then verify that $V(F)$ is a vector space or not?

Sol:- By def $(x_1,y_1)+(x_2,y_2) = (3y_1+3y_2, -x_1-x_2)$

$c(x_1,y_1) = (3cy_1, -cx_1)$.

1) 1.) closure axiom:- let (x_1,y_1) & $(x_2,y_2) \in V(F)$ then

$(x_1,y_1)+(x_2,y_2) = (3y_1+3y_2, -x_1-x_2) \in V(F)$.

$\therefore$ 'v' is closed.

2) Associative axiom:- let $x,y \in V(F)$, $x,y,z \in V(F)$ then

$(x+y)+z = ((x_1,y_1)+(x_2,y_2))+(x_3,y_3)$

$= (3y_1+3y_2, -x_1-x_2)+(x_3,y_3)$

$= (3(-x_1-x_2)+3y_3, -(3y_1+3y_2)-x_3)$

$= (-3x_1-3x_2+3y_3, -3y_1-3y_2-x_3)$.

$x+(y+z) = (x_1,y_1)+[(x_2,y_2)+(x_3+y_3)]$

$= (x_1,y_1)+(3y_2+3y_3, -x_2-x_3)$

$= [3y_1+3(-x_2-x_3), -x_1-(3y_2+3y_3)]$

$= (3y_1-3x_2-3x_3, -x_1-3y_2-3y_3)$.

$\therefore (x+y)+z \neq x+(y+z)$

$\therefore$ Hence 'v' is not associative.

$\therefore V(F)$ is not a field.

H.W

① Let 'v' be the set of all pairs (x,y) of real number and let 'f' be the field of real numbers. defined

1.) $(x,y)+(x_1,y_1) = (x+x_1, y+y_1)$

2.) $c(x,y) = (cx,y)$. Show that $V(F)$ is not a vector space.

Definition:- Let $(u_1, u_2 --- u_n)$ be a sequence of n-real numbers. The set of all such sequences is called n-space and is denoted R^n. u_1 is the first component of $(u_1, u_2, --- u_n)$. u_2 is the second component and so on.

Ex:- R^4 is the collection of all sets of four ordered real numbers.

Ex:- ① $(1, 2, 3, 4)$, ② $(5, 7, -8, 9)$ are elements of R^4.

Def:- Let $u = (u_1, --- u_n)$ and $v = (v_1, --- v_n)$ be two elements of R^n. We say that u and v are equal if $u_1 = v_1 --- u_n = v_n$.

Thus two elements of R^n are equal if their corresponding components are equal.

Def:- let $u = (u_1 ---- u_n)$ and $v = (v_1 ---- v_n)$ be elements of R^n and let 'c' be a scalar. Vector addition and scalar multiplication are performed as follows.

Addition:- $u+v = (u_1+v_1, ---- u_n+v_n)$

Scalar multiplication:- $cu = (cu_1, cu_2 ---- cu_n)$.

Zero vector:- the vector $(0, 0, --- 0)$ having 'n' zero components is called the zero vector of R^n and is denoted by 0.

Ex:- $(0, 0, 0)$ is a zero vector of R^3.

Negative vector:-

the vector $(-1)u$ is written $-u$ and is called the negative of u. It is a vector that has the same magnitude as 'u', but lies in the opposite direction to 'u'.

Subtraction:- Subtraction is performed on elements of R^n by subtracting corresponding components.

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Additional Sheet

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Vector Subspaces:-

Def:- let V(F) be a vector space and $W \subseteq V$ is said to be vector subspace of 'V' in 'W' it self is a vector space over F with the same operations of vector addition and scalar multiplication in V.

Theorem:- Let W(F) is a non-empty subset of a vector space V(F) then W(F) is a vector subspace of V(F) iff for any $a \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + \beta \in W$.

proof:- Neccessary condition:-

Let 'W' be a subspace of V(F) then 'W' is a vector space.

let $a \in F$, $\alpha \in W$ then $a\alpha \in W$ ($\because$ W is a v.s)

Now $a\alpha \in W$, $\beta \in W \Rightarrow a\alpha + \beta \in W$ ($\because$ W is closed).

Sufficient condition:-

Let 'W' be a non-empty subset of 'V' satisfying the given condition.

i.e $a \in F, \alpha \in W, \beta \in W \Rightarrow a\alpha + \beta \in W$.

i) Taking $a = -1$, for $\alpha \in W$, we have $(-1)\alpha + \alpha \in W$

$\Rightarrow -\alpha + \alpha \in W$

$\Rightarrow \bar{0} \in W$

the zero vector of 'V' is also the zero vector of W.

ii) Now $\bar{0} \in W$, $\alpha \in W$, $a \in F \Rightarrow a\alpha + \bar{0} \in W$

$\Rightarrow a\alpha \in W$.

$\therefore$ 'W' is closed under scalar multiplication.

iii) Taking $-1 \in F$ & $\alpha, \bar{0} \in W \Rightarrow (-1)\alpha + \bar{0} \in W$

$\Rightarrow -\alpha \in W$.

$\therefore$ Additive Inverse of 'V' is also in 'W'.

the remaining properties also hold in W. Since $W \subseteq V$.

Hence $W(F)$ is a vector subspace of V.

Ex:- prove that $W = \{(a,b,c,d) \in R^4 / a-b-3d=0\}$ is a vector subspace of vector space R^4.

Sol:- To prove that 'W' is subspace of R^4, $a \in F$.

$a \in F$, $\alpha, \beta \in R^4 \Rightarrow a\alpha + \beta \in W$.

$\alpha = (a_1, b_1, c_1, d_1)$, $\beta = (a_2, b_2, c_2, d_2)$

$a\alpha + \beta = a(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2)$

$= (aa_1, ab_1, ac_1, ad_1) + (a_2, b_2, c_2, d_2)$

$= (\underset{a}{aa_1 + a_2}, \underset{b}{ab_1 + b_2}, \underset{c}{ac_1 + c_2}, \underset{d}{ad_1 + d_2}) \in R^4$.

Now $a-b-3d = (aa_1 + a_2) - (ab_1 + b_2) - 3(ad_1 + d_2)$

$= (aa_1 + a_2 - ab_1 + b_2 - 3ad_1 - 3d_2)$

$= a(a_1 - b_1 - 3d_1) + a_2 + b_2 - 3d_2$

$= a(a_1 - b_1 - 3d_1) + (a_2 + b_2 - 3d_2)$

$= a(0) + 0$

$= 0$

$\therefore a\alpha + \beta \in W$

$\therefore$ For any $a \in F$, $\alpha, \beta \in R^4 \Rightarrow a\alpha + \beta \in W$.

Hence 'W' is a subspace of R^4.

Thm-2

Let $V(F)$ be a vector space and let $W \subseteq V$ the necessary and sufficient conditions for 'W' to be a subspace of 'V' are

1) $\alpha \in W$, $\beta \in W \Rightarrow \alpha - \beta \in W$.

2) $a \in F$, $\alpha \in W \Rightarrow a\alpha \in W$.

