Neuroprothetik Exercise 3
Mathematical Basics 2

Laura Bielenberg

22. Mai 2019

1 Solver Implementation

Implement the following numerical differential equation solvers as functions in Python or Matlab:

- Forward (Explicit) Euler
- Heun
- Exponential Euler

All three solvers have been implemented in Python. Their implementation is given in the file `solver.py`. To generate the plots execute `plot_all.py`.

2 Solve Functions

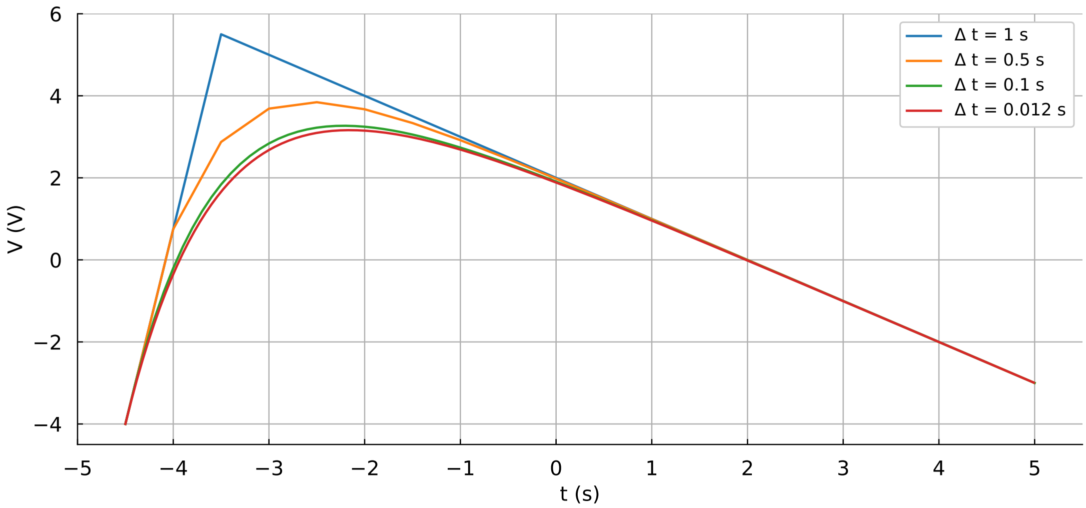
Solve the differential equation

$$\frac{dV}{dt} = 1 - V - t \tag{1}$$

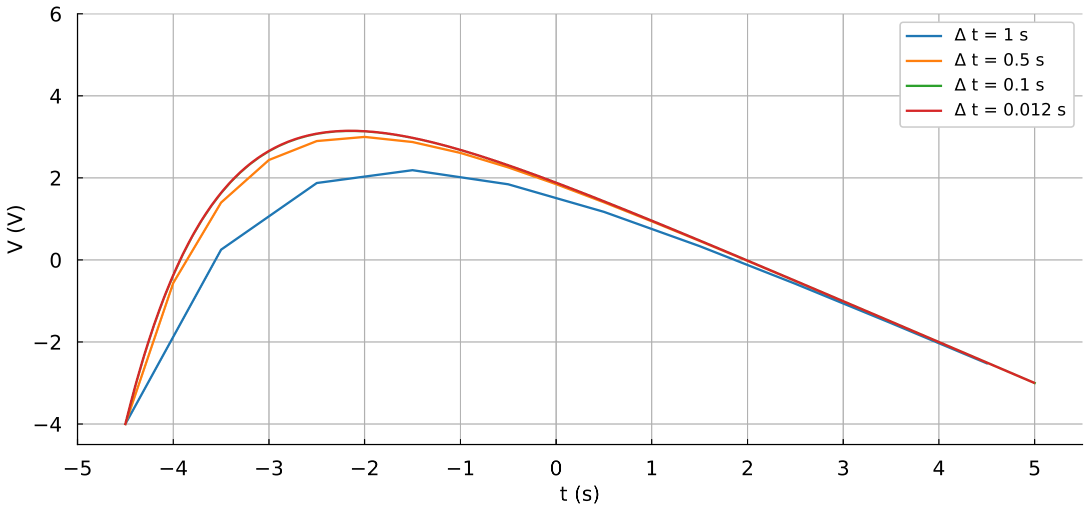
where $V(t = -4.5) = V_0 = -4$ with the solvers implemented above. Vary the stepsize (1s, 0.5s, 0.1s, 0.012s), plot the results and answer the following tasks:

