Advanced Mathematics - Linear Algebra

Chapter 4: Vector Geometry

Department of Mathematics
The FPT university

Chapter 4 Introduction

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Topics:

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4.1 Vectors and lines

4.2 Projections and Planes

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4.1 Vectors and lines

4.2 Projections and Planes

4.3 The cross product

4.4 Matrix transformation

4.5 An applications to computer graphics

4.1 Vectors and lines

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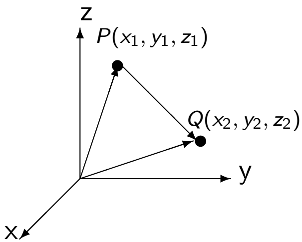

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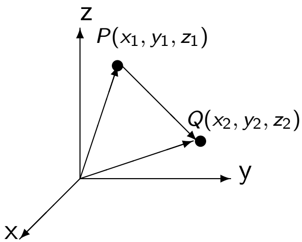

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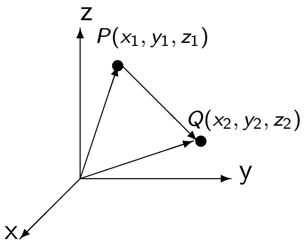

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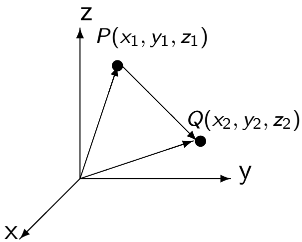

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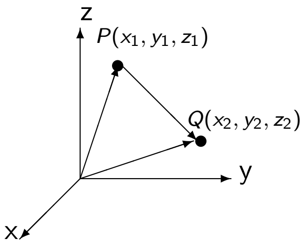

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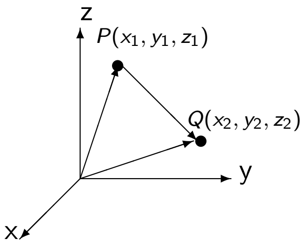

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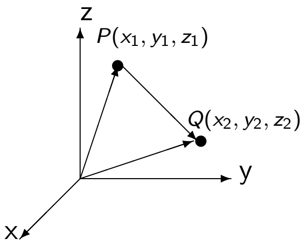

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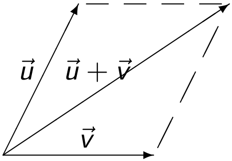

Equations of lines

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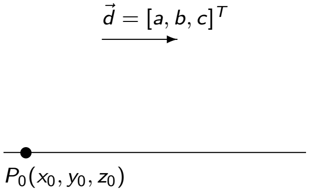

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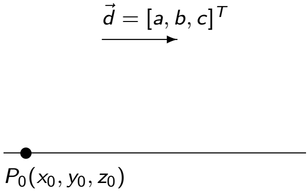

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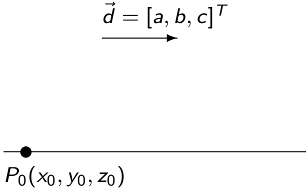

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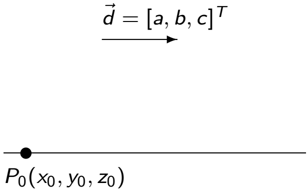

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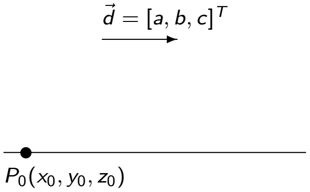

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4.2 Projections and plane

The dot product

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Let $\vec{u} = [u_1 \ \ u_2 \ \ u_3]^T, \vec{v} = [v_1 \ \ v_2 \ \ v_3]^T$. The dot product of $\vec{u}$ and $\vec{v}$, is

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Properties

- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $(k\vec{u}) \cdot \vec{v} = k(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (k\vec{v})$
- $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
- $||\vec{u}|| = \sqrt{\vec{u} \cdot \vec{u}}$
- Let θ be the angle between $\vec{u}$ and $\vec{v}$, then $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{||\vec{u}|| \, ||\vec{v}||}$
- $\vec{u}$ and $\vec{v}$ are orthogonal if and only if $\vec{u} \cdot \vec{v} = 0$

Projections

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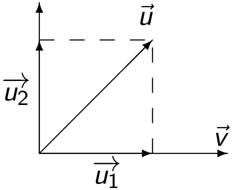

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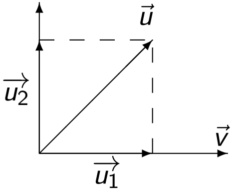

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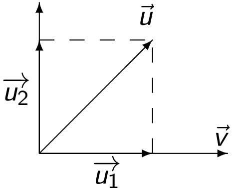

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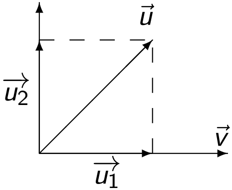
Example. Find the projection of the vector $\overrightarrow{u} = [1\ 2\ 3]^T$ on the vector $\overrightarrow{v} = [1\ \ -1\ 0]^T$.



