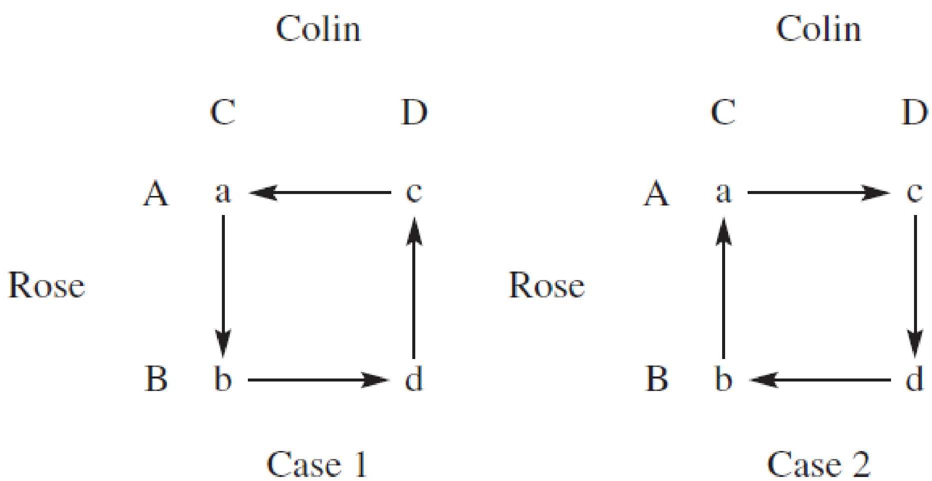
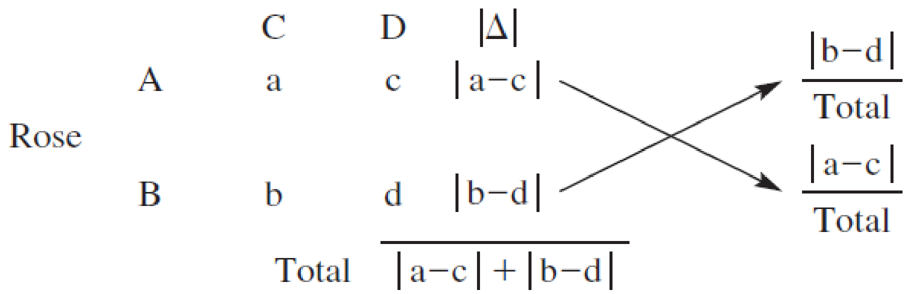
1. Method of Oddments Continued

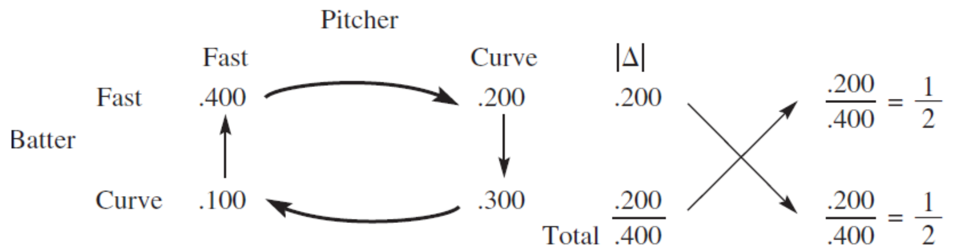


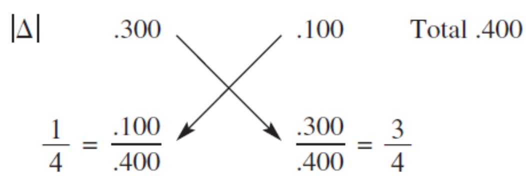
Looking back at the example what we had ended up with last time for Case 1 was:
$E(C) = ax + b(1-x) = cx + d(1-x) = E(D)$
$x = \frac{d-b}{(a-c)+(d-b)}$
$1 - x = \frac{a-c}{(a-c)+(d-b)}$
At this particular x, $E(C) = \frac{ad-bc}{(a-c)+(d-b)}$
For case 2 we get:
$E(C) = ax + b(1-x) = cx + d(1-x) = E(D)$, but this time we solve by bringing x to the opposite side (due to the ordering for case 2).
$x = \frac{b-d}{(c-a)+(b-d)}$
$1 - x = \frac{c-a}{(c-a)+(b-d)}$
At this particular x, $E(C) = \frac{bc-ad}{(c-a)+(b-d)}$
What this means is that no matter which case we are in we can have the solution if we look at absolute values.
$x = \frac{|b-d|}{|c-a|+|b-d|}$
$1 - x = \frac{|c-a|}{|c-a|+|b-d|}$
At this particular x, $E(C) = \frac{|bc-ad|}{|c-a|+|b-d|}$

