MATH 3310 Mathematical Modeling
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Homework 3
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Question 1.)

If first assuming that E, F, G, H all have the same probability of occurring and the goal is to maximize the expected value, you can just add up all of the expected values from each nature to get an estimate on which decision will be the best. For A, B, C, and D the results would all be 12, which doesn't help in determining a choice at all.

We can look at this in a different way. What if 3 out of the 4 natures did not occur? Which decision on average would be the best option then? Going in order, if E was the only activation:

A = 7	B = 3	C = 2	D = 6

If only F:

A = 10	B = 3	C = 2	D = 9

If only G:

A = -30	B = 3	C = 10	D = 10

If only H:

A = 25	B = 3	C = -2	D = -13

Because of the criteria we put in place for nature occurring, the decision is more obvious on the optimal solution. A is the most optimal outcome 3 out of 4 times, while C and D share a spot and B is never the most optimal outcome. The reason A is such a good decision is because of the 25% chance of G being picked. The higher the chance G is going to be picked, the much worse A becomes. For example, if instead a nature had a 75% chance of activating, A quickly becomes the WORST option to choose by far, where C would easily become the best.

D is just a less extreme version of A. Both choices have a ¼ chance losing value, a 2/4 chance of decent growth, and ¼ chance of large growth. So it could be argued D is a better choice because it is safer and you will lose less, but you will also gain less, so it's up for debate. B is consistent and a safe choice where you will never fail, and C seems to be around the same path for amount of gain.

Overall, my choice for optimal decisions would be A -> D -> C -> B where A is the best decision if the goal is to get the most points by the end.

Question 2)

I performed 3 separate methods in order to calculate the final results for each decision (described in code as well). I realized that my way of rationalizing the problem was technically incorrect twice and finally got it correct with the 3rd method.

I assumed simply choosing 1 out of 4 of the natures was equivalent to a 25% chance (because 1 out of 4) and based the calculations off that. Of course this is not how real random probabilities work. Sometimes the 25% can happen 3 out of 4 times and sometimes it won't happen at all. So because of this, I made the second method. However I then realized that the solution that was chosen needed to be the same for each possibility each iteration. As of now, the outcomes have been independent from each other. So finally a 3rd method was created.

The first method simply uses my original thought process. 1 random choice is chosen for each decision each time, essentially a "1 in 4 chance".

The second method actually determines each nature's chance of happening with a probability of 25% every time. This is how it would more realistically happen.

The third method tries each solution with a 25% chance AND applies that result to each of the choices. So if E is chosen, A, B, C, and D will all choose E.

For the first method:

```
===== METHOD 1 =====

After 12 iterations, the results for each choice are:
A: 72
B: 36
C: 20
D: 43
```

And after the second method:

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===== METHOD 2 =====

After 12 iterations, the results for each choice are:
A: 47
B: 27
C: 22
D: 25
```

The third and correct method:

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===== METHOD 3 =====

After 12 iterations, the results for each choice are:
A: -2
B: 27
C: 34
D: 71
```

My original thought process worked out well for the first 2 methods because those methods followed how I originally thought about the problem. However when I correctly implemented the model using the third method, it seems like my initial decision is completely wrong.

The regret table:

A = 25	B = 3	C = 10	D = 10

The total regret here is 73, because I chose A, I got a -2 when I could have chosen D and got a 71. Normally this should mean that decision A is the worst choice and from now on all bets should be placed on D. At face value, this conclusion is true, however, I found it hard to believe so I looked into my odds a little more and why A performed so poorly.

It turns out that the seed I originally chose was just very bad for A. The 4th nature 'H' never got chosen, which gave A a large boost in points while the 3rd nature 'G' got chosen more times than not. Because of this, it looks like A was really bad when it was actually just unlucky. Running the model again with a seed of 17 instead of 15 can show this easily:

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===== METHOD 3 =====

After 12 iterations, the results for each choice are:
A: 131
B: 36
C: 16
D: 12
```

If this was the result that had originally been shown, my prediction would have been validated and I would not have thought any deeper into it. I would not have looked deeper to get a better understanding of how the model was working. If I did not look further, I would have concluded that I was wrong and D should be chosen over A.

However, from this result, the regret value of choosing D when you could have chosen A is even greater than the original value; The difference being 119 vs. 73. Therefore, I will remain with my original thought and continue thinking A is the best choice, but there is always a gamble of whether or not it will pay off.