1. Continuous time Gauss-Markov Systems: Continuous Time Kalman Filter, Stationarity, Power Spectral Density and the Wiener Filter
   1. The continuous Time Kalman Filter

* Model

Where

* The innovation process

where the estimator, the estimator error

* The conditional estimator

where is the solution to the **Riccati** equation as

* In continuous time filter, there is no separation as Prediction and estimation.

%%% Kim’s comment

1. The problem is to find the **optimal estimator(optimal filter , optimal observer)** such that

The optimal solution is , which is given in (6.10)

1. **The model may be written in engineering textbook,** as

where as the white noise

Strictly speaking, the derivative of Brownian motion is not defined,(it we measure the position with a Brownian noise, the speed of the position, which is derivative of it, is not defined), hence this model is not correct. However, it is culture to write as it is.

1. In discrete system, it may be allowed since the time interval between the sampling is finite,

the difference of the Brownian motion is defined.

1. The Riccati equation is a non-linear differential equation, and in control engineering, it is important for another optimal problem.
2. What is another implication of ? This is the minimum mean variance estimator(or observer) , as
   1. Properties of the Continuous- Time Riccati Equation

* Hamiltonian Introduced

%%% HW\_1.1 : explain the Hamiltonian

%%%

Assume

Then from (6.12) the matrixes are satisfied following differential equations

where is called the Hamiltonian matrix

* Theorem 6.3 If there exists a such that

then the Hamiltonian is similar to

%% HW\_1.2

* What is properties of similar matrices?

%%%

* Theorem 6.4

If there exists a satisfying (6.19), then the real parts of are such that the eigenvalues of have negative real parts and the eigenvalues of are identical, except for having the opposite sign, that is

* Theorem 6.5 If the system is a minimal realization, then there is a unique solution to the ARE for which

%% HW\_1.3

What is a minimal realization? or what is not? give examples which is or not.

%%%

* 1. Stationarity
* Def.6.6 A random process is stationary if the joint probability distribution of

is the same as the joit probability distribution of

for all values of and

* Def. 6.7 We say that the random process is second-order stationary or **wide-sense stationary** if it has a constant mean

and if its correlation function

Is a function of only one time-argument-the difference in the two times at which the function is being examined.

* 1. Power Spectral Densities
     1. Fourier Transforms
* Remark on the scaler

where the coefficients and are such that

In general,

* Remark 6.10 - Parseval’s Theorem
  + 1. Fourier Analysis Applied to Random Process
* Autocorrelation (in sample sense)

where

* The Fourier transform of
* Energy / power signal
* Finite energy signal
* Finite power signal(average)
* Energy density function in

%% HW\_1.4

1. Prove
2. Find the fourier transform of , and

%%%

* The fourier Transform of the autocorrelation is the power spectrum density

The problem (6.30)’ is the F.T. of the random process …Using wide-sense stationary property

* Properties

2. If is real, then is real and even , and

* Ex. 6.11. Consider

Then

And

%% HW 1.5

Prove not using (1)’

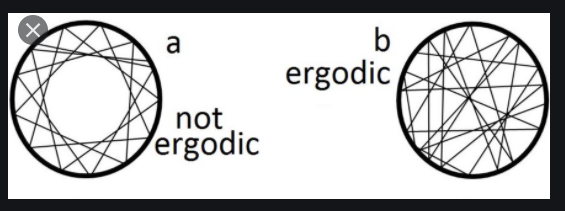
%%%

* + 1. Ergodic Random process

Ergodic process: ensemble(statistical) average ~ time average

%% Kim’s comment

* Ergodic : wiki

**“Ergodicity** is a property of the system; it is a statement that the system cannot be reduced or factored into smaller components.”

* Covariance :
* Correlation coefficient: page.44
* Correlation:=
* Auto-correlation:
  1. Continuous time linear system Driven by stationary signals
* Model

Or

1. The impulse response function
2. The PSD (power spectrum density)of input
3. The PSD of output

Let the fourier transform of the impulse response function as

.

Then

* Ex. 6.12

Suppose that the input spectrum is

And the filter spectrum is

Then the output PSD is

The mean-square output is

%% Kim’s comment

1. The state space representation of the filter is

where the input is the white noise whose correlation is **(see page 167)**

1. The PSD of is
2. If the input is impulse, the output is

so that the output total energy is

Compare this with (6.12)’

%%%

* Ex. 6.13

And the white noise has its correlation is

Then the PSD of the output is

%% HW\_1.6

1. prove the ex.6.13
2. Plot

%%%%%