Problem 1

a) if $a > b$ then that means $a - b > 0$. Since $c > 0$ this means that its positive, we then get $c(a-b) > c(0)$. From this we get $ac - bc > 0$ then $ac > bc$

b) if $a > b$ then that means $a - b > 0$. Since $c < 0$ this means that its negative, we then get $c(a-b) < c(0)$. From this we get $ac - bc < 0$ then $ac < bc$

c) assume that if $a > 0$ then $a^{-1} < 0$. if so then that would mean that $a \cdot a^{-1} < 0$. This is a contradiction because we know that $a \cdot a^{-1} = 1$ and that $1 < 0$ is not true. This means that if $a > 0$ then $a^{-1} > 0$

d)

$$0 < a < b$$

$$(0)a^{-1} < a \cdot a^{-1} < b \cdot a^{-1}$$

$$0 < 1 < b \cdot a^{-1}$$

$$(0)b^{-1} < (1)b^{-1} < b \cdot b^{-1} \cdot a^{-1}$$

$$0 < b^{-1} < a^{-1}$$

Problem 2

Problem 3

$$\lim_{n\to\infty} \frac{4n^2 - 3n + 1}{7n^3 - n^2 + 2n + 9} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{8n - 3}{21n^2 - 2n + 2} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{8}{42n - 2} = \frac{8}{\infty} = 0$$

The sequence a_n converges to $L = 0$

Problem 4

a) if $0 \leq a_n \leq b_n$ for any limit L. if $b_n \to 0$ as $n \to \infty$. Then that means $|b_n - L| < \epsilon$. If $L = 0$ then $b_n < 0$ (We can remove the absolute as b_n is positive). We can then say that $a_n < b_n < \epsilon$. This means that $a_n < \epsilon$. So if $L = 0$ then $|a_n - 0| < \epsilon$ so $|a_n - L| < \epsilon$. This proves that as $n- > \infty$. if $b_n \to 0$ then $a_n \to 0$ is also true.

b) Since both a_n and c_n converge toward L. We can say that $|a_n - L| < \epsilon$ and $|c_n - L| < \epsilon$ We can then say that $|a_n - L| < |c_n - L| < \epsilon$.

$$|a_n - L| < |c_n - L| < \epsilon$$

$$a_n < c_n < \epsilon + L$$

Since we know that $a_n \leq b_v \leq c_n$

$$a_n < b_n < c_n < \epsilon + L$$

$$b_n < \epsilon + L$$

$$|b_n - L| < \epsilon$$

THis proves that if a_n and c_n both converge towards the limit L. Then so does b_n

Problem 5

Proof. prove that $(1+h)^n \geq 1 + nh$
base case: $n = 0$

$$(1+h)^0 \geq 1 + h(0)$$
$$1 \geq 1$$
$$1 = 1$$

inductive step: assume that $(1+h)^n \geq 1 + nh$ is true. prove for $n+1$

$$(1+h)^{n+1} \geq 1 + (n+1)h$$
$$(1+h)^n \cdot (1+h) \geq 1 + nh + h$$
$$h + 1 \geq h$$

Following our assumption all we needed to do was prove the rest of the equation which was $h + 1 \geq h$ which is true.

$\square$

if $0 < r < 1$. Then $r = \frac{1}{h+1}$ such that $h > 0$. If we substitute r we will get

$$\lim_{n\to\infty} \left(\frac{1}{h+1}\right)^n = \frac{1}{(h+1)^n} = 0$$