An Alternative Method for Basic Feasible Solution of Transportation Problem

A project submitted for partial fulfillment of the requirements for the degree of Bachelor of Science in Applied Mathematics

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This work is dedicated to all of my beloved teachers.

Certificate

This is to certify that the project work entitled **“An Alternative Method for Basic Feasible Solution of Transportation Problem”** is an original project work carried out by **Riaz Mahmud** (Roll: ASH1506008M). The author has worked under my supervision for the partial fulfillment of the requirements for the degree of Bachelor of Science (Honours) in Applied Mathematics under the department of Applied Mathematics, Noakhali Science and Technology University, Sonapur-3814, Bangladesh.

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Declaration

I hereby declare that the project work carried out by me Riaz Mahmud and submitted to the Department of Applied Mathematics, Noakhali Science and Technology University, Sonapur-3814, Bangladesh for the partial fulfillment of the requirements for the degree of Bachelor of Science (Honours). The work incorporated in the present project is original except where due reference is made and has not been submitted to any University/Institution for the award of the degree.

Date: ……………

Riaz Mahmud

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ABSTRACT

This study deals with the optimality of transportation problem in LPP. All the standard existing methods for optimality of TP has been discussed here. A new method is proposed here which is more efficient than the Vogel's Approximation Method (VAM). Some advantages and limitations of the proposed method have been discussed. A C programming code for VAM and a Python code for new proposed method have been added in the appendix.

Keywords: LPP, TP, VAM, Optimization.

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List of Abbreviations

TP	Transportation Problem
LP	Linear Programming
FS	Feasible Solution
BFS	Basic Feasible Solution
IBFS	Initial Basic Feasible Solution
DBFS	Degenerate Basic Feasible Solution
NCM	North-west Corner Method
RMM	Row Minima Method
CMM	Column Minima Method
LCM	Least Cost Method
VAM	Vogel's Approximation Method

1. Introduction

Chapter One

INTRODUCTION

1.1 Introduction

In this era of competitive market, the companies need to find better ways to deliver the goods to the consumers efficiently at a minimum cost. The transportation problem gives a powerful framework to meet this challenge.

The transportation problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses or consumers (called demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales.

The transportation problem has an application in industry, communication network, planning, scheduling transportation and allotment etc.

1.2 A Brief History of Transportation Problem

In 1781 French mathematician Gaspard Monge first formulated a transportation problem by geometric means. In the 1920s A.N. Tolstoi was one of the first to study the transportation problem mathematically. In 1930, he published a paper named *'Methods of Finding the Minimal Kilometrage in Cargo-transportation in space'* in a book on transportation planning issued by the National Commissariat of Transportation of the Soviet Union. In this paper, he studied the transportation problem and described a number of solution approaches, including the, now well-known, idea that an optimum solution does not have any negative-cost.

Figure 1.2: Frank Lauren Hitchcock (1875-1957)

The modern era of transportation problem began in 1940s. American mathematician Frank Lauren Hitchcock first developed the basic transportation problem in 1941. He is considered as the pioneer of the modern transportation problem.

However, it could be solved for optimally as answers to complex business problem only in 1951, when George B. Dantizig applied the concept of linear programming in solving the transportation model. Dantzing (1951) gave the standard linear programming formulation transportation problem and applied the simplex method to solve it.

Since then the transportation problem has become the classical common subject in almost every textbook on operation research and mathematical programming.

1.3 Objectives of The Study

Let us consider a simple transportation problem as a product of a company to be transported from some factories to the consumers. The objective of the study is to minimize the transportation cost from each factory to each consumer and hence satisfy the demand.

2. Fundamentals of Transportation Problem

Chapter Two

Preliminaries and Fundamentals of Transportation Problem

2.1 Transportation Problem

Transportation problem is a particular class of linear programming, which associates with our day to day activities and it mainly deals with logistics. Transportation problem helps to distribute a product from a number of sources or origins to a number of destinations. The objective is to satisfy the demands at destinations from the supply constraints with minimum transportation cost possible.

2.2 Network Representation of Transportation Problem

A simple network diagram of transportation problem is illustrated in the following figure.

