ITMO UNIVERSITY

Adaptive and robust control

Dmitry N. Gerasimov
gerasimovdn@mail.ru, dngerasimov@itmo.ru

Saint-Petersburg, 2024

Lectures Outline

1. Introduction to Adaptive and Robust Control
2. Lyapunov Functions Method. Short Tutorial
3. Simple Example of Adaptive Controller Design
4. Simple Example of Robust Controller Design
5. Generalized Algorithm of Adaptive and Robust Controller Design
6. Standard Error Models
   6.1. Static Error Model
   6.2. Dynamic Error Model with Measurable State
   6.3. Dynamic Error Model with Measurable
7. Schemes №1, №2 of LTI plants Parameterization.
8. Adaptive Observer Design
9. Schemes №3 of LTI plants Parameterization
10. Model Reference Adaptive Control (MRAC) Schemes
11. Adaptation Algorithms with Improved Parametric Convergence
12. Adaptive Compensation of Multisinusoidal Disturbances
13. Adaptive Servotracking of Multisinusoidal Reference Signals
14. Adaptive Control of the Plants with Unmatched Uncertainties

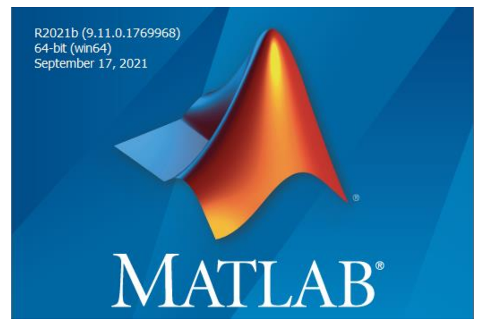
References

1. Narendra K.S. and Annaswamy A.M. *Stable Adaptive Systems*. — Englewood Cliffs. Courier Corporation, 2012. — 512 p.
2. Sastry S. and Bodson M., *Adaptive Control: Stability, Convergence and Robustness*. — Englewood Cliffs. N.J.: Prentice-Hall, 1989. — 377 p.
3. Ioannou P.A. and Sun J. *Robust adaptive control* — California: Prentice-Hall, 1996. — 848 p.
4. Khalil H. K., *Nonlinear Systems*, 3rd Edition, Prentice Hall, Upper Saddle River, 2002. —766 p.
5. Krstic M., Kanellakopoulos I., and Kokotovic P. V., *Nonlinear and Adaptive Control Design* — Wiley, 1995 — 563 p. (HNC book)
6. Gang Tao. *Adaptive control design and analysis*, Wiley-InterScience, 2003. — 618p.
7. Nikiforov V., Gerasimov D., *Adaptive regulation: reference tracking and disturbance rejection*. Lecture Notes in Control and Information Sciences — Springer Nature, 2022 — 376p.

Course structure

1. Lectures

2. Practical work/Labs



3. Exam/Credit

4. Additional optional homework

1. Introduction

Problems and motivation. Classical control theory

$$\dot{x} = Ax + bu,$$
$$y = c^T x,$$

where A, b, c are known precisely, y, x, and u are the measurable precisely output, state, and the control signals, respectively.

If y, x, and u are not accessible for measurement, they can be recovered exactly based on measurable signals.

1. Introduction

Problems and motivation. Classical control theory

$$\dot{x} = Ax + bu,$$
$$y = c^T x,$$

where A, b, c are known precisely, y, x, and u are the measurable precisely output, state, and the control signals, respectively.

If y, x, and u are not accessible for measurement, they can be recovered exactly based on measurable signals.

In real practical applications, this is not the case!

1. Introduction

Problems and motivation

mathematical models have limited accuracy over the whole range of plants operating

Aircraft

DC motors

U

α

DC motor dynamics

$$\dot{I} = -\frac{R}{L} I - \frac{k_E}{L}\omega + \frac{1}{L} U,$$

$$\dot{\omega} = \frac{k_M}{J} I - \frac{1}{J} M_L,$$

$$\dot{\alpha} = \omega$$

Spark ignition engines

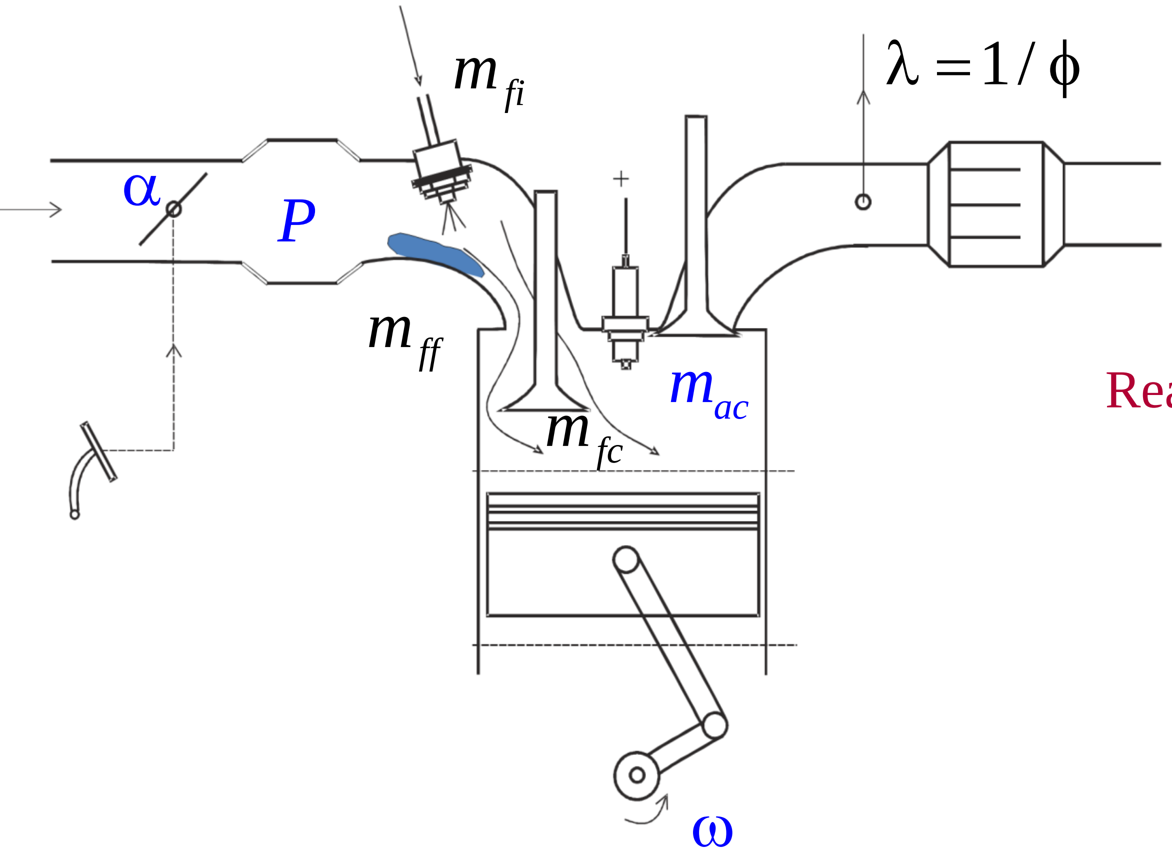
Fuel evaporation process dynamics

$$\dot{m}_{ff} = -\frac{1}{T} m_{ff} + \frac{K}{T} m_{fi}$$

$$m_{fc} = m_{ff} + (1-K) m_{fi}$$

Spark ignition engines

Scheme



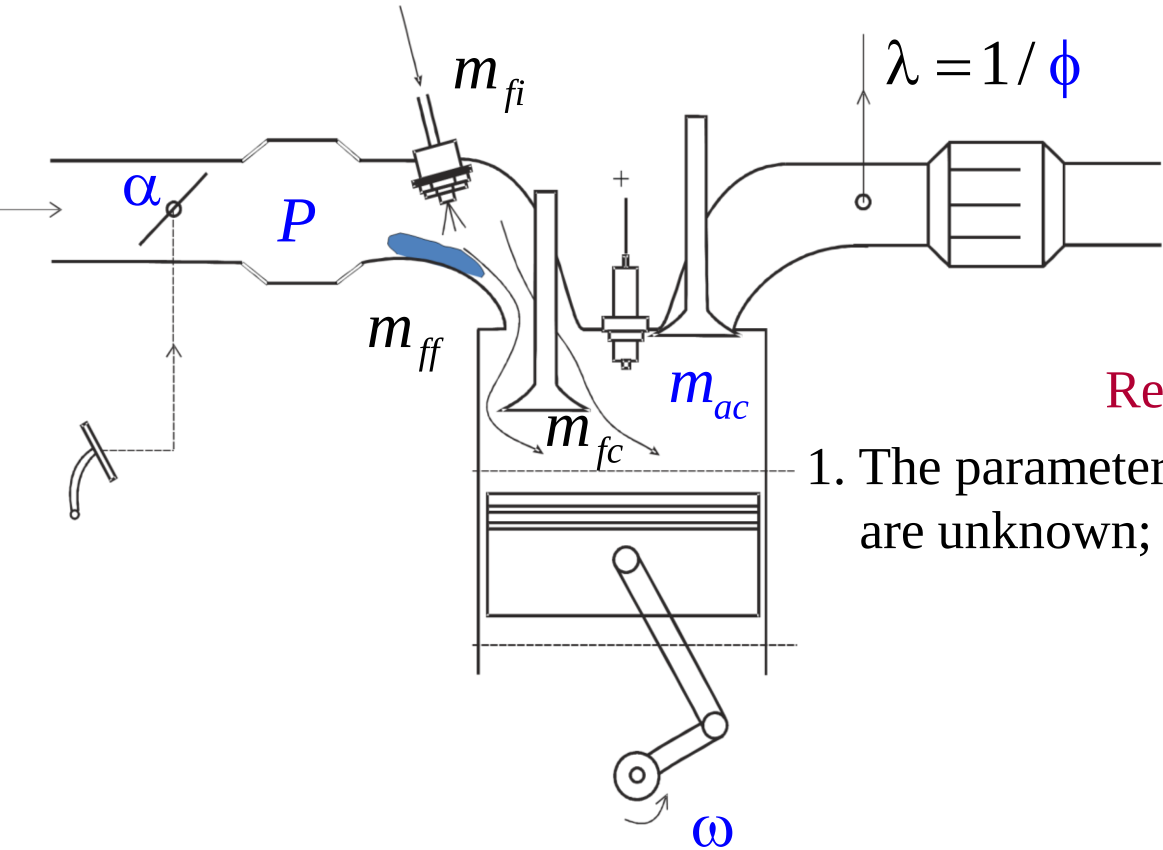
Fuel evaporation process dynamics

$$\begin{cases} \dot{m}_{ff} = -\dfrac{1}{T} m_{ff} + \dfrac{K}{T} m_{fi} \\ m_{fc} = m_{ff} + (1-K) m_{fi} \end{cases}$$

Real-life problems

Spark ignition engines

Scheme



Fuel evaporation process dynamics

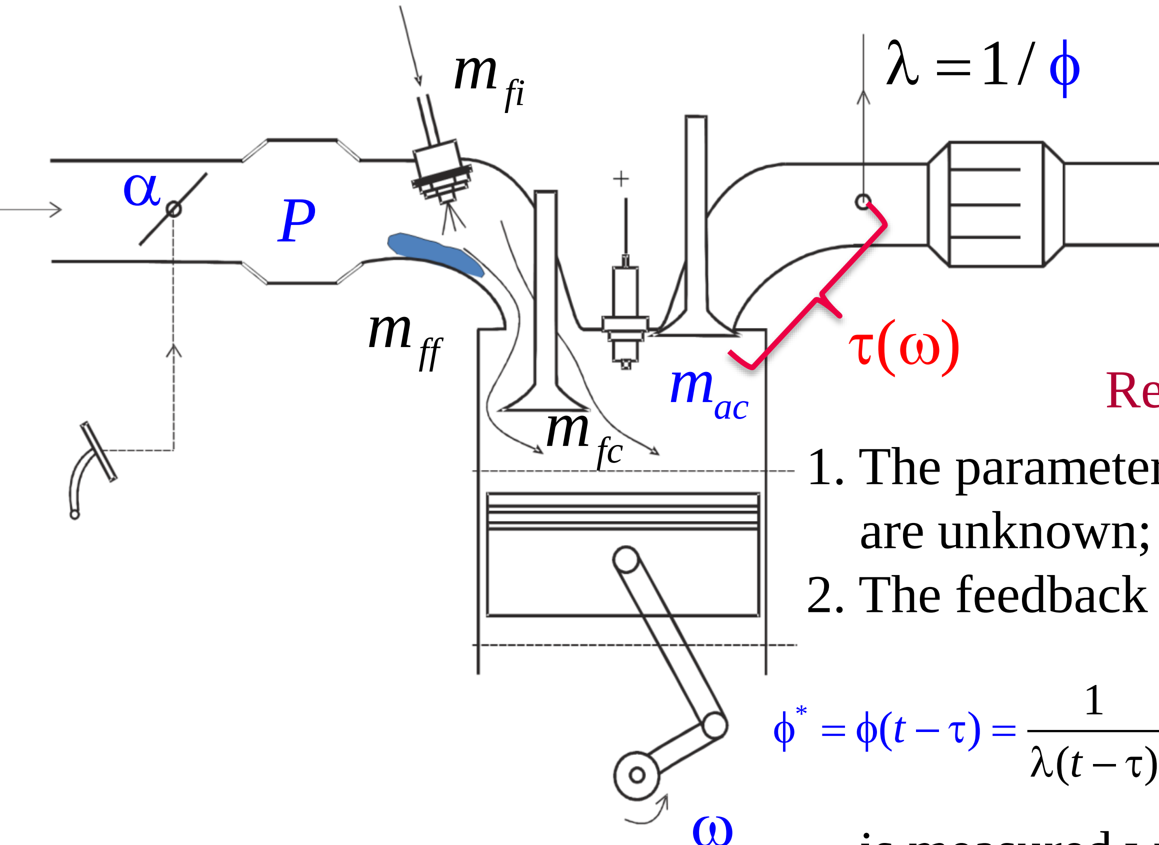
$$\begin{cases} \dot{m}_{ff} = -\dfrac{1}{T} m_{ff} + \dfrac{K}{T} m_{fi} \\ m_{fc} = m_{ff} + (1-K) m_{fi} \end{cases}$$

Real-life problems

1. The parameters of evaporation dynamics are unknown;

Spark ignition engines

Scheme



Fuel evaporation process dynamics

$$\begin{cases} \dot{m}_{ff} = -\dfrac{1}{T} m_{ff} + \dfrac{K}{T} m_{fi} \\ m_{fc} = m_{ff} + (1-K) m_{fi} \end{cases}$$

