# Simple Linear Equations

## Introduction to Equations

Consider the expressions 27 ÷ 3 and 5 + 4. Both give the same value i.e., 9.

Thus, 27 ÷ 3 = 5 + 4

In the above statement, value of the expression on either side of = sign is equal. Such a statement is known as **equality**.

Few more examples of equality are as follows:

5 × 3 = 8 + 7

46 – 20 = 2 × 13

6 × 4 = 72 ÷ 3

**Properties of equality:**

If we perform the same operation on both sides of the equality, then the values so obtained are equal.
Let us consider the equality 5 × 3 = 8 + 7 to verify these properties.

**Addition property of equality:**
**If the same number is added to both sides of the equality then the values so obtained are equal.**

For example,
(5 × 3) + 4 = (8 + 7) + 4
⇒ 15 + 4 = 15 + 4
⇒ 19 = 19

**Subtraction property of equality:**
**If the same number is subtracted from both sides of the equality then the values so obtained are equal.**

For example,
(5 × 3) – 10 = (8 + 7) – 10
⇒ 15 – 10 = 15 – 10
⇒ 5 = 5

**Multiplication property of equality:**
**If the same number is multiplied to both sides of the equality then the values so obtained are equal.**

For example,
$(5 \times 3) \times 6 = (8 + 7) \times 6$
$\Rightarrow 15 \times 6 = 15 \times 6$
$\Rightarrow 90 = 90$

**Division property of equality:**
**If both sides of the equality are divided by the same number then the values so obtained are equal.**

For example,
$(5 \times 3) \div 3 = (8 + 7) \div 3$
$\Rightarrow 15 \div 3 = 15 \div 3$
$\Rightarrow 5 = 5$

These properties can be generalised as follows:

If we have $p = q$, then

**(1) Addition property: $p + r = q + r$**

**(2) Subtraction property: $p - r = q - r$**

**(3) Multiplication property: $p \times r = q \times r$**

**(4) Division property: $p \div r = q \div r$**

These properties are very basic, but very useful.

Now, let us consider a situation.

Suppose Ritu has 12 marbles more than Raj. If Ritu has 40 marbles, then how many marbles does Raj have?

Let the number of marbles with Raj be $x$.

The mathematical statement for the given situation is:
$40 = x + 12$

This is an equation.

Thus, we can say

**"An equation is a condition to find the value of a variable".**

It is to be noted that an equation must have an 'equal sign' (=). The value on the right hand side (R.H.S.) and the left hand side (L.H.S.) of the 'equal sign' (=) must be equal, i.e., L.H.S. = R.H.S.

$x + 15 = 25$, $2y = 32$, $3p + 1 = 4$ etc. are the examples of equations.

**Note:** If L.H.S. is greater than R.H.S. or vice-versa, i.e., if L.H.S. > R.H.S. or L.H.S. < R.H.S., then it is not an equation.

For example, $x + 3 > 9$, $2y < 14$ etc. are not equations.

Let us now solve some examples to understand the concept of equations better.

**Example 1:**
**Shalini is 7 years younger than Sandhya. If Shalini is 18 years old, then what will be the equation for this situation?**

**Solution:**
Let Sandhya be $y$ years old.

According to the question,
Sandhya's age – 7 = Shalini's age
i.e., $y – 7 = 18$

**Example 2:**
**The length of a rectangular park is twice its breadth. If the breadth is 26 m, then what will be the equation for this situation?**

**Solution:**
Let the length of the rectangular park be $l$.
According to the question,

Length of the park = 2 × Breadth of the park
i.e., $l = 2 \times 26$

**Example 3:**
**Sanjay got 5 marks less than twice the marks that Vinay got in Mathematics. Sanjay got 81 marks. Which of the following equations satisfy this situation?**

**(i) $2m – 5 = 81$**
**(ii) $5m – 2 = 81$**
**(iii) $2m – 81 = 5$**
**(iv) $5m – 81 = 5$**

**Here, *m* is the marks obtained by Vinay.**

**Solution:**
Given, marks obtained by Vinay = $m$
Now, the equation for the above situation is as follows.

2 × Vinay’s marks – 5 = Sanjay’s marks
i.e., $2m – 5 = 81$

Thus, the first equation is the correct answer.

**Example 4:**
**Which of the following expressions are equations containing some variable? Also, write the variable present in those equations.**

**(i) $a + 5 = 11$**

**(ii) $2a + 5 < 2$**

**(iii) $2p + 18 = 14$**

**(iv)** $-\frac{x}{5} - 22 = 39$

**(v) $9 – 5 = 2 × 2$**

**(vi)** $25 – 5 > \frac{q}{2}$

**(vii) $3 = 4x$**

**Solution:**

Equation **(v),** $9 – 5 = 2 × 2$, contains only numbers. Therefore, this is a numerical equation without any variable.

