BRAD JONES Mid-Term Study Guide #1 10/18/2016

Ch 1: Functions:

Domain = inputs of the function
Range = outputs of the function

Eg.) $f(x) = 2x$

Domain	Range
$1 \rightarrow$	2
$2 \rightarrow$	4
$3 \rightarrow$	6
$4 \rightarrow$	8
$\infty \rightarrow$	∞

$f(x) + g(x) = (f+g)x$ = The domain of f plus the domain of g

$f(x) \div g(x) = (f/g)x$ = The domain of f plus domain of g; except where $g(x) = 0$

* When dividing functions, the denomonator cannot = 0 *

Domain cannot result in a denomonator of 0, or a negative Sqrt

- **Graphs of Functions:**

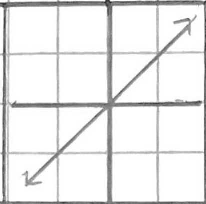
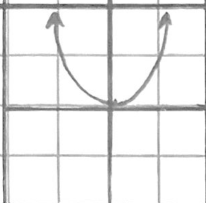
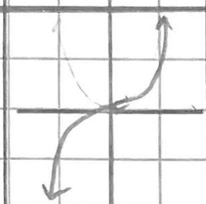
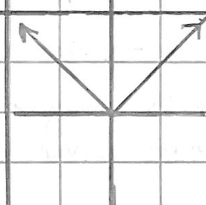
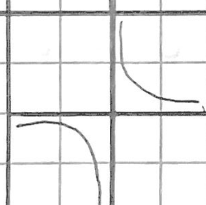
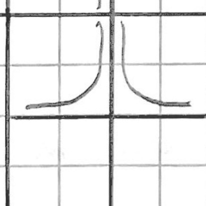
Most pass the vertical line test
Numbers in the domain cannot Result from multiple sources

- **Symmetry of Functions:**

Even Symmetry: Symmetry abot the y-axis (mirror flip)
$f(-x) = f(x)$

Odd Symmetry: Symmetry about the origin (180° rotation)
$f(-x) = -f(x)$

Library of functions:

$f(x) = x$		Domain: All $\mathbb{R}$	Symmetry: odd	
$f(x) = x^2$		Domain: All $\mathbb{R}$	Symmetry: even	
$f(x) = x^3$		Domain: All $\mathbb{R}$	Symmetry: odd	
$f(x) = \sqrt{x}$		Domain: All $\mathbb{R}$ where $x \geq 0$	Symmetry: even	
$f(x) = \lvert x \rvert$		Domain: All $\mathbb{R}$	Symmetry: even	
$f(x) = 1/x$		Domain: All $\mathbb{R}$ where $x \neq 0$	Symmetry: odd	
$f(x) = 1/x^2$		Domain: All $\mathbb{R}$ where $x \neq 0$	Symmetry: even	

Domain of a collection of functions:

$$f(x) = \begin{Bmatrix} -x-3, & x < -2 \\ 1/x, & -1 < x < 3 \end{Bmatrix}$$

Domain $= (-\infty, -2) \cup (-1, 3)$ plus $x \neq 0$

or

$(-\infty, -2) \cup (-1, 0) \cup (0, 3)$

Shifting graphs of functions:

Shifting vertically:

$f(x) \to f(x) \pm C$

Add or subtract outside of the function

eg. graph $\frac{-2}{(x+1)^2} + 3$

Stretching vertically:

$f(x) \to a \cdot f(x)$

Multiply outside of the function

Shifting horizontally:

$f(x) \to f(x+1)$

add or subtract within the function

* within the function, the graph moves opposite *

Stretching horizontally:

$f(x) \to f(\frac{1}{a}x)$

Shifting Order of Operations:

- horizontal shift
- flips + compression/stretching
- vertical shift

Chapter 2: Linear + Quadratic functions

$f(x) = mx + b$ general linear function

The domain of a line is always all $\mathbb{R}$ numbers unless $m = 0$. $m = 0$ is a horizontal line.

The dependant variable is m. if $m = 0$, then $y = b$

if $m =$ any $\mathbb{R}$, the range is any $\mathbb{R}$

Determine if a function is linear through data points:

eg

x	f(x)
1	2.5
3	4
5	5.5
7	7

1.) Determine rate of change between data points using $\Delta y / \Delta x = \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{4-2.5}{3-1} = \frac{1.5}{2}$$

$$\frac{5.5-4}{5-3} = \frac{1.5}{2}$$

$$\frac{7-5.5}{7-5} = \frac{1.5}{2}$$

A contant rate of change between all data points mean the function is linear

A quadratic function is linear when $a=0$

$ax^2 + bx + c$ = quadratic function

if $a=0$

$0 + bx + c$ = linear function

Finding the line of best fit:

eg

x	f(x)
1	2.5
3	4
5	5.5
7	7

Line of best fit formula:

$$m = \frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - \bar{x}^2}$$

* The average of all (x)(y), minus the avg of (x) times the avg. of (y). All over the avg of (x^2) minus the avg of $(x)^2$. *

Step 1: add 2 columns and 2 rows to the data point chart

	x	f(x)	x^2	xy
	x^1	y^1	x_1^2	$x_1 \cdot y_1$
	x^2	y^2	x_2^2	$x_2 \cdot y_2$
Sum				
Avg				

- Finding the line of best fit (cont.)
  - Step 2: Calculate the sum of all x and all y.
  - Step 3: Calculate the avg of all x and all y.
  - Step 4: Calculate the product of all x + y points.
  - Step 5: Calculate the squares of all x values.
  - Step 6: Calculate the sum of step 4 and step 5.
  - Step 7: Calculate the Avg. of Step 4 and Step 5
  - Step 8: Plug in values into formula for best fit.

