Douillard-Jacq Grégoire

ELSS

HOMEWORK

DYNAMIC STABILITY

Airplane: A-7 CORSAIR II - Case 1

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TABLE OF CONTENT

Flight conditions & Aircraft's infos

Geometric Data	
Wing Area (S)	34,84 m^2
Wingspan (b)	11,8 m
Aspect Ratio (A)	4
Mean Aerodynamic Chord ($\bar{c}$)	3.29 m

Lateral Control Derivatives	
$C_{y_{\delta_a}}$	-0,015
$C_{l_{\delta_a}}$	0,055
$C_{n_{\delta_a}}$	- 0,002
$C_{y_{\delta_r}}$	0,23
$C_{l_{\delta_r}}$	0,007
$C_{n_{\delta_r}}$	-0,105

Flight Conditions	
Altitude (H)	0 m
Air Density (r)	1,225 kg/m^3
Flight speed (U_0)	85 m/s
Mach Number	0,25
Initial Pitch Angle (θ_0)	11,2

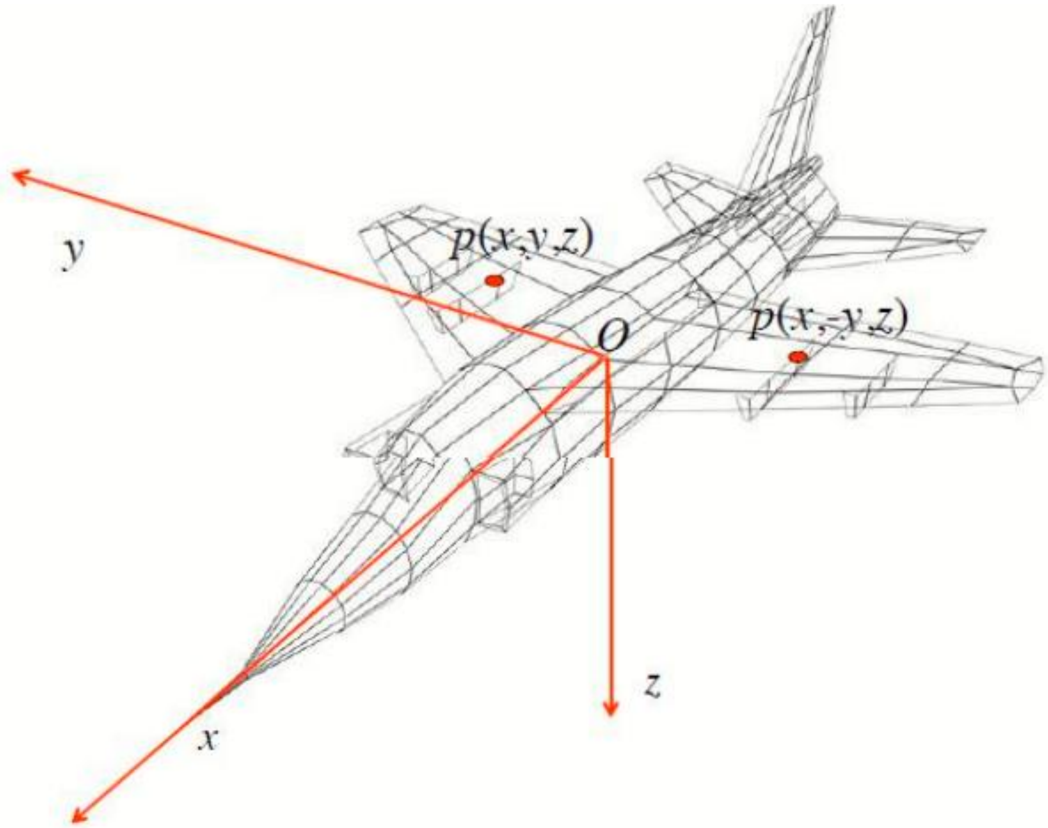
Mass & Inertial Data	
Aircraft Mass (m)	9926,7 kg
I_{xx}	18486.6 kg m^2
I_{yy}	68965 kg m^2
I_{zz}	91599 kg m^2
I_{xz}	3976.6 kg m^2

Longitudinal Stability Derivatives	
C_{D_u}	0,0105
C_{D_α}	1,52
C_{L_u}	0,04
C_{L_α}	3,95
$C_{L_{\dot{\alpha}}}$	0
C_{L_q}	0
C_{m_u}	0,012
C_{m_α}	-0,45
$C_{m_{\dot{\alpha}}}$	-0,7
C_{m_q}	-3,8

Lateral Stability Derivatives	
C_{y_β}	- 0,88
C_{y_p}	0
C_{y_r}	0
C_{l_β}	- 0,115
C_{l_p}	- 0,25
C_{l_r}	0,18
C_{n_β}	0,105
C_{n_p}	-0,01
C_{n_r}	-0,34

Initial Steady State Coefficients	
Lift Coefficient $(C_L)_0$	0,62
Drag Coefficient $(C_D)_0$	0,072

Longitudinal Control Derivatives	
$C_{D_{\delta_e}}$	0
$C_{L_{\delta_e}}$	0,6
$C_{m_{\delta_e}}$	-0,83



I – LongitudinalDynamic Stability of Airplane

The aircraft matrix A and control matrix B of aircraft in longitudinal motion

$$Along = I_n long^{-1} * A_n long$$

From the previous equation, we have the following matrices:

$$I_{n\,Long}{}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{(1 - Z_{\dot{w}})} & 0 & 0 \\ 0 & \dfrac{-M_{\dot{w}}}{(1 - Z_{\dot{w}})} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_{n\,Long} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta_0 \\ Z_u & Z_w & Z_q + U_0 & -g\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

It gives us :

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta_0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 \\ M_u + Z_u M_{\dot{w}} & M_w + Z_w M_{\dot{w}} & M_q + U_0 M_{\dot{w}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Thus, in our case using the initial conditions of the airplane and MATLAB, the matrix A in longitudinal motion is given by:

$$\mathbf{A} = \begin{bmatrix} -0.0282 & 0.0295 & 0 & -1.9915 \\ -0.2339 & -0.7349 & 85 & 9.6057 \\ 0.0013 & -0.0381 & -0.6405 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Now, let’s calculate the B matrix in longitudinal motion.

$$Blong = I_n long^{-1} * B_n long$$

By identifying we get:

$$B_{Long} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{(1-Z_{\dot{w}})} & 0 & 0 \\ 0 & \frac{-M_{\dot{w}}}{(1-Z_{\dot{w}})} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} & M_{\delta_T} \\ 0 & 0 \end{bmatrix}$$

$$B_{Long} = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} + M_{\dot{w}} Z_{\delta_e} & M_w + M_{\dot{w}} Z_{\delta_T} \\ 0 & 0 \end{bmatrix}$$

Thus, in our case using the initial conditions of the airplane and MATLAB, the matrix B in longitudinal motion is given by:

$$B = \begin{bmatrix} 0 & 0 \\ 0.1096 & 0 \\ -0.0719 & 0 \\ 0 & 0 \end{bmatrix}$$

The characteristic equation of the A matrix

To get the characteristic equation of the aircraft matrix, we must resolve the following equation:

$$det(\lambda * I - A_{long}) = 0$$

Using MATLAB function charpoly(), we directly get the characteristic equation’s coefficient.

$$\boldsymbol{det(\lambda * I - A_{long}) = \lambda^4 + 1.4037 * \lambda^3 + 3.7531 * \lambda^2 + 0.0459 * \lambda^2 + 0.0930 * \lambda}$$

The eigenvalues (roots of equation) of the system

We can now find the eigenvalues of the matrix A using eig():

$$\boldsymbol{\lambda_1 = -0.7004 + 1.7982i}$$

$$\boldsymbol{\lambda_2 = -0.7004 - 1.7982i}$$

$$\boldsymbol{\lambda_3 = -0.0015 + 0.1580i}$$

$$\boldsymbol{\lambda_4 = -0.0015 - 0.1580i}$$

Different modes of longitudinal stability

Using the standard form as follow:

$$(\lambda - \lambda_1) * (\lambda - \lambda_2) * (\lambda - \lambda_3) * (\lambda - \lambda_4) = 0$$

By identifying with the previous question, we get this equation:

$$\boldsymbol{(\lambda + 0.7004 - 1.7982i) * (\lambda + 0.7004 + 1.7982i) *}$$

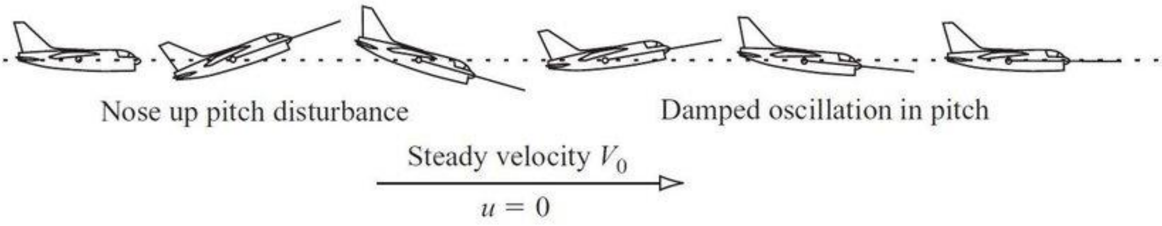
$$\boldsymbol{(\lambda + 0.0015 - 0.1580i) * (\lambda + 0.0015 + 0.1580i) = 0}$$

By multiplying the first and second terms together, and same for the third and fourth. The natural frequency and the damping ratio for these modes are given by:

$$(\lambda^2 + 2\xi_{sp}\omega_{n\,sp}\lambda + \omega_{n\,sp}^2)(\lambda^2 + 2\xi_p\omega_{n\,p}\lambda + \omega_{n\,p}^2) = 0$$

$$\boldsymbol{(\lambda^2 + 1.4006 * \lambda + 3.7324) * (\lambda^2 + \lambda * 0.000459 + 0.0249) = 0}$$

I. Short period mode (Natural Frequency, Damping Factor)



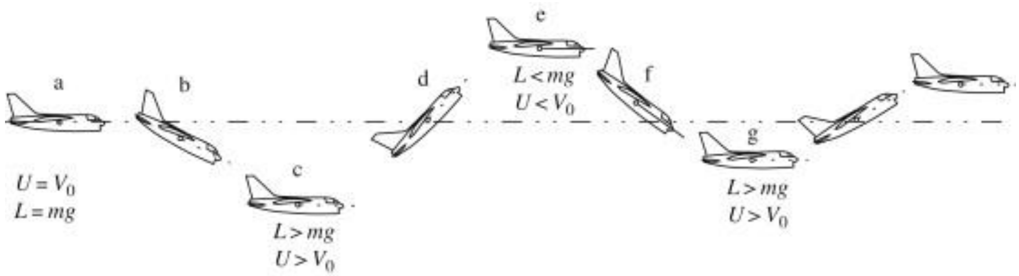
The first complex root represents a high-frequency, highly damped oscillation called the short-period mode:

$$\boxed{Damping\ factor\ (\xi_{sp})\ = \frac{1.4006}{2 * \omega_{s\,sp}}\ =\ 0.3629}$$

$$\boxed{Natural\ frequency\ (\omega_{n\,sp}) = \sqrt{3.7324} = 1.9298\ rad/sec}$$

Because Esp is greater than 0, it gives us the information that we are in stable mode. With this value it exhibits some degree of damping, which I generally desirable for stability.

II. Phugoid mode (Natural Frequency, Damping Factor)



the second complex root represents a low-frequency, lightly damped oscillation called the phugoid mode:

$$\boxed{Damping\ factor\ (\xi_{p})\ = \frac{0.000459}{2 * \omega_{s\,p}}\ = 0.0092}$$

$$\boxed{Natural\ frequency\ (\omega_{n\,p})\ = \sqrt{0.0249} =\ 0.1578\ rad/sec}$$

In phugoid mode, a damping factor that close to 0 implies extremely weak damping but the oscillations will last for a long time.

Curves of longitudinal motion

$$X = X_0\, e^{At} = \sum_{i=1}^{4} V_i\, e^{\lambda_i t}$$

From the above equation we get:

$$\begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{bmatrix} * \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ e^{\lambda_3 t} \\ e^{\lambda_4 t} \end{bmatrix}$$

With Δu, Δw, Δq, Δθ the **axial velocity**, **pitch rate**, **angle of attack** and **pitch angle** respectively.

Thus, we can write:

$$\Delta u = V_{11}e^{\lambda_1 t} + V_{12}e^{\lambda_2 t} + V_{13}e^{\lambda_3 t} + V_{14}e^{\lambda_4 t}$$
$$\Delta w = V_{21}e^{\lambda_1 t} + V_{22}e^{\lambda_2 t} + V_{23}e^{\lambda_3 t} + V_{24}e^{\lambda_4 t}$$
$$\Delta q = V_{31}e^{\lambda_1 t} + V_{32}e^{\lambda_2 t} + V_{33}e^{\lambda_3 t} + V_{34}e^{\lambda_4 t}$$
$$\Delta\theta = V_{41}e^{\lambda_1 t} + V_{42}e^{\lambda_2 t} + V_{43}e^{\lambda_3 t} + V_{44}e^{\lambda_4 t}$$

But from physical truth, we know that Δθ and Δu vary only on phugoid mode so we can get rid of the two first terms of each of their equation. It gives us:

$$\Delta u = V_{13}e^{\lambda_3 t} + V_{14}e^{\lambda_4 t}$$
$$\Delta\theta = V_{43}e^{\lambda_3 t} + V_{44}e^{\lambda_4 t}$$

Same for Δq and Δw, they only vary in short period mode so we can get rid of the two last terms of their equation. We get:

$$\Delta w = V_{21}e^{\lambda_1 t} + V_{22}e^{\lambda_2 t}$$
$$\Delta q = V_{31}e^{\lambda_1 t} + V_{32}e^{\lambda_2 t}$$

I. Axial velocity in function of time

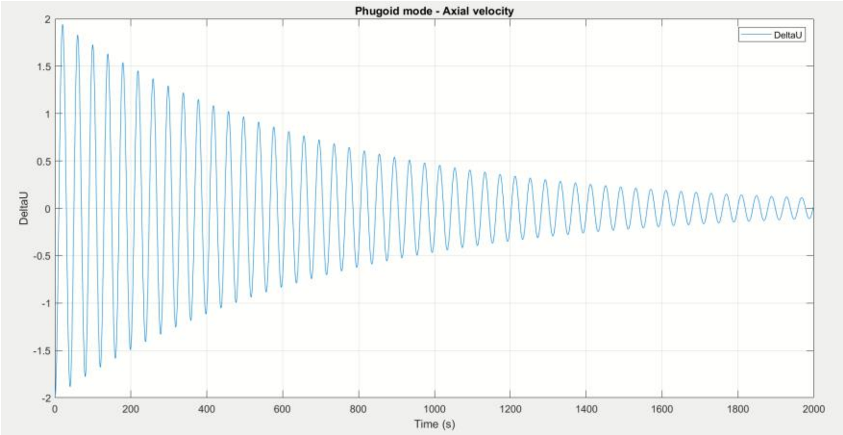


Image 4: Forward speed response after an impulse perturbation

II. Pitch rate

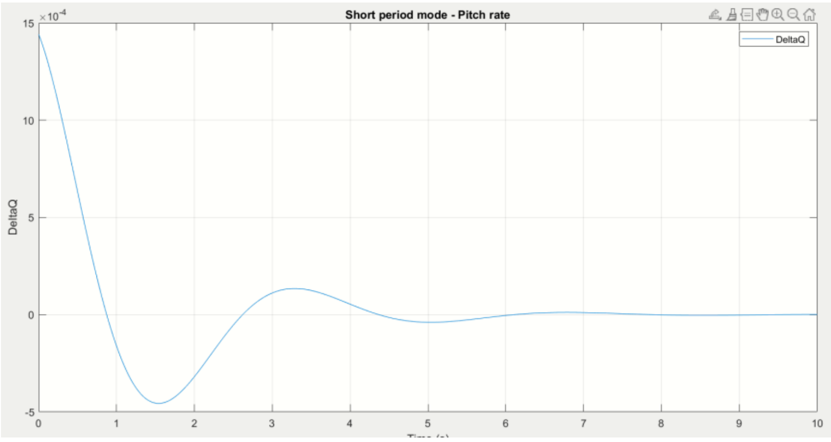


Image 5: AoA response after an impulse perturbation

- The **phugoid** stability mode of this aircraft is **stable.** The **axial velocity oscillates** and finally **tends toward zero** and the **pitch angle stays closed to zero** after the perturbation. We can say that after around **2000 seconds,** the plane has reached again his **initial flight conditions**.
- The **axial velocity varies more than the pitch angle** in the case of our plane.

