Back to the two competing hardware stores.

		Ace	
		Large City	Small City
Home Depot	Large City	60	68
	Small City	52	60

Lets look back at both minimax and maximin for this problem. For maximin from the perspective of Home Depot, we look at the minimums of all of the rows, then maximize those minimums. In this case the minimums of the two rows are 60 and 52 respectively. Therefore the maximum of these two minimums is 60. This represents that as long as Home Depot selects the Large City strategy, they are guaranteed a market share of 60. Therefore if we let S be the market share that Home Depot ends up with we have $S \geq 60$.
If we now look at the minimax from the perspective of Ace Hardware, where they are trying to minimize the market share of Home Depot, we look at the maximum of both columns. In this case 60 and 68 respectively. The minimum of these two maximums is 60. This represents that as long as Ace Hardware selects the Large City strategy, then Home Depot's market share is at most 60.
Combining both of these gets us that $60 \leq S \leq 60$, therefore $S = 60$.

When the maximin and minimax values are the same, this resulting outcome or payout is called a **saddle point**. If a game does have a saddle point, then that saddle point is the value of the game, as players can guarantee that they get at least this much of a payout.

What if we look at a larger model. Here each element in the matrix represents the probability of the prey actually escaping.

		Predator		
		Ambush	Pursue	Retreat
Prey	Hide	0.2	0.4	0.5
	Run	0.8	0.6	0.9
	Fight	0.5	0.5	0

What strategy does minimax suggest? What about maximin?

		Pitcher		Row minimum
		Fast	Curve	
Batter	Fast	.400	.200	**.200**
	Curve	.100	.300	.100
Column maximum		.400	**.300**	

If we look at this we can see that the batter can guarantee a batting average of at least .2, and the pitcher can guarantee a batting average of at most .3. Therefore in this case we have $.2 \leq A \leq .3$. From before we found the optimal strategy had the batting average of .25 which is indeed between .2 and .3.

		Predator				
		Ambush		Pursue		Retreat
			⟸	⟸	⟸	
Prey	Hide	0.2		0.4		0.5
		⇓		⇓		⇓
	Run	0.8	⇒	0.6	⇐	0.9
		⇑		⇑		⇑
	Fight	0.5	⟷	0.5	⇐	0

One important thing to note form the movement diagram for the predator and prey. In the bottom left we have two values that are both 0.5. How this is handles is a double ended arrow between the two of them, which basically means don't try to go this way to avoid an endless loop. Following everything else now, all arrows point to the middle cell making it the Nash Equilibrium.

1. Minimax Theorem

The Minimax Theorem states that every m x n two-person, total conflict game has a solution that is a unique number V , and there are optimal (pure or mixed) strategies for the two players such that (i) if Rose plays her optimal strategy, Rose's expected payoff will be $\geq V$ no matter what Colin does, and (ii) if Colin plays his optimal strategy, Rose's expected payoff will be $\leq V$ no matter what Rose does.

This theorem confirms what we mentioned earlier about having two condition lines crossing, sorting all the critical points and always taking the middle one, which would be the intersection point. What this means is if we can use the maximax and minimax for the maximization and minimization if the value or payout we get from both methods is the same then that is the pure strategy saddle point solution. If they are not the same then we set the two expected values of the opposing players strategies equal and solve.

To see how this works in an example, back to the batter and pitcher game.