1. *Interpret the impact of changing the stepsize:*
   In figures 1a to 1c we can see, that by reducing the stepsize all plots reach similar approximations.
   Especially when using a first order method, such as the Explicit-Euler, it can be seen that the stepsize has an nonneglectable impact on the approximation error. More precisely, the local error for the Explicit-Euler is proportional to Δt^2 and the global error is proporional to the Δt, meaning that by reducing the stepsize to a half the error is halfed, too. The other two methods, Heun and Exponential-Euler show similar step-size dependent global error behavior, where bisection of the stepsize leads to quartering of the global error for the Heun Method and halve the global error for the Exponential-Euler.

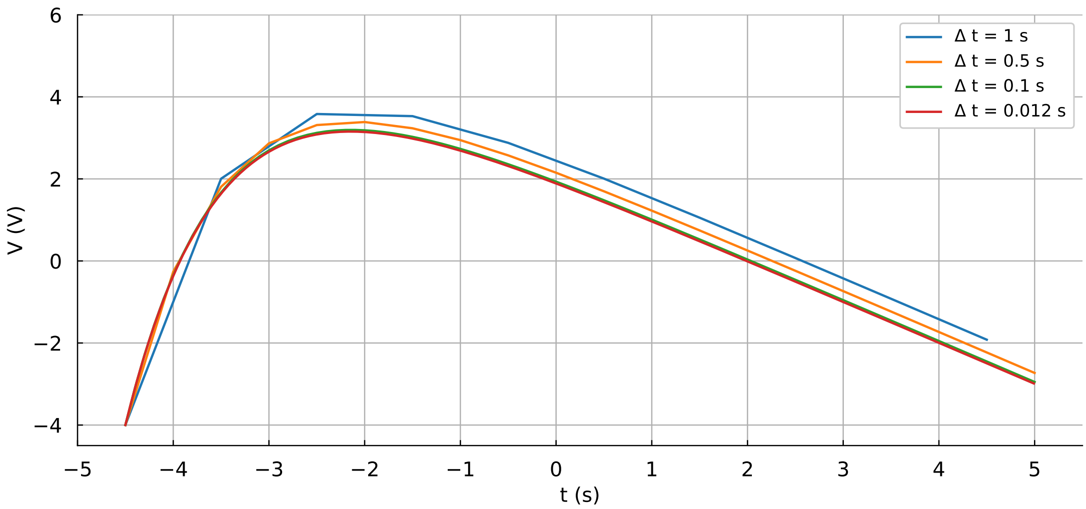
2. *Why not use an infinitesimal stepsize?*
   Using an infinitesimal stepsize would increase the computation time and effort considerably. Suitable approximations can already be obtained using suffitiantly small step-sizes.



(a)



(b)



(c)

Figure 1: Approximation of the differential equation given in 2 using different solvers and stepsizes. Figure 1a: Forward-Euler Method, figure 1b: Heun-Method, figure 1c: Exponential-Euler Method.

3 The Leaky Integrate and Fire Neuron

Implement a Leaky Integrate and Fire neuron model according to the following equation:

$$V_{n+1} = \begin{cases} V_n + \frac{\Delta t}{C_m}(-g_{leak}(V_n - V_{rest}) + I_{input}(t_n) & V_n < V_{thr} \\ V_{spike} & V_{thr} \leq V_n < V_{spike} \\ V_{rest} & V_{spike} \leq V_n \end{cases} \tag{2}$$

with

- V_m : cell membrane voltage
- $C_m = 1\,\mu\text{A}$: membrane capacity
- $g_{leak} = 100\,\mu\text{S}$: leak conductivity
- $V_{rest} = -60\,\text{mV}$: cell membrane resting voltage
- $V_{thr} = -20\,\text{mV}$: cell membrane spiking threshold voltage
- $V_{spike} = 20\,\text{mV}$: spiking voltage

