Zewail City of Science and Technology
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Probabilistic Analysis of Digital Signal Reliability

In Noisy Communication Channels — Term Project

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Presentation Roadmap

Section 1

Introduction & Project Aims

1.1 Description of the Problem

- In modern digital communications, data is transmitted as **binary bits** $(0, 1)$.
- Environmental interference acts as **"noise"** that can flip these bits.
- Studying the reliability of these channels is essential for engineering robust systems that can detect and correct errors [1].

1.2 Project Aims

1. Apply theoretical probability (**Bayes' Theorem**) to a practical problem.
2. Simulate a **Binary Symmetric Channel (BSC)** using Python.
3. Verify the **Law of Large Numbers** through experimental data.

Section 2

Mathematical Framework

2.1 Design Parameters

Table 1: Simulation Design Parameters

Parameter	**Value**
Source Probability $P(S_0), P(S_1)$	0.5 (50%)
Noise Probabilities (ϵ)	0.1 (10%) and 0.01 (1%)
Sample Size (N)	10,000 Bits

2.2 Theoretical Calculations

Total Probability Theorem:

$$P(R_1) = P(R_1|S_1)P(S_1) + P(R_1|S_0)P(S_0) = (0.9)(0.5) + (0.1)(0.5) = 0.5$$

Bayes' Theorem (Signal Reliability):

$$P(S_1|R_1) = \frac{P(R_1|S_1)P(S_1)}{P(R_1)} = \frac{0.45}{0.5} = 0.9$$

Conclusion: We are 90% confident in accuracy when $\epsilon = 0.1$.

Section 3

Numerical Simulation

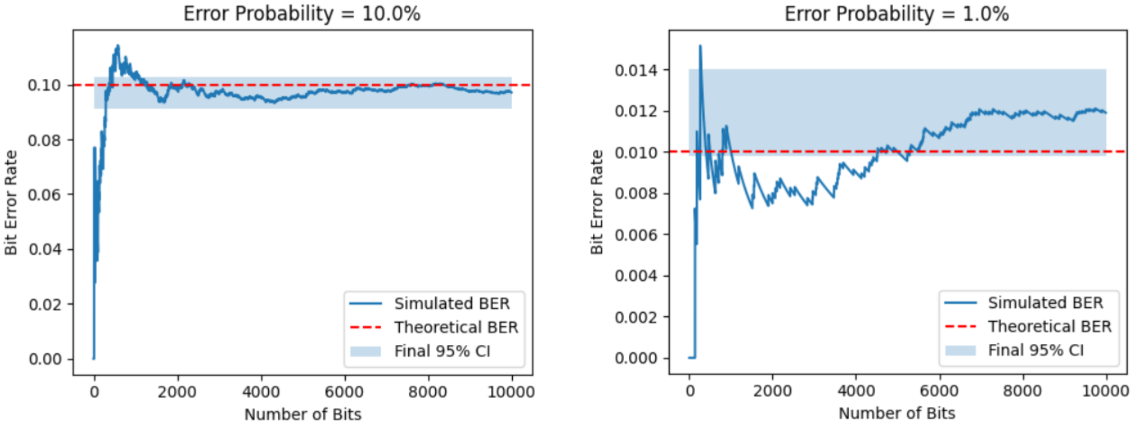
3.1 Python Code Implementation

```python
import numpy as np

def simulate_bsc(n_bits, error_prob):
    sent = np.random.randint(0, 2, n_bits)
    noise = np.random.rand(n_bits) < error_prob
    received = sent ^ noise

    errors = sent != received
    ber = np.cumsum(errors) / np.arange(1, n_bits + 1)
    return ber
```

Section 4

Analysis of Results

4.1 Convergence Visualization



4.2 Interpretation of Data

Law of Large Numbers (LLN) in Action

- **Small N:** The Bit Error Rate (BER) exhibits high volatility (*jitter*) because of high variance.
- **Large N:** As $N \to 10,000$, the empirical BER converges exactly to the theoretical crossover probabilities (ϵ).

Predictability: While individual bit flips are random, the aggregate behavior is mathematically predictable [3].

Section 5

Conclusion

5. Conclusion & Recommendations

- The simulation confirms that the **BSC model** accurately predicts error rates in digital systems.
- Posterior analysis proves that noise significantly impacts signal confidence.
- We recommend implementing **Forward Error Correction (FEC)** like Hamming codes to reduce error rates to near-zero.

Thank you! Questions?