Furthermore the value of this game is $V = \frac{ad-bc}{(a-c)+(d-b)}$.
How this looks with the table itself is:



Where that final element in row A or choice A is the proportion that the player or company on the left should pick choice A, and this hold similarly for B. This is a much faster way of evaluating the optimal performance.
Furthermore if we want to look at how Colin should play, we look at the differences in absolute value of the columns. $\Delta C = b - a$, $\Delta D = c - d$, $y = \frac{\Delta D}{\Delta C + \Delta D}$, so Colin's choices are solved for and handled similarly.





This is how this works with the pitcher and batter example with actual numbers instead of variables. Using the value of the game $V = \frac{ad-bc}{(a-c)+(d-b)}$ we get $V = \frac{(.4)(.3)-(.2)(.1)}{((.4)-(.1))+((0.3)-(0.2))} = .25$ which matches what we have calculated before.

		Country B		
		Disarm		Arm
	Disarm	3,3	$\Rightarrow$	1,4
Country A		$\Downarrow$		$\Downarrow$
	Arm	4,1	$\Rightarrow$	2,2

If we look back at the arms race example, we noted that this was not a total conflict game, since the sum of the payoffs for all the cells or entries are not equal. You can also see or test this by plotting the points by making one player the x coordinate and the other the y coordinate. If these points do not lay on a straight line, then it is not a total conflict game. Furthermore looking at this table, you can see that going from the bottom right cell (2,2) to the upper left cell (3,3) has both parties increase in their payout, which is not possible for a total conflict game.

The question now is, what is the objectives of the players in a partial conflict?

- **Maximize their own profit**. In this case each player plays only for themselves and doesn't care if the opponent ends up with nothing, and is just selfishly trying to get the best outcome possible.

- **Find a stable outcome**. Since this isn't a total conflict game, players may be interested in finding a stable outcome for both of them. This is often done by being able to find a Nash Equilibrium, from which neither player can unilaterally improve their payout.

- **Minimize their opponents profit**. Easiest way to talk about this is look at two competing corporations. Assuming that it is not total conflict the companies may start trying to maximize their own profits. At some point one of the company's may become unhappy with the current setup. At this point rather than just trying to increase their profits, they may want to limit their competitors profit. This could be due to revenge, or trying to force a company out of an industry, or trying to force a company to sell themselves cheaper to make more profit.

- **Find a mutually fair outcome (possibly through arbitration)**. In this case the best for one company may not be the best for the other and vice versa. If both companies try to maximize their own profits they could both end up with poor results. This can also happen in the case where if the companies would be bother detrimentally impacted if they both try to minimize the other company's profit.

	Country B			Player A: row minimum
	Disarm		Arm	
Disarm	(3, 3)	$\Rightarrow$	(1, 4)	1
Country A	$\Downarrow$		$\Downarrow$	
Arm	(4, 1)	$\Rightarrow$	(2, 2)	2
Player B: Column minimum	1		2	

With what we have been doing for the last while, we would simply have country A always arm as that row dominates the unarmed row since all the values for A are strictly larger. Similarly for B, it's maximum strategy will have it arm itself as well. This will get us to the payout of (2,2). Looking at the table, neither A nor B can move unilaterally to increase their payout. Making this point a Nash Equilibrium. However the option for both countries to disarm yields (3,3), which is better overall for bother countries.