III. Angle of attack

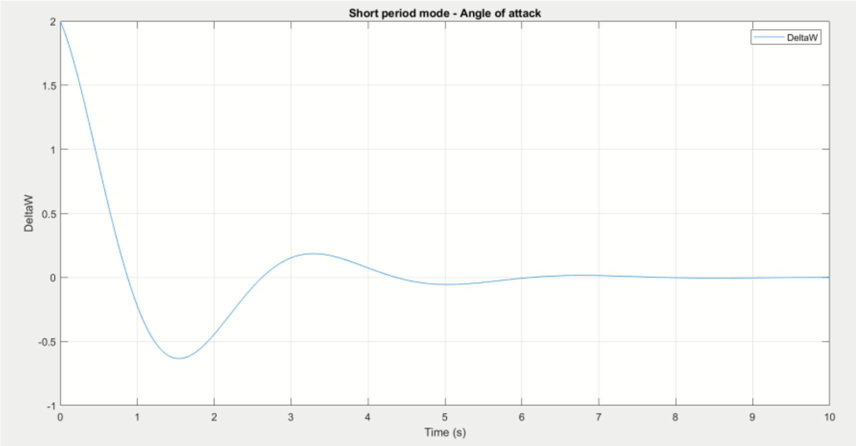
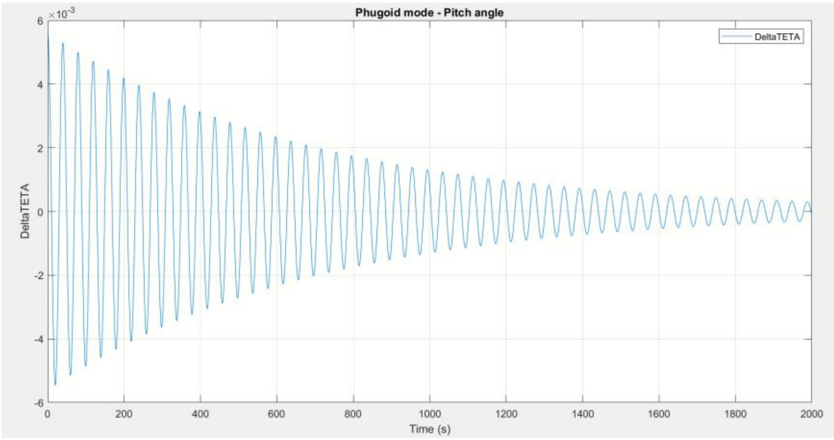


Image 6: AoA response after an impulse perturbation

IV. Pitch angle



- The **short period stability mode** of this aircraft is **stable**. Both angle of attack and pitch rate **oscillates** and finally tends toward **zero**. We can say that after around 2000 seconds, the plane has reached again his initial flight conditions.
- The **short period** read on the graph is **consistent** with the one found with calculation.

Transfer Functions of Each variable

We start by using the system state equation:

$$\dot{X} = A_{long}X + B_{long}\eta$$

Using Laplace transfer and by simplifying the writing as X(0) = 0, we obtain:

$$sX(s) - A_{long}X(s) = B_{long}\eta(s)$$

$$(s\boldsymbol{I} - \boldsymbol{A})\frac{\boldsymbol{X}(s)}{\boldsymbol{\eta}(s)} = \boldsymbol{B}$$

Finally, if the inverse matrix exists, it can be written as:

$$\boxed{\frac{X(s)}{\eta(s)} = \left(sI - A_{long}\right)^{-1} * B_{long}}$$

We call this quantity a Transfer function aircraft control response to elevator and throttle.

$$\begin{bmatrix} \frac{\Delta u(s)}{\Delta\delta_e(s)} & \frac{\Delta u(s)}{\Delta\delta_T(s)} \\ \frac{\Delta w(s)}{\Delta\delta_e(s)} & \frac{\Delta w(s)}{\Delta\delta_T(s)} \\ \frac{\Delta q(s)}{\Delta\delta_e(s)} & \frac{\Delta q(s)}{\Delta\delta_T(s)} \\ \frac{\Delta\theta(s)}{\Delta\delta_e(s)} & \frac{\Delta\theta(s)}{\Delta\delta_T(s)} \end{bmatrix} = (s\,\boldsymbol{I} - \boldsymbol{A})^{-1} * \begin{bmatrix} X_{\delta_e} & X_{\delta_T} \\ Z_{\delta_e} & Z_{\delta_T} \\ M_{\delta_e} + M_{\dot{w}}Z_{\delta_e} & M_{\delta_T} + M_{\dot{w}}Z_{\delta_T} \\ 0 & 0 \end{bmatrix}$$

To get the response to elevator we will only calculate the following equation:

$$\begin{bmatrix} \frac{\Delta u(s)}{\Delta\delta_e(s)} \\ \frac{\Delta w(s)}{\Delta\delta_e(s)} \\ \frac{\Delta q(s)}{\Delta\delta_e(s)} \\ \frac{\Delta\theta(s)}{\Delta\delta_e(s)} \end{bmatrix} = (s\,\boldsymbol{I} - \boldsymbol{A})^{-1} * \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} + M_{\dot{w}}Z_{\delta_e} \\ 0 \end{bmatrix}$$

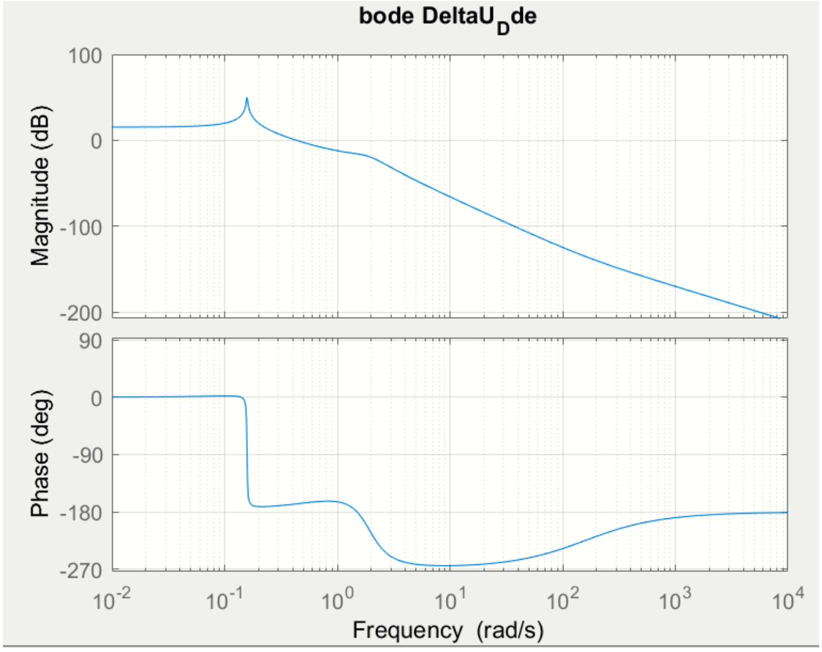
We'll use a bode diagram and analyse them to conclude about the stability of the aircraft.

I. *Axial velocity response to elevator:*

We can now express the axial velocity as follows:

$$\boxed{\frac{\Delta u(s)}{\Delta\delta_e(s)} = \frac{K_u\left(s + \frac{1}{T_u}\right)(s^2 + 2\xi_u\omega_u s + {\omega_u}^2)}{(s^2 + 2\xi_{Ph}\omega_{Ph}s + {\omega_{Ph}}^2)(s^2 + 2\xi_{Sp}\omega_{Sp}s + {\omega_{Sp}}^2)}}$$

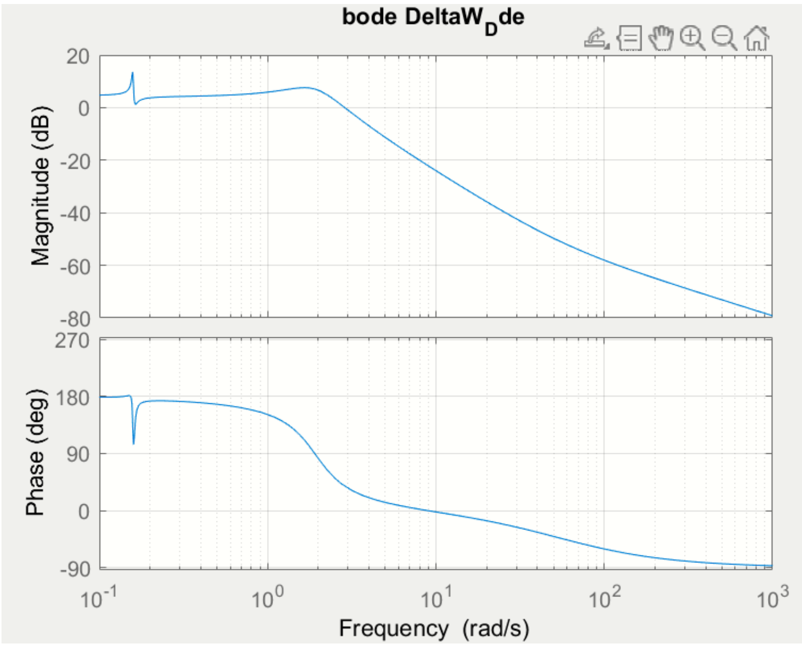
By identifying Ku, Tu, ξ_u and ω_u we get the following bode diagram:



II. *Vertical velocity response to elevator*

$$\frac{\Delta w(s)}{\Delta\delta_e(s)} = \frac{K_w\left(s+\frac{1}{T_w}\right)(s^2+2\xi_w\omega_w s+\omega_w{}^2)}{(s^2+2\xi_{Ph}\omega_{Ph}s+\omega_{Ph}{}^2)(s^2+2\xi_{Sp}\omega_{Sp}s+\omega_{Sp}{}^2)}$$

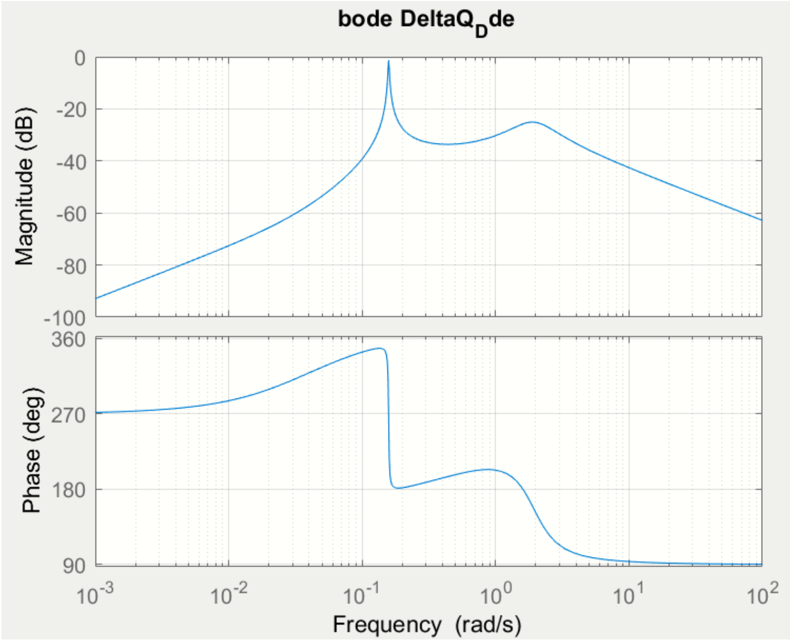
Here is the bode diagram of the vertical velocity:



III. *Pitch rate response to elevator*

$$\frac{\Delta q(s)}{\Delta\delta_e(s)} = \frac{K_q s\left(s+\frac{1}{T_{\theta_1}}\right)\left(s+\frac{1}{T_{\theta_2}}\right)}{(s^2+2\xi_{Ph}\omega_{Ph}s+\omega_{Ph}{}^2)(s^2+2\xi_{Sp}\omega_{Sp}s+\omega_{Sp}{}^2)}$$

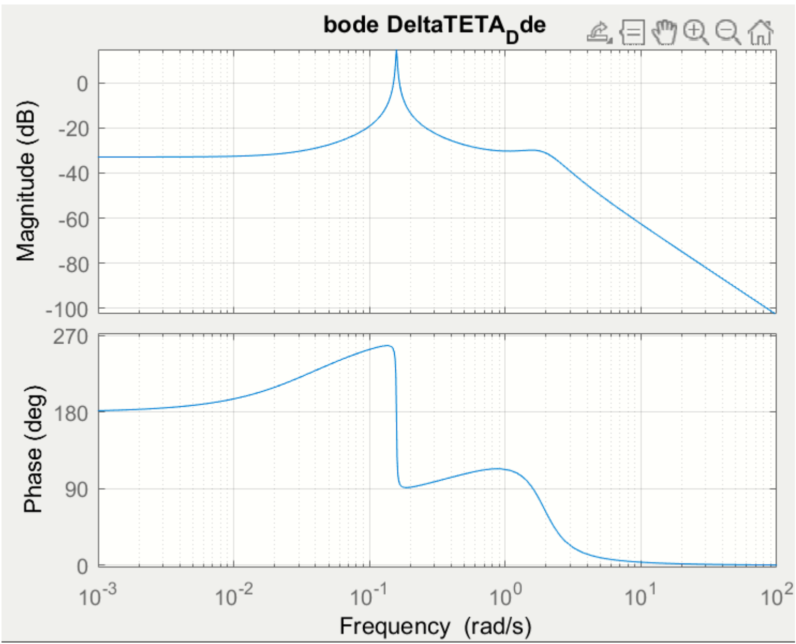
Here is the bode diagram of the pitch rate:



IV. Pitch angle response elevator

$$\frac{\Delta\theta(s)}{\Delta\delta_e(s)} = \frac{K_\theta\left(s+\frac{1}{T_{\theta_1}}\right)\left(s+\frac{1}{T_{\theta_2}}\right)}{\left(s^2+2\xi_{Ph}\omega_{Ph}s+\omega_{Ph}{}^2\right)\left(s^2+2\xi_{Sp}\omega_{Sp}s+\omega_{Sp}{}^2\right)}$$

Here is the bode diagram of the pitch angle:



At very low frequencies (ω < 0.01 rad/s):

The axial velocity, angle of attack and pitch angle follow the stick movement precisely. We can say that because there is no phase shift between the input and the output, and the gain remain constant. We can observe that the gain isn't constant for the **pitch rate**, and it means that in this flight conditions, the pilot must be aware of the **pitch attitude lag in response to the stick input**.