The expressions **(ii)**, $2a + 5 < 2$, and **(vi)**, $25 – 5 > \frac{q}{2}$, do not contain an ‘equal sign’ (=). They contain ‘>’ and ‘<’ signs. Therefore, **(ii)** and **(vi)** are not equations.

The equations **(i)**, **(iii)**, **(iv)** and **(vii)** are equations with a variable. The variables present in these equations are given in the following table.

| Equation | Variable |
|---|---|
| (i) a + 5 = 11<br>(iii) 2p + 18 = 14<br>(iv) $-\frac{x}{5}-22=39$<br>(vii) 3 = 4x | a<br>p<br>x<br>x |

## Solution of Equations by Transposing Terms

So, the most important point to remember about transposing terms is:

"**When we transpose a term from one side to the other side of the equation, the sign of the term changes.**"

We can also represent the solution of an equation graphically. Let us see how.

First of all, we need to find the solution of the given equation. Then, we need to represent it on a number line.

Let us start working with the equation $3x - 5 = -14$.

Here, we have

$3x - 5 = -14$

$\Rightarrow 3x = -14 + 5$ (Transposing 5 towards right side)

$\Rightarrow 3x = -9$

$\Rightarrow x = \frac{-9}{3}$ (Transposing 3 towards right side)

$\Rightarrow x = -3$

The solution is represented by a thick dot on the number line as shown below.


Let us now use this method of transposing the terms to solve some equations.

**Example 1:**

**Solve the following equations. Also, represent their solutions graphically.**

**1.** $z + \frac{5}{3} = 20$

**2.** $5(2a + 1) = 100$

**Solution:**

**1.** $z + \frac{5}{3} = 20$

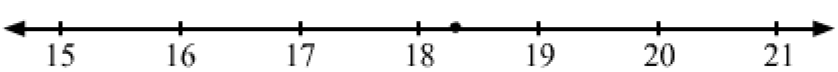
On transposing $\frac{5}{3}$ to R.H.S., we obtain

$z = 20 - \frac{5}{3}$

$\Rightarrow z = \frac{60-5}{3}$

$\Rightarrow z = \frac{55}{3} = 18\frac{1}{3}$

The solution is represented by a thick dot on the number line as shown below.




**2.** $5(2a + 1) = 100$

On transposing 5 to R.H.S., we obtain

$2a + 1 = \frac{100}{5}$

$\Rightarrow 2a + 1 = 20$

On transposing 1 to R.H.S., we obtain

$2a = 20 - 1$

⇒ $2a = 19$

⇒ $a = \frac{19}{2} = 9\frac{1}{2}$

The solution is represented by a thick dot on the number line as shown below.


**Example 2:**

**If 1 is added to one-third of a number, then it becomes 8. Find the number.**

**Solution:**

Let the number be *n*.

Now, one-third of *n* is $\frac{n}{3}$.

According to the question,

$\frac{n}{3} + 1 = 8$

On transposing 1 to R.H.S., we obtain

$\frac{n}{3} = 8 - 1$

⇒ $\frac{n}{3} = 7$

On transposing 3 to R.H.S., we obtain

$n = 7 \times 3$

⇒ $n = 21$

Thus, the required number is 21.

**Solution of Linear Equations That Contains Linear Expressions on Both Sides**

Suppose two children Arpit and Amit are playing a game. Arpit thinks of a number and says to Amit that if he subtracts 2 from that number, then the answer obtained is same as half of the original number. He asks Amit to find the original number, which he had thought in the beginning.

Can Amit find it? Is it possible to find the original number from the given situation?

Yes, it is. By using the concept of linear equations in one variable, we can find the solution for the above problem.

Now, we will first form a linear equation with the help of the given situation.

Let the original number be *x*. It is given that if Arpit subtracts 2 from the original number, then the answer obtained will be half of the original number.

i.e, $x-2=\frac{x}{2}$

In this equation, there is a variable on both the sides. How will we solve such an equation?

We will take the help of this video to understand the concept involved in solving this equation.

Now, let us solve some more problems to understand the concept better.

**Example 1**:

**Solve the following equations.**

1. $\mathbf{5z+\frac{17}{6}=3z+5}$
2. $\mathbf{3(z-3)=\frac{z}{2}-2}$
3. $\mathbf{2(x+20)=\frac{x}{3}+65}$

**Solution:**

1. $5z+\frac{17}{6}=3z+5$

Our first step is to rearrange the terms of the equation such that the variables are on one side and numbers are on the other side.

