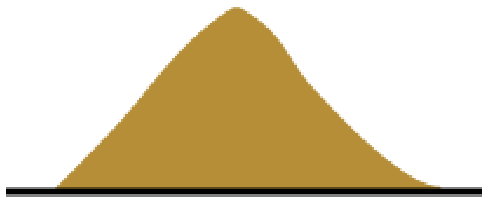
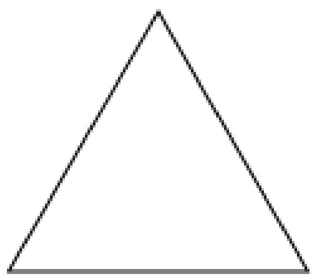
Triangles

Triangles and Their Attributes

Let us look at the figure of a heap of sand given below.



If we draw the above figure in a simple way, then it will look similar to the following figure.



We must have seen these types of shapes before. This is a three-sided polygon. It is called a **triangle.** We define a triangle as follows.

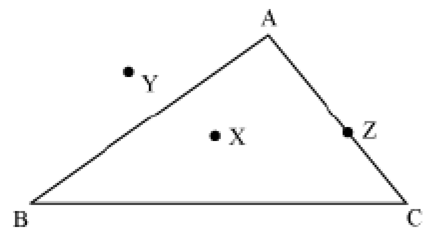
A polygon formed by three line segments is called a triangle i.e., a triangle is a polygon having three sides.

Let us discuss some characteristics of a triangle through various observations.

We know that a polygon has three regions.

1. Interior
2. Exterior
3. Boundary

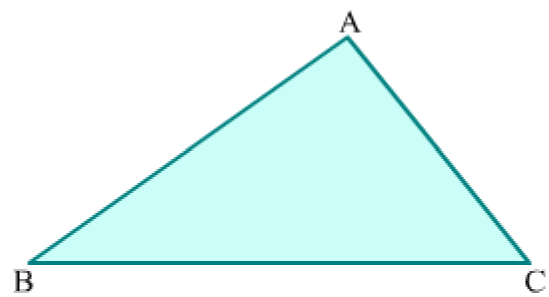
A triangle is a polygon, therefore, it also has the above three regions. It can be clearly understood with the help of the following figure.



In the above figure, point X lies in the interior of ΔABC; point Y lies in the exterior of ΔABC, while points A, B, C, and Z lie on the boundary of ΔABC.

The triangular area:

Look at the following triangle.

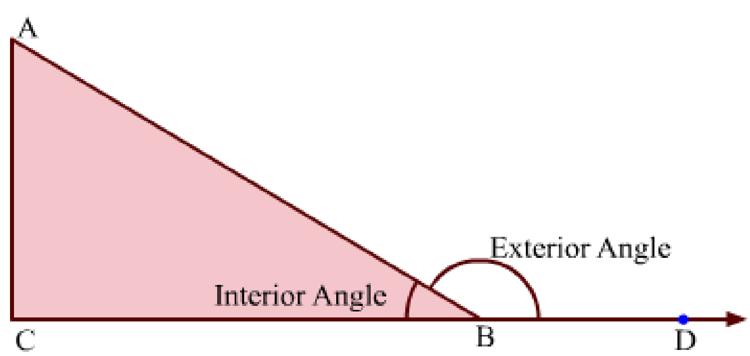


The interior of ΔABC is shaded. The whole shaded part of ΔABC along with its boundary is its area.

Thus, **interior and boundary of a triangle together form the area of triangle or triangular area.**

Exterior angle of a triangle:

Look at the triangle shown below.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

An exterior angle of a triangle can be defined as follows:

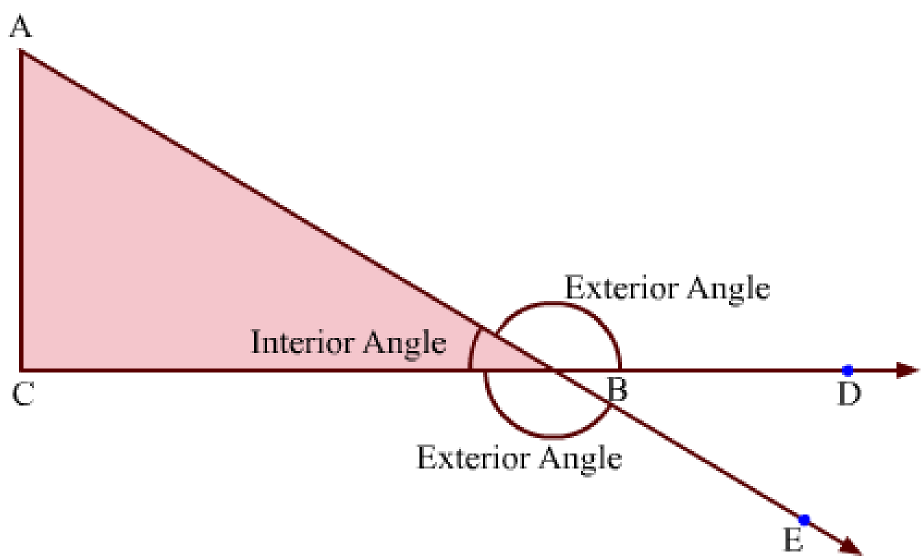
The angle formed by a side of a triangle with an extended adjacent side is called an exterior angle of the triangle.

Also, it can be seen that CBD is a line and hence, ∠ABC and ∠ABD form a linear pair at vertex B.

So, an exterior angle of a triangle can be defined in another way as follows:

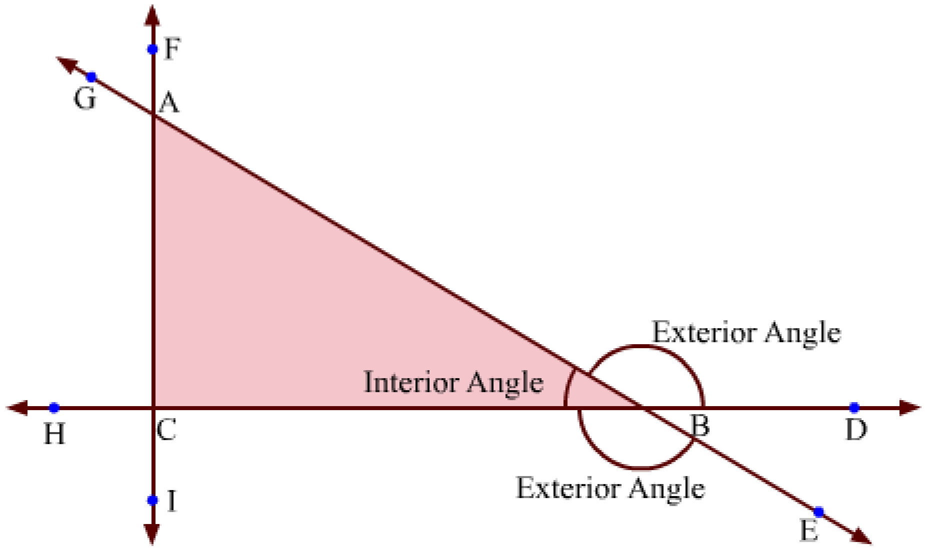
An angle forming linear pair with an interior angle of a triangle is known as exterior angle of that triangle.

Now, look at the following figure.



Here, two exterior angles such as ∠ABD and ∠CBE are formed at the vertex B.

Similarly, two exterior angles can be formed at each vertex of a triangle.



It can be seen that ∠BAF and ∠CAG are exterior angles formed at vertex A whereas, ∠ACH and ∠BCI are exterior angles formed at vertex C.