And simulate V_n for $50\,\mu\text{s}$ ($\Delta t = 25\,\mu\text{s}$) with I_{input} being

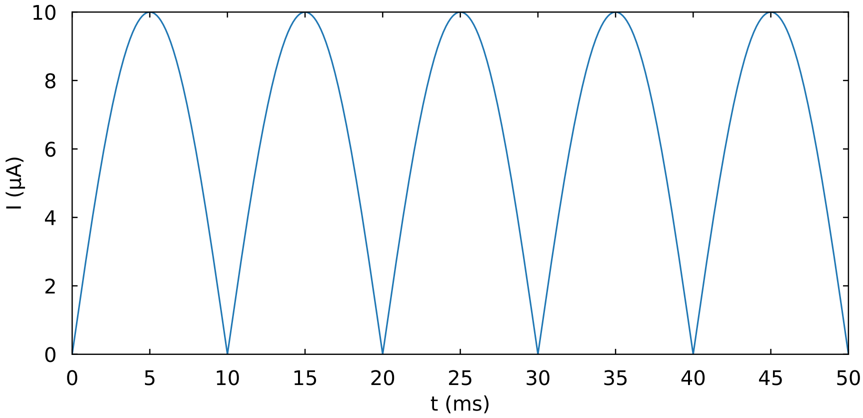
- constant $10\,\mu\text{A}$
- constant $20\,\mu\text{A}$
- rectified 50Hz sine with $10\,\mu\text{A}$ amplitude
- rectified 50Hz sine with $30\,\mu\text{A}$ amplitude

Plot and interpret the results.

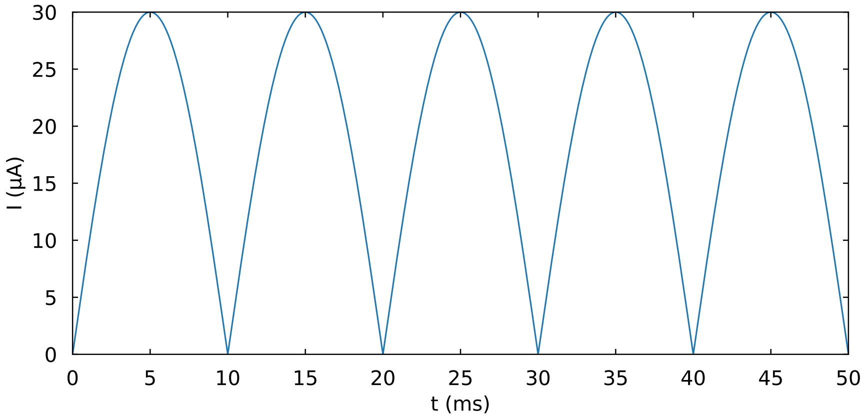
Interpretation:
In the case of a constant stimulation current I_{stim}, as given in figures 3a and 3b, it can be seen that the cell membrane voltage increases exponentially, until it hits the threshold V_{thr} at $-20\,\text{mV}$. At that point an action potential is generated where the voltage reaches V_{spike} and is then again reset to the initial resting potential. When comparing 3a and 3b we see that, by increasing I_{stim}, V_{stim} is reached earlier and thus, since there is no refractory period set for this LIF Neuron model to limit the neuronal firing rate, action potentials are being generated at a higher frequency.

When instead setting I_{stim} to a rectified 50 current, as given in figures 3c and 3d, we can see that the voltage increase is not anymore exponential, but depends on the value of I_{stim} at that point of time. Whenever I_{stim} reaches zero we can observe a small decline in the otherwise rising slope, probably caused by the leak conductivity. Apart from that, the action potential generation shows the same behaviour as when using a

constant current:
V_m rises $\rightarrow$ V_m hits V_{thr} $\rightarrow$ actionpotential is generated $\rightarrow$ reset to initial resting potential.

