Tribulations: A GCSE maths compilation

None of the diagrams are drawn accurately

Only use a calculator when specified

Trick questions do apply

Show all your working

The origins of the questions are linked at the end

Part 1: The Warm-up

What we call the “easy” questions

1. (a) Rationalise $\frac{1}{\sqrt{7}}$

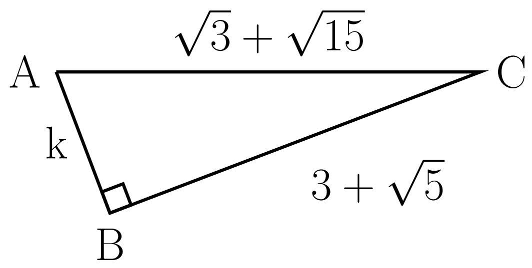
(b) Rationalise $\frac{33}{4-\sqrt{5}}$

(c) i. Expand and simplify

$$(\sqrt{3}+\sqrt{15})^2$$

Give your answer in the form $n+m\sqrt{5}$, where n and m are integers.

ii.



ABC is a right angled triangle. k is a positive integer.
Find the value of k.

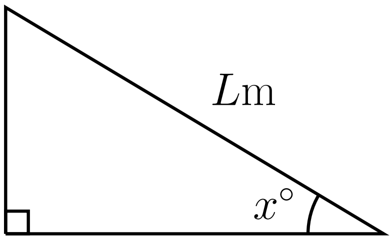
2. The Force, F, between two magnets is inversely proportional to the square of the distance, x, between them.
When $x = 3$, $F = 4$.

(a) Find an expression for F in terms of x.

(b) Calculate F when $x = 2$.

(c) Calculate x when $F = 64$.

3.



You may use a calculator.
Elliott did an experiment to find the value of g m/s^2, the acceleration due to gravity. He measured the time, T s, that a block took to slide L m down a smooth slope of angle $x°$.

He then used the formula $g = \frac{2L}{T^2 \sin(x°)}$ to calculate an estimate for g.

$T = 1.3$ correct to 1 decimal place. $L = 4.50$ correct to 2 decimal places.
$x = 30$ correct to the nearest integer.

(a) Calculate the lower bound and the upper bound for the value of g.
Give your answers correct to 3 decimal places.

(b) Use your answer to part (a) to write down the value of g to a suitable degree of accuracy.
Explain your reasoning.

4. Solve the simultaneous equations

$$x^2 + y^2 = 29$$
$$y - x = 3$$

5.

$$P = \frac{n^2 + a}{n + a}$$

Rearrange the formula to make a the subject.

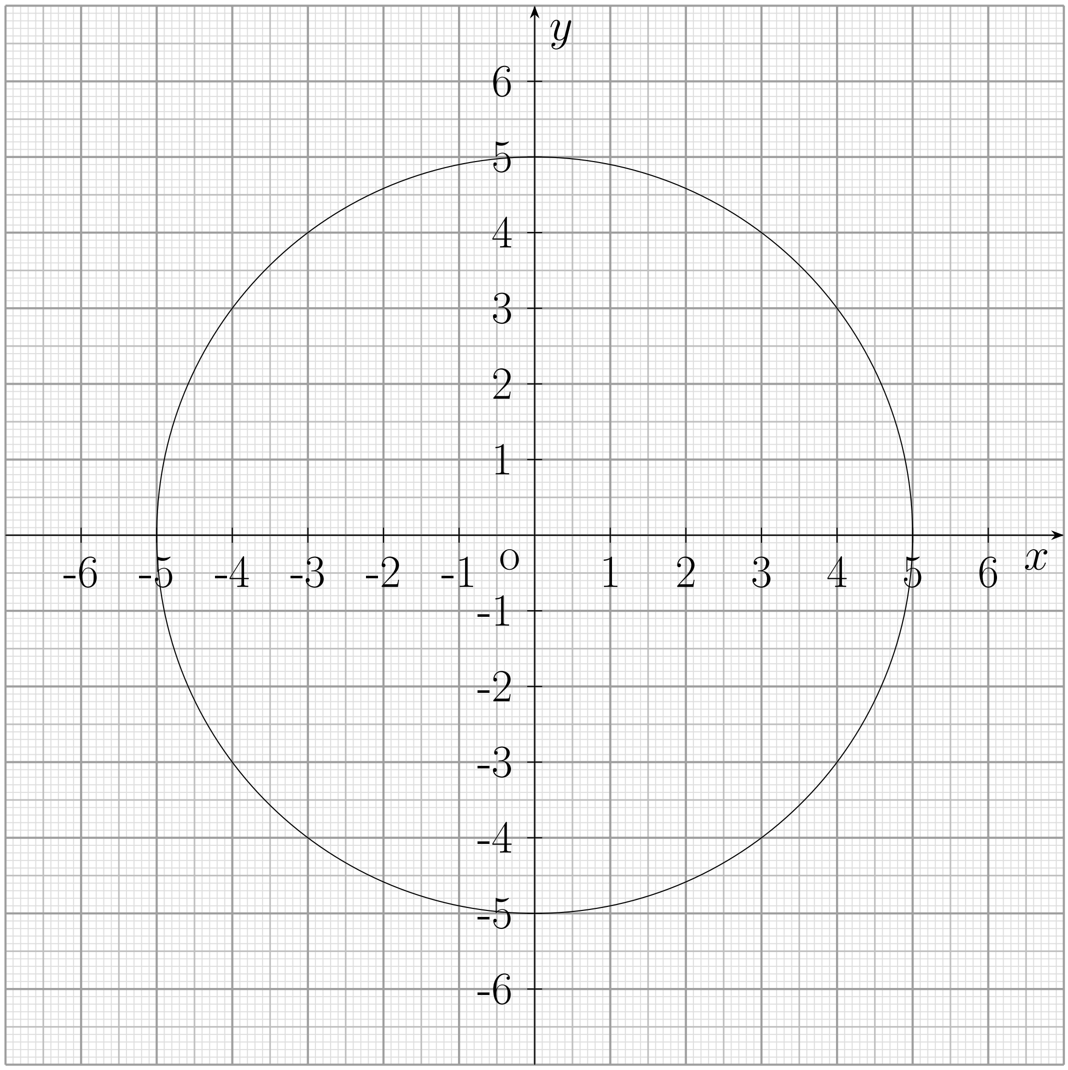
6. Simplify

$$\frac{4x^2 - 9}{2x^2 - 5x + 3}$$

7. Solve the equation

$$\frac{7}{x+2} + \frac{1}{x-1} = 4$$

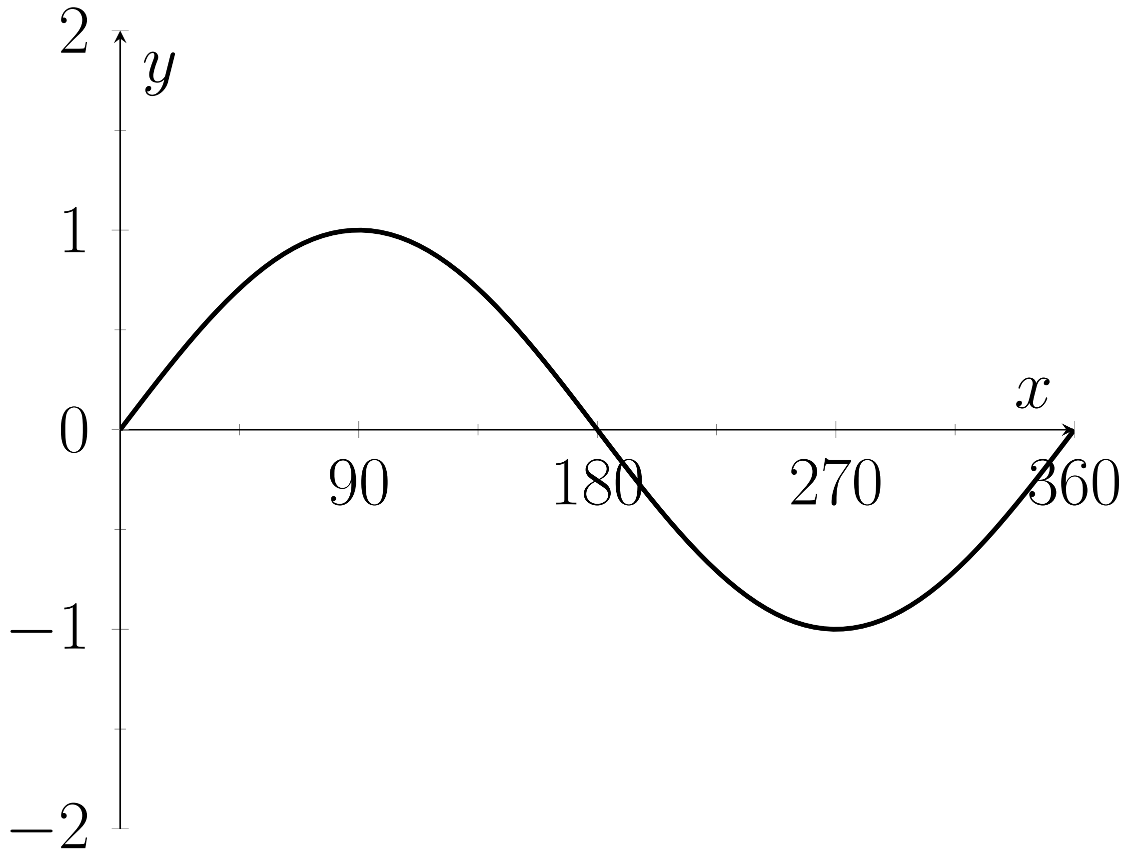
8. The diagram shows a circle of radius 5cm, centred at the origin.



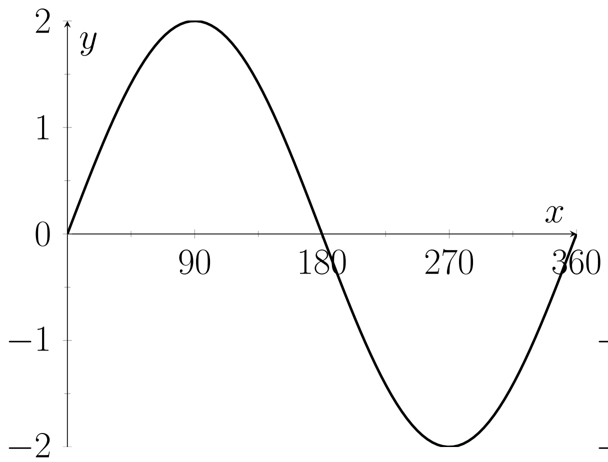
Draw a suitable straight line on the diagram to find estimates of the solutions to the pair of equations

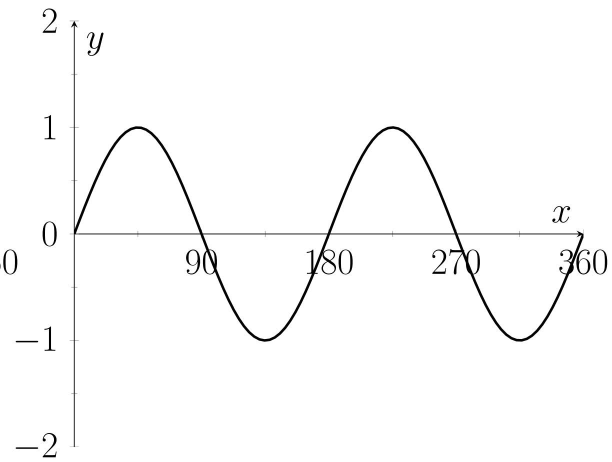
$$x^2 + y^2 = 25 \text{ and } y = 2x + 1$$

9. A sketch of the curve $y = \sin x°$ for $0 \leq x \leq 360$ is shown below.



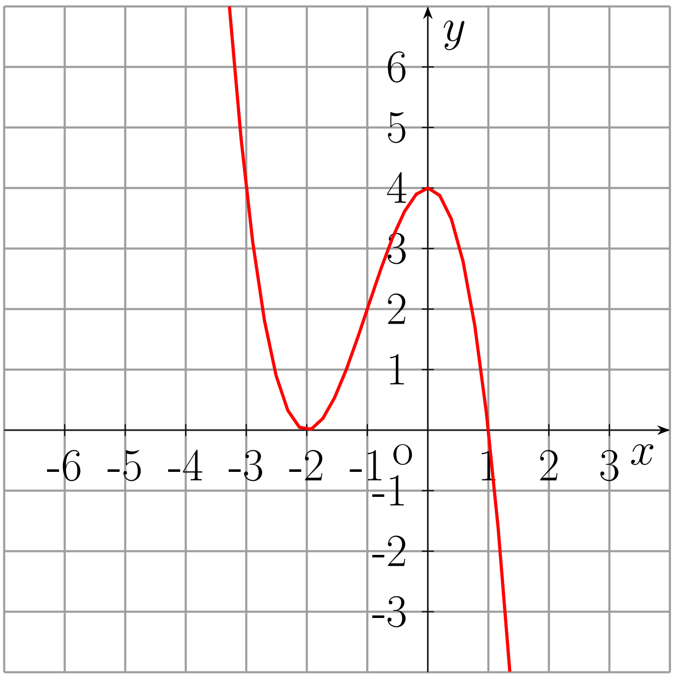
Using the sketch above, or otherwise, find the equations of the following curves.