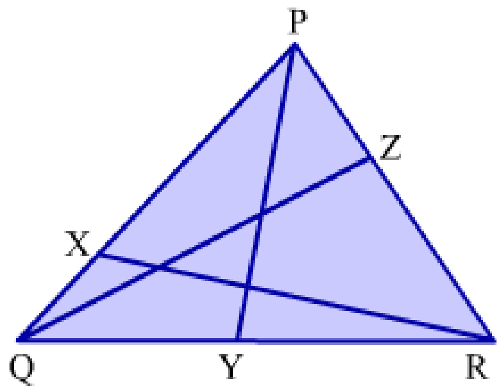
Thus, a triangle has six exterior angles.

In the above figure, it can be seen that there are three more angles such as ∠FAG, ∠DBE and ∠ICH which are vertically opposite angles of interior angles ∠BAC, ∠ABC and ∠ACB respectively. These angles are neither interior nor exterior angles of **Δ**ABC.

Cevians of a triangle:

A line segment joining a vertex of the triangle to any point on the opposite side (or its extension) is known as a cevian of that triangle.

Observe the give figure.



Here, line segments PY, QZ and RX all are cevians.

Note: Infinitely many cevians can be drawn from each vertex of a triangle. In other words, a triangle can have infinitely many cevians.

In a triangle there are few cevians such as **medians, altitudes** and **angle bisectors** are very special as these exhibit interesting properties.

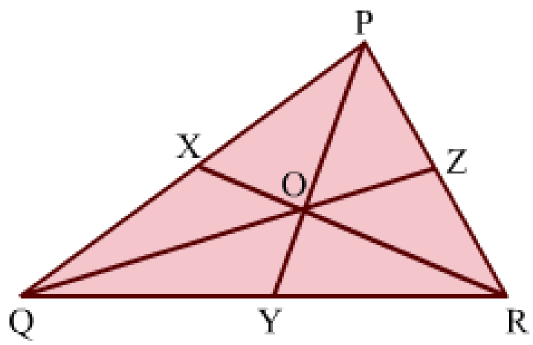
We know that all three medians are concurrent and the point of concurrence is known as centroid which divides each median in the ratio 2 : 1. Thus, centroid trisect each median. Also, centroid is the centre of gravity of any triangular lamina (cut out).

Also, all three altitudes are also concurrent and point of concurrence is known as orthocentre.

Similarly, all three angle bisectors are concurrent and the point of concurrence is known as incentre.

Ceva's Theorem:

A great Italian mathematician **Giovanni Ceva (Dec. 7, 1647 - June 15, 1734)** derived a very interesting property of cevians which is known as Ceva's theorem.



According to the theorem,

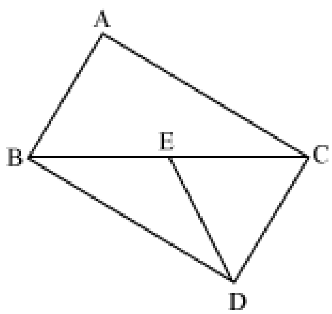
In ΔPQR, any three points X, Y and Z on the sides PQ, QR and RP respectively, the line segments PY, QZ and RX will be concurrent, if and only if

$$\frac{PX}{XQ}\cdot\frac{QY}{YR}\cdot\frac{RZ}{ZP}=1$$

Let us discuss the examples based on these concepts of a triangle.

Example 1:

With respect to the given figure, answer the following questions.



(a) How many triangles are there in the figure?

(b) Write the names of any nine angles.

(c) Write the names of all the line segments.

(d) Which two triangles have ∠DBE common?

(e) Which two triangles have line segment ED as the common side?

(f) Which two triangles have line segment BC as the common side?

Solution:

(a) There are four triangles, namely, ΔABC, ΔBCD, ΔBED, and ΔCED in the given figure.

(b) ∠BAC, ∠ABC, ∠ACB, ∠DBE, ∠BED, ∠BDE, ∠CED, ∠EDC, and ∠DCE are 9 angles of the given figure.

(c) The line segments are $\overline{AB}, \overline{BC}, \overline{CA}, \overline{BD}, \overline{DC}, \overline{BE}, \overline{ED}$, and $\overline{EC}$.

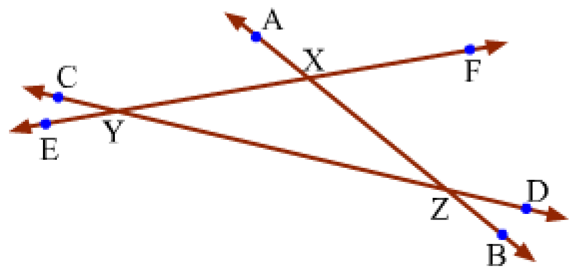
(d) ΔBCD and ΔBED have ∠DBE as common angle.

(e) ΔBED and ΔCED have line segment ED as the common side.

(f) ΔABC and ΔBCD have line segment BC as the common side.

Example 2:

With respect to the given figure, answer the following questions.



(a) Name all the exterior angles of ΔXYZ along with their respective vertices.

(b) Name all the angles which are neither exterior nor interior angles of ΔXYZ.

(c) Name the angles forming linear pair with ∠YXZ.

(d) Name the angle of ΔXYZ forming linear pair with ∠YZB.

Solution:

(a) The exterior angles of ΔXYZ along with their respective vertices are as follows:

∠YXA and ∠ZXF are formed at vertex X.

∠XYC and ∠ZYE are formed at vertex Y.

∠YZB and ∠XZD are formed at vertex Z.

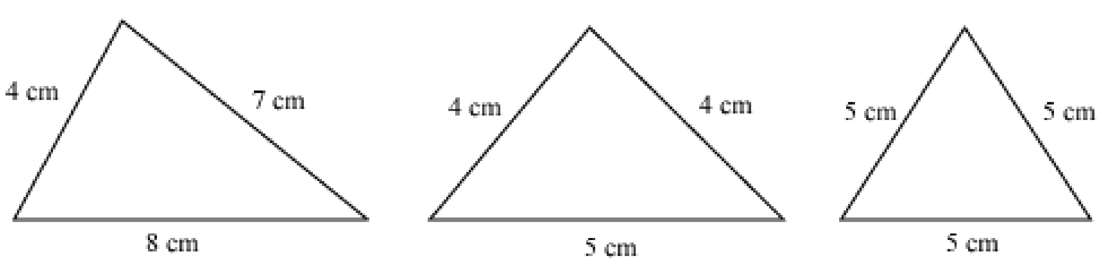
(b) ∠AXF, ∠DZB and ∠EYC are neither exterior nor interior angles of ΔXYZ.

(c) The angles forming linear pair with ∠YXZ are ∠YXA and ∠ZXF.

(d) ∠YZX forms linear pair with ∠YZB.

Classification of Triangles

Consider the following triangles.



Do you observe any difference among the given triangles?

In the first triangle, all sides are of different lengths.

In the second triangle, two sides are equal and the third one is of a different length.

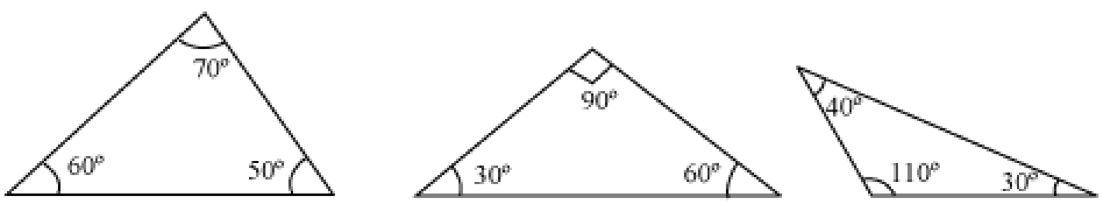
In the third triangle, all the sides are of equal lengths.