Peaks in the gain plot:

Peaks in the gain plot: We can notice **two peaks** on the gain plot, which correspond to the stability modes of the airplane. The first one indicates the **phugoid mode**, and the second one corresponds to the **short period mode**. We know that the **magnitude of the change** in gain and phase at a peak shows **the significance of the stability mode** in the response of the variable we are studying. We can also read in the bode diagram the **bandwidth frequency** of each variable, and it reveals the **quickness of response** achievable. So higher is the bandwidth frequency, lower is the time response to elevators movement.

So, we can see that the frequency response of **axial velocity** to elevators is clearly dominated by **phugoid**. Moreover, the gain drops as the frequency increases beyond the phugoid frequency, so the axial velocity **bandwidth frequency** is a little higher than phugoid frequency. In the same way, the **pitch angle** frequency response is also dominated by the **phugoid**, but not as clearly as the axial velocity. Indeed, we saw before in the plots that pitch angle oscillations in phugoid mode has low magnitude. According to **the angle of attack**, it is dominated by the **short period mode**, but like pitch angle, it is not evident on the bode diagram. The angle of attack bandwidth is a little higher than the short period frequency. To finish, the pitch rate is clearly dominated by short period mode. Its bandwidth frequency is **high**, and we can say that the pitch rate response is **almost instantaneous**.

Conclusion about bode diagram analysis:

The plot of the bode diagram showed us the **significance of the stability mode in the response of longitudinal variables**. We saw also that the **time response** of variables is **quicker for the short period** mode. But we must be conscious that we made our study assuming a **sinusoidal** elevator input to the aircraft, which is not the case. In the case of aggressive elevator movement, the frequency would exceed the bandwidth of the airplane and then it could not follow the pilot demand quickly enough. It could cause hazardous handling problems, which the pilot must be aware of.

STABILITY AUGMENTATION SYSTEM

We can now determine the longitudinal flying quality by comparing parameters of stability modes with theoretical one found in the following table:

$$\boxed{\boldsymbol{Damping\ factor\ (\xi_{sp})\ =\frac{1.4006}{2*\omega_{s\,sp}}\ =\ 0.3629}}$$

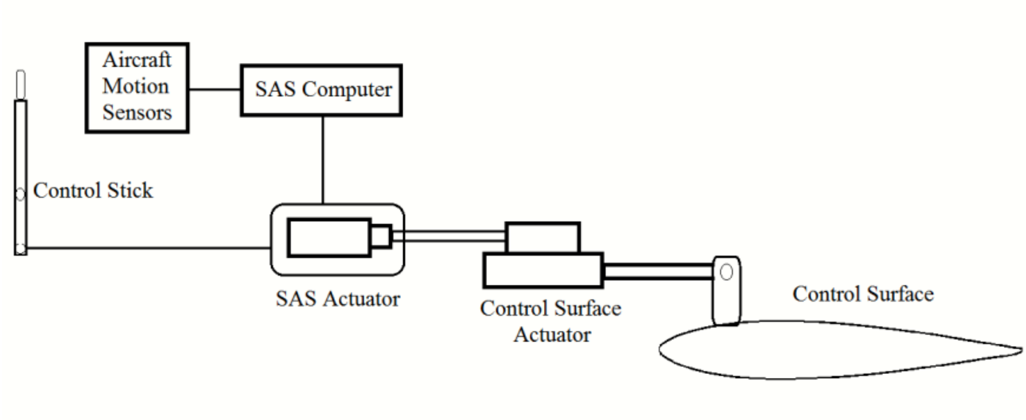
$$\boxed{\boldsymbol{Natural\ frequency\ (\omega_{n\,sp}) = \sqrt{3.7324} = 1.9298\ rad/sec}}$$

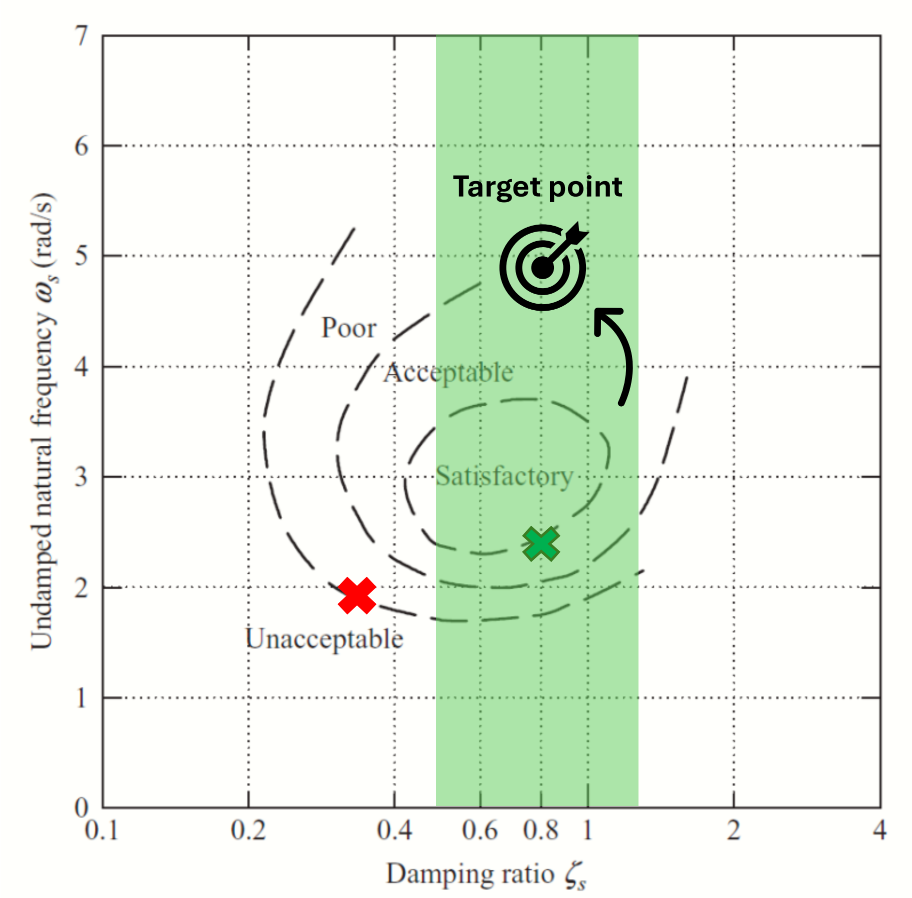
	Level 1		*Level 2*		*Level 3*
	ζ_s *min*	ζ_s *max*	ζ_s *min*	ζ_s *max*	ζ_s *min*
CAT A	0.35	1.30	0.25	2.00	0.10
CAT B	0.30	2.00	0.20	2.00	0.10
CAT C	0.50	1.30	0.35	2.00	0.25

The airplane we are studying is a fighter bomber airplane, so it is a type IV aircraft. We focus on "flight conditions" with altitude zero so we are in category C.

But according to the short period mode, εsp < 0.50 so this airplane hasn't acceptable flying qualities for this stability mode. It means that we will need to modify it by using stability augmentation systems.

We have to find a solution to get to the level 1 because actually we are at the limit between level 2 and 3 which isn't acceptable. To do so, we'll have to do a Stability Augmentation System (SAS) that is the first feedback control system designed to intended to improve dynamic stability characteristics of an aircraft. These systems generally an aircraft motion parameter, such as pitch rate, to provide a control deflection that opposed the motion and increased damping characteristics.





	Before SAS	Objective after SAS
Natural frequency	1.9298 rad/s	2.3 rad/s
Damping factor	0.3629	0.8

First, we must make sure that the system is state controllable. We have the controllability matrix V:

$$V_{long} = [B_{long} \;\; A_{long} \, B_{long}]$$

Using MATLAB we get:

$$A_{long}\,B_{long} = \begin{bmatrix} 0.0032 & 0 \\ -6.1962 & 0 \\ 0.0419 & 0 \\ -0.0719 & 0 \end{bmatrix}$$

We finally get:

$$V_{long} = \begin{bmatrix} 0 & 0 & 0.0032 & 0 \\ 0.1096 & 0 & -6.1962 & 0 \\ -0.0719 & 0 & 0.0419 & 0 \\ 0 & 0 & -0.0719 & 0 \end{bmatrix}$$

By applying the rank() function in MATLAB to V_{long}, we see that V_{long} is of rank 2, so the system is state controllable.

The desired characteristic equation for the closed-loop system can be written as:

$$\lambda^2 + 2\xi_{Sp}\omega_{Sp}\lambda + \omega_{Sp}{}^2 = 0$$

We substitute the values of ξ_{sp} and ω_{sp} with the one we want after the SAS. We find the following equation:

$$\lambda^2 + 3.68 * \lambda + 5.29 \;=\; 0$$

The augmented matrix with state feedback A* follows:

$$A* = \; A \;-\; B * k^T$$

It gives us:

$$A* = \; A \;-\; B * k^T \;=\; \begin{bmatrix} -0.7349 & 85.00 \\ -0.0381 & -0.6405 \end{bmatrix} - \begin{bmatrix} 0.1096 \\ -0.0719 \end{bmatrix} * \begin{bmatrix} k1 & k2 \end{bmatrix}$$

$$A* = \begin{bmatrix} -(137 * k1)/1250 \;-\; 0.7349 & 85 \;-\; (137 * k2)/1250 \\ (719 * k1)/10000 \;-\; 0.0381 & (719 * k1)/1000 \;-\; 1281/2000 \end{bmatrix}$$

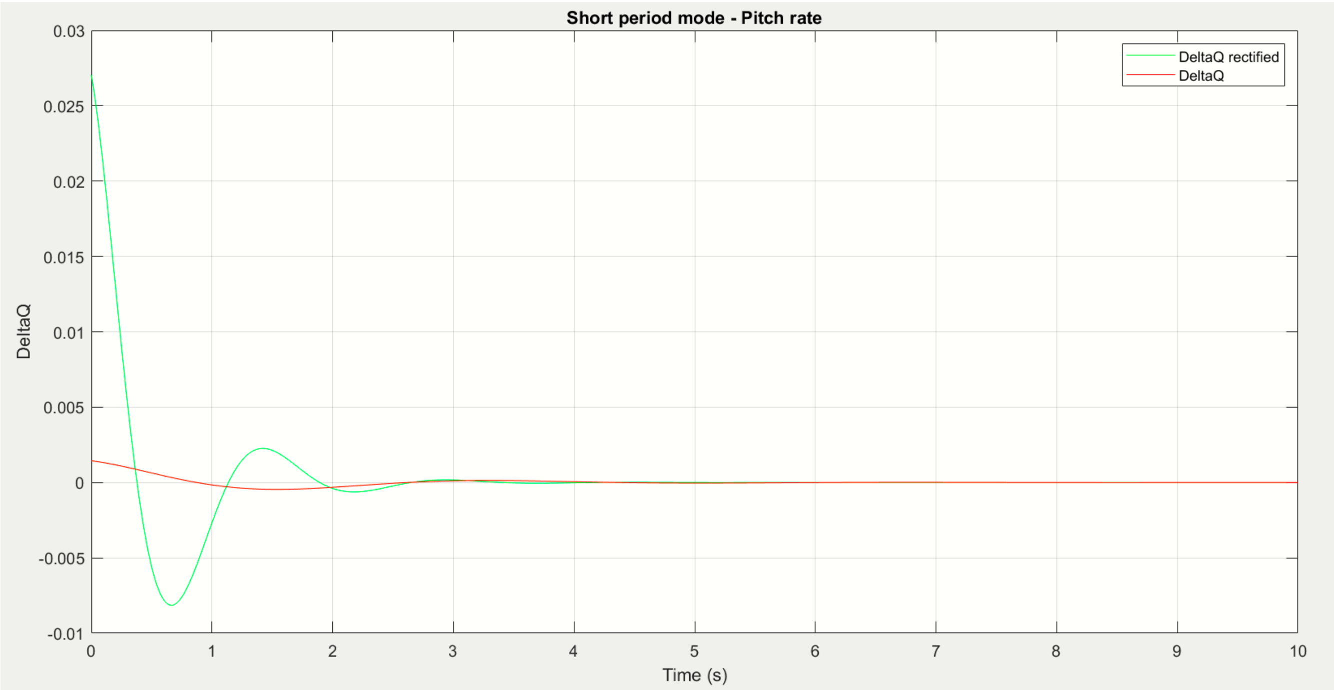
By resolving it we find the vector K's values:

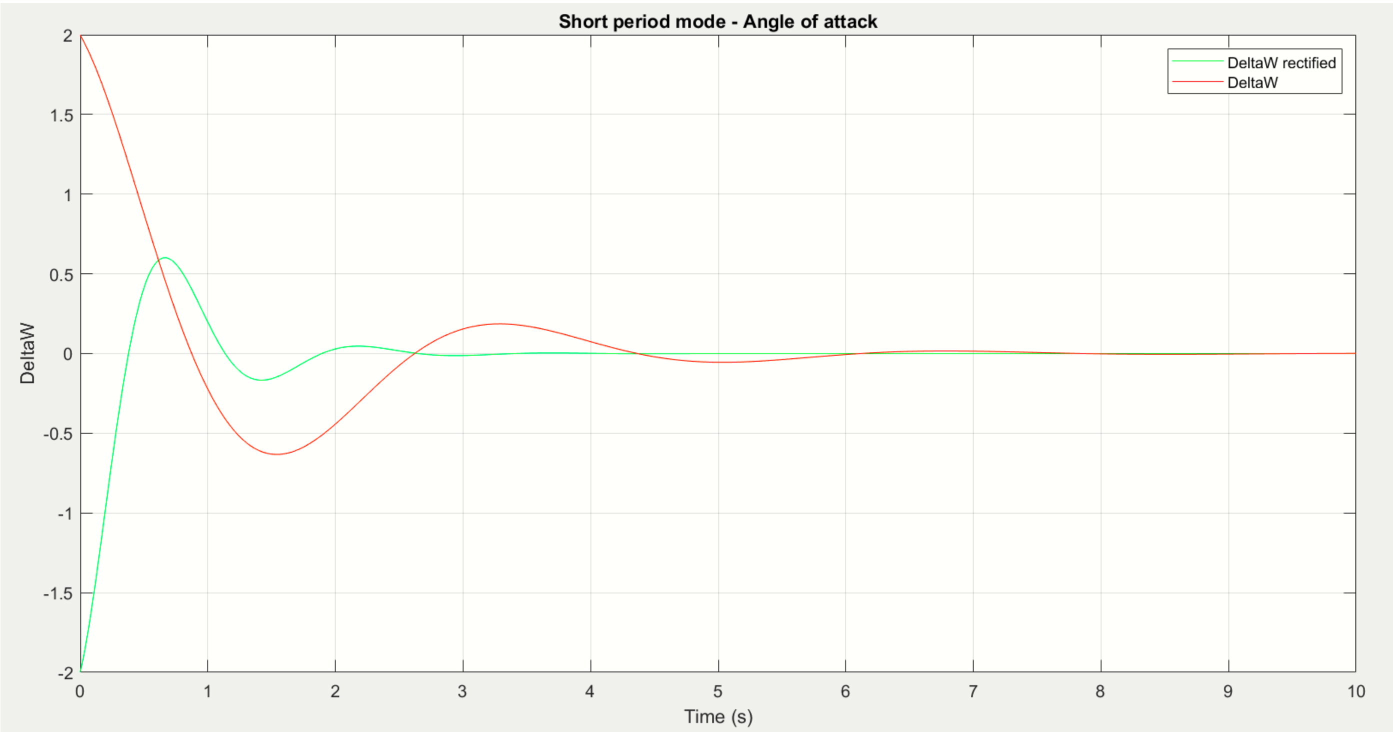
$$K = \begin{bmatrix} -2.4015 \\ -31.2615 \end{bmatrix}$$

We apply those changes to A_{long} and B_{long} to finally get the matrix of A that correspond to with the augmentation stability system:

$$A_{rectified} = \begin{bmatrix} -0.0282 & 0.0295 & 0 & -9.6232 \\ -0.2339 & -0.4716 & 88.4274 & -1.9054 \\ 0.0013 & -0.2109 & -2.8898 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

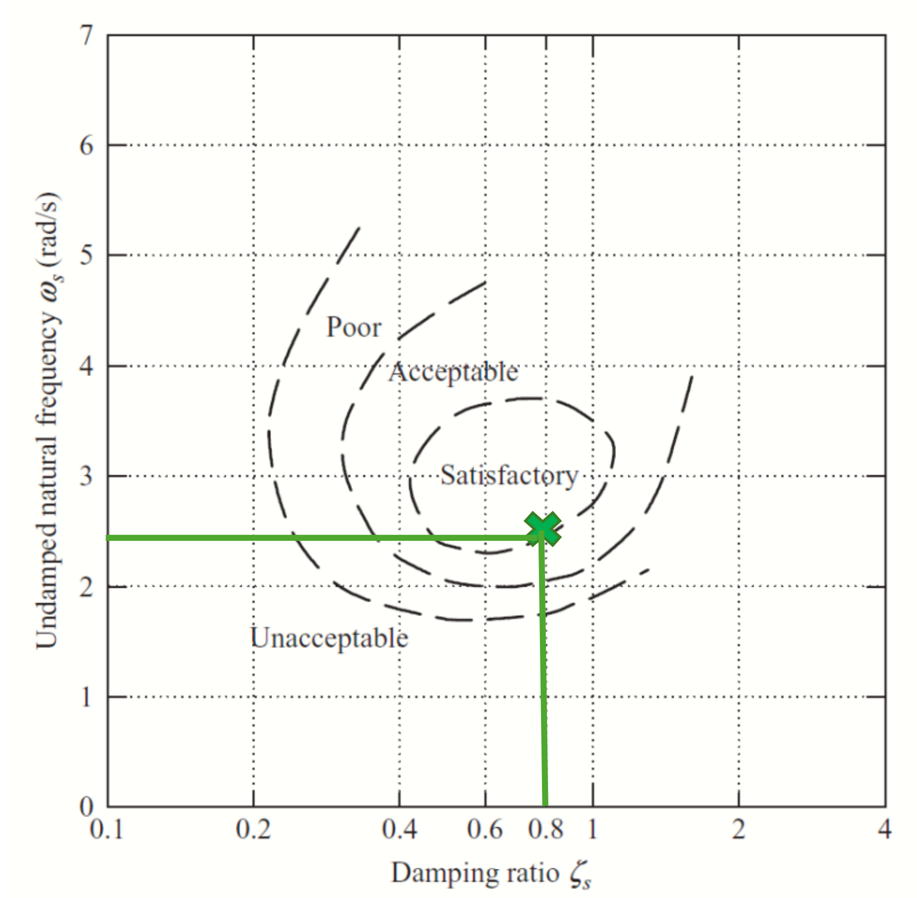
We can compare at the same scale, the short period mode with and without augmentation stability system:





These curves allow us to say that:

- The new short period stability mode is **stable**.
- The curves **return faster to the initial state** it means that the mode is **mode stable** because after the perturbation, the **aircraft isn't influenced that much by this perturbation**.



So, we can conclude that by adding a **gain matrix on rudders**, threw stabilization augmentation systems, we reached the **level 1 of longitudinal stability**, and make the **manoeuvring of the airplane easier for the pilot.**

I – Lateral Dynamic Stability of Airplane

The aircraft matrix A and control matrix B of aircraft in lateral motion

$$\boxed{Alat = I_n lat^{-1} * A_n lat}$$

From the previous equation, we have the following matrix:

$$A_{Lateral} = \begin{bmatrix} Y_v & Y_p & -(U_0 - Y_r) & g\cos\theta_0 \\ L_v & L_p & L_r & 0 \\ N_v & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Thus, in our case using the initial conditions of the airplane and MATLAB, the matrix A in lateral motion is given by:

$$A = \begin{bmatrix} -0.1608 & 0 & -85 & 9.6232 \\ -0.1331 & -1.7077 & 1.2296 & 0 \\ 0.0245 & -0.0138 & -0.4687 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Now, let's calculate the B matrix in lateral motion.

$$B_{Lateral} = \begin{bmatrix} Y_{\delta_r} & Y_{\delta_a} \\ L_{\delta_r} & L_{\delta_a} \\ N_{\delta_r} & N_{\delta_a} \\ 0 & 0 \end{bmatrix}$$

$$B_{lateral} = \begin{bmatrix} 0.420 & -0.0027 \\ 0.0081 & 0.0637 \\ -0.0245 & -0.005 \\ 0 & 0 \end{bmatrix}$$

The characteristic equation of the A matrix in lateral motion

To get the characteristic equation of the aircraft matrix, we must resolve the following equation:

$$\boldsymbol{det(\lambda * I \ - \ A_{lateral}) \ = \ 0}$$

Using MATLAB function charpoly(), we directly get the characteristic equation's coefficient.

$$\boldsymbol{det(\lambda * I - A_{lateral}) = \lambda^4 + 2.3373 * \lambda^3 + 3.2529 * \lambda^2 + 5.1302 * \lambda^2 + 0.3103 * \lambda}$$

The eigenvalues (roots of equation) of the system

Like in the study of longitudinal motion, we first focus on the free response case to determine the characteristics of stability modes:

$$\dot{X} = A_{lat}X$$

We can now find the eigenvalues of the matrix A using eig():

$$\boldsymbol{\lambda_1 = -1.9682}$$

$$\boldsymbol{\lambda_2 = -0.1531 \ + \ 1.5760i}$$

$$\boldsymbol{\lambda_3 = -0.1531 \ - \ 1.5760i}$$

$$\boldsymbol{\lambda_4 = -0.0629}$$

The corresponding eigenvector is:

-0.8451 + 0.0000i	0.9974 + 0.0000i	0.9974 + 0.0000i	0.8728 + 0.0000i
-0.4766 + 0.0000i	-0.0471 + 0.0356i	-0.0471 - 0.0356i	-0.0304 + 0.0000i
0.0094 + 0.0000i	0.0028 - 0.0154i	0.0028 + 0.0154i	0.0538 + 0.0000i
0.2421 + 0.0000i	0.0253 + 0.0275i	0.0253 - 0.0275i	0.4841 + 0.0000i

We can now determine the different modes of lateral stability.

Different modes of lateral stability

Using the standard form as follow:

$$(\lambda - \lambda_1) * (\lambda - \lambda_2) * (\lambda - \lambda_3) * (\lambda - \lambda_4) = 0$$

By identifying with the previous question, we get this equation:

$$(\boldsymbol{\lambda} + \mathbf{1.9682}) * (\boldsymbol{\lambda} + \mathbf{0.1531} - \mathbf{1.5760}\boldsymbol{i}) *$$

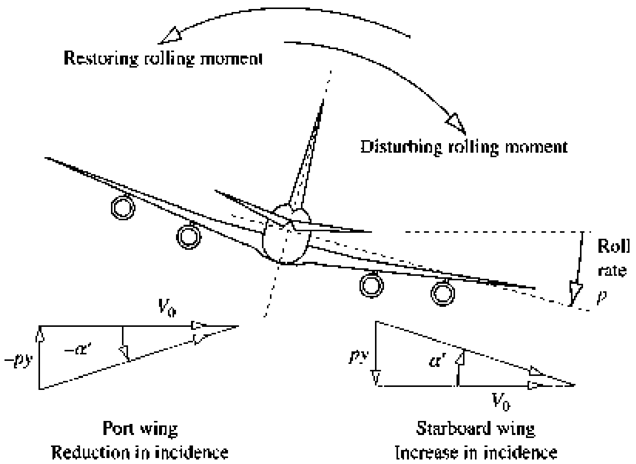
$$(\boldsymbol{\lambda} + \mathbf{0.1531} + \mathbf{1.5760}\boldsymbol{i}) * (\boldsymbol{\lambda} + \mathbf{0.0629}) = \mathbf{0}$$

By multiplying the first and second terms together, and same for the third and fourth.

$$(\lambda - \lambda_{\text{roll}}) * (\lambda - \lambda_{\text{spiral}}) * (\lambda^2 + 2 * \varepsilon_{\text{dutch roll}}\, \omega_{\text{dutch roll}} * \lambda + \omega_{\text{dutch roll}}{}^2) = 0$$

So, by identification we have the following parameters of lateral stability modes:

I. Rolling mode:



We have:

$$\boxed{\boldsymbol{\lambda}_{\mathbf{roll}} = -\mathbf{1.9682}}$$

Let's calculate the rolling time constant:

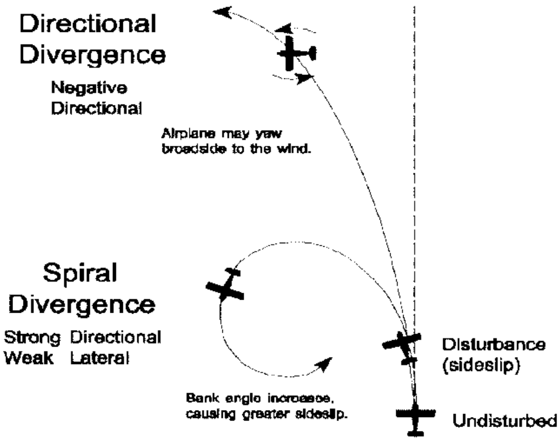
$$\boldsymbol{Roll\ time\ constant\ T_1} = \frac{\mathbf{1}}{|\boldsymbol{\lambda}_{\mathbf{Roll}}|}$$

$$\boxed{\boldsymbol{T_1} = \mathbf{0.508}\ \boldsymbol{sec}}$$

	Maximum value of T_r (seconds)		
	Level 1	Level 2	Level 3
A, C	1.0	1.4	—
A, C	1.4	3.0	—
B	1.4	3.0	—

To finish, we can say thanks to the previous table that the rolling mode of this aircraft is level 1, because the value that we got is under the minimum value of the level1 one (1.0). It means that flying qualities are clearly adequate to accomplish the mission flight phase.

II. Spiral mode



We have:

$$\boxed{\lambda_{\text{roll}} = -0.0629}$$

Spiral time constant:

$$\textit{Spiral time constant } T_1 = \frac{1}{|\lambda_{\text{Spiral}}|}$$

$$\boxed{T_1 = 15.898\ sec}$$

Time to double:

$$\textit{Time to double } T_2 = \frac{Ln\,\dot{2}}{|\lambda_{\text{Spiral}}|}$$

$$\boxed{T_1 = 11.006\ sec}$$

	Minimum value of T_2 (seconds)		
	Level 1	Level 2	Level 3
A, C	12	8	5
B	20	8	5

The values we have got satisfies the minimum required in the table, that means that the spiral stability mode's qualities are level 2 because this board is for time to double variable. It means that they are adequate to accomplish the missions flight phase, with some increase in pilot workload or degradation of mission effectiveness.

III. Dutch roll mode

$$\boxed{\omega_{dutch\ roll} = \sqrt{2.484} = 1.576\ rad/sec}$$

$$\boxed{Damping\ factor\ (\xi_p) = \frac{0.000459}{2 * \omega_{s\,p}} = 0.09669}$$

	Minimum values							
	Level 1			Level 2			Level 3	
	ζ_d	$\zeta_d\omega_d$	ω_d	ζ_d	$\zeta_d\omega_d$	ω_d	ζ_d	ω_d
CAT A	0.19	0.35	1.0	0.02	0.05	0.5	0	0.4
CAT A	0.19	0.35	0.5	0.02	0.05	0.5	0	0.4
CAT B	0.08	0.15	0.5	0.02	0.05	0.5	0	0.4
CAT C	0.08	0.15	1.0	0.02	0.05	0.5	0	0.4
CAT C	0.08	0.10	0.5	0.02	0.05	0.5	0	0.4

We can say thanks to the previous table that the Dutch roll mode of this aircraft is level 1. Indeed, we have $\varepsilon dutch\ roll * \omega dutch\ roll$ = 0.152 rad/s < 0.15 so this stability mode isn't level 2. It means that flying qualities are clearly adequate to accomplish the mission flight phase.

