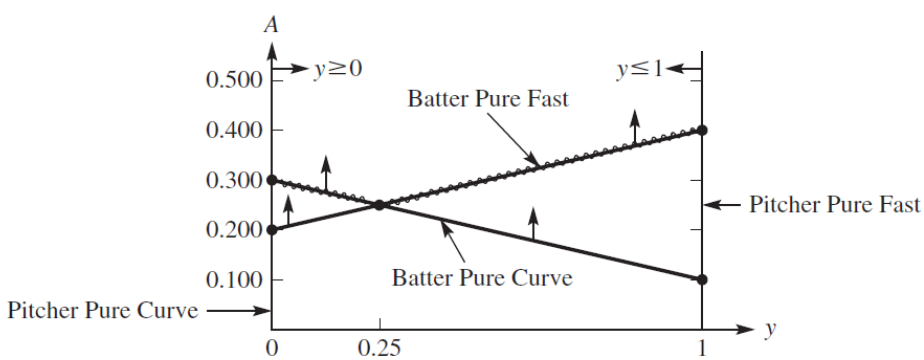
		Pitcher	
	Fastball		Curve
Fastball	.400	⇨	.200
Batter	⇧		⇩
Curve	.100	⇦	.300

Now we want to look at how does this game work from the Pitcher's perspective. One thing to note when we did look at the batter it was assumed that the Pitcher makes the best choice possible, which means we already know that the batting average should end up as 0.25. Let y be the proportion of fast balls, then (1-y) is the proportion of curve balls. Let A be the batting average obtained by the batter, then in this case we want to minimize A. Based on the image above our constraint lines are now found going across the row rather than down a column like it was for the batter. Therefore we up with

$A \geq .4y + .2(1 - y) = .2y + .2$

$A \geq .1y + .3(1 - y) = -.2y + .3$



Things to realize form this graph:

The y values (on the x axis) represent the probability of throwing fastballs, so in this case the line y=1 would be throwing only fastballs and the line y=0 would be throwing only curve balls. The line that goes up and to the right is the batter guessing fastballs, so as more and more fastballs are thrown, the higher the batting average becomes. Since we are trying to minimize the batting average since we are looking at the pitchers perspective, our feasibility region is the top part of this graph. The conclusion of this is that the pitcher should throw 25 percent fastballs, and 75 percent curve balls, and no matter what strategy the batter decides to use they can only hit 25 percent at best.

Furthermore another way of looking at this is simply to look at all of the points and determine which of those are feasible, and which of those lead to the minimum batting average. We can look at where these lines intersect each other, and where they intersect the lines $y = 0$ and $y = 1$.

y	A	Feasible?
0	.2	No
0	.3	Yes
1	.1	No
1	.4	Yes
.25	.25	Yes

Now since we are minimizing we simply take the smallest of these values that is in the feasibility region, which in this case will simply be $A = .25$ which occurs when $y = 0.25$.

Table 10.1 Expected values for the batter-pitcher duel at 0.05 increments

		x													
		0	0.05	0.1	...	0.2	0.25	0.3	...	**0.5**	0.55	...	0.95	1
	0	.300	.295	.29	...	.28	.275	.27	...	**.25**	.245	...	.205	.2
	0.05	.290	.286	.282	...	.274	.270	.266	...	**.25**	.246	...	.214	.21
y	0.1	.280	.277	.274	...	.268	.265	.262	...	**.25**	.247	...	.223	.22
	...	...	...	...	...	...	...	...	...	**.25**	...	...	...	...
	0.2	.260	.259	.258	...	.256	.255	.254	...	**.25**	.249	...	.241	.24
	0.25	**.250**	**.25**	**.25**	**...**	**.25**	**.25**	**.25**	**...**	**.25**	**.25**	**...**	**.25**	**.25**
	...	...	...	...	...	...	...	...	...	**.25**	...	...	...	...
	0.95	.110	.124	.138	...	.166	.180	.194	...	**.25**	.264	...	.376	.390
	1	.100	.115	.130	...	.160	.175	.190	...	**.25**	.265	...	.385	.400

This table shows up some of the values in increments of 0.05. This table can be broken up into 4 quadrants. Assume that it's like the coordinate axis, where we have quadrants 1,2,3,4. If we are currently in quadrant 1, then the bating average is too low, and the batter will adjust their strategy to hit more often pulling the value to the left until it hits .25. If the batter keep adjusting the strategy pulling it even more to the left, then the pitcher will adjust their strategy pulling it down to .25. Basically this looks like a circular path where once the solution is no longer optimal, the person who doesn't have the benefit will update to make it more into their benefit. Eventually if these adjustments can be done as quickly as possible you will eventually end up at the equilibrium $(.5, .25)$ getting a batting average of .25.

