}{dt} \qquad ...(17)$$

Where L, the constant of proportionality, has come to be called self inductance of the circuit. The self inductance (or simply inductance) is the property of a coil by which it opposes any change of current. It is well known that the unit of inductance is Henry.

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t \,.\, dt$$

Integrating both sides

$$\int di = \int \frac{V_m}{L} \sin \omega t \; dt$$

$$i = \frac{V_m}{L}\left(-\frac{\cos \omega t}{\omega}\right) = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(18)$$

where $X_L = \omega L$ (opposition offered to flow of alternating current by a pure inductance) and is called inductive reactance. It is given in ohm if L is in Henry and ω is in radian/second.

The maximum value of current is given when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$.

$$\therefore I_m = \frac{V_m}{X_L} \qquad \dots (19)$$

Substituting this value in equation (18), we get

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \dots (20)$$

Figure (2.9) (a) shows voltage and current waveforms for purely inductive load fed by AC source. Here, the current is said to lag by 90° with respect to voltage.

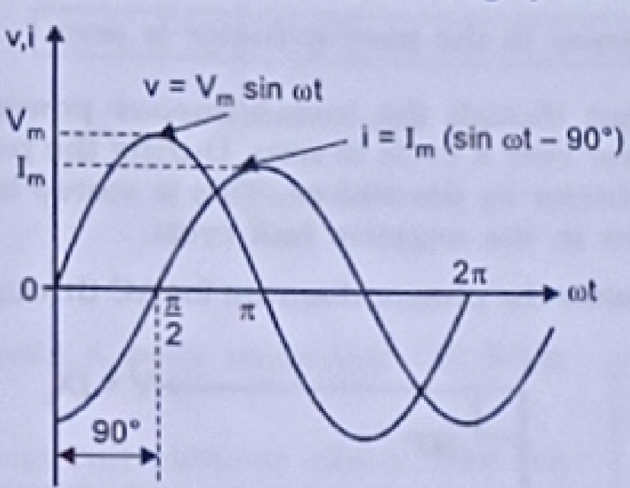


Figure 2.9 (a)

Average Power

Figure 2.9 (b) shows the instantaneous voltage, current and power waveforms.

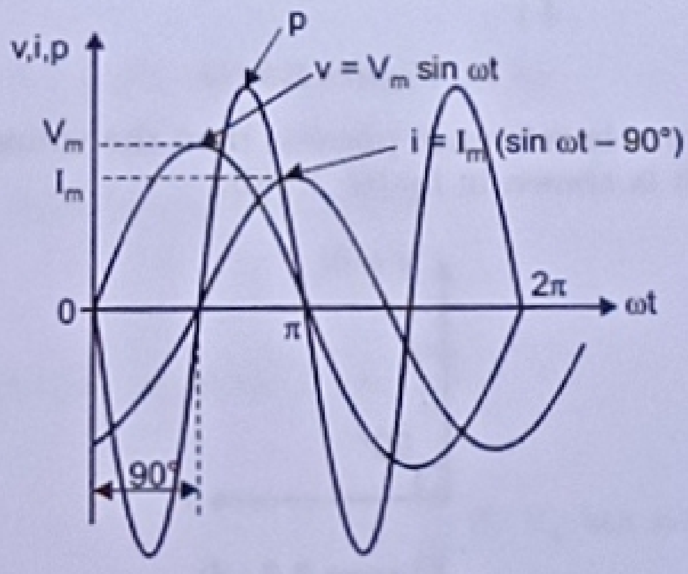


Figure 2.9 (b)

Instantaneous power $p = v.i$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} 2 \sin \omega t \cos \omega t$$

$$= -\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$\therefore$ Power for the whole cycle,

$$P = \frac{-V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \int_0^{2\pi} \sin 2\omega t = 0$$

$$\boxed{P_{av} = 0} \quad \cdots (21)$$

$\boxed{\text{Average power in the pure inductor is zero}}$

This implies that though the instantaneous power in an inductor is not zero, the average power over a cycle is zero. During the positive half cycle, energy is delivered to the inductor by the source. This is stored in the magnetic field and returned to the source in the negative half cycle.

Figure 2.9 (c) shows the phasor diagram for AC through pure inductor circuit.

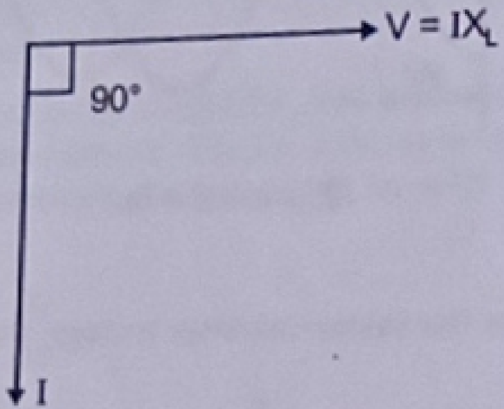


Figure 2.9 (c)

Suppose, current is reference phasor, then the voltage leads by 90° to with respect to current. It is shown in figure 2.9(d).

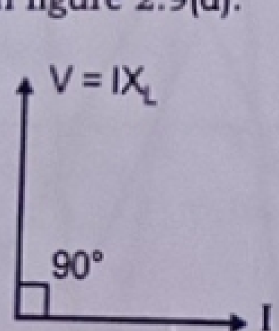


Figure 2.9 (d)

Important points in a pure inductance circuit is shown AC supply.

1. Rms current $I = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi fL}$
2. Input current I always lags behind the input voltage by 90°
3. Average power consumed by pure inductor is zero.

Variation of X_L and f:

Inductance reactance $X_L = \omega L = 2\pi fL$

Here if L is constant, then $X_L \alpha f$

By increasing the frequency, the inductive reactance also linearly increases. It is shown in figure 2.9(e).

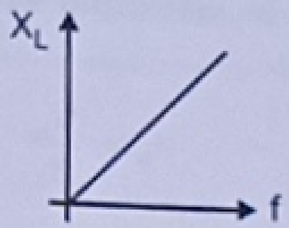


Figure 2.9 (e)

2.8.3 Capacitor

Figure 2.10 shows a pure capacitor fed from AC source.

A capacitor is a circuit element which, like the inductor, stores energy during periods of time and returns the energy during others. In the capacitor, storage takes place in an electric field unlike the inductor where storage takes place in a magnetic field.

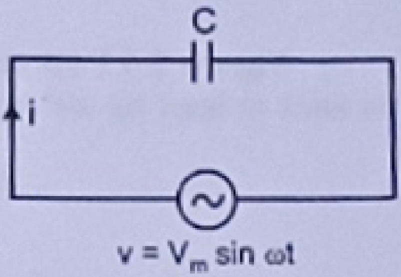


Figure 2.10

A capacitor is formed by two parallel plates separated by an insulating medium.

The AC voltage fed to the capacitor is given by

$$v = V_m \sin \omega t \qquad \text{...(22)}$$

Charge on the capacitor at any instant,

$$q = Cv \qquad \text{...(23)}$$

Current through the capacitor $i = \frac{dq}{dt}$

$$= \frac{d}{dt}(C\,V_m \sin \omega t) = \omega C\,V_m \cos \omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \cdots (24)$$

$$i = \frac{V_m}{X_c} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \cdots (25)$$

where

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \cdots (26)$$

X_C stands for capacitive reactance. It opposes the flow of alternating current by a pure capacitor. It is given in ohms if C is in Farad and ω in radian/second.

The value of current will be maximum when $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$

$$\therefore \quad I_m = \frac{V_m}{X_c} \quad \cdots (27)$$

Substituting this value in equation we get

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \cdots (28)$$

Figure 2.11 (a) shows the voltage and current waveform. Here, the current is said to lead by 90° with respect to voltage.

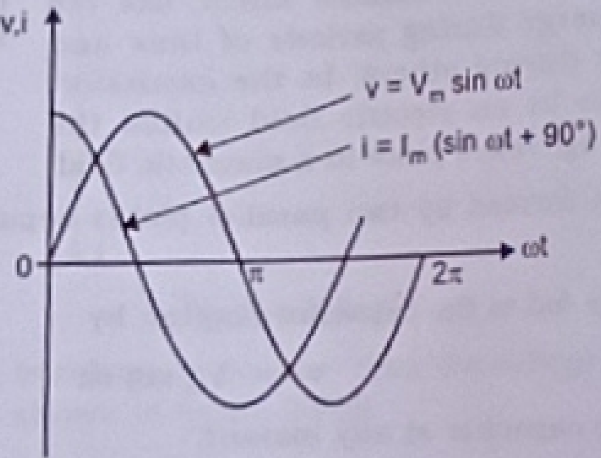


Figure 2.11 (a)

Figure 2.11 (b) shows the phasor diagram for AC through pure capacitor circuit.

