Amplitude embedding with Walsh series

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Abstract

We implement an approximate quantum state preparation algorithm described by Zylberman and Debbasch (2023) and use it to demonstrate amplitude embedding. The algorithm decomposes a normalized feature vector with non-negative entries into Walsh functions and implements the decomposition with a controlled unitary using an ancillary qubit. Extracting the state is done using a repeat-until-success strategy, requiring a tradeoff between approximation to the input and the success probability. We run simulations to study the average behaviour numerically and execute it on IBM Quantum hardware to demonstrate the feasibility of the algorithm.

Keywords: Quantum state preparation, Walsh functions, amplitude embedding, repeat-until-success

1. Introduction

We define the amplitude embedding problem as the following. We are given a normalized feature vector $\{x_1, x_2, \ldots, x_N\}$ with $N = 2^n$ non-negative entries. Given n qubits, we must create the state

$$|\psi\rangle = \sum_{i=0}^{N-1} x_i |i\rangle$$

Arbitrary state preparation can be done exactly, for example the algorithm by Mottonen et al. (2004). However, for larger numbers of qubits, the number of gates required scales exponentially. We can get around this by using algorithms which use less gates at the cost of getting an approximation to the desired state. Zylberman and Debbasch (2023) describe an algorithm to achieve this using Walsh series.

2. The algorithm, in short

Given an input vector of size 2^n, we can classically compute its Walsh coefficients using a sequency-ordered fast Walsh-Hadamard transform in $O(n2^n)$. Now, one can efficiently implement a circuit to create the Walsh operators using CNOTs and an RZ rotation. Additionally, we can cancel CNOTs by using Gray code ordering to reach a gate count of $2^{(n+1)} - 3$ elementary gates (Welch et al. (2014)). The original paper suggests keeping only the largest few terms of a Walsh decomposition, but in our work we will keep all of them. We will call the result of this as the *Walsh unitary.*

Now we describe the Walsh Series Loader. First, we apply Hadamard gates on all the initial qubits including the ancilla qubit, resulting in

$$|\psi\rangle = H^{\otimes(n+1)}|0\rangle = \frac{1}{\sqrt{2}}\left(|s\rangle\, 0 + |s\rangle\, 1\right) \tag{1}$$

Next, we apply a controlled Walsh unitary using the ancilla qubit as the control, resulting in

$$|\psi\rangle == \frac{1}{\sqrt{2}}\left(|s\rangle|0\rangle + e^{-i\hat{f}\epsilon}|s\rangle|1\rangle\right) \tag{2}$$

where ϵ is some strictly positive constant (which relates to the approximation vs success probability tradeoff). Also, the zeroth order RZ gate has to be added to the ancilla qubit because global phases in a unitary matter when it is turned into a controlled unitary.

Then, we apply a Hadamard, followed by the $S^\dagger$ gate to get,

$$|\psi\rangle = \frac{\hat{I} + e^{-i\hat{f}\epsilon}}{2}|s\rangle|0\rangle - i\frac{\hat{I} - e^{-i\hat{f}\epsilon}}{2}|s\rangle|1\rangle \tag{3}$$

Now we can measure the ancilla qubit. It turns out that the collapsed state is approximately $|f\rangle + O(\epsilon)$. We will measure $|1\rangle$ with a probability $O(\epsilon^2)$.

These are theoretical bounds based on sampling continuous functions, however we have a discrete function, of which there are no theoretical guarantees. We implemented this and ran it.

3. Results

In our implementation, we didn't implement the optimal circuit construction, rather we have ordered in terms of decreasing Walsh coefficients. This is because we initially intended to study how well we can approximate functions.

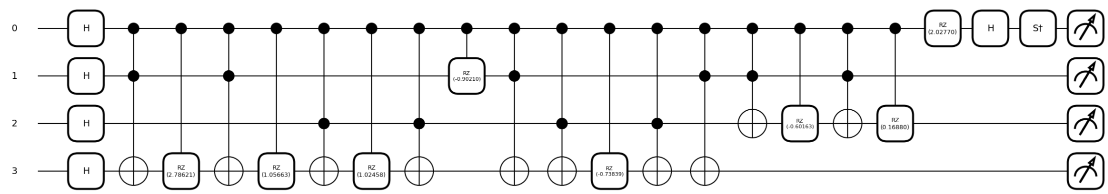

Figure 1: An example circuit generated by our code for 3 qubits.