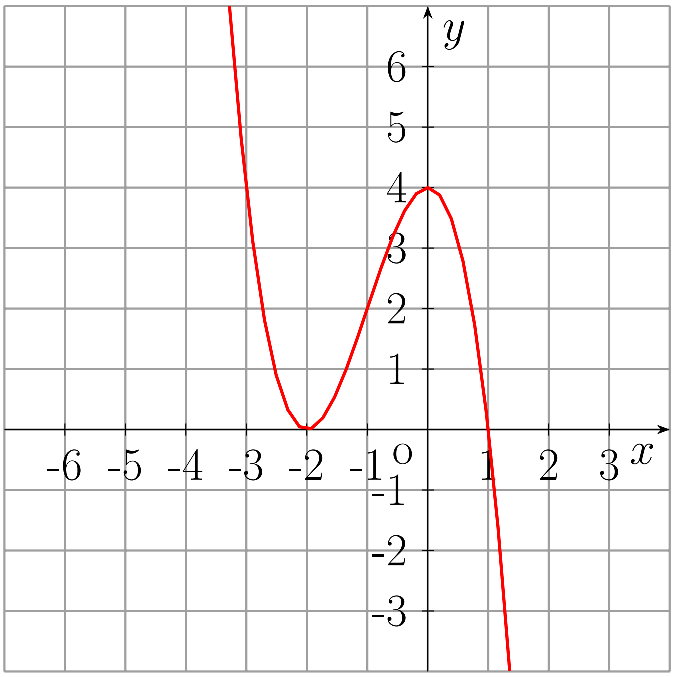


10. The graph of $y = f(x)$ is shown on the grids.

(a) On this grid, sketch the graph of $y = f(x) + 1$



(b) On this grid, sketch the graph of $y = f(\frac{x}{2})$

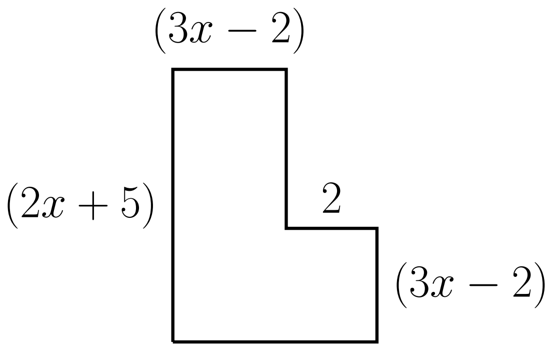


11. (a) Show that

$$(2a-1)^2-(2b-1)^2=4(a-b)(a+b-1)$$

(b) Prove that the difference between the squares of any two odd numbers is a multiple of 8.
(Hint: an odd number is a multiple of 2 minus 1)

12. The diagram below shows an irregular hexagon.
All the corners are right angles.



The area of the shape is 25.
Show that $6x^2 + 17x - 39 = 0$

13. The expression $8x - x^2$ can be written in the form $p - (x - q)^2$, for all values of x.

(a) Find the value of p and the value of q.

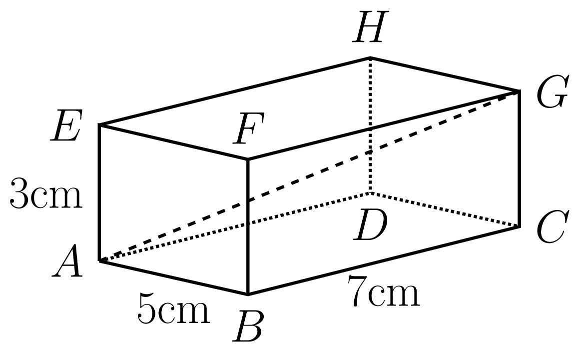
(b) The expression $8x - x^2$ has a maximum value.

i. Find the maximum value of $8x - x^2$.

ii. State the value of x for which this maximum value occurs.

14. You may use a calculator.

The diagram represents a cuboid $ABCDEFGH$

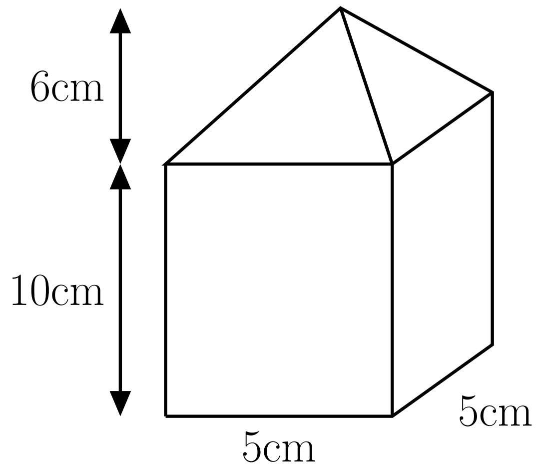


$AB = 5\text{cm}$. $BC = 7\text{cm}$. $AE = 3\text{cm}$.

(a) Calculate the length of AG.
Give your answer correct to 3 significant figures.

(b) Calculate the size of the angle between AG and the face $ABCD$.
Give your answer correct to 1 decimal place.

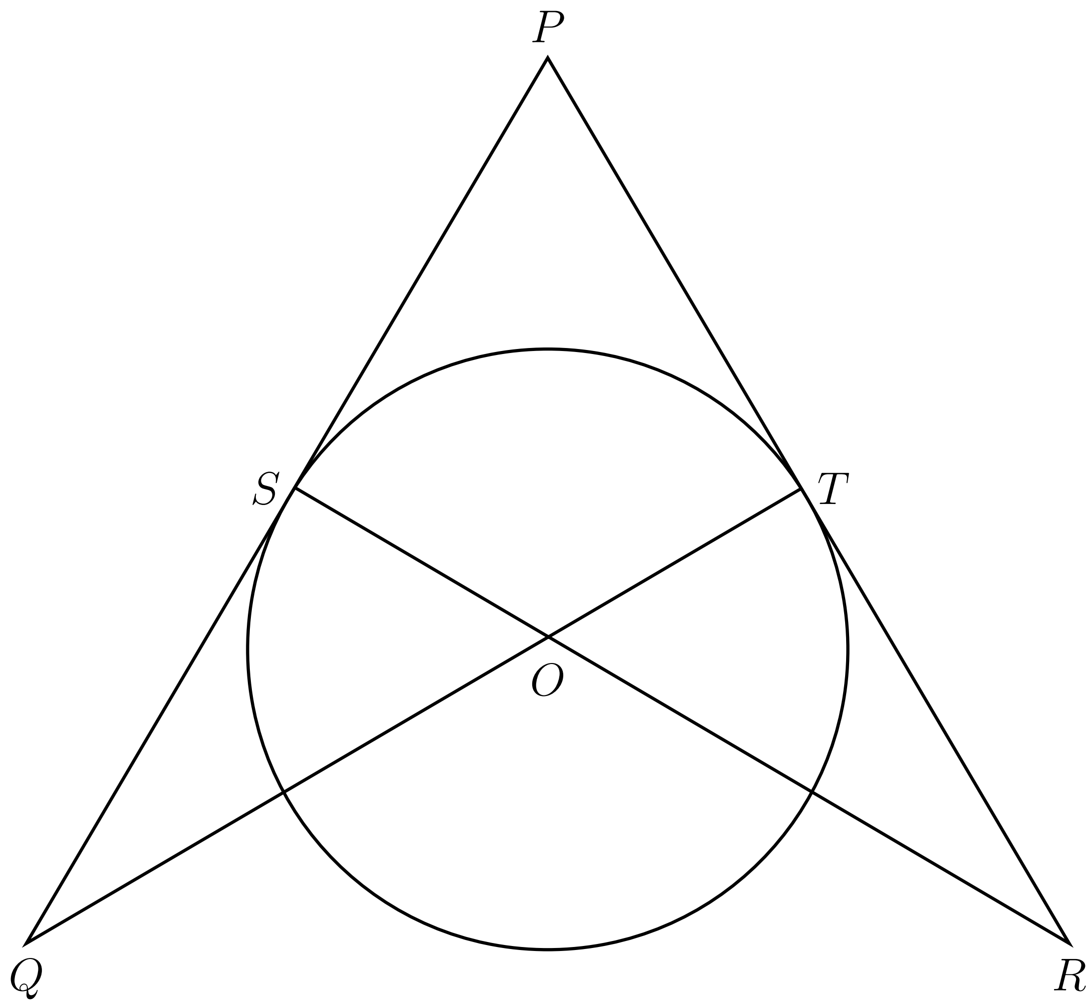
15. The shape is a model of a concrete post. It is a cuboid with a pyramid on top.



(a) Calculate the volume of the model.

(b) The model is built to a scale of 1:30 and the surface area is 290cm^2.
Calculate the surface area of the actual post.
Give your answer in square metres.

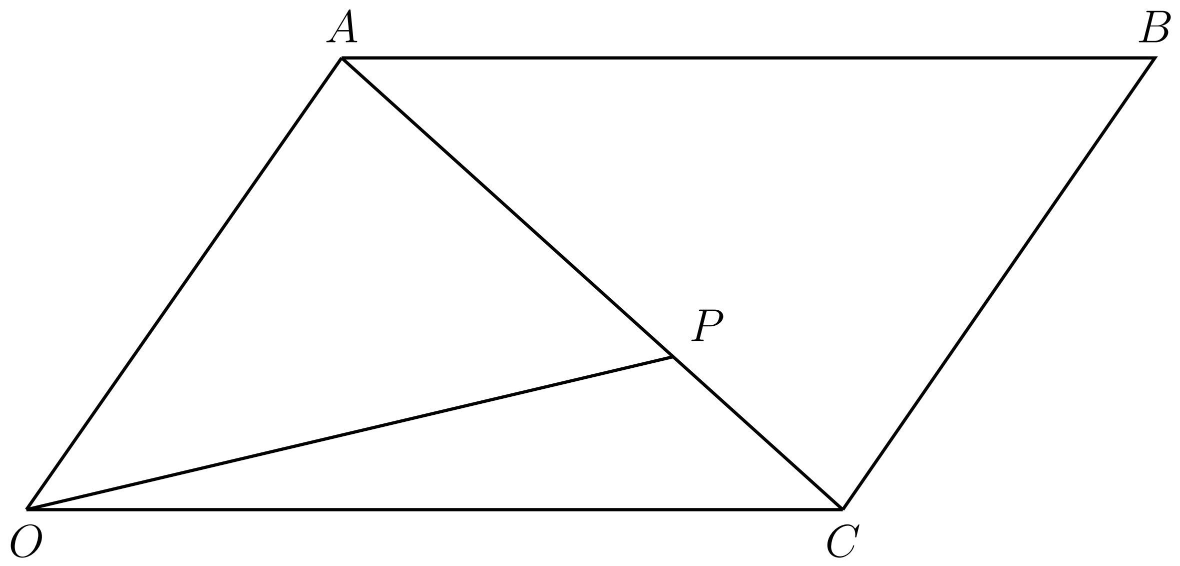
16.



S and T are points on a circle, centre O.
PSQ and PTR are tangents to the circle.
SOR and TOQ are straight lines.

Prove that triangle PQT and triangle PRS are congruent.

17.



$OABC$ is a parallelogram.

P is the point on AC such that $AP = \frac{2}{3}AC$.

$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

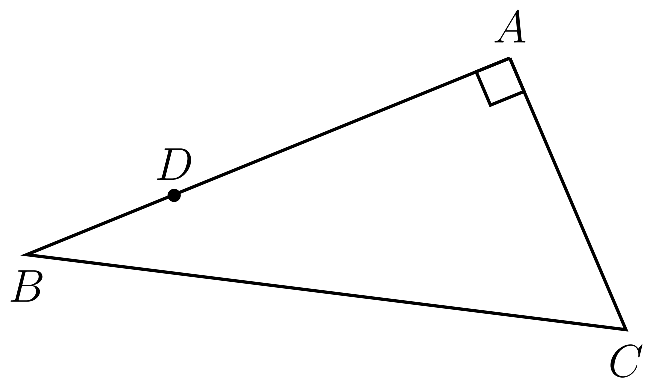
(a) Find the vector $\overrightarrow{OP}$.

Give your answer in terms of $\mathbf{a}$ and $\mathbf{c}$.

The midpoint of CB is M.

(b) Prove that OPM is a straight line.

18.

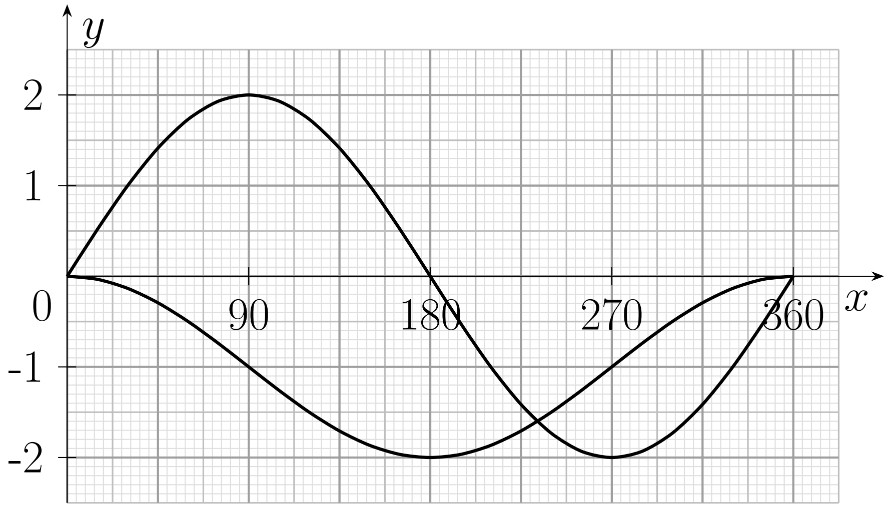


ABC is a right angled triangle.
D is the point on AB such that $AD = 3DB$.
Additionally $AC = 2DB$.

