Using Gradient Descent to Optimize Paths for Sustaining Wireless Sensor Networks

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***Abstract*—A structural-health wireless sensor network (WSN) should last for decades, but traditional disposable batteries cannot sustain such a network. Energy is the major impediment to sustainability of WSNs. Most energy is consumed by (i) wireless transmissions of perceived data, and (ii) long-distance multi-hop transmissions from the source sensors to the sink. This paper explores how to exploit emerging wireless power transfer technology by using robotic unmanned vehicles (UVs) to service the WSNs. These UVs cut data transmissions from long to short-distances, collect sensed information, and replenish WSN's energy. This paper presents path-planning and path optimization algorithms for sustaining WSNs.**

Keywords*—Wireless sensor networks, wireless recharge, robot, unmanned vehicles

I. Introduction

New wireless sensor technologies have enabled wireless sensor networks (WSNs) to proliferate in many different fields (e.g., battlefield surveillance, environmental sensing, biomedical observation) [1], [17], [20], [30]. Although advances in processing and computing designs can endow sensors with a multitude of sensing modalities (temperature, pressure, light, magnetometer, infrared, etc.), advances in battery technology have been more modest. Energy constraints on battery-powered sensors limits the sustainability of WSNs. In WSNs, the majority of energy is consumed by (i) wireless transmission of perceived data [18], [29], [30], and (ii) long-distance multi-hop transmissions from source sensors to the sink. Research efforts to address WSN energy concerns have focused on energy conservation [8], environmental energy harvesting [12], [28] and incremental sensor deployment [38]. However, energy conservation schemes only slow energy consumption, not compensate energy depletion. Harvesting environmental energy, such as solar, wind and vibration, is subject to their availability, and is often uncontrollable. Incremental sensor deployment makes WSNs neither sustainable nor environmentally friendly, since most disposable sensors' batteries contain cadmium, lead, mercury, copper, zinc, manganese, lithium, or potassium [10]. These heavy metals "*can leach into soil and water, polluting lakes and streams, making them unfit for drinking, swimming, fishing, and supporting wildlife, and even posing hazards to human health*" [9].

Fortunately, recent breakthroughs in the area of wireless power transfer technologies (e.g. inductive coupling, magnetic resonant, and RF energy harvesting) [24] provide promising alternatives for deploying such WSNs. *Magnetic resonant wireless power transfer* [24] has the ability to wirelessly transfer electric power from the energy storage device to the receiving device efficiently within medium range (e.g., 40% within 2 meters). It is also insensitive to the neighboring environment and does not require a line of sight between the charging and receiving devices. Researchers proposed that a mobile unmanned vehicle (UV) carrying a wireless charging device could visit and recharge each sensor to sustain a WSN [36].

However, one UV may not be able to visit every sensor if the WSN is deployed in harsh environments/terrains (e.g. dense forest, mountains, underwater), or the WSN is large-scale, consisting of a great number of sensors. Although these seminal studies replenished sensor energy, most of the energy was still wasted by long-distance wireless transmissions of perceived data, especially by relaying sensors. Due to charging and travel time of the UV, some bottleneck sensors may drain their residual energy while waiting for the UV. Great unsolved challenges on control remain, including how to select the optimal path for the UV to travel within WSNs and how to efficiently dispatch multiple UVs to recharge WSNs.

Assigning sensors to UVs using matching theory often assumes that energy costs due to power transmission greatly exceed the UV's transportation costs. This assumption might not fit for WSNs spread over large geographic areas, or terrain with obstacles, or where transportation costs are high, such as subsea or aerial UVs. This paper focuses on algorithms that make such WSNs sustainable by focusing on path-planing, trajectory optimization, and responding to dynamic network conditions.

II. Related Work

The path-planning problem for UVs has been investigated from several angles. To minimize path length, the authors in [7] survey the multiple-Traveling Salesman Problem, itself a generalization of the vehicle routing problem [11]. Servicing a WSN is closely related to coverage problems, recent work includes methods for optimizing speed along given routes [31], and techniques to continually improve existing routes [32]. Much work has focused on the data ferrying problem, from minimizing the latency between visits to nodes [2], to maximizing the total data rate from sensors to sink using UVs [19], to minimizing overall delay while sharing bandwidth [16], to having a set schedule and opportunistically deviating from it [15].