These are actually different types of triangles. One of the ways of classifying triangles is on the basis of the lengths of their sides.

Watch this video to understand the various types of triangles on the basis of the length of their sides.

Is the length of the sides the only criterion for the classification of triangles?

No. Let us first consider the following triangles.



What do you observe in these triangles?

Observe the following points.

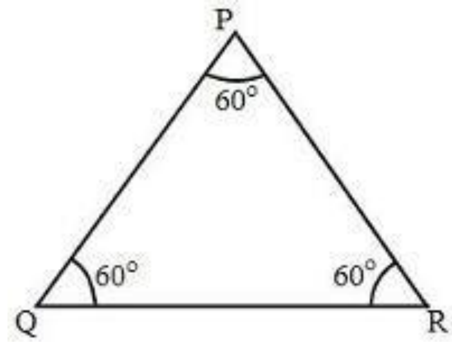
In the first triangle, all angles are less than 90°.

In the second triangle, only one angle is a right angle and the other two angles are less than 90°.

In the third triangle, one angle is more than the right angle and the other two angles are less than 90°.

These are all different types of triangles, which can be classified according to the measures of their angles.

Equiangular triangle:
An equiangular triangle is the triangle whose each angle is equal to each other, or we can say that whose each angle is equal to 60°.

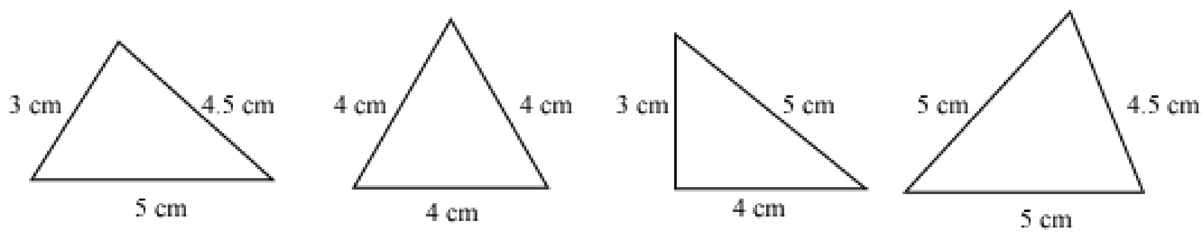


ΔPQR is an equiangular triangle as its each angle measures 60°.
i.e., $\angle P = \angle Q = \angle R = 60°$

Let us now look at some more examples to understand the concept better.

Example 1:

Classify the following triangles based on the nature of their sides.



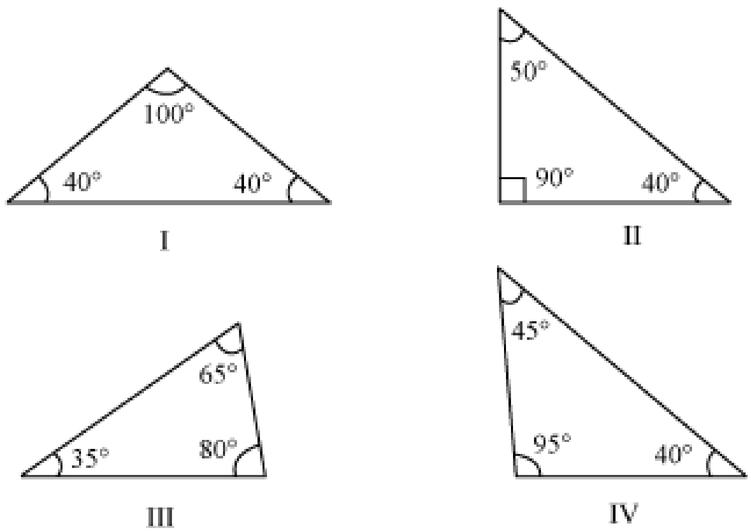
(i) **(ii)** **(iii)** **(iv)**

Solution:

1. Since all the sides are of different lengths, it is a scalene triangle.
2. Since all the sides are of same lengths, it is an equilateral triangle.
3. Since all the sides are of different lengths, it is a scalene triangle.
4. Since two sides of the given triangle are equal, it is an isosceles triangle.

Example 2:

Classify the following triangles based on the nature of their angles.

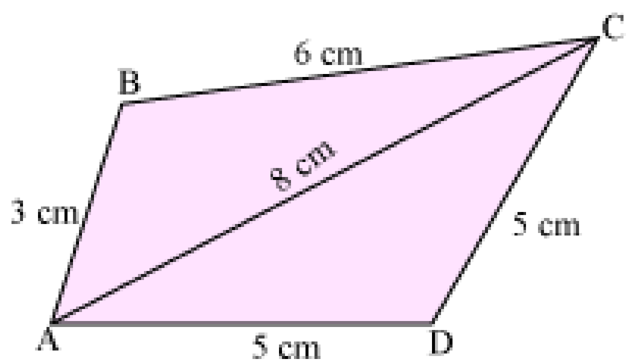


Solution:

1. Since one of the three angles is greater than a right angle, it is an obtuse-angled triangle.
2. Since one angle is a right angle, it is a right-angled triangle.
3. Since all the angles are less than 90°, it is an acute-angled triangle.
4. Since one of the angles is greater than a right angle, it is an obtuse-angled triangle.

Example 3:

Classify the triangles ABC and ADC based on the nature of their sides.



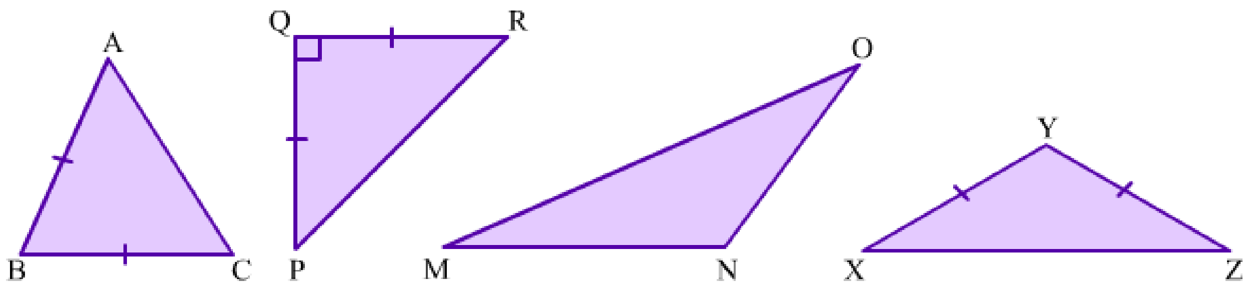
Solution:

ΔABC is a scalene triangle because all the three sides are of different lengths.

ΔACD is an isosceles triangle because two of its sides, AD and CD, are of equal lengths.

Example 4:

Classify the following triangles based on the nature of their angles and sides both.



Solution:

ΔABC has two equal sides and all of its angles are acute angles, so it is an acute angled isosceles triangle.

ΔPQR is a right angled triangle having two equal sides, so it is a right angled isosceles triangle.

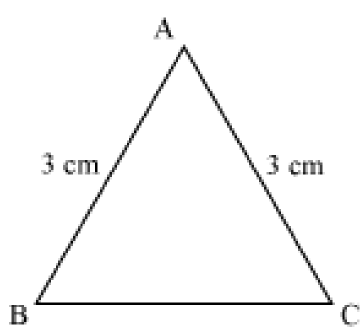
ΔMNO is an obtuse angled triangle having three unequal sides, so it is an obtuse angled scalene triangle.