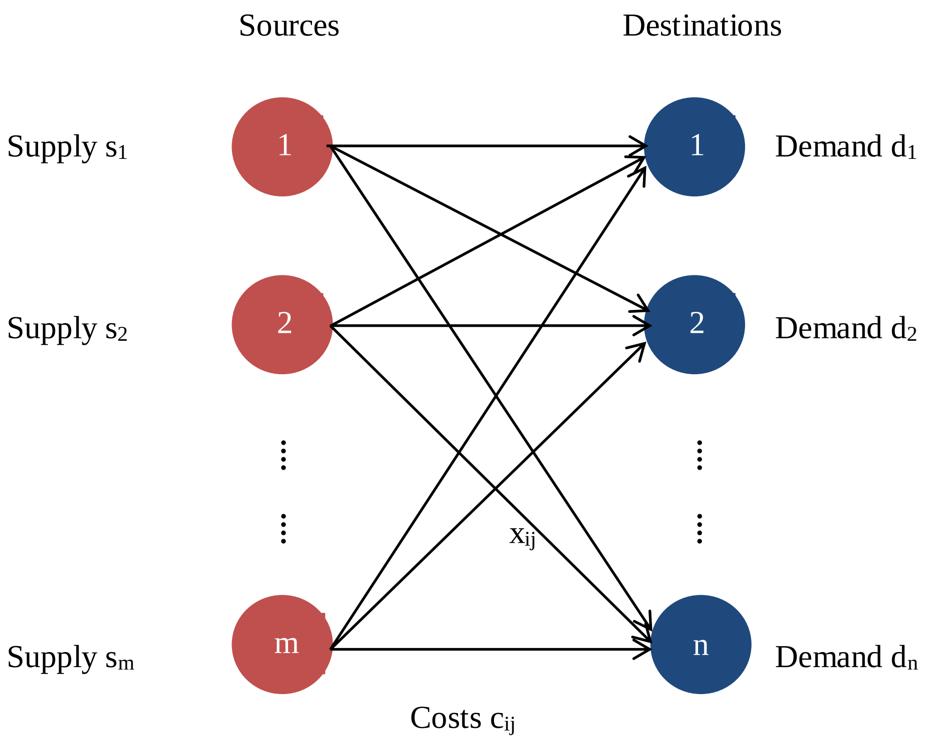

Figure 2.2: Network Representation of Transportation Problem

2.3 Classifications of Transportation Problem

Balanced Transportation Problem: A Transportation Problem is said to be balanced Transportation Problem if total number of supply is same as total number of demand.

Unbalanced Transportation Problem: A Transportation Problem is said to be unbalanced Transportation Problem if total number of supply is not same as total number of demand.

In this project work, we only considered the balanced transportation problem

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2.4 Tabular Representation of Transportation Problem

A balanced transportation problem having m sources of supply $s_1, s_2, \dots \dots, s_m$ with

$a_i(i = 1,2, \dots, m)$ unit of supplies and n destinations $d_1, d_2, \dots \dots, d_n$with $b_j(j = 1,2, \dots, n)$ unit of requirements can be represented in a table as follows.

To From	d_1	d_2	……	d_n	Supply (a_i)
s_1	c_{11}	c_{12}	……	c_{1n}	a_1
s_2	c_{21}	c_{22}	……	c_{2n}	a_2
⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮	⋮ ⋮
s_m	c_{m1}	c_{m2}	……	c_{mn}	a_m
Demand (b_j)	b_1	b_2	……	b_n	$\sum a_i = \sum b_j$

Table 2.4: Tabular representation of transportation problem

2.5 Mathematical Formulations of Transportation Problem

Mathematically a transportation problem is nothing but a special linear programming problem in which the objective function is to minimize the cost of transportation subjected to the demand and supply constraints.

It applies to situations where a single commodity is transported from various sources of supply (origins) to various demands (destinations).

Let there be m sources of supply $s_1, s_2, \dots \dots, s_m$ having $a_i(i = 1,2, \dots, m)$ units of supplies respectively to be transported among n destinations $d_1, d_2, \dots \dots, d_n$with $b_j(j = 1,2, \dots, n)$ units of requirements respectively.

Let c_{ij} be the cost of shipping one unit of commodity from source i to destination j for each route. If x_{ij} represent the units shipped per route from source i, to destination j, then the problem is to determine the transportation schedule which minimizes the total transportation cost of satisfying supply and demand conditions.

$$\text{Minimize } z = \sum_i^m \sum_j^n c_{ij} x_{ij}$$

Subject to the constraints,

$$\sum_j^n x_{ij} = a_i \quad , i = 1,2,\dots,m \ (supply\ constraints)$$

$$\sum_i^m x_{ij} = b_j \quad , j = 1,2,\dots,n \ (demand\ constraints)$$

$$x_{ij} \geq 0 \text{ for all i and j}$$

2.5.1 Understanding Assumptions

1. Only a single type of commodity is being shipped from an origin to a destination.
2. Total supply is equal to the total demand.
   $\sum_i^m a_i = \sum_j^n b_j$, a_i (supply) and b_j (demand) are all positive integers.
3. The unit transportation cost of the item from all sources to destinations is certainly and preciously known.
4. The objective is to minimize the total cost.

2.6 Basic Definitions and Terminologies

Feasible Solution (FS): A set of non-negative allocation $x_{ij} \geq 0$ which satisfies the row and column restriction is known as Feasible Solution.

Basic Feasible Solution (BFS): A feasible solution to an *m*-origins and *n*-destination problem is said to be Basic Feasible Solution if the number of positive allocation are (m+n−1). If the number of allocations in basic feasible solution are less than (m+n−1), it is called Degenerate Basic Feasible Solution (DBFS) otherwise non-degenerate.

Degenerate Basic Feasible Solution: A basic feasible solution that contains less than m+n-1 non-negative allocation.

Optimal Solution: A feasible solution (not necessarily basic) is said to be Optimal if minimizes the total transportation cost.

Optimality Test: Optimality test can be performed if the number of allocation cells in an initial basic feasible solution = m+n-1 (no. of rows + no. of columns -1).Otherwise the optimality test cannot be performed.

Cell: It is a small compartment in the transportation tables.

Circuit: A circuit is a sequence of cells (in the balanced transportation tables) such that

(i) It starts and ends with the same cell.

(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tables.

Allocation: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tables.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints.

