Week\_7 Linear Quadratic Gaussian(LQG)

1. Re-visit LQR problem in steady state

Consider steady state LQR problem as the path constraint,

Then

where

* To implement the optimal control law, we need the full state – feedback, i.e., we need the number of sensors = the dimension of
* We implement sensors 🡪 it is bulky and sometimes it is not possible to measure some state in physically.
* We may design optimal estimators for unmeasured states.

1. Controllability / Observability [1]
   1. Controllability

Given , it is controllable if for any state is reachable in a finite time with some

1. Controllability matrix test

is a full column rank

1. Popov-Belevitch—Hautus test: must have column rank n for all
2. Gramian test :
   1. Observability

* Fact: Observability

Let the measurement states as

If A system is observable if it is possible to estimate any state from a time-history of the measurement

* Test for the Observability

1. The observability matrix: is a full row rank
2. Popov-Belevitch—Hautus test: must have row rank n for all
3. Gramian test :
4. Kalman Filter(Estimator) : Full-State Estimation
   1. Problem

Find the estimator

%%% Kim’s comment

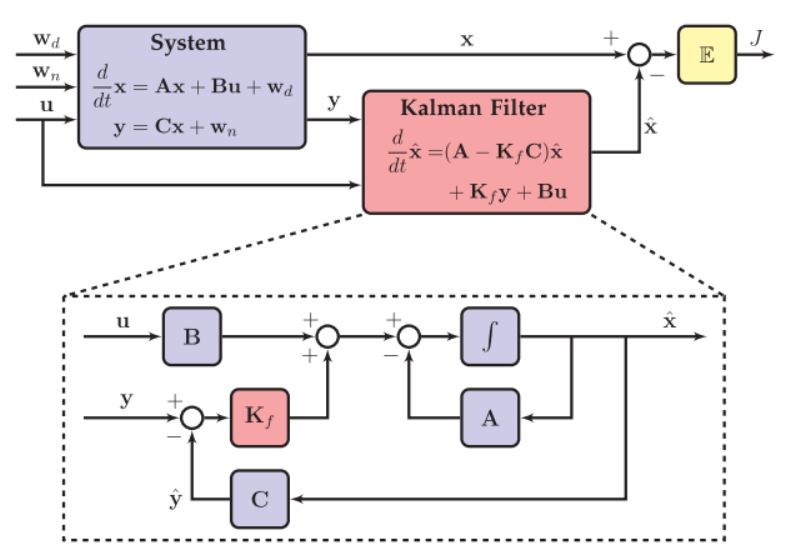
is the minimum mean square error estimator for

%%%%

* 1. Kalman Gain

where

* 1. Block Diagram



1. The closed kalman system

From (1) and (2)

Now the estimation error

Therefore if is asymptotically stable,

%%% Kim’s comment

As you may see, the e-values of the estimator may be bigger negative than that of the controller. %%%%

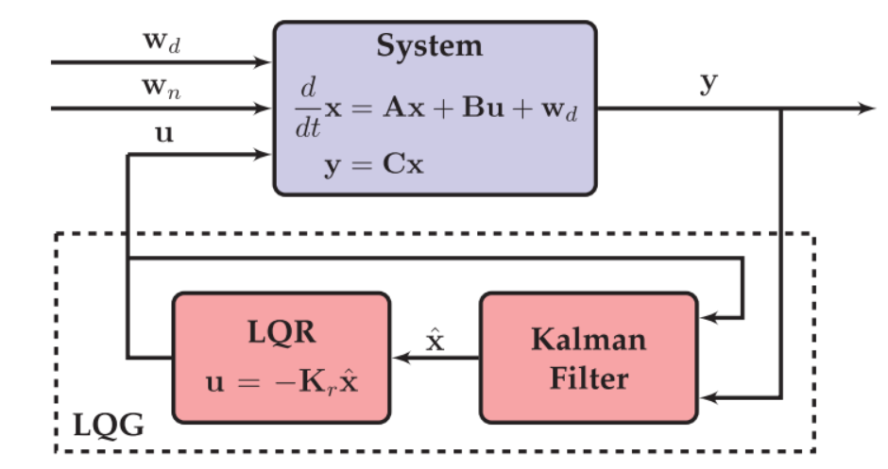
1. Optimal Sensor-based Control(output): Linear Quadratic Gaussian(LQG)[1]

Given the system

Find the optimal controller to minimize

* 1. Solution

The optimal output feedback controller is



%%% Kim’s comment

The full state optimal feedback for LQR is

Since only output are avaialble, we may design the optimal estimater as

So that

%%%

* 1. Separation Principle[2]

As before the estimatrion error , then the augumented stste equation may be

The eigenvalues of

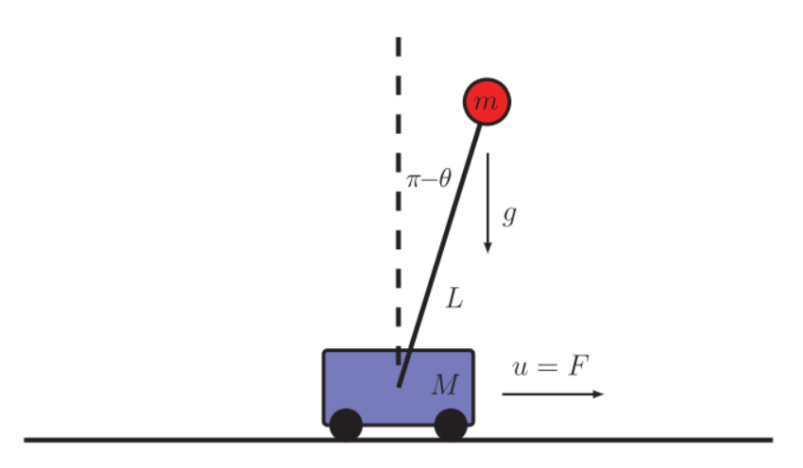
are decomposed into two parts as

wich implies

the regulator and estimator can be designed separatedly.