Therefore, on subtracting 5*z* from both the sides, we obtain

$$5z+\frac{17}{6}-5z=3z+5-5z$$
$$\Rightarrow 5-2z=\frac{17}{6}$$

Now, on subtracting 5 from both the sides, we obtain

$$5-2z-5=\frac{17}{6}-5$$
$$\Rightarrow -2z=\frac{17-30}{6}$$
$$\Rightarrow -2z=\frac{-13}{6}$$
$$\Rightarrow 2z=\frac{13}{6}$$

On dividing both the sides by 2, we obtain

$$z=\frac{13}{12}$$

2. $3(z-3)=\frac{z}{2}-2$

$$3z - 9 = \frac{z}{2} - 2$$

On subtracting $3z$ from both the sides, we obtain

$$\Rightarrow 3z - 9 - 3z = \frac{z}{2} - 2 - 3z$$

$$\Rightarrow -9 = \frac{-5z}{2} - 2$$

On adding 2 to both the sides, we obtain

$$\Rightarrow -9 + 2 = \frac{-5z}{2} - 2 + 2$$

$$\Rightarrow -7 = \frac{-5z}{2}$$

$$\Rightarrow \frac{5z}{2} = 7$$

On multiplying both the sides by $\frac{2}{5}$, we obtain $z = \frac{14}{5}$

3. $2(x + 20) = \frac{x}{3} + 65$

$$2x + 40 = \frac{x}{3} + 65$$

On subtracting $2x$ from both the sides, we obtain

$$\Rightarrow 2x + 40 - 2x = \frac{x}{3} + 65 - 2x$$

$$\Rightarrow 40 = \frac{-5x}{3} + 65$$

On subtracting 65 from both the sides, we obtain

$$\Rightarrow 40 - 65 = \frac{-5x}{3} + 65 - 65$$

$$\Rightarrow -25 = \frac{-5x}{3}$$

$$\Rightarrow \frac{5x}{3} = 25$$

On multiplying both the sides by 3/5, we obtain

$$x = 25 \times \frac{3}{5}$$

⇒ $x = 5 \times 3$

⇒ $x = 15$

**Example 2:**

**The difference between the ages of Raj and Mohini is 20 years. Ten years later, Raj's age will be half of Mohini's age. Find their present ages.**

**Solution:**

Let Raj's age be $x$. The difference between their ages is 20 years. Now, we are given that Raj's age after 10 years will be half of Mohini's age. Therefore, Mohini is older than Raj.

We can write Mohini's age as $20 + x$.

After 10 years, Raj's age will be $x + 10$ and Mohini's age will be $20 + x + 10 = 30 + x$

According to the given condition, we obtain

$$x + 10 = \frac{1}{2}(30 + x)$$

On multiplying both the sides by 2, we obtain

$2(x + 10) = 30 + x$

⇒ $2x + 20 = 30 + x$

On subtracting $2x$ from both the sides, we obtain

$2x + 20 - 2x = 30 + x - 2x$

⇒ $20 = 30 - x$

On subtracting 30 from both the sides, we obtain

$20 - 30 = 30 - x - 30$

$\Rightarrow -10 = -x$

$\Rightarrow x = 10$

$\therefore$ Raj's age = 10 years

Mohini's age = 20 + 10 = 30 years

## Mathematical Expressions Of Word Problems

Suppose Rahul has Rs 2100 with him. He goes to a market and purchases five shirts. Now the money left with him is Rs 100.

How can we write this situation mathematically?

Let us look at some more examples now.

**Write the following statements in the form of a linear equation.**

**1. The sum of two consecutive even numbers is 46.**

**2. One-fourth of a number plus 5 is 30.**

**3. When 20 is subtracted from *m*, the result is 16.**

**4. The perimeter of an equilateral triangle is 27 cm.**

**5. Mohit is 5 years older than Rohit and the sum of their ages is 35.**

**Solution:**

1. Let one even number be $2x$.
   Then, the other even number will be $(2x + 2)$.
   The linear equation is
   $2x + (2x + 2) = 46$
   $\Rightarrow 4x + 2 = 46$

2. Let the number be $z$.

   One-fourth of the number = z/4

   According to the given statement,

$$\frac{z}{4} + 5 = 30$$

3. The difference between *m* and 20 is 16.
   Therefore,
   $m - 20 = 16$

4. We know that in an equilateral triangle, all sides are equal in length.
   Let the length of one side be *x*.

   The perimeter is the sum of all sides of the triangle.
   $\Rightarrow x + x + x = 27$
   $\Rightarrow 3x = 27$

5. Let the age of Rohit be *x* years.
   Mohit's age will be $(x + 5)$ years.

   The sum of their ages is 35,
   $\therefore x + (x + 5) = 35$
   $\Rightarrow 2x + 5 = 35$