Real-life problems

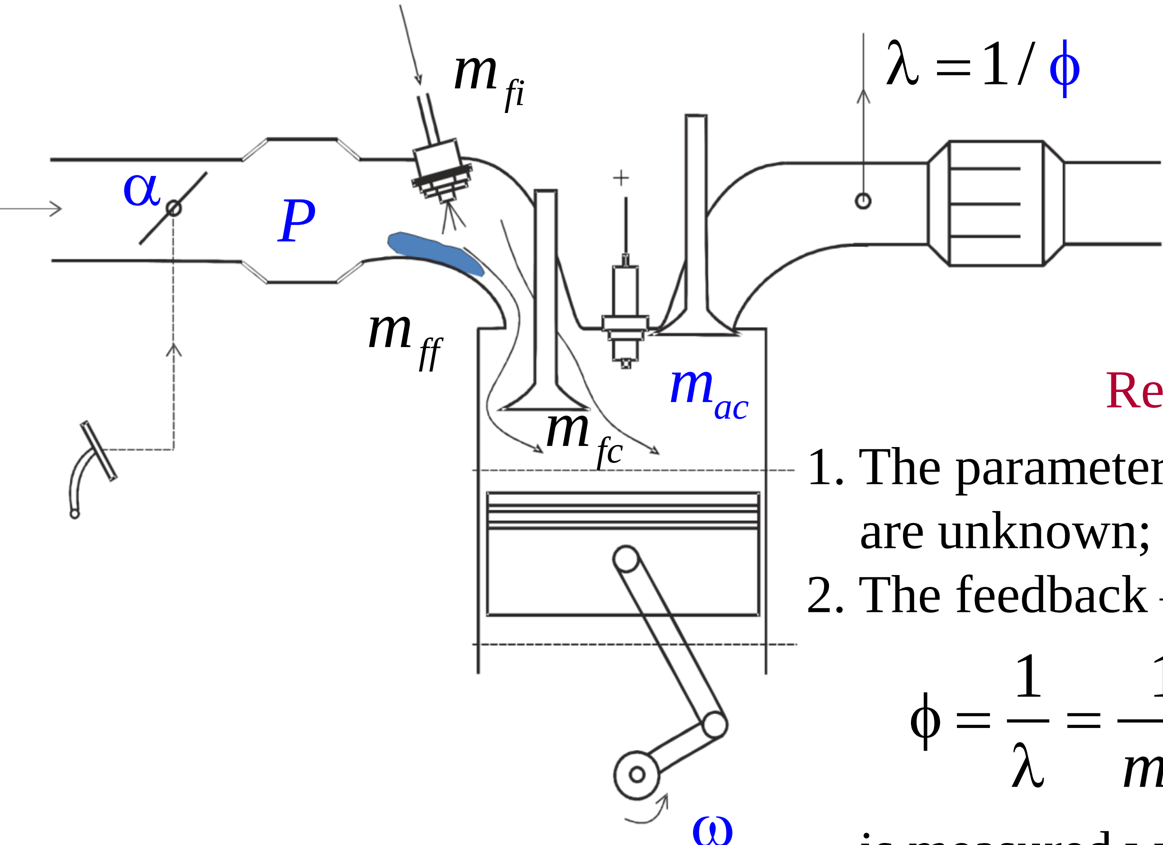
1. The parameters of evaporation dynamics are unknown;
2. The feedback — air-to-fuel ratio AFR (FAR)

$$\phi^* = \phi(t-\tau) = \frac{1}{\lambda(t-\tau)} = \frac{1}{m_{ac}(t-\tau)}\left(m_{ff}(t-\tau) + (1-K) m_{fi}(t-\tau)\right)$$

is measured with time-varying unknown delay;

Spark ignition engines

Scheme



Fuel evaporation process dynamics

$$\begin{cases} \dot{m}_{ff} = -\dfrac{1}{T} m_{ff} + \dfrac{K}{T} m_{fi} \\ m_{fc} = m_{ff} + (1-K) m_{fi} \end{cases}$$

Real-life problems

1. The parameters of evaporation dynamics are unknown;
2. The feedback — air-to-fuel ratio AFR (FAR)

$$\phi = \frac{1}{\lambda} = \frac{1}{m_{ac}} \left(m_{ff} + (1-K) m_{fi} \right)$$

   is measured with time-varying unknown delay;
3. The amount of air in cylinders m_{ac} is not known exactly.

$$m_{ac} = c\eta(P, \omega) / \omega.$$

Blood system

u

y

Blood pressure dynamics with delay

$$y(t) = \frac{Ke^{-T_1 s}\left(1 + ae^{-T_2 s}\right)}{1 + \tau s}\left[u(t)\right]$$

y is the deviation of mean arterial pressure from normal;

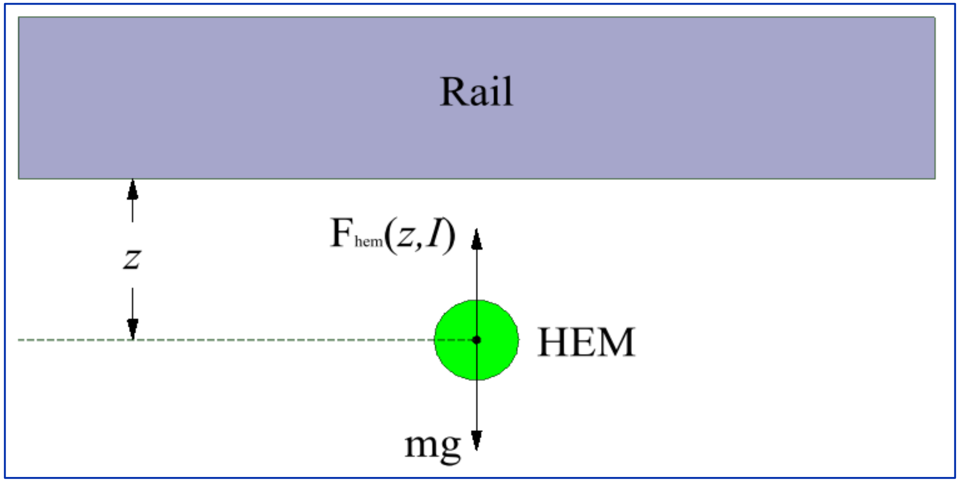
u is the infusion rate of drug (nitroprusside);

T_1, T_2 are the initial and recirculation transport delays;

K is the sensitivity to the drag;

a is the recirculation fraction; τ is the lag.

Maglev system



$$\begin{cases} \dot{z} = v \\ \dot{v} = a \\ \dot{a} = \varphi_v^T \theta_v v + \varphi_u u \\ \dot{I} = u \end{cases}$$

$F_{HEM}(z, I) = \varphi^T \theta$ *– is the hybrid electromagnetic force*

In this context, the approaches of control theory that can come up with the problems of plants uncertainties are of special interest.

Can the control system choose the correct control to improve the performance of the plant operating in the presence of uncertainties?

In this context, the approaches of control theory that can come up with the problems of plants uncertainties are of special interest.

Can the control system choose the correct control to improve the performance of the plant operating in the presence of uncertainties?

In this context, the approaches of control theory that can come up with the problems of plants uncertainties are of special interest.

Can the control system choose the correct control to improve the performance of the plant operating in the presence of uncertainties?

How to design an adaptive control?

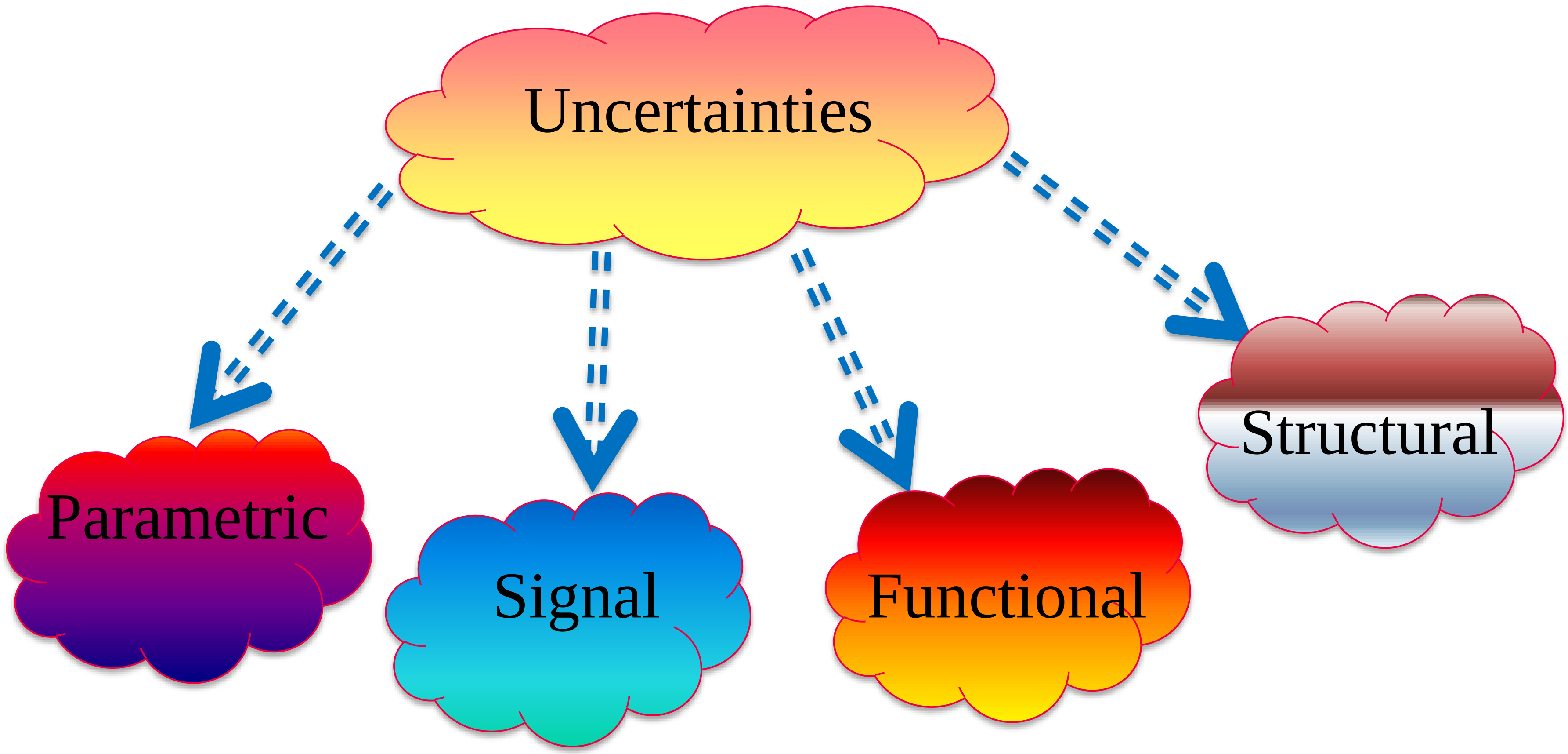
Some definitions

1. **Uncertainties** are unknown or not known precisely characteristics, structure or parameters of the plant;

2. Uncertainties of the plant = uncertainties of the model;

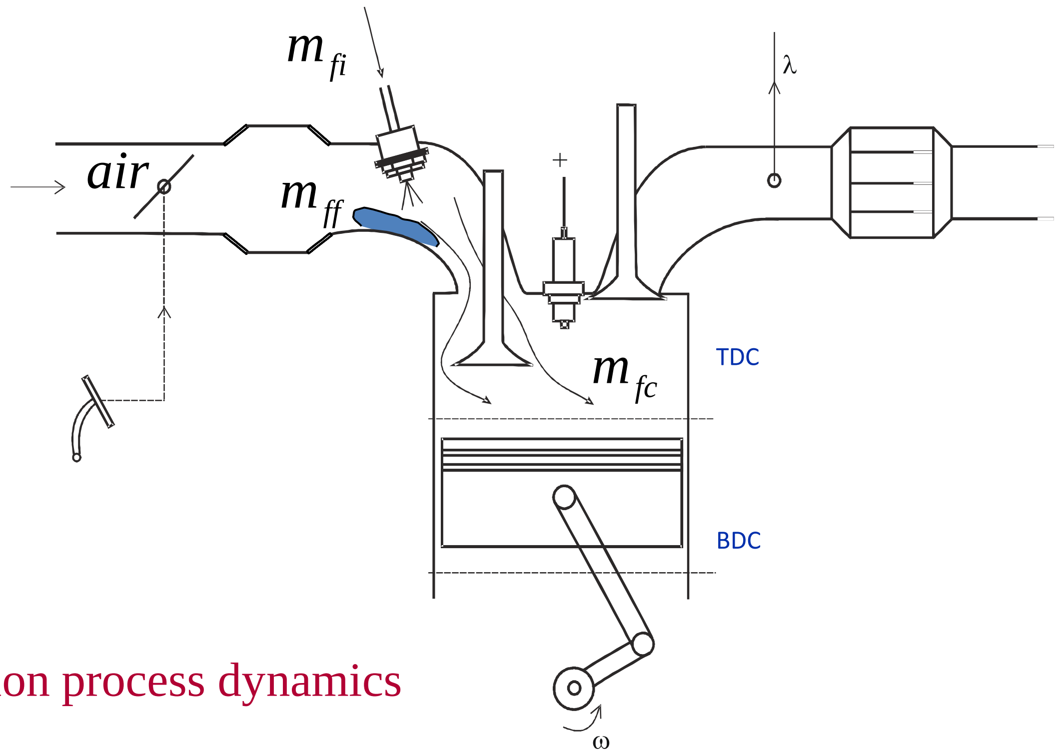
3. Model with uncertainties **used for controller design** is called **nominal**;

4. Characteristics, structure or parameters of the nominal model are called **nominal**.


Uncertainties
Parametric
Signal
Functional
Structural

Parametric uncertainties imply that the parameters of the plant model are constant and unknown.

Spark ignition engines



Fuel evaporation process dynamics

$$\dot{m}_{ff} = -\frac{1}{T} m_{ff} + \frac{K}{T} m_{fi}$$

$$m_{fc} = m_{ff} + (1-K) m_{fi}$$

Signal uncertainties imply that the plant model contains unknown functions of time.

DC motors

DC motor dynamics

$$\dot{I} = -\frac{R}{L} I - \frac{k_E}{L}\omega + \frac{1}{L}U,$$

$$\dot{\omega} = \frac{k_M}{J} I - \frac{1}{J} M_L,$$

$$\dot{\alpha} = \omega$$

$$\boxed{R = R(temperature) = R(time)}$$

Functional uncertainties imply that plant model contains unknown functions of state.

Tail-shaft dynamics equation

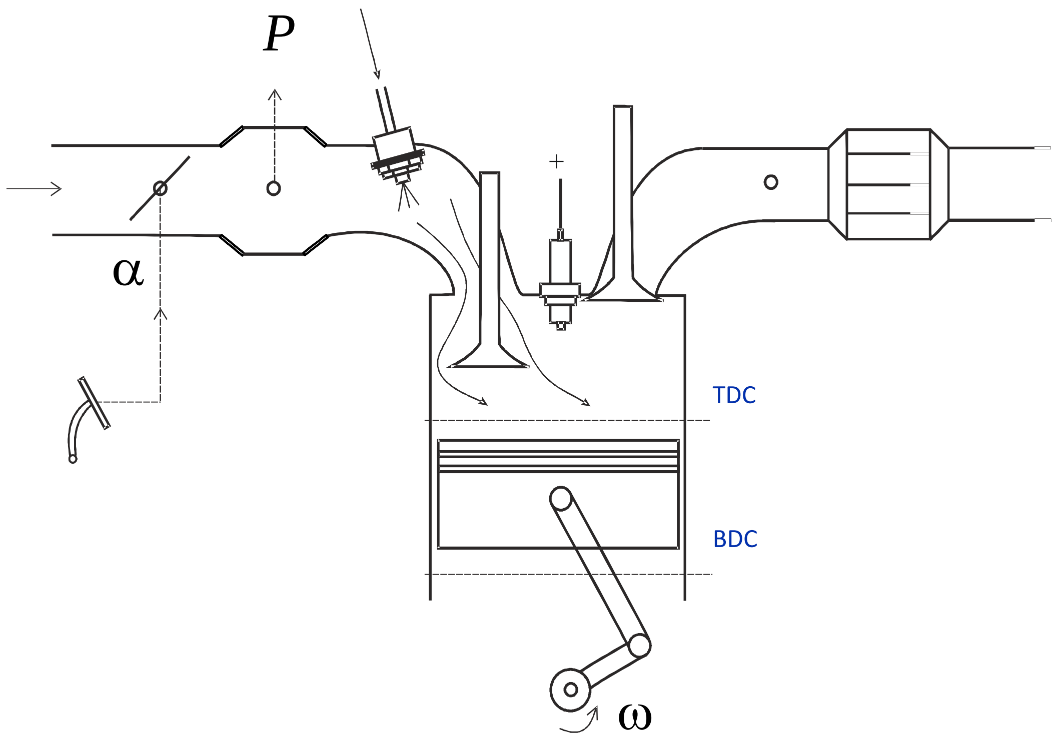
$$J\dot{\omega} = M - M_V$$

M is the engine effective torque

M_V is the viscous friction

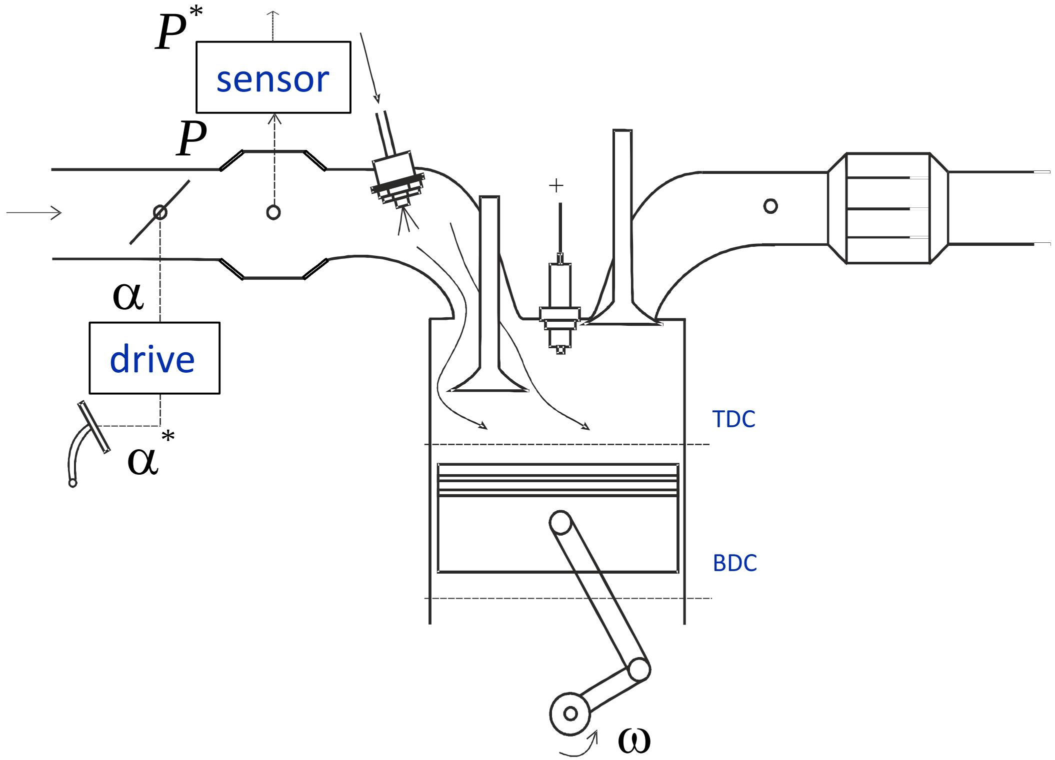
$$\boxed{M_V = M_V(\omega) \approx c_0 + c_1\omega + c_2\omega^2}$$

Structural uncertainties imply that the plant model contains unknown structures.