Next, we simulated it for different values of ϵ and studied the probability of success and the maximum error in Pennylane.

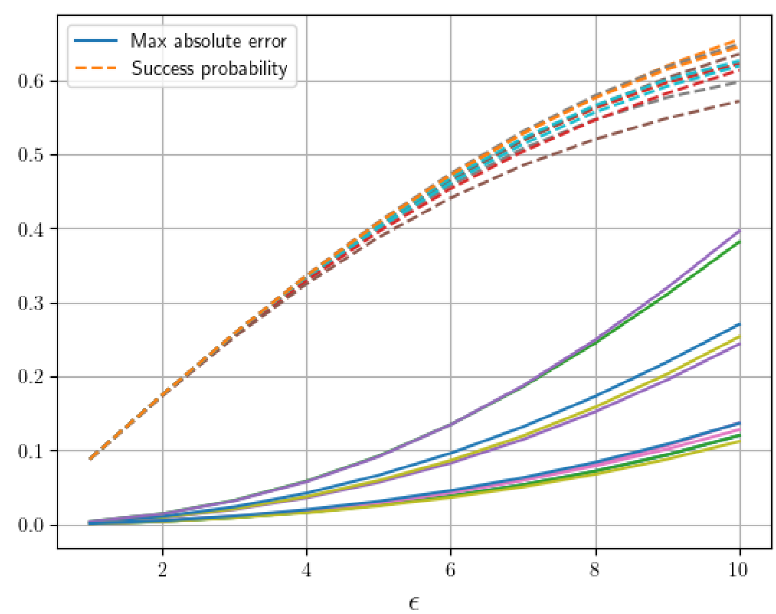

Figure 2: Simulated for 10 random vectors of length 2^5 for 5 qubits on Pennylane.

We can see that the error grows quickly as we increase ϵ, along with an increased probability of success. According to the paper, the success probability should scale as $O(\epsilon^2)$, however that is not what we observe.

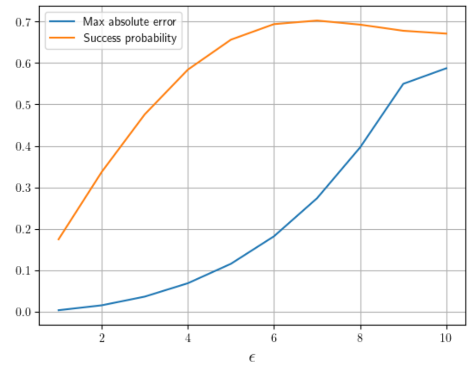

Figure 3: Simulated for 3 qubits on Pennylane.

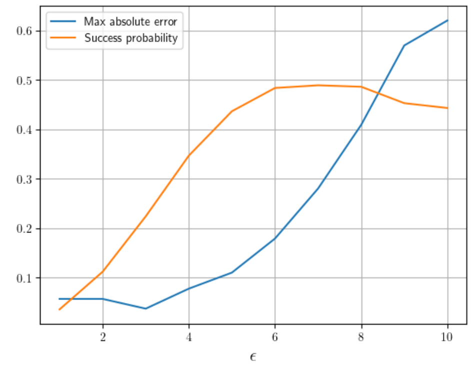

Figure 4: Simulated for 3 qubits on `simulator_statevector`.

Next, we simulated it on IBMQ's `simulator_statevector` for the input vector {0.12787755, 0.02232474, 0.29006564, 0.26871596, 0.65356168 0.53247637, 0.33595946, 0.05709531} and took counts. We also simulated it on Pennylane and calculated the theoretical success probability from the statevector.

The maximum absolute error is roughly the same, however the success rates are lower on `simulator_statevector`. We do not know why.

For $\epsilon = 5$ for the same input vector, this is the simulated output.