Curves of lateral motion:

We can have a better visualization of the results we have got previously by plotting on MATLAB different relevant curves. For lateral motion, we have for each stability mode:

$$\begin{bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta \varphi \end{bmatrix} = \begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{bmatrix} * \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ e^{\lambda_3 t} \\ e^{\lambda_4 t} \end{bmatrix}$$

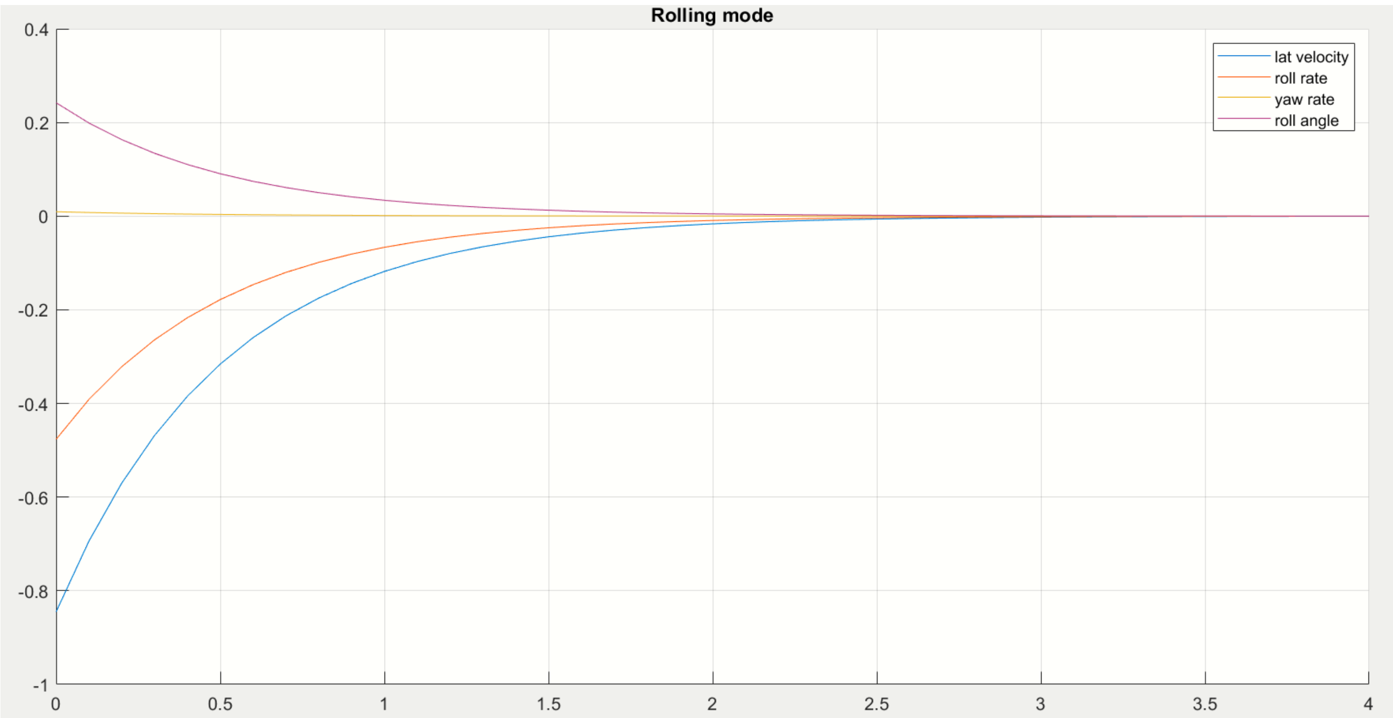
I. Rolling mode

We have the following expressions in rolling mode:

$$\Delta v = V_{11} e^{\lambda_1 t} for\ rolling\ mode$$
$$\Delta p = V_{21} e^{\lambda_1 t} for\ rolling\ mode$$
$$\Delta r = V_{31} e^{\lambda_1 t} for\ rolling\ mode$$
$$\Delta \varphi = V_{41} e^{\lambda_1 t} for\ rolling\ mode$$

With Vij the terms of the 4x4 matrix composed by the eigen vectors of $Alat$, and λi the terms of the column matrix composed by the eigenvalues of $Alat$.

We obtain this curve:



These curves allow us to say that:

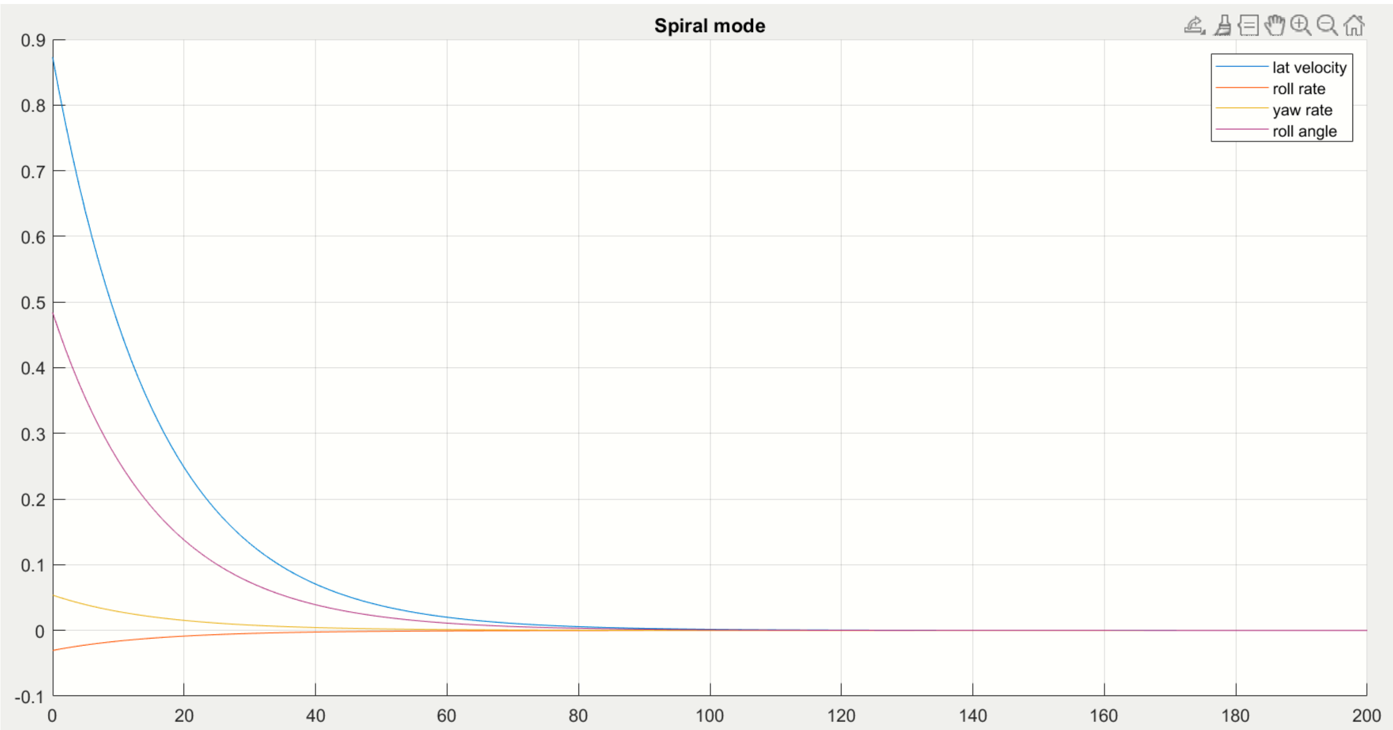
- The **rolling stability mode** of this aircraft is **stable**. All the parameters (side velocity, roll rate, yaw rate, and roll angle) **tend toward zero** with time.
- All the parameters reach **zero almost 2.5 seconds** after the perturbation.
- The **yaw rate** is not really impacted and doesn't vary a lot during rolling mode.

II. Spiral mode

We have the following expressions in spiral mode:

$$\Delta v = V_{12} e^{\lambda_2 t} \; for \; Spiral \; mode$$
$$\Delta p = V_{22} e^{\lambda_2 t} for \; Spiral \; mode$$
$$\Delta r = V_{32} e^{\lambda_2 t} for \; Spiral \; mode$$
$$\Delta \varphi = V_{42} e^{\lambda_2 t} for \; Spiral \; mode$$

We obtain this curve:



These curves allow us to say that:

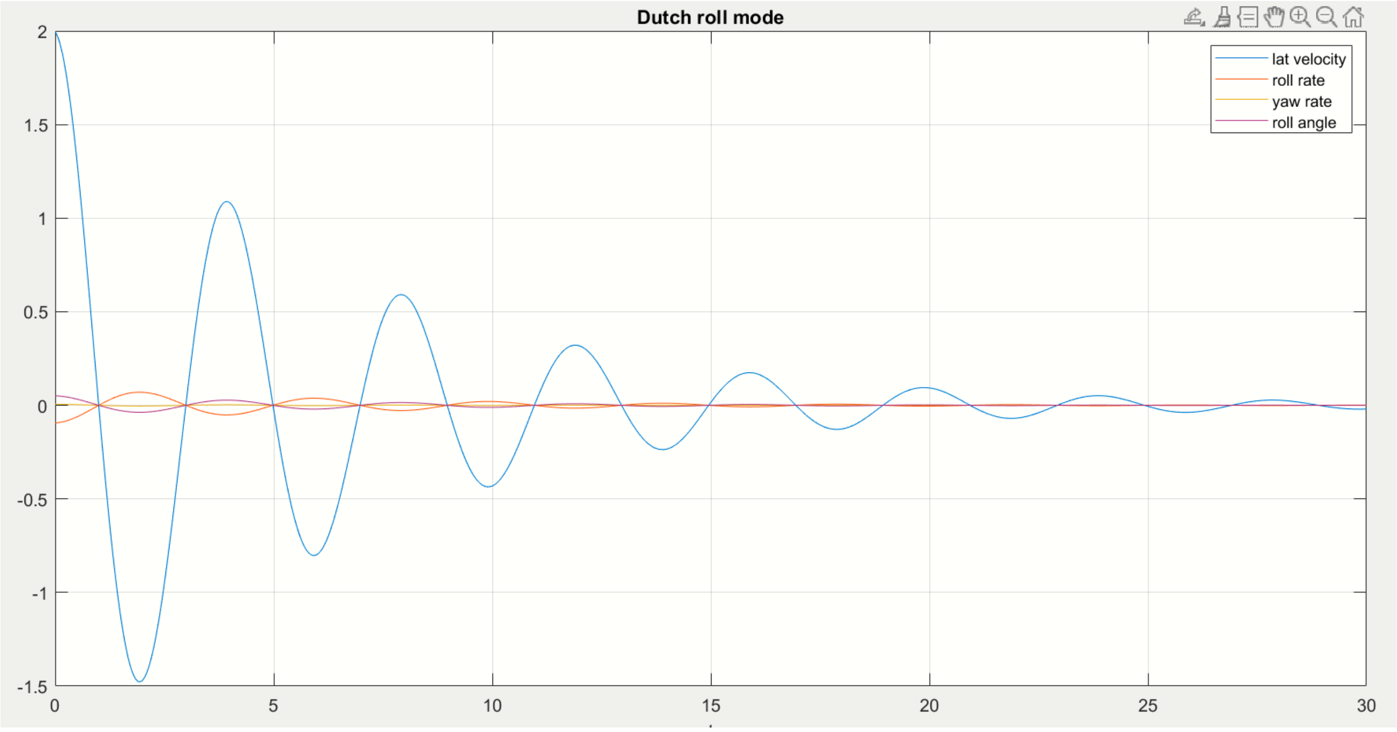
- The **spiral stability mode** of this aircraft is **stable**. All the parameters (side velocity, roll rate, yaw rate, and roll angle) **tend toward zero** with time.
- All the parameters **reach zero almost 80 seconds** after the perturbation.
- The **roll rate** and **yaw rate** aren't really impacted and don't vary a lot during spiral mode.

III. Dutch roll mode

We have the following parameters in dutch roll mode:

$$\Delta v = V_{13} e^{\lambda_3 t} + V_{14} e^{\lambda_4 t} \text{ for Dutch roll mode}$$
$$\Delta p = V_{23} e^{\lambda_3 t} + V_{24} e^{\lambda_4 t} \text{for Dutch roll mode}$$
$$\Delta r = V_{33} e^{\lambda_3 t} + V_{34} e^{\lambda_4 t} \text{for Dutch roll mode}$$
$$\Delta \varphi = V_{43} e^{\lambda_3 t} + V_{44} e^{\lambda_4 t} \text{for Dutch roll mode}$$

We obtain the following curve:



These curves allow us to say that:

- The **Dutch roll stability mode** of this aircraft is **stable**. All the parameters (side velocity, roll rate, yaw rate, and roll angle) **tend toward zero** with time.

- All the parameters **reach zero almost 30 seconds** after the perturbation.
- **The lateral velocity** is the variable which varies the most during Dutch roll.