Show that $\sin C = \frac{k}{\sqrt{20}}$, where k is an integer.

Write down the value of k.

19.



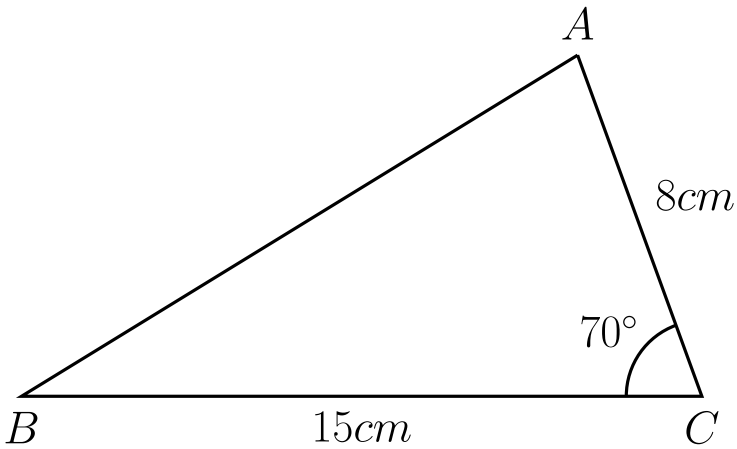
The diagram shows part of 2 graphs.
The equation of one graph is $y = a\sin(x^\circ)$
The equation of the other graph is $y = \cos(x^\circ) + b$

(a) Use the graphs to find the value of a and the value of b.

(b) Use the graphs to find the values of x in the range $0^\circ \le x \le 720^\circ$ when $a\sin(x^\circ) = \cos(x^\circ) + b$.

(c) Use the graphs to find the value of $a\sin(x^\circ) - (\cos(x^\circ) + b)$ when $x = 450^\circ$.

20.



You may use a calculator.

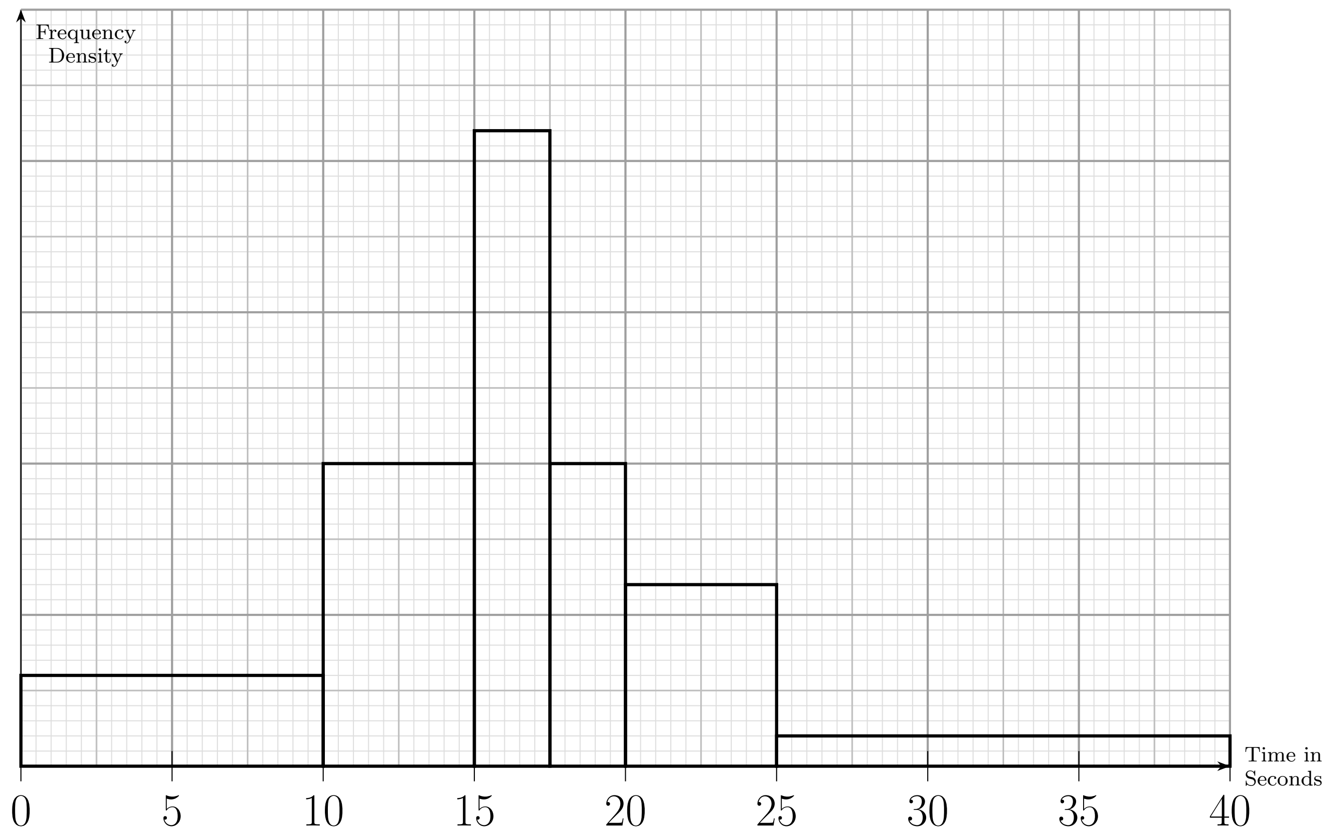
(a) Calculate the length of AB.
Give your answer correct to 3 significant figures.

(b) Calculate the size of angle BAC.
Give your answer correct to 1 decimal place.

21. The radius of a sphere is 3cm.
The radius of the base of a cone is also 3cm.
The volume of the sphere is 3 times the volume of the cone.

Work out the curved surface area of the cone.
Give your answer as a multiple of π.

22. You may use a calculator.



The histogram shows information about the time it took some children to connect to the internet.
None of the children took more than 40 seconds to connect to the internet.

110 children took up to 12.5 seconds to connect to the internet.
Work out the number of children who took 21 seconds or more to connect to the internet.

23. Stefan's drawer contains 5 white socks and 3 black socks.

He takes out two socks at random.

Work out the probability that Stefan takes out two socks of the same colour.

Part 2: The Test

The woes begin

1. You may use a calculator. Island X is a small island.

   In the winter of 1986 the ratio of natives to tourists on the island was $7 : 1$.

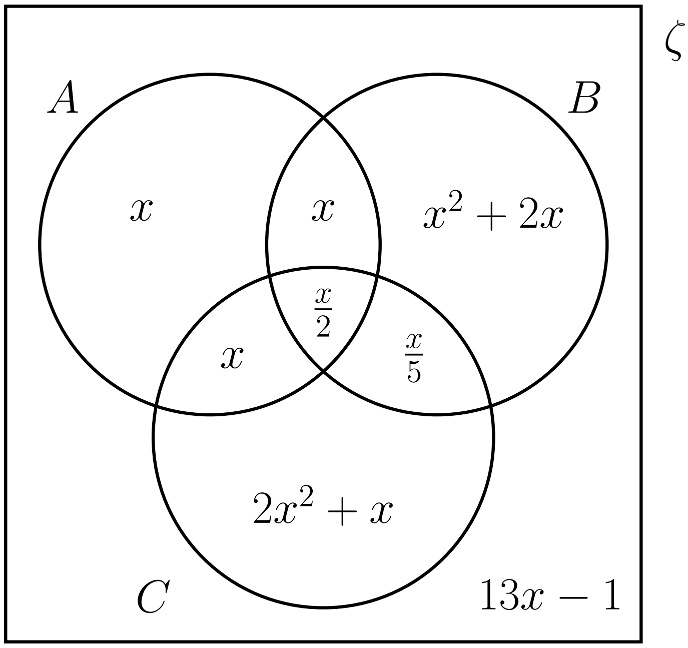
   In the summer of 1987 the ratio of natives to tourists on the island was $155 : 69$.

   The number of natives on the island decreased by 100 from the winter of 1986 to summer of 1987.

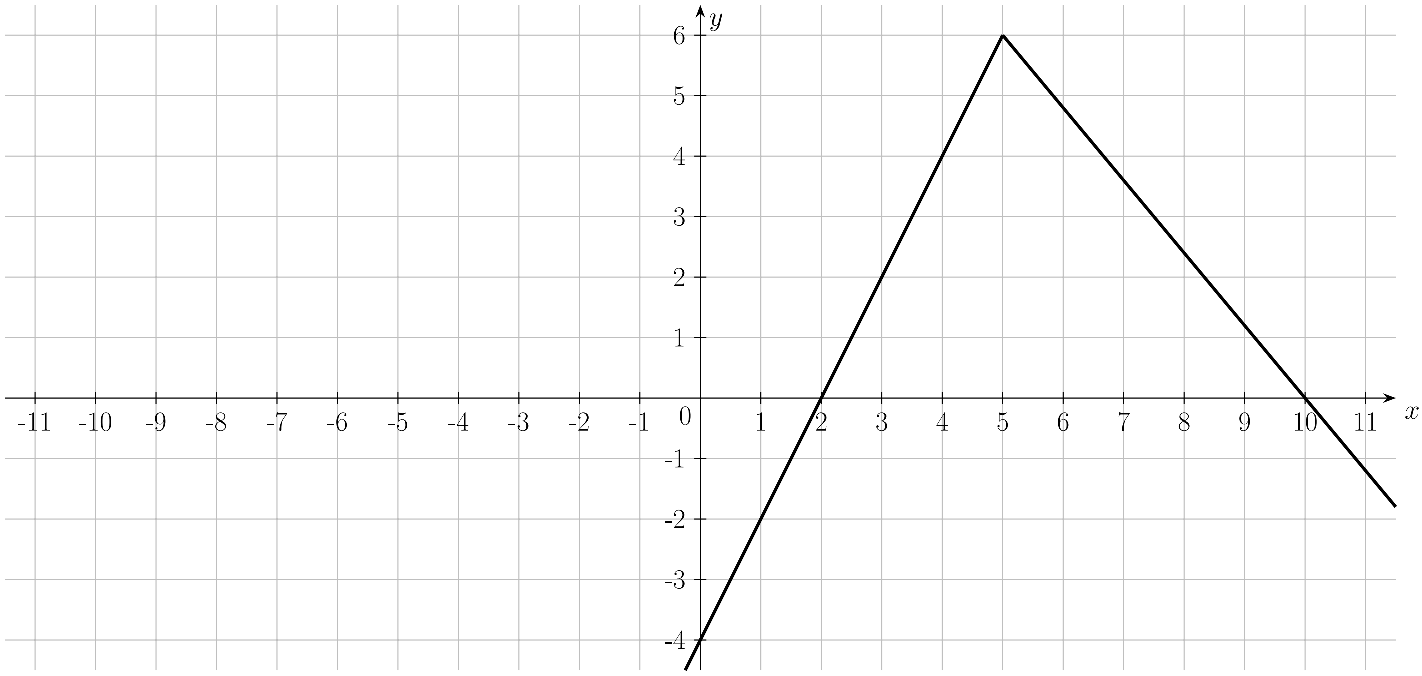
   The number of tourists on the island increased by 220 from the winter of 1986 to summer of 1987.

   Find the number of tourists that were there on Island X in the winter of 1986.