Manifold air pressure dynamics equation: $\dot{\boldsymbol{P}} + k_1\eta_c(\omega)\boldsymbol{P} = k_2\eta_t(\boldsymbol{P})\varphi_1(\boldsymbol{P})\varphi_2(\boldsymbol{\alpha})$

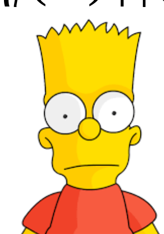
Structural uncertainties imply that the plant model contains unknown structures.



Manifold air pressure dynamics equation: $\dot{\boldsymbol{P}} + k_1\eta_c(\omega)\boldsymbol{P} = k_2\eta_t(\boldsymbol{P})\varphi_1(\boldsymbol{P})\varphi_2(\boldsymbol{\alpha})$

Pressure sensor dynamics: $\dot{\boldsymbol{P}}^* = -a\boldsymbol{P}^* + b\boldsymbol{P}$

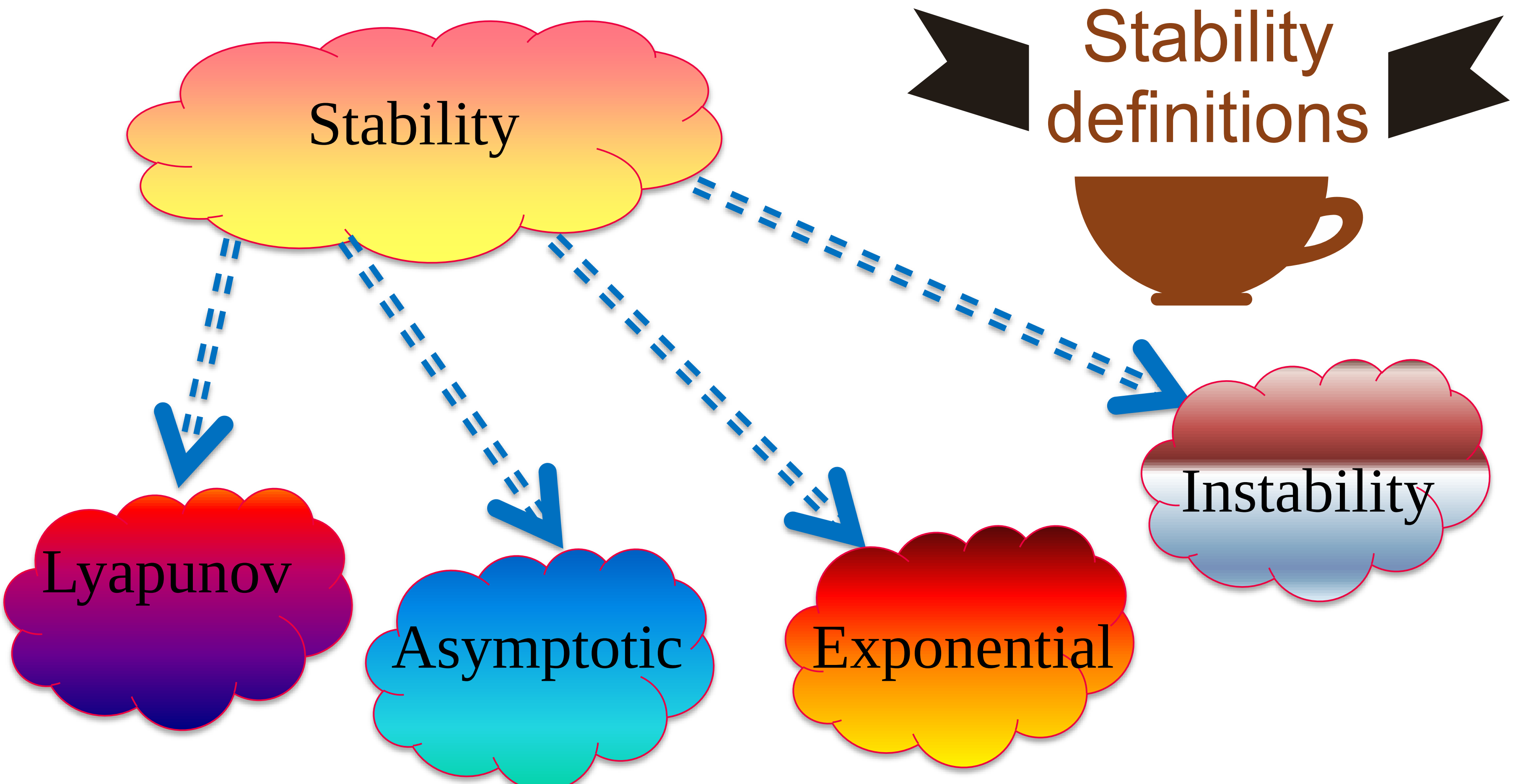
Throttle drive dynamics: $\dot{\alpha} = -c\alpha + d\alpha^*$



Adaptive and robust control definitions

1. adaptive control is the control that tunes its own parameters in a changing environment to provide acceptable performance. AC implies the compensation of uncertainties.
2. robust control is a control that is insensitive with respect to uncertainties and disturbances. RC typically does not imply the compensation of uncertainties, but uses a sufficiently high feedback gains.

2. Lyapunov Functions Method. Short tutorial



Consider an autonomous nonlinear system

$$\dot{x} = f(x), \quad x(0),$$

at equilibrium x^*, where $x \in \mathbb{R}^n$ is the state vector, $f(x) \in \mathbb{R}^n \cap C^1$ is a nonlinear mapping.

Consider an autonomous nonlinear system

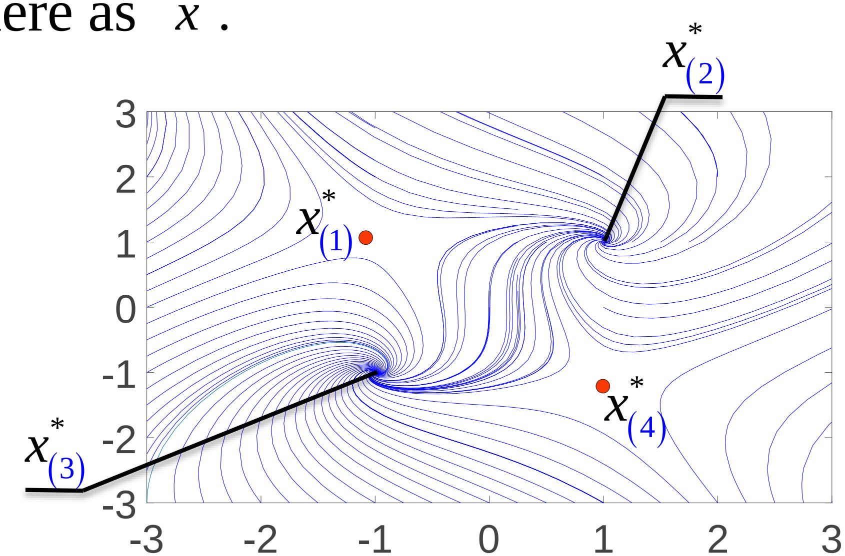
$$\dot{x} = f(x), \quad x(0),$$

at equilibrium x^*, where $x \in \mathbb{R}^n$ is the state vector, $f(x) \in \mathbb{R}^n \cap C^1$ is a nonlinear mapping.

***Remark 2.1*:** there can be several equilibrium points with different stability properties, hence the need to investigate the stability in a particular equilibrium point denoted here as x^*.

Example 2.1:

$$\begin{cases} \dot{x}_1 = x_1^2 - x_2^2 \\ \dot{x}_2 = x_1^2 + x_2^2 - 2 \end{cases}$$



Lyapunov stability

1. The system $\dot{x} = f(x)$ is Lyapunov stable at the equilibrium x^* if

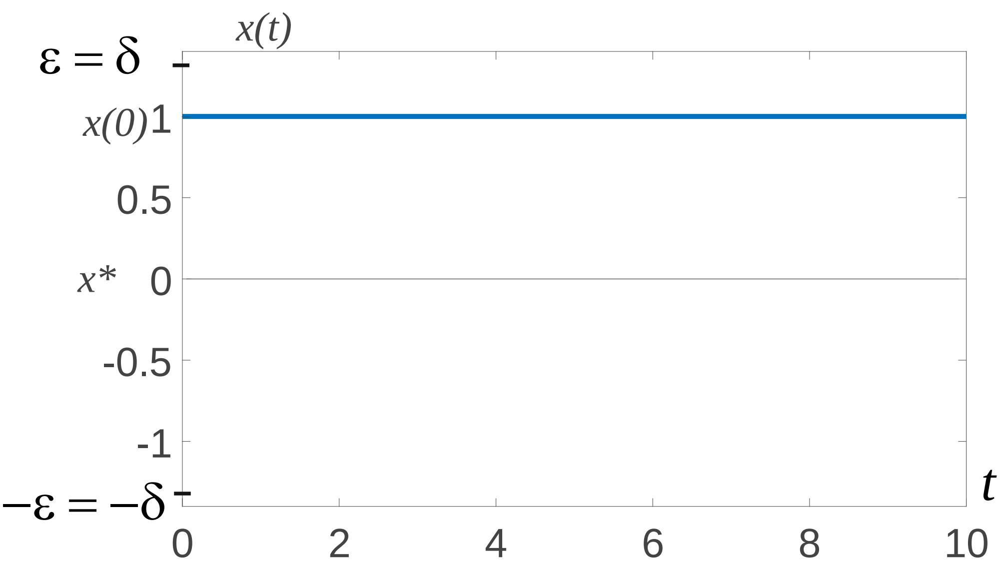
   $\forall\ \varepsilon > 0\ \ \exists\ \delta(\varepsilon) > 0:$ if $\left\| x(0) - x^* \right\| \le \delta$, then $\left\| x(t) - x^* \right\| \le \varepsilon\ \ \forall t > 0.$

Lyapunov stability

1. The system $\dot{x} = f(x)$ is Lyapunov stable at the equilibrium x^* if $\forall\ \varepsilon > 0\ \exists\ \delta(\varepsilon) > 0:$ if $\|x(0) - x^*\| \le \delta$, then $\|x(t) - x^*\| \le \varepsilon\ \ \forall t > 0.$

Example 2.2:

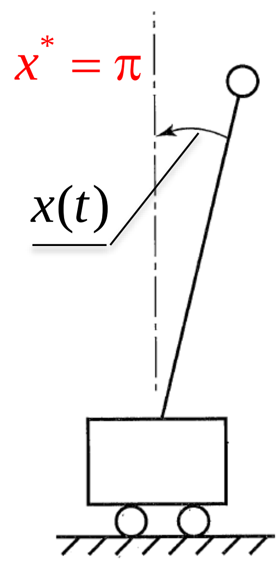
$\dot{x} = 0$

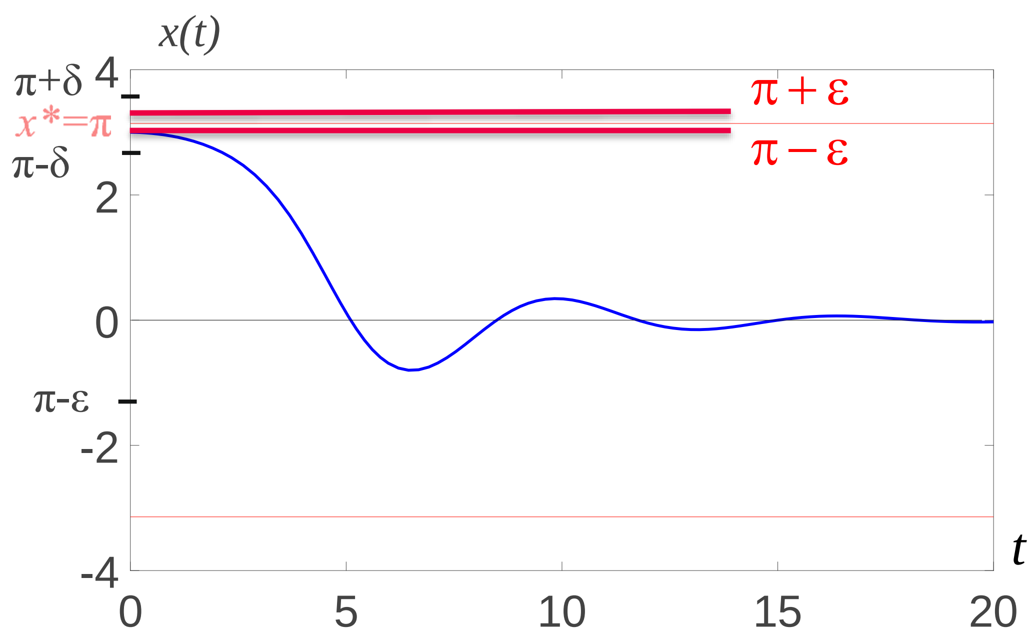


Lyapunov stability

1. The system $\dot{x} = f(x)$ is Lyapunov stable at the equilibrium x^* if
$\forall\ \varepsilon > 0\ \exists\ \delta(\varepsilon) > 0:$ if $\left\| x(0) - x^* \right\| \le \delta$, then $\left\| x(t) - x^* \right\| \le \varepsilon\ \ \forall t > 0.$

What the difference between unstable and Lyapunov stable system???