The Constraints: The constraints are the conditions that force supply and demand needs to be satisfied. In a Transportation Problem, there is one constraint for each node.

i) The supply at each source must be used:

$$\sum_j^n x_{ij} = a_i \quad , i = 1,2,\dots,m$$

ii) The demand at each destination must be met:

$$\sum_i^m x_{ij} = b_j \quad , j = 1,2,\dots,n$$

iii) Non negativity:

$$x_{ij} \geq 0 \text{ for all i and j}$$

3. Existing Methods for Solving Transportation Problem

Chapter Three

Existing Methods for Solving Transportation Problem

3.1 Introduction

There are some different linear programming methods and their algorithms for solving the transportation problems. Here, we will discuss about the existing methods of solving transportation problem to find the optimal solution.

3.2 Some Existing Methods of Transportation Problem

The models given below are always used for solving the transportation problems.

1. North-west Corner Method (NCM)
2. Row Minima Method (RMM)
3. Column Minima Method (CMM)
4. Least Cost Method (LCM)
5. Vogel's Approximation Method (VAM)

3.2.1 North-west Corner Method (NCM)

The so-called Northwest corner rule appears in virtually every text-book chapter on the transportation problem. It is a standard method for computing a basic feasible solution and it does so by fixing the values of the basic variables one by one and starting from the Northwest corner of matrix.

The North-west corner rule is very simple and easy to use and apply. However, it is not sensitive to costs and consequently yields to poor initial solutions. The algorithm of North-west Corner method is given below:

Step 1:

Select the upper left-hand corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand,

i.e. $min(s_1, d_1)$

Step 2:

Adjust the supply and demand numbers in the respective rows and columns.

Step 3:

If the demand for the first cell is satisfied, then move horizontally to the next cell in the second column.

Step 4:

If the supply for the first row is exhausted, then move down to the first cell in the second row.

Step 5:

If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.

Step 6:

Continue the process until all supply and demand values are exhausted.

3.2.1.1 Example 1

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 3.2.1.1(a): Transportation table of example 1

Solution:

To From	A	B	C	D	E	Supply
P	~~55~~/25	~~30~~/10	~~40~~/05	50	50	~~40~~/~~15~~/~~05~~/0
Q	35	30	~~100~~/15	~~45~~/05	60	~~20~~/~~05~~/0
R	40	60	95	~~35~~/25	30	~~40~~/~~15~~/0
Demand	~~25~~/0	~~10~~/0	~~20~~/~~15~~/0	~~30~~/~~25~~/0	~~15~~/0	100

Table 3.2.1.1(b): Solution of example 1

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$55 \times 25 + 30 \times 10 + 45 \times 05 + 100 \times 15 + 45 \times 5 + 35 \times 25 + 35 \times 15 = 5325$$

3.2.1.2 Example 2

Find a feasible solution for the transportation table:

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 3.2.1.2(a): Transportation table of example 2

Solution:

To Form	D	E	F	Supply
A	~~6~~/20	~~4~~/30	1	~~50~~/~~30~~/0
B	3	~~8~~/40	7	~~40~~/0
C	4	~~4~~/25	~~2~~/35	~~60~~/~~35~~/0
Demand	~~20~~/0	~~95~~/~~65~~/~~25~~/0	35/0	150

Table 3.2.1.2(b): Solution of example 2

Number of basic variables = m + n – 1 = 3+3-1 = 5

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$6 \times 20 + 4 \times 30 + 8 \times 40 + 4 \times 25 + 2 \times 35 = 730$$

3.2.1.3 Example 3

Find a feasible solution for the transportation problem

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 3.2.1.3(a): Transportation table of example 3

Solution:

To From	A	B	C	D	E	Supply
P	~~4~~/22	~~1~~/38	3	4	4	~~60~~/~~38~~/0
Q	2	~~3~~/7	~~2~~/20	~~2~~/8	3	~~35~~/~~28~~/~~08~~/0
R	3	5	2	~~4~~/10	~~4~~/30	~~40~~/~~30~~/0
Demand	~~22~~/0	~~45~~/~~07~~/0	~~20~~/0	~~18~~/~~10~~/0	~~30~~/0	135

Table 3.2.1.3(b): Solution of example 3

Number of basic variables = m + n – 1 = 5+3-1=7

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$4 \times 22 + 1 \times 38 + 3 \times 7 + 2 \times 20 + 2 \times 8 + 4 \times 10 + 4 \times 30 \; = \; 363$$

3.2.2 Row Minima Method (RMM)

In this method we allocate maximum possible in the lowest cost cell of the first row. The idea is to exhaust either the capacity of the first source or the demand at destination center is satisfied or both. Continue the process for the other reduced transportation costs until all the supply and demand conditions are satisfied. The minimum transportation cost can be obtained by following the steps given below:

Step 1:

In this method, we allocate as much as possible in the lowest cost cell of the first row, i.e. allocate $min(s_i, d_j)$.

Step 2:

a. Subtract this min value from supply s_i and demand d_j.

b. If the supply s_i is 0, then cross (strike) that row and if the demand d_j is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step 3:

Repeat this process for all uncrossed rows and columns until all supply and demand values are 0.

