Section 2.7

1) For every real number x, $x^2 > 0$. Statement is **false** since $0^2 > 0$ is not true.

2) For every real number x, there exists some natural number n, $x^n \geq 0$. Statement is **true**

3) There exists a real number a for which $ax = a$ for every real number x. **true**

4) for all elements of the power set of the natural numbers X, is a subset of the real numbers.**true**

8) For all integers n, there exists some subset of the natural numbers X such that $|X| = n$, **true**

9) For every integer n there is another integer m such that $m = n + 5$. **true**

10) There exists an integer m such that for all integers n, $m = n + 5$ **true**

Section 3.2

2) There are 26 letters in the alphabet. So the total number of 3 letter combinations is $26 \times 26 \times 26 = 17,576$

4) There are 2 types of coffee, 3 different sizes, and two options of where to have the coffee. So $2 \times 3 \times 2 = 12$

8) 10 coins, each coin there are two options, heads or tails. So the answer is 2^{10}

Section 3.3

2) There are 4 suits, each suit has 13 cards. So we have $4 \times 13 \times 12 \times 11 \times 10 \times 9$

4) There are 4 queens, so we have 48 cards left. Furthermore, the one queen can have 5 positions on the lineup. So we have $5 \times 48 \times 47 \times 46 \times 45$

12) Six math books = 6! arrangements. Four physics books = 4! arrangements. Three chemistry books = 3!. Out of the 3 groups there are 6 ways to arrange them. So total we have $6! \times 4! \times 3! \times 6 = 622{,}080$

Section 3.6

1) 1 10 45 120 210 252 210 120 45 10 1

2) 13C5 = 1287

3) same as number 2

4) $9c3 \times 3^6 \times -2^3$

Section 3.7

1) There are 100 students studying math, so there are $100 - 33 = 67$ students studying **only** math. So there are $523 - 67 = 456$ students studying history.

10) Out of 6 digits, there are 9 possible options for the first digit, 10 options for the next 4 digits, and for the last one there are 6 options(0, 2, 4, 5, 6, 8). We then have $9 \times 10^4 \times 6 = 540,000$

Section 3.9

1) For any number divided by 5, there are 6 possible remainders(0, 1, 2, 3, 4). So if there are six integers, and 5 possible remainders, by the pigeonhole principle, there has to be at least two numbers with the same remainder.

2) 13 cards, the worst case scenario is to draw 4 cards from each suit, so 12 cards, and the last card will guarantee at least 5 cards of the same suit.