Thm 3:- Let $V(F)$ be a vector space. A non-empty set $W \subseteq V$. The necessary and sufficient conditions for W to be a subspace of V is $\Rightarrow a, b \in F$ ~~&~~ $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

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Ex2:- Verify that $W = \{a, b \in R \,|\, (a, a^2, b)\}$ is a vector subspace of a vector space R^3 or not?

Sol:- To prove that 'w' is subspace of R^3; $p \in R$.

$u, v \in R^3 \Rightarrow pu + v = w$. ($\because a\alpha + \beta \in w$) form

$u = (a_1, a_1^2, b_1)$, $v = (a_2, a_2^2, b_2) \in R^3$.

$pu + v = p(a_1, a_1^2, b_1) + (a_2, a_2^2, b_2)$

$= (pa_1, pa_1^2, b_1) + (a_2, a_2^2, b_2)$

$= (pa_1 + a_2, pa_1^2 + a_2^2, b_1 + b_2)$ which is not of the form (a, a^2, b).

Hence $pu + v \notin w$.

$\therefore$ 'w' is not v.subspace of R^3.

Problems:-

① The set w of order to triads $(x, y, 0)$ where $x, y \in F$ is subspace of $V_3(F)$.

Sol:- Given that the set $w = \{(x, y, 0) \,|\, x, y \in F\}$

let $\alpha, \beta \in w$, where $\alpha = (x_1, y_1, 0)$, $\beta = (x_2, y_2, 0)$.

for $x_1, y_1, x_2, y_2 \in F$.

let $a, b \in F$, $\alpha, \beta \in w$

then $a\alpha + b\beta = a(x_1, y_1, 0) + b(x_2, y_2, 0)$

$= (ax_1, ay_1, 0) + (bx_2, by_2, 0)$

$a\alpha + b\beta = (ax_1 + bx_2, ay_1 + by_2, 0)$.

Since $ax_1 + bx_2$, $ay_1 + by_2 \in F$ then $a\alpha + b\beta \in w$.

$\therefore$ 'w' is a subspace of $V_3(F)$.

H.W

(2) Let p, q, r be the fixed elements of a field F. Show that the set 'W' of all triads (x, y, z) of element of F, such that $px+qy+rz=0$ is a vector subspace of $V_3(F)$ ($a\alpha+b\beta$ ...)

(3) Let 'R' be the field of real number. Show the set of triads that
($\because a\alpha+b\beta$)

1.) $\{(x, 2y, 3z) \mid x, y, z \in R\}$

2.) $\{(x, x, x) \mid x \in R\}$ form the subspace of $R^3(R)$

(4) Let $V=R^3=\{(x, y, z) \mid x, y, z \in R\}$ and 'W' be the set of triads (x, y, z) such that $x-3y+4z=0$. Show that 'W' be a subspace of $V_3(R)$

(5) Let $u=(-1, 4, 3, 7)$ and $v=(-2, -3, 1, 0)$ be vectors in R^4. Find $u+v$ and $3v$.

Sol:- $u=(-1, 4, 3, 7)$, $v=(-2, -3, 1, 0)$

$u+v=(-3, 1, 4, 7)$.

$3u=3(-1, 4, 3, 7)$

$=(-3, 12, 9, 21)$.

(6) Let $u=(2, 5, -3)$, $v=(-4, 1, 9)$, $w=(4, 0, 2)$. Determine the vector $2u-3v+w$

Sol:- $2u-3v+w=2(2, 5, -3)-3(-4, 1, 9)+(4, 0, 2)$

$=(4, 10, -6)-(-12, 3, 27)+(4, 0, 2)$

$=(4+12+4, 10-3+0, -6-27+2)$

$=(20, 7, -31)$.

(7) Let 'C' denote the complex numbers and 'R' denotes real numbers. Is 'C' a vector space over R under ordinary addition and multiplication? Is R a vector space under 'C'?

Sol:- 'C' is a vector space over R. But 'R' is not a a vector space over C. Since 'R' is not closed under scalar multiplication over 'C'.

⑧ Prove that $x = \{(a, 2a-3b, 5b, a+2b, a) \mid a, b \in R\}$ is a vector space of R^5.

Sol:- Let $\alpha \in F$, and $u, v \in R^5$.

$u = (a_1, 2a_1-3b_1, 5b_1, a_1+2b_1, a_1)$

$v = (a_2, 2a_2-3b_2, 5b_2, a_2+2b_2, a_2)$.

$\alpha u + v = \alpha(a_1, 2a_1-3b_1, 5b_1, a_1+2b_1, a_1) + (a_2, 2a_2-3b_2, 5b_2, a_2+2b_2, a_2)$

$= (\alpha a_1 + a_2, \alpha(2a_1-3b_1) + (2a_2-3b_2), \alpha(5b_1) + 5b_2, \alpha(a_1+2b_1) + (a_2+2b_2), \alpha a_1 + a_2)$.

$\therefore$ 'x' is a vector space of R^5.

H.W Theorem:- Let $X \subseteq V$, $Y \subseteq V$ be a vector subspaces of a vector space $V(F)$. Then their intersection $X \cap Y$ is also a vector subspace of V.

Theorem:- The intersection of any family of subspaces of a vector space is also a subspace.

Proof:- Let $V(F)$ be a vector space.

Let $W_1, W_2, W_3 \ldots W_n$ be n-spaces of $V(F)$.

Let $W = \bigcap_{i=1}^{n} W_i$

$\bar{0} \in W_i$ for $i = 1, 2, \ldots n \Rightarrow \bar{0} \in \bigcap_{i=1}^{n} W_i$

$\Rightarrow \bigcap_{i=1}^{n} W_i \neq \phi$.

Let $\alpha, \beta \in \bigcap_{i=1}^{n} W_i \Rightarrow \alpha, \beta \in W_n, \forall n$.

Since each W_n is a subspace, we have $a, b \in F$, & $\alpha, \beta \in W_n$.

$\Rightarrow a\alpha + b\beta \in W_n, \forall n$.

$\Rightarrow a\alpha + b\beta \in \bigcap_{i=1}^{n} W_i$

$\therefore$ Hence $\bigcap_{i=1}^{n} W_i$ is a subspace of $V(F)$.

Theorem:- Let $V(F)$ is a vector space and let $V_1 \in V$ and $V_2 \in V$ be vector subspaces. prove that if $V_1 \cup V_2$ is a vector subspace of V, then either $V_1 \subseteq V_2$ (or) $V_2 \subseteq V_1$.

Sol:- If $V_1 \subseteq V_2$, or $V_2 \subseteq V_1$ then it is trivial that $V_1 \cup V_2$ is a subspace of V.

Suppose $V_1 \not\subseteq V_2$ or $V_2 \not\subseteq V_1$

So, $U_1 \in V_1$ and $U_2 \in V_2$ then $U_1 + U_2$ cannot be V_1 and V_2.

If $U_1 + U_2 \in V_1$ then $(U_1+U_2) - U_1 = U_2 \in V_1$.

$U_1 \cup U_2$ is not closed w.r.to vector addition.