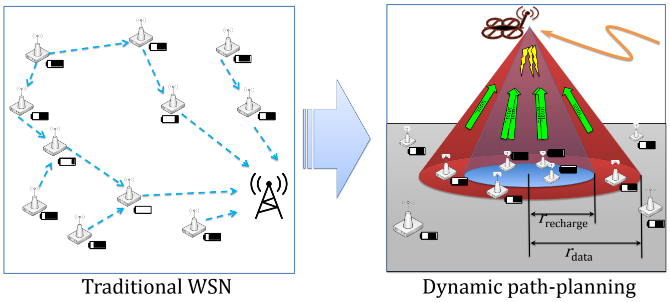


Fig. 1: Evolution from traditional wireless sensor networks (WSNs) to servicing WSN with UV(s). We present techniques that use unmanned vehicles (UVs) to gather aggregated data and recharge sensors using one or more vehicles, and design energy-optimal control policies for the UVs.

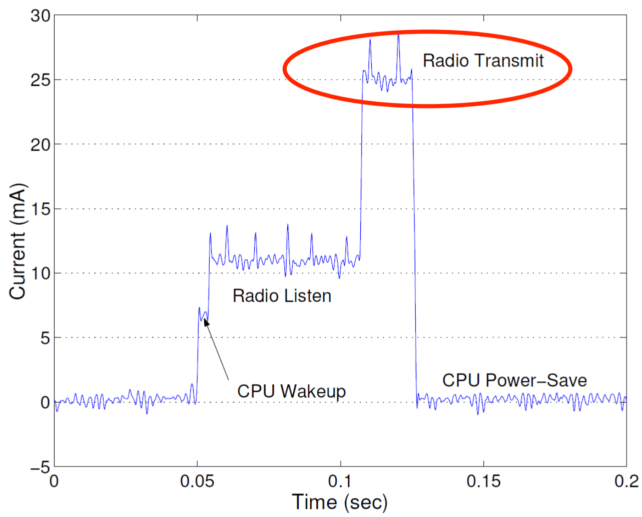


Fig. 2: Power usage in a wireless sensor node is dominated by transmission costs and listening costs. Figure modified from [26].

Using unmanned aerial vehicles to recharge other robots or sensor nodes has focused on physical design, which includes direct contact, such as swapping batteries [34], [35] or direct recharge [27], wireless resonant coupling [14], [21], [22], and electromagnetic radiation [37], and algorithmic improvements using graph theory [26], linear programming [31], and gradient descent optimization [32].

Finally, data aggregation and recharging shares similarities with persistent robotic tasks, such as cleaning, mowing, observing, and patrolling [?], [?], [?], [?], [?], [5], [31], [32]

III. Overview

This paper's goal is to explore path-optimization techniques. Previous work often uses optimization/matching theory to assign one UV/multiple UVs to WSN nodes, and use a Hamiltonian cycle to visit each node. This is reasonable if recharging nodes is the largest component of a UV's energy budget: $E_{\text{recharge}}|\text{nodes}| \gg \text{E}_{\text{movement}} * \text{path\_length}$. If this assumption is violated, path-planning becomes the key concern. A simplified form of this decision could be written

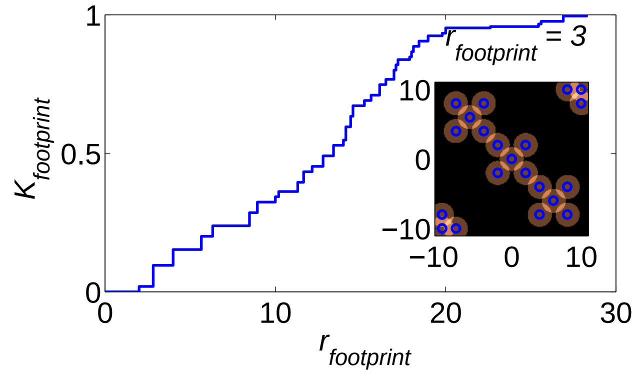


Fig. 3: As the recharging footprint or data-transfer footprint $r_{footprint}$ grows, more sensors can be recharged simultaneously. The plot above shows $K_{footprint}(r_{footprint})$, and given by (2).