The definition of the **Prison's Dilemma** is a two person partial conflict game in which each player has two strategies, defect or cooperate; defect dominates cooperate for both players even though the mutual defection outcome, which is the unique Nash equilibrium in the game, is worse for both players than the mutual cooperation outcome.

Another common form of the Prison's Dilemma is the game of chicken. Assume that two drivers, Colin and Rose, are both driving at each other. Whoever swerves first loses. Furthermore assume that the driver's cannot communicate with each other before the game actually starts. The worst outcome for this game is both drivers are stubborn and neither one wants to swerve causing both cars to crash into each other. This game has 4 different levels of outcomes. If neither driver swerves, then the outcome is 1. The first driver to swerve has an outcome of 2, which the driver who didn't swerve gets an outcome of 4. If both driver's swerve at the same time then the outcome is 3. In this game the higher the value of the outcome the more it is worth.

	Colin			Rose: row minimum
	Swerve		Not Swerve	
Swerve	(3, 3)	$\Rightarrow$	(2, 4)	**2**
Rose	$\Downarrow$		$\Uparrow$	
Not swerve	(4, 2)	$\Leftarrow$	(1, 1)	1
Colin: column minimum	**2**		1	

This looks similar to the arms race we just had but now the off diagonal elements are worth more for failing, and the cell (2,2) element is worth the least since double non swerve will likely result in death or severe injuries. Because of this the minimum for the row choice is now larger in the first row, and the minimum for the column choice has also similarly moved to column 1 instead of column 2. Based on the movement diagram that we have, neither player had a dominate strategy. Furthermore (3,3) in this case is not a Nash Equilibrium, as either person deciding to not swerve is a higher outcome. However if both players switch to not swerving we end up at (1,1) which is lower. What this basically means is both players have a interest in not swerving but if both actually take up this better interest they end up with a catastrophe. In this example both off diagonal elements are a Nash Equilibrium. Another way of seeing why they are a nash equilibrium is to do the following:
Player 1's best response against each of player 2's strategy = red circle.
Player 2's best response against each of player 1's strategy = blue circle.
For cells which have both a red and blue circle, they are Nash Equilibrium.

The definition of the game of **Chicken** is a two person partial conflict game in which each player has two strategies: swerve to avoid a crash, or not swerve to attempt to win the game. Neither player has a dominant strategy. If both players choose swerve, the outcome is not a Nash equilibrium and therefore unstable. There are two Nash equilibria where one of the two players chooses swerve and the second player chooses not swerve.

Now looking at the game of chicken with communication. Assume that Rose is able to act first and tell Colin what strategy she is doing. If she says she will not swerve no matter what, then Colin's best interest is to actually swerve netting a payout of (4,2). Colin could also end up telling Rose that he is also not going to swerve so she should change her strategy. This is called Issuing a Threat. A threat has to satisfy the following conditions:

- Colin communicates that he will play a certain strategy contingent on a previous action of Rose.
- Colin's action is harmful to Rose.
- Colin's action is harmful to Colin.

We can see that Colin telling Rose that if she doesn't swerve, he won't swerve either is detrimental to both Colin and Rose. If Rose believes this threat, then she should swerve making her outcome go from 1 to 2.

Another strategy with communication is Issuing a Promise. A promise has to satisfy the following conditions:

- Colin communicates that he will play a certain strategy contingent on a previous action of Rose.
- Colin's action is harmful to Colin.
- Colin's action is beneficial to Rose.

If Colin chooses to not swerve, then Rose's maximum possible outcome is 2. If Rose swerves, then Colin should not swerve getting a payout of 4. For Colin to harm himself, he must choose to swerve as well. Thus the promise looks like "If you swerve I'll swerve also" which generates a payout of (3,3) for both of them which is more than they would get if the other person refuses to actually swerve. However even if Colin promises to swerve if Rose swerves, Rose can agree and simply not swerve netting herself an outcome of 4 instead of 3. If Colin issues a threat, he can eliminate (4, 2) and obtain (2, 4). A promise by Colin eliminates (2, 4) but results in (4, 2), which does not improve the (3, 3) likely outcome without communication.