ΔXYZ is an obtuse angled triangle having two equal sides, so it is an obtuse angled isosceles triangle.

Isosceles Triangles

We can classify triangles into different categories according to its sides (or angles). Now, let us study about a special type of triangle known as isosceles triangle.

"A triangle in which two sides are of equal length is called isosceles triangle."



In ΔABC, two sides AB and AC are of equal length. Therefore, ΔABC is an isosceles triangle.

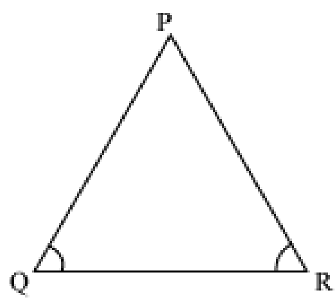
"In an isosceles triangle, angles opposite to equal sides are also equal."

In the above figure, ∠B and ∠C are the angles opposite to equal sides AC and AB respectively.

∴ ∠B = ∠C

Therefore, we can also define isosceles triangle in terms of angles.

"If two angles of a triangle are equal in measure, then the triangle is called isosceles triangle."



In the given figure,

∠Q = ∠R

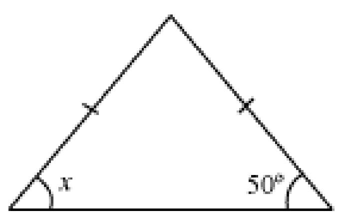
∴ ΔPQR is an isosceles triangle.

Now, let us solve some examples involving isosceles triangles.

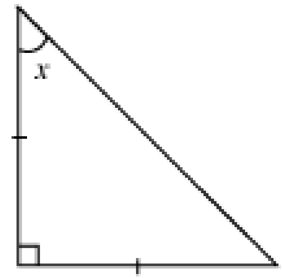
Example 1:

Find the value of *x* for the following figures.

(i)



(ii)



Solution:

(i) The given triangle is an isosceles triangle.

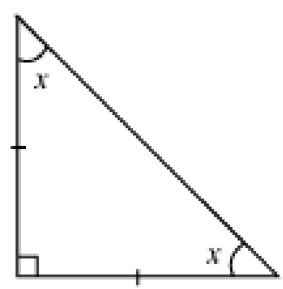
The angles opposite to equal sides are equal.

∴ $x = 50°$

(ii) The given triangle is an isosceles triangle.

The angles opposite to equal sides are equal.

Thus, we have



Now, using angle sum property of triangles, we obtain

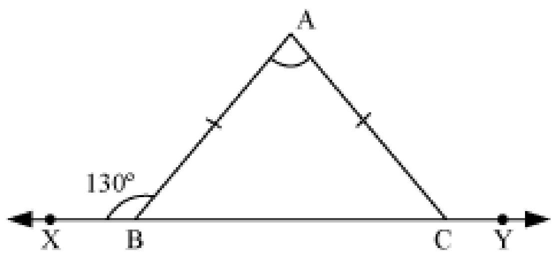
$x + x + 90° = 180°$

$2x = 90°$

$x = 45°$

Example 2:

In the given figure, find ∠BAC.



Solution:

It is given that ΔABC is an isosceles triangle. We know that in an isosceles triangle, angles opposite to equal sides are equal.

∴ ∠ABC = ∠ACB (1)

Now, ∠ABC = 180° − ∠ABX (By linear pair axiom)

= 180° − 130°

= 50°

From equation (1), we obtain

∠ACB = 50°

Using angle sum property in ΔABC, we obtain

∠ABC + ∠ACB + ∠BAC = 180°

50° + 50° + ∠BAC = 180°

∠BAC = 180° − 100°

∠BAC = 80°

Thus, the measure of ∠BAC is 80°.

Properties Of Equilateral Triangles

An equilateral triangle is a special case of a triangle. And why is it special? Watch the video and you will soon come to know about its special properties.

The three most important properties of equilateral triangles are as follows.

All the sides are of equal length.

All the angles are equal and the measure of each angle is 60°.

All the exterior angles are equal and the measure of each exterior angle is 120°.

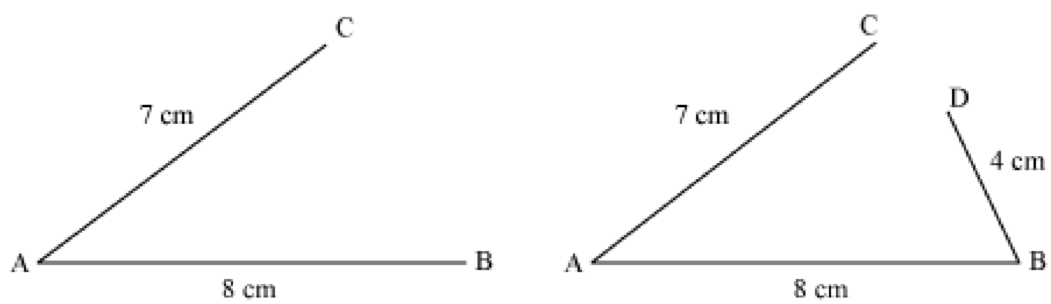
Construction of a Triangle when the Lengths of its Sides Are Given

Suppose if someone asks us to draw a triangle. The first question that strikes us is that what are the lengths of the sides of the triangle which is to be drawn?

Therefore, if the three sides of a triangle are given to us, then can we draw the triangle?

If we try to draw the triangle only with the help of a ruler, then it is not possible to draw it. With the help of a ruler, we can draw two sides of the triangle very easily. However, when we try to draw the third side, it may or may not intersect the third side.

Let us assume that we are asked to draw a triangle and the sides of the triangle are 8 cm, 7 cm, and 4 cm. Firstly, we draw the two sides of the triangle, which are 8 cm and 7 cm and then the third side of length 4 cm as shown in the following figure.



In these figures, we can see that a triangle is not formed.

Thus, we cannot draw a triangle only with the help of a ruler, but by using the ruler and compass.

Now, let us see the construction of a triangle using ruler and compass.

Before constructing a triangle, we should check whether the triangle is possible with the given sides or not.

In a triangle, the sum of any two sides must be greater than the third side.

For example: Can we draw a triangle with sides of length 6 cm, 9 cm, and 2 cm?

Here, 6 cm + 2 cm = 8 cm < 9 cm

i.e., the sum of the lengths of two sides is less than the length of the third side.

Therefore, we cannot draw a triangle with sides of given lengths.

Let us solve some examples based on the construction of triangles.

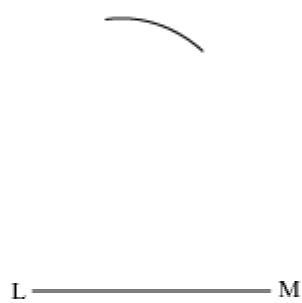
Example 1:

Construct an isosceles triangle such that the two equal sides are of lengths 9 cm each and the unequal side is of length 4 cm.

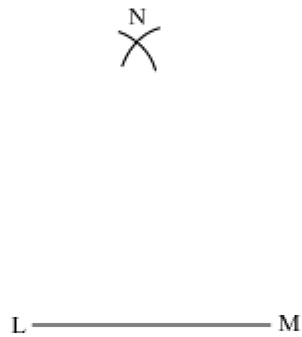
Solution:

Firstly, we draw a line-segment $\overline{LM}$ of length 4 cm using a ruler.