Asymptotic stability

2. The system $\dot{x} = f(x)$ is asymptotically stable at the equilibrium x^* if

a) it is Lyapunov stable at x^*;

b) the equilibrium point is attractive, i.e.

$$\lim_{t\to\infty} \left\| x(t) - x^* \right\| = 0.$$

Asymptotic stability

2. The system $\dot{x} = f(x)$ is asymptotically stable at the equilibrium x^* if

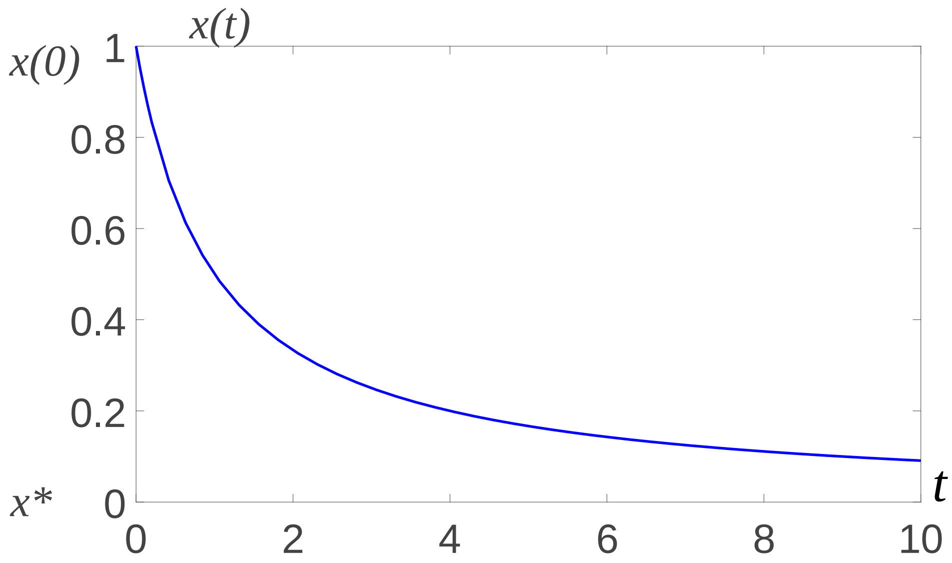
a) it is Lyapunov stable at x^*;

b) the equilibrium point is attractive, i.e.

$$\lim_{t\to\infty}\left\|x(t) - x^*\right\| = 0.$$

Example 2.3:

$$\dot{x} = -\frac{1}{t+1}x$$



Exponential stability

3. The system $\dot{x} = f(x)$ is exponentially stable at the equilibrium x^* if

a) it is asymptotically stable at x^*;

b) there exist such coefficients $\alpha > 0,\ \beta = \beta(x(0), x^*) > 0$ that

$$\left\| x(t) - x^* \right\| \le \beta e^{-\alpha t}.$$

Exponential stability

3. The system $\dot{x} = f(x)$ is exponentially stable at the equilibrium x^* if

a) it is asymptotically stable at x^*;

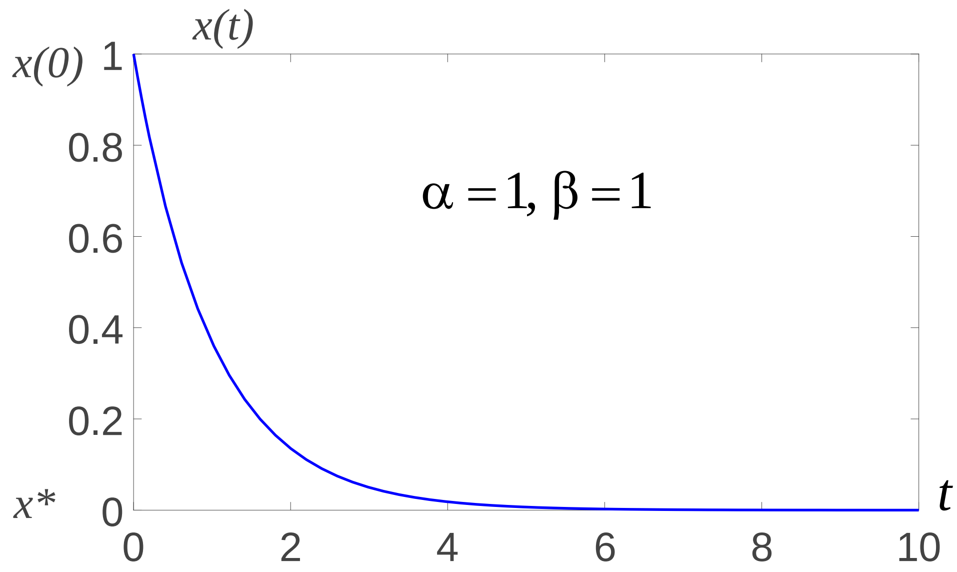
b) there exist such coefficients $\alpha > 0,\ \beta = \beta(x(0), x^*) > 0$ that

$$\left\| x(t) - x^* \right\| \le \beta e^{-\alpha t}.$$

Example 2.4:

$$\dot{x} = -x$$

$$\left(x(t) = e^{-t} x(0) \right)$$



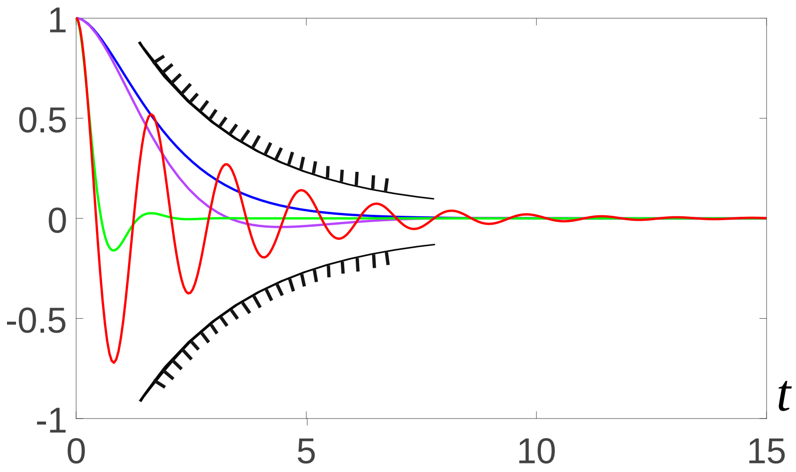
Instability

4. The system $\dot{x} = f(x)$ is unstable at x^* when it is not stable at x^*.

Why the exponential stability is more preferable?

Why the exponential stability is more preferable?

1. **it makes possible to regulate the transient time and overshooting**



Why the exponential stability is more preferable?

1. it makes possible to regulate the transient time and overshooting;
2. the property of exponential stability is robust with respect to disturbances, i.e. it is preserved in the presence of small (in some sense) variations of the system parameters or of small disturbances.

Why the exponential stability is more preferable?

1. it makes possible to regulate the transient time and overshooting;
2. the property of exponential stability is robust with respect to disturbances, i.e. it is preserved in the presence of small (in some sense) variations of the system parameters or of small disturbances.

***Example 2.5*: Given asymptotically and exponentially stable systems**

$$\dot{x} = -\frac{1}{t+1}x \qquad\qquad \dot{x} = -x$$

Why the exponential stability is more preferable?

1. **it makes possible to regulate the transient time and overshooting;**
2. **the property of exponential stability is robust with respect to disturbances, i.e. it is preserved in the presence of small (in some sense) variations of the system parameters or of small disturbances.**

***Example 2.5*: Given asymptotically and exponentially stable systems**

$0 < \mu << 1$ is the parameter perturbation

$$\dot{x} = -\frac{1}{t+1}x \qquad \dot{x} = -x$$

$$\dot{x} = -\frac{1}{t+1}x + \mu x \qquad \dot{x} = -x + \mu x$$

Why the exponential stability is more preferable?

1. **it makes possible to regulate the transient time and overshooting;**
2. **the property of exponential stability is robust with respect to disturbances, i.e. it is preserved in the presence of small (in some sense) variations of the system parameters or of small disturbances.**

***Example 2.5*: Given asymptotically and exponentially stable systems**

$0<\mu<<1$ is the parameter perturbation

Unstable after $t>\frac{1}{\mu}-1$!!!

$$\dot{x}=\left(-\frac{1}{t+1}+\mu\right)x$$

Stable for any $t\geq 0$

$$\dot{x}=-(1+\mu)x$$

2. Lyapunov Functions Method. Short tutorial

Universal approach of stability analysis for autonomous plants

$$\dot{x} = f(x), \quad x(0), \tag{2.1}$$

at equilibrium x^*, where $x \in \mathbb{R}^n$ is the state vector, $f \in \mathbb{R}^n$ is the continuous nonlinear mapping.

Lyapunov functions $V(x)$ **:**

1. $V(x)$ is monotonic ;
2. $V(x) > 0 \quad if \quad \|x\| \neq 0$;

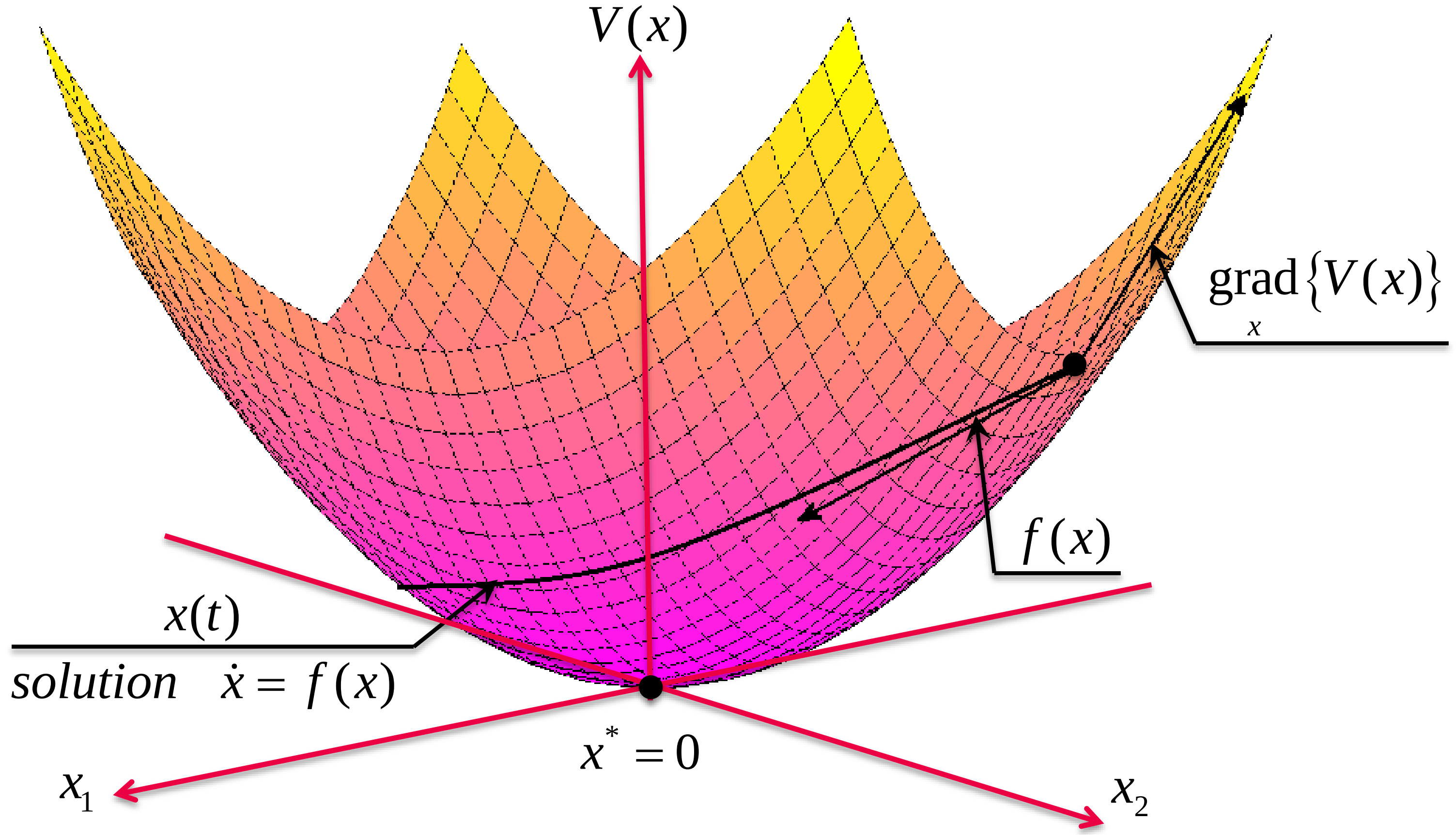
   $V(0) = 0$;
3. $V(x) \in C^1$ (continuous and differentiable in x and t)

Time derivative of Lyapunov function in view of (2.1):

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\dot{x}$$

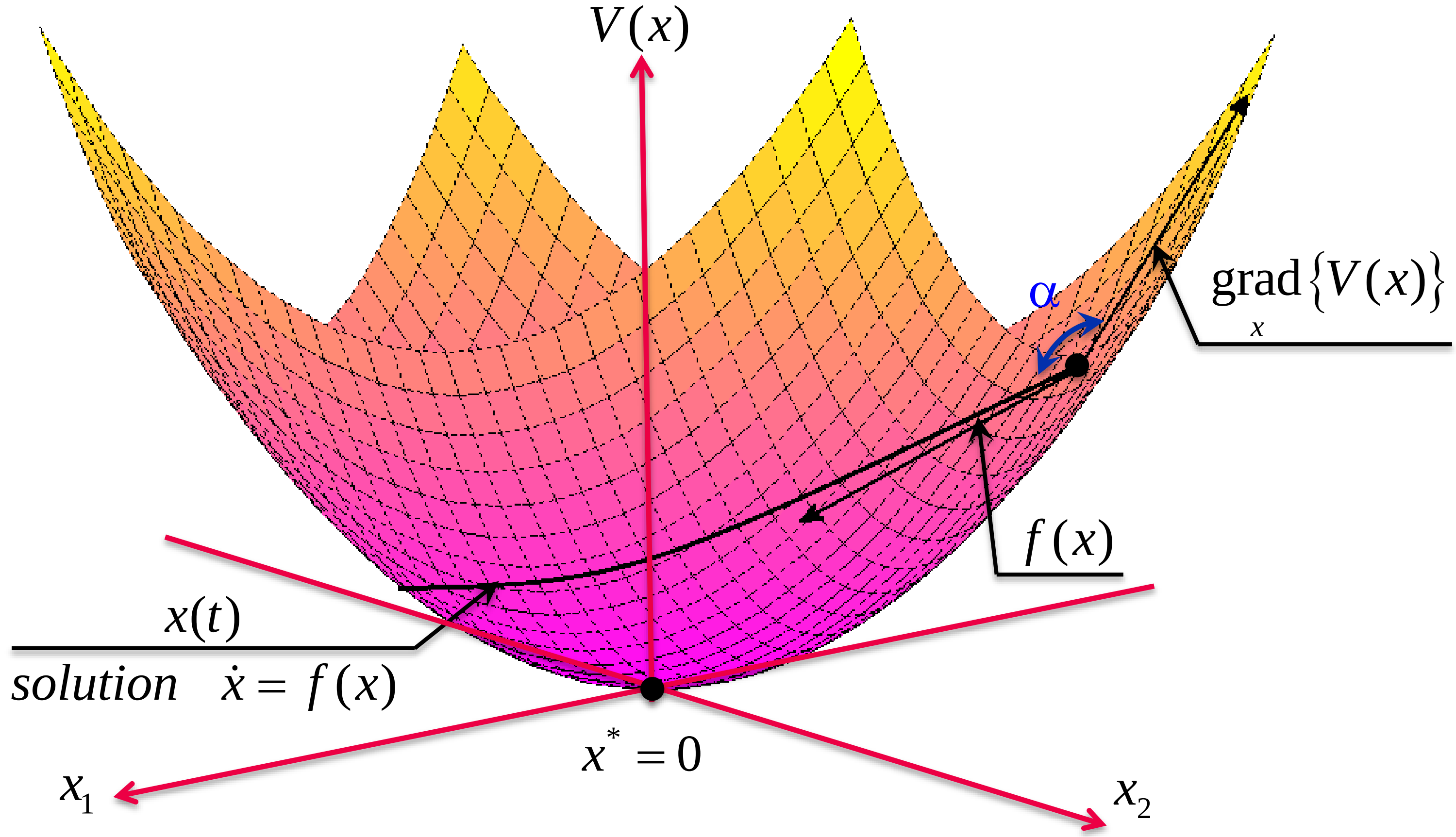
Time derivative of Lyapunov function in view of (2.1):

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\dot{x} = \frac{\partial V(x)}{\partial x} f(x) = \underset{x}{\operatorname{grad}}\{V(x)\} f(x)$$



Time derivative of Lyapunov function in view of (2.1):