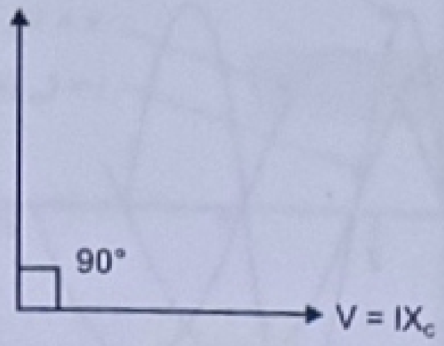


Figure 2.11 (b)

Suppose, current is reference phasor, then the voltage lags behind the current by 90°. It is shown in figure 2.11 (c).

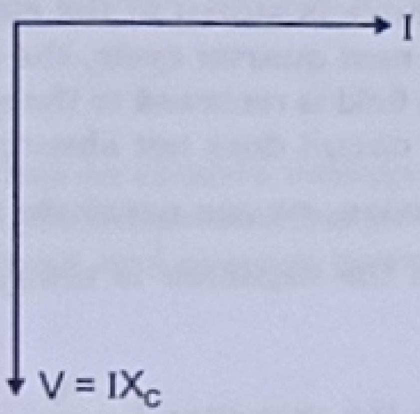


Figure 2.11 (c)

Average Power

Instantaneous power $p = v.i$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t \ \cos \omega t$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t$$

$\therefore$ Power for the whole cycle,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \int_0^{2\pi} \sin \ 2\omega t$$

$$\boxed{P_{av} = 0} \qquad \text{...(21)}$$

The power waveform shown in figure 2.11 (d).

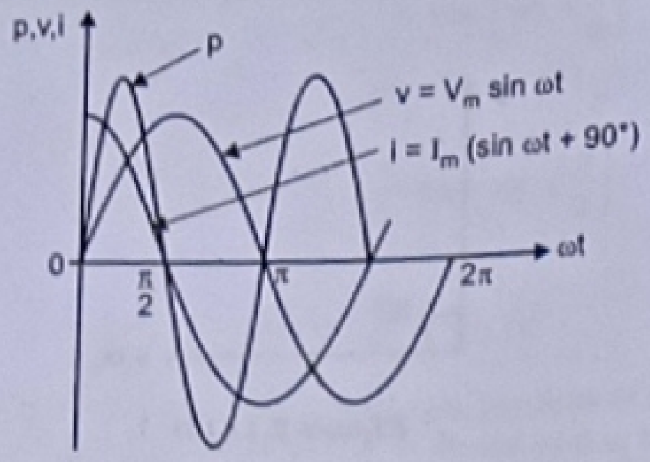


Figure 2.11 (d)

From this waveform, during the first quarter cycle, what so ever power or energy is supplied by the source is stored in the electric field set-up between the capacitor plates. During the next quarter cycle, the electric field collapses and the power or energy stored in the field is returned to the source. The process is repeated in each alternation and this circuit does not absorb any power.

From the above discussion, we can conclude that

1. Current through the capacitor is always lead by 90° with respect to supply voltage.
2. Current through the capacitor $I = \frac{V}{X_C}$
3. Average power consumed is zero.

Variation of X_C and f.

Capacitance reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Now , C kept constant then $X_c \; \alpha \; \frac{1}{f}$

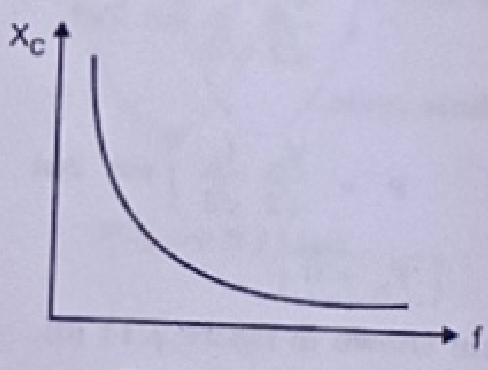


Figure 2.11 (e)

Figure 2.11 (e) shows the variation of X_C and f. As the frequency increases, X_C decreases, so the current increases.

2.8.4 Use of Complex operator "j"

The use of operator "j" helps greatly in solving problems in AC circuits. Recall that any quantity multiplied by "j" means that the quantity is rotated through 90° in the counter clockwise direction. Thus $j = 1\angle 90°$.

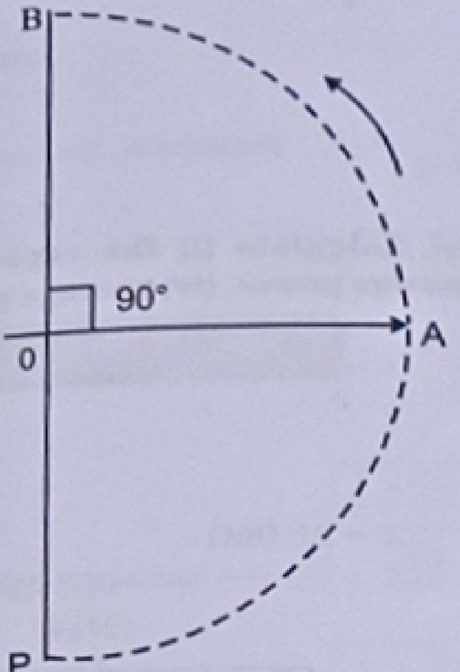


Figure 2.12

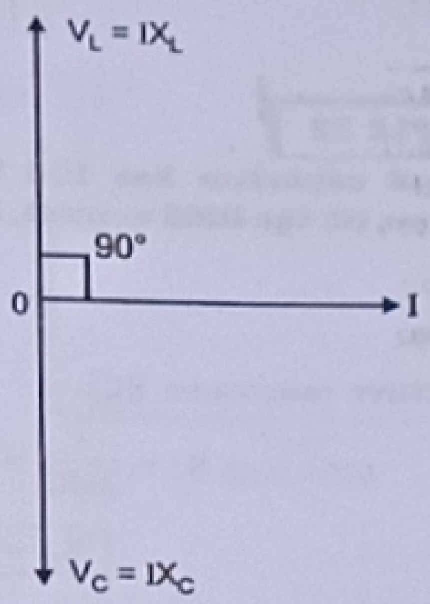


Figure 2.13

Referring to figure 2.12

$$OB = j(OA) \quad \text{and} \quad OP = -j(OA)$$

In figure 2.13, I is the current applied to an inductor of reactance X_L Ohms. Voltage across the inductance $V_L = IX_L$.

If we now write V_l as

$$V_L = I(jX_L)$$

We immediately know that V_L is not along the horizontal but 90° ahead (vertically up). similarly when I is applied to capacitor, we write $V_c = I(-jX_L)$.

This is because in a capacitor, the current leads (voltage lags) the voltage. Hence we shall introduce the operator + j and X_L and − j for X_C.

EXAMPLE 23

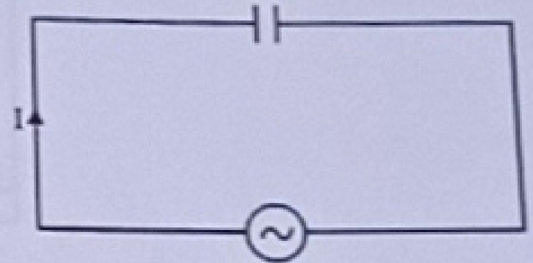
A 318 μF capacitor is connected across a 230 V, 50 Hz system. Determine 1. the capacitive reactance; 2. rms value of current; 3. equations for voltage and current.

(AU/ECE - June 2005)

Solution:

1. Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}}$

$$\boxed{X_C = 10\Omega}$$



2. RMS current $I = \frac{V}{X_C} = \frac{230}{10} = 23\text{ A}$

$$\boxed{I = 23\text{ A}}$$

3. Voltage equation $v = \sqrt{2} \times 230 \sin(2\pi \times 50)t$

$$\boxed{v = 325.2 \sin 314t}$$

Current equation $i = I_m \sin(\omega t + \pi/2)$

$$I_m = \sqrt{2}I = \sqrt{2} \times 23 = 32.5\text{ A}$$

$$\boxed{i = 32.5 \sin(314t + \pi/2)}$$

2.9 RL SERIES CIRCUIT

Figure 2.14 shows RL series circuit. Here, resistor and inductor are connected in series.

where R = Resistance in Ω

L = Inductance in Henry

I = Current flowing in RL series circuit

V = Input voltage

V_R = voltage drop across R = IR

V_L = Voltage drop across L = IX_L

X_L = Inductive reactance in Ω = $2\pi fL$

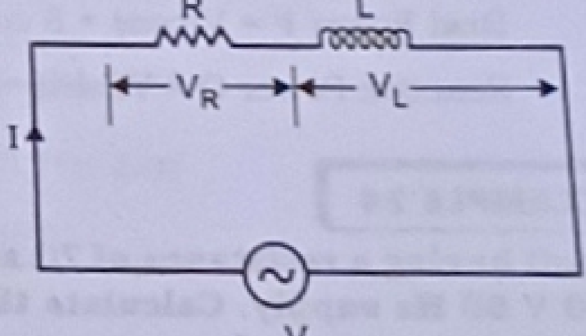


Figure 2.14

First we consider, resistor only. In a resistive circuit, V_R and I are inphase. This is shown in figure 2.15.

