Game Theory is basically decision theory where there are multiple people playing the game. Here the outcomes will not only depends on the persons choice but possibly all the other people's choices as well. There are a bunch of famous game theory examples and the simplest one is the arms race. Suppose we have two countries (A and B) who are currently in an arms race. Suppose that they have a meeting between delegates and it is decided that both countries should disarm. Suppose as well that if one of the countries does disarm, and the other country does not, than the country that disarmed will lose the war. This yields 4 possible scenarios.

- Country A disarms, Country B does not disarm, and Country B wins.
- Country A disarms, Country B does disarm, and we have peace.
- Country A does not disarm, Country B does disarm, and Country A wins.
- Country A does not disarm, Country B does not disarm, and we are stuck in war.

What happens here is country A needs to calculate what they think the odds of country B actually disarming, and if that is high enough to actually aim for peace.

1. EXAMPLE

Suppose we have two competing hardware stores, Home Depot, and Ace Hardware. In North America, Ace has far more stores than Home Depot, but each store is typically smaller. Furthermore assume that these are the only two hardware stores for our area of interest. Both companies are looking at opening a store in two different locations. One is a large city and one is a smaller city. Suppose we have the following table for market share from these two decisions for Home Depot:

	Ace	
	Large City	Small City
Large City	60	68
Home Depot	$\Uparrow$	$\Uparrow$
Small City	52	60

The way to read this table is that the 1st row is for Home Depot building in the large city, which the bottom row is for Home Depot building in the small city. The first column is for Ace Hardware building in a large city, and the final column is for Ace Hardware building in a small city. The actual values in the table is how much market share Home Depot would have under these choices. Since we are dealing with market share and are assuming these are the only two hardware stores, this is a **total conflict** game, since every 1 market share that Home Depot gains is a market share that Ace Hardware loses. Looking at the table that we have, Home Depot has a **Dominant Strategy**, which is simply to build in a large city. This option is best for them no matter what Ace Hardware does. This can be seen by the arrows in the table. If Ace Hardware knows this data, then it knows for sure that Home Deport will open in the large city, and then can deside if opening in the large city themselves is worth the extra 8 percent market share for themselves. Ace's table is simply the table with 100 minus all of these values. Their table would have arrows going left, as for them Large City is also the most favorable. In this case both companies build in the Large City as that is the dominate strategy for both companies. Putting both sets of the arrows together, we get that all arrows point to Large Large, and this is known as a **Nash Equilibrium**, where neither player or company can do any better by making any other choice.

What if we don't get a Nash Equilibrium? Suppose we have the following table for a baseball batter vs a patcher.

	Pitcher		
	Fastball		Curve
Fastball	.400	$\Rightarrow$	.200
Batter	$\Uparrow$		$\Downarrow$
Curve	.100	$\Leftarrow$	.300

How to read this table is that if the batter assumes that the pitcher is going to throw a fastball, and the pitcher actually does throw a fastball, then he has a 40 percent chance to hit the ball. On the other hand, if the batter assumes that the pitcher is going to throw a fastball and instead the pitcher throws a curve, the batter only has a 20 percent chance of hitting the ball. Notice that in this diagram we don't have a Nash Equilibrium, as there is no element that has arrows pointing to it in both the horizontal and vertical direction. In this example the pitcher needs both the fastball and curve strategy as if he picks and sticks with just one he ends up being worse off once batters catch on to what he throws. In this case we say that **Mixed Strategy** is best.