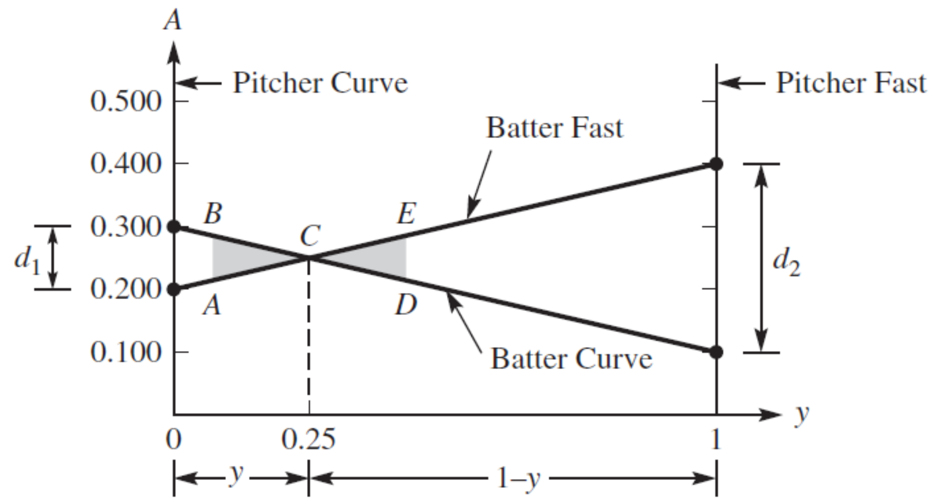
		Pitcher		Row minimum
		Fast	Curve	
Batter	Fast	.400	.200	**.200**
	Curve	.100	.300	.100
Column maximum		.400	**.300**	

As we can see from before .2 and .3 are not equal. So if we want to solve for the optimal batter strategy, we have to set both of the pitcher strategy expected values to be equal and solve. Let x again be the proportion of fastballs guesses, then we end up with:
$E(PF) = .4x + .1(1 - x) = .3x + .1$
$E(PC) = .2x + .3(1 - x) = .3 - .1x$
Setting these two equal we get $.4x = .2$ or $x = .5$, which matches what we had from before.
Similarly for the pitcher we have to set the strategies of the batter equal to each other. In this case we let y be the probability of throwing a fastball then we end up with:
$E(BF) = .4y + .2(1 - y) = .2y + .2$
$E(BC) = .1y + .3(1 - y) = .3 - .2y$

Setting these two equal we get $.4y = .1$ or $y = .25$, which also matches with what we had before.

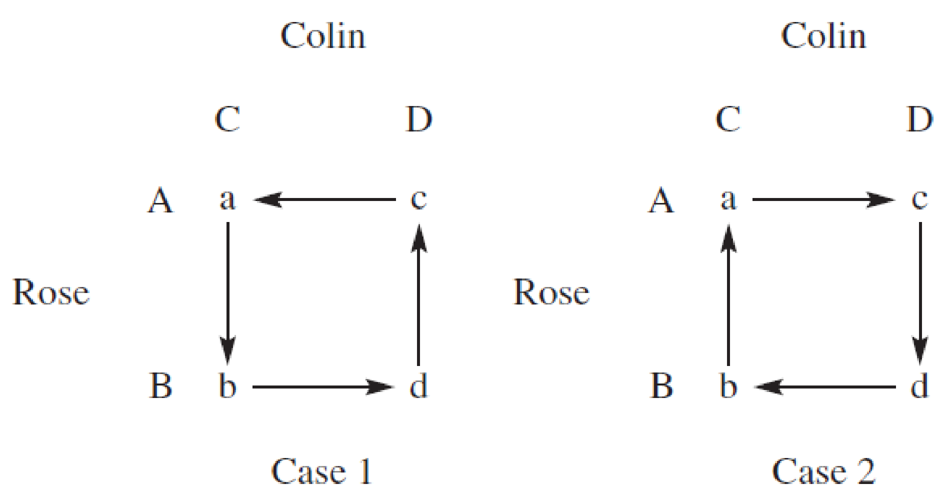
2. method of Oddments

As usual, back to the batter and pitcher example.



Looking at the picture, triangle ABC is similar to triangle DEC. More specifically we can do $\frac{.1}{.3} = \frac{d_1}{d_2} = \frac{y}{1-y}$. Solving this yields $y = .25$.

How this works in a more complicated example:



Let x represent the probability that Rose plays game A, therefore $(1-x)$ represents the probability that Rose plays game B. Looking at the arrows, there isn't going to be a pure strategy that works, so to solve this like before we set the expected values of Colin's strategies to be equal and solve:

$E(C) = ax + b(1-x) = cx + d(1-x) = E(D)$
$x = \frac{d-b}{(a-c)+(d-b)}$
$1 - x = \frac{a-c}{(a-c)+(d-b)}$
At this particular x, $E(C) = \frac{ad-bc}{(a-c)+(d-b)}$