1. Decision Theory Revisited: Games against Nature

In the last part we saw how the batter could find an optimized strategy, which didn't matter on what the pitcher did, to get a guaranteed outcome. If the batter was able to see what the pitcher's optimal strategy is, then the batter can make a counter strategy to the optimal strategy. Furthermore if the batter notices that the pitcher is not playing optimally, then they can adjust their own strategy to get a batting average even higher than .25. In this sense, the pitcher's optimal strategy represents a breakpoint for the batter. In the real world, there is no guarantee that the pitcher is a rational player.

What we now want to look at is still a 2 player game, but in this case player one is a company, and player two is now going to be the economy. We are going to break this into parts. First off like the batter, we are going to have the company see how it can maximize it's return if it has no idea how the economy will react. Next the firm will look at this from the economy's standpoint and see how it can minimize the company's profit. The company can now play its maximization strategy, and guarantee an outcome regardless of the economy. Or, the company can determine (observe or predict) whether the economy will be better or worse than the economy's optimal strategy. If it is anything other than optimal, then the company can change from it's maximization strategy to another strategy which is worth more, just like where the batter would switch from 50 percent guess of fastballs if the pitcher is only pitching fastballs. We call games where the second player is not necessarily a reasoning player **games against nature**.

2. EXAMPLE

		Nature: Economy	
		Poor	Good
Firm	Small	\$500	\$300
	Large	\$100	\$900

In this case the company has a choice between a large amount of production or a small amount of production. Depending on the two different economy levels, Poor or Good, the payout for these choices will be different. To start this off lets look at this from the firm perspective.
Let V be the total amount of profit.
Let x be the portion of time that the company chooses to have small production.
Therefore $1 - x$ is the portion of time that the company chooses to have large production.
What we look at first again is the two extremes of the economy. The economy can always be poor (EP), or the economy can always be good (EG). This leads to the following for the firm:

Maximize V
Subject to:
$V \leq E(EP) = 500x + 100(1 - x)$
$V \leq E(EG) = 300x + 900(1 - x)$
$0 \leq x$
$x \leq 1$
And the following for the economy:
Minimize V
Subject to:
$V \geq E(EP) = 500x + 100(1 - x)$
$V \geq E(EG) = 300x + 900(1 - x)$
$0 \leq x$
$x \leq 1$
Which we can solve for both simultaneously.

```
syms x y
y1=500*x+100*(1-x);
y2=300*x+900*(1-x);
x=solve((500*x+100*(1-x))-(300*x+900*(1-x)));
V(1)=subs(y1); %subs updates the function with the new values for x
x=0;
V(2)=subs(y1);
V(3)=subs(y2);
x=1;
V(4)=subs(y1);
V(5)=subs(y2);
Firm=max([V(1),V(2),V(5)]);
Economy=min([V(1),V(3),V(4)]);
%OR
W=sort(V);
Firm2=W(ceil(end/2));
Economy2=W(ceil(end/2));
```

Few things to note from this code:
Assuming these two condition lines actually cross, then what's really going to happen is that the maximum in the feasibility region will be the middle value of the array for the maximization problem and the minimum in the feasibility region will be the middle value of the array for the minimization problem. This leads to the OR at the bottom of the code, instead of realizing which values to minimize and maximize you can simply sort the array, and then find the middle value of the array. Note that with only two condition lines that cross we are guaranteed to get 5 points for this array, and $ceil(\frac{5}{2}) = 3$ which is the middle value of this array.
The subs function tells matlab to upgrade the variable with the new update input values. So subs(y1) really means use the new x values to recalculate y1.

3. Investment Strategy

	Fast Growth	Normal Growth	Slow Growth
Stocks	10000	6500	-4000
Bonds	8000	6000	1000
Savings	5000	5000	5000

What makes this one different is we are now dealing with 3 strategies instead of 2. So the problem is modified:
Let V again be the profit.
Let x be the proportion invested in Stocks.
Let y be the proportion invested in Bonds.
Then $1 - x - y$ is the proportion invested in Savings.
Furthermore lets divide everything in the table by 1000 and we can then just take the final answer and multiply it by 1000 at the end.

Maximize V
Subject to:
$V \leq 10x + 8y + 5(1 - x - y) = 5 + 5x + 3y$
$V \leq 6.5x + 6y + 5(1 - x - y) = 5 + 1.5x + y$
$V \leq -4x + y + 5(1 - x - y) = 5 - x - 4y$
$x, y, (1 - x - y) \leq 1$
$0 \leq x, y, (1 - x - y)$
In this case we can actually solve this easier if we look at this from the perspective of the economy. No matter what the firm chooses, the economy can punish the firm the most by always picking slow growth. Which means that the company should invest everything into savings, making a return of 5000.