3.2.2.1 Example 1

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 3.2.2.1(a): Transportation table of example 1

Solution:

To From	A	B	C	D	E	Supply
P	55	~~30~~/10	~~40~~/20	~~50~~/10	50	~~40/30/10~~/0
Q	~~35~~/20	30	100	45	60	~~20~~/0
R	~~40~~/5	60	95	~~35~~/20	~~30~~/15	~~40/25/5~~/0
Demand	~~25/5~~/0	~~10~~/0	~~20~~/0	~~30/20~~/0	~~15~~/0	100

Table 3.2.2.1(b): Solution of example 1

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$30\times10+40\times20+50\times10+35\times20+40\times5+35\times20+30\times15=3650$$

3.2.2.2 Example 2

Find a feasible solution for the transportation table:

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 3.2.2.2(a): Transportation table of example 2

Solution:

To Form	D	E	F	Supply
A	6	~~4~~/15	~~1~~/35	~~50/15~~/0
B	~~3~~/20	~~8~~/20	7	~~40/20~~/0
C	4	~~4~~/60	2	~~60~~/0
Demand	~~20~~/0	~~95/80/60~~/0	~~35~~/0	150

Table 3.2.2.2(b): Solution of example 2

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$4\times15+1\times35+3\times20+8\times20+4\times60=555$$

3.2.2.3 Example 3

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 3.2.2.3(a): Transportation table of example 3

Solution:

To From	A	B	C	D	E	Supply
P	4	~~1~~/45	~~3~~/15	4	4	~~60/15~~/0
Q	~~2~~/22	3	2	~~2~~/13	3	~~35/13~~/0
R	3	5	~~2~~/5	~~4~~/5	~~4~~/30	~~40/35/5~~/0
Demand	~~22~~/0	~~45~~/0	~~20/5~~/0	~~18/5~~/0	~~30~~/0	135

Table 3.2.2.3(b): Solution of example 3

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$1\times45+3\times15+2\times22+2\times13+2\times5+4\times5+4\times30=310$$

3.2.3 Column Minima Method (CMM)

In this method, we start with the first column and allocate as much as possible in the lowest cost cell of column, so that either the demand of the first destination center is satisfied or the capacity of the 2nd is exhausted or both. The minimum transportation cost can be obtained by following the steps given below:

Step 1:

In this method, we allocate as much as possible in the lowest cost cell of the first Column, i.e. allocate $min(s_i, d_j)$.

Step 2:

a. Subtract this min value from supply s_i and demand d_j.

b. If the supply s_i is 0, then cross (strike) that row and If the demand d_j is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step 3:

Repeat this process for all uncrossed rows and columns until all supply and demand values are 0.

3.2.3.1 Example 1

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 3.2.3.1(a): Transportation table of example 1

Solution:

To From	A	B	C	D	E	Supply
P	55	~~30~~/10	~~40~~/20	50	~~50~~/10	~~40/30/10~~/0
Q	~~35~~/20	30	100	45	60	~~20~~/0
R	~~40~~/5	60	95	~~35~~/30	~~30~~/5	~~40/35/5~~/0
Demand	~~25/5~~/0	~~10~~/0	~~20~~/0	~~30~~/0	~~15/10~~/0	100

Table 3.2.3.1(b): Solution of example 1

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$30\times10+40\times20+50\times10+35\times20+40\times5+35\times30+30\times5=3700$$

3.2.3.2 Example 2

Find a feasible solution for the transportation table:

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 3.2.3.2(a): Transportation table of example 2

Solution:

To Form	D	E	F	Supply
A	6	~~4~~/35	~~1~~/15	~~50/15~~/0
B	~~3~~/20	8	~~7~~/20	~~40/20~~/0
C	4	~~4~~/60	2	~~60~~/0
Demand	~~20~~/0	~~95/35~~/0	~~35/20~~/0	150

Table 3.2.3.2(b): Solution of example 2

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$4\times35+1\times15+3\times20+7\times20+4\times60=595$$

3.2.3.3 Example 3

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 3.2.3.3(a): Transportation table of example 3

Solution:

To From	A	B	C	D	E	Supply
P	4	~~1~~/45	3	~~4~~/5	~~4~~/10	~~60/15/10~~/0
Q	~~2~~/22	3	2	~~2~~/13	3	~~35/13~~/0
R	3	5	~~2~~/20	4	~~4~~/20	~~40/20~~/0
Demand	~~22~~/0	~~45~~/0	~~20~~/0	~~18/5~~/0	~~30/10~~/0	135

Table 3.2.3.3(b): Solution of example 3

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$1\times45+4\times5+4\times10+2\times22+2\times13+2\times20+4\times20=295$$

3.2.4 Least Cost Method (LCM)

The Least Cost Method is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost. The minimum transportation cost can be obtained by following the steps given below:

Step 1:

Select the cell having minimum unit cost c_{ij} and allocate as much as possible, i.e. $min\ (s_i, d_j)$

Step 2:

a. Subtract this min value from supply s_i and demand d_j.
b. If the supply s_i is 0, then cross (strike) that row and if the demand d_j is 0 then cross (strike) that column.
c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible.

Step 3:

Repeat this steps for all uncrossed rows and columns until all supply and demand values are 0.