2. The Arms Race

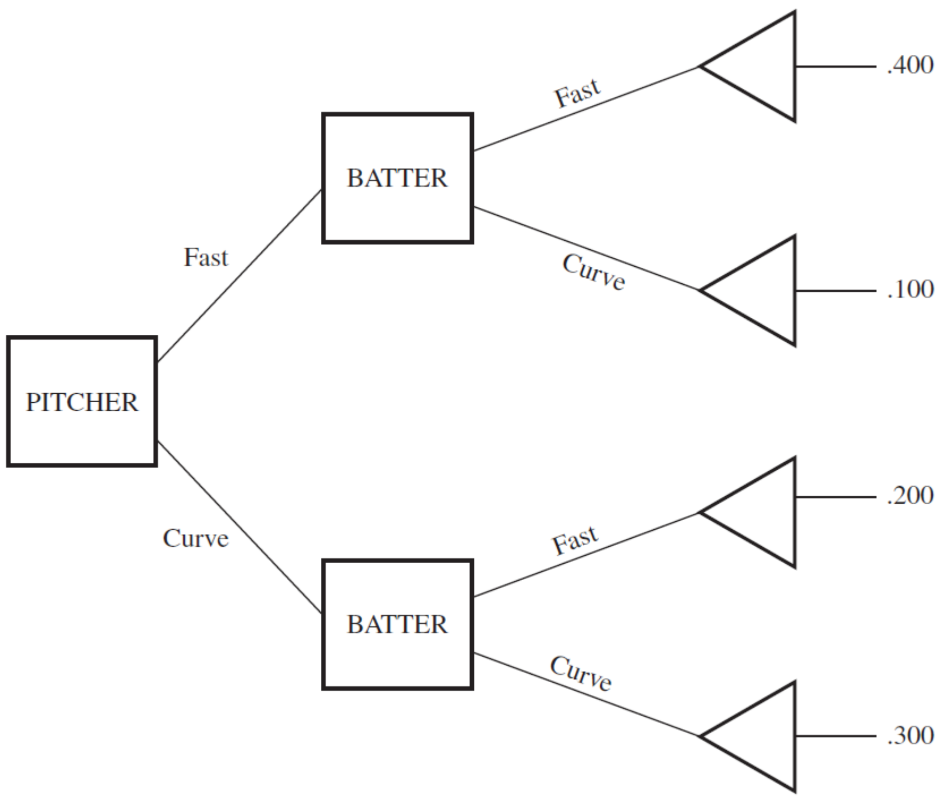
Suppose that we now assign outcome values to each of the events for the arms race. If country A stays armed and B disarms, we have an outcome of (3,1), where the payoff for country A is 3 and the payoff for country B is 1. If country A stays armed and B also stays armed, we have an outcome of (2,2). If country A disarms and B also stays armed, we have an outcome of (1,3). If both countries disarm we have a payout of (3,3). This leads to the following table:

		Country B		
		Disarm		Arm
Country A	Disarm	3,3	$\Rightarrow$	1,4
		$\Downarrow$		$\Downarrow$
	Arm	4,1	$\Rightarrow$	2,2

Notice the (2,2) option has arrows going to it from both sides, as each countries does better for itself if it goes from being overwhelmed (unarmed versus armed) into armed vs armed. However this is not a Nash Equilibrium, as there exists a point in the table where both countries have a payout worth more than (2,2), namely the (3,3) option. If for each possible outcome the payoffs to each player do not sum to the same constant, the game is classified as **partial conflict**. If we look at the sums of this table we have 6,5,5,4, so if we are trying to maximize the overall payouts, having both countries disarm is best for both countries, however it's in their own interest to backstab the other country to make their payout 1 higher. In this type of game it is extremely important to know do the two players have any communication with each other. If they do for this example they can decide to both disarm and be happy with the (3,3) payout. If they don't they have to figure out how likely the other country would be to actually disarm.

Another name for this arms race game is the Prisoner's Dilemma. Consider that the police have arrested two people, A and B. The Police don't have enough evidence to actually attempt to convict either person, but are trying hard at interrogating each of them individually. The way this works is that which ever person rats out the party first gets a light sentence, while the other person gets a harsh sentence. However if neither person rats out the party they will both walk away with no charges. In this case it's in the parties interest to have neither person rat out the party, however it is often suggested to immediately rat out the party so you end up with a light sentence.

		Pitcher	
	Fastball		Curve
Fastball	.400	⇒	.200
Batter	⇑		⇓
Curve	.100	⇐	.300



If we want to see how this table is filled out, we need to know the pitcher probabilities of the pitcher, and the guessing probabilities of the batter. Once these are known we can simply fill in this decision entirely. What we use to actually solve this is we let the batting average be A and look at what happens when the picture does a pure fastball and a pure curve strategy with the batters guesses being x for a fastball and $(1-x)$ for a curve respectively. This will set up two inequalities, four if you count that $0 \leq x \leq 1$. Assume that the pitcher only pitches fastballs, then the batter guess that it is a fastball with probability x and a curve with probability $1-x$, so the expected value of a hit will simply be $A \leq .4x + (.1)(1-x)$. Similarly we do the same thing for a pure curve strategy. $A \leq .2x + .3(1-x)$.

This leads us to an optimization problem with

$A \leq .4x + (.1)(1-x)$

$A \leq .2x + .3(1-x)$

$x \leq 1$

$0 \leq x$

Which we can look at by drawing in both of the constraint lines, $y = .1 + .3x$ and $y = .3 - .1x$. Setting these two equations as equal we get $x = \frac{1}{2}$, $y = .25$.