as

$$K_{dist} = \frac{E_{\text{movement}} * \text{path\_length}}{E_{\text{recharge}}|\text{nodes}|}. \tag{1}$$

Here K_{dist} represents the *tipping point*, the variable where the decision problem becomes fundamentally different. If K_{dist} is small, path-planning is inconsequential, and almost any solver is sufficient. However, when K_{dist} is large path-planning becomes the key consideration. Our eventual goal is to design full trajectories that optimize the path of each UV, by servicing multiple nodes simultaneously. However, even just the path-planning component is NP-hard [5]. To make progress, this paper focuses on path optimization techniques.

A UV has an associated recharging footprint and a data-transfer footprint, which can often be modeled as disks of radius $r_{recharge}$ and r_{data}, as illustrated in Fig. 4. If sensor nodes are clustered, a UV can service multiple clients simultaneously.

We represent the fraction of sensors that are clustered as

$$K_{footprint} = \frac{2}{N^2 - N} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \left(\|p_i - p_j\|_2 \leq r_{footprint} \right). \tag{2}$$

Here, p_i is the position of the ith node, and there are N nodes.

In general, energy-efficient recharging requires closer proximity than data transmission, so this implies there are two tipping points related to node density, $K_{recharge}$, and K_{data}. Correspondingly, the WSN recharge problem has three regimes with differing solutions. Before the tipping points, nodes are sparse and not clustered. In this regime optimal paths are straight lines from node to node, and the optimal solution is a variant of the traveling salesman problem. As sensors get closer together, the optimal path may be *between* one or more sensors. In Fig. 4, path **A** is designed to visit each node, but path **B** is designed to recharge all nodes. Here, the optimal solution is often to weave between clusters of nodes. The third regime is when many nodes are close enough for transfer data, as shown in path **C**. The simulations in this paper take advantage of the non-zero $r_{footprint}$ to allow the UVs to pass near sensors without requiring them to visit each node.

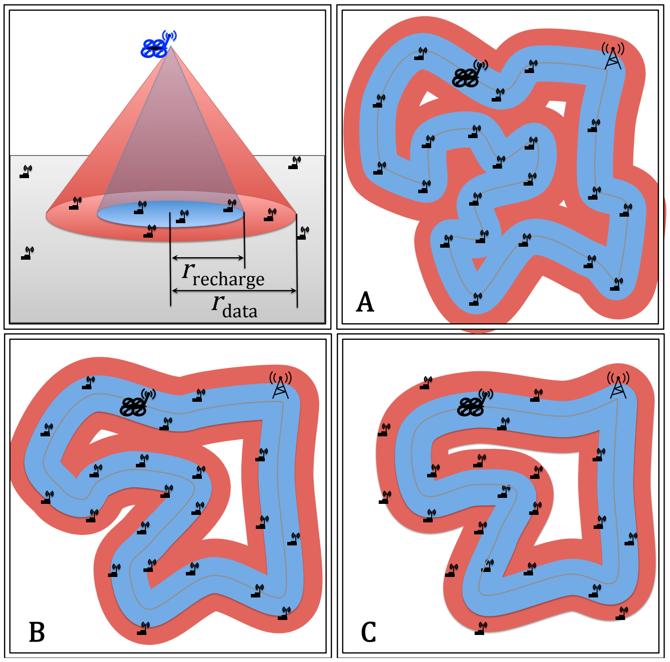


Fig. 4: A UV has an associated recharging footprint and a data-transfer footprint, which can often be modeled as disks of radius $r_{recharge}$ and r_{data}. Path **A** visits each node, but path **B** is shorter because it is designed merely to recharge all nodes. Path **C** is the least tortuous because it is designed to transfer data from all nodes, and $r_{data} > r_{recharge}$.