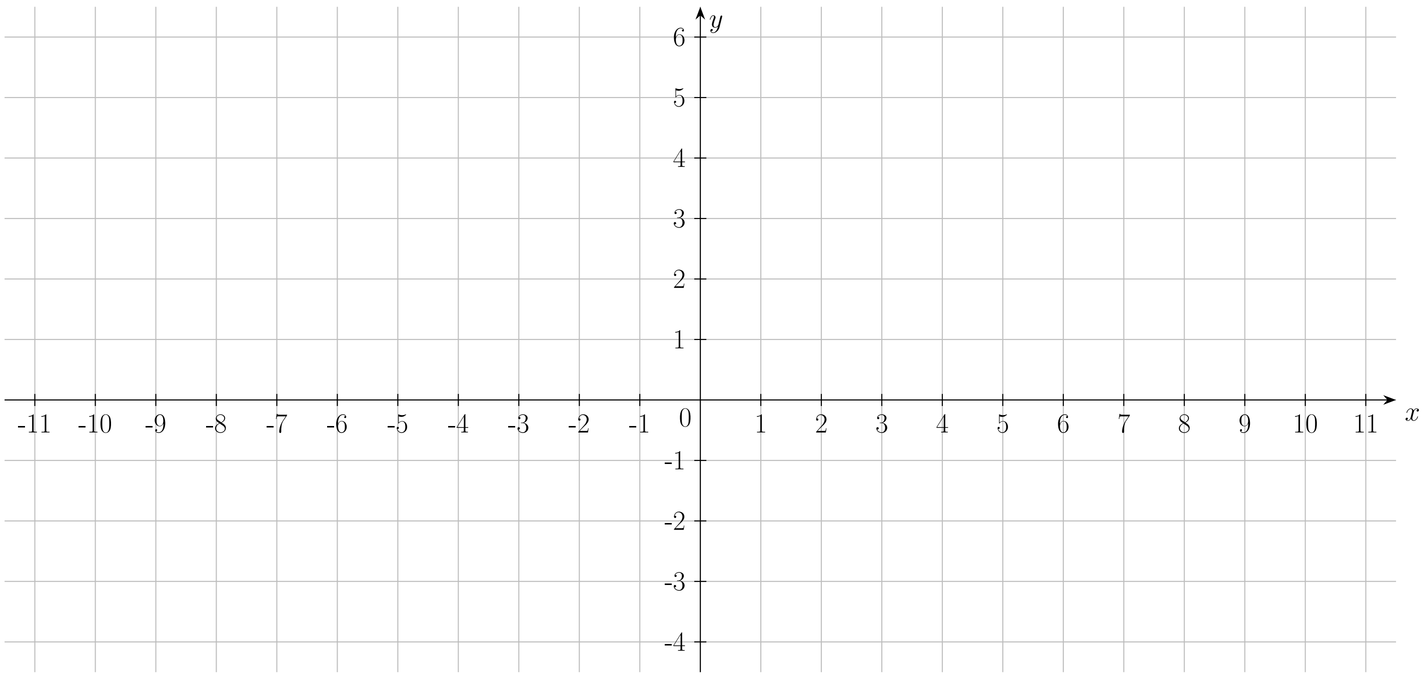
2. You may use a calculator. Find $P(B \cap \bar{A}\ |C)$.
Give your answer as a fraction in the form $\frac{a}{b}$ where a and b are integers.



3. The diagram below shows part of the graph of $y = 2f(x-1)$



On the grid below, draw the graph of $y = -f(-x)$



4. Using algebra, show that part of the line $3x + 4y = 0$ is a diameter of the circle with equation $x^2 + y^2 = 25$.

5. Sue is making a toy rocket in her science lesson which is to be launched from the ground.
The flight path of the toy rocket can be modelled by the equation
$h = -2t^2 + 6t + 1$.
h is the height in metres the rocket reaches above the ground.
t is the time in seconds after the rocket is launched.

Find the maximum height above the ground that the rocket reaches and the time it takes to reach this height.

6. Shape A is a regular polygon.
The ratio of the size of the interior angle to the size of the exterior angle is $7 : 2$. The ratio of the side length to the number of sides is $5 : 3$.
Find the perimeter of shape A.

7. You may use a calculator. Jamal is going to paddle between 3 points on a large lake. Jamal will start from Point A and paddle on a bearing of 050° until he reaches Point B. Once at Point B, Jamal will then paddle 1.6km on a bearing of 142° to reach Point C. Finally, Jamal will paddle directly to Point A from Point C where he will finish. The bearing of Point C from Point A is 098°.

   Given that Jamal can paddle at an average speed of 7.2kph, find the time it will take him to paddle directly back to Point A from Point C. Give your answer to the nearest minute.

8. Triangle ABC is an isosceles triangle.
   The points X and Y lie on the line AC.
   $AB = BC$.
   $AY = 3AX$.
   $AC = 4AX$.
   Prove that triangle ABX and triangle CBY are congruent.

9. Company T are designing a toy to be sold online.
The toy will be made up of a hemisphere with radius $X cm$ and a right cone with radius $X cm$ and heigh $Y cm$.
The cone will be attached to the top of the hemisphere.

Given that the total mass of the toy is 100π grams and the density of the toy is $60g/cm^3$, express Y in terms of X. Give your answer in the simplest form.

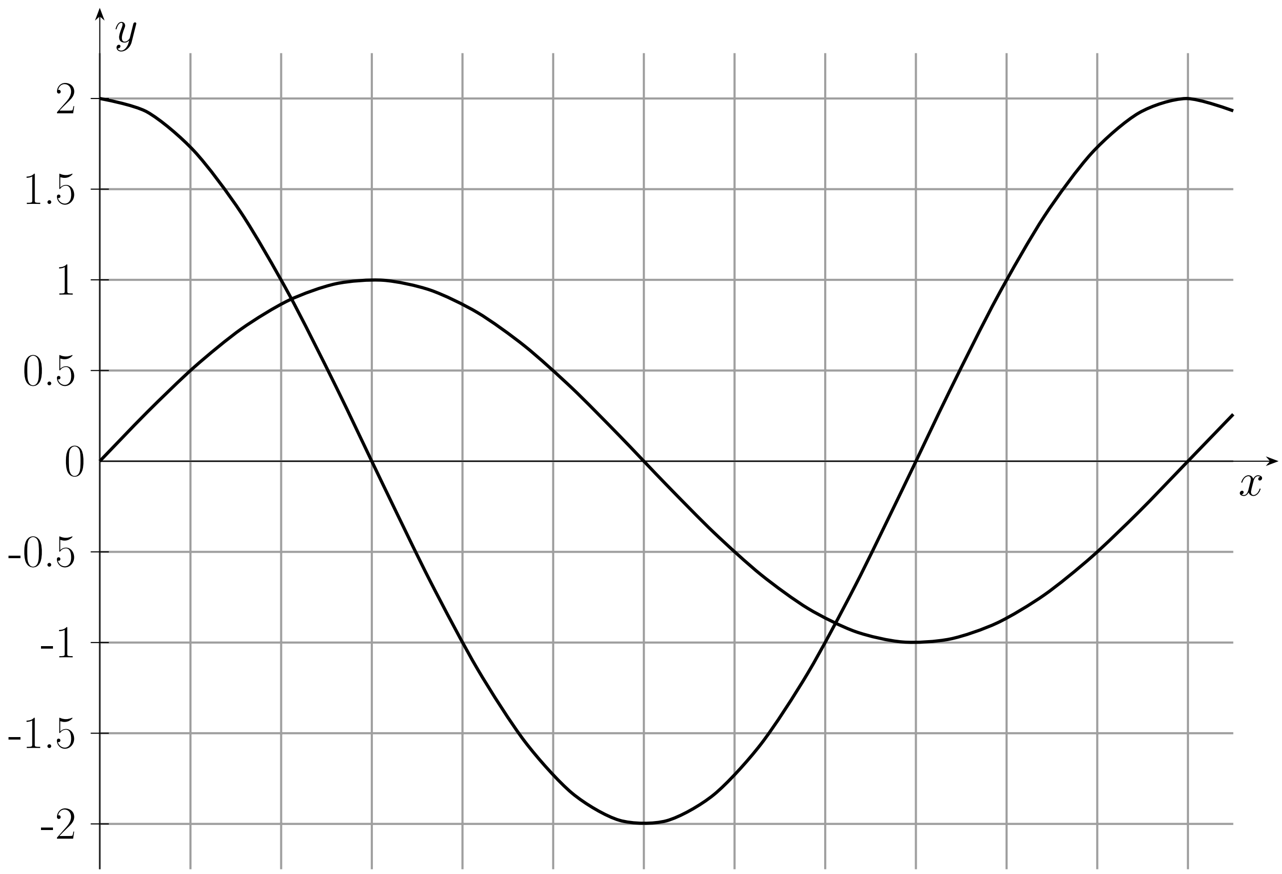
10. There are N boys in a class at school.
For every 2 boys in the class there are 3 girls in the class.
3 students are chosen at random and taken out of the class.

Given that the probability of choosing 3 boys is $\frac{1}{30}$,
show that $23N^2 - 114N + 88 = 0$

11. Consider 2 lines. Line A and Line B.
    Line A has gradient 2.
    Line B is perpendicular to Line A.
    Line A and Line B intersect at the Point $P(p, q)$.
    Line B crosses the y-axis at Point B.

    Show that the coordinates of Point B can be written as $(0, \frac{p}{2} + q)$.

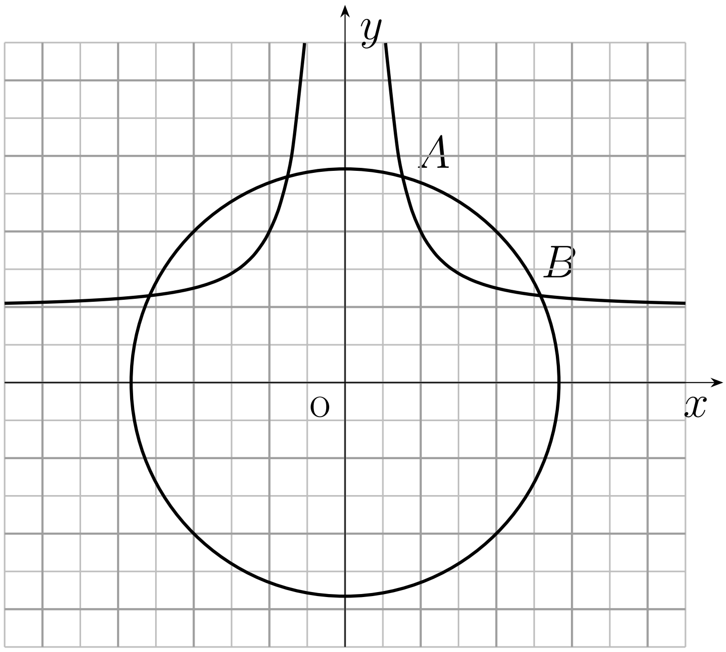
12. The graphs of $y = 2\cos x$ and $y = \sin x$ are shown in the diagram below for $0 \leq x \leq 360°$.



Use the graphs to find estimates for the solutions of the equation:

$$\sin x - 2\cos x = 0 \text{ for } 0 \leq x \leq 360°$$

13. The diagram below shows the graph of the equation $x^2 + y^2 = 8$ and part of the graph of the equation $y = \frac{1}{x^2} + 1$.



The points $A(p, q)$ and $B(r, s)$ are two of the points where the graphs intersect.

(a) Using the graphs above, find the solutions to the simultaneous equations:

$$y = \frac{1}{x^2} + 1$$
$$x^2 + y^2 = 8.$$

Giving your answer in terms of p,q,r, and s.

There are 2 real solutions to the simultaneous equations: $x^2 + y^2 = 8$ and $x = a$.

(b) Find the set of values of a giving your answer in simplified surd form.

14. The students in Class X and Class Y sat the same maths exam.
Information is given about the performance of each class in the table below.

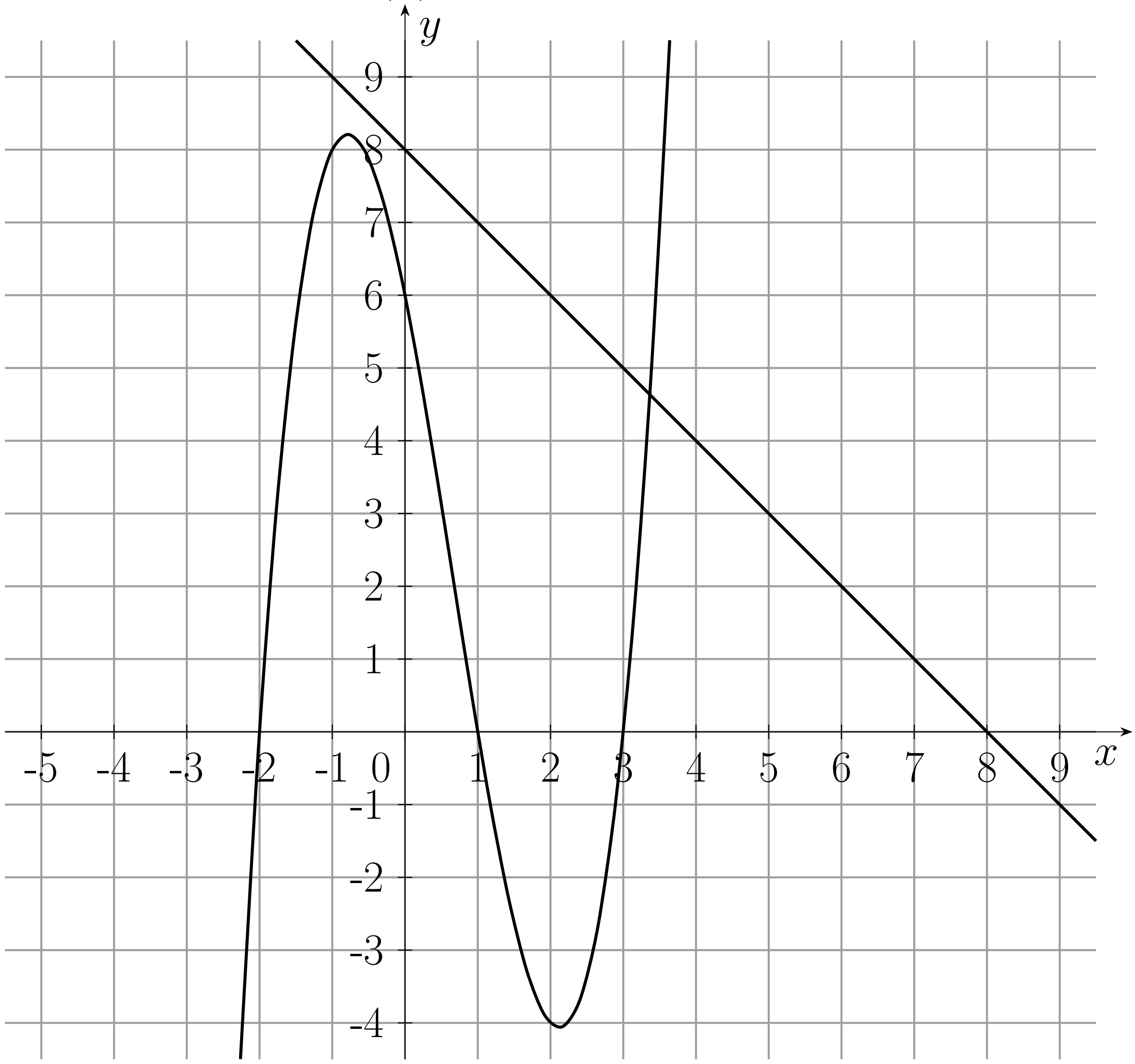
	X	Y
Lowest Score	$x-1$	$y+1$
Lower Quartile	$x+2$	$2(y+1)$
Median	x^2-3	$y(y-1)$
Upper Quartile	$4x+2$	$3y+1$
Highest Score	$2(x^2+2)$	$5y-4$

The median score for Class X was half the median score for Class Y.
The interquartile range for Class X was three times the interquartile range for Class Y.

