Problem 1

Let use define the set of *finite* subsets of $\mathbb{N}$ as S. In order for the subsets of $\mathbb{N}$ to be finite, the subsets must be contained within some $\{0, 1, 2, ..., n\}$ such that $n \in \mathbb{N}$. This would mean that S is a finite set as each subset can only appear once. Which we can then create a bijective map $S \to \mathbb{N}$. This means that we can say S is countable.

Problem 2

If A is uncountable and B is countable. Then A/B is uncountable.

Proof. Assume that A/B is countable. If so, the union of countable sets creates another countable set. Which would mean that $(A/B) \cup B$ is also countable. However, $A \subset (A/B) \cup B$. We know that A is an uncountable set. This creates a contradiction. Which means that A/B has to be an uncountable set. □

Problem 3

We know that the set $f^{-1}(k)$ is essentially a subset of A given some domain $k \in \mathbb{N}$.We can then say that

$$A = \bigcup_{k \in \mathbb{N}} f^{-1}(k)$$

Since the union of countable sets results in another countable set. We can then say that A is finite or countable.

Problem 4

Proof. If X is a countable set, then we can say that $X/\sim$ is also countable. In class we discussed that the equivalence relation $\sim$ creates partitions of X. Each of these partitions or non-empty subsets of X, which would also make them countable. Since $X/\sim$ is the set of all equivalence classes (as discussed in class). $X/\sim$ is the union of all these countable sets should still be countable in the end. □

Problem 5

case 1: $x = y$

$$|x| \leq |y| + |x - y|$$

$$|x| \leq |y| + 0$$

$$|x| = |y|$$

case 2: $x < y$

$$|x| \leq |y| + |x - y|$$

since $|x-y|$ will be positive so this statement will always be true since $x<y$
case 3: $x>y$

$$|x| \leq |y| + |x-y|$$

The $|x-y|$ will be added to the $|y|$. Since the $|x-y|$ will be the difference between x and y. This equation will always end up with $|x|=|y|$

Assuming the proof is correct and $|x| \leq |y| + |x-y|$ is true. We can then say that $|x|-|y| \leq |x-y|$ We can also then say that $|x|-|y| \leq |x-y|$ is true through the triangle inequality. Since it is symmetric, this implies that $||x|-|y|| \leq |x-y|$ is true