IV. Path Optimization Algorithm

Our solution designs a closed-loop path that intersects the origin for each UV. These paths are composed of waypoints connected by straight-line segments. The following sections describe how this path is initialized (IV-A), and then how the path is optimized by switching between a gradient descent optimization routine that finds local minimas (IV-B), and a genetic algorithm that rearranges the order of waypoints to find better paths (IV-C).

A. Initializing Path with Hilbert Curve

It is important to have an initial path that fills the map. This ensures the UVs will visit every node. We adapt the space-filling *Hilbert Curve*, which creates a fractal path that fills up a unit area space and serves as an initial path for the first iteration [13]. If there are multiple UVs, the Hilbert curve is scaled by a scalar to ensure the waypoints are unique.

B. Gradient Descent on a Path Composed of Waypoints

The following algorithm is derived from [32]. Consider $\mathbf{N}(r = 1 \ldots \mathbf{N})$ UVs servicing a Wireless Sensor Network in a convex, bounded area $\mathbf{Q} \subset \mathbf{R^2}$ and let $\mathbf{p}_i^r$ be the position of the $\mathbf{i}^{th}(\mathbf{i} \in (1 \ldots n(r)))$ waypoint of the $\mathbf{r}^{th}$ UV. Servicing includes recharging the nodes and collecting a part of the data that the nodes are about to transmit to the sink there by reducing the power expenditure in the sensor nodes. The algorithm help in formulating an optimal path to visit the sensor nodes in the WSN depending on the interesting regions. Waypoints are a set of points that define the path for each UV $(\mathbf{1}:\mathbf{r})$, the UV travels in a straight line in between two neighboring waypoints. At each step, we compute the Voronoi partition($\mathbf{V_i^r}$) defined by the waypoints, with one partition assigned to each waypoint. We define an interesting function $\phi(\mathbf{q})$ that commands the usefulness of a location on the map for servicing the WSN $(\mathbf{q} \in Q | \phi(\mathbf{q}) > 0)$.

$$\mathbf{H} = \sum_{r=1}^{\mathbf{N}} \sum_{i=1}^{\mathbf{n(r)}} \int_{V_i^r} \frac{\mathbf{W_s}}{2} \|q - p_i^r\|^2 \phi(\mathbf{q}) d\mathbf{q} \quad + \sum_{r=1}^{\mathbf{N}} \sum_{i=1}^{\mathbf{n(r)}} \frac{\mathbf{W_n}}{2} \|p_i^r - p_{i+1}^r\|^2 \tag{3}$$

The above mentioned equation is the cost-function of the algorithm we aim at minimizing the value. The first part of the equation indicates sensing in regions far away from the interesting regions is costly and the second part indicates having neighboring points far away is also costly. This help is producing a concise path that mostly travels only on the interesting region there by minimizing the cost of travel. $\mathbf{W_s}, \mathbf{W_n}$ are positive scalar constants that are used to weight the sensing and neighbor distance respectively depending on the experimental setup.

We compute the mass, mass-moment, and centroid of the V_i^r (Voronoi partition for $\mathbf{i}^{th}$ waypoint of the $\mathbf{r}^{th}$ UV) as follows:

$$M_i^r = \int_{V_i^r} \phi(\mathbf{q}) d\mathbf{q}, \mathbf{L}_i^r = \int_{V_i^r} \mathbf{q}\phi(\mathbf{q}) d\mathbf{q}, \quad \mathbf{C}_i^r = \frac{\mathbf{L}_i^r}{M_i^r} \tag{4}$$

The control law for each waypoint is the summation of forces that pulls the waypoint toward the centroid of the Voronoi partition (weighted by $\phi(\mathbf{q})$)

$$\mathbf{u}_i^r = \frac{K_i^r (M_i^r \mathbf{e}_i^r + \boldsymbol{\alpha}_i^r)}{\beta_i^r} \tag{5}$$

Here, $\mathbf{K_i^r}$ is a positive definite matrix (potentially-time varying). $\mathbf{e}_i^r = \mathbf{C}_i^r - \mathbf{p}_i^r$, the error introduces the first primitive by obtaining the difference between the waypoint position and the weighted centroid. This tries to move the waypoint towards the interesting region there by reshaping the path of the robot. The second term $\boldsymbol{\alpha}_i^r = W_n(\mathbf{p}_{i+1}^r + \mathbf{p}_{i-1}^r - 2\mathbf{p}_i^r)$ introduces the second primitive which binds the neighboring waypoints together in order to obtain a closed path. $\boldsymbol{\beta}_i^r = \mathbf{M}_i^r + 2\mathbf{W}_n$ normalizes the weight distribution between servicing interesting regions and staying close to neighbors.