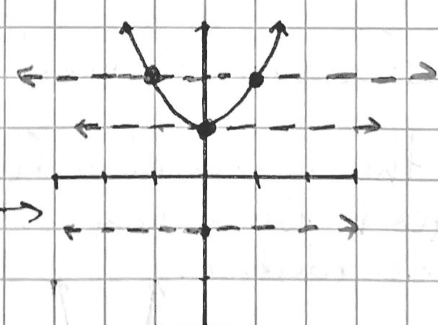
- Quadratic Functions:

A Quadratic function has form ax^2+bx+c

A Quadratic function has 0, 1, or 2 solutions

when $f(x)=2$; 2 solutions →
when $f(x)=1$; 1 solution →

when $f(x)<1$; no solutions →



The root of a Quadratic Function is found by setting $f(x)=0$

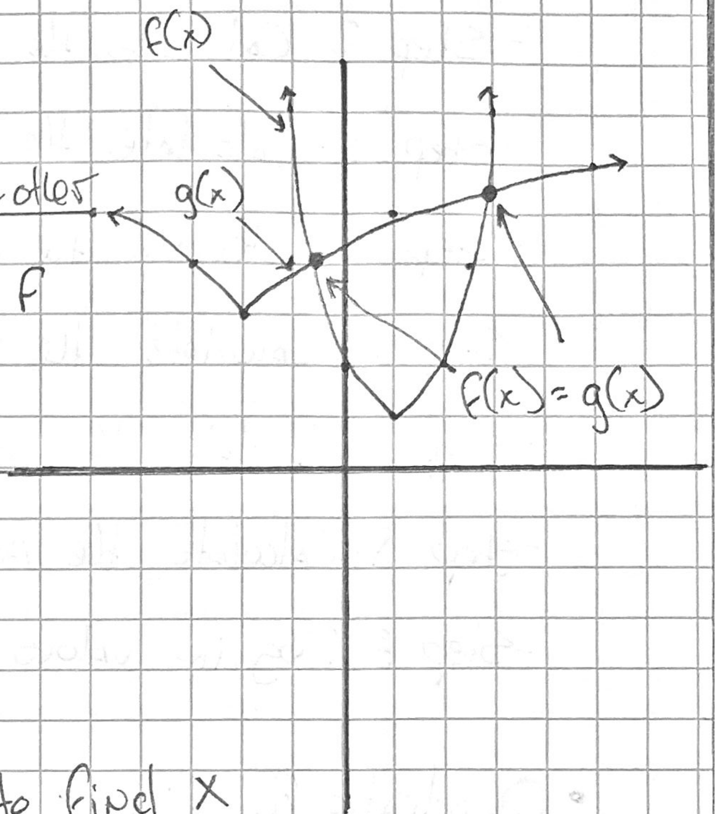
- $ax^2+bx+c=0$

Solve using the Quadratic formula:

- $x = \dfrac{-b \pm \sqrt{(b)^2-4(a)(c)}}{2a}$ Plug in the A, B, and C values and solve

- Finding the intersection point of 2 quadratic functions:

$$f(x) = Ax^2 + bx + C$$
$$g(x) = Dx^2 + Ex + F$$



- Set the functions equal to each other

$$Ax^2 + bx + C = Dx^2 + Ex + F$$

- Move all values to one side

$$Ax^2 + bx + C - Dx^2 - Ex - F = 0$$

- Combine like terms

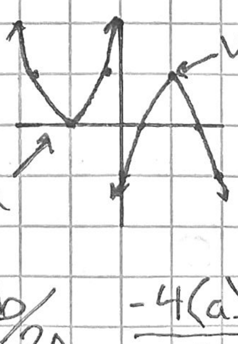
$$ADx^2 + BEx + CF = 0$$

- Use the quadratic formula to find X

- Plug X into the original equations and check for the same y value

- Properties of Quadratic Functions

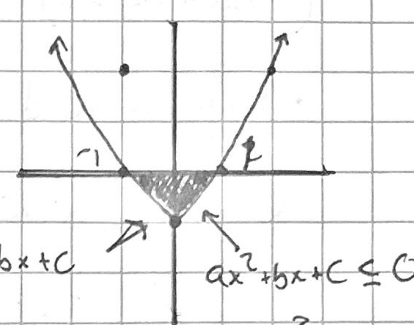
  - Finding the vertex of a function
    the vertex is the highest (or lowest) point of a quadratic function.