Michael scored 17 marks in his maths exam.
Complete the following sentence;

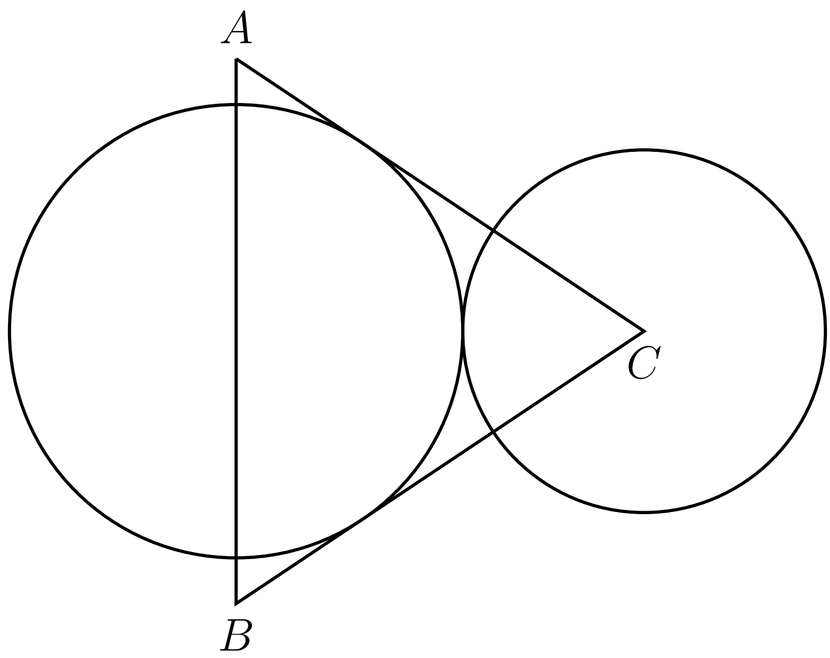
"Michael was in the top ___ % of performers in Class ___"

15. The diagram below shows parts of the cubic graph of $y = f(x)$ and the linear graph of $y = g(x)$.



Find the integer value of $gff(-2)$.

16. You may use a calculator. The diagram below shows two touching circles, Circle L and Circle R.



Circle L has radius 5cm. Circle R has radius 4cm.
C is the centre of Circle R.
The line AB lies on the diameter of Circle L.
The line AB is perpendicular to a line passing through the centre of both circles.
AC and BC are tangents to Circle L and pass through C.

Find the area of triangle ABC.
Give your answer to 3 significant figures.

17. You may use a calculator. The area of a parallelogram is $15cm^2$ correct to the nearest integer.
The shortest side of the parallelogram is $4.5cm$
correct to 2 significant figures.
The longest side of the parallelogram is $7.1cm$
correct to 2 significant figures.

Find the largest size of the two acute angles in the parallelogram.
Give your answer correct to 3 decimal places.

Part 3: The Exam

A great filter

1. Given that $x(a+bx)(a-bx) = 25x - 4x^3$, Solve for b^{-a}.

2. Freda plays the lottery.

   There are 49 balls to choose from.
   The balls are numbered 1 - 49.

   Freda chooses the following 6 numbers in order in which they appear:
   3, 4, 7, 12, 19, 28.
   John believes the numbers were chosen randomly.
   Is it possible that John is not correct? Why?

3. Triangle ABC has the following properties:

$AC = x$

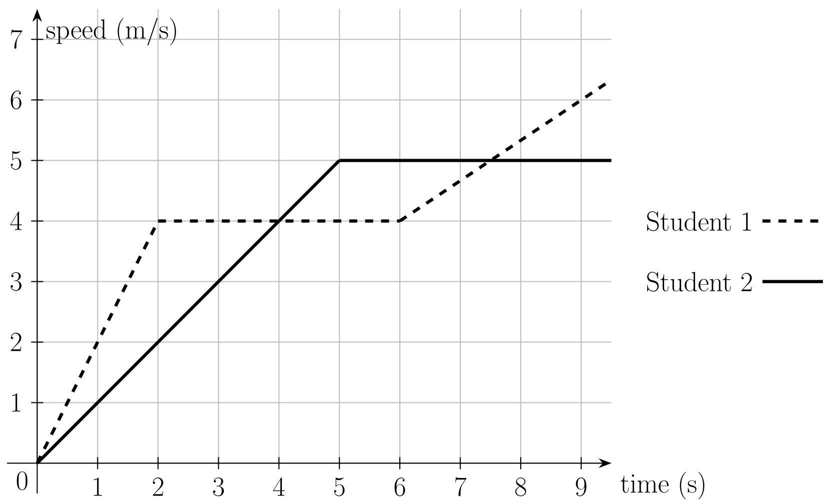
$BC = 3x$

Angle $ACB = 60°$

Write down the perimeter of triangle ABC in terms of x.

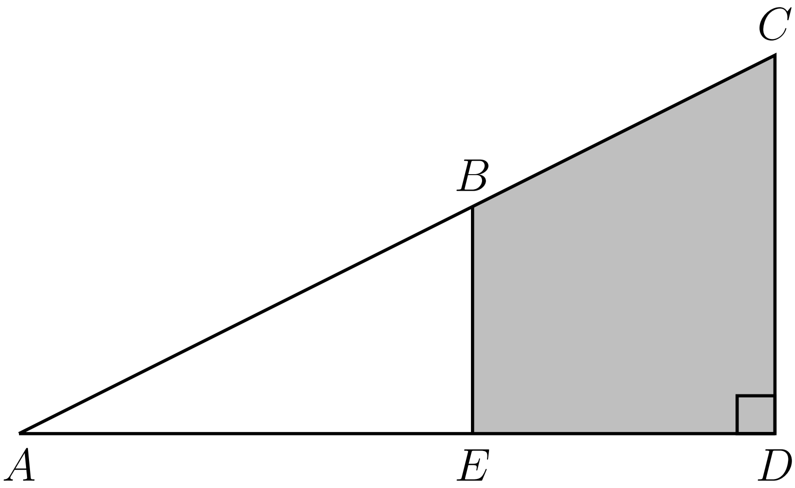
4. Find the value of $\left(\frac{1}{0.16}\right)^{1.5}$

5. Two students walk together along a road, starting at the same time. The speed-time graph below shows the first 9 seconds of the walk.



The ratio of the distance covered by Student 1 to the distance covered by Student 2 in the first 9 seconds of the walk can be written in the form $m : n$ where m and n are double digit integers.
Find the value of m and the value of n.

6. Triangle ACD is shown in the diagram below.

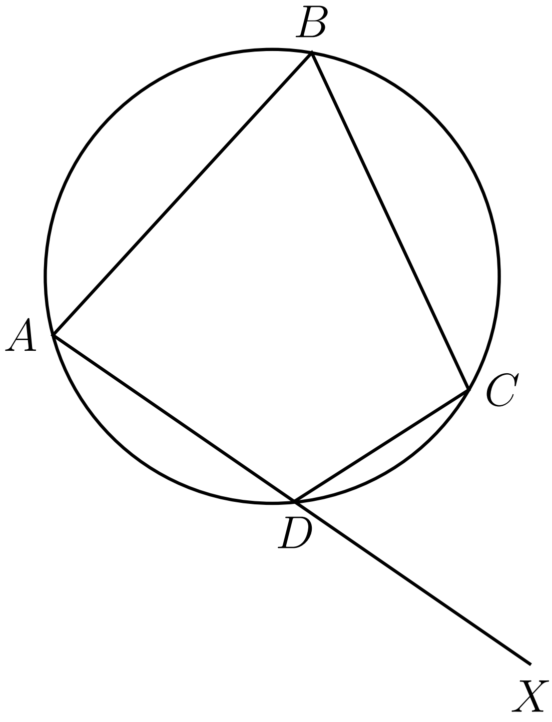


AED is a straight line
$AB = 3\sqrt{5}$
$AE = 2BE$
$3AD = 5AE$
BE and CD are parallel.

Find the area of the shaded quadrilateral $BCDE$.

7. Evaluate: $\left(\frac{\cos(60^\circ)}{\sin(60^\circ)} + \frac{10}{\sqrt{12}}\right)^2$

8. You may use a calculator.
A, B, C, and D are all points on the circumference of a circle as shown in the diagram below.



$\angle DAB = x^2 - 5x - 8$
$\angle BCD = x^2 + 4x - 88$
$\angle CDA = y^2 - 15y + 90$
$\angle ABC = 5y - 6$
$\angle CDX = x^2 - 70$

Prove that ADX is a straight line.

9. The first five terms of an arithmetic sequence are:

$$x + 1, \quad 2x, \quad \frac{2(2x+3)}{6-x}, \quad x^2 - 2, \quad 5x - 3$$

Show that the term $4x^2 - 3$ is not in the sequence.

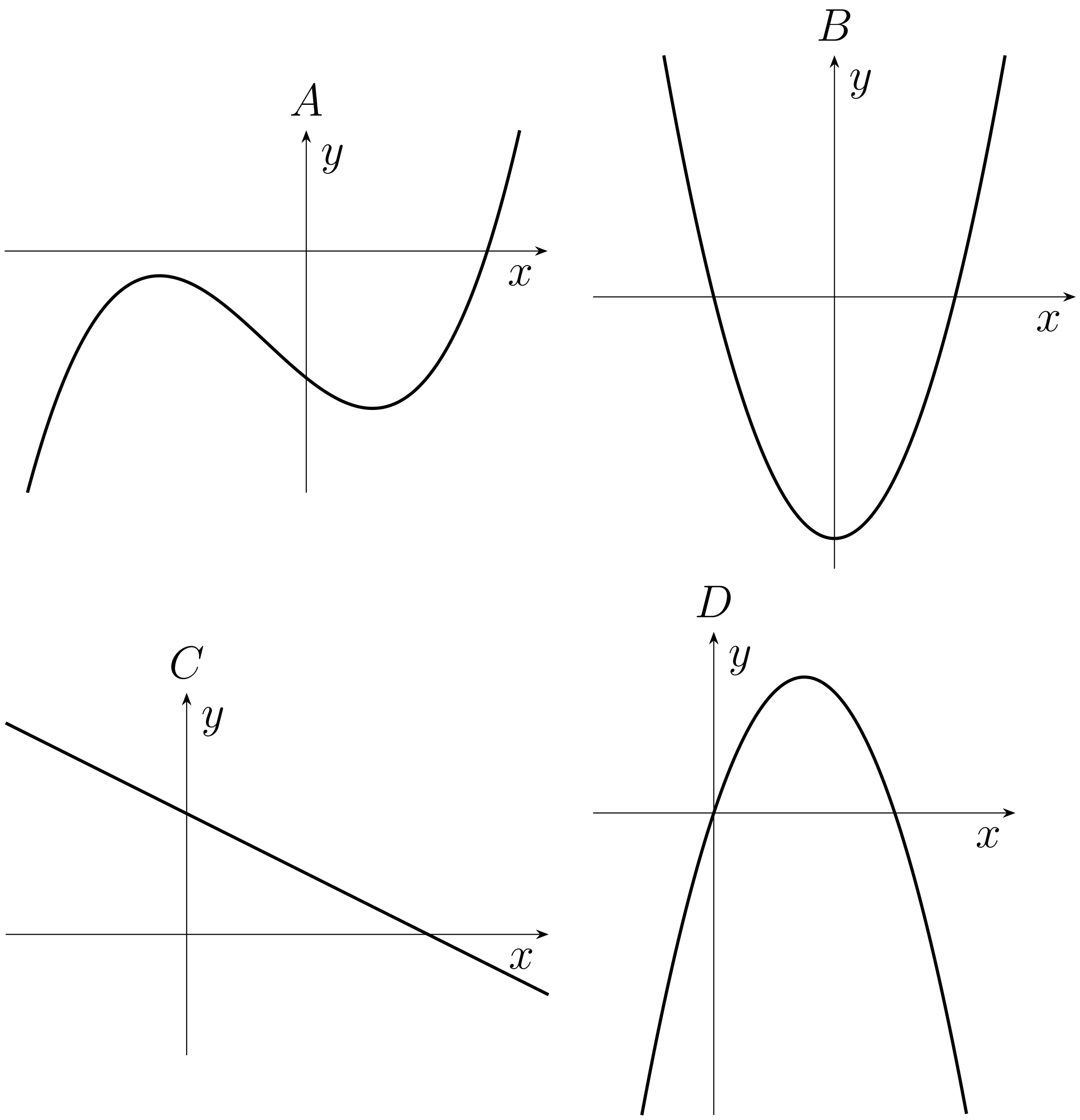
10. A solid hemisphere piece of gold with diameter 1.2×10^2cm is to be melted into identical solid cuboids.
The dimensions of the cuboids are 30cm, 12cm, and 10πcm.
Find the number of cuboids that can be made from the hemisphere.