The control is then applied to each waypoint and it's position is updated:

$$\mathbf{p}_i^r(k) = \mathbf{p}_i^r(k-1) + \mathbf{u}_i^r \tag{6}$$

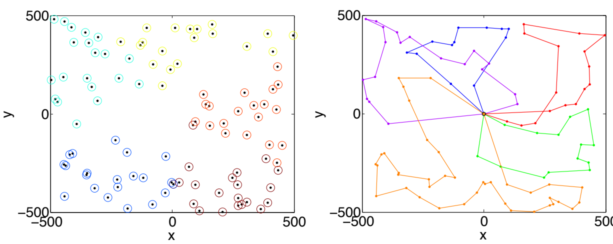
C. multiple-Traveling Salesman Problem (mTSP)

The gradient descent algorithm can get stuck in local minima. To further improve the path we input the location of the waypoints obtained after running the gradient descent algorithm into a multiple-Traveling Salesman Problem (mTSP) search algorithm. Given a list of cities to visit, the classic *traveling salesman problem* (TSP) attempts to find an ordering

Algorithm 1 IC (Interesting Closed) path controller for the i^{th} waypoint $\mathbf{p}_i^r$ in robot r's path in a known environment (from [31], implemented at [33]).

Require: Ability to calculate Voronoi partition
Require: Knowledge of the location of neighboring waypoints $\mathbf{p}_{i-1}^r$ and $\mathbf{p}_{i+1}^r$

loop
    Compute the waypoints Voronoi partition
    Compute $\mathbf{C}_i$ according to (4)
    Obtain neighbor waypoint locations $\mathbf{p}_{i-1}^r$ and $\mathbf{p}_{i+1}^r$
    Compute $\mathbf{u}_i^r$ according to (5)
    Update $\mathbf{p}_i^r$ according to (6)



(a) nodes partitioned by angle (b) near-optimal solution.

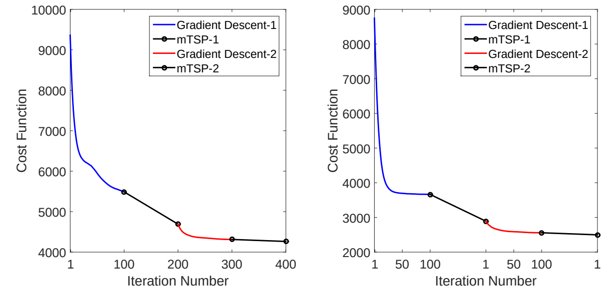
Fig. 5: Screenshots from mTSP solver aided by heuristic. Left, nodes partitioned according to angle from sink. Right, near-optimal solution from a mTSP solver aided by our heuristic.

of the cities that minimizes the total distance on a tour that visits all the cities once [3]. The solution is the shortest Hamilton cycle. By labelling our sensor nodes as cities, the solution to the traveling salesman problem gives the shortest length path. The mTSP solution straightens out loops in the path and can reduce the cost function. This often moves the solution out of the local minimum obtained after the executing the gradient algorithm.

This problem is NP-hard, but many powerful heuristics are available, and software packages can provide answers for tens of thousands of nodes (e.g., the Concorde TSP Solver [4]). A solution with multiple salesman is called a mTSP. The mTSP is an NP-hard (Non-Deterministic Polynomial) problem [5], so the solutions returned by the search algorithm may not be the global optimum.