Transfer Functions of Each variable

We can know study the stability of the aircraft in controlled response case. We have now the general equation:

$$\dot{X} = A_{lat}X + B_{lat}\eta$$

By following the same steps as in longitudinal stability study, we have:

$$\frac{X(s)}{\eta(s)} = (sI - A_{lat})^{-1} * B_{lat}$$

So, the control response to ailerons and rudders can be studied by using this expression:

$$\begin{bmatrix} \frac{\Delta v(s)}{\Delta\delta_r(s)} & \frac{\Delta v(s)}{\Delta\delta_a(s)} \\ \frac{\Delta p(s)}{\Delta\delta_r(s)} & \frac{\Delta p(s)}{\Delta\delta_a(s)} \\ \frac{\Delta r(s)}{\Delta\delta_r(s)} & \frac{\Delta r(s)}{\Delta\delta_a(s)} \\ \frac{\Delta\varphi(s)}{\Delta\delta_r(s)} & \frac{\Delta\varphi(s)}{\Delta\delta_a(s)} \end{bmatrix} = \frac{Adj(sI - A)}{\det(sI - A)} \times \begin{bmatrix} X_{\delta r} & X_{\delta a} \\ L_{\delta r} & L_{\delta a} \\ N_{\delta r} & N_{\delta a} \\ 0 & 0 \end{bmatrix}$$

Response to rudders

I. Side velocity response to rudders

Right below is the theoretical formula for the side velocity to rudders:

$$\frac{\Delta v(s)}{\Delta\delta_r(s)} = \frac{K_v\left(s + \frac{1}{T_{\beta_1}}\right)\left(s + \frac{1}{T_{\beta_2}}\right)}{\left(s + \frac{1}{T_s}\right)\left(s + \frac{1}{T_r}\right)\left(s^2 + 2\xi_{Dr}\omega_{Dr}s + \omega_{Dr}{}^2\right)}$$

Using MATLAB we get:

```
   0.042027 (s+50.05) (s+1.82) (s-0.06629)
 ---------------------------------------------
 (s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

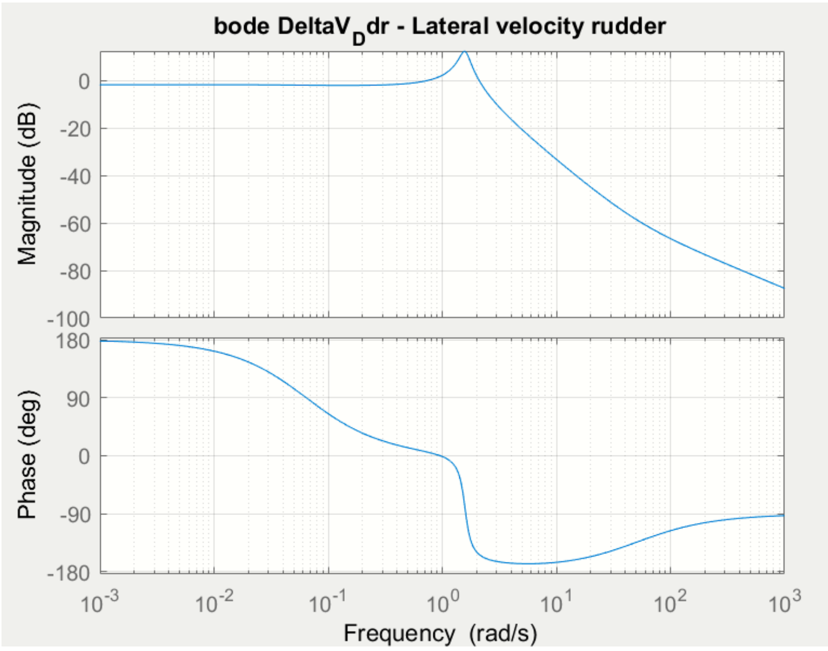
And, by identifying we find the following parameters:

$$K_v = 0.042027 \qquad \frac{1}{T_{\beta 1}} = 50.05 \;\text{->}\; T_{\beta 1} = 0.01998$$

$$\frac{1}{T_{\beta 2}} = 1.82 \;\text{->}\; T_{\beta 2} = 0.549 \qquad \frac{1}{T_{\beta 3}} = 0.06629 \;\text{->}\; T_{\beta 3} = 15.085$$

$$\frac{1}{T_s} = 1.968 \;\text{->}\; T_s = 0.508 \qquad \frac{1}{T_r} = 0.06288 \;\text{->}\; T_r = 15.903$$

We get the following curves:



II. Roll velocity response to rudders

Right below is the theoretical formula for the roll velocity to rudders:

$$\frac{\Delta p(s)}{\Delta \delta_r(s)} = \frac{K_p\, s\left(s^2 + 2\xi_\varphi \omega_\varphi s + {\omega_\varphi}^2\right)}{\left(s + \frac{1}{T_s}\right)\left(s + \frac{1}{T_r}\right)\left(s^2 + 2\xi_{Dr}\omega_{Dr} s + {\omega_{Dr}}^2\right)}$$

Using MATLAB we get:

```
        0.0081045 s (s-7.929) (s+4.145)
  ------------------------------------------------
  (s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$$K_p = 0.0081045$$

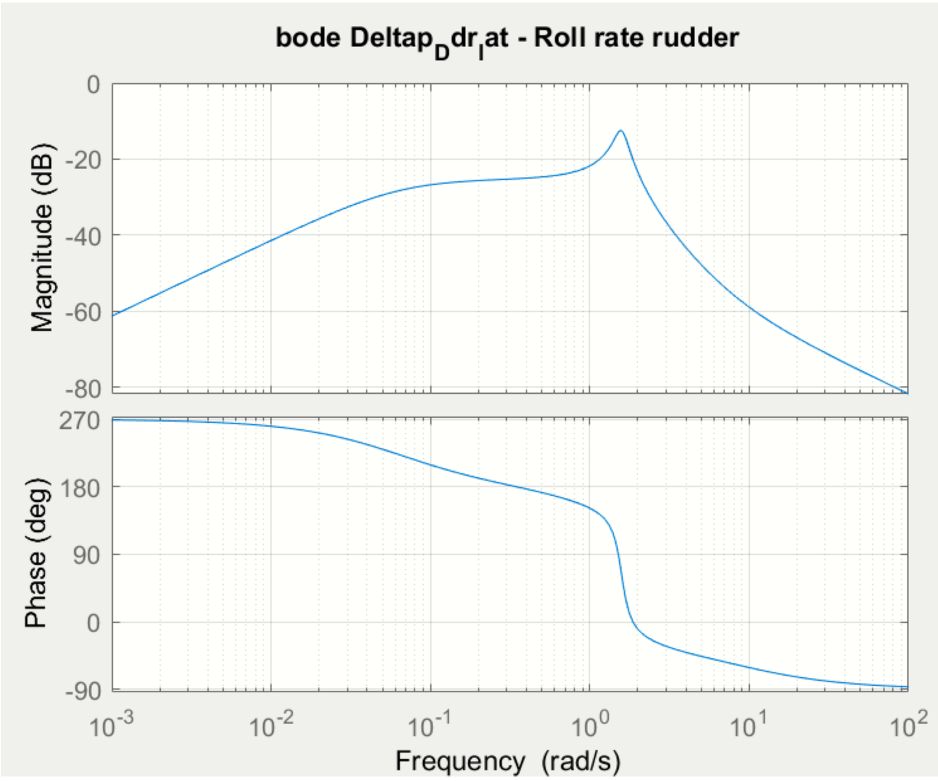
$$\frac{1}{T_r} = 0.06288 \;\text{->}\; T_r = 15.903$$

$$\frac{1}{T_s} = 1.968 \;\text{->}\; T_s = 0.508$$

$$\omega_\varphi\text{^}2 = 32.866 \;\text{->}\; \omega_\varphi = 5.733$$

$$2 * \xi_\varphi * \omega_\varphi = 3.784 \;\text{->}\; \xi_\varphi = 0.33$$

We get the following curves:



We can identify one peak for the magnitude that is positive.

III. Yaw rate response to rudders

Right below is the theoretical formula for the yaw rate response to rudders:

$$\frac{\Delta r(s)}{\Delta \delta_r(s)} = \frac{K_r\left(s+\frac{1}{T_\psi}\right)(s^2+2\xi_\psi\omega_\psi s+\omega_\psi{}^2)}{\left(s+\frac{1}{T_s}\right)\left(s+\frac{1}{T_r}\right)(s^2+2\xi_{Dr}\omega_{Dr}s+\omega_{Dr}{}^2)}$$

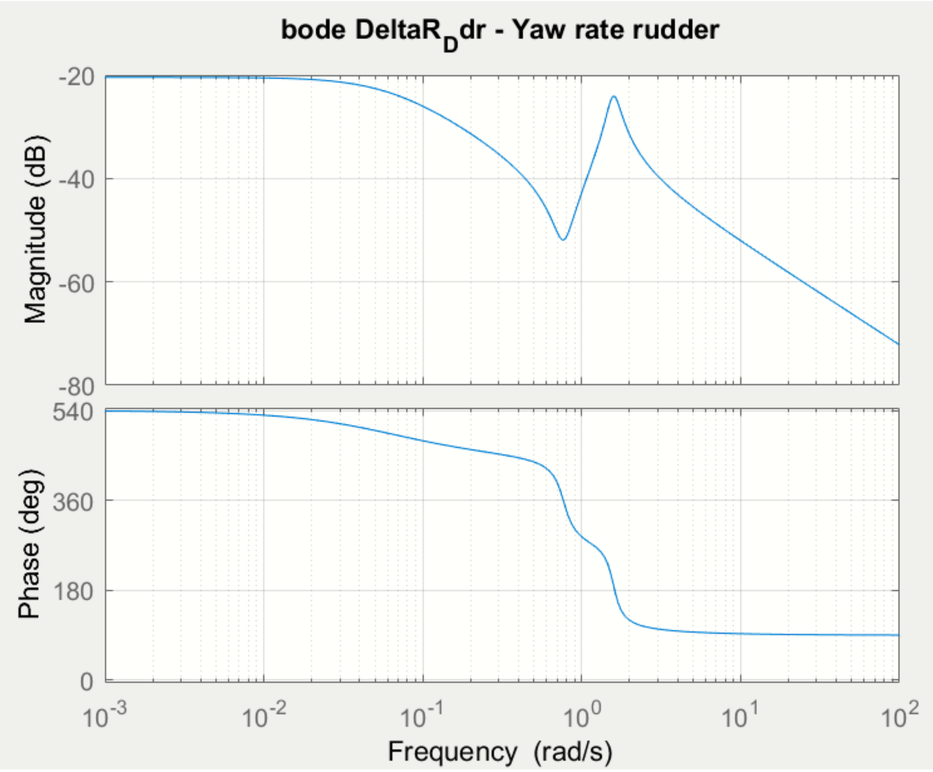
Using MATLAB we get:

```
-0.024535 (s+2.025) (s^2 - 0.1944s + 0.5941)
---------------------------------------------
(s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$K_p = -0.024535$	$\omega_\varphi\text{^}2 = 32.866$ -> $\omega_\varphi = 5.733$
$\frac{1}{T_r} = 0.06288$ -> $T_r = 15.903$	$2 * \xi_\varphi * \omega_\varphi = 3.784$ -> $\xi_\varphi = 0.33$
$\frac{1}{T_s} = 1.968$ -> $T_s = 0.508$	$\frac{1}{T_\psi} = 2.025$ -> $T_\psi = 0.494$

We get the following curves:



We can identify two different peaks for the magnitude, one positive and 1 negative. The phase impact a lot the magnitude's value.

IV. Side angle response to rudder

Right below is the theoretical formula for the side angle response to rudders:

$$\frac{\Delta\varphi(s)}{\Delta\delta_r(s)} = \frac{K_\varphi\left(s^2 + 2\xi_\varphi\omega_\varphi s + {\omega_\varphi}^2\right)}{\left(s+\frac{1}{T_s}\right)\left(s+\frac{1}{T_r}\right)\left(s^2 + 2\xi_{Dr}\omega_{Dr}s + {\omega_{Dr}}^2\right)}$$

Using MATLAB we get:

```
      0.0081045 (s-7.929) (s+4.145)
-------------------------------------------
(s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$$\boldsymbol{K_\varphi = 0.0081}$$

$$\boldsymbol{\frac{1}{T_r} = 0.06288 \;\text{->}\; T_r = 15.903}$$

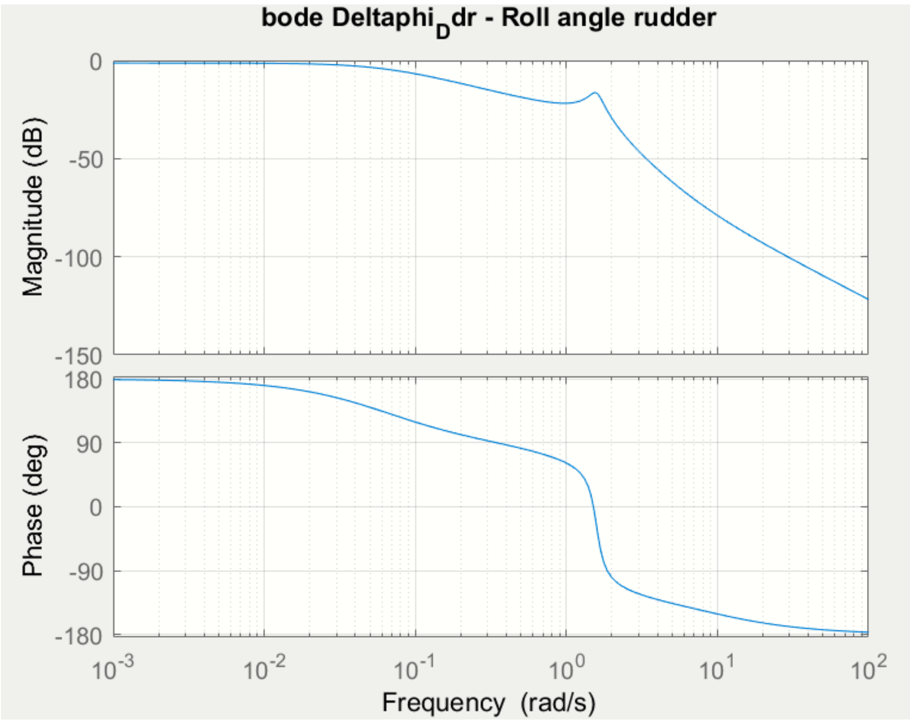
$$\boldsymbol{\frac{1}{T_s} = 1.968 \;\text{->}\; T_s = 0.508}$$

$$\boldsymbol{\omega_\psi\text{^}2 = 0.5941 \;\text{->}\; \omega_\psi = 0.771}$$

$$\boldsymbol{2 * \xi_\psi * \omega_\psi = -0.1944 \;\text{->}\; \xi_\psi = 0.126}$$

$$\boldsymbol{\frac{1}{T_\psi} = 2.025 \;\text{->}\; T_\psi = 0.494}$$

We get the following curves:



Response to ailerons

V. Side velocity response to ailerons

Right below is the theoretical formula for the side velocity response to ailerons:

$$\frac{\Delta v(s)}{\Delta \delta_a(s)} = \frac{K_v \left(s + \frac{1}{T_{\beta_1}}\right)\left(s + \frac{1}{T_{\beta_2}}\right)}{\left(s + \frac{1}{T_s}\right)\left(s + \frac{1}{T_r}\right)(s^2 + 2\xi_{Dr}\omega_{Dr}s + \omega_{Dr}{}^2)}$$