3.2.4.1 Example 1

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 3.2.4.1(a): Transportation table of example 1

Solution:

To From	A	B	C	D	E	Supply
P	~~55~~/5	~~30~~/10	~~40~~/20	~~50~~/5	50	~~40~~/~~30~~/~~10~~/~~5~~/0
Q	~~35~~/20	30	100	45	60	~~20~~/0
R	40	60	95	~~35~~/25	~~30~~/15	~~40~~/~~25~~/0
Demand	~~25~~/~~5~~/0	~~10~~/0	~~20~~/0	~~30~~/5/0	~~15~~/0	100

Table 3.2.4.1(b): Solution of example 1

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$55\times5+30\times10+40\times20+50\times5+35\times20+35\times25+30\times15=3650$$

3.2.4.2 Example 2

Find a feasible solution for the transportation table:

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 3.2.4.2(a): Transportation table of example 2

Solution:

To Form	D	E	F	Supply
A	6	~~4~~/15	~~1~~/35	~~50/15~~/0
B	~~3~~/20	~~8~~/20	7	~~40/20~~/0
C	4	~~4~~/60	2	~~60~~/0
Demand	~~20~~/0	~~95/35/20~~/0	~~35~~/0	150

Table 3.2.4.2(b): Solution of example 2

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$4\times15+1\times35+3\times20+8\times20+4\times60=555$$

3.2.4.3 Example 3

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 3.2.4.3(a): Transportation table of example 3

Solution:

To From	A	B	C	D	E	Supply
P	4	~~1~~/45	3	~~4~~/5	~~4~~/10	~~60/15/5~~/0
Q	~~2~~/22	3	2	~~2~~/13	3	~~35/13~~/0
R	3	5	~~2~~/20	4	~~4~~/20	~~40/20~~/0
Demand	~~22~~/0	~~45~~/0	~~20~~/0	~~18/5~~/0	~~30~~/10	135

Table 3.2.4.3(b): Solution of example 3

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$1\times45+4\times5+4\times10+2\times22+2\times13+2\times20+4\times20=295$$

3.2.5 Vogel's Approximation Method (VAM)

The Vogel's Approximation Method (VAM) is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense. The minimum transportation cost can be obtained by following the steps given below:

Step 1:

Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.

Step 2:

Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.

Step 3:

Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell.
If there is a tie in the values of penalties then select the cell where maximum allocation can be possible

Step 4:

Adjust the supply & demand and cross out (strike out) the satisfied row or column.

Step 5:

Repeat this steps until all supply and demand values are 0.

3.2.5 .1 Example 1

Find a feasible solution for the transportation table:

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 3.2.5.1(a): Transportation table of example 1

Solution:

To From	A	B	C	D	E	Supply	Row Penalty
P	~~55~~/5	~~30~~/10	~~40~~/20	~~50~~/5	50	40	10 | 20 | 20 | 5 | 5 | 5 | 50 |
Q	~~35~~/20	30	100	45	60	20	5 | 5 | 5 | 10 | 10 | -- | -- |
R	40	60	95	~~35~~/25	~~30~~/15	40	5 | 5 | 5 | 5 | -- | -- | -- |
Demand	25	10	20	30	15	100	
Column Penalty	5 5 5 5 20 55 --	0 0 0 -- -- -- --	55 -- -- -- -- -- --	10 10 10 10 5 50 50	20 20 -- -- -- -- --		

Table 3.2.5.1(b): Solution of example 1

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$55\times5+30\times10+40\times20+50\times5+35\times20+35\times25+30\times15=3650$$

3.2.5 .2 Example 2

Find a feasible solution for the transportation table:

To Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 3.2.5.2(a): Transportation table of example 2

Solution:

To Form	D	E	F	Supply	Row Penalty
A	6	~~4~~/15	~~1~~/35	50	3 | 3 | 4 | 4 | 4 |
B	~~3~~/20	~~8~~/20	7	40	4 | 1 | 8 | -- | -- |
C	4	~~4~~/60	2	60	2 | 2 | 4 | 4 | -- |
Demand	20	95	35	150	
Column Penalty	1 -- -- -- --	0 0 0 0 4	1 1 -- -- --		

Table 3.2.5.2(b): Solution of example 2

Number of basic variables = m + n – 1 = 3+3-1 = 05

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$4\times15+1\times35+3\times20+8\times20+4\times60=555$$

3.2.5 .3 Example 3

Find a feasible solution for the transportation table

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 3.2.5.3(a): Transportation table of example 3

Solution:

<table>
<tr><td>To

From</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>Supply</td><td>Row Penalty</td></tr>
<tr><td>P</td><td>4</td><td>~~1~~/45</td><td>3</td><td>4</td><td>~~4~~/15</td><td>60</td><td>2 | 1 | 1 | 0 | 0 | 4 | -- |</td></tr>
<tr><td>Q</td><td>~~2~~/17</td><td>3</td><td>2</td><td>~~2~~/18</td><td>3</td><td>35</td><td>0 | 0 | 0 | 1 | -- | -- | -- |</td></tr>
<tr><td>R</td><td>~~3~~/5</td><td>5</td><td>~~2~~/20</td><td>4</td><td>~~4~~/15</td><td>40</td><td>1 | 1 | 1 | 1 | 1 | 4 | 4 |</td></tr>
<tr><td>Demand</td><td>22</td><td>45</td><td>20</td><td>18</td><td>30</td><td>135</td><td></td></tr>
<tr><td>Column Penalty</td><td>1
1
1
1
1
--
--</td><td>2
--
--
--
--
--
--</td><td>0
0
0
--
--
--
--</td><td>2
2
--
--
--
--
--</td><td>1
1
1
1
0
0
4</td><td></td><td></td></tr>
</table>

Table 3.2.5.3(b): Solution of example 3

Number of basic variables = m + n – 1 = 5+3-1 = 07

The total transportation cost is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

$$1\times45+4\times15+2\times17+2\times18+3\times5+2\times20+4\times15=290$$

4. An Alternative Proposed Method

Chapter Four

An Alternative Proposed Method

4.1 Introduction

So far the existing methods for solving the transportation problem have been discussed. In this chapter, we proposed an alternative method to solve the transportation problem which gives an initial basic feasible solution as well as the optimal or near optimal solution.