Figure 2.15

Secondly we consider, inductor only. Here, current I is said to be lagging by 90° with respect to V_L. It is shown in figure 2.16.

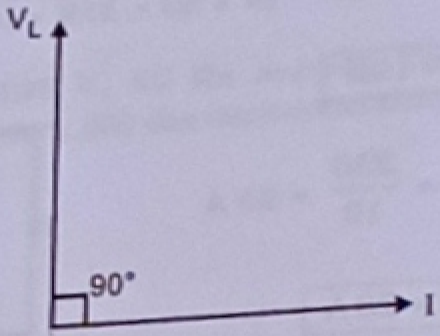


Figure 2.16

Combining these two figure (2.15) and (2.16), we can get phasor diagram for RL series circuit. It is shown in figure 2.17.

Impendance $Z = R + jX_L$

$= \sqrt{R^2 + X_L^2}$

$Z = \frac{V}{I}$

Phase angle $\phi = \tan^{-1}\frac{X_L}{R}$

Power factor $= \cos\phi$ (lag) $= \frac{R}{Z}$

Apparent power $S = VI$

Real Power $P = VI\cos\phi = S\cos\phi$

Reactive Power $Q = VI\sin\phi = S\sin\phi$

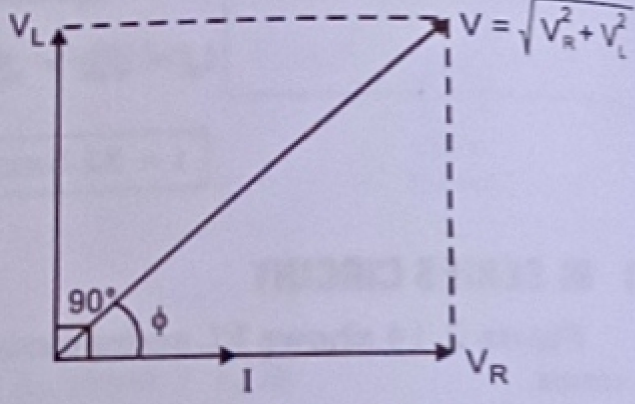


Figure 2.17

2.10 RC SERIES CIRCUIT

Figure 2.18 shows RC series circuit. Here, resistor and capacitor are connected in series.

where

R = Resistance in Ω

C = Capacitance in F

I = Current flowing through the RC circuit

V = Input voltage

V_R = Voltage drop across resistance = IR

V_C = Voltage drop across capacitance = IX_C

$X_C = \dfrac{1}{2\pi fC}$ = Capacitive reactance

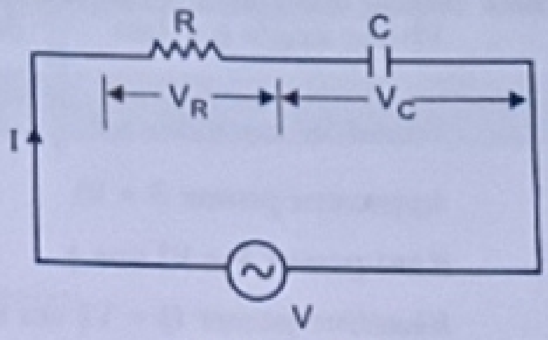


Figure 2.18

First, we consider resistor only. In a resistor, V_R and I are in phase. It is shown in figure 2.19.

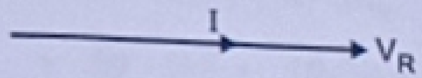


Figure 2.19

Secondly, we consider capacitor only. Here, current I is said to be leading by 90° with respect to V_C. It is shown in figure 2.20.

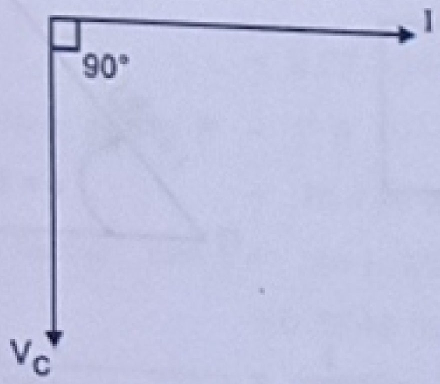


Figure 2.20

Combining these two figures (2.19) and (2.20), we can get phasor diagram for RC series circuit. It is shown in figure 2.21.

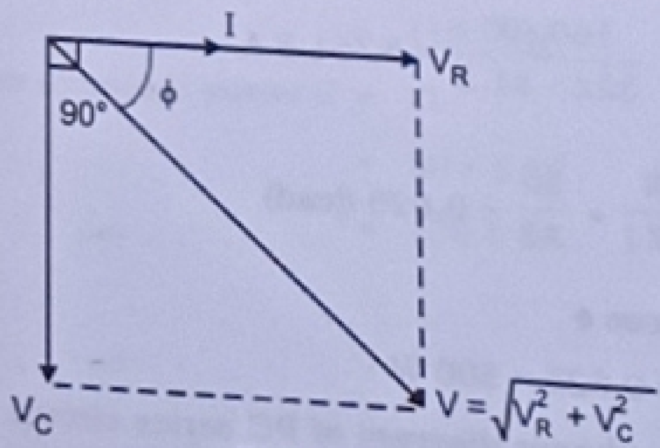


Figure 2.21

In this RC circuit,

Impedance $Z = R + (-jX_c) = \sqrt{R^2 + X_c^2}$

Phase angle $\phi = \tan^{-1}\frac{X_c}{R}$ (lead)

Power factor $= \cos\phi = \frac{R}{Z}$

Apparent power $S = VI$

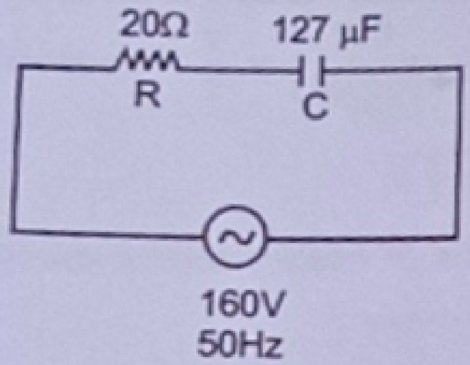
Real power $P = VI\cos\phi$

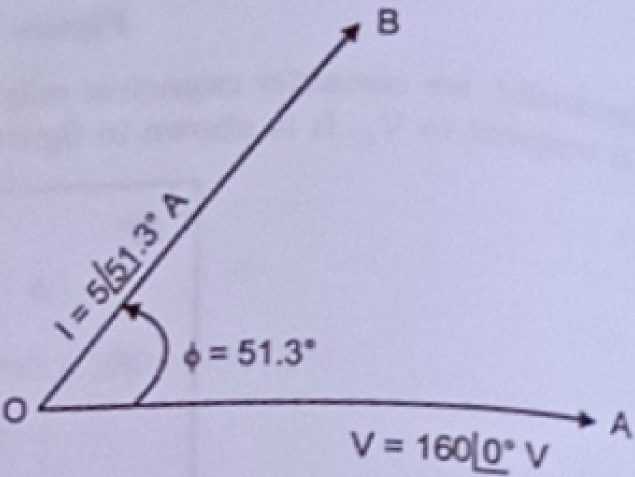
Reactive power $Q = VI\sin\phi$

EXAMPLE 27

A series R - C circuit with R = 20 Ω and C = 127 μF has 160 V, 50 Hz supply connected to it. Find (a) the impedance (b) current (c) power factor (d) power. Draw the phasor diagram.

Solution:





Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 127 \times 10^{-6}} = 25\Omega$

Impedance $Z = R - jX_C = 20 - j25 = 32\angle -51.3°\Omega$

Current $I = \frac{V}{Z} = \frac{160\angle 0°}{32\angle -51.3} = 5\angle 51.3$ A

Power factor $= \frac{R}{|Z|} = \frac{20}{32} = 0.625$ (lead)

Power $= |V|\ |I| \cos\phi$

$= 160 \times 5 \times 0.625 = 500$ W

Figure shows the phasor diagram of RC series circuit.

2.11 RLC SERIES CIRCUIT

This section deals with RLC series circuit.

