Kick Start 2022 - Round B

Hamiltonian Tour

Problem

Hamilton is a Canadian city near Toronto, and a nice place to take a walking tour.

In this problem, Hamilton is represented by a grid of unit cells with $2\mathbf{R}$ rows and $2\mathbf{C}$ columns, where each cell is either empty (represented by `*`) or contains a building (represented by `#`). The cell on the i-th row and j-th column is represented by $A_{i,j}$ where $1 \leq i \leq 2\mathbf{R}$ and $1 \leq j \leq 2\mathbf{C}$. It is not possible to enter cells containing buildings and you can only move to an adjacent cell that shares a side with the current cell (not just a corner). The grid is such that if it is divided evenly into 2×2 blocks of unit cells, then in each of those blocks, either all four cells are empty, or all four cells are occupied by a building. Let us represent the block formed by $A_{2i-1,2j-1}$, $A_{2i-1,2j}$, $A_{2i,2j-1}$, and $A_{2i,2j}$ cells as $\mathbf{B_{i,j}}$ where $1 \leq i \leq \mathbf{R}$ and $1 \leq j \leq \mathbf{C}$.

Grace is a tourist in Hamilton and wants to visit all the empty cells in Hamilton. Grace is currently in cell $A_{1,1}$. Visiting the same cell twice could be boring for her. Hence, Grace wants to visit each of the empty cells exactly once and finally end in cell $A_{1,1}$. Can you help Grace by providing a string (consisting of directional moves {`N`, `E`, `S`, `W`} representing the unit moves to the north, east, south, or west respectively) which Grace can follow to visit every empty cell once and end again in $A_{1,1}$.

Input

The first line of the input gives the number of test cases, $\mathbf{T}$. $\mathbf{T}$ test cases follow.
The first line of each test case contains two integers $\mathbf{R}$ and $\mathbf{C}$.
The next $\mathbf{R}$ lines of each test case contains $\mathbf{C}$ characters each.

The j-th character on the i-th of these lines represents the block $\mathbf{B_{i,j}}$ formed by the following four cells: $A_{2i-1,2j-1}$, $A_{2i-1,2j}$, $A_{2i,2j-1}$, and $A_{2i,2j}$.
If $\mathbf{B_{i,j}} = $ `#`, all four of the cells in $\mathbf{B_{i,j}}$ are occupied by a building.
Otherwise, if $\mathbf{B_{i,j}} = $ `*`, all four of the cells in $\mathbf{B_{i,j}}$ are empty.

Output

For each test case, output one line containing `Case #`x`: `y, where x is the test case number (starting from 1) and y is the answer to the problem as follows.

If there is no solution to the problem, y should be `IMPOSSIBLE`. Otherwise, y should be a sequence of characters from the set {`N`, `E`, `S`, `W`}, representing the unit moves (to the north, east, south, or west respectively) in a valid route, starting from $A_{1,1}$, as described in the statement above.

Note that your last move should take you to $A_{1,1}$; this move does not count as visiting the same cell twice.

If there are multiple valid solutions, you may output any one of them.

Limits

Time limit: 25 seconds.
Memory limit: 1 GB.
$1 \leq \mathbf{T} \leq 100$.
$1 \leq \mathbf{R} \leq 200$.
$1 \leq \mathbf{C} \leq 200$.
All characters in the grid are from the set {`#`,`*`}.
The first character of the first line of the input grid for each test case is a `*` character, i.e. $\mathbf{B_{1,1}} =$ `*`.

Test Set 1

A block contains buildings if and only if the row number and column number of it are divisible by 2. i.e. $\mathbf{B_{i,j}} =$ `#` $\iff ((i \bmod 2 = 0)$ and $(j \bmod 2 = 0))$.

Test Set 2

No extra constraints.

Sample

Note: there are additional samples that are not run on submissions down below.

Sample Input

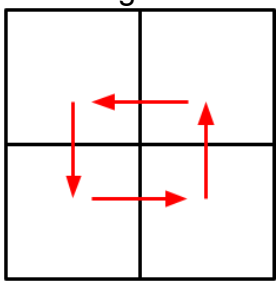
```
3
1 1
*
2 2
**
*#
3 4
****
*#*#
****
```

Sample Output

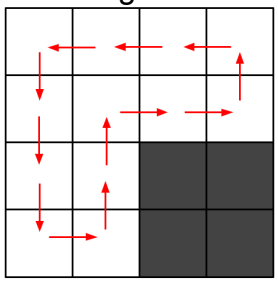
```
Case #1: SENW
Case #2: SSSENNEENWWW
Case #3:
ESSSSEEENNNWWNEEEEESWWSSSEESWWWW
```

The sample output displays one set of answers to the sample cases. Other answers may be possible.

In Sample Case #1, Grace will follow the route $A_{1,1}$, $A_{2,1}$, $A_{2,2}$, $A_{1,2}$, and finally $A_{1,1}$. Note that `ESWN` is the only other possible valid answer.
The image below shows one of the possible routes for Sample Case #1.



The image below shows one of the possible routes for Sample Case #2.



Additional Sample - Test Set 2

The following additional sample fits the limits of Test Set 2. It will not be run against your submitted solutions.

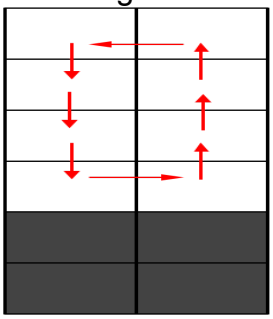
Sample Input

```
3
3 1
*
*
#
1 3
*#*
3 4
**#*
**#*
****
```

Sample Output

```
Case #1: SSSENNNW
Case #2: IMPOSSIBLE
Case #3:
ESSSSENNNNESSSSEEENNNNESSSSSWWWW
```

The image below shows one of the possible routes for Sample Case #1.



In Sample Case #2, it is impossible for Grace to travel to any cell in $\mathbf{B_{1,3}}$ from $A_{1,1}$.