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Problem 1

For any $x \in A$. we can say
$$0x = x0$$

Through the properties of 0
$$0x = x(0 + 0)$$

Using distribution
$$0x = 0x + 0x$$

Plus, with additive inverses we can get

$$0x + (-0x) = (0x + 0x) + (-0x)$$

By associativity we get
$$0x + (-0x) = 0x + (0x + (-0x))$$
$$0 = 0x + 0$$

So this means that $0 = 0x$ for all $x \in A$

Problem 2

Since we know that $x \neq 0$. This means that x has an inverse, and that $x^{-1}x = xx^{-1}x = 1$. So we can say that.

$$xy = xz$$
$$x^{-1}(xy) = x^{-1}(xz)$$
$$(x^{-1}x)y = (x^{-1}x)z$$
$$1 \cdot y = 1 \cdot z$$
$$y = z$$

This shows that $y = z$

Problem 3

a) $\{\bar{1}, \bar{2}\}$
b) $\{\bar{1}, \bar{3}\}$
c) $\{(\bar{1}, \bar{1})\}$
d) $\{\bar{1}, \bar{5}, \bar{7}, \bar{11}\}$
e) $\{(\bar{1}, \bar{1}), (\bar{1}, \bar{3}), (\bar{2}, \bar{1}), (\bar{2}, \bar{3})\}$
f) $\{(\bar{1}, \bar{1}, \bar{1}), (\bar{1}, \bar{1}, \bar{2})\}$

Problem 4(A)

The first difference between addition and multiplication is that addition only happens once, whereas multiplication is repeated addition. for example, $1 + 3 = 4$. But then 1 x 3 is = $1 + 1 + 1 = 3$. Another difference is the use of negative signs. For example if you were to do 2 - 2, you would get 0. But if you multiply 2 x -2, you get -4. The most obvious and third difference is the different sign that multiplication uses. Addition uses the regular + sign whereas multiplication can use an x symbol, a dot or just parenthesis together.

Problem 5

The two rings $\mathbb{Z}/10\mathbb{Z}$ and $\mathbb{Z}/5\mathbb{Z}$ are abelian under addition. We can say that for the map $\varphi : \mathbb{Z}/10\mathbb{Z} \to \mathbb{Z}/5\mathbb{Z}$, that $\varphi(1) = 2$. With that we know that there is a homomorphism between them as groups. However, since we are using $\varphi(1) = 2$, this means that $\varphi(1) \neq 1$ which can not make this a ring homomorphism.

Problem 6(B)

ev_a is a one-to-one function just like our regular pre-calculus functions. Also , when you do addition between two functions you only get one value. In regular function you most likely would get a numerical value, whereas in rings you would get whatever A is. The difference between them is that you would use ev_a for only polynomials, whereas in earlier mathematics we did these types of operations on any functions, whether it be polynomials or linear functions.