** Hence $V_1 \cup V_2$ is not a subspace of V.

Thm:- The union of two subspaces is a subspace iff one is contain in the other that is $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$.

Sol:- Let W_1 & W_2 be two subgroups of subspaces of $V(F)$.

Neccessary condition:-

Let $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$

then $W_1 \cup W_2 = W_2$ or W_1

$\therefore W_1 \cup W_2$ is a subspace of $V(F)$.

Sufficient condition:-

let $W_1 \cup W_2$ be a subspace.

let us suppose $W_1 \not\subseteq W_2$ & $W_2 \not\subseteq W_1$

Now $W_1 \not\subseteq W_2 \Rightarrow \exists\, x \in W_1$ & $x \notin W_2 \rightarrow (1)$

$W_2 \not\subseteq W_1 \Rightarrow \exists\, y \in W_2$ & $y \notin W_1 \rightarrow (2)$

$\therefore x \in W_1 \cup W_2$ & $y \in W_1 \cup W_2$.

$\Rightarrow x+y \in W_1 \cup W_2$ ($\because W_1 \cup W_2$ is a subspace)

Now $x+y \in W_1$, $x \in W_1$, $x+y \in W_2$. ($\because W_1$ is a subspace $ax+by \in W_1$ a, b is 1, -1 $\in W_1$)

$x+y \in W_1$, $x \in W_1 \Rightarrow 1(x+y) + (-1)x \in W_1$

$\Rightarrow x+y-x \in W_1 \Rightarrow y \in W_1 \rightarrow (3)$

Ily $x+y \in W_2$, $y \in W_2 \Rightarrow 1(x+y)+(-1)y \in W_2$

$\Rightarrow x+y-y \in W_2$ ($\because W_2$ is a sub space) $a\alpha+b\beta \in W_2$

$\Rightarrow x \in W_2 \rightarrow (4)$

$\therefore$ from (3) and (4), contradictions (2) and (1)

either $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$

Linear sum of two subspace:-

Let W_1 and W_2 be two subspaces of vector space $V(F)$ then the linear sum of the subspaces W_1 and W_2 denoted by W_1+W_2 is the set of all sums $\alpha_1+\alpha_2$ such that $\alpha_1 \in W_1$, $\alpha_2 \in W_2$, That is $W_1+W_2 = \{\alpha_1+\alpha_2 \mid \alpha_1 \in W_1, \alpha_2 \in W_2\}$.

Thm:- W_1 and W_2 are any two subspace of vector space $V(F)$ then i) W_1 and W_2 is a subspace of $V(F)$.

ii) $W_1 \subseteq W_1+W_2$ & $W_2 \subseteq W_1+W_2$, $W_1 \cup W_2 \subseteq W_1+W_2$

proof:-

i) Let W_1 & W_2 be two subspaces of $V(F)$.

let $\alpha, \beta \in W_1+W_2$ then

$\alpha = \alpha_1+\alpha_2$ & $\beta = \beta_1+\beta_2$, where $\alpha_1, \beta_1 \in W_1$ & $\alpha_2, \beta_2 \in W_2$.

let $a, b \in F$ then $a\alpha_1, b\beta_1 \in W_1$ ($\because W_1$ is a subspace).

$a\alpha_1+b\beta_1 \in W_1$

$a\alpha_2, b\beta_2 \in W_2$ ($\because W_2$ is a subspace)

$a\alpha_2+b\beta_2 \in W_2$

$$a\alpha+b\beta = a(\alpha_1+\alpha_2)+b(\beta_1+\beta_2)$$
$$= (a\alpha_1+a\alpha_2+b\beta_1+b\beta_2)$$
$$= (a\alpha_1+b\beta_1)+(a\alpha_2+b\beta_2) \in W_1+W_2$$

$a, b \in F$ & $\alpha, \beta \in W_1+W_2 \Rightarrow a\alpha+b\beta \in W_1+W_2$.

ii) let $\alpha_1 \in W_1$ & $\bar{0} \in W_2 \Rightarrow \alpha_1 + \bar{0} \in W_1 + W_2$.

$\Rightarrow \alpha_1 \in W_1 + W_2$

Now $\alpha \in W_1$ & $\alpha_1 \in W_1 + W_2 \Rightarrow W_1 \subseteq W_1 + W_2$.

lly Suppose $\alpha_2 \in W_2$, $\bar{0} \in W_1 \Rightarrow \alpha_2 + \bar{0} \in W_2 + W_1$

$\Rightarrow \alpha_2 \in W_1 + W_2$.

Now $\alpha_2 \in W_2$, $\alpha_2 \in W_1 + W_2 \Rightarrow W_2 \subseteq W_1 + W_2$

Hence $W_1 \cup W_2 \subseteq W_1 + W_2$.

Linear Combination of vectors

Linear combination of vectors :-

Def :- Let $V(F)$ be a vector space. If $\alpha_1, \alpha_2 \ldots \alpha_n \in V$ then any vector $\gamma = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n$; where $a_1, a_2 \ldots a_n \in F$ is called linear combination of the vectors $\alpha_1, \alpha_2, \ldots \alpha_n$.

Linear span of a set :-

Let 'S' be a non-Empty subset of a vector space $V(F)$. The linear span of 'S' is the set of all possible linear combination of finite subsets of 'S'. The linear span of 'S' is denoted by $L(S)$.

$\therefore L(S) = \{\gamma : \gamma = \sum a_i \alpha_i \ ; \ a_i \in F, \ \alpha_i \in S\}$.

Theorem :- The linear span $L(S)$ of any subset 'S' of a vector space $V(F)$ is a subspace of $V(F)$.

Proof :- Let $\alpha, \beta \in L(S)$ & $a, b \in F$.

then $\alpha = a_1\alpha_1 + a_2\alpha_2 + \cdots + a_m\alpha_m$

$\beta = b_1\beta_1 + b_2\beta_2 + \cdots + b_n\beta_n$.

where a_i's, b_i's $\in F$, & α_i's, β_i's $\in L(S)$

Now $a\alpha + b\beta = a(a_1\alpha + a_2\alpha_2 + \cdots + a_m\alpha_m) + b(b_1\beta_1 + b_2\beta_2 + \cdots + b_n\beta_n)$.

$a\alpha+b\beta = aa_1\alpha_1 + aa_2\alpha_2 + \cdots + aa_m\alpha_m + bb_1\beta_1 + bb_2\beta_2 + \cdots + bb_n\beta_n$.

$= (aa_1)\alpha_1 + (aa_2)\alpha_2 + \cdots + (aa_m)\alpha_m + (bb_1)\beta_1 + (bb_2)\beta_2 + \cdots + (bb_n)\beta_n$

$a\alpha+b\beta$ is a linear combination of finite set $\alpha_1, \alpha_2 \cdots \alpha_m$, $\beta_1, \beta_2, \cdots \beta_n$ of element of S.

$\therefore a\alpha+b\beta \in L(S)$

$\therefore$ for $a, b \in F$, $\alpha, \beta \in L(S)$ we have $a\alpha+b\beta \in L(S)$

Hence $L(S)$ is a subspace of $V(F)$.