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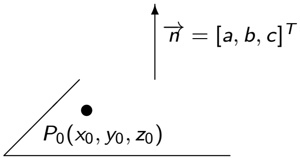

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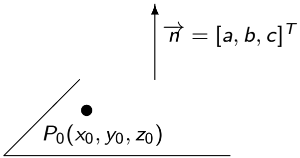

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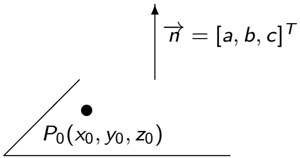

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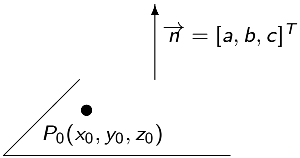

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Example. Write an equation for the plane passing through $P(1, 2, 1)$ and perpendicular to the line

$$x = 2 + t, y = -1 + 3t, z = -2 - t.$$

Shortest distance problems

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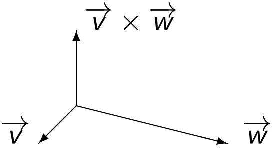

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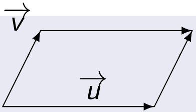

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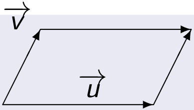

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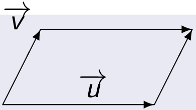

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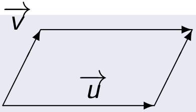

The volume of the parallelepiped determined by $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ is $|\overrightarrow{u} \cdot (\overrightarrow{v} \times \overrightarrow{w})|$.

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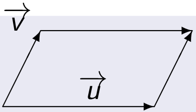

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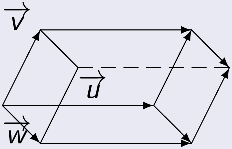

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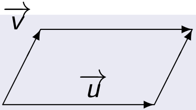

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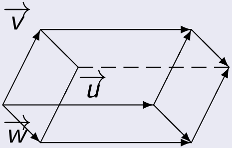

Note.

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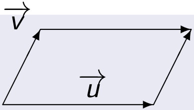

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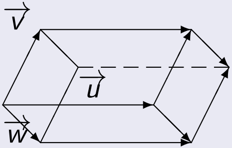

Note. If $\overrightarrow{u} = [x_0\ y_0\ z_0]^T, \overrightarrow{v} = [x_1\ y_1\ z_1]^T, \overrightarrow{w} = [x_2\ y_2\ z_2]^T$ then the volume of the parallelepiped determined by $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ is the *absolute value* of the determinant of the matrix $\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ z_0 & z_1 & z_2 \end{bmatrix}$

4.4 Matrix transformations II

Transformations in plane

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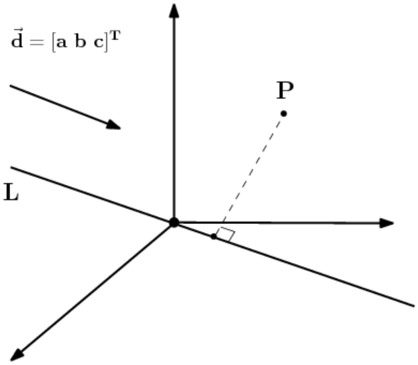

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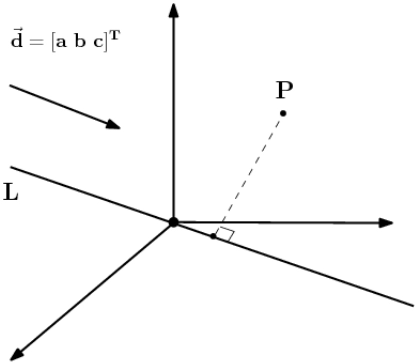

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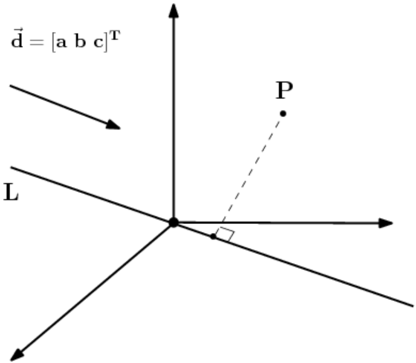

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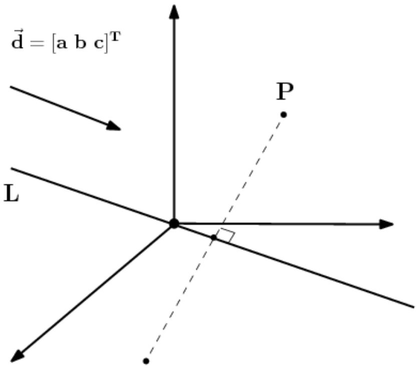
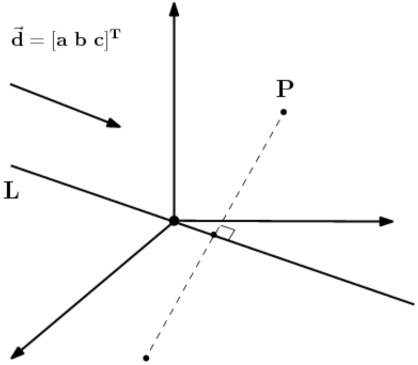
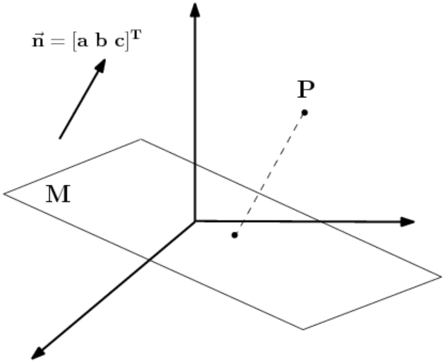