Using MATLAB we get:

```
  -0.0027409 (s-23.96) (s+11.26) (s+0.3808)
  -------------------------------------------
  (s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$$K_v = -0.0027409$$

$$\frac{1}{T_r} = 0.06288 \;\; \text{->} \;\; T_r = 15.903$$

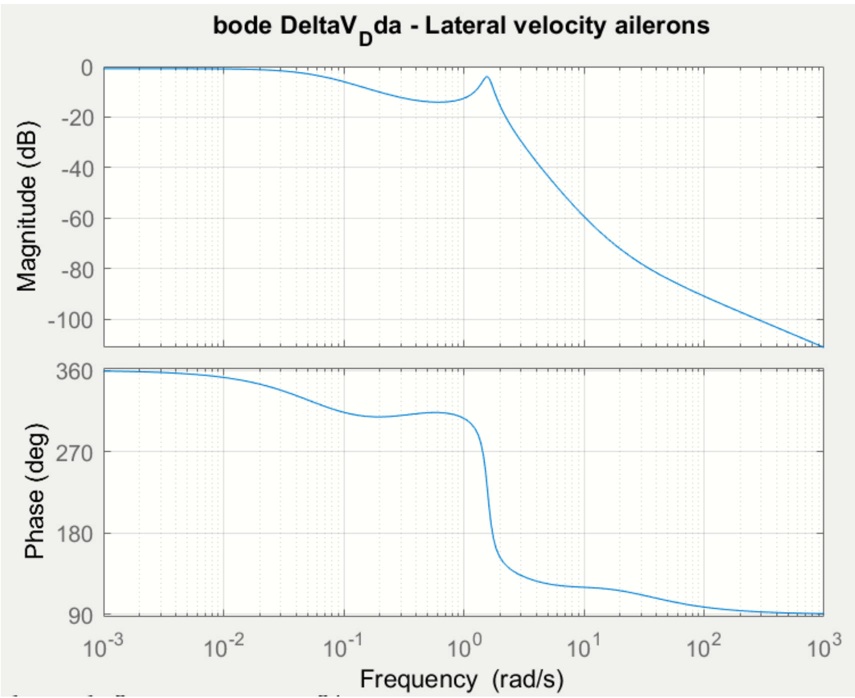
$$\frac{1}{T_s} = 1.968 \;\; \text{->} \;\; T_s = 0.508$$

$$\frac{1}{T_{\beta 1}} = 23.96 \;\; \text{->} \;\; T_{\beta 1} = 0.042$$

$$\frac{1}{T_{\beta 2}} = 0.06288 \;\; \text{->} \;\; T_{\beta 2} = 15.903$$

$$\frac{1}{T_{\beta 3}} = 0.3808 \;\; \text{->} \;\; T_{\beta 3} = 2.626$$

We get the following curves:



VI. *Roll velocity response to ailerons*

Right below is the theoretical formula for the roll velocity to ailerons:

$$\frac{\Delta p(s)}{\Delta \delta_a(s)} = \frac{K_p\, s(s^2 + 2\xi_\varphi \omega_\varphi s + \omega_\varphi{}^2)}{\left(s+\frac{1}{T_s}\right)\left(s+\frac{1}{T_r}\right)(s^2 + 2\xi_{Dr}\omega_{Dr}s + \omega_{Dr}{}^2)}$$

Using MATLAB we get:

```
      0.063678 s (s^2 + 0.6262s + 2.078)
  ---------------------------------------------
  (s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$K_p = 0.063678$

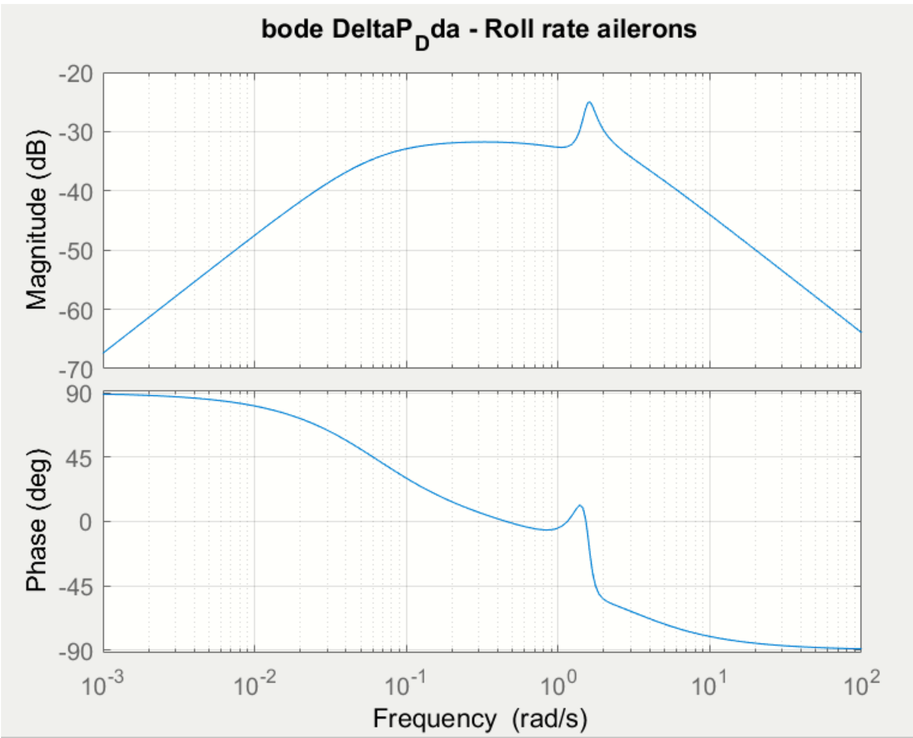
$\frac{1}{T_r} = 0.06288$ -> $T_r = 15.903$

$\frac{1}{T_s} = 1.968$ -> $T_s = 0.508$

ω_φ^2 $= 2.078$ -> $\omega_\varphi = 1.44$

$2 * \xi_\varphi * \omega_\varphi = 3.784$ -> $\xi_\varphi = 1.314$

We get the following curves:



VII. Yaw rate response to ailerons

Right below is the theoretical formula for the yaw rate response to ailerons:

$$\frac{\Delta r(s)}{\Delta\delta_a(s)} = \frac{K_r\left(s+\frac{1}{T_\psi}\right)\left(s^2+2\xi_\psi\omega_\psi s+\omega_\psi{}^2\right)}{\left(s+\frac{1}{T_s}\right)\left(s+\frac{1}{T_r}\right)\left(s^2+2\xi_{Dr}\omega_{Dr}s+\omega_{Dr}{}^2\right)}$$

Using MATLAB we get:

```
-0.00046733 (s-2.187) (s^2 + 6.078s + 14.12)
---------------------------------------------
(s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$$K_r = -0.00467$$

$$\frac{1}{T_r} = 0.06288 \text{ -> } T_r = 15.903$$

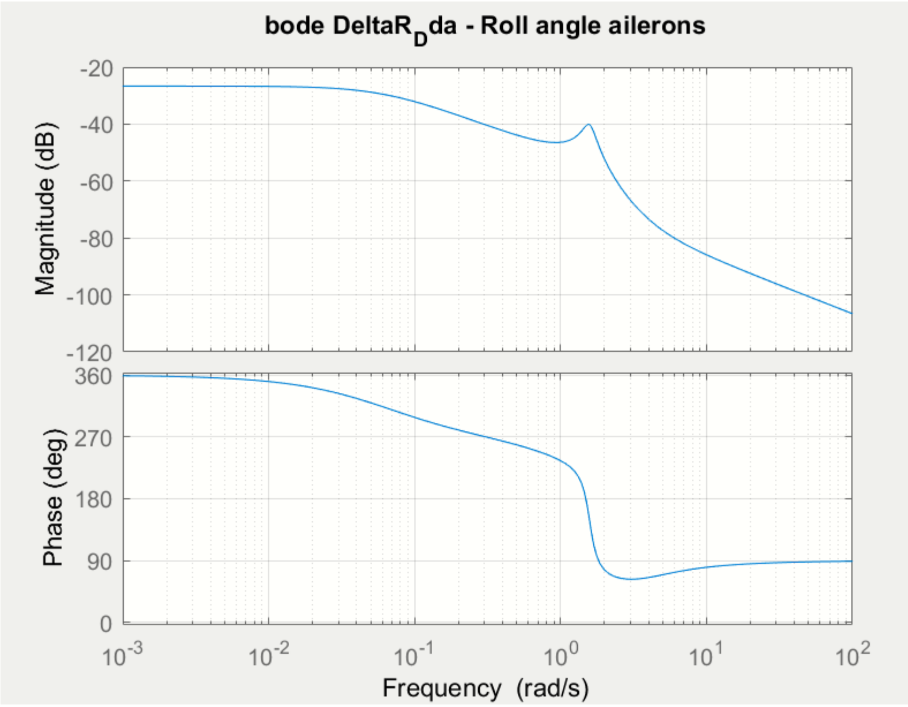
$$\frac{1}{T_s} = 1.968 \text{ -> } T_s = 0.508$$

$$\omega_\psi\text{^}2 = 14.12 \text{ -> } \omega_\psi = 3.758$$

$$2 * \xi_\psi * \omega_\psi = 3.784 \text{ -> } \xi_\psi = 0.503$$

$$\frac{1}{T_\psi} = 2.187 \text{ -> } T_\psi = 0.457$$

We get the following curves:



VIII. Side angle response to ailerons

Right below is the theoretical formula for the side angle response to rudders:

$$\frac{\Delta\varphi(s)}{\Delta\delta_a(s)} = \frac{K_\varphi \left(s^2 + 2\xi_\varphi \omega_\varphi s + {\omega_\varphi}^2\right)}{\left(s + \frac{1}{T_s}\right)\left(s + \frac{1}{T_r}\right)\left(s^2 + 2\xi_{Dr}\omega_{Dr}s + {\omega_{Dr}}^2\right)}$$

Using MATLAB we get:

```
       0.063678 (s^2 + 0.6262s + 2.078)
----------------------------------------------
(s+1.968) (s+0.06288) (s^2 + 0.3062s + 2.507)
```

And, by identifying we find the following parameters:

$$\boldsymbol{K_\varphi = 0.063678}$$

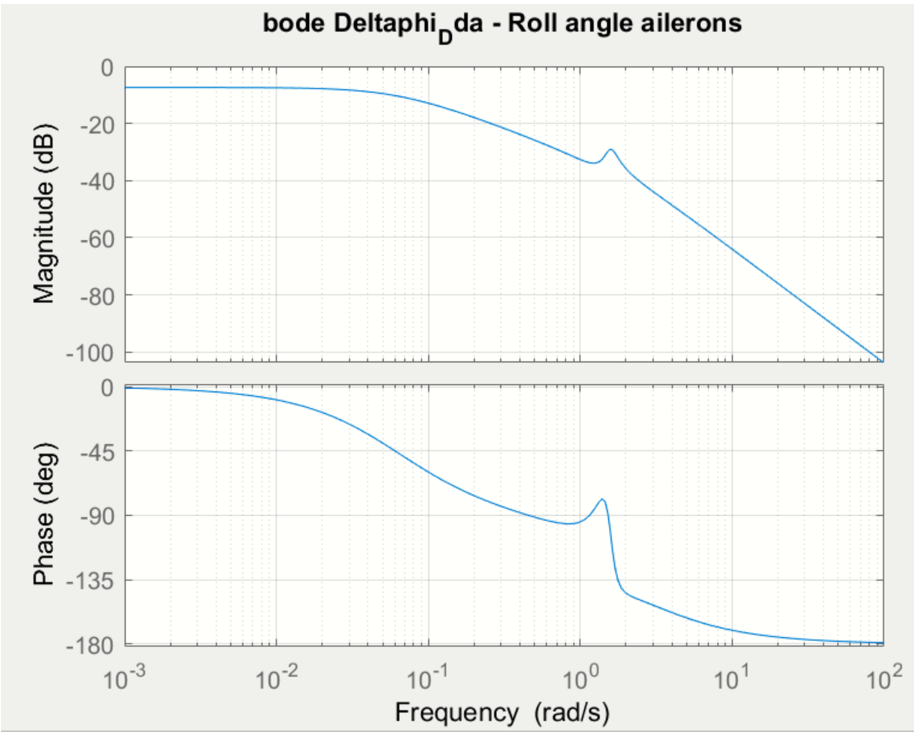
$$\boldsymbol{\frac{1}{T_r} = 0.06288 \ \text{->}\ T_r = 15.903}$$

$$\boldsymbol{\frac{1}{T_s} = 1.968 \ \text{->}\ T_s = 0.508}$$

$$\boldsymbol{\omega_\varphi\text{^}2 = 2.078 \ \text{->}\ \omega_\varphi = 1.442}$$

$$\boldsymbol{2 * \xi_\varphi * \omega_\varphi = 0.6262 \ \text{->}\ \xi_\varphi = 0.217}$$

We get the following curves:



Conclusion of the stability study

This stability study led us to analyse the longitudinal and the lateral stability of a fighter bomber aircraft. Thanks to a MATLAB code, we were able to determine the parameters of stability modes. For the longitudinal stability, we identified a short period and a phugoid stability mode, which are both lightly damped. According to the lateral analysis, we identified the spiral, rolling and Dutch roll stability modes.

Because this airplane is a fighter bomber, its manoeuvrability is more important than its stability, that's why we found that for some stability modes like short period, Dutch roll and rolling mode, the flying qualities are poor (level 2, 3 or lower). To make the workload of the pilot less important during steady flight conditions like cruise, we determined gain matrix which can be added threw stability augmentation systems applied on rudders.

This study also showed us the importance of stability of an aircraft when it encounters perturbations. If the plane isn't stable, or if the pilot doesn't balance the effects of it, it could induce degradations of flight conditions, and then human or materials loss.

APPENDIX

LONGITUDINAL MOTION

Airplane's characteristics and flight conditions

```
% GEOMETRIC DATA
S = 34.84; % WingArea in m^2
b = 11.8;% Wingspan in m
A = 4; % Aspect Ratio
c_bar = 3.29; % Mean aerodynamic chord in m
e = 0.85;

% MASS &  INERTIAL DATA
m = 9926.7; % Aircraft mass in kg
Ixx = 18486.6; % kgm^2
Iyy = 68965; % kgm^2
Izz = 91599; %kgm^2
Ixz = 3976.6; %kgm^2
g = 9.81;

% FLIGHT CONDITIONS
H = 0; %Altitude in m
rho = 1.225; % Air Density kg/m^3
U0 = 85; % Flight Speed in m/s
Mach = 0.25;
TETA0 = 11.2*(pi/180); % Initial Pitch Angle

% INITIAL STEADY STATE COEFFICIENTS
Cl0 = 0.62; % Lift Coefficient
Cd0 = 0.072; % Drag Coefficient
```

```
% LONGITUDINAL STABILITY DERIVATIVES
CDu = 0.0105;
CDalpha = 1.52;
CLu = 0.04;
CLalpha = 3.95;
CLalphadot = 0;
CLq = 0;
CMu = 0.012;
CMalpha = -0.45;
CMalphadot = -0.7;
CMq = -3.8;

% LONGITUDINAL CONTROL DERIVATIVES
CDdelta_e = 0;
CLdelta_e = 0.6;
CMdelta_e = -0.83;

% LATERAL STABILITY DERIVATIVES
Cy_beta = -0.88;
Cy_p = 0;
Cy_r = 0;
Cl_beta = -0.115;
Cl_p = -0.25;
Cl_r = 0.18;
Cn_beta = 0.105;
Cn_p = -0.01;
Cn_r = -0.34;

% LATERAL CONTROL DERIVATIVES
CYdelta_a = -0.015;
CLdelta_a = 0.055;
CNdelta_a = -0.002;
CYdelta_r = 0.23;
CLdelta_r = 0.007;
CNdelta_r = -0.105;
```

The aircraft matrix A in longitudinal motion