4.2 Algorithm of The Proposed Alternative Method

The steps of the proposed alternative method are discussed below:

Step: 1

Find the least two cost of each row and column from transportation problem table.

Step: 2

Calculate their mean values.

Step: 3

Identify the row or column that contains the least mean value. If two mean values tie then choose the row or column which contains the least cost.

Step: 4

Identify the least cost of the row or column that was selected in the step-3. If two or more same least costs tie then choose the cost that corresponds the most allocation. If two or more costs correspond the same allocation then choose a random one.

Step: 5

Assign the demanded value to the cost that is chosen in the step-4. If the demand is satisfied then round off the corresponding column, otherwise round off the corresponding row.

Step: 6

For the other values of transportation table apply the steps from 1 to 5 until the demand is satisfied.

4.3 Numerical Examples

In this section we will discuss some detailed numerical examples to illustrate the steps of the proposed alternative method.

4.3.1 Example 1

Find a feasible solution for the following transportation table.

To From	A	B	C	D	E	Supply
P	55	30	40	50	50	40
Q	35	30	100	45	60	20
R	40	60	95	35	30	40
Demand	25	10	20	30	15	100

Table 4.3.1(a): Transportation table of example 1

Solution:

Iteration-1

Calculate mean value of the least two costs for each row and column.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55	30	40	50	50	40	35
Q	35	30	100	45	60	20	32.5
R	40	60	95	35	30	40	32.5
Demand	25	10	20	30	15	100	
Mean of Least Two Costs	37.5	30	67.5	40	40		

Table 4.3.1(b): Iteration-1 of example 1

Iteration-2

Here 30 is the least mean value, so select the second column and then select the least cost 30. Here two least costs tie so select any of them randomly and then assign 10 into it and then round off the second column. Let us select the least cost of the first row. Round off the first column and hence calculate the mean values of least two costs for other rows and columns.

To / From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55	30 10	40	50	50	40/30	45
Q	35	30	100	45	60	20	40
R	40	60	95	35	30	40	32.5
Demand	25	10/0	20	30	15	100	
Mean of Least Two Costs	37.5		67.5	40	40		

Table 4.3.1(c): Iteration-2 of example 1

Iteration-3

Now the third row contains the least mean value. So select the third row and then select the least cost 30 then assign 15 into it. Since demand is fulfilled so round off the fifth column. Calculate the mean values of least two costs for the remaining rows and columns.

To / From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55	30 10	40	50	50	40/30	45
Q	35	30	100	45	60	20	40
R	40	60	95	35	30 15	40/25	40
Demand	25	10/0	20	30	15/0	100	
Mean of Least Two Costs	37.5		67.5	40			

Table 4.3.1(d): Iteration-3 of example 1

Iteration-4

Now the first column contains the least mean value. So select the first column and then select the least cost 35. Assign 20 into 35 and then round off the second row. Calculate the mean values of least two costs for the remaining rows and columns.

To / From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55	30 10	40	50	50	40/30	45
Q	35 20	30	100	45	60	20/0	
R	40	60	95	35	30 15	40/25	40
Demand	25/5	10/0	20	30	15/0	100	
Mean of Least Two Costs	47.5		67.5	40			

Table 4.3.1(e): Iteration-4 of example 1

Iteration-5

Now the third row contains the least mean value. So select the third row and then select the least cost 35. Assign 25 to the least cost and then round off the third row.

To / From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55	30 10	40	50	50	40/30	45
Q	35 20	30	100	45	60	20/0	
R	40	60	95	35 25	30 15	40/25/0	
Demand	25/5	10/0	20	30/5	15/0	100	
Mean of Least Two Costs	27.5		20	25			

Table 4.3.1(f): Iteration-5 of example 1

Iteration-6

Now only first row remains. So the next three allocations will be 20, 5 and 5 into the first, third and fourth cells of the first row respectively.

To / From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	55 5	30 10	40 20	50 5	50	40/30/10/5/0	
Q	35 20	30	100	45	60	20/0	
R	40	60	95	35 25	30 15	40/25/0	
Demand	25/5/0	10/0	20/0	30/5/0	15/0	100	
Mean of Least Two Costs							

Table 4.3.1(g): Iteration-6 of example 1

So the total cost:

$$30 \times 10 + 30 \times 15 + 35 \times 25 + 35 \times 20 + 40 \times 20 + 50 \times 5 + 55 \times 5 = 3650\ units$$

The solutions of this problem obtained by using NCM, RMM, CMM, LCM and VAM are 5325, 3650, 3700, 3650 and 3650. So the proposed method gives an optimal feasible solution.

4.3.2 Example 2

Find a feasible solution for the following transportation table:

To / Form	D	E	F	Supply
A	6	4	1	50
B	3	8	7	40
C	4	4	2	60
Demand	20	95	35	150

Table 4.3.2(a): Transportation table of example 2

Solution:

Iteration-1

Calculate mean value of the least two costs for each row and column.

To Form	D	E	F	Supply	Mean of Least Two Costs
A	6	4	1	50	2.5
B	3	8	7	40	5
C	4	4	2	60	3
Demand	20	95	35	150	
Mean of Least Two Costs	3.5	4	1.5		

Table 4.3.2(b): Iteration-1 of example 2

Iteration-2

Here 1.5 is the least mean value so select the third column and then select the least cost 1. Assign 35 to this cost and round off the third column. Calculate the mean values of least two costs for other rows and columns.