Theorem:- Let $v_1, v_2 \cdots v_m$ be vectors in a vector space V. Let 'U' be the set consisting of all linear combinations of the $v_1, v_2 \cdots v_m$. Then 'U' is a subspace of 'V' spanned by the vectors $v_1, v_2 \cdots v_m$. 'U' is said to be the vector space generated by $v_1, v_2 \cdots v_m$.

proof:- Let $u_1 = a_1v_1 + \cdots + a_mv_m$ and

$u_2 = b_1v_1 + \cdots + b_mv_m$

then $u_1+u_2 = (a_1v_1 + \cdots + a_mv_m) + (b_1v_1 + \cdots + b_mv_m)$.

$= (a_1+b_1)v_1 + \cdots + (a_m+b_m)v_m$.

u_1+u_2 is a linear combination of $v_1, v_2 \cdots v_m$.

thus u_1+u_2 is in U. in vector addition.

Let 'c' be an arbitrary scalar.

then $cu_1 = c(a_1v_1 + a_2v_2 + \cdots + a_mv_m)$

$= (ca_1)v_1 + (ca_2)v_2 + \cdots + (ca_m)v_m$.

cu_1 is a linear combination of $v_1, v_2 \cdots v_m$.

$\therefore cu_1$ is in U.

'U' is closed under scalar multiplication.

thus 'U' is a subspace of V.

By the def of 'U', every vector in 'U' can be written as a linear combination of $v_1, v_2 \ldots v_m$.

thus $v_1, v_2 \ldots v_m$ span 'U'.

problems:-

① Express the vector $\alpha = (1,-2,5)$ as a linear combination of the vectors given that $e_1 = (1,1,1)$, $e_2 = (1,2,3)$ and $e_3 = (2,-1,1)$.

Sol:- Given that $\alpha = (1,-2,5)$, & $e_1 = (1,1,1)$, $e_2 = (1,2,3)$

$e_3 = (2,-1,1)$.

let $\alpha = xe_1 + ye_2 + ze_3$.

$(1,-2,5) = x(1,1,1) + y(1,2,3) + z(2,-1,1)$

$(1,-2,5) = (x+y+2z,\ x+2y-z,\ x+3y+z)$

comparing on both sides, we get.

$x+y+2z = 1 \rightarrow ①$

$x+2y-z = -2 \rightarrow ②$

$x+3y+z = 5 \rightarrow ③$

solving ① & ②, we get

$$\begin{array}{r} \cancel{x}+y+2z = 1 \\ \cancel{x}+2y-z = -2 \\ - \quad - \quad + \quad + \\ \hline -y+3z = 3 \rightarrow ④ \end{array}$$

solving ④ & ⑤, we get

$$\begin{array}{r} -\cancel{y}+3z = 3 \\ \cancel{y}+2z = 7 \\ \hline 5z = 10 \Rightarrow \boxed{z=2} \end{array}$$

solving ② & ③, we get

$$\begin{array}{r} \cancel{x}+3y+z = 5 \\ \cancel{x}+2y-z = -2 \\ - \quad - \quad + \quad + \\ \hline y+2z = 7 \rightarrow ⑤ \end{array}$$

sub 'z' value in ④, we get

$-y+3(z) = 3$

$-y+3(2) = 3$

$-y = 3-6$

$\cancel{-}y = \cancel{-}3 \Rightarrow \boxed{y=3}$

Sub y and z values in Eq ①, we get

$x+y+2z=1$

$x+3+4=1$ (It is Consistent Unique Solution)

$x+7=1 \Rightarrow x=1-7 \Rightarrow x=-6.$

$\therefore$ the eq'n have the solution $x=-6, y=3, z=2$

Hence $\alpha=-6e_1+3e_2+2e_3$.

$(1,-2,5)=-6(1,1,1)+3(1,2,3)+2(2,-1,1)$.

② Show that the vector $\alpha=(2,-5,3)$ in R^3 cannot be expressed as linear combination of the vector $e_1=(1,-3,2)$, $e_2=(2,-4,-1)$ $e_3=(1,-5,7)$

Sol:- Given that $\alpha=(2,-5,3)$.

$e_1=(1,-3,2)$, $e_2=(2,-4,-1)$, $e_3=(1,-5,7)$.

the linear combination $\alpha=xe_1+ye_2+ze_3$.

$(2,-5,3)=x(1,-3,2)+y(2,-4,-1)+z(1,-5,7)$.

$(2,-5,3)=(x+2y+z, -3x-4y-5z, 2x-y+7z)$

$x+2y+z=2$

$-3x-4y-5z=-5$

$2x-y+7z=3$.

$$[A\ B]=\begin{bmatrix}1 & 2 & 1 & 2\\ -3 & -4 & -5 & -5\\ 2 & -1 & 7 & 3\end{bmatrix}$$

$R_2 \to R_2+3R_1$

$R_3 \to R_3-2R_1$

$$\begin{bmatrix}1 & 2 & 1 & 2\\ 0 & 2 & -2 & 1\\ 0 & -5 & 5 & -1\end{bmatrix}$$

$R_3 \to 2R_3+5R_2$
$$\begin{bmatrix}1 & 2 & 1 & 2\\ 0 & 2 & -2 & 1\\ 0 & 0 & 0 & 3\end{bmatrix}$$

$x+2y+z=2$

$2y-2z=1$

$(0\neq 3)$

the above system is inconsistent.

$\therefore$ there is no solution.

R^3 cannot be expressed as linear combination of the vector.

'α' is cannot linear combination.

3) The subset $S=\{(1,0,0),(0,1,0),(0,0,1)\}$ of $V_3(R)$ Generators the entire vectorspace $V_3(R)$.

Sol:- Given that $S=\{(1,0,0),\ (0,1,0),\ (0,0,1)\}$.

Let $(a,b,c) \in V_3(R)$ then

$(a,b,c) = a(1,0,0)+b(0,1,0)+c(0,0,1)\}$

$(a,b,c) \in L(S)$.

Now $(a,b,c) \in V_3(R)$ & $(a,b,c) \in L(S)$ of $L_3(R) \subseteq L(S)$

W.K.T $L(S) \subseteq V_3(R)$.

$\therefore L(S) = V_3(R)$.

$\therefore$ the given subset generates the entire vectorspace $V_3(R)$.

H.W

4) Determine whether or not the vector $(-1,1,5)$ is a linear combination of vectors $(1,2,3)$, $(0,1,4)$ and $(2,3,6)$

5) Express the vector $(4,5,5)$ as a linear combination of vectors $(1,2,3)$, $(-1,1,4)$ and $(3,3,2)$.

Sol:- Given that $\alpha=(4,5,5)$.

$v_1=(1,2,3)$, $v_2=(-1,1,4)$, $v_3=(3,3,2)$.

$c_1v_1+c_2v_2+c_3v_3=\alpha$.

$c_1(1,2,3)+c_2(-1,1,4)+c_3(3,3,2)=(4,5,5)$.