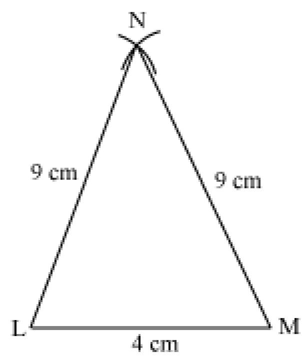
Then using compass, we draw an arc of radius 9 cm taking L as the centre.



Again, taking M as the centre, we draw another arc of radius 9 cm. Now, both the arcs intersect each other at a point N.



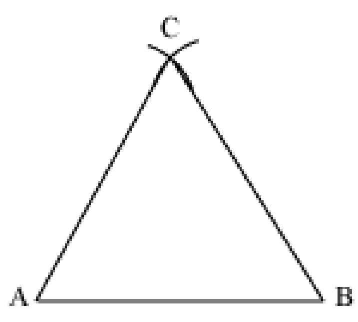
Now, we join the line segments $\overline{LN}$ and $\overline{MN}$.



Hence, ΔLMN is the required isosceles triangle with sides of given lengths.

Example 2:

A line segment $\overline{AB}$ is drawn. Then, two arcs with radius equal to the length of $\overline{AB}$ are drawn taking A and B as centres. The arcs intersect at C as shown in the given figure.



What type of a triangle is ΔABC?

Solution:

ΔABC is an equilateral triangle. As the two arcs have been drawn with radius equal to the length of $\overline{AB}$, therefore, $\overline{AC}$ and $\overline{BC}$ both are equal to $\overline{AB}$ i.e., all the three sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ of ΔABC are equal.

Example 3:

Construct an equilateral triangle of side 4.9 cm.

Solution:

We know that to construct a triangle, we require the measure of the lengths of all its three sides.

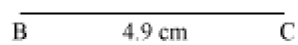
Now, here, we are required to construct an equilateral triangle of side 4.9 cm.

To construct the required triangle, we will use a property of an equilateral triangle.

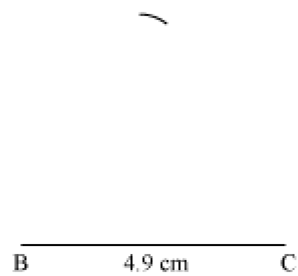
We know that all sides of an equilateral triangle are of equal length. So, we have to construct a triangle ABC with AB = BC = CA = 4.9 cm.

The steps of construction are as follows:

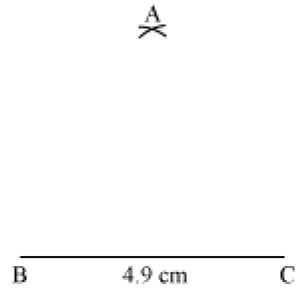
1. Draw a line segment BC of length 4.9 cm.



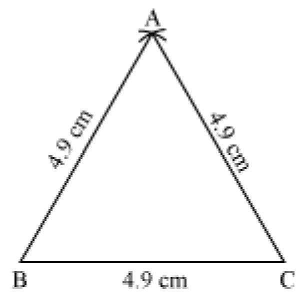
2. Taking point B as centre draw an arc of 4.9 cm radius.



2. Taking point C as centre draw an arc of 4.9 cm radius to meet the previous arc at the point A.



2. Join A to B and A to C.



ABC is the required equilateral triangle.

Construction of a Triangle when Two Angles and the Length of Side Between Them Are Given

Riya had studied in a book that if the measure of two angles of a triangle and the length of included side are given, then a unique triangle can be constructed based on this information.

She wants to try and see if this is really true or not? So, she tries to construct a triangle ABC such that two angles ∠C = 80°, ∠B = 40° and one side $\overline{BC} = 6$ cm are given.

Thus, to construct a triangle when the measure of two angles and the length of the included side are given, first we draw the side whose length is given and then we draw two rays making given angles with this side.

Let us solve some examples using this method.

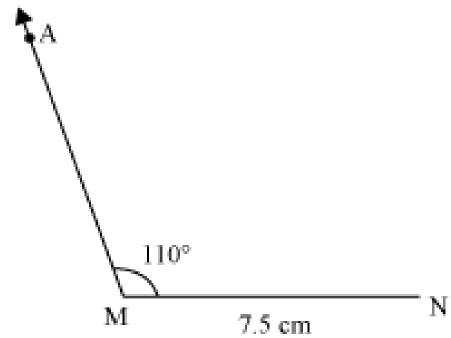
Example 1:

Construct a triangle such that the measures of two of its angles are 30° and 110° and the length of the side included between these two angles is 7.5 cm.

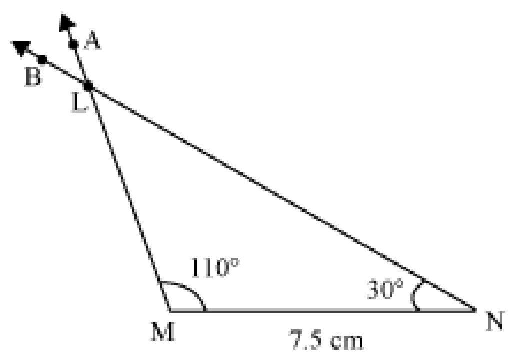
Solution:

First we draw a line segment $\overline{MN}$ of length 7.5 cm using a ruler.

Next we draw a ray $\overrightarrow{MA}$ from point M making an angle of measure 110° with the line segment $\overline{MN}$.



Now, we draw another ray $\overrightarrow{NB}$ from the point N, making an angle of 30° with $\overline{MN}$. Let it intersect the previously drawn ray $\overrightarrow{MA}$ at point L.



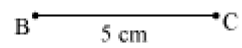
Thus, ΔLMN is the required triangle with the given measures.

Example 2:

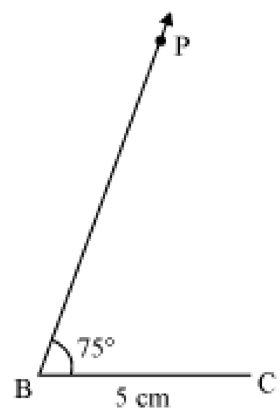
Construct an isosceles triangle such that the length of its unequal side is 5 cm and each of the two angles opposite to the equal sides is of measure 75°.

Solution:

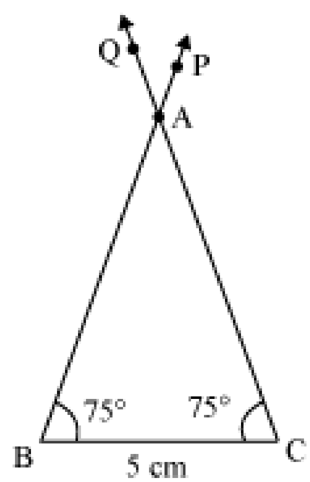
First we draw a line segment $\overline{BC}$ of length 5 cm using a ruler.



Now, we draw a ray $\overrightarrow{BP}$ from point B making an angle of measure 75° with $\overline{BC}$.



Again, we draw another ray $\overrightarrow{CQ}$ from point C making an angle of measure 75° with $\overline{BC}$. Let it intersect the previously drawn ray $\overrightarrow{BP}$ at point A.

