# Problem 1

## a)

| P | Q | R | $(Q \land R)$ | $(P \land Q)$ | $P \land (Q \land R)$ | $(P \land Q) \land R$ |
|---|---|---|---|---|---|---|
| F | F | F | F | F | F | F |
| F | F | T | F | F | F | F |
| F | T | F | F | F | F | F |
| F | T | T | T | F | F | F |
| T | F | F | F | F | F | F |
| T | F | T | F | F | F | F |
| T | T | F | F | T | F | F |
| T | T | T | T | T | T | T |

## b)

| P | Q | R | $(Q \lor R)$ | $(P \lor Q)$ | $P \lor (Q \lor R)$ | $(P \lor Q) \lor R$ |
|---|---|---|---|---|---|---|
| F | F | F | F | F | F | F |
| F | F | T | T | F | T | T |
| F | T | F | T | T | T | T |
| F | T | T | T | T | T | T |
| T | F | F | F | T | T | T |
| T | F | T | T | T | T | T |
| T | T | F | T | T | T | T |
| T | T | T | T | T | T | T |

## c)

| P | Q | R | $(P \iff Q)$ | $(Q \iff R)$ | $(P \iff Q)$ and $(Q \iff R)$ |
|---|---|---|---|---|---|
| F | F | F | T | T | T |
| F | F | T | T | F | F |
| F | T | F | F | F | F |
| F | T | T | F | T | F |
| T | F | F | F | T | F |
| T | F | T | F | F | F |
| T | T | F | T | F | F |
| T | T | T | T | T | T |

| P | Q | R | $(P \implies Q)$ | $(Q \implies R)$ | $(R \implies P)$ | $(P \implies Q) \wedge (Q \implies R) \wedge (R \implies P)$ |
|---|---|---|---|---|---|---|
| F | F | F | T | T | T | T |
| F | F | T | T | T | F | F |
| F | T | F | T | F | T | F |
| F | T | T | T | T | F | F |
| T | F | F | F | T | T | F |
| T | F | T | F | T | T | F |
| T | T | F | T | F | T | F |
| T | T | T | T | T | T | T |

## Problem 2

### a)

| P | Q | $(P \implies Q)$ | $(\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (P \wedge Q)$ |
|---|---|---|---|
| F | F | T | T |
| F | T | T | T |
| T | F | F | F |
| T | T | T | T |

### b)

| P | Q | $(P \implies Q)$ | $\neg (P \implies Q)$ | $(P \wedge \neg Q)$ |
|---|---|---|---|---|
| F | F | T | F | F |
| F | T | T | F | F |
| T | F | F | T | T |
| T | T | T | F | F |

## Problem 3

*Proof.* Base case $n = 2$

$$\neg(P_1 \wedge P_2) = (\neg P_1) \vee (\neg P_2)$$

Works through demorgans law
Assume case $n = k$

$$\neg(P_1 \wedge P_2 \wedge ... \wedge P_k) = (\neg P_1) \vee (\neg P_2) \vee ... \vee (\neg P_k)$$

Inductive step $n = k + 1$

$$\begin{aligned} \neg(P_1 \wedge P_2 \wedge ... \wedge P_{k+1}) &= \neg(P_1 \wedge P_2 \wedge ... \wedge P_k \wedge P_{k+1}) \\ &= \neg((P_1 \wedge P_2 \wedge ... \wedge P_k) \wedge (P_{k+1})) \\ &= \neg(P_1 \wedge P_2 \wedge ... \wedge P_k) \vee \neg(P_{k+1}) \\ &= ((\neg P_1) \vee (\neg P_2) \vee ... \vee (\neg P_k)) \vee (\neg P_{k+1}) \\ &= (\neg P_1) \vee (\neg P_2) \vee ... \vee (\neg P_k) \vee (\neg P_{k+1}) \end{aligned}$$

□

## Problem 4

**a)** $(3|1) \vee (3|2) \vee (3|3) \vee (3|4) \vee ...$
**b)** $\forall n \in \mathbb{N}(2|n) \iff \neg(2|n + 1)$
**c)** $\forall n \in \mathbb{N}, \exists m \in \mathbb{N}\ (m \geq n)$

## Problem 5

**a)**

$$\begin{aligned} x \in A \cup (B \cup C) \\ x \in A \text{ or } x \in (B \text{ or } C) \\ x \in A \text{ or } x \in B \text{ or } x \in C \\ x \in (A \text{ or } B) \text{ or } x \in C \\ x \in (A \cup B) \cup C \end{aligned}$$

This process can go the other way. This then shows that $A \cup (B \cup C) \subset (A \cup B) \cup C$ and $(A \cup B) \cup C \subset A \cup (B \cup C)$ This means that $A \cup (B \cup C) = (A \cup B) \cup C$

**b)**

$$\begin{aligned} x \in A \cap (B \cap C) \\ x \in A \text{ and } x \in (B \text{ and } C) \\ x \in A \text{ and } x \in B \text{ and } x \in C \\ x \in (A \text{ and } B) \text{ and } x \in C \\ x \in (A \cap B) \cap C x \in (A \cap B) \cap C \end{aligned}$$

Similar to the last question, this can be shown in the other direction easily. This then shows that $A \cap (B \cap C) \subset (A \cap B) \cap C$ and $(A \cap B) \cap C \subset A \cap (B \cap C)$ This means that $A \cap (B \cap C) = (A \cap B) \cap C$

**c)** $A \cap (B \cup C)$ This means that $x \in A$ and $x \in B$ or $C$. This means that $x \in A$ and $B$ or $x \in A$ and $C$. This means that $x \in (A \cap B) \cup (A \cap C)$ This means that $A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$. We can also prove this the other way which would then show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

**d)** $A \cup (B \cap C)$ This means that $x \in A$ or $x \in B$ and $C$. This means that $x \in A$ or $B$ and $x \in A$ or $C$. This means that $x \in (A \cup B) \cap (A \cup C)$ This means that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$. Prove this the other way would then show that $A \cup (B \cap C) = (A \cup B) \cup (A \cup C)$.

**e)** $x \in A \cap (B/C)$ this means that $x \in A \cap B$ and $x \notin A \cap C$ or vice versa. We can then rewrite this as $x \in (A \cap B)/(A \cap C)$