%% Kim’s comment

If the input is a white noise, sometimes we design a filter (low, high..) . See the model as

Since the input is a white noise as

Then the output PSD is

which is a lowpass filter output PSD. The magnitude of the output PSD is preserved in the form of absolute magnitude, but the phase information is lost.

6.6. Discrete – Time Linear System Driven by Stationary Ransom Processes

* Model

Assume : and is stable

* Correlation

-steady state

-if wide sense stationary

Hence

%%% Kim’s comment

Given a deterministic discrete linear system

The lyapunov equation for stability is

Then is asymptotic stable, i.e., . compare this with (6.48) %%%

%% HW 2.1

Prove the above equation (1’) %%%

* PSD

And

6.7 Steady State Kalman Filter : The Wiener Filter

In the steady state, i.e., , Wiener developed the optimal Estimator, which is same to the steady state Kalman estimator, except some conditions

6.7.1 (skip)

6.7.2 (skip)

6.7.3 (skip) non-causal filter

%%% kim’s comment : Non-causal filter??

If a system is non-causal, then it is not realizable. For example given time indexed measured data

Then you may design a filter as

which is the present output is dependent of the future input. How? It is violate that the effect should follow the cause.

However, in the batch mode, you may rearrange the data order as

Then the re-arranged date is time reversed, hence you may design a filter as a causal.

What is the merits of non-causal filter, see following open comments <http://www.mechanicalvibration.com/filtfilt_Causal_versus_non_.html> %%%

%%% HW 2.2

1. Write a code in the comments, run your code to check the result as in (2’)
2. Is it a realtime filtering? %%%
3. The extended Kalman filter

%%% Kim’s comments

Consider a non-linear differential equation

1. The stationary points

The points so that implies the trajectory is stationary at that point.

Hence

1. At the stationary points, find the nominal solution

For a small perturbation , the taylor series of is at the stationary points

1. At

hence around

The small perturbation is governed by

which gives solution as

is stable.

In conclusion near the trajectories solution to (2’) converge to %%%

%%% HW 2.3

1. How about the other stationary points?
2. Draw the phase portraits
3. At the initial point draw the trajectory.
4. At the initial point , draw the trajectory. %%%
   1. Linearized Kalman filtering
      1. Continuous time theory

Assume a trajectory as

Then

Define perturbations away from this state to be

By Taylor series

Take the first term of this series

Here

The linearized (7.2) is

The output is also non-linear as

The linearized trajectory is

The final linear model is

The estimate of the full state is

%%% Kim’s comments

* What is the nominal solutions? If you know the nominal solutions (i.e., near stationary points or trajectory are already known ) it may be good. Or initial points should be dependents. In general , the nominal solutions are unknown and the estimates on initial points may be incorrect, linearized kalman filter may be not good, i.e., it is divergent.

%%%

* + 1. Skip - Discrete Time
  1. The extended Kalman Filter
* Solution (1970)

Where

* It is linear and non-linear, i.e., it is non-linear equation.
* Is it optimal ?... not clear…
* Better performance to linearized Kalman filter

%%% Extended Kalman Filter in discrete version

<https://en.wikipedia.org/wiki/Extended_Kalman_filter#:~:text=In%20estimation%20theory%2C%20the%20extended,the%20current%20mean%20and%20covariance>.

* Modeling
* Predict
* Update(Correction / Estimation)
* Innovation:
* (Near-Optimal) Kalman gain
* Update

1. It is suboptimal
2. The gain formula equation is different from the previous ones(last semester) but the result is same %%%
3. This formulae is oftenly referred as the extended Kalman filter

7.1