11. Two functions are given below:

$$f(x) = (x + p)(x + q)$$

$$g(x) = \frac{r}{x}, \quad x \neq 0$$

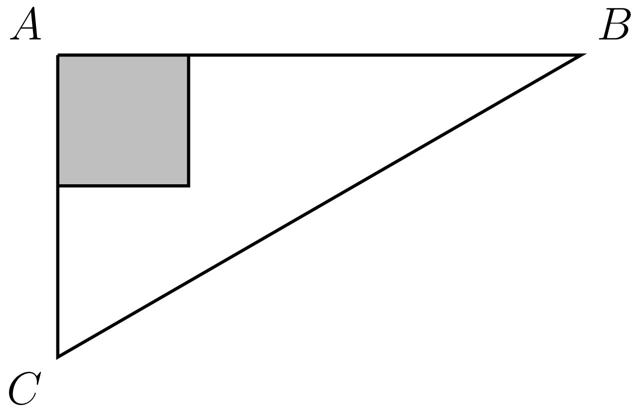
where p, q, and r are constants.
State which of the following graphs could be used to solve the equation $f(x) = g(x)$.



12. A is inversely proportional to $B^{\frac{1}{3}}$ and C is directly proportional to the square of B.
When $A = 0.5$, $B = 64$.
When $C = 15$, $B = 5$.
Express C in terms of A.

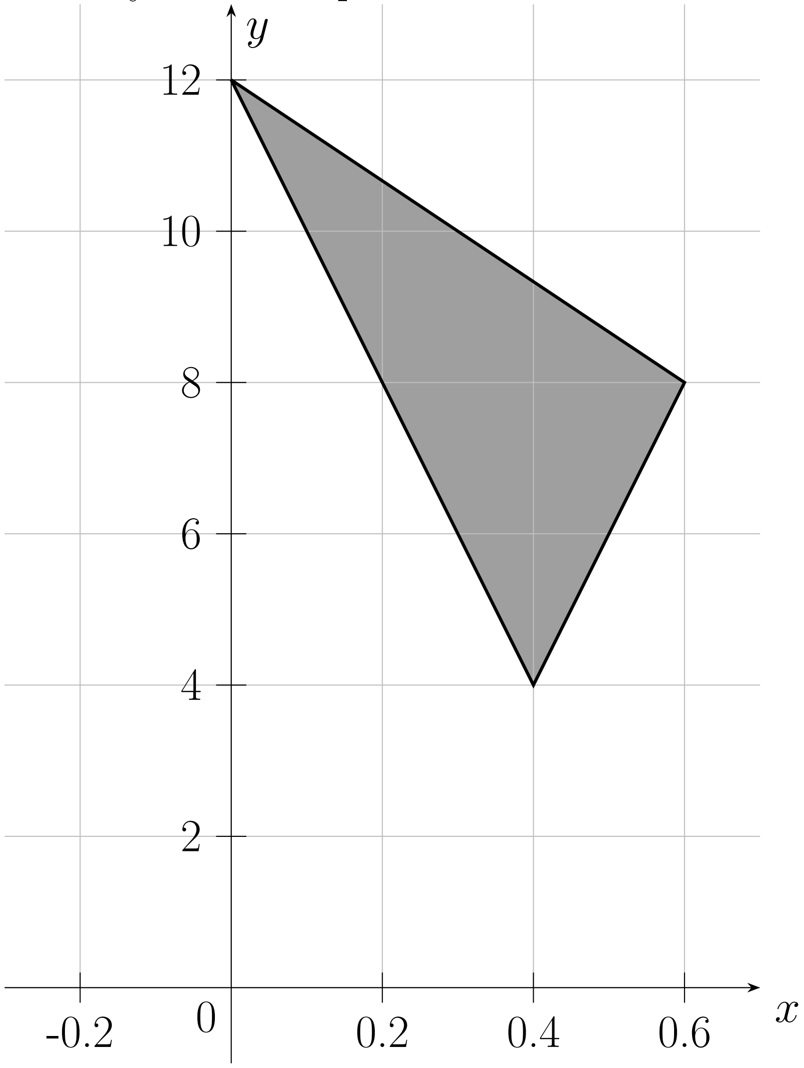
13. You may use a calculator. Mr Lucky plays two games.
Playing Game A and playing Game B are independent events.
The probability that Mr Lucky wins both games is $\frac{9}{25}$.
The probability that Mr Lucky wins Game B is four times greater than the probability of him losing Game A.
Find the probability that Mr Lucky wins only one of the two games he plays.

14.



Triangle ABC is half of an equilateral triangle, and $\angle BAC = 90°$.
The shaded square touches the lines AB and AC and the side length of the square is quarter that of AB.
Show that the ratio of the area of the triangle to the area of the square is $8 : \sqrt{3}$.

15. You may use a calculator. The shaded region in the diagram below is defined by three inequalities.



Find the three inequalities in the form $ay + bx \geq c$ where a, b, and c are integers.

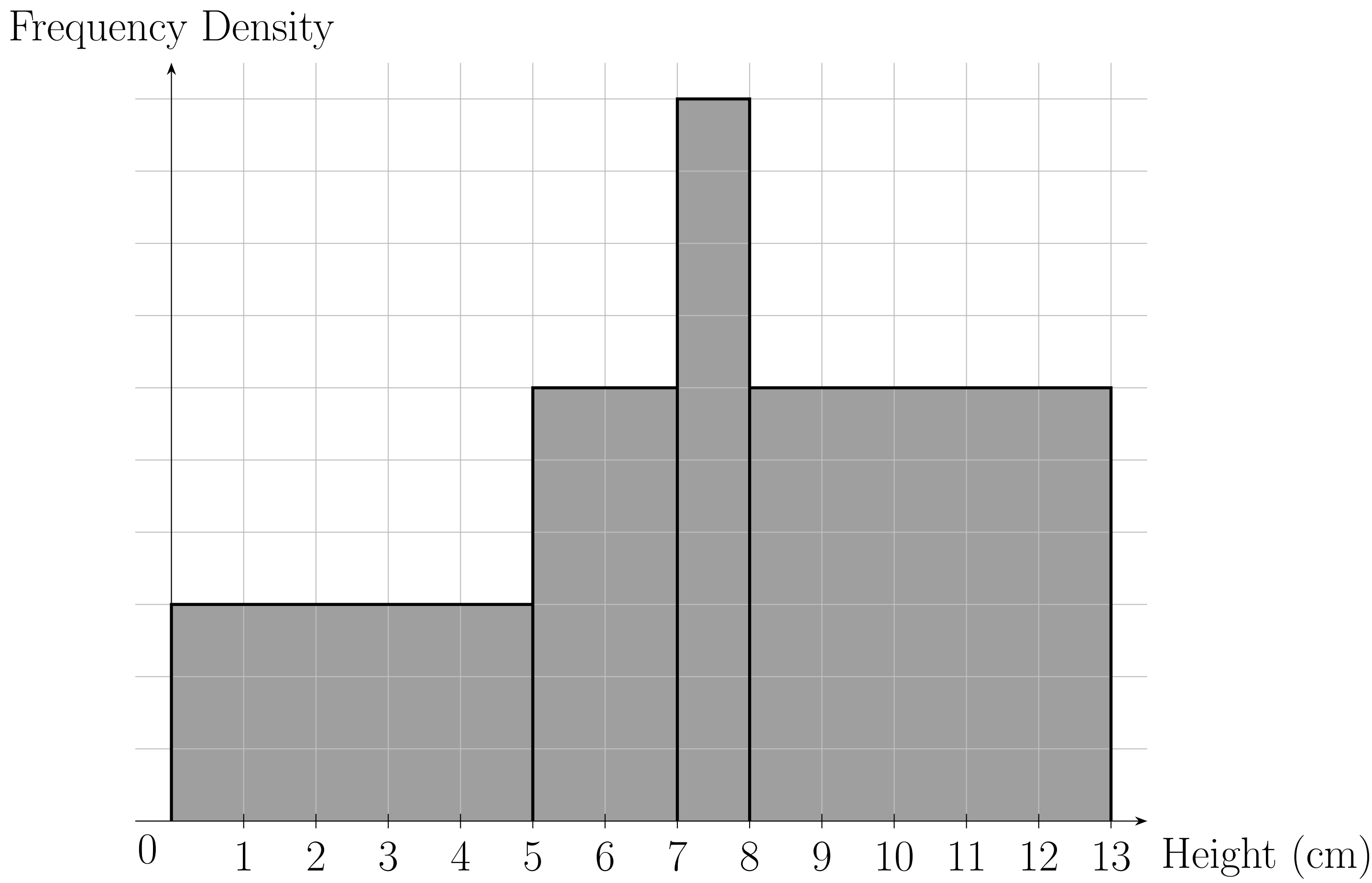
16. Two vectors are defined as follows:

$$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Find the value of $\cos(ACB)$ in its simplest form.

17. You may use a calculator. The histogram below shows information about the height (cm) of a number of plants.



There are 40 plants between 7 and 8cm tall.
Michael takes two plants at random from the sample and doesn't replace them.
He writes down his calculations for the probability and its answer as:

$$\frac{30}{67} \times \frac{16}{89} = \frac{480}{5963}.$$

Write down the minimum height of each of the plants Michael chooses.

Part 4: The Challenge

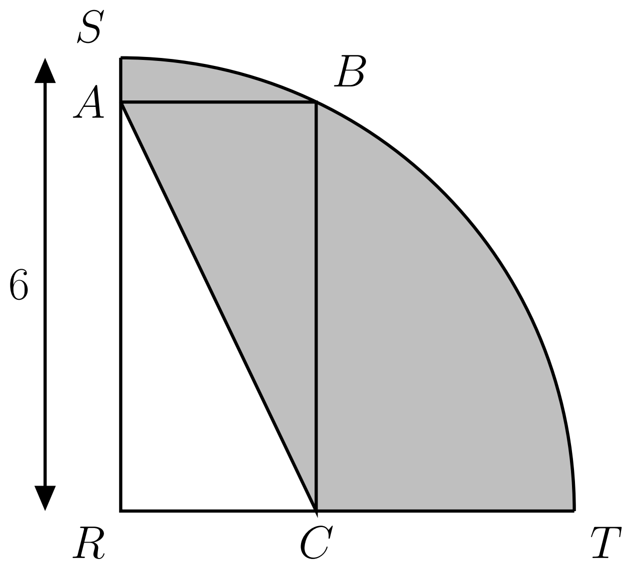
An exercise to the reader

1. If $(ax + 2)(bx + 7) = 15x^2 + cx + 14$ for all values of x, and $a + b = 8$, what are the two possible values for c?

2. The numbers a, b, and c are positive integers.
An apple costs £a, a banana costs £b, and a cherry costs £c.
The costs of b apples, b bananas. and $a + b$ cherries is £77.
What would the cost be for one apple, two bananas, and one cherry?

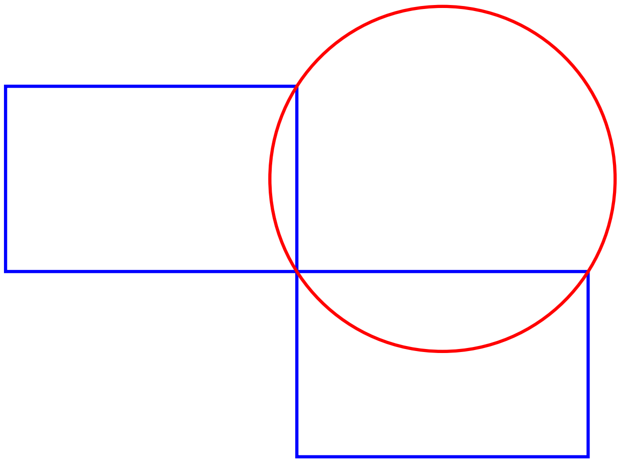
3.

In the figure, arc SBT is one quarter of a circle with centre R and radius 6. If the length plus width of rectangle $ABCR$ is 8, then what is the perimeter of the shaded region?

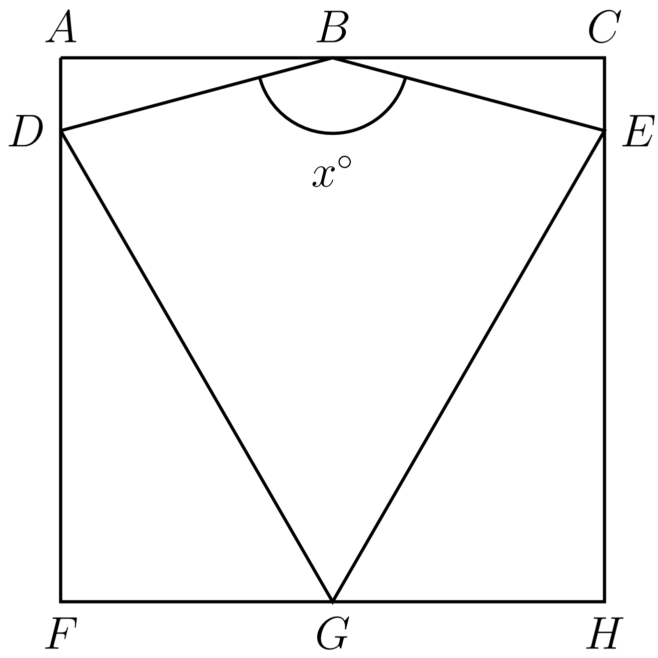


4.