In [] based on heuristics you get an approximate solution, it depends on the execution time of the algorithm. The waypoints position after $\mathbf{n}$ iteration is given to the mTSP solver which considers these to be city locations and formulates a path depending on the execution time. These algorithms are repeated in succession to obtain a minimal value of cost function depending on the computational time available.

A good heuristic can increase TSP solver performance. In our numerical simulations, priming an open-source genetic algorithm solver [23] by sorting the nodes by angle from the sink and dividing the sorted list equally between the UVs decreased path costs by 20%. Figure 5 shows results from our simulation with 100 nodes and 5 UVs.



(a) Cost-function plotted for the single robot case. (b) Cost-function plotted for the two robot case.

Fig. 8: Cost function indicates a decreasing trend approving the optimization algorithm.

V. Results

We developed a MATLAB simulation using the three algorithms described in IV. The code is available at [33]. The next two sections describe the results with one UV and with multiple UVs.

A. One UV

A single UV system was simulated using MATLAB in Fig. 6. The initial path at iteration 0 was set to be a space-filling Hilbert's curve, in order to identify the location of the sensor nodes. The UV follows this initial path there by getting access to the whole map and identifying the interesting points. A waypoint at $[0, 0]$ is stationary, it acts as a sink for the UV to recharge and to unload the data collected while servicing the sensor nodes. For the first 100 iterations the Algorithm with Gradient Descent is used for simulation. This uses the Voronoi diagram for moving the waypoints to locations where there is high sensory information and uses the Gradient Descent for optimization. The path achieved after 100 iterations is a local-optimum and iterating further does not decrease the cost function. This sub-optimal solution obtained is not impressive. Thus to optimize our path further and to snap out of this local-optimum the path is inputted into a mTSP (multiple-Traveling Salesman Problem) solver for the next 100 iterations, this straightens the loops by reconnecting the waypoints without changing the waypoint location($\mathbf{p}_i^r$). After 200 iterations the cost function has deceased due to straightening of the loops by the mTSP solver. These two algorithms are called in succession to optimize the cost function depending on the time available for calculation or until the cost function converges asymptotically. The cost function has also been plotted which is strictly decreasing reassures the significance of the Algorithms used.

B. Multiple UVs

A two UV system was simulated on MATLAB. As shown in 7, UVs service different sets of nodes in the WSN. A multi-UV system is apt in a practical sense since a single UV might not be able to handle a large network.Similar to the one UV case a space filling hilbert's curve is used for the same reasons stated. A waypoint at $[0, 0]$ is stationary, it acts as a sink for both the UVs to recharge and to unload the data collected while

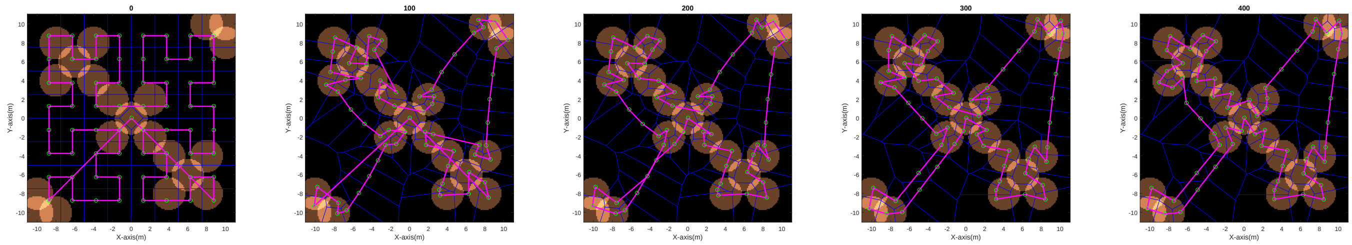


Fig. 6: Simulation results of optimization algorithm in Section IV for one UV. 1.) Iteration 0: Hilbert's Space Filling Curve. 2.) Iteration 100: gradient descent algorithm (first). 3.) Iteration 200: mTSP solver (first). 4.) Iteration 300: gradient descent algorithm (second). 5.) Iteration 400: mTSP solver (second). The waypoints are indicated by a set of linked ○ markers, the associated Voronoi diagram is in blue, the magenta lines represent the path the UV follows for servicing the sensor nodes, and the underlying density plot represents the interesting regions generated by the sensor nodes.