Consider the circuit in figure 2.22. Let the current be $i = I_m \sin \omega t$ or $I\angle 0$. Then we know

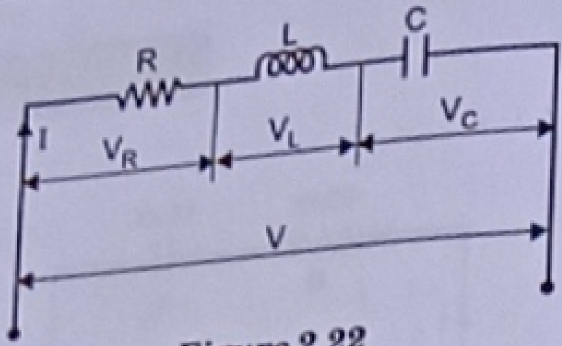


Figure 2.22

V_R, the rms voltage across $R = IR\angle 0°$

V_L, the rms voltage across $L = IX_L\angle 90° = jIX_L$

V_C, the rms voltage across $C = IX_C\angle -90° = -jIX_C$

The total rms voltage (rms) $V = V_R + V_L + V_C$

This summation is not arithmetic, but phasorial

Hence $V = IR + jIX_L + (-jIX_C)$

$= I(R + j(X_L - X_C))$

When this is compared with the basic Ohm's law equation in chapter 1, it is at once clear that the quantity $[R + j(X_L - X_C)]$ must be something similar to resistance. This is called the impedance of the circuit and is given the symbol Z.

$$Z = R + j(X_L - X_C)\ \Omega \quad \cdots (31)$$

Converting to the polar form, Z can be written in magnitude and angle form.

$$Z = |Z| \angle\phi \quad \cdots (32)$$

Where $$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \quad \cdots (33)$$

and $$\phi = \tan^{-1}\frac{(X_L - X_C)}{R} \quad \cdots (34)$$

It is clear that, if I is given, then

$$V = IZ \quad \cdots (35)$$

and if V were given

$$I = \frac{V}{Z} \quad \cdots (36)$$

This is illustrated in the phasor diagram of figure 2.23, where we have assumed $X_L > X_C$. If,on the other hand, $X_C > X_L$ then ϕ will be negative. The operations represented by equations (35) and (36) should be done with due regard to the magnitudes and phase angles of the quantities concerned. If one or more of the parameters (R, L and C) are absent, equations (33) and (34) get modified suitably. The power (average power) in such a circuit is given by the power in the resistor alone as the average power is zero in either L or C.

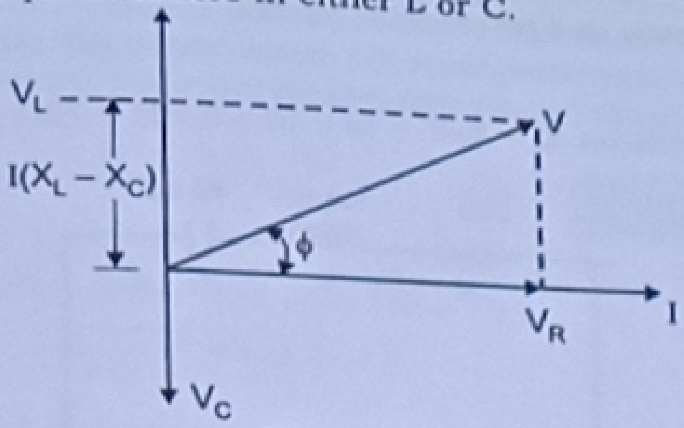


Figure 2.23

Hence power in the circuit $= |I|^2R$

$$= \frac{|V|^2}{|Z|^2}R = |V|\frac{|V|}{|Z|}\frac{R}{|Z|}$$

$$= |V|\,|I|\cos\phi \qquad \text{...(37)}$$

as $$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Power may be calculated as $\frac{|V_R|^2}{R}$

Equation (37) implies that the product |V| |I| does not give the power in an AC circuit. This product must be further multiplied by the factor cos ϕ to get the power in the AC circuit. The product |V| |I| is called volt amperes, or apparent power in a circuit.

$$|V|\ |I| = \text{volt amps} = \text{apparent power} \qquad \text{...(38)}$$

$$|V|\ |I|\cos\phi = \text{real power} \qquad \text{...(39)}$$

The term $$\cos\phi = \frac{\text{Real Power}}{\text{Apparent Power}} \qquad \text{...(40)}$$

is called power factor.

$$|V|\ |I|\sin\phi = \text{reactive power} \qquad \text{...(41)}$$

Equation (40) defines the power factor of a circuit. Power factor can also be calculated as

$$\text{Power factor} = \frac{R}{|Z|}$$

It is in effect the cosine of the angle between the phasors V (total voltage) and the current I in the circuit.

2.12 THREE PHASE CIRCUITS

2.12.1 Introduction

For household applications, we use single phase AC supply (230 V, 50 Hz). But industries or big consumers are consuming large amount of power. Single phase is not sufficient for producing large amount of power. The large amount of power can be obtained from three phase AC supply, (440 V, 50 Hz). There are several advantages of using three phase supply systems as compared to single phase systems. Some of them are given below.

1. In a three phase circuit, the total power is more nearly uniform unlike in a single phase circuit, where the power varies widely.
2. For a given power rating, a three phase alternator is smaller in size leading to saving in copper and other material.
3. Three phase induction motors are self starting unlike single phase induction motors.
4. Three phase machines have better power factor and efficiency.
5. For the same size, the capacity of a three phase machine is higher.
6. Generation, transmission and utilization of power is more economical in three phase systems compared to single phase systems.

2.12.2 Generation of three phase EMF

A three phase alternator has three separate windings in its stator. The three windings are displaced from one another by 120°. The three voltages will then differ by 120°. If RR', YY', BB' are the three windings, then the voltages in the

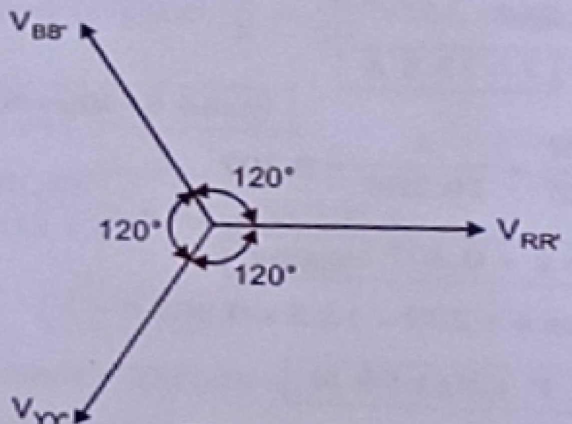


Figure 2.24

winding $V_{RR'}$, $V_{YY'}$ and $V_{BB'}$ (all rms) differ by 120° each and are shown in the phasor diagram of figure (2.24).

Figure 2.25 shows three phase sinusoidal voltage waveform.

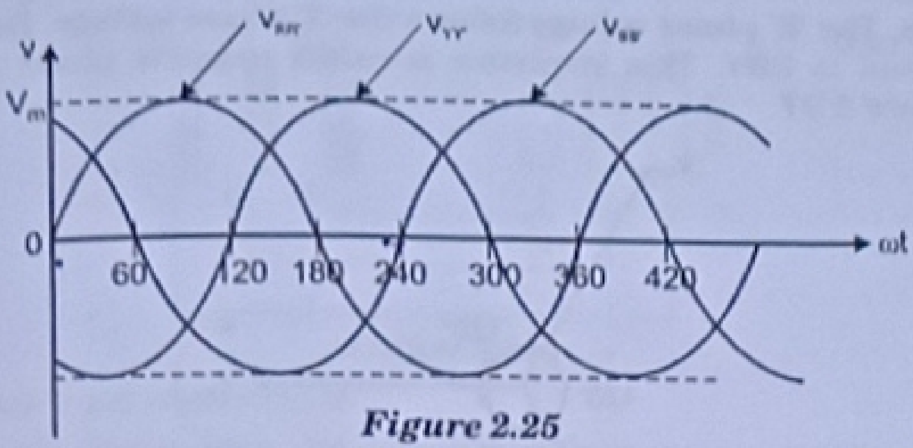


Figure 2.25

Since the three windings are identical, the magnitudes of the three voltages are equal. We may write

$v_{RR'} = V_m \sin\omega t$ ...(1)

$v_{YY'} = V_m \sin(\omega t - 120°)$ ...(2)

$v_{BB'} = V_m \sin(\omega t + 120°)$ or $V_m \sin(\omega t - 240°)$ ...(3)

In polar form we can write

$$V_R = V_{RR'} = V\angle 0°, \ V_Y = V_{YY'} = V\angle -120°$$

$$V_B = V_{BB'} = V\angle -120°$$

$$|V_{RR'}| = |V_{YY'}| = |V_{BB'}| = V = \frac{V_m}{\sqrt{2}}$$

2.12.3 Phase Sequence

From the phasor diagram of figure (2.26) as well as from the equations (1, 2, 3) it is evident that the three voltages are not in phase. There exists a phase difference of 120° between each of them. The 'Y' phase voltage follows the 'R' phase voltage. The 'B' phase voltage follows the 'Y' phase voltage. Hence we say the phase sequence is RYB. The RYB sequence in the anticlockwise direction defines the positive phase sequence. It is shown in figure 2.26.