You may use a calculator.
Two congruent rectangles are shown.
The are of the rectangle is 1260.
The perimeter of the rectangle is 146.
Find the circumference of the circle.



5.



The kite $GDBE$ is placed in the square $ACHF$.
$DG = GB = EG$.

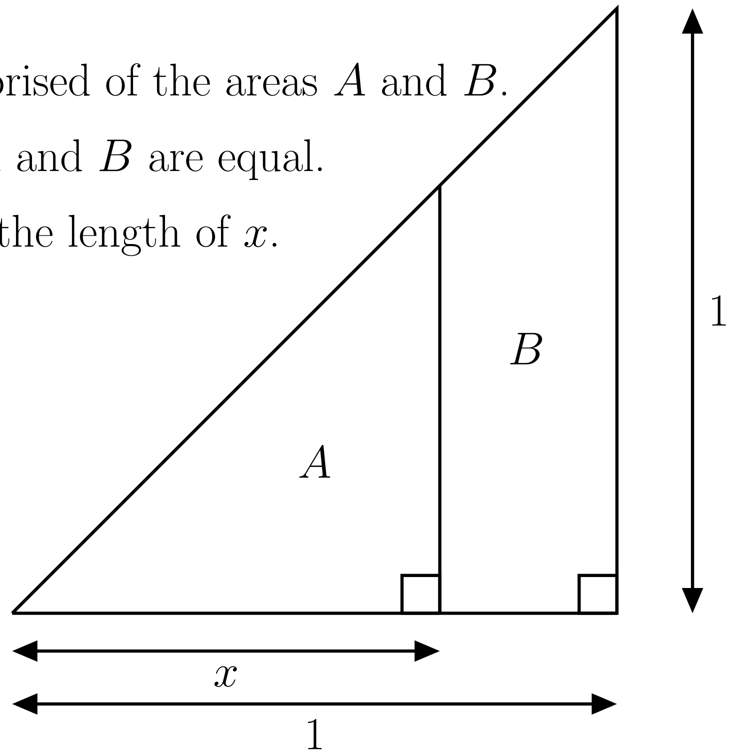
Calculate the size of x, the angle DBE.

6.

A triangle is comprised of the areas A and B.

Areas A and B are equal.

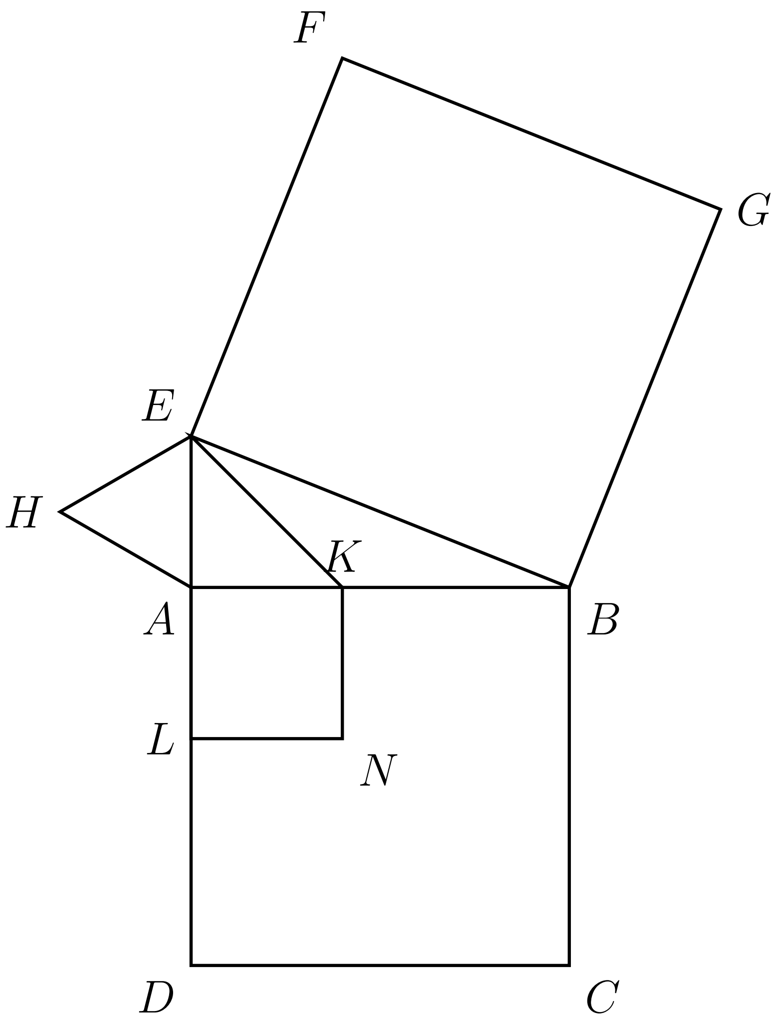
Find the length of x.



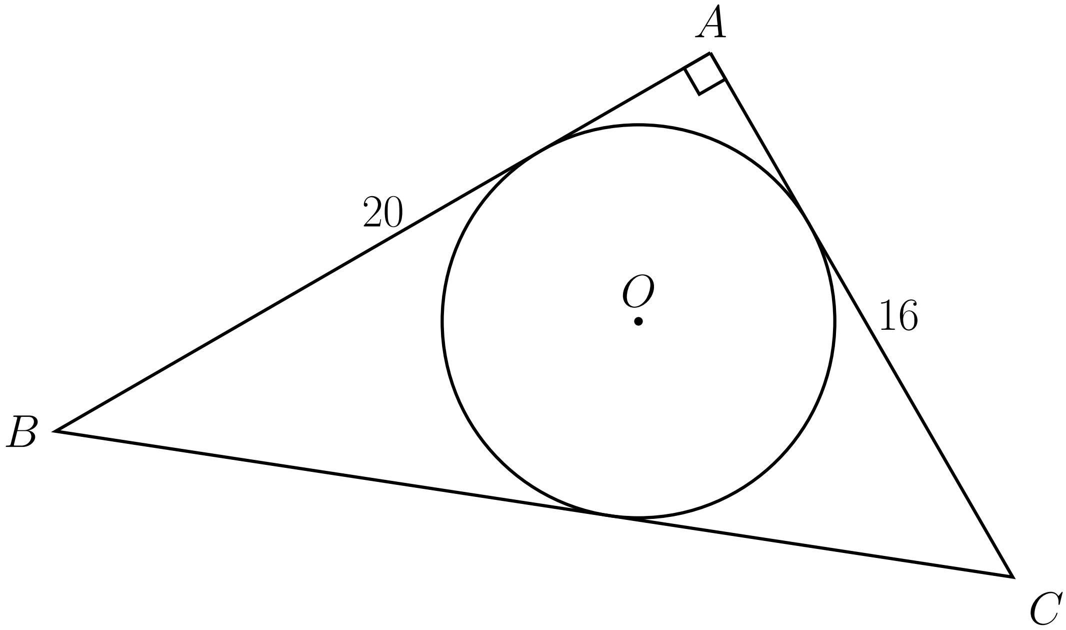
7\.

The diagram is not drawn accurately.
$ABCD$, $EFGB$, and $ALNK$ are squares,
and HEA is an equilateral triangle.
Find the length of EK.

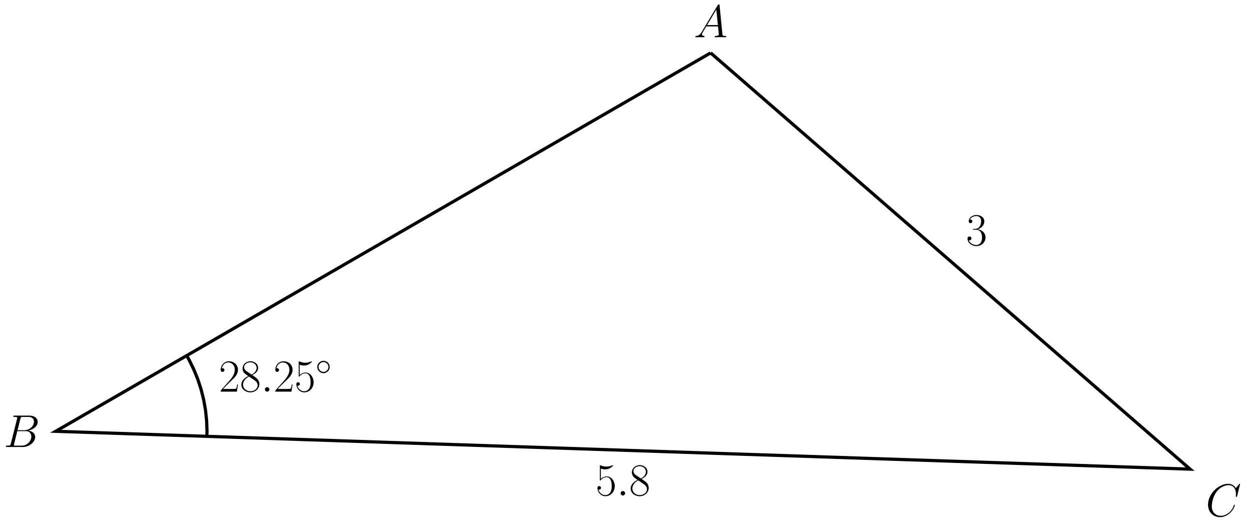
Area of $EFGB$ is 36cm^2.
Area of HEA is $27\sqrt{\frac{3}{16}}\text{cm}^2$.
Area of $BCDLNK$ is 25cm^2.



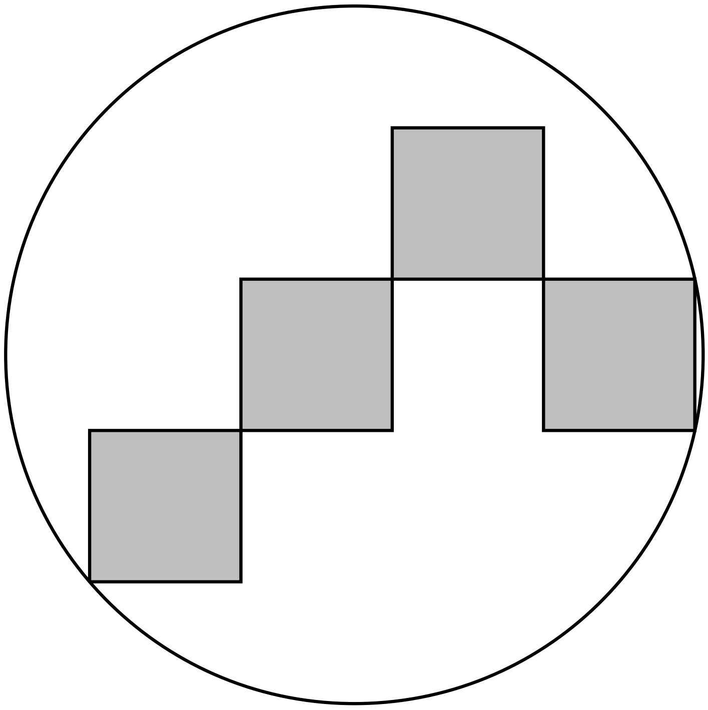
8. Find the length of the radius of the circle.