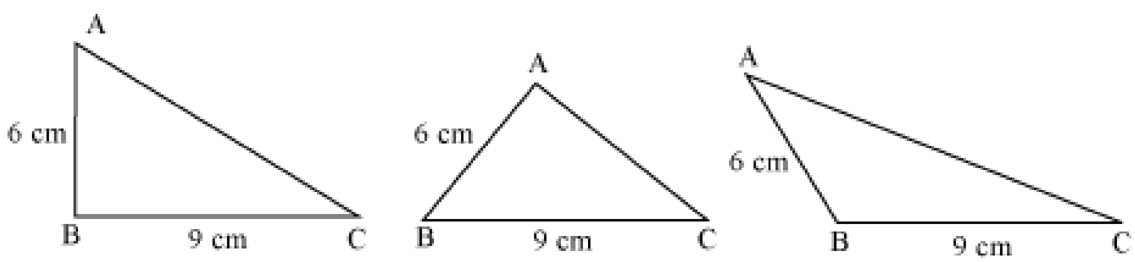


Thus, ΔABC is the required isosceles triangle with the given measures.

Construction of a Triangle when the Lengths of Two Sides and Angle Between Them Are Given

Abhijit constructed ΔABC. He told Ravi that the lengths of two sides of ΔABC are $\overline{AB}$ = 6 cm and $\overline{BC}$ = 9 cm. Then, he asked Ravi to construct a triangle with the same dimensions.

Ravi started constructing the triangle and observed that more than one triangle could be constructed using the same dimensions as shown in the following figures.



He asked Abhijit to tell something else about the triangle also. Therefore, Abhijit told him the measure of the angle between the two known sides.

Now, can Ravi construct a unique triangle based on the information given by Abhijit?

Yes, Ravi can construct a unique triangle because with the given information, one and only one triangle can be constructed.

The point to remember here is that

> **'If the lengths of any two sides and the measure of the angle between them are given, then a unique triangle can be constructed".**

Let us take an example. Assume that we have to construct a triangle whose lengths of any two sides are 7 cm and 5 cm and the measure of the angle between them is 45°.

Let us look at some more examples.

Example 1:

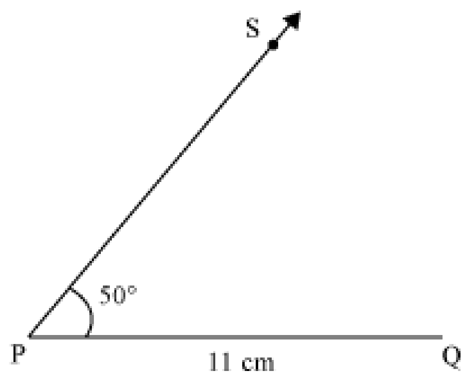
Construct a triangle PQR in which $\overline{PQ}$ = 11 cm, $\overline{PR}$ = 9 cm, and ∠QPR = 50°.

Solution:

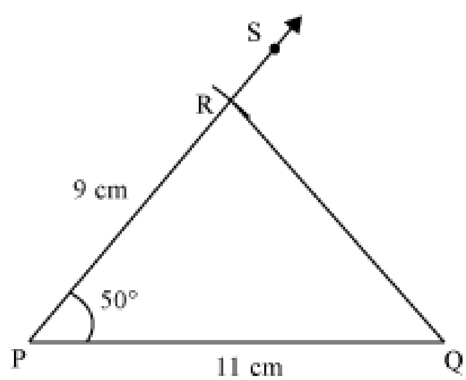
First, we draw a line segment $\overline{PQ}$ of length 11 cm using a ruler.

This is one of the sides of the triangle.

Now, we draw a ray $\overrightarrow{PS}$ from point P making an angle of measure 50° with the line segment $\overline{PQ}$.



Next, we draw an arc of radius 9 cm taking point P as the centre, which cuts $\overrightarrow{PS}$ at a point R. Then, we join the points Q and R to obtain the line segment $\overrightarrow{QR}$.



Thus, ΔPQR is the required triangle.

Example 2:

Construct a triangle ABC, where $\overline{AB}$ = 6 cm, $\overline{AC}$ = 2 $\overline{AB}$, and ∠BAC = 110°.

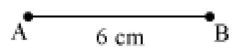
Solution:

Given, ∠BAC = 110°

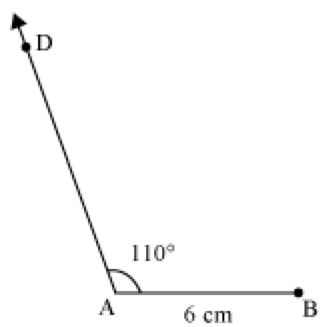
$\overline{AB}$ = 6 cm

$\overline{AC}$ = 2 $\overline{AB}$ = 2 × 6 cm = 12 cm

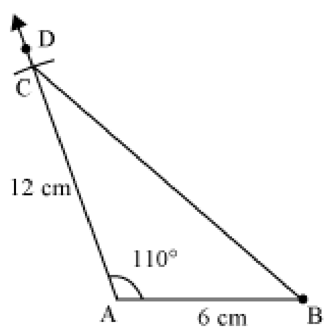
First, we draw a line segment $\overline{AB}$ of length 6 cm.



Then, we draw a line segment $\overrightarrow{AD}$ making an angle of measure 110° with $\overline{AB}$ such that ∠BAD = 110°.



Now, using compass, we draw an arc of radius 12 cm taking A as the centre, which cuts $\overrightarrow{AD}$ at a point C. Then, we join points B and C.



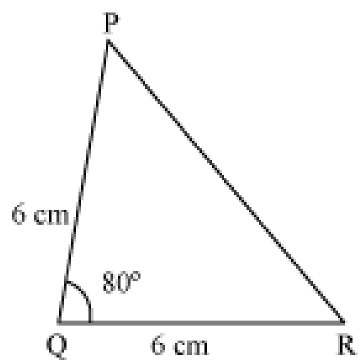
Thus, ΔABC so obtained is the required triangle.

Example 3:

Construct an isosceles triangle in which the length of each of its equal sides is 6 cm and the angle between them is 80°.

Solution:

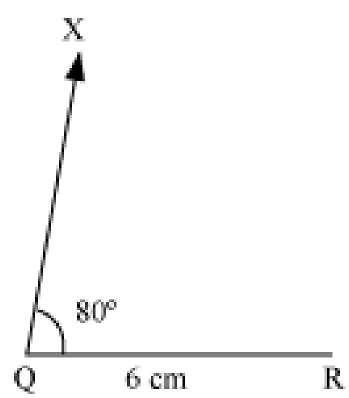
We have to construct an isosceles triangle PQR with PQ = QR = 6 cm. A rough sketch of the required triangle may be drawn as follows:



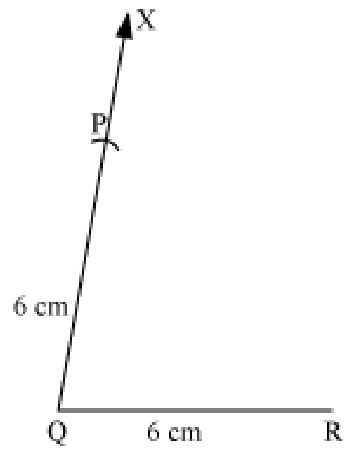
The steps of construction are as follows:

1. Draw the line segment QR of length 6 cm.

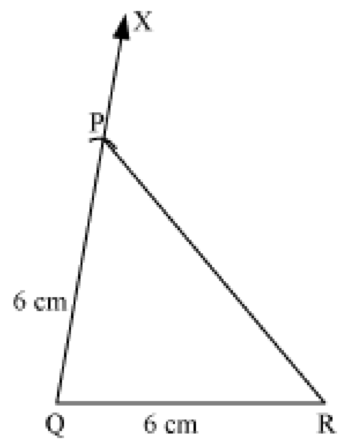
2. At point Q, draw a ray QX making an angle 80° with QR.



3. Taking Q as centre, draw an arc of 6 cm radius. It intersects QX at the point P.



iv. Join P to R to obtain the required triangle PQR.