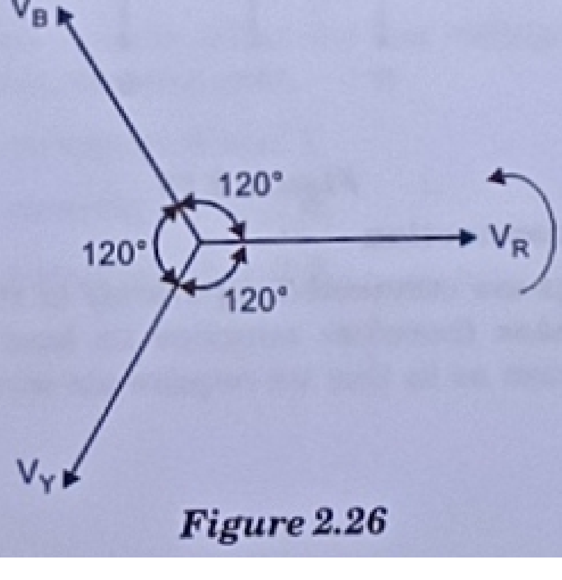


Figure 2.26

For the same system, if the phase sequence is given as RBY, it indicates 'B' phase voltage follows the 'R' phase voltage. The 'Y' phase voltage follows the 'B' phase voltage. The 'R' phase voltage follows the 'Y' phase voltage. Hence we say the phase sequence is RBY. This sequence is called negative phase sequence. It is shown in figure 2.27.

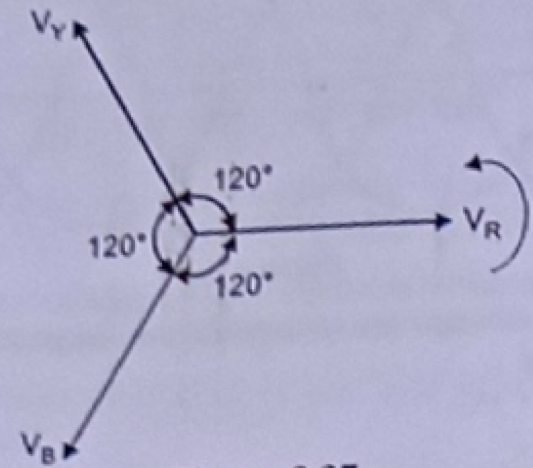


Figure 2.27

2.12.4 Inter connection of windings

The three windings of a 3-phase alternator are shown in figure 2.28 for simplicity.

These three windings are connected in the following connection.

1. Independent connection
2. Star connetion
3. Delta connection

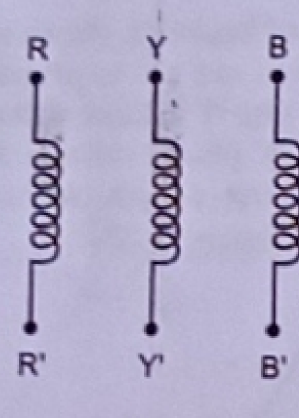


Figure 2.28

2.12.4.1 Independent connection

The three windings are connected separately to the three loads. It is shown in figure 2.29. Each phase therefore supplies its load separately. This is not a commonly used connection as in this we require six wires, a pair of wires for each phase.

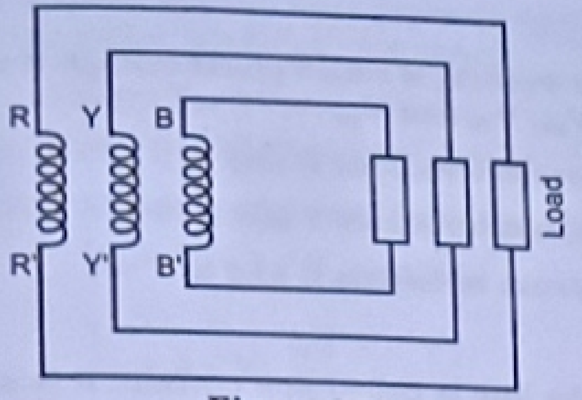


Figure 2.29

2.12.4.2 Star (or Wye) connection

Figure 2.30 shows that the three windings are connected in star. The terminals R', Y' and B' are connected together to form the star point, also called the neutral (N). The lines R, Y and B are connected to the load. If the neutral is connected to the neutral of the load, it becomes a 3-phase, 4 wire system. Otherwise it is a 3-phase, 3 wire system.

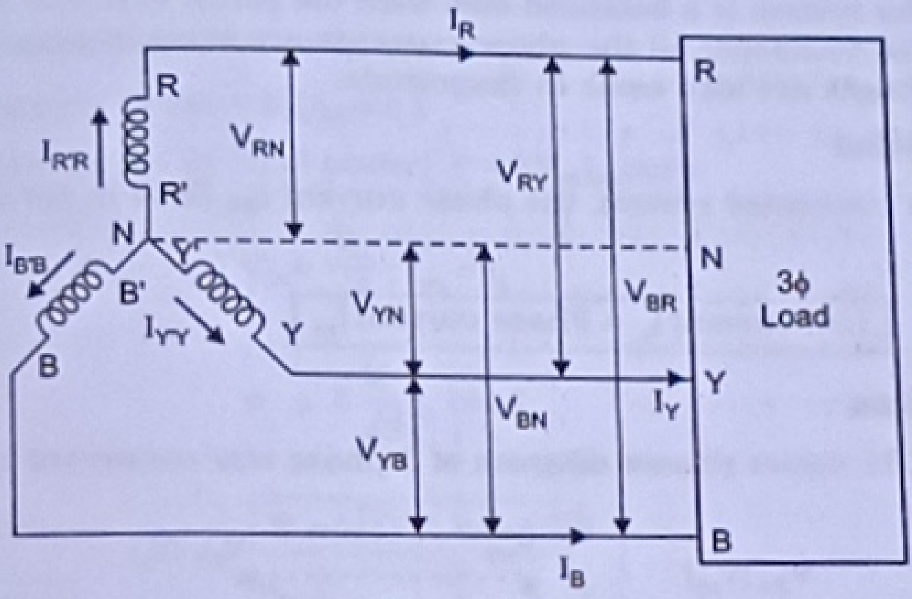


Figure 2.30

Line voltage (V_L)

Voltage across any two lines is called the line voltage. Here the line voltages are V_{RY}, V_{YB} and V_{BR}. It is also denoted as V_L.

V_{RY} = Voltage across terminals R and Y

V_{YB} = Voltage across terminals Y and B

V_{BR} = Voltage across terminals B and R

Phase voltage (V_{ph})

Voltage across the winding is called phase voltage. It is denoted as V_{ph}. Here the phase voltages are V_{RN}, V_{YN} and V_{BN}.

V_{RN} = Voltage across terminals R and N.

V_{YN} = Voltage across terminals Y and N.

V_{BN} = Voltage across terminals B and N.

Line current (I_L)

Current through the line is called line current. It is denoted as I_L. Here the line currents are I_R, I_Y and I_R.

Phase current (I_{ph})

Current flowing in any phase winding is called phase current. It is denoted as I_{ph}. Here the phase currents are

$I_{RR'}$, $I_{YY'}$ and $I_{BB'}$.

When the system is a balanced one, then the phase currents and the phase voltages will be balanced. All the phase currents are same in magnitude and all the phase voltages are also same in magnitude.

Current equation

In a star connected system, the phase current $I_{RR'}$ flows in the line R also, so that $I_R = I_{RR'}$.

$$\boxed{\text{Line current } I_L = \text{Phase current } I_{ph}}$$

Voltage equation

Figure 2.31 shows phasor diagram of 3-phase star connected system.

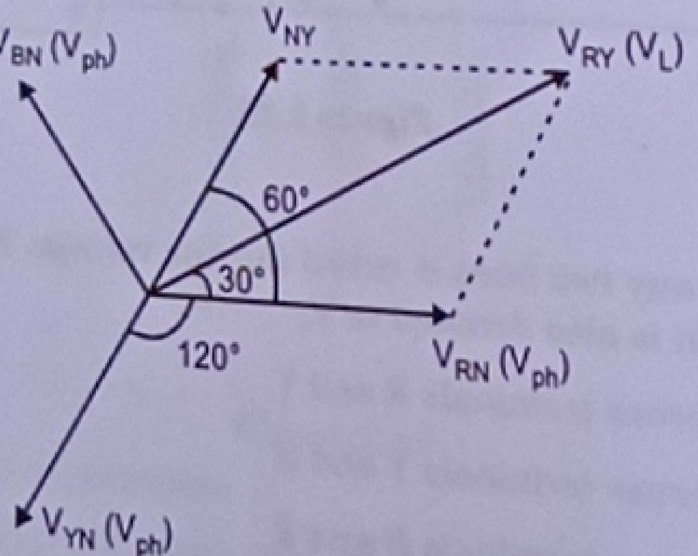


Figure 2.31

Potential difference between any two lines depends on the potential difference between the two ends of the phase windings concerned.

The line voltage $\quad V_{RY} = V_{RN} + V_{NY}$

The phasor diagram shows the addition of V_{RN} and V_{NY}.

The magnitude of line voltage is obtained from the phasor diagram.

$$V_{RY} = V_L = 2 \times V_{ph} \cos 30^\circ$$

$$= 2 \times V_{ph} \frac{\sqrt{3}}{2}$$

$$\boxed{V_L = \sqrt{3} V_{ph}}$$

Hence, the line voltage in 3-phase star connected system is $\sqrt{3}$ times the phase voltage.

Power equation

Let $\cos \phi$ be the power factor at which the power is delivered to a load.