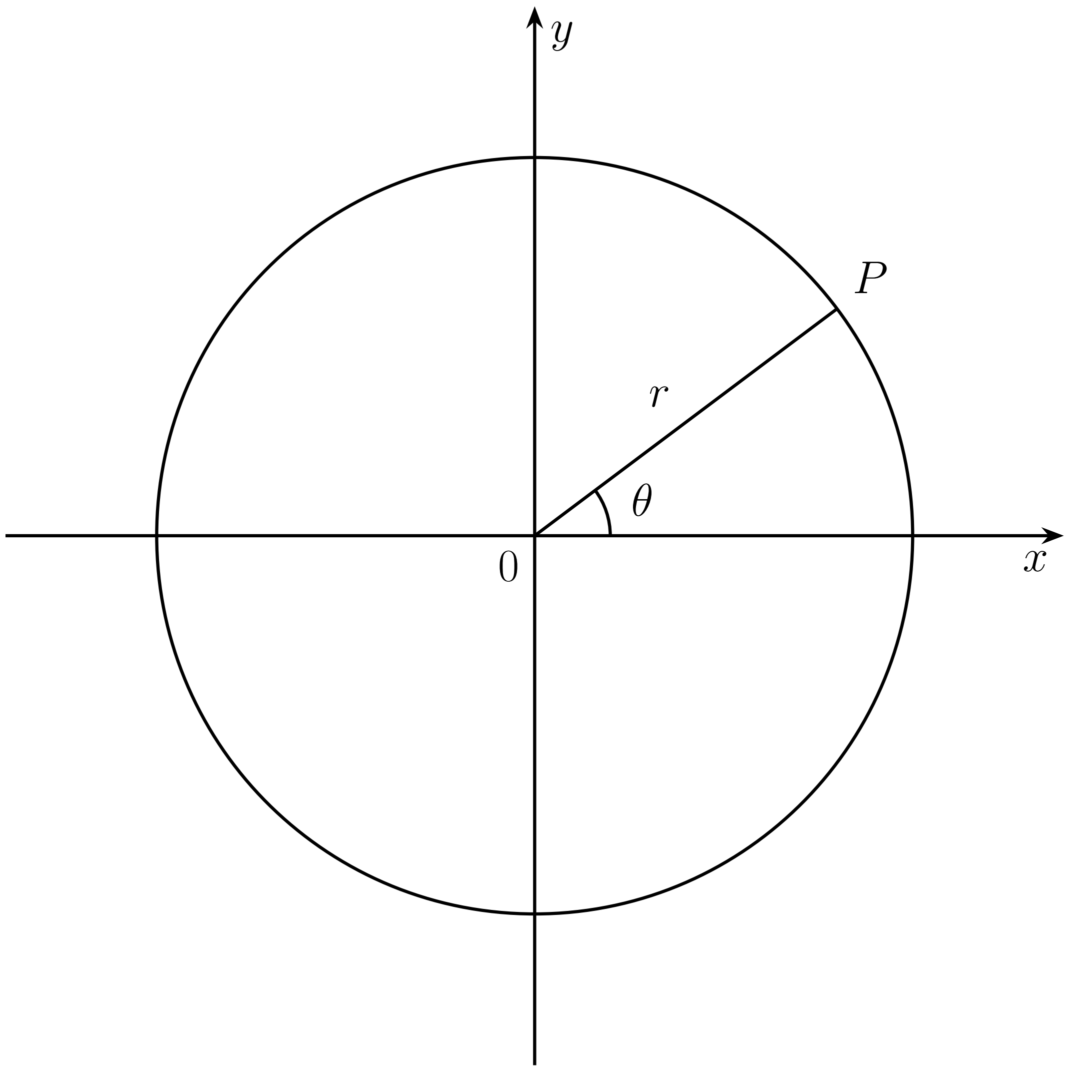
9. Find the size of the obtuse angle at A.



10. Find the area of the circle. Each square has an area of 16.



11. Find the x and y coordinates of point P in terms of radius r and angle θ.

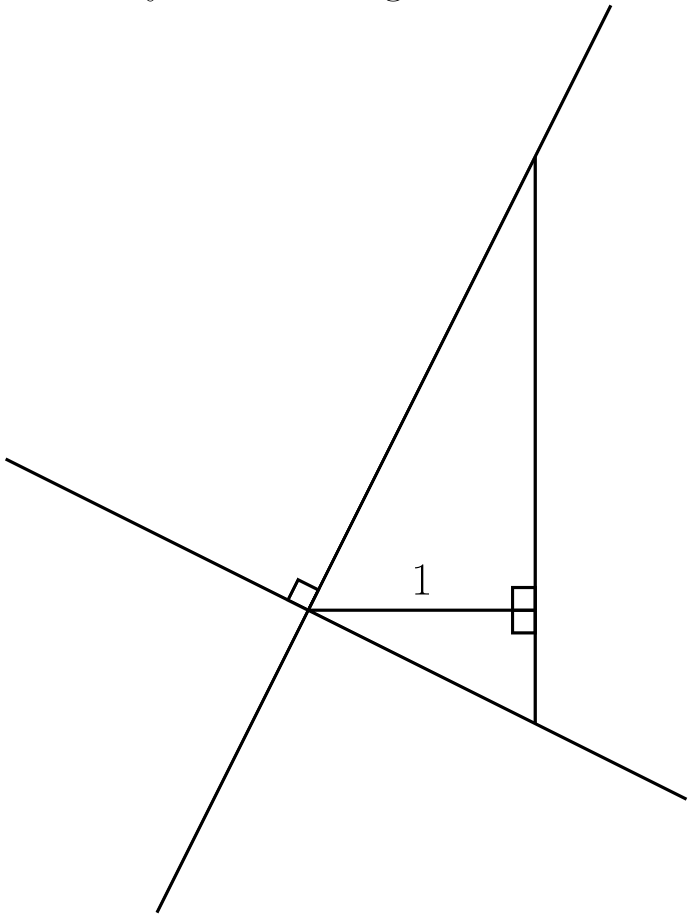


12.

$$\text{Prove that} \qquad \left(x+\sqrt{x^2-1}\right)^2 = \frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} \qquad .$$

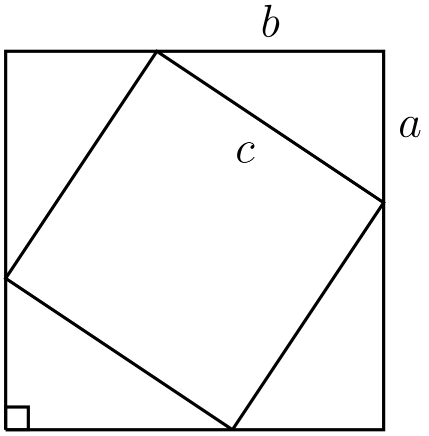
13. Prove the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

14. Show the relation between the gradients of 2 perpendicular lines.
You may use the diagram below.



15. Prove the Pythagorean Theorem, $a^2 + b^2 = c^2$.

You may use the diagram below, which displays a shape with rotational symmetry of order 4.



16. Prove that $\sqrt{2}$ is irrational.

17. Prove that the shape traced out by a quadratic, the parabola, is always symmetrical around a certain axis.
Hint: You may assume that a function $f(x)$ is symmetrical around the $x = 0$ axis when $f(x) = f(-x)$.

18. You may use a calculator.

Elliott the alien is brewing themselves a cup of coffee. To brew the perfect coffee, they follow this simple formula:

$$\xi = \frac{24\zeta - 4\zeta^2 - 27}{(-2\Xi + 3)(\Xi - 2)}$$

where ξ is the perfection level of the coffee, ζ is the amount of Florp, and Ξ is how much sugar is in the coffee (the units are alien, so you don't need to worry about them).

Elliott measures out each of the 2 ingredients to get

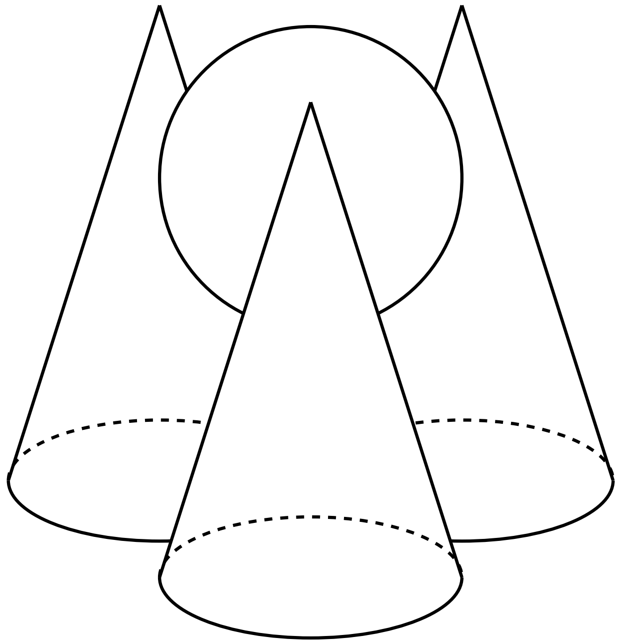
$\zeta = 2.8$ to the nearest 0.7,

and $\Xi = 1.95$ to the nearest 0.05.

Calculate the maximum value for how perfect Elliott's coffee is.

19. You may use a calculator.

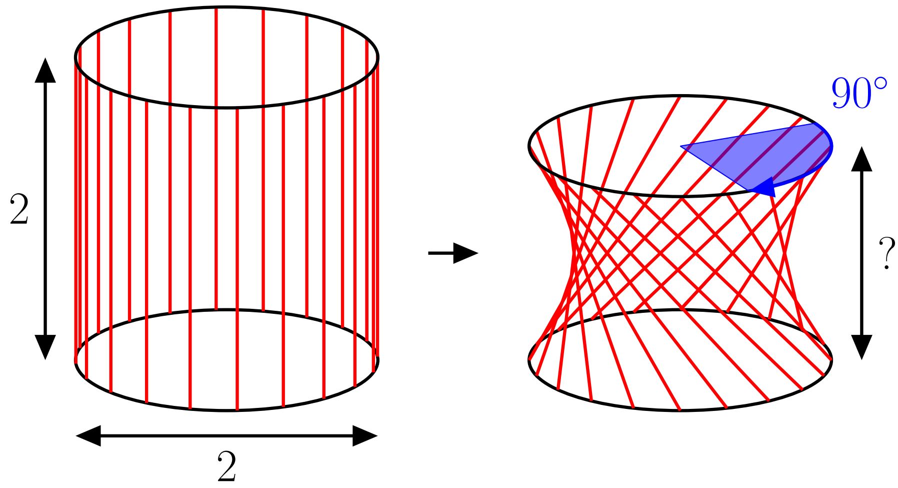
Three identical cones each have a radius of 50 and a height of 120. The cones are placed such that their circular bases touch one another. Then a sphere is placed such that it rests in the space created by the three cones, as shown.



If the top of the sphere is the same height as the top of the cones, what is the radius of the sphere to three decimal places?

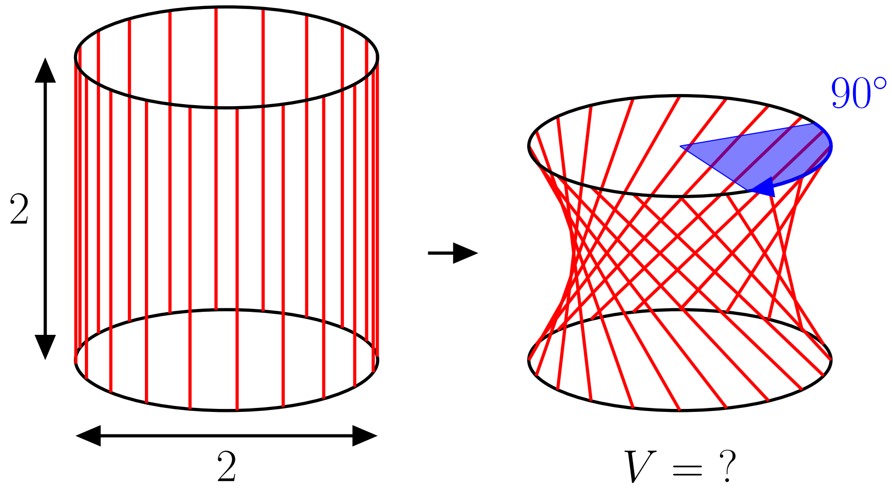
20. Two unit circles are joined by many equally spaced strings of length 2 to form a cylinder. The upper circle is rotated by 90° while the lower is fixed. Naturally, the two circles come closer together.

What is the new distance between them?



21. You may use any tool and any means, good luck.
Two unit circles are joined by many equally spaced strings of length 2 to form a cylinder. The upper circle is rotated by 90° while the lower is fixed. Naturally, the two circles come closer together.

What is the volume inside this 3D figure?



The Sources
(that I could find)

All of Part 1 comes from the Edexcel GCSE mathematics A* paper (not for the faint of heart): https://pbs.twimg.com/media/DBFJheDXYAAnLxO?format=jpg&name=4096x4096

All of Part 2 comes from the GCSE 9-1 mathematics Higher Tier grade 9 "tough paper" Paper 2: https://m4ths.com/uploads/3/5/2/1/35219558/new_9_to_1_grade_9_paper_2_calc.pdf

All of Part 3 comes from the GCSE 9-1 mathematics Higher Tier grade 9 "tough paper" Paper 1: https://m4ths.com/uploads/3/5/2/1/35219558/357.pdf

Part 4 question 1 comes from Part 3 question 15 of the SAT practice test 1: https://cdn2.hubspot.net/hubfs/360031/PrepScholar-sat-practice-t
pdf

Part 4 question 2 comes from an early round of the norwegian mathematical olympiad, detailed in a Mind your decisions video: https://www.youtube.com/watch?v=yBW-saaH-PQ

Part 4 question 3 comes from the official SAT study guide, detailed in a Mind your decisions video: https://www.youtube.com/watch?v=Cy0hrcZsXmg&t=19s

Part 4 question 4 does not have a known source, please let me know if you know the origins of this problem.

Part 4 question 5 comes from the NZQA Level 1 mathematics and statistics Exam 2017, detailed in a Mind your decisions video: https://www.youtube.com/watch?v=z3zjyCZFzDo

Part 4 question 7 is a modified version of a problem I can't find the source of, but the original problem was not solvable (I added the equilateral triangle and its area)

Part 4 question 8 comes from the CCEA A-level Maths past papers June 2017 Module C2:AS Core Mathematics 2 AMC21: https://revisionmaths.com/level-maths/level-maths-past-papers/ccea-level-maths-past-pa

Part 4 question 9 was a question I argued with my teacher about when I was still doing my GCSEs, if you have seen this problem before, please tell me its origin.

Part 4 question 10 comes from an Andy maths video: https://www.youtube.com/watch?v=ROK__9RcF78

Part 4 question 19, question 20, and question 21 all come from Brilliant daily problems, which have been discontinued

All questions not mentioned in the list above come from my strange mind.