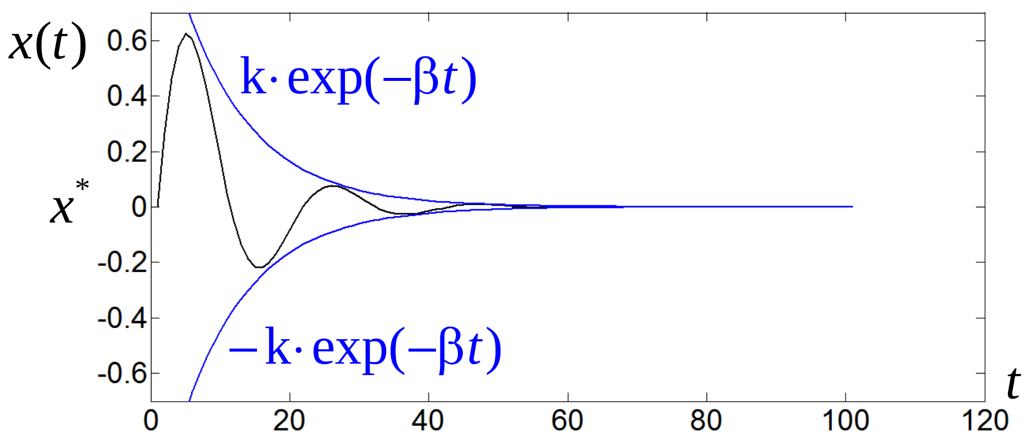
$$\dot{V}(x) = \frac{\partial V(x)}{\partial x}\dot{x} = \frac{\partial V(x)}{\partial x} f(x) = \operatorname*{grad}_x \{V(x)\} f(x) = \left\| \operatorname*{grad}_x \{V(x)\} \right\| \| f(x) \| \cos\alpha$$



Stability criterias:

1. If $\dot{V}(x) \le 0$, then the equilibrium $x^* = 0$ is Lyapunov stable;
2. If $\dot{V}(x) < 0$, then the equilibrium $x^* = 0$ is asymptotically stable;
3. If $c_1\|x\|^2 \le V(x) \le c_2\|x\|^2$ and $\dot{V}(x) \le -c_3\|x\|^2$, $c_1, c_2, c_3 > 0$, then the equilibrium $x^* = 0$ is exponentially stable.

$$\dot{V}(x) \le -\frac{c_3}{c_1}V(x) \quad \Rightarrow \quad V(t) \le \mathrm{e}^{-\frac{c_3}{c_1}t}V(0) \quad \Rightarrow \quad \|x(t)\|^2 \le \frac{c_2}{c_1}\mathrm{e}^{-\frac{c_3}{c_1}t}\|x(0)\|^2$$



Examples of Lyapunov functions:

1. Linear system

$$\dot{x} = Ax, \quad x(0) \tag{2.2}$$

where A is the time-invariant matrix.

Lyapunov function candidate

$$V(x) = x^T Px, \tag{2.3}$$

where $P = P^T \succ 0$ is the time-invariant matrix.

$$\dot{V}(x) = \dot{x}^T Px + x^T P\dot{x} = x^T A^T Px + x^T PAx =$$

$$= x^T \left(A^T P + PA\right)x = -x^T Qx < 0$$

Conclusion: if for any $Q = Q^T \succ 0$ there exists $P = P^T \succ 0$ such that

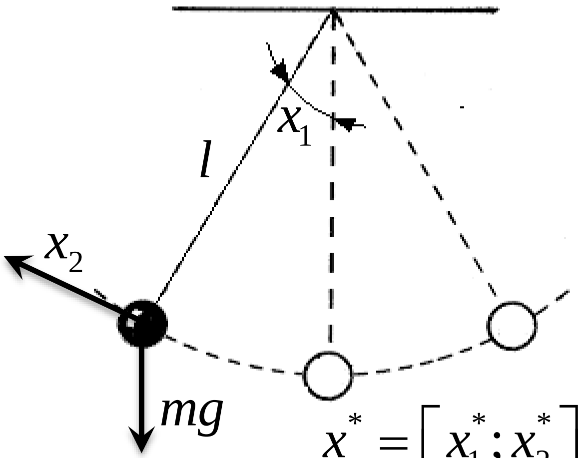
$$A^T P + PA = -Q, \tag{2.4}$$

where $Q = Q^T \succ 0$, system (2.2) is asymptotically stable.

2. Pendulum

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\frac{g}{l}\sin(x_1) - \frac{k}{m}x_2 \tag{2.5}$$

where g is the gravity acceleration, l is the length of rod, k is the friction coefficient.



Lyapunov function candidate: **sum of potential and kinetic energy**

$$V(x) = mgl\left(1-\cos(x_1)\right) + \frac{mx_2^2}{2}l^2. \tag{2.6}$$

Time derivative:

$$\dot{V}(x) = mgl\sin(x_1)\dot{x}_1 + mx_2\dot{x}_2 l^2 = mgl\sin(x_1)x_2 + mx_2\left(-\frac{g}{l}\sin(x_1) - \frac{k}{m}x_2\right)l^2.$$

or

$$\dot{V}(x) = -l^2kx_2^2 < 0 \tag{2.7}$$

Conclusion: pendulum is asymptotically stable at the equilibrium $x^* = [0;0]$.

3. Simple Example of Adaptive Controller Design

Motivation

3. Simple Example of Adaptive Controller Design

Motivation

Can the classic theory come up with the problems of uncertainties?

3. Simple Example of Adaptive Controller Design

Motivation

Problem statement:

Plant:

$$\dot{x} = \theta x + u, \tag{3.1}$$

where x is the scalar state, u is the control, θ is the known parameter.

Objective is to design a control providing the following limiting equality:

$$\lim_{t\to\infty} x(t) = 0. \tag{3.2}$$

3. Simple Example of Adaptive Controller Design

Motivation

Problem statement:

Plant:

$$\dot{x} = \theta x + u, \tag{3.1}$$

where x is the scalar state, u is the control, θ is the known parameter.

Objective is to design a control providing the following limiting equality:

$$\lim_{t \to \infty} x(t) = 0. \tag{3.2}$$

Solution:

$$u = -\theta x - \lambda x, \tag{3.3}$$

where λ is the positive constant parameter.

$$u = -\theta x - \lambda x, \qquad \dot{x} = \theta x + u \quad \Rightarrow \quad \dot{x} = -\lambda x \quad \Rightarrow \quad x(t) = \exp(-\lambda t)\, x(0). \tag{3.4}$$

$$u = -\theta x - \lambda x, \quad \dot{x} = \theta x + u \quad \Rightarrow \quad \dot{x} = -\lambda x \quad \Rightarrow \quad x(t) = \exp(-\lambda t)\, x(0). \tag{3.4}$$

Let us the design parameter is $\lambda = 1$ and plant parameter $\theta = 5$,

i.e.

control: $u = -6x$

system $\dot{x} = -x$ is stable.

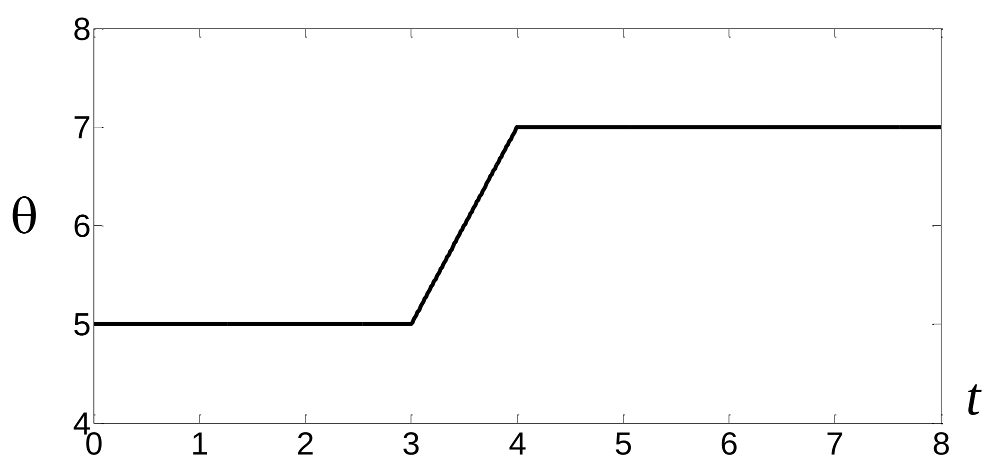
$$u = -\theta x - \lambda x, \quad \dot{x} = \theta x + u \quad \Rightarrow \dot{x} = -\lambda x \Rightarrow x(t) = \exp(-\lambda t)\,x(0). \tag{3.4}$$

Let us the design parameter is $\lambda = 1$ and plant parameter $\theta = 5$, i.e.

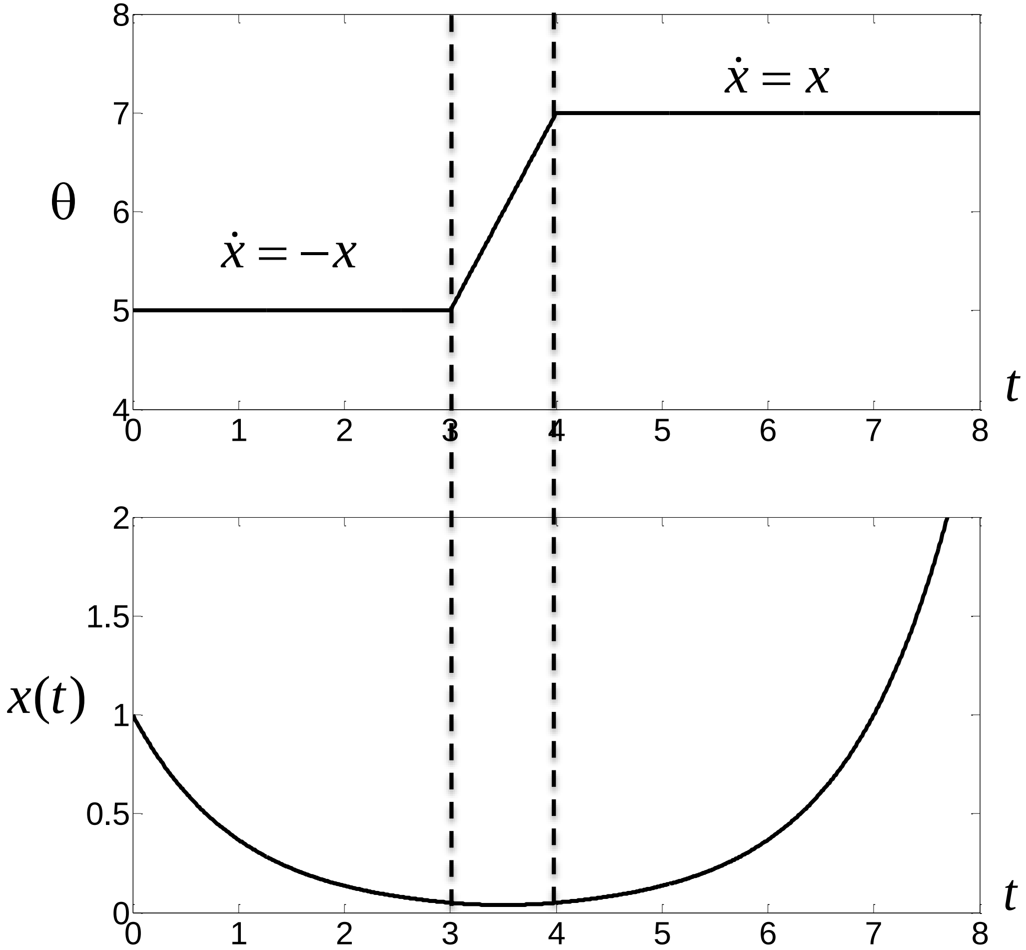
control: $u = -6x$

system $\dot{x} = -x$ is stable.

Now let us imagine, the plant parameter θ unpredictably changes from 5 to 7:



$$Control: \; u = -6x$$



Classical control is not reliable and does not work properly in presence of uncertainties

Problem statement of adaptive control:

Plant:

$$\dot{x} = \theta x + u, \tag{3.5}$$

where θ is the unknown parameter.

Objective is to design a control providing the following limiting equality:

$$\lim_{t\to\infty}\left(x_M(t) - x(t)\right) = 0, \tag{3.6}$$

where x_M is the output of reference model

$$\dot{x}_M = -\lambda x_M + \lambda g, \tag{3.7}$$

g is the piece-wise continuous and bounded reference signal, λ is the positive parameter responsible for transient time.

Solution:

1. Let the parameter θ be known.

Form the error signal $\varepsilon = x_M - x$ and take its derivative in view of plant and reference model equations:

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + u\right)$$

Solution:

1. Let the parameter θ be known.

Form the error signal $\varepsilon = x_M - x$ and take its derivative in view of plant and reference model equations:

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + u\right)$$

What control to choose to provide the exponential decaying of error?

Solution:

1. Let the parameter θ be known.

Form the error signal $\varepsilon = x_M - x$ and take its derivative in view of plant and reference model equations:

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + u\right)$$

Let $\dot{\varepsilon} \triangleq -\lambda\varepsilon = -\lambda x_M + \lambda x \;\Rightarrow\; \varepsilon(t) = e^{-\lambda t}\varepsilon(0)$. Therefore

$$\left(-\lambda x_M + \lambda g\right) - \left(\theta x + u\right) = -\lambda x_M + \lambda x$$

$$\Downarrow$$

$$\boxed{u = -\theta x - \lambda x + \lambda g} \qquad (3.8)$$

Solution:

2. Let the parameter θ be unknown. Therefore the control

$$u = -\theta x - \lambda x + \lambda g$$

is not implementable. Substitute estimate $\hat{\theta}$ for θ and obtain implementable adjustable control:

$$\boxed{u = -\hat{\theta}(t)x - \lambda x + \lambda g} \tag{3.9}$$

Solution:

2. Let the parameter θ be unknown. Therefore the control

$$u = -\theta x - \lambda x + \lambda g$$

is not implementable. Substitute estimate $\hat{\theta}$ for θ and obtain implementable adjustable control:

$$\boxed{u = -\hat{\theta}(t)x - \lambda x + \lambda g} \tag{3.10}$$

Replace (3.10) in the plant equation $\dot{x} = \theta x + u$:

$$\dot{x} = \theta x - \hat{\theta}x - \lambda x + \lambda g, \tag{3.11}$$

Solution:

2. Let the parameter θ be unknown. Therefore the control

$$u = -\theta x - \lambda x + \lambda g$$

is not implementable. Substitute estimate $\hat{\theta}$ for θ and obtain implementable adjustable control:

$$\boxed{u = -\hat{\theta}(t)x - \lambda x + \lambda g} \tag{3.10}$$

Replace (3.10) in the plant equation $\dot{x} = \theta x + u$:

$$\dot{x} = \theta x - \hat{\theta}x - \lambda x + \lambda g, \tag{3.11}$$

Take the derivative of the error

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x - \hat{\theta}x - \lambda x + \lambda g\right)$$

Signal Error Model

$$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x,} \tag{3.12}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parametric error.

Solution:

3. Let us choose the algorithm generating estimate $\hat{\theta}$:

$$\boxed{\dot{\hat{\theta}}(t) = \Omega(t)} \tag{3.13}$$

where $\Omega(t)$ is implementable (measurable) function.

Solution:

3. Let us choose the algorithm generating estimate $\hat{\theta}$:

$$\boxed{\dot{\hat{\theta}}(t) = \Omega(t)} \tag{3.13}$$

where $\Omega(t)$ is implementable (measurable) function.

Taking into account that $\tilde{\theta} = \theta - \hat{\theta}$ and

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

we get

Parametric Error Model

$$\boxed{\dot{\tilde{\theta}}(t) = -\Omega(t)} \tag{3.14}$$

Solution:

3. Let us choose the algorithm generating estimate $\hat{\theta}$:

$$\boxed{\dot{\hat{\theta}}(t) = \Omega(t)} \quad (3.13)$$

where $\Omega(t)$ is implementable (measurable) function.

Taking into account that $\tilde{\theta} = \theta - \hat{\theta}$ and

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

we get

Parametric Error Model

$$\boxed{\dot{\tilde{\theta}}(t) = -\Omega(t)} \quad (3.14)$$

How to choose the function $\Omega(t)$???