$c_1-c_2+3c_3=4$, $2c_1+c_2+3c_3=5$, $3c_1+4c_2+2c_3=5$.

using gauss - Jordan method

The Augmented matrix $[A\ B] = \begin{pmatrix} 1 & -1 & 3 & 4 \\ 2 & 1 & 3 & 5 \\ 3 & 4 & 2 & 5 \end{pmatrix}$

$R_2 \to R_2-2R_1$
$R_3 \to R_3-3R_1$
$\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -3 & -3 \\ 0 & 7 & -7 & -7 \end{bmatrix}$

$R_3 \to 3R_3 - 7R_2$ $\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{array}{cccc} 0 & 21 & -21 & -21 \\ -0 & -21 & 21 & 21 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$R_2 \to \frac{R_2}{3}$ $\begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x - y + 3z = 4$
$y - z = -1$
$z = k$

$\Rightarrow y - k = -1$
$y = -1 + k$
$y = k - 1$

$x - (k-1) + 3k = 4$
$x - k + 1 + 3k = 4$
$x + 2k + 1 = 4$
$x = 4 - 1 - 2k$
$\boxed{x = 3 - 2k}$

the system of linear eq'n has infinitely many solutions.

the system is consistent.

$x = 3 - 2k$, $y = k-1$, $z = k$.

the vector $(4,5,5)$ can be expressed in many ways as a linear combinations of vectors $(1,2,3)$, $(-1,1,4)$ and $(3,3,2)$.

$(4,5,5) = (3-2k)(1,2,3) + (k-1)(-1,1,4) + k(3,3,2)$

Ex: $k=3$ gives

$(4,5,5) = (-3)(1,2,3) + (2)(-1,1,4) + 3(3,3,2)$.

⑥ In each the following determine whether the first function is a linear combination of the functions that follows

ⓐ $f(x) = 3x^2 + 2x + 9$
$g(x) = x^2 + 1$
$h(x) = x + 3$

H.W ⓑ $f(x) = x^2 + 4x + 5$
$g(x) = x^2 + x - 1$
$h(x) = x^2 + 2x + 1$.

Sol: a) $f(x) = 3x^2 + 2x + 9$, $g(x) = x^2 + 1$, $h(x) = x + 3$.

To express $f(x)$ is a linear combination of $g(x)$ and $h(x)$, we have to examine following identity

$c_1 g(x) + c_2 h(x) = f(x)$, c_1 & c_2 are scalars

$c_1(x^2+1) + c_2(x+3) = 3x^2+2x+9.$

$c_1x^2 + c_1 + c_2x + 3c_2 = 3x^2+2x+9.$

$c_1x^2 + c_2x + (c_1+3c_2) = 3x^2+2x+9$

Equating corresponding coefficients

we get $c_1 = 3$, $c_2 = 2$, $c_1+3c_2 = 9$ ..

thus $f(x)$ is a linear combination of $g(x)$ and $h(x)$.

i.e $f(x) = 3g(x) + 2h(x)$

$(3x^2+2x+9) = 3(x^2+1) + 2(x+3)$

⑨ Let $\vec{v}_1$ and $\vec{v}_2$, $\vec{v}$ be vectors in a vector space $V(F)$. Let $\vec{v}$ be a linear combination of $\vec{v}_1$ and $\vec{v}_2$. If 'c_1' and 'c_2' are non-zero scalars show that $\vec{v}$ is also linear combination of $c_1\vec{v}_1$ & $c_2\vec{v}_2$.

Show that $\vec{v}$ is also linear combination of $\vec{v}_1$ and $\vec{v}_2$.

Sol:- Given that $\vec{v}$ is a linear combination of $\vec{v}_1$ and $\vec{v}_2$.

there exists scalars 'a_1' and 'a_2' such that

$\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 \rightarrow ①$

let 'c_1' and 'c_2' are non-zero scalars, now we have to show that $\vec{v}$ is a linear combination of $c_1\vec{v}_1$ & $c_2\vec{v}_2$.

for this from eq'n ①

$$\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2$$

$$= \left(\frac{a_1}{c_1}\right)(c_1\vec{v}_1) + \frac{a_2}{c_2}(c_2\vec{v}_2) \quad ; \because c_1 \neq 0, \ \& \ c_2 \neq 0.$$

$$= b_1(c_1\vec{v}_1) + b_2(c_2\vec{v}_2) \quad \left(\because \frac{a_1}{c_1} = b_1, \ \frac{a_2}{c_2} = b_2\right)$$

$\Rightarrow \vec{v}$ is linear combination of $c_1\vec{v}_1 + c_2\vec{v}_2$

⑧ Theorem:- Let $\vec{v}_1$ and $\vec{v}_2$ span a subspace 'U' of a vector space V. let 'k_1' and 'k_2' be non-zero scalars. show that $k_1\vec{v}_1$ and $k_2\vec{v}_2$ also spans 'U'.

Sol:- Given that $\vec{v}_1$ and $\vec{v}_2$ spans a subspace U.

$\Rightarrow$ 'U' is the set of all linear combination of $\vec{v}_1$ and $\vec{v}_2$

let $\vec{v} \in U$.

$\Rightarrow \vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2$, where a_1 & a_2 are non-zero scalars

$\vec{v} = a_1 \frac{k_1}{k_1} \vec{v_1} + \frac{a_2}{k_2} \cdot k_2 \vec{v_2}$, where k_1 & k_2 are non-zero elements

$= \left(\frac{a_1}{k_1}\right) k_1 \vec{v_1} + \left(\frac{a_2}{k_2}\right) k_2 \vec{v_2}$

'v' is a linear combination of $k_1\vec{v_1}$ and $k_2\vec{v_2}$

i.e $\vec{v} \in U$ and $\vec{v}$ is a linear combination of $k_1\vec{v_1}$ and $k_2\vec{v_2}$

$\Rightarrow$ 'U' is spanned by $k_1\vec{v_1}$ & $k_2\vec{v_2}$

$\Rightarrow$ $k_1\vec{v_1}$ and $k_2\vec{v_2}$ spans a subspace 'U'.

Hence the theorem is proved.

(9) Let 'U' be the vector space generated by the functions $f(x) = x+1$ and $g(x) = 2x^2 - 2x + 3$. Show that function $h(x) = 6x^2 - 10x + 5$ lies in U.

Sol:- Given that

'U' be the vector space generated by the functions

$f(x) = x+1$, $g(x) = 2x^2 - 2x + 3$

Now, we have to show that the function $h(x) = 6x^2 - 10x + 5$ lies in U.

For this it is enough to show that $h(x)$ is a linear combination of $f(x)$ & $g(x)$.

Let us consider for some scalars 'c_1' and 'c_2' we have

$$c_1 f(x) + c_2 g(x) = h(x).$$

$\Rightarrow c_1(x+1) + c_2(2x^2 - 2x + 3) = 6x^2 - 10x + 5$

$\Rightarrow c_1 x + c_1 + 2c_2 x^2 - 2c_2 x + 3c_2 = 6x^2 - 10x + 5$

$\Rightarrow 2c_2 x^2 + (c_1 - 2c_2)x + (c_1 + 3c_2) = 6x^2 - 10x + 5$.