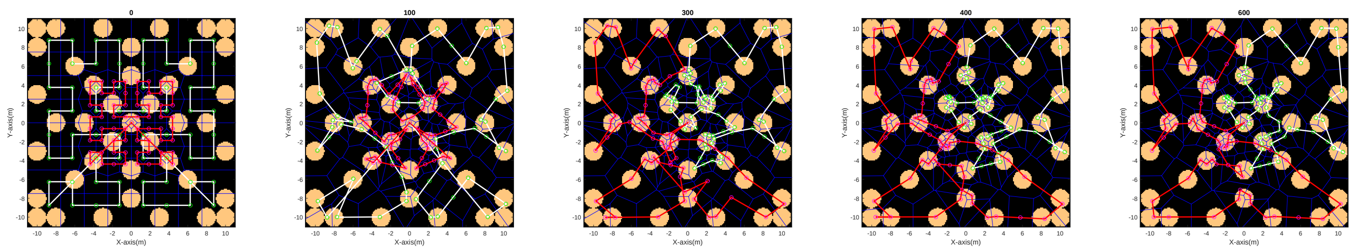


Fig. 7: Simulation results of optimization algorithm in Section IV for two UVs. 1.) Iteration 0: Hilbert's Space Filling Curve. 2.) Iteration 100: gradient descent algorithm (first). 3.) Iteration 200: mTSP solver (first). 4.) Iteration 300: gradient descent algorithm (second). 5.) Iteration 400: mTSP solver (second). The waypoints are indicated by a sets of linked red and yellow○ markers, the associated Voronoi diagram is in blue, the white and cyan lines represent the path the UVs follow for servicing the sensor nodes, and the underlying density plot represents the interesting regions generated by the sensor nodes.

servicing the sensor nodes. The sensor nodes were placed in random locations to verify the robustness of the algorithm. The Algorithm proceeds in a similar fashion to the one UV case. The simulation results show the optimization of the path and the minimization of the cost function.

VI.Conclusion

An optimized path-planning algorithm was simulated to service a WSN. The path constructed is adaptive to the sensor node locations. Future work should extend our simulation to handle non-stationary sensor nodes, improve convergence rate, and use our mTSP code to escape local minimal. We are in the process of implementing the algorithm on mobile-robots, with eventual implementation with a set of quadcopters.

A. Static vs. dynamic environments

The simulations above used a static WSN, but often sensor data transmission is dependent on transient phenomena. For example, a swarm of subsea sensors may track a school of fish, the progress of an oil slick, or seasonal drift of ocean currents. These are time-varying phenomena, and so the UV servicing the sensors should be able to adapt.

The same local optimization techniques can iteratively adapt the paths of UVs. A schematic of our the adaptive control law is shown in Fig. 9. Recent research has focused on local optimization techniques that gradually improve the paths followed by robots during persistent tasks [32]. These techniques are amenable to WSN. The base technique is a variant of Lloyd's algorithm [6], [25]. Each path is represented by a finite number of waypoints, and these waypoints are both attracted to the centroid of all sensor nodes within their Voronoi cell, and attracted to their neighboring waypoints. Our MATLAB implementation is available at mathworks.com/matlabcentral/fileexchange/49863 [33].

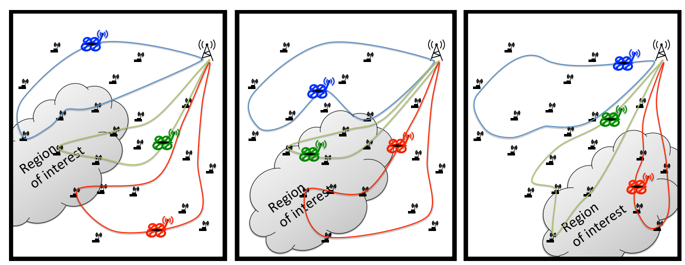


Fig. 9: The grey cloud represents a time-varying region of interest. Allowing the UVs to dynamically modify their routes in a distributed manner enables a robust response to changing conditions while maintaining service.

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