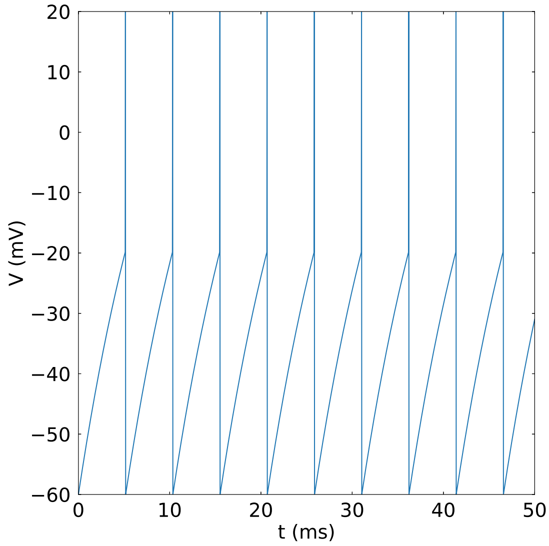


(a) Rectified Sine-Input for the LIF model with an amplitude of 10 µA.

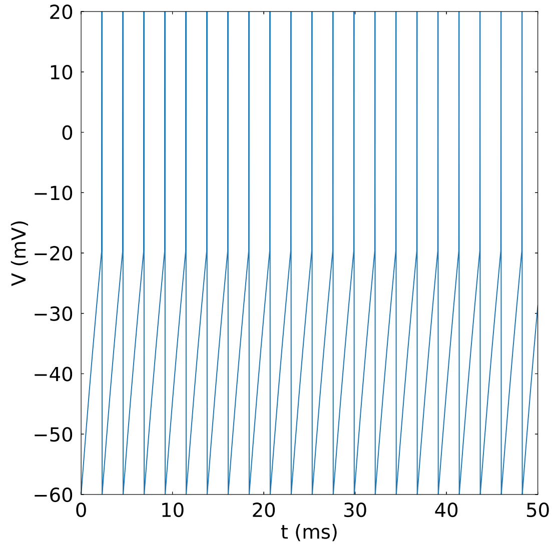


(b) Rectified Sine-Input for the LIF model with an amplitude of 30 µA.

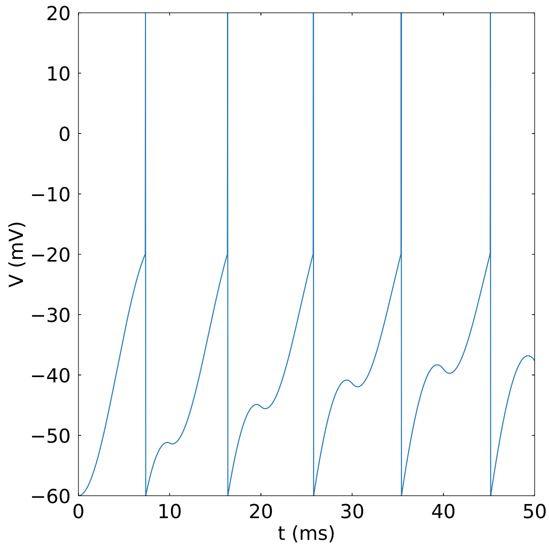
Figure 2: Current inputs for the LIF-Model, outputs visible in figures 2a and 2b .



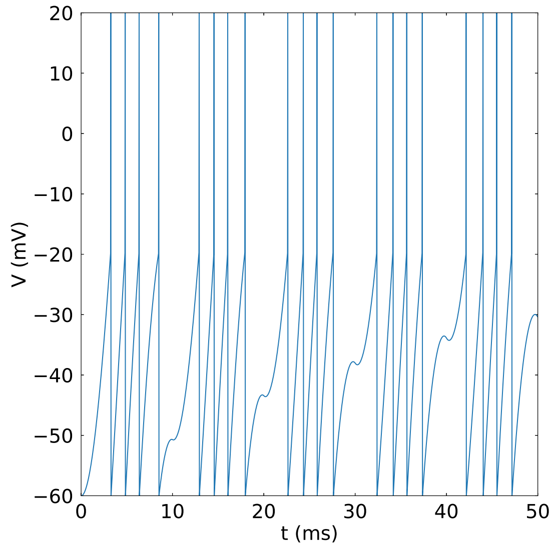
(a) 10 µA constant input current.



(b) 20 µA constant input current.



(c) Rectified input with 50 Hz and 10 µA amplitude.



(d) Rectified input with 50 Hz and 30 µA amplitude.

Figure 3: Cell membrane voltage of a LIF-Model using different current inputs.