Now, we equate corresponding co-efficients on both sides

$2c_2 = 6 \Rightarrow c_2 = 3$ | $c_1 - 2c_2 = -10$ | $c_1 + 3c_2 = 5$

$c_1 + 3c_2 = 5$

$c_1 + 3(3) = 5$

$c_1 = 5 - 9$

$c_1 = -4$; $c_2 = 3$

Thus $h(x)$ is a linear combination of $f(x)$ & $g(x)$.

$\Rightarrow h(x) = 4f(x) + 3g(x)$.

$6x^2 - 10x + 5 = 4(x+1) + 3(2x^2 - 2x + 3)$.

(10) Given three vectors in the subspace of R^3 generated by vectors $(1,2,3)$, $(1,2,0)$.

Sol:- Let $\vec{v_1} = (1,2,3)$, $\vec{v_2} = (1,2,0)$

The following '3' vectors are generated by $\vec{v_1}$ and $\vec{v_2}$

(1) $\vec{v_1} + \vec{v_2} = (1,2,3) + (1,2,0) = (1+1, 2+2, 3+0) = (2,4,3)$

(2) $\vec{v_1} - \vec{v_2} = (1,2,3) - (1,2,0) = (1-1, 2-2, 3-0) = (0,0,3)$.

(3) $2\vec{v_1} + 3\vec{v_2} = 2(1,2,3) + 3(1,2,0) = (2,4,6) + (3,6,0)$

$= (2+3, 4+6, 6+0)$

$= (5, 10, 6)$.

(11) Show that the vectors $(1,2,0)$, $(0,1,-1)$, and $(1,1,2)$ span R^3.

Sol:- Given $v_1 = (1,2,0)$, $v_2 = (0,1,-1)$, $v_3 = (1,1,2)$. span R^3. it is enough to show that $\alpha = (a,b,c)$ is a linear combination of v_1, v_2, v_3.

Let $(a,b,c) \in R^3$; $S = \{(1,2,0), (0,1,-1), (1,1,2)\}$.

for any scalars x, y, z, we have.

Let us consider $\alpha = (x)v_1 + y v_2 + z v_3$.

$(a,b,c) = x(1,2,0) + y(0,1,-1) + z(1,1,2)$

$(a,b,c) = (x+z, 2x+y+z, -y+2z)$

$x + z = a \rightarrow (1)$ (using elimination method also).

$2x + y + z = b \rightarrow (2)$

$-y + 2z = c \rightarrow (3)$

By gauss - Jordan elimination method.

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Additional Sheet

ID NUMBER:

Invigilator signature

$$[A\ B] = \begin{bmatrix} 1 & 0 & 1 & a \\ 2 & 1 & 1 & b \\ 0 & -1 & 2 & c \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \sim \begin{bmatrix} 1 & 0 & 1 & a \\ 0 & 1 & -1 & b-2a \\ 0 & -1 & 2 & c \end{bmatrix}$

$R_3 \to R_3 + R_2 \sim \begin{bmatrix} 1 & 0 & 1 & a \\ 0 & 1 & -1 & b-2a \\ 0 & 0 & 1 & c+b-2a \end{bmatrix}$

$R_1 \to R_1 - R_3 \sim \begin{bmatrix} 1 & 0 & 0 & a-c-b+2a \\ 0 & 1 & -1 & b-2a \\ 0 & 0 & 1 & c+b-2a \end{bmatrix}$

$R_2 \to R_2 + R_3 \sim \begin{bmatrix} 1 & 0 & 0 & 3a-b-c \\ 0 & 1 & 0 & b-2a+c+b-2a \\ 0 & 0 & 1 & c+b-2a \end{bmatrix}$

$[A\ B] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3a-b-c \\ 0 & 1 & 0 & -4a+2b+c \\ 0 & 0 & 1 & -2a+b+c \end{array}\right]$

$x = 3a-b-c,\ y = -4a+2b+c\ ;\ z = -2a+b+c$

another method

solving eq'n

$x+z=a \to ①$

$2x+y+z=b \to ②$

$-y+2z=c \to ③$

② & ③.

$2x+y+z=b$

$-y+2z=c$

$2x+3z=b+c \to ④$

$2\times$① & ④

$2x+2z=2a$

$2x+3z=b+c$

$-z = 2a-b-c$

$\boxed{z=-2a+b+c}$

① $\Rightarrow x-2a+b+c=a$

$\boxed{x=3a-b-c}$

③ $\Rightarrow -y+2z=c$

$y = 2z-c$

$= -4a+2b+2c-c$

$\boxed{y=-4a+2b+c}$

$(a,b,c) = x(1,2,0) + y(0,1,-1) + z(1,1,2)$

$(a,b,c) = (3a-b-c)(1,2,0) + (-4a+2b+c)(0,1,-1) + (-2a+b+c)(1,1,2)$.

The vectors $(1,2,0)$, $(0,1,-1)$, and $(1,1,2)$ span R^3.

(12) Determine $(a,b,c) \in L(S)$

Now $(a,b,c) \in R^3$, $(a,b,c) \in L(S) \Rightarrow R^3 \subseteq L(S)$.

W.K.T $L(S) \subseteq R^3$. ($\because L(S)$ is c.m of a v.s.s R^3 is v.s)

Hence $L(S) = R^3$.

(12) Determine whether the following vectors span $R^3(R)$.

***

a) $\{(2,1,0), (-1,3,1), (4,5,0)\}$

b) $\{(1,2,1), (-1,3,0), (0,5,1)\}$

c) $\{(1,0,0), (1,1,0), (1,1,1)\}$

d) $\{(1,2,3), (0,1,2), (0,0,1)\}$

e) $\{(1,2,1), (2,1,0), (1,-1,2)\}$.

13) In the following sets of vectors, determine whether the first vector is a linear combination of other vector.

$\{(-3,3,7); (1,-1,2), (2,1,0), (-1,2,1)\}$.

Sol:- Let $\bar{\alpha} = (-3,3,7)$.

And $v_1 = (1,-1,2)$, $v_2 = (2,1,0)$, $v_3 = (-1,2,1)$

To express $\bar{\alpha}$ is a linear combination of v_1, v_2, v_3 or not, we get the following system of linear eq'n of the linear combination.

$$\alpha = x v_1 + y v_2 + z v_3.$$

$$(-3,3,7) = x(1,-1,2) + y(2,1,0) + z(-1,2,1)$$

$$(-3,3,7) = (x+2y-z, -x+y+2z, 2x+z).$$

$$x+2y-z = -3 \rightarrow (1)$$

$$-x+y+2z = 3 \rightarrow (2)$$

$$2x+z = 7 \rightarrow (3)$$

Using gauss Jordan elimination method.

$$[AB] = \begin{bmatrix} 1 & 2 & -1 & -3 \\ -1 & 1 & 2 & 3 \\ 2 & 0 & 1 & 7 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & 3 & 1 & 0 \\ 0 & -4 & 3 & 13 \end{bmatrix}$$