Solution:

4. Models

Signal Error Model $$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x,} \qquad (3.12)$$

Parametric Error Model $$\boxed{\dot{\tilde{\theta}}(t) = -\Omega(t)} \qquad (3.14)$$

Choose the Lyapunov function candidate

$$V(\varepsilon,\tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \qquad \gamma > 0 \qquad (3.15)$$

and take its time derivative using (3.12) and (3.14):

$$\dot{V}(\varepsilon,\tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \frac{1}{\gamma}\tilde{\theta}\Omega(t)$$

Solution:

4. Models

Signal Error Model $\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x,}$ (3.12)

Parametric Error Model $\boxed{\dot{\tilde{\theta}}(t) = -\Omega(t)}$ (3.14)

Choose the Lyapunov function candidate

$$V(\varepsilon,\tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \qquad \gamma > 0 \qquad (3.15)$$

and take its time derivative using (3.12) and (3.14):

$$\dot{V}(\varepsilon,\tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \frac{1}{\gamma}\tilde{\theta}\Omega(t)$$

Answer?

Solution:

4. Models

Signal Error Model $$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x,} \tag{3.12}$$

Parametric Error Model $$\boxed{\dot{\tilde{\theta}}(t) = -\Omega(t)} \tag{3.14}$$

Choose the Lyapunov function candidate

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \qquad \gamma > 0 \tag{3.15}$$

and take its time derivative using (3.12) and (3.14):

$$\dot{V}(\varepsilon, \tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \frac{1}{\gamma}\tilde{\theta}\Omega(t)$$

If $\Omega(t) = -\gamma x\varepsilon$ then $\dot{V}(\varepsilon, \tilde{\theta}) = -\lambda\varepsilon^2 < 0$

$$\boxed{\dot{\hat{\theta}} = -\gamma x\varepsilon} \tag{3.16}$$

Summary

Adjustable controller:

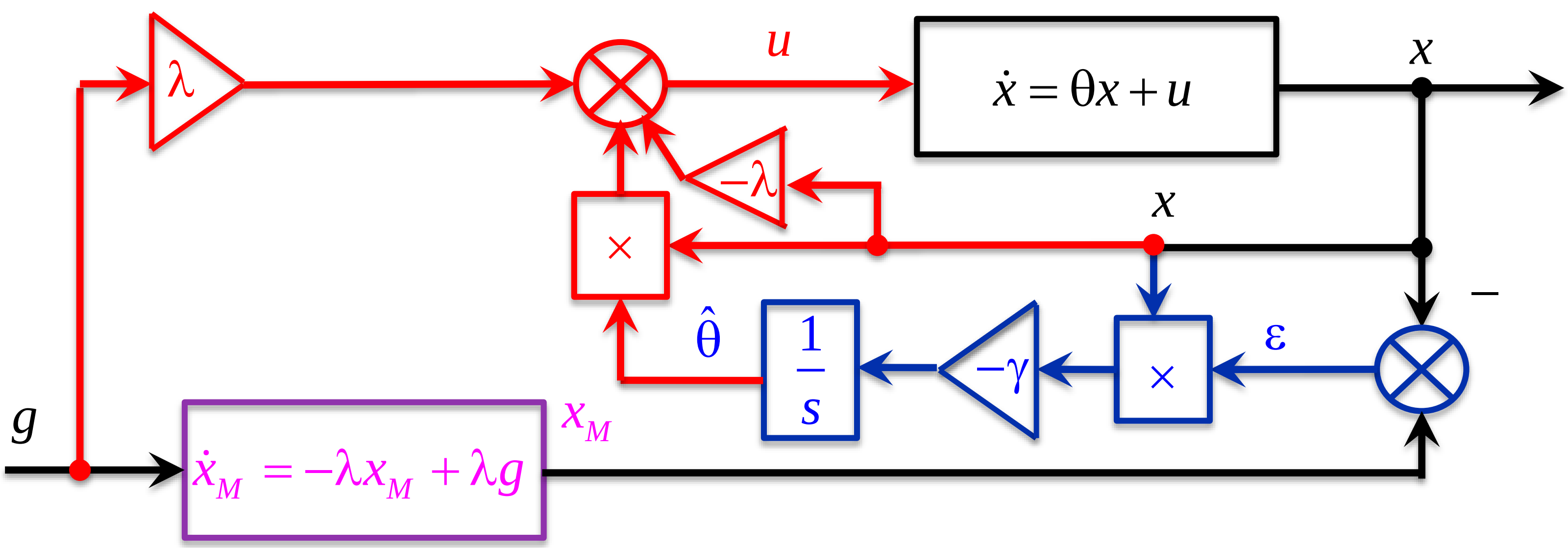
$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{3.10}$$

Adaptation algorithm:

$$\dot{\hat{\theta}} = -\gamma x \varepsilon \tag{3.16}$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g. \tag{3.7}$$



Summary

Properties of the closed-loop system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ asymptotically tends to zero;
3. Parametric error $\tilde{\theta} = \theta - \hat{\theta}$ in general case tends to a constant;

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2,$$

$$\dot{V}(\varepsilon, \tilde{\theta}) = -\lambda\varepsilon^2 < 0$$

4. There is an optimal adaptation gain γ corresponding the fastest parametrical convergence;

Summary

Properties of the closed-loop system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ asymptotically tends to zero;
3. Parametric error $\tilde{\theta} = \theta - \hat{\theta}$ in general case tends to a constant;

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2,$$

$$\dot{V}(\varepsilon, \tilde{\theta}) = -\lambda\varepsilon^2 < 0$$

4. There is an optimal adaptation gain γ corresponding the fastest parametrical convergence;

5. There can be parametric drift phenomena in presence of disturbance, i.e., if

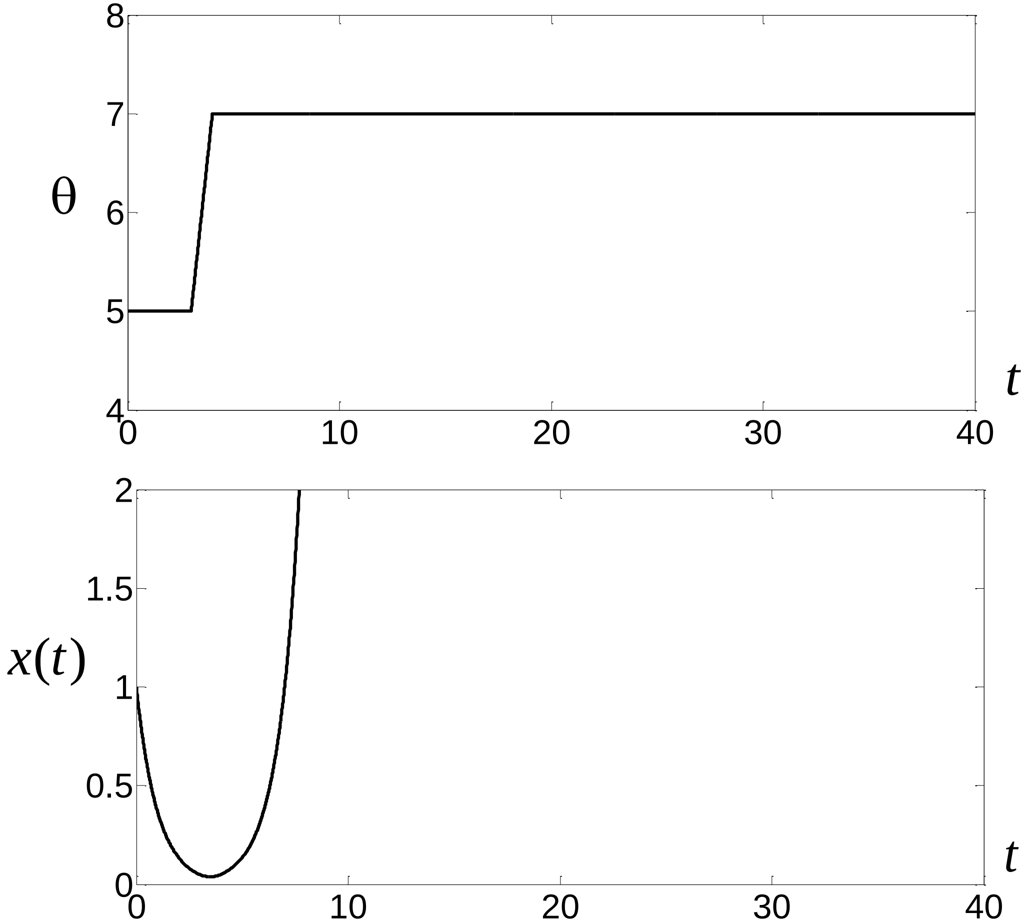
$$\dot{x} = \theta x + u + \delta,$$

where δ is bounded disturbance, $\hat{\theta}(t) \to \infty$.*as* $t \to \infty$

Example: Classical stabilizing control for unstable plant

$$\dot{x} = 5x + u$$

$$u = -6x$$

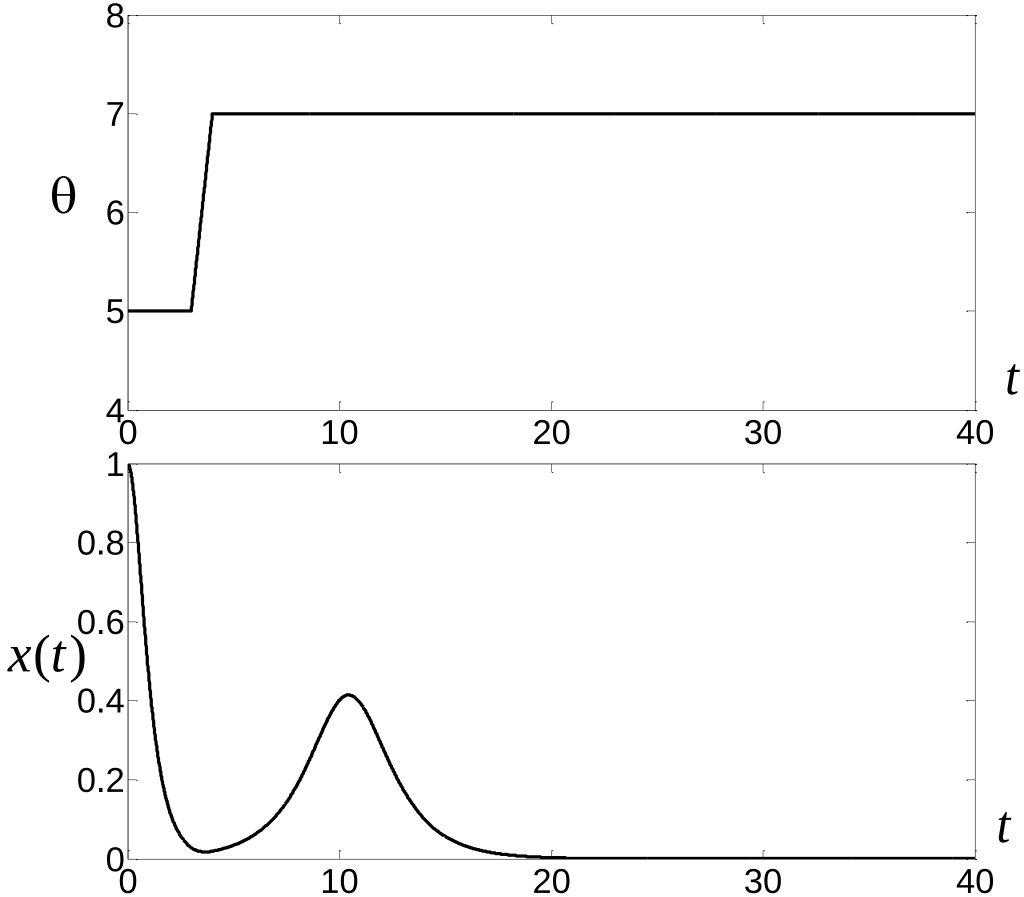


Example: Adaptive stabilizing control for unstable plant $\dot{x} = 5x + u$

$$u = -6x - \hat{\theta}x,$$

$$\dot{\hat{\theta}} = -2x\varepsilon, \qquad \varepsilon = x_M - x,$$

$$\dot{x}_M = -6x_M$$



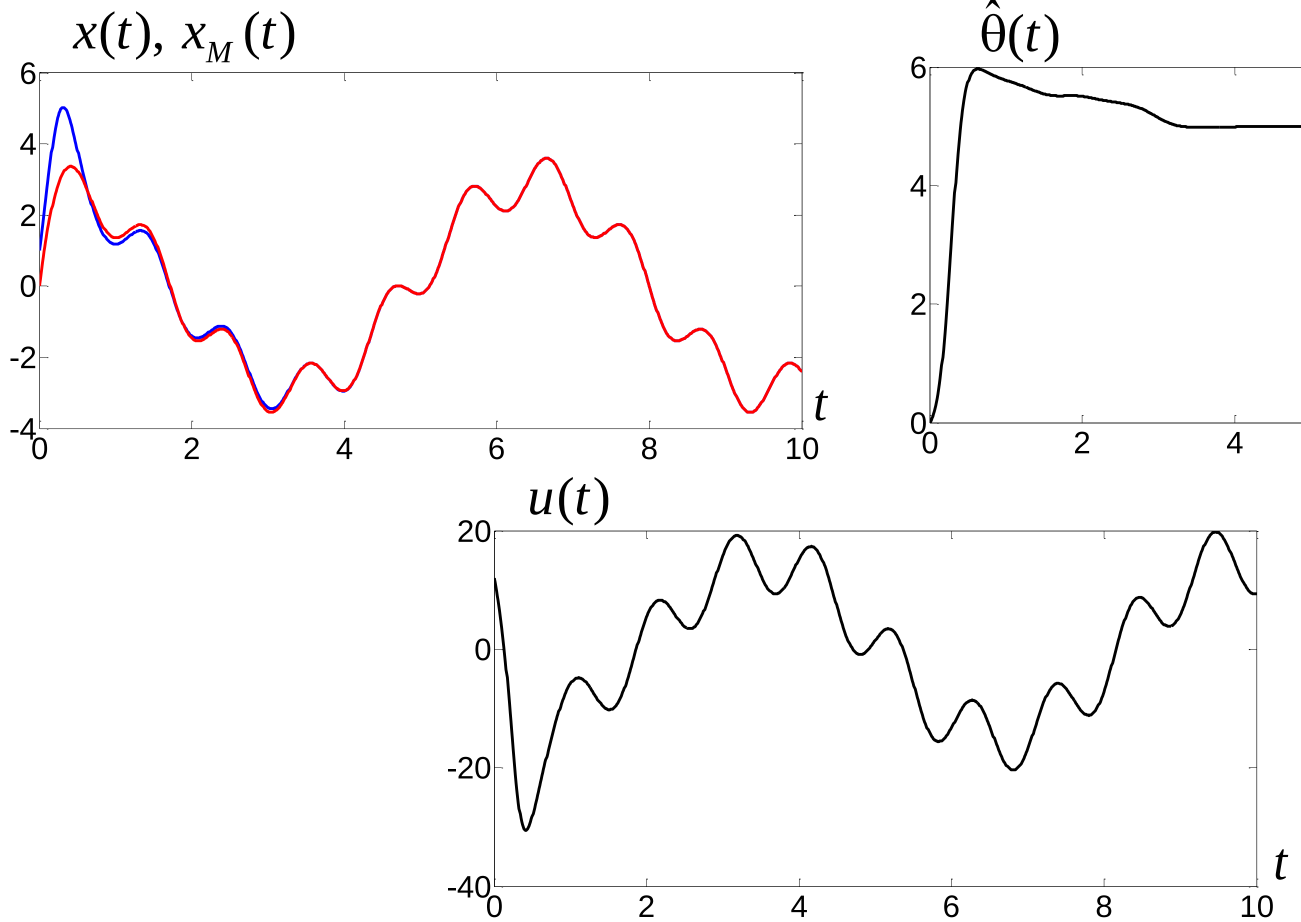
Example: Adaptive tracking control for unstable plant

$$\dot{x} = 5x + u$$

$$u = -6x - \hat{\theta}x + 6g,$$

$$\dot{\hat{\theta}} = -2x\varepsilon, \qquad \varepsilon = x_M - x,$$

$$\dot{x}_M = -6x_M + 6g, \qquad g(t) = \sin 6t + 3\cos t$$



Definitions:

Adaptive and robust control are the controls providing desired performance of the plant operating in presence of uncertainties:

1. adaptive control is the control that tunes its own parameters in a changing environment to provide acceptable performance. AC implies the compensation of uncertainties.
2. robust control is a control that is insensitive with respect to uncertainties and disturbances. RC typically does not imply the compensation of uncertainties, but uses a sufficiently high feedback gains.