Example 4:

Construct a triangle ABC, given that AB = 5 cm, BC = 6.5 cm and $\angle B = 60°$ **.**

Solution:

The steps of construction are as follows:

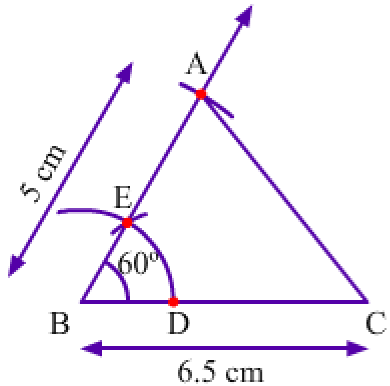
1. Draw a line segment BC of length 6.5 cm.

2. At B, using a compass draw an angle ∠EBC = 60 º.

3. With B as centre and radius 5 cm, draw an arc intersecting BE at A.

4. Join AC.

Thus, we get the required triangle ABC as shown below.



Construction of a Right-angled Triangle when the Length of One Leg and Hypotenuse Are Given

We know what a right-angled triangles is. Also, we know that it has two perpendicular sides and a longest side which is known as the hypotenuse.

Let us try to construct a right-angled triangle using the least information.

"A right-angled triangle can be constructed if the length of one of its sides or arms and the length of its hypotenuse are known".

Note: We can also construct a triangle if the lengths of its two arms are given.

The stepwise method to construct a right-angled triangle, when the length of one of the perpendicular sides and the length of hypotenuse is given, is as follows.

1. Firstly we draw one of the perpendicular sides of the triangle.
2. Then we draw the perpendicular on one of its end points.
3. Then we draw an arc from its other end point taking radius as the length of the hypotenuse to intersect the perpendicular. This point of intersection gives the third vertex of the right triangle.

Now let us see one more example to understand the method of construction better.

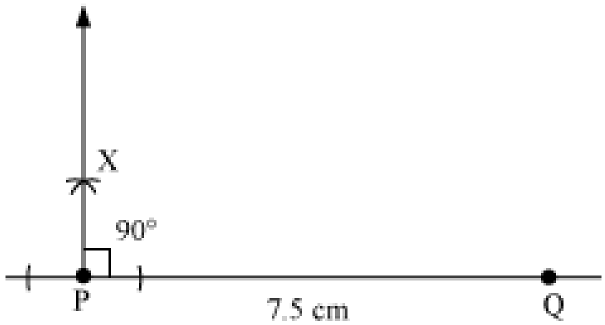
Example1:

Construct a right-angled triangle such that the lengths of its hypotenuse and one of its sides are 10 cm and 7.5 cm respectively.

Solution:

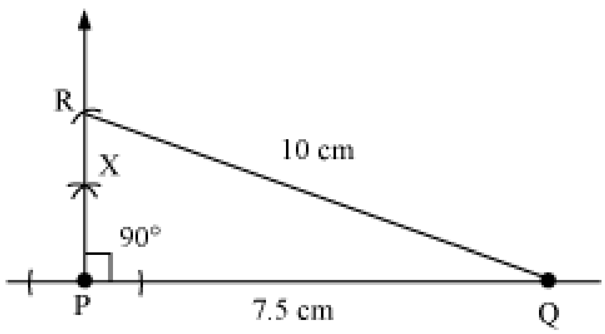
Firstly, we draw a line segment PQ of length 7.5 cm using a ruler.

Now, we draw the perpendicular PX to the line segment $\overline{PQ}$ at point P using compasses. Therefore, ∠QPX so formed is a right angle.



Now, we draw an arc of radius 10cm taking Q as the centre, which cuts $\overline{PX}$ at a point R.

Q and R are joined to make the hypotenuse.



Now, ΔPQR so obtained is the required right-angled triangle.

Construction of Circumcircle of a Triangle

We know that a circle which passes through all the vertices of ΔABC is called circumcircle of ΔABC.

Now, suppose the sides of a ΔABC are given as AB = 5 cm, BC = 5.4 cm, and CA = 6 cm and we are required to construct a circumcircle of ΔABC.

Let us now look at one more example to understand the construction of circumcircle more clearly.

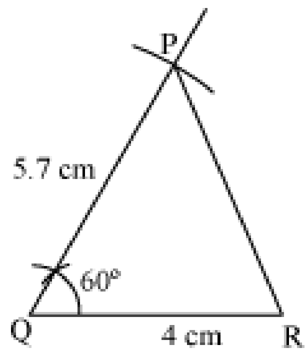
Example:

Construct a ΔPQR such that ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm. Also, construct the circumcircle of this triangle.

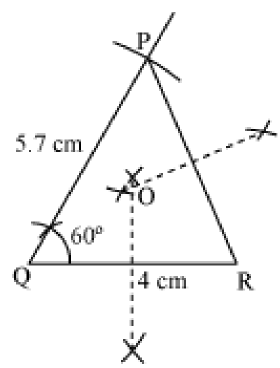
Solution:

The steps of construction are as follows:

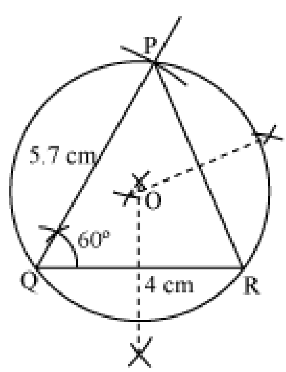
(1) Draw a triangle PQR with ∠Q = 60°, QR = 4 cm, and QP = 5.7 cm



(2) Draw perpendicular bisector of any two sides, say QR and PR. Let these perpendicular bisectors meet at O.



(3) With O as centre and radius equal to OP, draw a circle. The circle so drawn passes through the points P, Q, and R, and is the required circumcircle of ΔPQR.



Construction of Incircle of a Triangle

Suppose we have to construct an incircle of a triangle ABC and the following information about the ΔABC is given:

∠B = 60°, ∠C = 60°, and BC = 5.5 cm

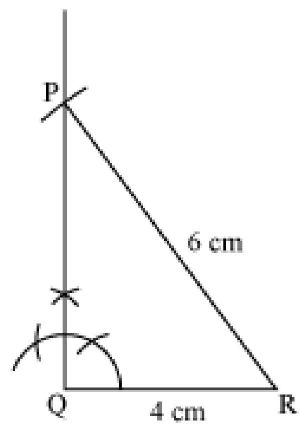
Example:

Construct a right triangle ΔPQR, right angled at Q, such that QR = 4 cm and PR = 6 cm. Also construct the incircle of ΔPQR.

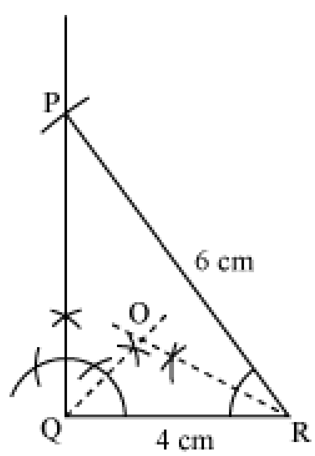
Solution:

The steps of construction are as follows:

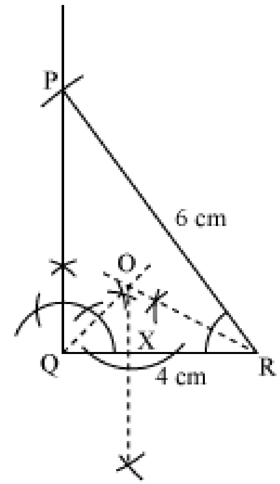
(1) Draw a ΔPQR right-angled at Q with QR = 4 cm and PR = 6 cm



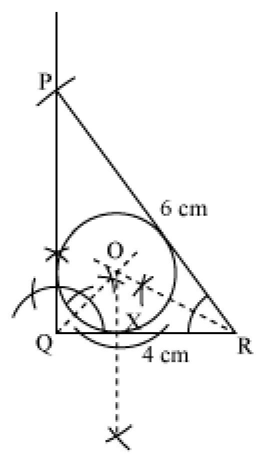
(2) Draw bisectors of ∠Q and ∠R. Let these bisectors meet at the point O.