$R_3 \to 3R_3 + 4R_2$

$$\begin{bmatrix} 1 & 2 & -1 & -3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 13 & 39 \end{bmatrix}$$

$$\begin{array}{cccc} 0 & -12 & 9 & 39 \\ 0 & 12 & 4 & 0 \\ \hline 0 & 0 & 13 & 39 \end{array}$$

$R_3 \to \frac{R_3}{13}$ $\begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

$R_1 \to 3R_1 - 2R_2$ $\begin{pmatrix} 3 & 0 & -5 & -9 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

$$\begin{array}{cccc} 3 & 6 & -3 & -9 \\ 0 & -6 & -2 & 0 \\ \hline 3 & 0 & -5 & -9 \end{array}$$

$R_1 \to R_1 + 5R_3$
$R_2 \to R_2 - R_3$ $\begin{pmatrix} 3 & 0 & 0 & 6 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

$R_1 \to \frac{R_1}{3}$
$R_2 \to \frac{R_2}{3}$ $\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix}$; It is reduced echelon form

By this $x=2, y=-1, z=3$.

thus, the vector $(-3,3,7)$ is a linear combination of vectors $(1,-1,2)$, $(2,1,0)$, and $(-1,2,1)$.

i.e., $(-3,3,7) = 2(1,-1,2) + (-1)(2,1,0) + 3(-1,2,1)$.

(14) Let 'U' be the subspace generated by the vectors $(1,2,0)$ and $(-3,1,2)$. Let V be the subspace generated by the vectors $(-1,5,2)$ and $(4,1,-2)$. Show that $U=V$.

Sol:- Given that

(i) $(1,2,0)$ and $(-3,1,2)$ span U.

(ii) $(-1,5,2)$ and $(4,1,-2)$ span V.

Now, we have to show that $U=V$.

Let $\vec{\alpha}$ be a vector in U. Let us show that $\vec{\alpha}$ is in V.

Since $\vec{\alpha}$ is in U, there exists scalars a & b such that

$\vec{\alpha} = a(1,2,0) + b(-3,1,2) \to ①$

$\vec{u} = (a, 2a, 0) + (-3b, b, 2b)$

$\vec{u} = (a-3b, 2a+b, 2b) \rightarrow (2)$

Let us see if we can write $\vec{\alpha}$ as a linear combination of

$(-1, 5, 2)$ and $(4, 1, -2)$

$\therefore$ there exists some scalars 'p' & 'q' such that

$\vec{u} = p(-1, 5, 2) + q(4, 1, -2) \rightarrow (3)$

$\vec{u} = (-p, 5p, 2p) + (4q, q, -2q)$

$= (-p+4q, 5p+q, 2p-2q) \rightarrow (4)$

from (2) & (4)

such p, q would have to satisfy

$-p+4q = a-3b \rightarrow (5)$

$5p+q = 2a+b \rightarrow (6)$

$2p-2q = 2b \rightarrow (7)$

$p-q = b$

Solve (6) & (7)

$5p+\cancel{q} = 2a+b$

$p-\cancel{q} = b$

$6p = 2a+2b$

$\boxed{p = \frac{a+b}{3}}$

from (7)

$p-q = b$

$\frac{a+b}{3} - q = b$

$q = \frac{a+b}{3} - b$

$= \frac{a+b-3b}{3}$

$\boxed{q = \frac{a-2b}{3}}$

from (3), $\bar{u} = \left(\frac{a+b}{3}\right)(-1, 5, 2) + \left(\frac{a-2b}{3}\right)(4, +1, -2)$

$\vec{u}$ is a vector in 'V'.

Conversely, let $\vec{v}$ be a vector in V. Similar to above we can show that $\vec{v}$ is in U.

$\therefore \boxed{U = V}$

(15) Let 'U' be the subspace of R^3 generated by the vectors $(3, -1, 2)$ and $(1, 0, 4)$. Let V be the subspace of R^3 generated by the vectors $(4, -1, 6)$ and $(1, -1, -6)$. Show that $U = V$.

$0 = (0, 0, 0 \dots 0_n)$ identity
$-\alpha = (-x_1, -x_2, -x_3 \dots -x_n)$ inverse

④ show that the set of all triades (x_1, x_2, x_3) where x_1, x_2, x_3 real numbers forms a vector space over the field of real numbers with respect to the operations of addition and scalar multiplication defined as

1) $(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$

2) $c(x_1, x_2, x_3) = (cx_1, cx_2, cx_3)$

⑤ let V be the set of all pairs (x, y) of real numbers and R ~~due to~~ is a field of real numbers. show that with the operation i) $(x_1 + y_1) + (x_2, y_2) = (x_1 + x_2, 0)$

ii) $a(x_1, y_1) = (ax_1, y_1)$ then $V(R)$ is or not a vector space?

let V = The set of all pairs

$= \{ \alpha = (x_1, y_1) / \alpha = (x_1, y_1) \in V, x_1, y_1 \in R \}$

let $\alpha = (x_1, y_1)$, $\beta = (x_2, y_2)$, $\gamma = (x_3, y_3) \in V$ and $x_1, y_1, x_2, y_2, x_3, y_3 \in R$

I) prove that $(V, +)$ is an abelian rule

i) $\forall \alpha, \beta \in V \Rightarrow \alpha + \beta = (x_1, y_1) + (x_2, y_2)$

$= (x_1 + x_2, 0) \in V$

$(x_1, x_2 \in R \Rightarrow x_1 + x_2 \in R)$

ii) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$

Take $(\alpha + \beta) + \gamma = [(x_1, y_1) + (x_2, y_2)] + (x_3, y_3)$

$= [(x_1 + x_2), 0] + (x_3, y_3) = [(x_1 + x_2 + x_3), 0]$

$\alpha+(\beta+\gamma) = (x_1, y_1) + [(x_2, y_2) + (x_3, y_3)]$

$= (x_1, y_1) + [(x_2 + x_3), 0]$

$= [(x_1 + x_2 + x_3), 0]$ by (1)

$(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$

iii) Identity $\alpha + 0 = \alpha$

let $\alpha = (x_1, y_1)$, $0 = (0, 0)$

$(x_1, y_1)(0, 0) = ((x_1 + 0), 0)$

$= (x_1, 0)$

~~It is~~ $\neq \alpha$ In this Identity element is not exis

It is not satisfy the abelian rule.

~~Beco~~ Because $V(R)$ is ~~a~~ not a vector space.

(c) let v be the set of all ordered pairs ...

from I, II, III the given R^n is a vector space.