Definitions:

Adaptive and robust control are the controls providing desired performance of the plant operating in presence of uncertainties:

1. adaptive control is the control that tunes its own parameters in a changing environment to provide acceptable performance. AC implies the compensation of uncertainties.
2. robust control is a control that is insensitive with respect to uncertainties and disturbances. RC typically does not imply the compensation of uncertainties, but uses a sufficiently high feedback gains.

4. Simple Example of Robust Controller Design

Problem statement of adaptive control:

Plant:

$$\dot{x} = \theta x + u + \delta, \qquad |\delta| \le \bar{\delta} \tag{4.1}$$

where θ is the unknown parameter, $\delta(t)$ is unmeasurable bounded disturbance.

Objective is to design a control providing the following inequality:

$$|x_M(t) - x(t)| \le \Delta \text{ for } any\ t \ge T, \tag{4.2}$$

where x_M is the output of reference model

$$\dot{x}_M = -\lambda x_M + \lambda g, \tag{4.3}$$

g is the piece-wise continuous and bounded reference signal, λ is the positive parameter responsible for the transient time.

How to prevent unbounded growth of the estimates $\hat{\theta}$ in presence of disturbance in the adaptation algorithm $\dot{\hat{\theta}} = -\gamma x \varepsilon$?

Solution #1

Adjustable controller:

$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.4}$$

~~Adaptation algorithm:~~ $\longrightarrow$ **Nonlinear static feedback:**

$$\hat{\theta} = -\gamma x \varepsilon \tag{4.5}$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g.$$

Solution #1

Adjustable controller:

$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.4}$$

~~Adaptation algorithm:~~ $\longrightarrow$ **Nonlinear static feedback:**

$$\hat{\theta} = -\gamma x \varepsilon \tag{4.5}$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g.$$

Substitution of (4.5) into (4.4) gives “high-gain” type controller

$$u = \gamma x^2 \varepsilon - \lambda x + \lambda g.$$

Solution #1

Adjustable controller:

$$u = -\hat{\theta}x - \lambda x + \lambda g \qquad (4.4)$$

~~Adaptation algorithm:~~ $\longrightarrow$ **Nonlinear static feedback:**

$$\hat{\theta} = -\gamma x \varepsilon \qquad (4.5)$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g.$$

Substitution of (4.5) into (4.4) gives "high-gain" type controller:

$$u = \gamma x^2 \varepsilon - \lambda x + \lambda g.$$

Then substitute this control into disturbed plant $\dot{x} = \theta x + u + \delta.$

$$\dot{x} = \theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta.$$

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

$$\boxed{\dot{\varepsilon} = -\lambda \varepsilon - \theta x - \gamma x^2 \varepsilon - \delta} \qquad (4.6)$$

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = (-\lambda x_M + \lambda g) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

$$\boxed{\dot{\varepsilon} = -\lambda \varepsilon - \theta x - \gamma x^2 \varepsilon - \delta} \qquad (4.6)$$

Lyapunov function candidate???

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

$$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \theta x - \gamma x^2 \varepsilon - \delta} \tag{4.6}$$

Choose the Lyapunov function candidate

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2$$

and take its time derivative using (4.6):

$$\dot{V}(\varepsilon, \tilde{\theta}) = \varepsilon\dot{\varepsilon} = -\lambda\varepsilon^2 - \theta x\varepsilon - \gamma x^2 \varepsilon^2 - \delta\varepsilon$$

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

$$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \theta x - \gamma x^2 \varepsilon - \delta} \qquad (4.6)$$

Choose the Lyapunov function candidate

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2$$

and take its time derivative using (4.6):

$$\dot{V}(\varepsilon, \tilde{\theta}) = \varepsilon\dot{\varepsilon} = -\lambda\varepsilon^2 - \theta x\varepsilon - \gamma x^2\varepsilon^2 - \delta\varepsilon = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \theta x\varepsilon - \gamma x^2\varepsilon^2 - \delta\varepsilon =$$

Solution #1

Again, take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\theta x + \gamma x^2 \varepsilon - \lambda x + \lambda g + \delta\right)$$

$$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \theta x - \gamma x^2 \varepsilon - \delta} \qquad (4.6)$$

Choose the Lyapunov function candidate

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2$$

and take its time derivative using (4.6):

$$\dot{V}(\varepsilon) = \varepsilon\dot{\varepsilon} = -\lambda\varepsilon^2 - \theta x\varepsilon - \gamma x^2\varepsilon^2 - \delta\varepsilon = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \theta x\varepsilon - \gamma x^2\varepsilon^2 - \delta\varepsilon =$$

$$= -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon \pm \frac{1}{2\lambda}\delta^2 - \gamma x^2\varepsilon^2 - \theta x\varepsilon \pm \frac{\theta^2}{4\gamma}$$

Solution #1

$$\dot{V}(\varepsilon) = -\frac{\lambda}{2}\varepsilon^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \left(\sqrt{\gamma}x\varepsilon + \frac{\theta}{2\sqrt{\gamma}}\right)^2 + \frac{\theta^2}{4\gamma}$$

Solution #1

$$\dot{V}(\varepsilon) = -\frac{\lambda}{2}\varepsilon^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \left(\sqrt{\gamma}x\varepsilon + \frac{\theta}{2\sqrt{\gamma}}\right)^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\delta^2 + \frac{\theta^2}{4\gamma}$$

Solution #1

$$\dot{V}(\varepsilon) = -\frac{\lambda}{2}\varepsilon^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \left(\sqrt{\gamma}x\varepsilon + \frac{\theta}{2\sqrt{\gamma}}\right)^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \leq -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\delta^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \leq -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

$$|\delta(t)| \leq \bar{\delta}$$

Solution #1

$$\dot{V}(\varepsilon) = -\frac{\lambda}{2}\varepsilon^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \left(\sqrt{\gamma}x\varepsilon + \frac{\theta}{2\sqrt{\gamma}}\right)^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\delta^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

$$|\delta(t)| \le \bar{\delta}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \bar{\Delta}$$

$$\bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

Solution #1

$$\dot{V}(\varepsilon) = -\frac{\lambda}{2}\varepsilon^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \left(\sqrt{\gamma}x\varepsilon + \frac{\theta}{2\sqrt{\gamma}}\right)^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\delta^2 + \frac{\theta^2}{4\gamma}$$

$$\dot{V}(\varepsilon) \le -\frac{\lambda}{2}\varepsilon^2 + \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

$$|\delta(t)| \le \bar{\delta}$$

$$\bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

$$\bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

$$\boxed{\dot{V}(\varepsilon) \le -\lambda V(\varepsilon) + \bar{\Delta}}$$

$$V(\varepsilon) = \frac{1}{2}\varepsilon^2 \quad (4.7)$$

Solution #1

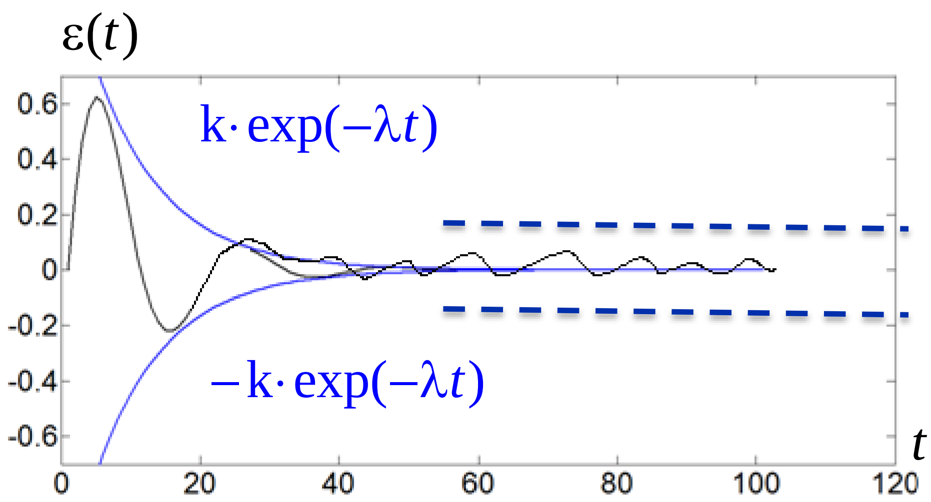
$$\dot{V}(\varepsilon) \le -\lambda V(\varepsilon) + \bar{\Delta} \quad \Rightarrow \quad V(t) \le e^{-\lambda t} V(0)\left(1 - \frac{\bar{\Delta}}{\lambda}\right) + \frac{\bar{\Delta}}{\lambda} V(0)$$

Exponential convergence of ε to bounded set is proved.

Solution #1

$$\dot{V}(\varepsilon) \le -\lambda V(\varepsilon) + \bar{\Delta} \quad \Rightarrow \quad V(t) \le e^{-\lambda t} V(0)\left(1 - \frac{\bar{\Delta}}{\lambda}\right) + \frac{\bar{\Delta}}{\lambda} V(0)$$

Exponential convergence of ε to bounded set is proved.



Solution #1

Summary

Adjustable controller:

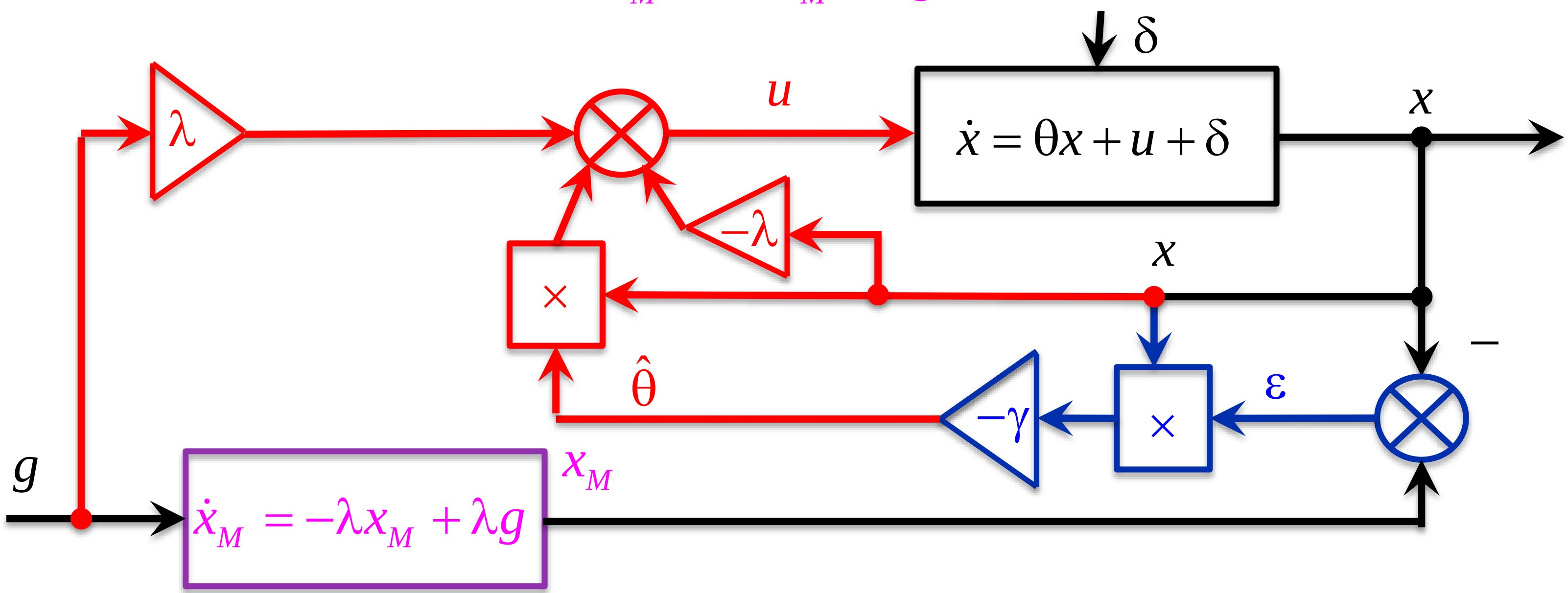
$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.4}$$

Nonlinear static feedback :

$$\dot{\hat{\theta}} = -\gamma x \varepsilon \tag{4.5}$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g. \tag{4.3}$$



Solution #1

Summary

Properties of the closed-loop robust system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ exponentially tends to the neighborhood of zero;
3. The radius of neighborhood can be arbitrary reduced by

How?

$$\dot{V}(\varepsilon) \leq -\lambda V(\varepsilon) + \bar{\Delta} \quad \text{where} \quad \bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

Solution #1

Summary

Properties of the closed-loop robust system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ exponentially tends to the neighborhood of zero;
3. The radius of neighborhood can be arbitrary reduced by

λ  *or* γ 

$$\dot{V}(\varepsilon) \leq -\lambda V(\varepsilon) + \bar{\Delta} \quad \text{where} \quad \bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

Solution #1

Summary

Properties of the closed-loop robust system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ exponentially tends to the neighborhood of zero;
3. The radius of neighborhood can be arbitrary reduced by

λ  *or* γ 

$$\dot{V}(\varepsilon) \le -\lambda V(\varepsilon) + \bar{\Delta} \quad \text{where} \quad \bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\theta^2}{4\gamma}$$

4. There is no compensation of uncertainty!

Even, if the plant is not disturbed ($\delta = \bar{\delta} = 0$), the error $\varepsilon = x_M - x$ does not go to zero!