To From	D	E	F	Supply	Mean of Least Two Cost
A	6	4	1 35	50	5
B	3	8	7	40	5.5
C	4	4	2	60	4
Demand	20	95	35	150	
Mean of Least Two Cost	3.5	4			

Table 4.3.2(c): Iteration-2 of example 2

Iteration-3

Now 3.5 is the least mean value so select the first column and then select the least cost 3. Assign 20 to this least cost and round off the first column. Calculate the mean values of least two costs for the remaining rows and columns.

To / From	D	E	F	Supply	Mean of Least Two Cost
A	6	4	1 [35]	50/15	2
B	3 [20]	8	7	40/20	
C	4	4	2	60	
Demand	20/0	95	35/0	150	
Mean of Least Two Cost		4			

Table 4.3.2(d): Iteration-3 of example 2

Iteration-4

Now only second column remains. So the next three allocations will be 60, 15 and 20 into the third, first and second cells of the second column respectively.

To / From	D	E	F	Supply	Mean of Least Two Cost
A	6	4 [15]	1 [35]	50/15/0	
B	3 [20]	8 [20]	7	40/20/0	
C	4	4 [60]	2	60/0	
Demand	20/0	95/35/20/0	35/0	150	
Mean of Least Two Cost					

Table 4.3.2(e): Iteration-4 of example 2

So the total cost:

$$1 \times 35 + 3 \times 20 + 4 \times 60 + 4 \times 15 + 8 \times 20 = 555\ units$$

The solutions of this problem obtained by using NCM, RMM, CMM, LCM and VAM are 730, 555, 595, 555 and 555. So the proposed method gives an optimal feasible solution.

4.3.3 Example 3

Find a feasible solution for the following transportation table:

To From	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	135

Table 4.3.3(a): Transportation table of example 3

Solution:

Iteration-1

Calculate mean values of two least costs for each row and column.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1	3	4	4	60	2
Q	2	3	2	2	3	35	2
R	3	5	2	4	4	40	2.5
Demand	22	45	20	18	30	135	
Mean of Least Two Costs	2.5	2	2	3	3.5		

Table 4.3.3(b): Iteration-1 of example 3

Iteration-2

Here first & second rows and second & third columns tie with same least mean value 2. But first row & second column contain the least cost 1 so we can choose a random one. Let us choose the first row and do the first allocation in the least cost of first row. Assign 45 into 1. Since the demand is satisfied so round off the second column. Calculate the mean values of least two costs for other rows and columns.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 45	3	4	4	60/15	3.5
Q	2	3	2	2	3	35	2
R	3	5	2	4	4	40	2.5
Demand	22	45/0	20	18	30	135	
Mean of Least Two Costs	2.5		2	3	3.5		

Table 4.3.3(c): Iteration-2 of example 3

Iteration-3

Now second row and third column corresponds the same least mean value 2. But we can allocate more in the second row so we choose the second row. The least cost is 2. The allocation will be in the first column of second row. So assign 22 in the least cost and round off the first column. Calculate the mean values of least two costs for the remaining rows and columns.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 45	3	4	4	60/15	3.5
Q	2 22	3	2	2	3	35/13	2
R	3	5	2	4	4	40	3
Demand	22/0	45/0	20	18	30	135	
Mean of Least Two Costs			2	3	3.5		

Table 4.3.3(d): Iteration-3 of example 3

Iteration-4

Next allocation will be in third row and third column as it contains the least mean and as well as more allocation. Round off the third column as the demand is satisfied. Calculate the mean values of least two costs for the remaining rows and columns.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 45	3	4	4	60/15	4
Q	2 22	3	2	2	3	35/13	2.5
R	3	5	2 20	4	4	40/20	4
Demand	22/0	45/0	20/0	18	30	135	
Mean of Least Two Costs				3	3.5		

Table 4.3.3(e): Iteration-4 of example 3

Iteration-5

Next allocation will be in second row and fourth column as it contains the least mean and as well as more allocation. Round off the third row. Calculate the mean values of least two costs for the remaining rows and columns.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 45	3	4	4	60/15	4
Q	2 22	3	2	2 13	3	35/13/0	
R	3	5	2 20	4	4	40/20	4
Demand	22/0	45/0	20/0	18/5	30	135	
Mean of Least Two Costs				4	4		

Table 4.3.3(f): Iteration-5 of example 3

Iteration-6

Now all the least mean values and costs are same. So the next allocation will be in the third row and fifth column since it contains the most allocation. Round off the third row.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 (45)	3	4	4	60/15	4
Q	2 (22)	3	2	2 (13)	3	35/13/0	
R	3	5	2 (20)	4	4 (20)	40/20/0	
Demand	22/0	45/0	20/0	18/5	30/10	135	
Mean of Least Two Costs							

Table 4.3.3(g): Iteration-6 of example 3

Iteration-7

Now only one row remains. So the next allocations are in the fifth and fourth rows of the first column respectively.

