Review Calculus of Variations.

1. Mathematical tools
   1. Lagrange multiplier – Optimal with equality constraints
2. Find s.t.
3. The adjoin function
4. The necessary conditions: find the stationary point as
   1. Calculus of variations w/o constraints. – Euler equation
5. Find
6. Necessary con. (**Euler Equation**)

%%% Kim’s comment. Is the following make-sense?

However,

make-sense. In this case, we need calculus of variations %%%

* 1. Calculus of variations: Hamlitonian -Jacobian-Bellman (HJB) Euler equation

1. Find

Subject to

1. Define Hamiltonian
2. Necessary and Sufficient condition –HJB equation
3. Example of HJB
   1. Problem – linear quadratic regulator (LQR)

With the path constraints

Find to minimize

* 1. Hamiltonian
  2. For optimal control
* Since 🡪 a global minimum
  1. Solve for

Assume

Then

The HJB is

Substituting these into HJB:

which is satisfied for all , Hence the following matrix differential equation is satisfied

If , then h terminal condition as

which implies the final condition of (a) is

1. Some comments:
   1. Riccati equation (a)

The equation (a) is called as Riccati equation which is important in the optimal control theory. Remember the matrix differential equation (a) is with no initial condition but the final condition( as we may expect since the cost function is with the fixed the final conditions not the initial condition.

%%% Kim’s comment

In the Stochastic course, Riccati is introduced as (Last semester Week\_12\_Continuous)

where .

Then the best linear mean square estimator is

where is the solution to the **Riccati** equation as

The two equation are very similar, but

-the constraints are different : final condition / initial condition

- Let , they are in general different

%%%%

* 1. The full state feedback

Since the optimal control law is

In order to utilize the optimal controller, we need observe full states compared to the PID controllers.

1. Selection of the weighting matrix and
   1. The heuristic way

The weighting matrixes in the cost functional are

For example, the state weighting matrix results is

Now if the desired trajectories are to be , then the weighting function may be selected as

* 1. Another : Considering the amplitude constraints

A good rule of thumb[1]:

where represent the largest desired response/control input and

* 1. In general to choose are dependent of designers skill!!

1. The Steady state solution – linear time-invariant system
   1. the steady state solution to the Riccati equation

Consider a continuous Riccati equation(CRE)

And the closed loop system is

* Theorem 3.7 (Sivan, page 237, [1])

Consider the algebraic Riccati equation (ARE)

If is stabilizable, then ARE has the unique non negative solution

* 1. Properties of steady state solution for the LQR controller

1. The closed loop system is

is asymptotically stable.

1. For any , the closed loop system is asymptotically stable!
2. Discrete LQR [3]
   1. Problem
   2. The optimal solution

* The optimal controller
* Discrete Riccati equation

%%% Kim’s comment

In the same as continuous system, Riccati equation is backward %%%

* 1. Steady state solution

%%% Kim’s comment

The Riccati equation looks complicate!!

1. Continuous Lyapunov equation

🡪

1. Discrete Lyapunov equation
2. There are several functions for lqr in Matlab as

* lqr
* dlqr
* dlqry
* lqrd

so that you may choose the functions properly to get the optimal controllers. However all are steady state, if you find the finite time, you may code by your self.

%%%%

%% HW\_week\_6

1. Given a linear system
2. What is the controllability condition?
3. Give an example of a controllable system and uncontrollable system
4. Consider the transfer function
   1. find the equivalent state space form of the given transfer function
   2. Is it controllable?
5. Let the transfer function as where are no common factor.
   1. If the system is transform to equivalent state space model, is it controllable?
   2. Consider . Transform each system to the equivalent state space model and check the controllability.
6. Consider following state space systems. Find the transfer function and check the controllability[4]

[1] MIT OpenCourseWare, “Principles of Optimal Control , lecture4

[2] “Linear Optimal Control system”, Sivan & Kwaternaak, page 169

[3] [https://stanford.edu/class/ee363/lectures/dlqr.pdf - 2008](https://stanford.edu/class/ee363/lectures/dlqr.pdf%20-%202008)

[4] “Data-Driven Science and Engineering, Machine learning and Dynamic Systems and Control”, Steven L.Bruton, 2019