Power per phase $= V_{ph} I_{ph} \cos \phi$

[Three phase power or Total power] $P = 3 V_{ph} I_{ph} \cos \phi$

$$V_{ph} = \frac{V_L}{\sqrt{3}}\ ;\ I_{ph} = I_L$$

$$\therefore \quad P = 3 \frac{V_L}{\sqrt{3}} I_L \cos \phi$$

Total power $\quad \boxed{P = \sqrt{3} V_L I_L \cos \phi}$ W

Total reactive power $\quad \boxed{Q = \sqrt{3} V_L I_L \sin \phi}$ VAR

Total Apparent power $\quad S = \sqrt{P^2 + Q^2}$

$$= \sqrt{\left(\sqrt{3} V_L I_L \cos \phi\right)^2 + \left(\sqrt{3} V_L I_L \sin \phi\right)^2}$$

$$\boxed{S = \sqrt{3} V_L I_L} \text{ VA}$$

2.12.4.3 Delta connection

Figure 2.32 shows delta connections of three phase winding. In this figure, the end of R phase winding R' is connected to the start of the next phase winding Y. The end of the Y phase winding Y' is connected to the start of the next phase winding B. B' and R terminals are connected. This connections is for delta or mesh connection. This delta connection gives 3-phase 3-wire system only.

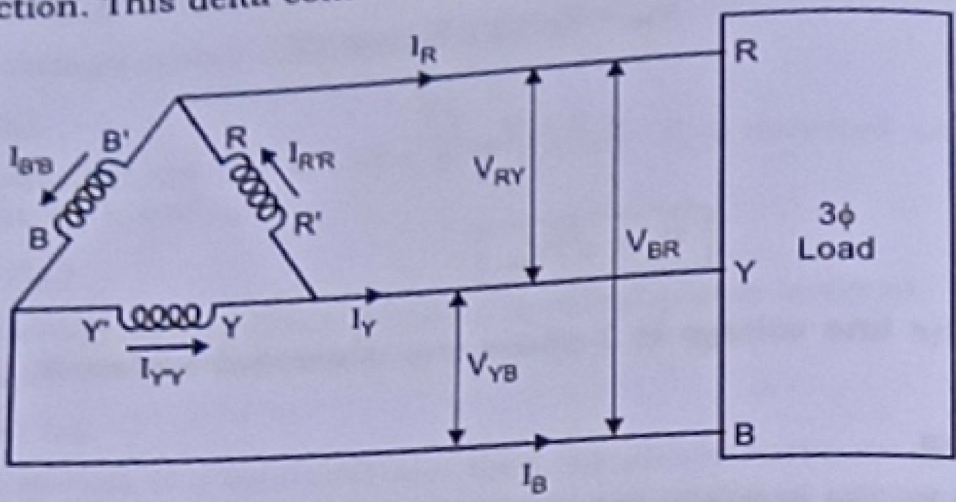


Figure 2.32

Voltage equation

In the delta connection, the phase volage $V_{RR'}$ is also the line voltageV_{RY}. Hence, in the connection, the line voltage is equal to phase voltage.

$$V_{RY} = V_{RR'}$$

$$\boxed{V_L = V_{ph}}$$

Current equation

Figure 2.33 shows phasor diagram of delta connection.

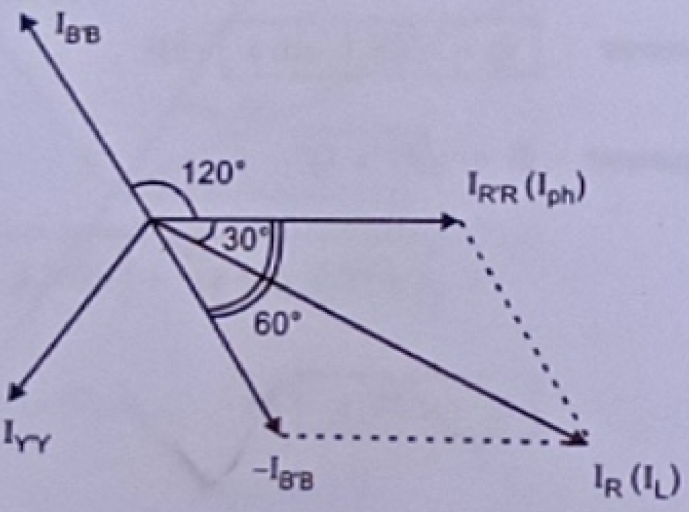


Figure 2.33

From this phasor diagram, the line current $I_R = I_{R'R} + I_{BB'}$

The magnitude of the line current

$$I_L = I_R = 2\, I_{ph} \cos 30°$$

$$= 2 I_{ph} \frac{\sqrt{3}}{2}$$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

Hence, the line current in 3-phase delta connected system is $\sqrt{3}$ times the phase current.

Power equations

Per phase power $P = V_{ph} I_{ph} \cos\phi$

Total power $P = 3 V_{ph} I_{ph} \cos\phi$

$$= 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}\ \text{W}$$

Reactive power $\boxed{Q = \sqrt{3}\, V_L I_L \sin\phi}$ VAR

Apparent power $\boxed{S = \sqrt{3}\, V_L I_L}$ VA

EXAMPLE 31

Each phase of a three-phase alternator generates a voltage of 3810 volts and carries a maximum current of 300 amps. Find the line currents, line voltage and total kVA capacity if the alternator is (a) star (b) delta connected.

Solution:

a) Star connection

Line voltage $V_L = \sqrt{3} \times V_{ph}$

$$= \sqrt{3} \times 3810 = 6600 \text{ V}$$

Line current $I_L = I_{ph} = 300$ A

Capacity $= 3\, V_{ph} I_{ph}$ or $\sqrt{3}\, V_L I_L$

$$= 3 \times 3810 \times 300 = 3429000 \text{ VA or } 3429 \text{ kVA}$$

b) Delta connection

Line voltage $V_L = V_{ph} = 3810$ V

Line current $I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 300 = 519.62$ A

$$\text{Capacity} = 3V_{ph}I_{ph}$$
$$= 3 \times 3810 \times 300$$
$$= 3429 \text{ kVA}$$

EXAMPLE 32

A 220 V 3 phase motor has a full load output of 7.355 kW, Power factor 0.85 and efficiency 95%. Find the phase and line currents if the motor is (a) star connected (b) delta connected.

Solution:

a) Star connection

Full load ouput = $1000 \times 7.355 = 7355$ W

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

$$\therefore \text{Input power} = \frac{7355}{0.95} = 7742 \text{ W}$$

$$\text{Input power} = \sqrt{3}\, V_L I_L \cos\phi = 7742 \text{ W}$$

$$\text{Therefore, } I_L = \frac{7742}{\sqrt{3} \times 220 \times 0.85} = 23.9 \text{ A}$$

$I_L = I_{phase}$ (for star connection)

b) Delta connection

$I_L = 23.9$ A

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{23.9}{\sqrt{3}} = 13.8 \text{ A}$$

2.12.5 Three phase balanced Load

A 3-phase load has three separate load impedances which may be connected in star or delta. When the three impedances are identical, then the three phase load is called balanced load. A balanced three-phase load is treated as 3 identical single-phase cases. We first draw the single phase equivalent and then work out the solution on per phase values.