1. Example : inverted pendulum
   1. Physical model [1], [2],[3]

Purpose: design the controller to stabilize the inverted pendulm.



There are several modelings and different control strategies.In [1], [2], and [3].

At least in simulation they succeeded to stablize the unstable inverted pendulum( In [3] it was realized to show the control performance). However they are a little bit different as followd.

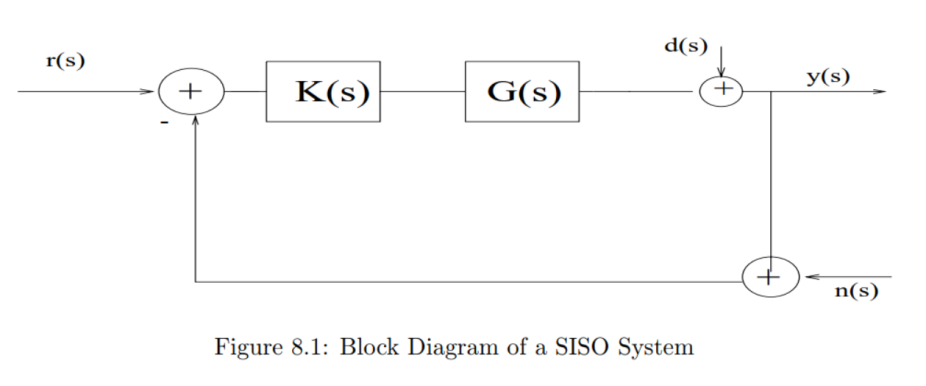
1. Deterministic model[3] / Stochastic model[1], [2]
2. The number of output measurement [1], [2]
3. In [3], it was realized to show the control performance!
4. In [1], the simualtion is performed the system model as non-linear dynamic system to be simulated in C program

So which one is correct or better than others? I think it depends on system environment(noise in there?) and system specifications, and so on..So far in real system is not so simple in the text book. And the optimal controlled, which is grobally optimal, is not optimal in real situation. It should be closely checked.

* 1. The simulation will be done in the laboratory[1]

1. Modern Control- Robust Control [4]

7.1 SISO



* Notation:

The input of command

The CompensatorThe plant

the control command

The output

The disturbance

The sensor noise

the error signal

* 1. Single input Single output
* Definition

Return Difference Transfer Function

Sensitivity T.F

Complememtary Sensitivity T.F

Feedforwardloop gain

* Fact:
  1. Control Objective : in SISO case [4]

Sensitivity to Model Uncertainty, Command Following, Disturbance Rejection, Noise Progagation, Stability and Robust

1. Sensitivity to Model Error

Assume :

1. Open loop case

Hence sensitivity due to model uncertainty is

1. Closed loop

Then

* It is **better is large**

1. Command Following

* It is **better is large**

1. Disturbance rejection

If , then

* It is **better is large**. One Comment should be put in here. In general the disturances are in the low frequencies

1. Noise reduction

If , then

* **It is better is small.** . One Comment should be put in here. In general the noises are in the high frequencies

1. **Stability and Robust**

* The nominal Plant should be stable. i.e., is stable
* Let the real plant be modelled as

multiplicative uncertainty

* Assumptions  
  A.1) # of unstable poles of = those of

A.2)

A.3) The controller is chosen so that the nominal closed loop is stable.

%%% kim’s Comment [5]

For the sensitivity / robust there is a **good tool of Nyquist plot**. %%%

* Fact to stability / robust problem using Nyquist plot

The closed loop systme is stable if

The encirclements of plots of around (-1,0) point is not changed for all allowable from the nominal system .

* Check the robust in scaler case
* It will be satisfied if

Hence

or

* Implications

In general

* For robust due to model uncertainty, it is better

1. **In summary**

Now since the sensitivity f.t. and the complementary t.f.

Hence

+ is small 🡪 is large is small

+ is large 🡪 is small is large

+ For all frequenccies,

Mag.

**To satisfy the control objective the controller may be designed as**

1. In small frequencies range, to select to S(s) is small for compensating

* Model error
* Command following
* Disturbance rejection
* For robust

1. In large frequencies range, to select to T(s) is small for compensating

* Noise reduction
* For robust

1. **Roughly speaking the LQR optimal controller satisfies these condition. However, LQG controller may not satisfy these.**
   1. Control objective: in MIMO case

In MIMO case, things are complicate so that in modern control after Kalman was introduced. First

You may notice that (output) = T(s) (input) , hence even if in the scaler case it is more logical to write as

Second, in MIMO the transfer function is not scaler but matrix which is function of frequency. It is difficult to apply Nyquist plot in MIMO. For example, i.e.,

Then is is not directly applicable Fact introduce in scaler case. What tool is to measure of ? One of the popular tools is singular value decompostion.

HW\_Week\_7 [5]

1. In Section 7, derive HW-1
2. Following systems find stability and gain margin

Reference:

[1] “Data-Driven Science and Engineering, Machine learning and Dynamic Systems and Control”, Steven L.Bruton, 2019

[2] “Linear Optimal Control system”, Sivan & Kwaternaak, page 391

[3] <https://www.youtube.com/watch?v=D3bblng-Kcc>, “Lecture 26, Feedback Example: The inverted Pendulum |MIT RES.6.007 Signals and Systems, Spring 2011

[4] <http://mae2.eng.uci.edu/~fjabbari//me270b/chap8.pdf>

[5] <https://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlFrequency#26>