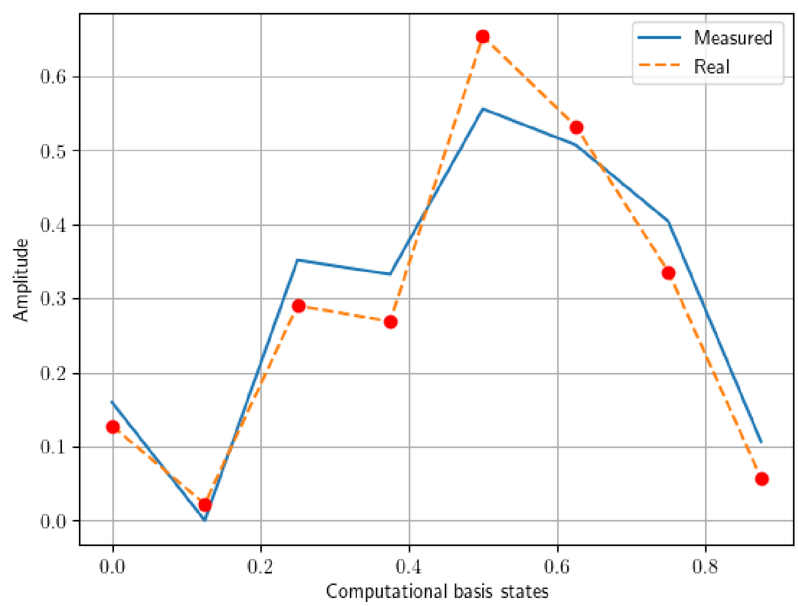

Figure 5: Simulated for 3 qubits on `simulator_statevector`.

We tried running it on `ibm_osaka` with $\epsilon = 5$, but this is what we got.

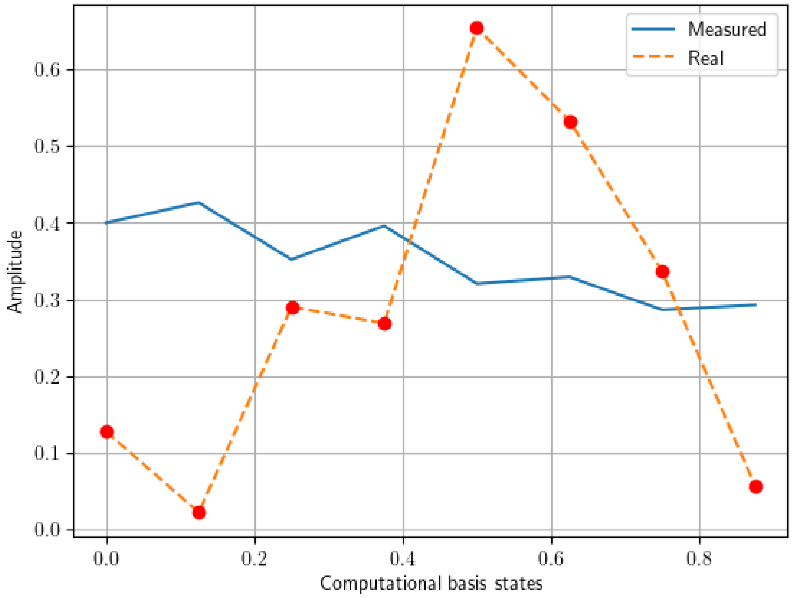

Figure 6: 3 qubits on `ibm_osaka`.

We inspected the depth of the circuit, and it was 482. The failed result may be due the large number of CNOT gates involved along with the depth of the circuit. It may also be because the right value of the ϵ is hardware dependent. However, we cannot make any further guesses without first optimising the circuit to using the $2^{n+1} - 3$ gate construction as well as studying the maximum error and success probability curves as a function of ϵ. We did not have enough time to do this.

4. Conclusions

For practical usage of the algorithm, we can do the following.

Figure 2 suggests that there is only a small range of ϵ values required for achieving a particular success probability. We also realised that this value is dependent on the number of qubits. So we can first run this experiment to collect "calibration" data of the optimal ϵ values for fixed target success probability and fixed number of qubits, and then use the found ϵ values to run all state preparations afterwards.

Also, we do not need a very high value of success probability to get a successful state preparation quickly, on average. The expected value of tries until a success for a process with success probability p is $\frac{1}{p}$.

The optimised gate construction must also be implemented for the circuit because noise due to the CNOTs will destroy any state we try to create otherwise.

Overall, simulations show promising results for the algorithm. However it would have been great if we had time to run it on current hardware.

References

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