Star Connection

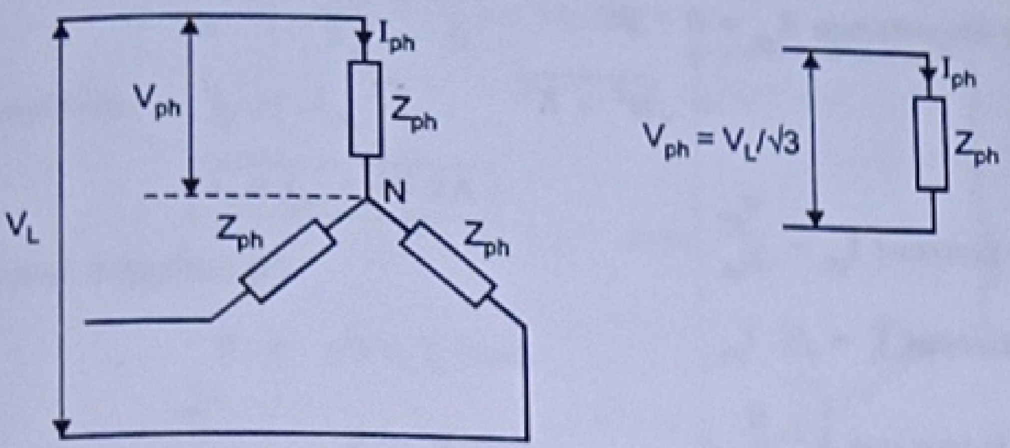


Figure 2.24 Star connection

Phase voltage $V_{ph} = \dfrac{V_L}{\sqrt{3}}$

Phase impedance $Z_{ph} = R + jX = \sqrt{R^2 + X^2}$

Phase current $I_{ph} = \dfrac{V_{ph}}{Z_{ph}}$

Line current $I_L = I_{ph}$

Power factor $\cos\phi = \dfrac{R}{Z}$

Per phase power $P = V_{ph}I_{ph}\cos\phi$

Total power $P = \sqrt{3}\,V_L I_L \cos\phi$

Reactive power per phase $Q = V_{ph}I_{ph}\sin\phi$

Total reactive power $Q = \sqrt{3}\,V_L I_L \cos\phi$

Apparent power per phase $= V_{ph}I_{ph}$

Total apparent power $S = \sqrt{3}\,V_L I_L$

Delta connection

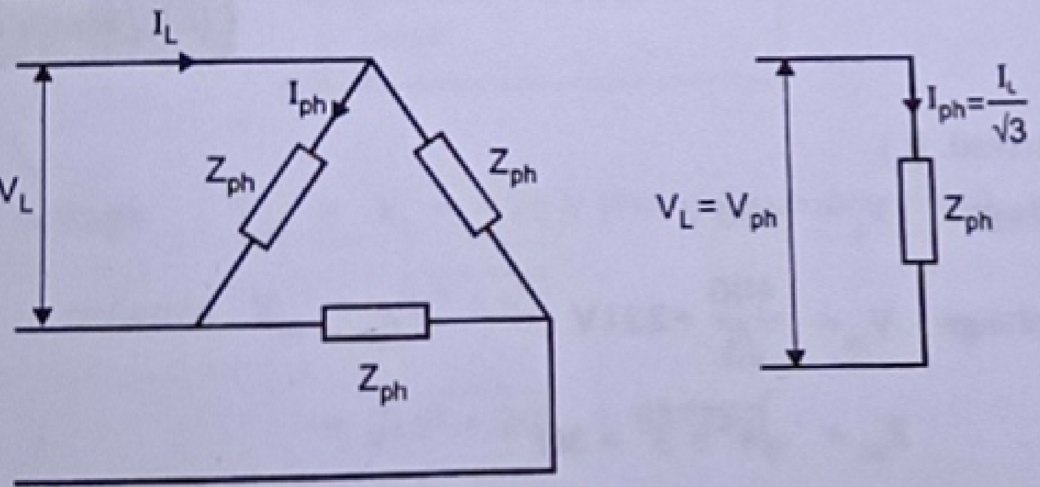


Figure 2.35

Phase voltage $V_{ph} = V_L$

Phase impedance $Z_{ph} = R + jX$

$$= \sqrt{R^2 + X^2}$$

Phase current $I_{ph} = \dfrac{V_{ph}}{Z_{ph}}$

Line current $I_L = \sqrt{3}\ I_{ph}$

Power factor $\cos\phi = \dfrac{R}{Z}$

Per phase power $P = V_{ph}I_{ph}\cos\phi$

Total power $P = \sqrt{3}\, V_L I_L \cos\phi$

Reactive power per phase $Q = V_{ph}I_{ph}\sin\phi$

Total reactive power $Q = \sqrt{3}\, V_L I_L \sin\phi$

Apparent power per phase $= V_{ph}I_{ph}$

Total apparent power $S = \sqrt{3}\, V_L I_L$

Note : A 3 ϕ balanced load when connected in delta across a 3 ϕ balanced supply. The total power in 3ϕ delta connected load is equal to the three times of power in star connected load. i.e.,

Total power in delta connected system = 3 × total power in star connected system

EXAMPLE 33

400 V (line to line) is applied to three phase star connected identical impedances each containing of a 4Ω resistance in series with 3Ω inductive reactance. Find (i) line current (ii) total power supplied.

(AU/Mech - Dec 2005)

Solution:

i) Line Current (I_L)

Line voltage $V_L = 400\,V$

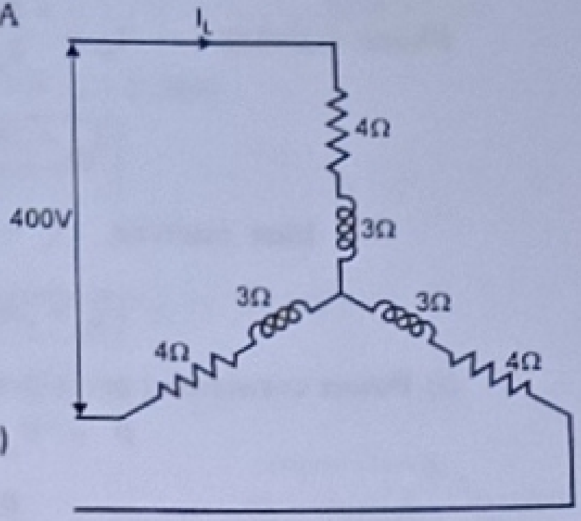
Phase voltage $V_{ph} = \dfrac{400}{\sqrt{3}} = 231\,V$

$$Z_{ph} = \sqrt{4^2 + 3^2} = 5\Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5} = 46.2\text{A}$$

In star connection, $I_L = I_{ph}$

$$\boxed{I_L = I_{ph} = 46.2\text{A}}$$



ii) Total power supplied (P)

$$P = \sqrt{3}V_L I_L \cos\phi$$

$$\cos\phi = \frac{R}{Z_{ph}} = \frac{4}{5} = 0.8\,(\text{lag})$$

$$P = \sqrt{3} \times 400 \times 46.2 \times 0.8$$

$$\boxed{P = 25606.6\,\text{W}}$$

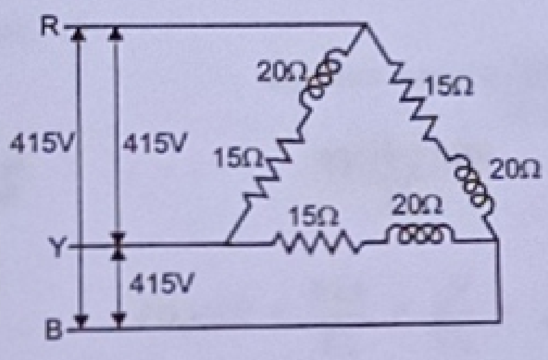
EXAMPLE 34

A 415 V, 3 phase voltage is applied to a balance delta connected load of phase impedances each equal to 15 + j20Ω. Find i) Phase and line values of current, ii)power consumed per phase.

(AU/Mech - May 2003)

Solution:

Figure shows delta connected load.



i) I_{ph}, I_L

Phase voltage $V_{ph} = V_L = 415$ V (For delta connection)

Phase impedance $Z_{ph} = \sqrt{R^2 + X_L^{\,2}}$

$$= \sqrt{15^2 + 20^2} = 25\,\Omega$$

Phase current $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{25} = 16.6A$

$$\boxed{I_{ph} = 16.6A}$$

Line current $I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 16.6 = 28.75A$

$$\boxed{I_L = 28.75A}$$

ii) Power consumed per phase

$$P = V_{ph} I_{ph} \cos\phi$$

$$\cos\phi = \frac{R}{Z_{ph}} = \frac{15}{25} = 0.6(\text{lag})$$

$$P = 415 \times 16.6 \times 0.6 = 4133.4\,W$$

$$\boxed{P = 4133.4W}$$

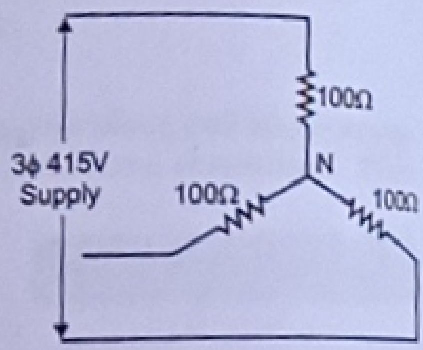
EXAMPLE 35

Three 100Ω resistors are connected first in star and then in delta across 415V, 3-phase supply. Calculate the line and phase currents in each case and also the power taken from the source.

(AU/Mech – Dec 2007)

Solution:

Star Connection:



In star connected, phase voltage

$$= \frac{\text{line voltage}}{\sqrt{3}}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6V$$

Phase current $I_{ph} = \frac{V_{ph}}{R} = \frac{239.6}{100} = 2.396A$

$$\boxed{I_{ph} = 2.396A}$$

Line current $\boxed{I_L = I_{ph} = 2.396A}$

Power taken from the source P $= \sqrt{3} V_L I_L$ [$\cos\phi = 1$]

$$= \sqrt{3} \times 415 \times 2.396$$

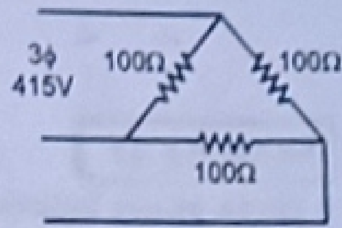
$$\boxed{P = 1722.2W}$$

Delta connection:

Phase voltage V_{ph} = Line voltage V_L = 415 V

Phase current $I_{ph} = \frac{V_{ph}}{R} = \frac{415}{100} = 4.15A$

Line current $I_L = \sqrt{3} I_{ph}$



$$= \sqrt{3} \times 4.15 = 7.188A$$

Power taken from the source $P = \sqrt{3} V_L I_L$

$$= \sqrt{3} \times 415 \times 7.188 = 5166.7W$$

EXAMPLE 36

Each phase of a 3-phase alternator produces a voltage of 6351V and can carry a current of 315A. Find the line voltage, maximum line current and total kVA capacity of the alternator if it is

i) Star connected

ii) Delta connected

(AU/Mech – Dec 2003)