To From	A	B	C	D	E	Supply	Mean of Least Two Costs
P	4	1 (45)	3	4 (5)	4 (10)	60/15/5/0	
Q	2 (22)	3	2	2 (13)	3	35/13/0	
R	3	5	2 (20)	4	4 (20)	40/20/0	
Demand	22/0	45/0	20/0	18/5/0	30/10/0	135	
Mean of Least Two Costs							

Table 4.3.3(h): Iteration-7 of example 3

So the total cost:

$$1 \times 45 + 2 \times 22 + 2 \times 20 + 2 \times 13 + 4 \times 20 + 4 \times 10 + 4 \times 5 = 295\ units$$

The solutions of this problem obtained by using NCM, RMM, CMM, LCM and VAM are 363, 310, 295, 295 and 290. So the proposed method gives a near optimal feasible solution.

4.4 Advantages of The Proposed Alternative Method

The prime advantage of the proposed alternative method is that it constructed of very easy logical and arithmetical algorithm. Therefore, it is very easy to understand and hence requires less arithmetic calculations than other existing methods. In most of the cases, the initial basic feasible solution obtained by the proposed alternative method is optimal or near optimal comparing with other existing standard methods.

4.5 Limitations of The Proposed Alternative Method

Though most of the time the proposed alternative method gives an optimal or near optimal solution, but when the dispersion amongst the costs is too high, the proposed method does not give an optimal solution. This is the major limitation of the proposed method.

5. Summary, Conclusion and Scope of Future Work

Chapter Five
Summary, Conclusion and Scope of Future Work

5.1 Summary

In this project work, we discussed all the existing standard methods for basic feasible solution of transportation problem with some relevant numerical examples. Hence we discussed an alternative proposed method for basic feasible solution of transportation problem along with its advantages and limitations. We illustrated some detailed numerical examples to explain the alternative proposed method more elaborately.

5.2 Conclusion

There are many attempts have been made by the research scholars to find an efficient technique for optimal solution of transportation problem. Though there are some effective methods out there but there is no such method that gives ultimate optimal solution.

The proposed alternative method gives an optimal or near optimal solution of transportation problem at most of the cases. So this method can be useful for minimizing the cost of transportation problem.

5.3 Scope of Future Work

Though the solution obtained by the proposed alternative method is optimal or near optimal at most of the time but is not always optimal. Especially when the dispersion among the costs is too high, the proposed alternative method does not hold optimality.

Further work can be done in future to resolve these limitations and find a better efficient way to find the optimal solution of transportation problem.

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Appendix

Python Code for the Alternative Proposed Method

```
def peak_element_position(means, demands, supplies):
    min_cnt = 0
    min_mean = means[0][0]

    for i in range(1, len(means)):
        if means[i][0] == min_mean:
            min_cnt += 1

    position_li = []
    min_cost = 1e9
    for i in range(min_cnt + 1):
        x = min(means[i][3])
        if x < min_cost:
            min_cost = x

    for i in range(min_cnt + 1):
        li = means[i][3]
        for j in range(len(li)):
            if li[j] == min_cost:
                if means[i][2] == 'col':
                    col = means[i][1]
                    row = j
                else:
                    row = means[i][1]
                    col = j

                supply = supplies[row]
                demand = demands[col]
                if demand <= supply:
```

```
            a = demand
        else:
            a = supply

        position = [row, col]
        position_li.append([a, position])

    position_li = sorted(position_li)
    # print(position_li)
    position = position_li[-1]
    return position



def brain(costs, demands, supplies):
    f = len(costs)
    s = len(costs[0])
    tr_costs = [[costs[j][i] for j in range(f)] for i in range(s)]

    means = []

    # print(costs)
    # print(tr_costs)

    for i in range(f):
        x = sorted(costs[i])
        if s >= 2:
            m = (x[0] + x[1])/2
        else:
            m = x[0]

        means.append([m, i, 'row', costs[i]])
```

```
    for i in range(s):
        x = sorted(tr_costs[i])
        if f >= 2:
            m = (x[0] + x[1])/2
        else:
            m = x[0]

        means.append([m, i, 'col', tr_costs[i]])

    means = sorted(means)

    # print(sorted(means))

    position = peak_element_position(means, demands, supplies)
    # print(position)
    return position


def main():
    print("Enter Number of Factories and Stories: ")
    f, s = map(int, input().split())

    print("Enter the cost Matrix: ")
    costs = [[int(i) for i in input().split()] for j in range(f)]

    print("Enter supplies: ")
    supplies = [int(i) for i in input().split()]

    print("Enter Demands: ")
    demands = [int(i) for i in input().split()]
```

```
total_supplies = sum(supplies)
total_demands = sum(demands)
d = 0
total_min_cost = 0

if total_demands != total_supplies:
    print("Here Demands and supplies are not balanced!")
    return

while d != total_demands:
    position = brain(costs, demands, supplies)
    a = position[0]
    row = position[1][0]
    col = position[1][1]
    total_min_cost += a * costs[row][col]

    supply = supplies[row]
    demand = demands[col]
    if demand <= supply:
        demands[col] = 0
        supplies[row] = supply - a
    else:
        supplies[row] = 0
        demands[col] = demand - a

    if supplies[row] == 0:
        del costs[row]
        del supplies[row]
    else:
        # print(costs)
```

```
        f = len(costs)
        s = len(costs[0])
        for i in range(f):
            for j in range(s):
                if j == col:
                    del costs[i][j]
        del demands[col]

    # print("The new cost matrix: ")
    # print(costs)
    # print(supplies)
    # print(demands)
    # print(total_min_cost)

    d += a

print('Total minimum cost of the transportation problem: ', total_min_cost)

if __name__ == "__main__":
main( )
```