$\vec{d} = [a\ b\ c]^T$
P
L

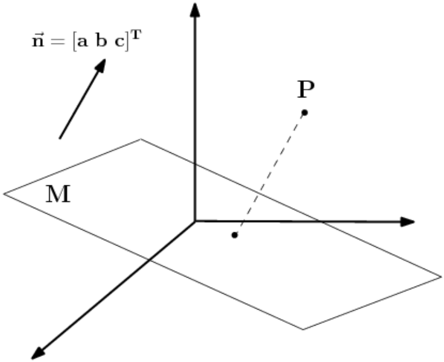


- The matrix of reflection in L is

$$\frac{1}{a^2+b^2+c^2}\begin{bmatrix} a^2-b^2-c^2 & 2ab & 2ac \\ 2ab & b^2-a^2-c^2 & 2bc \\ 2ac & 2bc & c^2-a^2-b^2 \end{bmatrix}$$

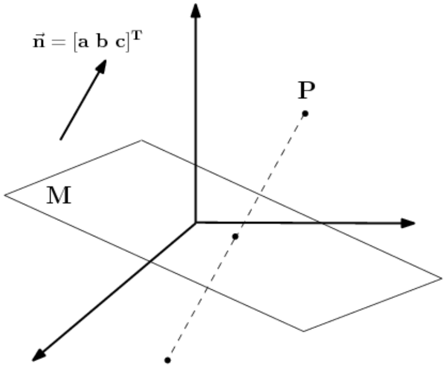


$\vec{n} = [a\ b\ c]^T$
P
M



- The matrix of projection on M is

$$\frac{1}{a^2+b^2+c^2}\begin{bmatrix} b^2+c^2 & -ab & -ac \\ -ab & a^2+c^2 & -bc \\ -ac & -bc & a^2+b^2 \end{bmatrix}$$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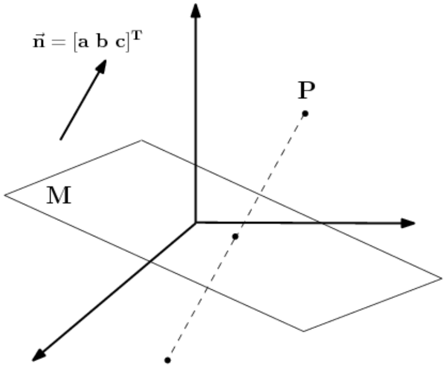
P
M



- The matrix of reflection in M is

$$\frac{1}{a^2+b^2+c^2}\begin{bmatrix} b^2+c^2-a^2 & -2ab & -2ac \\ -2ab & a^2+c^2-b^2 & -2bc \\ -2ac & -2bc & a^2+b^2-c^2 \end{bmatrix}$$

4.5 Applications in computer graphics

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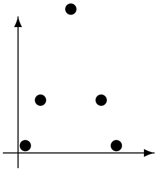
$$D = \begin{bmatrix} 0 & 6 & 5 & 1 & 3 \\ 0 & 0 & 3 & 3 & 9 \end{bmatrix}$$



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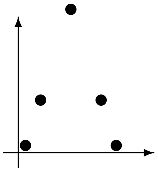

To rotate the letter A about the origin through $\pi/6$, we can multiply the data matrix D by the matrix $R_{\pi/6} = \begin{bmatrix} \cos(\pi/6) & -\sin(\pi/6) \\ \sin(\pi/6) & \cos(\pi/6) \end{bmatrix}$. This gives

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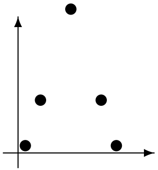

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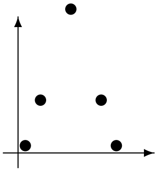

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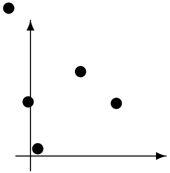

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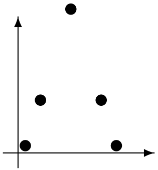

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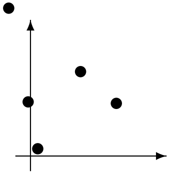

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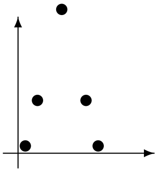

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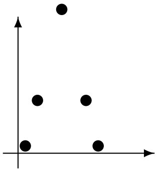

Practice problems

- Slant the letter to the right
- Turn the letter upside down
- Rotate the letter through $\pi/6$ about the point $(1, 2)$.