  - The vertex is $\left(\frac{-b}{2A}, \frac{-4(a)(c)}{2A}\right)$

  - The Range of a Quadratic is $\left(-b/2A, \text{ to } \infty, \text{ or } -\infty\right)$

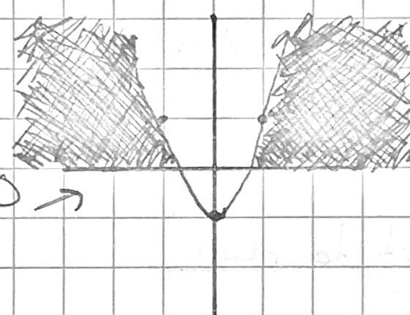
- Quadratic inequalities:
- $Ax^2 + bx + C \leq 0$
- Set equal to $0 \rightarrow Ax^2 + bx + C = 0$
- Solve for x and graph



- Evaluate: when is ax^2+bx+c less than or equal to 0?

inside the parabola; $f(x) \leq 0$ os

$\{-1 \leq x \leq 1\}$



- $Ax^2 + bx + C \geq 0 \rightarrow$

- Solving Quadritic Models

- finding a Quadratic function from data points

eg

x	$f(x)$	
-1	6	$= a + (-1)b + C$
1	0	$a + b + C$
3	2	$a(3)^2 + b(3) + C$

put the x values for the points in the quadratic equation and set equal to y

- Use Elimination and substitution to solve the set

eg subtract the first and second equation

$$\begin{array}{r} a - b + C = 6 \\ -\quad a + b + C = 0 \\ \hline -2b = 6 \end{array}$$

and subtract the third and second equation:

$$\begin{array}{r} 9a + 3b + C = 2 \\ -\quad a + b + C = 0 \\ \hline 8a + 2b = 2 \end{array}$$

• Solving a Quadratic Model (cont.)

$$\begin{Bmatrix} -2b = 6 \\ 8a + 2b = 2 \end{Bmatrix} = \begin{Bmatrix} b = -3 \\ a = 1 \end{Bmatrix}$$

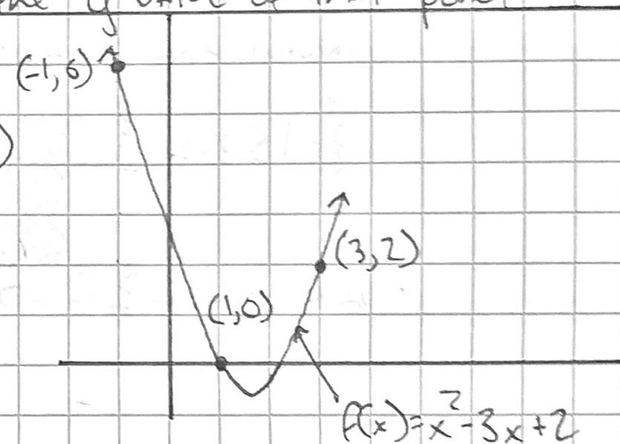
• Plug a and b into the original equations and solve c.

$$(-1, 6) \Rightarrow \overset{a}{(1)}x^2 - \overset{b}{(-3)}x + c = 6$$

$$c \quad -1 + 3 = 6$$
$$\quad -4 \quad\quad -4$$
$$\underline{c = 2}$$

• plug in the a and b value we found, use the x value of the point your using and set it equal to the y value of that point

• So $\begin{Bmatrix} a = 1 \\ b = -3 \\ c = 2 \end{Bmatrix} = x^2 - 3x + 2 = f(x)$



• Plug in each point to check your answer.

• Completing the Square

eg $\quad 2x^2 + 6x + 2$

Step 1: Factor out the leading coefficent

$2(x^2 + 3x) + 2$

Step 2: find ½ of b

$3/2$

Step 3: Square ½ of b

$(3/2)^2 = 9/4$

Remember! the square is completed w $(\frac{1}{2}b)^2$

Step 4: Add and subtract $(\frac{1}{2}b)^2$ from original equation:

$2(\underbrace{x^2 + 3x + 9/4}_{\text{square}} - 9/4) + 2$

Step 5: Factor the perfect Square

$2(x + 3/2)^2 - 9/4 + 2$

Step 6: Combine like terms

$\quad -9/4 + 8/4$

$2(x + 3/2)^2 - 1/4$

Step 7: Solve as normal

- Proportional Variables within Quadratics

when x is directly proportional to y

$y = (k)x$ - y equals x times an unknown constant

when x is inversely proportional to y

$y = k/x$ y equals x divided by an unknown constant.

- Formulas to remember

Rate of Change/slope $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Area of:

Circle $= (2r\pi)^2$ or $(d\pi)^2$

Square $= L(w)$

Triangle $= \frac{1}{2}b(h)$

Volume: $(L)(w)(D)$

Distance:

$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$a^2 + b^2 = c^2$

Vertex:

$\left(\frac{-b}{2a}, f(x)\right)$

Quadratic:

$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$