Solution:

V_{ph} = 6351 V

I_{ph} = 315 A

i) Star connected

Line voltage $V_L = \sqrt{3} V_{ph} = \sqrt{3} \times 6351 = 11000V$

Line current $I_L = I_{ph} = 315$ A

$$I_{max} = \sqrt{2} \times I_L = \sqrt{2} \times 315 = 445.47 \text{ A}$$

Total kVA capacity $= \sqrt{3} V_L I_L$

$$= \sqrt{3} \times 11000 \times 315 = 6 \text{ MVA}$$

ii) Delta connected

Line voltage $V_L = V_{ph}$

$= 6351 \text{ V}$

Line current $I_L = \sqrt{3} I_{ph}$

$= \sqrt{3} \times 315 = 545.6 \text{A}$

Maximum line current $= \sqrt{2} \times I_L = \sqrt{2} \times 545.6$

$= 771.5 \text{ A}$

Total kVA capacity $= \sqrt{3} \times V_L I_L$

$= \sqrt{3} \times 6351 \times 545.6 = 6 \text{ MVA}$

EXAMPLE 37

A three phase balanced delta connected load of $(4 + j8)\Omega$ is connected across a 400V, 3ϕ balanced supply. Determine the phase current and line current. Assume the RYB phase sequence also calculate the power drawn by the load.

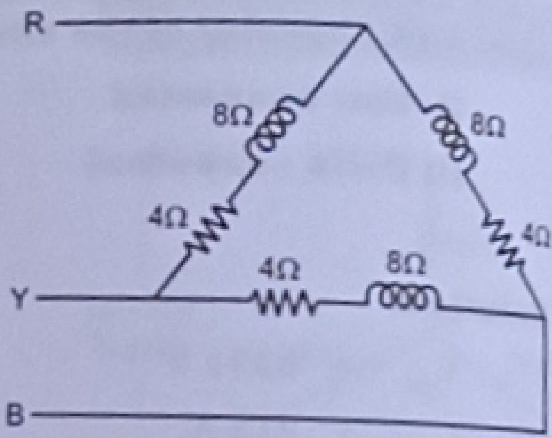
(AU/EEE – May 2004)

Solution:

Impedance $Z_{ph} = \sqrt{R^2 + X_L^2} = \sqrt{4^2 + 8^2} = 8.94\Omega$

Phase voltage V_{ph} = Line voltage V_L

$= 400 \text{ V}$



Phase current $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{8.94}$

$\boxed{I_{ph} = 44.74\text{A}}$

Line current $I_L = \sqrt{3} I_{ph}$

$= \sqrt{3} \times 44.74 = 77.5 \text{ A}$

$\boxed{I_L = 77.5\text{A}}$

Power drawn by the load $P = \sqrt{3} V_L I_L \cos\phi$

$$\cos\phi = \frac{R}{Z_{ph}} = \frac{4}{8.94} = 0.447$$

$$P = \sqrt{3} \times 400 \times 77.5 \times 0.447$$

$\boxed{P = 24\text{kW}}$

TWO MARK QUESTIONS AND ANSWERS

1. **What is meant by alternating quantity?**

   It is one in which the magnitude and direction change with respect to time.

2. **What is meant by cycle?**

   One complete set of positive and negative values of an alternating quantity.

3. **Define time period.** **(AU/Mech – Dec 2007)**

   The time taken to complete one cycle is called the time period of the quantity (p).

4. **Define frequency.** **(AU/Mech – Dec 2007)**

   The number of cycles occuring per second is called frequency $f = \frac{1}{T} Hz$

5. **Define amplitude.**

   The maximum value, either positive or negative, of an alternating quantity is called amplitude.

6. **What is meant by average value?**

   $$\text{Average value} = \frac{\text{Area under the curve over one complete cycle}}{\text{Base (Time period)}}$$

7. **Define RMS value.** **(AU/Mech – May 2007)**

   The effective value of an alternating current is that value of steady direct current which produces the same heat as that produced by the alternating current when passed through the same resistor for the same interval of time.

   $$RMS\ value = \sqrt{\frac{Area\ the\ square\ curve\ for\ one\ cycle}{period}}$$

8. **Define form factor.** **(AU/CSE – May 2007)**

   $$\text{Form factor} = \frac{RMS\ value}{Average\ value}$$

9. **Define crest (peak) factor.** **(AU/CBE/CSE – May 2007)**

   $$\text{Crest (peak) factor} = \frac{Maximum\ value}{RMS\ value}$$

10. **Give the voltage and current equations for a purely resistive circuit.**

    $v = V_m \sin \omega t.$

    $i = I_m \sin \omega t.$

    where v, i are instantaneous values of voltage and current respectively.

    V_m , I_m are maximum voltage and current respectively.

    ω - angular velocity, T – Time period.

11. Define inductance.

When a time varying current passes through a circuit, varying flux is produced. Because of this change in flux, a voltage is induced in the circuit proportional to the time rate of change of flux or current i.e., emf induced $\alpha \frac{di}{dt} = L\frac{di}{dt}$

where L, the constant of proportionality has come to be called self inductance of the circuit. The self inductance (or simply inductance) is the property of a coil by which it opposes any change of current. It is well known that the unit of inductance is Henry.

12. Define capacitance.

A capacitor is a circuit element which, like the inductor, stores energy during periods of time and returns the energy during others. In the capacitor, storage takes place in an electric field unlike the inductance where storage is in a magnetic field. A capacitor is formed by two parallel plates separated by an insulating medium. The emf across capacitor is proportional to the charge in it, i.e., e α q or $e = \frac{q}{C}$

where "C" the constant is called capacitance.

13. Define power factor. **(AU/Mech - May 2007)**

The power factor is the cosine of the phase angle between voltage and current.

$$\cos\phi = \frac{\text{Resistance}}{\text{Impedance}}$$

$$\cos\phi = \frac{\text{Real Power}}{\text{Apparent Power}}$$

14. What are the three types of power used in a.c. circuit?

i) Real or active power $P = VI\cos\phi$.

ii) Reactive power $Q = VI\sin\phi$.

iii) Apparent power, $S = VI$

15. Define real power.

The actual power consumed in an a.c. circuit is called real power. If V and I are rms values of voltage and current respectively and ϕ is the phase angle between V and I, then, $P = VI\cos\phi$.

16. Define reactive power.

The power consumed by a pure reactance (X_L or X_C) in an a.c. circuit is called reactive power. The unit is VAR. $Q = VI\sin\phi$.

17. Define apparent power.

It is given by the product of rms values of applied voltage and circuit current. The unit is Volt-Amperes (VA).

$S = VI$

18. Give the advantages of three phase system? **(AU/Mech – May 2008)**

1. In a three phase circuit, the total power is more nearly uniform unlike in a single-phase circuit, where the power varies widely.
2. For a given power rating, a three phase alternator is smaller in size leading to saving in copper and other material.
3. Polyphase motors (induction motors) are self starting unlike single-phase induction motors.
4. Polyphase machines have better power factor and efficiency.
5. For the same size, the capacity of a polyphase machine is higher.
6. Generation, transmission and utilization of power is more economical in polyphase systems compared to single-phase systems.

19. What is phase sequence?

The order in which the voltages in the three phases reach their maximum value or minimum value is called the phase sequence.

20. What is meant by balanced system?

A balanced system means that the currents in the three phases are equal in magnitude and are displaced from one another by 120°.

21. Define phase voltage and phase current.

Phase voltage is nothing but voltage across each winding. The current flowing in the phases is called phase current. (I_{ph}).

22. Define line voltage and line current.

The line voltage is nothing but voltage across any two lines (E_L). The current flowing in the lines is called line current (I_L).

23. What is the relation between line voltage and phase voltage for star and delta connection?

For star connection

$V_L = \sqrt{3}\, V_{ph}$

For delta connection

$V_L = V_{ph}$

where V_L = Line voltage, V_{ph} = Phase voltage

24. **What is the relation between line current and phase current for star and delta connections?**

For star connection

$I_L = I_{ph}$

For delta connection

$I_L = \sqrt{3}\, I_{ph}$

where I_L = Line current, I_{ph} = Phase current

25. **Define balanced load.**

A load is said to be a balanced load, if the power factors and phase currents in the 3-phases are equal.

26. **Define unbalanced load.**

A load is said to be unbalanced load, if the power factors and phase currents are unequal.

27. **Write down the expression for real, reactive and apparent power in a three phase system.**

Real power $P = 3\, V_{ph}\, I_{ph} \cos\phi.$

$= \sqrt{3}\, V_L\, I_L \cos\phi$

Reactive power $Q = 3\, V_{ph}\, I_{ph} \sin\phi.$

$= \sqrt{3}\, V_L\, I_L \sin\phi$

Apparent power $S = 3\, V_{ph}\, I_{ph}$

$= \sqrt{3}\, V_L\, I_L$