

Special Formulas:

1. $\int x^m e^{nx} dx = x^m \cdot \frac{e^{nx}}{n} - \frac{d}{dx}(x^m) \cdot \frac{e^{nx}}{n^2} + \frac{d^2}{dx^2}(x^m) \cdot \frac{e^{nx}}{n^3} - \frac{d^3}{dx^3}(x^m) \cdot \frac{e^{nx}}{n^4} - \dots$
2. $\int x^n \sin x dx = x^n (-\cos x) - nx^{n-1} (-\sin x) + n(n-1)x^{n-2} (\cos x) - n(n-1)(n-2)x^{n-3} (\sin x) + \dots$
3. $\int x^n \cos x dx = x^n (\sin x) - nx^{n-1} (\cos x) + n(n-1)x^{n-2} (-\sin x) - n(n-1)(n-2)x^{n-3} (\cos x) + \dots$
4. $\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1} \right)$
5. $\int e^{ax} \cos bxdx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$
6. $\int e^{ax} \sin bxdx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$
7. $\int e^{ax} \{af(x) + f'(x)\} dx = e^{ax} f(x) + c \Rightarrow \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$

Example: $\int x \ln x dx$

Example: $\int x \sin x dx$

Example-1. Integrate $\int x^2 e^x dx$ [C. 04; S. 13, 09]

Example-2. Integrate $\int \cos^{-1} x dx$ [RB-'10; JB-'08; CB-'14, '05; ChB-'06, '12; SB-'12]

Example 3. Find $\int \ln x dx$; $x > 0$ [C. 06; Ch. 08, 04; B. 04]

Example-4. Integrate $\int x \tan^{-1} x dx$ [R. 06; J. 06; C. 10; S. 08,04; B. 11; Dj. 11]

Example-5. Integrate $\int e^{ax} \cos bxdx$

A special formula: $\int e^{ax} \{af(x) + f'(x)\} dx = e^{ax} f(x) + c$

Example-6. Integrate $\int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx$

Example-7. Integrate $\int \frac{xe^x}{(x+1)^2} dx$ [D. 11; R.; J. 09, 03; Ch. 13,11,05,04; S. 06; C. 11]

Exercise 10(E)

Find the following integrals (1-5)

Type-I

1. (i) $\int x^2 e^{-3x} dx$
- (ii) $\int x^3 e^{x^2} dx$
- (iii) $\int x \ln x dx$ [D. 13,06; J. 03; R. 13; B. 06]
- (iv) $\int (\ln x)^2 dx$ [J. 14,05; Ch. 07]
- (v) $\int x^2 (\ln x)^2 dx$ [E.A.'05]
- (vi) $\int x^n \ln x dx$ [E.A.'93]
- (vii) $\int \frac{\ln(\ln x)}{x} dx$

(viii) $\int x \ln x^2 dx$ [RB-2019]

Type-II

2. (i) $\int x^2 \sin x dx$

(ii) $\int x^2 \cos x dx$

(iii) $\int x \cos x dx$

(iv) $\int e^{2x} \cos e^x dx$

(v) $\int x \sin x \cos x dx$

(vi) $\int x \sin x \sin 2x dx$

(vii) $\int x \sec^2 x dx$ [Ch. 14]

(viii) $\int \frac{x}{\cos^2 x} dx$

(ix) $\int x \cos^2 x dx$ [S. 07]

(x) $\int x \sin^2 \frac{x}{2} dx$ [J.'02]

(xi) $\int x \tan^2 x dx$ [R. 05; S. 05]

(xii) $\int x \cos^3 x dx$

Type-III

3. (i) $\int \sin^{-1} x dx$ [D. 14; J. 10,03; B. 12]

(ii) $\int \tan^{-1} x dx$ [D. 04; J. 13; Dj. 12; B. 10]

(iii) $\int \sec^{-1} x dx$

(iv) $\int x \sin^{-1} x dx$ [D. 07]

(v) $\int x \cos^{-1} x dx$ [M. 11]

(vi) $\int x \sin^{-1} x^2 dx$ [D. 05; R. 13, 06]

(vii) $\int x \cos^{-1} x^2 dx$

4. (i) $\int \tan^{-1} \frac{2x}{1-x^2} dx$

(ii) $\int \sin^{-1} \frac{2x}{1+x^2} dx$

(iii) $\int \cos^{-1} \frac{1-x^2}{1+x^2} dx$ [Ch. 06; C. 05]

(iv) $\int \frac{\ln(\sec^{-1} x)}{x\sqrt{x^2-1}} dx$ [D. '08]

(v) $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$ [Ch. 10; C. 11]

5. (i) $\int e^x \cos x dx$ [Dj. 10; E.A.'04,'06]

(ii) $\int e^x \sin x dx$ [D. 12; C. 13,08,03; R. 14; Dj. 14]

(iii) $\int e^x \sin 2x dx$ [D. 03; R. 09, 04; Dj. 09; S. 10]

(iv) $\int e^{-3x} \cos 4x dx$

(v) $\int e^x (\sin x + \cos x) dx$ [D. 10; S. 11,05; B. 06]

(vi) $\int e^x \left\{ \tan x - \ln (\cos x) \right\} dx$ [C.'02; E.A. 92]

(vii) $\int e^x \left\{ \tan x + \ln (\sec x) \right\} dx$ [R. 03; Ch. 13; J. 11]

(viii) $\int e^x \left(\ln x + \frac{1}{x} \right) dx$ [J.'07; Di.'13]

(ix) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

(x) $\int e^{-x} \left\{ \frac{1}{x} + \frac{1}{x^2} \right\} dx$

(xi) $\int e^{5x} \left\{ 5 \ln x + \frac{1}{x} \right\} dx$ [Ct.'09]

(xii) $\int \frac{e^{m \tan^{-1} x}}{(1+x^2)^2} dx$