```
q_bar = 0.5*rho*U0^2;
Xu = -((q_bar*S)/(m*U0))*(2*Cd0 + CDu);
Xw = ((q_bar*S)/(m*U0))*(Cl0*(1-(2*CLalpha/(pi*e*A))));
Zu = -((q_bar*S)/(m*U0))*(2*Cl0 + CLu);
Zw = -((q_bar*S)/(m*U0))*(Cd0 + CLalpha);
Mu = ((q_bar*S*c_bar*CMu)/(Iyy* U0));
Mw = ((q_bar*S*c_bar*CMalpha)/(Iyy* U0));
Mw_dot = ((q_bar*S*c_bar^2*CMalphadot)/(2*Iyy* U0^2));
Mq = ((q_bar*S*c_bar^2*CMq)/(Iyy*U0*2));

Along = [Xu          Xw          0           -g*cos(TETA0);
     Zu          Zw          U0          -g*sin(TETA0);
     Mu+Zu*Mw_dot Mw+Zw*Mw_dot Mq+U0*Mw_dot 0;
     0           0           1           0           ];
```

The aircraft matrix B in longitudinal motion

```
Xdelta_t = 0;
Zdelta_t = 0;
Mdelta_t = 0;
Xdelta_e = ((q_bar*S)/(m*U0))*(CDdelta_e);
Zdelta_e = ((q_bar*S)/(m*U0))*(CLdelta_e);
Mdelta_e = ((q_bar*S*c_bar)/(Iyy*U0))*(CMdelta_e);

B = [Xdelta_e                 Xdelta_t
     Zdelta_e                 Zdelta_t
     Mdelta_e+Zdelta_e*Mw_dot Mdelta_t+Zdelta_t*Mw_dot
     0                        0];
```

The characteristic equation

```
eq_char = charpoly(Along);
```

The eigenvalues (roots of equation) of the system

```
[eigenvectors, eigenvalues] = eig(Along);
```

Different modes of longitudinal stability:

- *Short period mode (Natural Frequency, Damping Factor)*

```
%I. Short period mode (Natural Frequency, Damping Factor)
fprintf("\nSHORT PERIOD MODE");
WNsp = sqrt(eigenvalues(1,1)*eigenvalues(2,2));
Esp = (-eigenvalues(1,1)-eigenvalues(2,2))/(2*WNsp);

fprintf('\nWNsp = %f', WNsp);
fprintf('\nEsp = %f', Esp);

if Esp >0
    fprintf('\nDamping factor positive ==> MODE STABLE\n')
else
    fprintf('\nDamping factor positive ==> MODE UNSTABLE\n')
end
```

- ## *Phugoid mode (Natural Frequency, Damping Factor)*

```
%Period
Tsp = (2*pi)/(WNsp*sqrt(1-Esp^2));

% b.    Phugoid mode (Natural Frequency, Damping Factor)
fprintf("\nPHUGOID MODE");

WNp = sqrt(eigenvalues(3,3)*eigenvalues(4,4));
Ep = (-eigenvalues(3,3)-eigenvalues(4,4))/(2*WNp);

fprintf('\nWNp = %f', WNp);
fprintf('\nEp = %f', Ep);

if Ep >0
    fprintf('\nDamping factor positive ==> MODE STABLE\n')
else
    fprintf('\nDamping factor positive ==> MODE UNSTABLE\n')
end
```

Curves of longitudinal motion

- ## *Axial velocity*

```
t = 0:1:2000;

% Axial velocity
DeltaU = real(eigenvectors(1, 3))*exp(eigenvalues(3,3)*t) + real(eigenvectors(1, 4))*exp(eigenvalues(4,4)*t);

plot(t, DeltaU);
xlabel('Time (s)');
ylabel('DeltaU');
title('Phugoid mode - Axial velocity');
legend('DeltaU');
grid();
```

- ## *Pitch rate*

```
t_sp = 0:0.01:10;

% Pitch rate
DeltaQ = real(eigenvectors(3, 1))*exp(eigenvalues(1, 1)*t_sp) +  real(eigenvectors(3, 2))*exp(eigenvalues(2, 2)*t_sp);

figure(1)
plot(t_sp, DeltaQ);
xlabel('Time (s)');
ylabel('DeltaQ');
title('Short period mode - Pitch rate');
legend('DeltaQ');
grid();
```

- ## *Angle of attack*

```
t_sp = 0:0.01:10;

% Angle of attack
DeltaW = real(eigenvectors(2, 1))*exp(eigenvalues(1, 1)*t_sp) +  real(eigenvectors(2, 2))*exp(eigenvalues(2, 2)*t_sp);

plot(t_sp, DeltaW);
xlabel('Time (s)');
ylabel('DeltaW');
title('Short period mode - Angle of attack');
legend('DeltaW');
grid();
```

- *Pitch angle*

```
t = 0:1:2000;

% Pitch angle
DeltaTETA = real(eigenvectors(4, 3))*exp(eigenvalues(3, 3)*t) + real(eigenvectors(4, 4))*exp(eigenvalues(4, 4)*t);

plot(t, DeltaU);
xlabel('Time (s)');
ylabel('DeltaU');
title('Phugoid mode - Axial velocity');
legend('DeltaU');
grid();
```

Transfer Functions of Each variable

```
syms delta;

s = tf('s');
TF = minreal(inv(s*eye(4)-Along)*B, 1e-4);

DeltaU_Dde = zpk(TF(1,1));
DeltaU_Ddt = zpk(TF(1,2));
DeltaW_Dde = zpk(TF(2,1));
DeltaW_Ddt = zpk(TF(2,2));
DeltaQ_Dde = zpk(TF(3,1));
DeltaQ_Ddt = zpk(TF(3,2));
DeltaTETA_Dde = zpk(TF(4,1));
DeltaTETA_Ddt = zpk(TF(4,2));

figure(3)
bode(DeltaU_Dde)
grid on;
title("bode DeltaU_Dde");

figure(4)
bode(DeltaW_Dde)
grid on;
title("bode DeltaW_Dde");

figure(5)
bode(DeltaQ_Dde)
grid on;
title("bode DeltaQ_Dde");

figure(6)
bode(DeltaTETA_Dde)
grid on;
title("bode DeltaTETA_Dde");
```

Stability augmentation system

```
syms k1 k2

% Given matrices and values
M1 = [-0.7349 85.00; -0.0381 -0.6405];
M2 = [0.1096; -0.0719];
K = [k1 k2]; % Assuming k1 and k2 are symbolic variables

% Compute the matrix operation
result = M1 - M2 * K;

% Définir les équations du système
eq1 = 0.1096*k1 - 0.0719*k2 + 1.3755 == 3.36;
eq2 = 3.7092675 - 6.0413012*k1 -0.05702226*k2 == 20;

% Résoudre le système
solution = solve([eq1, eq2], [k1, k2]);
k1_decimal = double(solution.k1);
k2_decimal = double(solution.k2);

disp(k1_decimal);
disp(k2_decimal);

K =[0 k1_decimal k2_decimal 0; 0 0 0 0];
A_de = Along -B*K;
```

LATERAL MOTION

The aircraft matrix A in lateral motion

```
Yv = (q_bar*S)/(m*U0)*Cy_beta;
Yp = (q_bar*S*b)/(2*m*U0)*Cy_p;
Yr = (q_bar*S*b)/(2*m*U0)*Cy_r;
Lv = (q_bar*S*b)/(Ixx*U0)*Cl_beta;
Lp = (q_bar*S*b^2)/(2*Ixx*U0)*Cl_p;
Lr = (q_bar*S*b^2)/(2*Ixx*U0)*Cl_r;
Nv = (q_bar*S*b)/(Izz*U0)*Cn_beta;
Np = (q_bar*S*b^2)/(2*Izz*U0)*Cn_p;
Nr = (q_bar*S*b^2)/(2*Izz*U0)*Cn_r;

Alateral = [Yv          Yp          -(U0-Yr)        g*cos(TETA0);
            Lv          Lp          Lr              0;
            Nv          Np          Nr              0;
            0           1           0               0           ];
```

The aircraft matrix B in lateral motion

```
Ydelta_r = (q_bar*S)/(m*U0)*CYdelta_r;
Ydelta_a = (q_bar*S)/(m*U0)*CYdelta_a;
Ldelta_r = (q_bar*S*b)/(Ixx*U0)*CLdelta_r;
Ldelta_a = (q_bar*S*b)/(Ixx*U0)*CLdelta_a;
Ndelta_r = (q_bar*S*b)/(Izz*U0)*CNdelta_r;
Ndelta_a = (q_bar*S*b)/(Izz*U0)*CNdelta_a;

Blateral = [Ydelta_r                Ydelta_a
     Ldelta_r               Ldelta_a
     Ndelta_r               Ndelta_a
     0                      0];
```

The characteristic equation

```
eq_char_lat = charpoly(Alateral);
```

The eigenvalues (roots of equation) of the system

```
[eigenvectors2, eigenvalues2] = eig(Alateral);
```

Different modes of longitudinal stability:

- <u>*Spiral mode & Roling mode & Dutch roll mode*</u>

```
% Identification des valeurs propres pour chaque mode
rolling_eigenvalue = eigenvalues2(1, 1);
spiral_eigenvalue = eigenvalues2(2, 2);
dutch_roll_eigenvalue = eigenvalues2(3, 3);

% Calcul des fréquences naturelles et des facteurs d'amortissement
rolling_natural_frequency = abs(imag(rolling_eigenvalue));
rolling_damping_ratio = -real(rolling_eigenvalue) / abs(rolling_eigenvalue);

spiral_natural_frequency = abs(imag(spiral_eigenvalue));
spiral_damping_ratio = -real(spiral_eigenvalue) / abs(spiral_eigenvalue);

dutch_roll_natural_frequency = abs(imag(dutch_roll_eigenvalue));
dutch_roll_damping_ratio = -real(dutch_roll_eigenvalue) / abs(dutch_roll_eigenvalue);

% Affichage des résultats
disp('Rolling Mode:');
disp(['Natural Frequency: ', num2str(rolling_natural_frequency), ' rad/s']);
disp(['Damping Ratio: ', num2str(rolling_damping_ratio)]);

disp('Spiral Mode:');
disp(['Natural Frequency: ', num2str(spiral_natural_frequency), ' rad/s']);
disp(['Damping Ratio: ', num2str(spiral_damping_ratio)]);

disp('Dutch Roll Mode:');
disp(['Natural Frequency: ', num2str(dutch_roll_natural_frequency), ' rad/s']);
disp(['Damping Ratio: ', num2str(dutch_roll_damping_ratio)]);
```

Curves of lateral motion:

- <u>*Dutch roll mode*</u>

```
% Time vector
t = 0:0.01:30;
r_lat = roots(eq_char_lat);

% Dutch roll mode
v_dutch = real(eigenvectors2(1,3))*exp(eigenvalues2(3,3)*t)+real(eigenvectors2(1,2))*exp(eigenvalues2(2,2)*t);
figure (7)
plot(t,v_dutch)
hold on
p_dutch = real(eigenvectors2(2,3))*exp(eigenvalues2(3,3)*t)+real(eigenvectors2(2,2))*exp(eigenvalues2(2,2)*t);
plot(t,p_dutch)
hold on
r_dutch = real(eigenvectors2(3,3))*exp(eigenvalues2(3,3)*t)+real(eigenvectors2(3,2))*exp(eigenvalues2(2,2)*t);
plot(t,r_dutch)
hold on
phi_dutch = real(eigenvectors2(4,3))*exp(eigenvalues2(3,3)*t)+real(eigenvectors2(4,2))*exp(eigenvalues2(2,2)*t);
plot(t, phi_dutch)
xlabel ('t')
title('Dutch roll mode')
grid()
legend('lat velocity', 'roll rate', 'yaw rate', 'roll angle')
```

- *Spiral mode & Roll mode*

```
%roll
figure(8)
t=0:0.1:4;
hold on;
v = real(eigenvectors2(1,1))*exp(t*r_lat(1));
plot(t,v)
hold on;
p = real(eigenvectors2(2,1))*exp(t*r_lat(1));
plot(t,p)
hold on;
r = real(eigenvectors2(3,1))*exp(t*r_lat(1));
plot(t,r)
hold on;
phi = real(eigenvectors2(4,1))*exp(t*r_lat(1));
plot(t,phi)
xlabel ('t')
title('Rolling mode')
grid()
legend('lat velocity', 'roll rate', 'yaw rate', 'roll angle')
```

```
%spiral
figure(9)
t=0:200;
x_sp = exp(r_lat(4)*t);
hold on;
v_sp = real(eigenvectors2(1,4))*exp(t*r_lat(4));
plot(t,v_sp)
hold on;
p_sp = real(eigenvectors2(2,4))*exp(t*r_lat(4));
plot(t,p_sp)
hold on;
r_sp = real(eigenvectors2(3,4))*exp(t*r_lat(4));
plot(t,r_sp)
hold on;
phi_sp = real(eigenvectors2(4,4))*exp(t*r_lat(4));
plot(t,phi_sp)
xlabel ('t')
title('Spiral mode')
grid()
legend('lat velocity', 'roll rate', 'yaw rate', 'roll angle')
```

Transfer functions of each variable

```
syms delta;

s = tf('s');
TF_lat = minreal(inv(s*eye(4)-Alateral)*Blateral, 1e-4);

DeltaV_Ddr = zpk(TF_lat(1,1));
DeltaV_Dda = zpk(TF_lat(1,2));
DeltaP_Ddr = zpk(TF_lat(2,1));
DeltaP_Dda = zpk(TF_lat(2,2));
DeltaR_Ddr = zpk(TF_lat(3,1));
DeltaR_Dda = zpk(TF_lat(3,2));
Deltaphi_Ddr = zpk(TF_lat(4,1));
Deltaphi_Dda = zpk(TF_lat(4,2));


figure(11)
bode(DeltaV_Ddr)
grid on;
title("bode DeltaV_Ddr - Lateral velocity rudder");

figure(12)
bode(DeltaP_Ddr)
grid on;
title("bode Deltap_Ddr_lat - Roll rate rudder");

figure(13)
bode(DeltaR_Ddr)
grid on;
title("bode DeltaR_Ddr - Yaw rate rudder");

figure(14)
bode(Deltaphi_Ddr)
grid on;
title("bode Deltaphi_Ddr - Roll angle rudder");
```

```
figure(15)
bode(DeltaV_Dda)
grid on;
title("bode DeltaV_Dda - Lateral velocity ailerons");

figure(16)
bode(DeltaP_Dda)
grid on;
title("bode DeltaP_Dda - Roll rate ailerons");

figure(17)
bode(DeltaR_Dda)
grid on;
title("bode DeltaR_Dda - Roll angle ailerons");

figure(18)
bode(Deltaphi_Dda)
grid on;
title("bode Deltaphi_Dda - Roll angle ailerons");
```