(3) From O, draw OX perpendicular to the side QR.

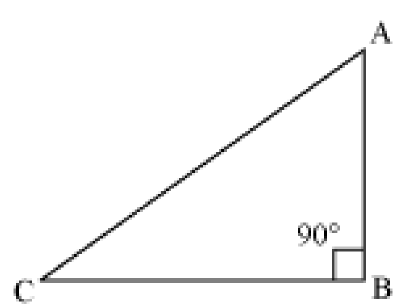


(4) With O as centre and radius equal to OX, draw a circle. The circle so drawn touches all the sides of ΔPQR and is the required incircle of ΔPQR.



Pythagoras Theorem and Its Converse

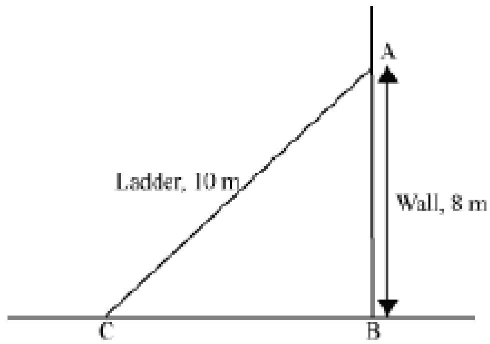
Look at the following right-angled triangle ABC. Here, m∠ABC = 90°.



In a right-angled triangle, special names are given to the sides. The side opposite to the right angle is termed as **hypotenuse** and the remaining two sides are termed as the **legs** of the right-angled triangle.

In real life, we come across many situations where a right angle is formed. Let us consider such a situation.

A 10m long ladder is placed on a wall such that the ladder touches the wall at 8m above the ground. This situation can be shown geometrically as follows.



In the above figure, AB is the wall of height 8 m and AC is the ladder of length 10 m. We know that a wall is perpendicular to the floor, i.e. AB is perpendicular to BC.
Thus, ∠ABC is a right angle.

Now, can we calculate the distance of the foot of the ladder from the base of the wall?

In this way, we can use Pythagoras theorem in many situations where right-angled triangle is formed.

In the reverse case, we can say that *for a triangle to be right-angled, the sum of the squares of two sides must be equal to the square of the third side.*

This is the converse of Pythagoras theorem. Using this converse, we can check whether a triangle is right-angled or not.

For example, let ABC be a triangle where $\overline{AB}$ = 10 cm, $\overline{BC}$ = 6 cm, and $\overline{AC}$ = 8 cm. Let us find whether ΔABC is a right-angled triangle or not.

Here, $\left(\overline{BC}\right)^2 + \left(\overline{AC}\right)^2 = 6^2 + 8^2$

$= (6 \times 6) + (8 \times 8)$

$= 36 + 64$

$= 100$

Again, $\left(\overline{AB}\right)^2 = 10^2$

$= 10 \times 10$

$= 100$

Therefore, $\left(\overline{AB}\right)^2 = \left(\overline{BC}\right)^2 + \left(\overline{AC}\right)^2$

Using converse of Pythagoras theorem, we can say that ΔABC is a right-angled triangle.

Here, AB = 10 cm is the **hypotenuse** of the triangle.

Let us now solve some more examples based on Pythagoras theorem and its converse.

Example 1:

Check whether the triangles having sides of given lengths are right-angled or not. Also identify the hypotenuse and the right angle in the right-angled triangles.

1. 5 cm, 6 cm, and 8 cm
2. 5 cm, 13 cm, and 12 cm

Solution:

1. $5^2 + 6^2 = 25 + 36 = 61$

   $8^2 = 8 \times 8 = 64$

   As, $5^2 + 6^2 \neq 8^2$

   Therefore, the triangle having the given sides is not a right-angled triangle.

2. $(5)^2 + (12)^2 = 25 + 144 = 169$

   $(13)^2 = 13 \times 13 = 169$

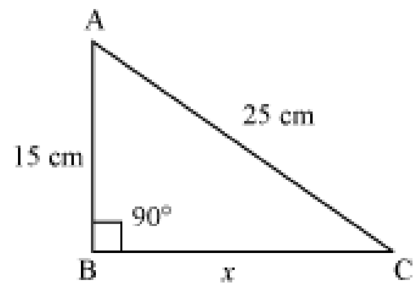
   As, $(5)^2 + (12)^2 = (13)^2$

Therefore, the triangle having the given sides is a right-angled triangle.

Here, the side of length 13 cm is the hypotenuse and the angle opposite to the hypotenuse is the right angle.

Example 2:

Find the value of x in the given figure.



Solution:

ΔABC is a right-angled triangle, right-angled at B.

Therefore, using Pythagoras theorem,

$15^2 + x^2 = 25^2$

On transposing 15^2 from L.H.S. to R.H.S., we obtain

$x^2 = 25^2 - 15^2$

$= 625 - 225$

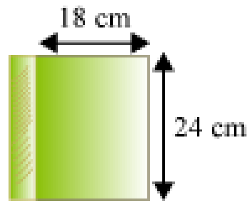
$= 400$

$= 20^2$

$\therefore x = 20$ cm

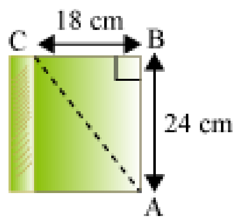
Example 3:

The lengths of two adjacent edges of a book are 18 cm and 24 cm. Find the length of the diagonal of the book.



Solution:

Let AC be the diagonal of the book as shown in the following figure.



Here, we can see that ∠ABC is a right angle. Therefore, using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 24^2 + 18^2$

$\Rightarrow AC^2 = 576 + 324$

$\Rightarrow AC^2 = 900$

$\Rightarrow AC = 30$

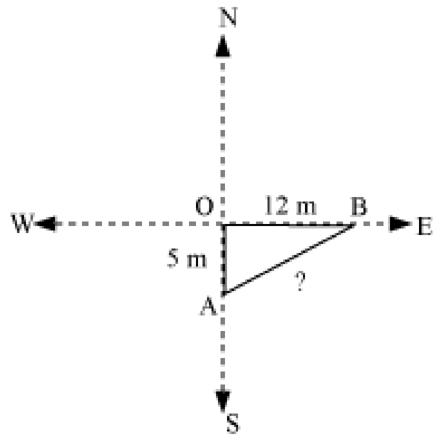
Therefore, the length of the diagonal of the book is 30 cm.

Example 4:

Gopal and Khushboo are friends. They are standing in front of a pole. Gopal is at a distance of 5 m south of the pole and Khushboo is standing 12 m east of the pole. What is the distance between Gopal and Khushboo?

Solution:

Let O, A, and B be the positions of the pole, Gopal, and Khushboo.



Using Pythagoras theorem, we obtain

$AB^2 = OA^2 + OB^2$

$AB^2 = 5^2 + 12^2$

$AB^2 = 169$

$AB^2 = 13^2$

$AB = 13$

Therefore, the distance between Gopal and Khushboo is 13 m.