(4)

A) $V = \{ \alpha = (x_1, x_2, x_3) / x_1, x_2, x_3 \in R \}$

i) $(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$

2) $c(x_1, x_2, x_3) = (cx_1, cx_2, cx_3)$

I To prove abelian rule $(V, +)$

i) closero

$\alpha = (x_1, x_2, x_3), \beta = (y_1, y_2, y_3) \in V$

$\alpha + \beta = (x_1, x_2, x_3) + (y_1, y_2, y_3)$

$= (x_1+y_1, x_2+y_2, x_3+y_3) \in V$

ii) associated

$\alpha = (x_1, x_2, x_3), \beta = (y_1, y_2, y_3), \gamma = (z_1, z_2, z_3)$

$(\alpha+\beta)+\gamma = \alpha + (\beta+\gamma)$

$(\alpha+\beta)+\gamma = [(x_1, x_2, x_3) + (y_1, y_2, y_3)] + [z_1, z_2, z_3]$

$= [(x_1+y_1, x_2+y_2, x_3+y_3)] + [z_1, z_2, z_3]$

$= (x_1+y_1+z_1, x_2+y_2+z_2, x_3+y_3+z_3)$

$\alpha + (\beta+\gamma) = (x_1, x_2, x_3) [(y_1, y_2, y_3) + [z_1, z_2, z_3]]$

$= (x_1, x_2, x_3) [y_1+z_1, y_2+z_2, y_3+z_3]$

$= (x_1+y_1+z_1), (x_2+y_2+z_2), (x_3+y_3+z_3)$

$(\alpha+\beta)+\gamma = \alpha + (\beta+\gamma)$

iii) Identity

$\alpha = (x_1, x_2, x_3) \in V$ and $0 \in V$

$\alpha + 0 = (x_1, x_2, x_3) + (0, 0, 0)$

$= (x_1, x_2, x_3)$

$= \alpha$

0 is identity element

iv) Inverse property

$\alpha = (x_1, x_2, x_3)$, $-\alpha = (-x_1, -x_2, -x_3)$

$\alpha + (-\alpha) = (-\alpha) + \alpha = 0$

take $\alpha + (-\alpha) = (x_1, x_2, x_3) + (-x_1, -x_2 - x_3)$

$= 0$

$-\alpha$ ~~ad~~ is inverse additive property.

v) commutative

$\alpha = (x_1, x_2, x_3)$, $\beta = (y_1, y_2, y_3) \in V$

$\alpha + \beta = \beta + \alpha$

$\alpha + \beta = (x_1, x_2, x_3) + (y_1, y_2, y_3)$

$= (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

$\beta + \alpha = (y_1, y_2, y_3) + (x_1, x_2, x_3)$

$= (x_1 + y_1, x_2 + y_2 + x_3 + y_3)$

$\alpha + \beta = \beta + \alpha$

II <u>Scalar multiplication</u>

by ② the given vector is verify scalar multiplication

III <u>distribution property</u>

① $a(\alpha + \beta) = a\alpha + a\beta$

$\alpha = (x_1, x_2, x_3)$, $\beta = (y_1, y_2, y_3) \in V$

$a(\alpha + \beta) = a[(x_1, x_2, x_3) + (y_1, y_2, y_3)]$

$= a[(x_1 + y_1, x_2 + y_2, x_3 + y_3)]$

$= a(x_1 + y_1), a(x_2 + y_2) + a(x_3 + y_3)$

$a\alpha + a\beta = a(x_1, x_2, x_3) + a(y_1, y_2, y_3)$

$= ax_1, ax_2, ax_3 + ay_1, ay_2, ay_3$

$= a(x_1 + y_1) + a(x_2 + y_2) + a(x_3 + y_3)$

$a(\alpha + \beta) = a\alpha + a\beta$

② $(a+b)\alpha = a\alpha + b\alpha$

$\alpha = (x_1, x_2, x_3) \in V$

$(a+b)\alpha = (a+b)(x_1, x_2, x_3)$

$= (a+b)x_1, (a+b)x_2, (a+b)x_3$

$a\alpha + b\alpha = a(x_1, x_2, x_3) + b(x_1, x_2, x_3)$

$= ax_1, ax_2, ax_3 + bx_1, bx_2, bx_3$

$= (a+b)x_1, (a+b)x_2, (a+b)x_3$

③ $(ab)\alpha = a(b\alpha)$

$\alpha = (x_1, x_2, x_3) \in V$

$(ab)\alpha = ab(x_1, x_2, x_3)$

$= abx_1, abx_2, abx_3$

$a(b\alpha) = a[b(x_1, x_2, x_3)]$

$= a[bx_1, bx_2, bx_3]$

$= abx_1, abx_2, abx_3$

$(ab)\alpha = a(b\alpha)$

④ $1.\alpha = \alpha$

$\alpha = (x_1, x_2, x_3)$

$1.\alpha = 1(x_1, x_2, x_3) = (x_1, x_2, x_3)$

$= \alpha$

from I, II, III the given V is vector space.

① The set W of ordered triads $(x, y, 0)$ where $x, y \in F$ is a sub space of $V_3(F)$. given that $V_3(F)$ is a vector space and W = The set of all triads $= \{(x, y, 0) / x, y \in R\}$ here $W \subseteq V$.

(i) let $\alpha = (x_1, y_1, 0), \beta = (x_2, y_2, 0) \in W$ and x_1, y_1, x_2, y_2, a

Now we prove that $\alpha, \beta \in W$ and $a, b \in F$ then $a\alpha + b\beta \in W$!

A) take $a\alpha + b\beta = a(x_1, y_1, 0) + b(x_2, y_2, 0)$

$= (ax_1, ay_1, a0) + (bx_2, by_2, b0)$

$= [(ax_1 + bx_2), (ay_1 + by_2), 0] \in W$

$x_1, x_2, y_1, y_2, a, b \in F \Rightarrow ax_1 + bx_2, ay_1 + by_2 \in F$

$\therefore W$ is a sub space of $V(F)$

2) let p, q, r be the fixed element a field F. show that the set of all triads (x, y, z) of element of n, such that ~~px+qy~~ $+r$ $px+qy+rz=0$ is a vector subspace of $V_3(F)$.

A) given that F is field and $p, q, r \in F$

$w =$ The set of all triads $- \{(x,y,z) / x,y,z \in F$

given that $(x,y,z) \in w$ and $p, q, r \in F$ then

such that $px+qy+rz=0$

let $\alpha = (x_1, y_1, z_1) \in w$ then $px_1 + qy_1 + rz_1 = 0$ by 1.

$\beta = (x_2, y_2, z_2) \in w$ then ~~a, b ∈ F then aα+bβ ∈ w~~

$px_2 + qy_2 + rz_2 = 0$ by 1

Now we prove that $\alpha, \beta \in w$ and $a, b \in F$ then $a\alpha + b\beta \in w$.

Take $a\alpha + b\beta = a(x_1, y_1, z_1) + b(x_2, y_2, z_2)$

$= (ax_1, ay_1, az_1) + (bx_2, by_2, bz_2)$

$= (ax_1 + bx_2, ay_1 + by_2, az_1 + bz_2)$

by deff - 1

$(ax_1+bx_2, ay_1+by_2, az_1+bz_2) = p(ax_1+bx_2) + q(ay_1+by_2) + r(az_1+bz_2)$

$= pax_1 + pbx_2 + qay_1 + qby_2 + raz_1 + rbz_2$

$= a(px_1 + qy_1 + rz_1) + b(px_2 + qy_2 + rz_2)$

$= a(0) + b(0)$ by 1

$= 0$

$a\alpha + b\beta \in w$, w is a subspace of V

basis and