Review the last lecture(week\_3\_Unscented Transform)

Given a non-linear transform

Fine the mean of

1. Find the pdf of
2. Monte Carlo simulation
3. Linearizing the non-linear transform
4. Without PDF, apply a small perturbation to get 🡪 The unscented transform

Once more the Unscented Transform is

* Unscented Transform[2]

1. Problem : given the following non-linear system

Find the mean and the covariance of

1. Define sigma points with weighting function

where

1. Instantiate (projection) each points as
2. **The mean of**
3. **The covariance of**

%%% Kim’s comment

* The number of sigma points = , which is small, compared to Monte Carlo.
* In order to get the mean in 4), **it does not need the Probability Density function of y.** For the covariance of **it does not need either! See 4), and 5) !!** In general to get the pdf of a non-linear transform is difficult so that the linearization may be used.
* The deficiency of linearization is it is necessary to calculate the Jacobian , so that some system it is difficult and it may not be linearized,.i.e.,
* **Why does not UT need PDF to get the mean and variance? see [3]**
* Example of UT in

Consider a non-linear system

1. Define sigma points

Since n=1,

Where

1. Projection these points thru
2. Calculating to get

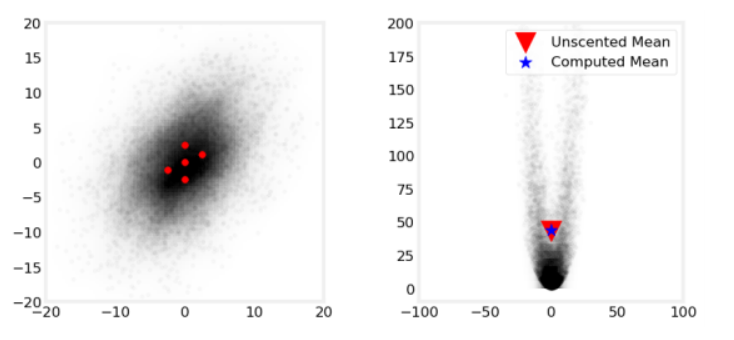
which is independent of the tuning parameter , , which is the same of ““TRUE mean of y in 2.1 and 2.3 inWeek\_3\_Nonlinear filter”.

1. Calculating the variance of

Which is the same of “TRUE variance of y in 2.1 and 2.3 in Week\_3\_Nonlinear filter ” if .

* Example of UT in multivariable case [1]

In the previous example of “error in linearization”, if UT is applied, then



The mean errors compared to Monte Carlo, are

which is much small to linearization method,

%%% Kim’s comment:

If the states are random, and a non linear function of the states is analyzed using UT not Linearized method! There are several reasons to select UT. %%%

* Basic Ideas of Unscented Kalman Filter

1. Recall the linear system

Then The Kalman filter is

where

are the conditional mean(expectation)

Now Consider the non-linear system

In this case we need the mean of . If we know the pdf of ,

Or if we define sigma points to get unscented transform ,

With a properly chosen weighting function . In this way, we may get the mean of using sigma points unscented transform

**In summary**

|  |  |  |
| --- | --- | --- |
| Linear | Non-linear | Remarks |
|  |  |  |
| Prediction |  | Find sigma points of  Find the mean of |
| Correction |  | Find the mean of |

1. A new algorithm for : cross covariance

In the linear or extended KF, the Kalman gain is

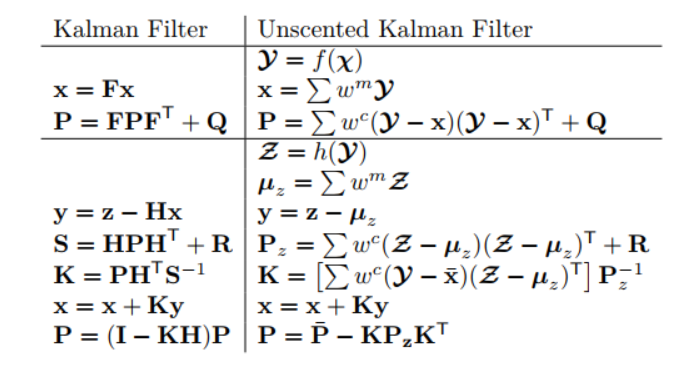
Hence to get Kalman Gain , we need the measurement matrix, . Since the measurement is non-linear , to get “H” matrix we may linearize or using the cross covariance between the states and the measurement as

The Kalman gain is equivalent to

Here to get , we may use

* The UKF [1]

First to find the sigma points of , . then



%%% Kim’s comment

In the recursive procedure, for notation simplicity we may use as

* Without the subscription in the matlab programming

For i= 1:N

X = F\*X;

X =H\*X

End

%%%%

Reference

[1]” Kalman and Bayesian Filters in Python”, ch.10

[2]” A New Extension of the Kalman Filter to Non-linear System”

[3] “A General Method for Approximating Nonlinear Transformations of Probability Distributions”

[4] <https://www.youtube.com/watch?v=LkHBR7efKQw>