Programming Project – 2019-08-28

# MATB21: Analysis in several variables 1

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This assignment has 3 tasks.

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The goal of this project is to demonstrate how with help of programming skills examples for a more theoretical course can be generated. A side-effect of this project is that your programming skills are kept alive.

The projects follows the line of a previous exam in the course MATB21 (Aug, 2017)

*For this project you should work in* ***groups of two or three****. Solve the project tasks during the course and upload your project answers as a single file having one of the file types* `*.py` *or* `*.ipynb` *in Canvas.*

*Deadline: October 30, 2019. You will get an appointment for an oral presentation of your results to one of the teaching assistants. This presentation is a mandatory part of the project.*

All questions and discussion with regards to the tasks should be done using Canvas.

## Task 1

This is related to Task 1 in the above mentioned exam:

**1.** Determine local extreme values for the function $f(x, y) = 8xy - 4x^2y - 2xy^2 + x^2y^2$.

Determine the local extremal values graphically in a contour plot of this function and compare your results with your theoretical results.

Take a method from `scipy.optimize.fmin` and numerically compute some of the local minima. If the extreme value is a maximum, find the minimum of $-f$ instead. Trace in the contour plot the iterates which this method produces. Hint: a related figure is in the plot section of the course book of the course NUMA01.

## Task 2

This is related to Task 3 in the above mentioned exam:

**3.** Compute the length of the plane curve $\gamma : (x, y) = (t^2, t^3)$, $-2 \leq t \leq 1$.

Solve this task numerically with `quad`

## Task 3

This is related to Task 6 in the above mentioned exam:

**6.** Show that the equation $x + 2y + z + e^{2z} = 1$ has a smooth solution $z = z(x, y)$ defined in a neighbourhood of the origin $x = y = z = 0$. Find the Taylor polynomial of degree 2 of the function $z(x, y)$ about the point $x = y = 0$.

- Illustrate the smoothness of $z(x, y)$ in a neighborhood of the origin using a 3D plot. Hint: For a given $x$ and $y$ you can solve the equation for $z$ using `scipy.optimize.fsolve`.
- Determine the coefficients of the Taylor polynomial $P_2(x, y)$ of degree 2 of $z(x, y)$ about the point $x = y = 0$. Hint: Express the derivatives in terms of $z(x, y)$ or use numerical differentiation (try $h = 10^{-8}$ for simple and $h = 10^{-4}$ for second derivatives).
- Make a 3D plot of $P_2(x, y)$.
- Plot the relative error

$$e(x, y) := \frac{|z(x, y) - P_2(x, y)|}{|z(x, y)|}.$$