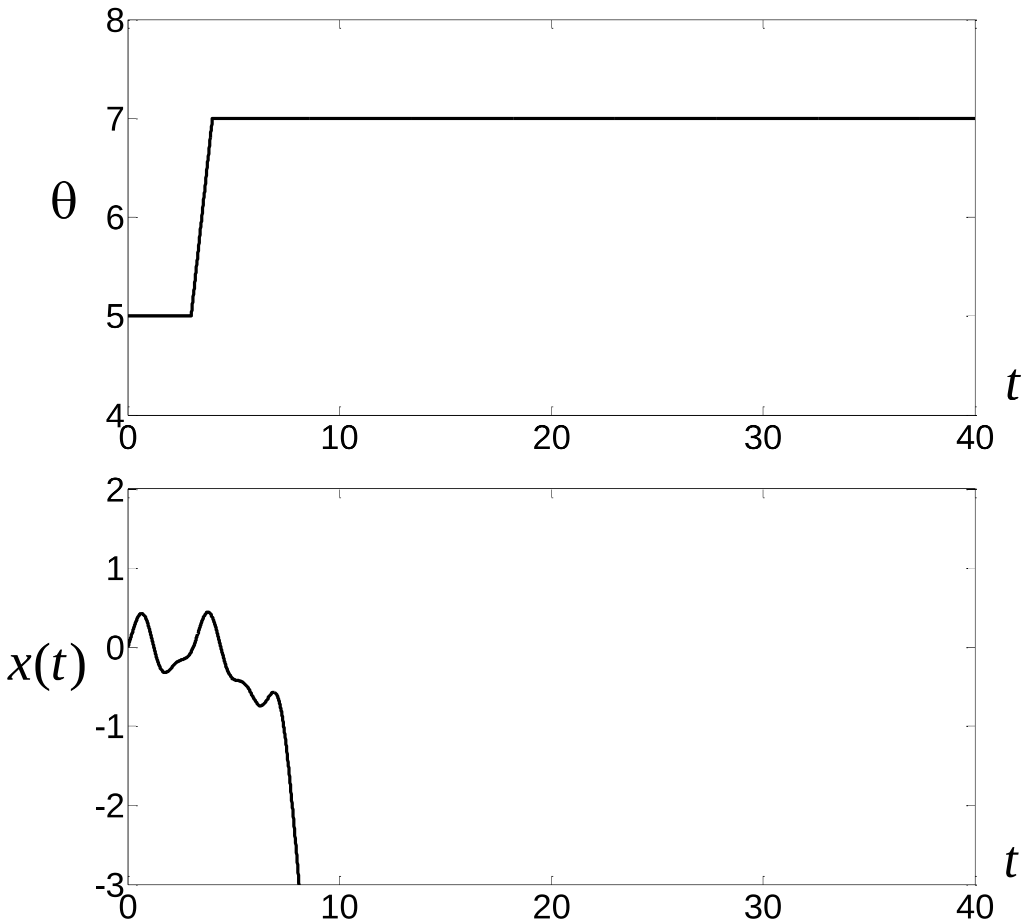
Example: Classical stabilizing control for unstable plant

$$\boxed{\dot{x} = \theta x + u + \delta}$$

$$u = -6x$$

$$\boxed{\theta = 5}$$

$$\boxed{\delta(t) = 0,5\sin(4t) + 0,75\cos(2t)}$$



Example: Robust stabilizing control for unstable plant

$$\boxed{\dot{x} = \theta x + u + \delta}$$

$$u = -6x - \hat{\theta}x,$$

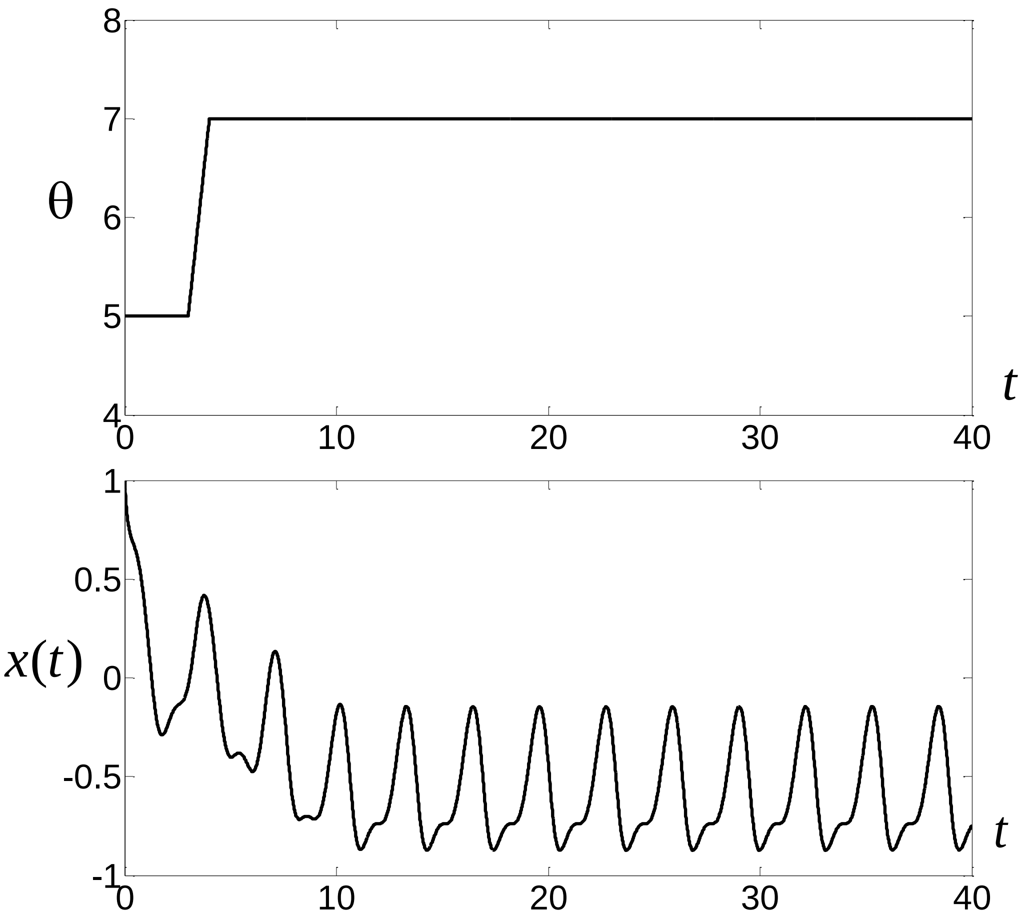
$$\boxed{\theta = 5}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon, \qquad \varepsilon = x_M - x,$$

$$\boxed{\delta(t) = 0,5\sin(4t) + 0,75\cos(2t)}$$

$$\dot{x}_M = -6x_M$$

$\gamma = 2$



Example: Robust stabilizing control for unstable plant

$$\boxed{\dot{x} = \theta x + u + \delta}$$

$$u = -6x - \hat{\theta}x,$$

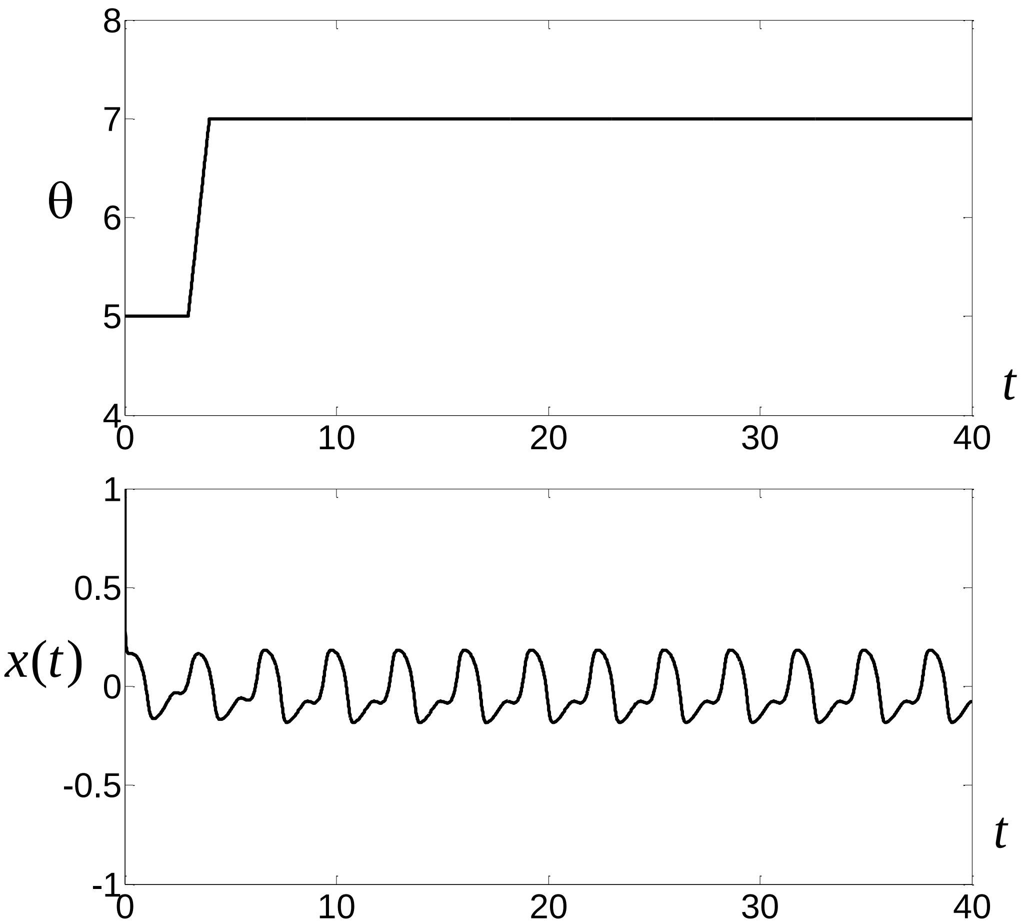
$$\boxed{\theta = 5}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon, \qquad \varepsilon = x_M - x,$$

$$\boxed{\delta(t) = 0{,}5\sin(4t) + 0{,}75\cos(2t)}$$

$$\dot{x}_M = -6x_M$$

$\gamma = 200$



Example: Robust tracking control for unstable plant

$$\boxed{\dot{x} = \theta x + u + \delta}$$

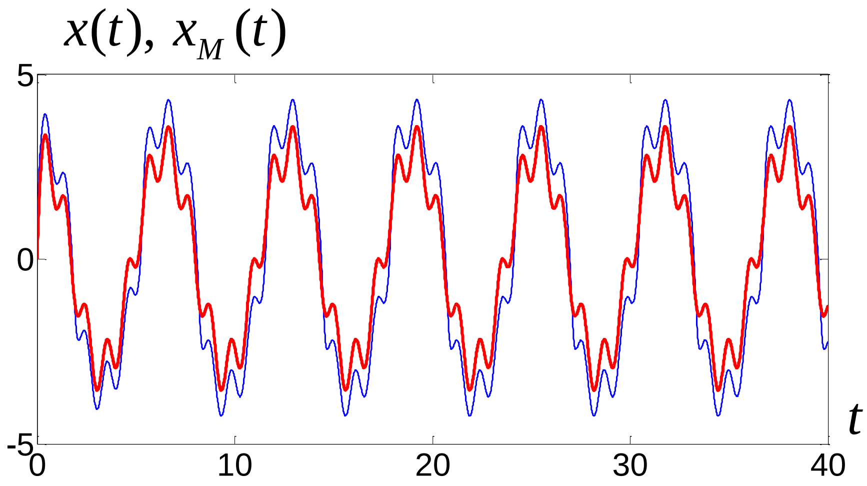
$$u = -6x - \hat{\theta}x + 6g,$$

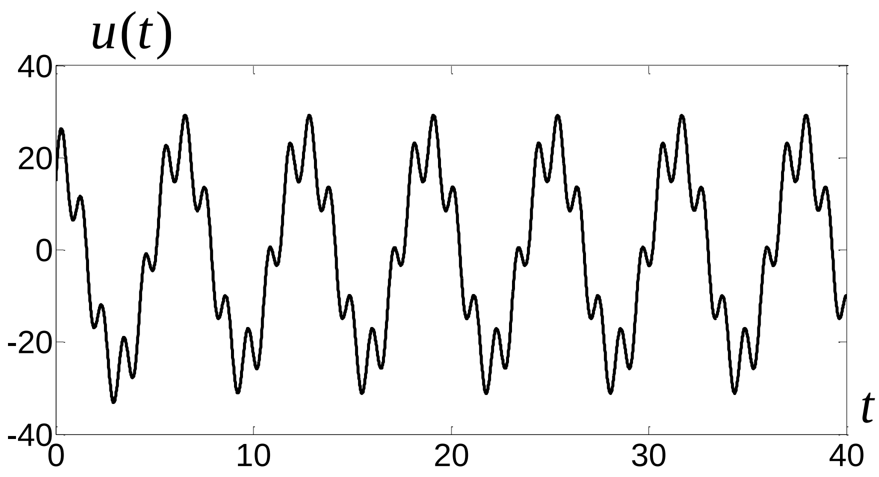
$$\boxed{\theta = 5}$$

$$\dot{\hat{\theta}} = -2x\varepsilon, \qquad \varepsilon = x_M - x,$$

$$\boxed{\delta(t) = 0{,}5\sin(4t) + 0{,}75\cos(2t)}$$

$$\dot{x}_M = -6x_M + 6g, \qquad g(t) = \sin 6t + 3\cos t$$





Adaptive control provides the complete compensation of the uncertainties, but it is not robust w.r.t. disturbance condition

$$\hat{\theta}(t) \to \infty, \; t \to \infty$$

Robust control guarantees the strongest exponential stability, but does not compensate of the uncertainties, therefore $\varepsilon(t) \not\to 0,\ t \to \infty$

Adaptive control provides the complete compensation of the uncertainties, but it is not robust w.r.t. disturbance condition

$$\hat{\theta}(t) \to \infty, \ t \to \infty$$

trade off ?

Robust control guarantees the strongest exponential stability, but does not compensate of the uncertainties, therefore $\varepsilon(t) \not\to 0, \ t \to \infty$

Adjustable controller:

$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.8}$$

~~**Adaptation algorithm:**~~ $\longrightarrow$ **Robust modification of AA:**

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma\hat{\theta} \tag{4.9}$$

where σ is a positive feedback gain,

$\varepsilon = x_M - x$, x_M is the output of reference model

$$\dot{x}_M = -\lambda x_M + \lambda g.$$

Solution #2

Adjustable controller:

$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.8}$$

~~**Adaptation algorithm:**~~ $\longrightarrow$ **Robust modification of AA:**

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma\hat{\theta} \tag{4.9}$$

where σ is a positive feedback gain,

$\varepsilon = x_M - x,$ x_M is the output of reference model

$$\dot{x}_M = -\lambda x_M + \lambda g.$$

Then substitute control (4.8) into disturbed plant $\dot{x} = \theta x + u + \delta.$

$$\dot{x} = \theta x - \hat{\theta}x - \lambda x + \lambda g + \delta.$$

$$\dot{x} = \tilde{\theta}x - \lambda x + \lambda g + \delta. \qquad (\tilde{\theta} = \theta - \hat{\theta})$$

Solution #2

Again, form take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\tilde{\theta} x - \lambda x + \lambda g + \delta\right)$$

Solution #2

Again, form take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\tilde{\theta}x - \lambda x + \lambda g + \delta\right)$$

Signal Error Model

$$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x - \delta} \tag{4.10}$$

Solution #2

Again, form take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\tilde{\theta} x - \lambda x + \lambda g + \delta\right)$$

Signal Error Model $$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta} x - \delta} \tag{4.10}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma\hat{\theta} \longrightarrow \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

Parametric Error Model $$\boxed{\dot{\tilde{\theta}} = \gamma x \varepsilon + \sigma\hat{\theta}} \tag{4.11}$$

Solution #2

Again, form take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\tilde{\theta} x - \lambda x + \lambda g + \delta\right)$$

Signal Error Model $$\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta} x - \delta} \tag{4.10}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma\hat{\theta} \longrightarrow \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

Parametric Error Model $$\boxed{\dot{\tilde{\theta}} = \gamma x \varepsilon + \sigma\hat{\theta}} \tag{4.11}$$

Lyapunov function candidate???

Solution #2

Again, form take the derivative of the error $\varepsilon = x_M - x$

$$\dot{\varepsilon} = \dot{x}_M - \dot{x} = \left(-\lambda x_M + \lambda g\right) - \left(\tilde{\theta} x - \lambda x + \lambda g + \delta\right)$$

Signal Error Model

$$\boxed{\dot{\varepsilon} = -\lambda \varepsilon - \tilde{\theta} x - \delta} \tag{4.10}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma \hat{\theta} \longrightarrow \dot{\tilde{\theta}} = -\dot{\hat{\theta}}$$

Parametric Error Model

$$\boxed{\dot{\tilde{\theta}} = \gamma x \varepsilon + \sigma \hat{\theta}} \tag{4.11}$$

Choose the Lyapunov function candidate

$$V(\varepsilon, \tilde{\theta}) = \frac{1}{2}\varepsilon^2 + \frac{1}{2\gamma}\tilde{\theta}^2, \qquad \gamma > 0 \tag{4.12}$$

Solution #2

Take the time derivative of Lyapunov function using (4.10) and (4.11):

Signal Error Model $\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x - \delta}$

Parametric Error Model $\boxed{\dot{\tilde{\theta}} = \gamma x\varepsilon + \sigma\hat{\theta}}$

$$\dot{V}(\varepsilon, \tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \frac{1}{\gamma}\tilde{\theta}\Omega(t)$$

Solution #2

Take the time derivative of Lyapunov function using (4.10) and (4.11):

Signal Error Model $\boxed{\dot{\varepsilon} = -\lambda\varepsilon - \tilde{\theta}x - \delta}$

Parametric Error Model $\boxed{\dot{\tilde{\theta}} = \gamma x\varepsilon + \sigma\hat{\theta}}$

$$\dot{V}(\varepsilon,\tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = -\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \frac{1}{\gamma}\tilde{\theta}\Omega(t)$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = \varepsilon\dot{\varepsilon} + \frac{1}{\gamma}\tilde{\theta}\dot{\tilde{\theta}} = \left(-\lambda\varepsilon^2 - \tilde{\theta}x\varepsilon - \delta\varepsilon\right) + \frac{1}{\gamma}\tilde{\theta}\left(\gamma x\varepsilon + \sigma\hat{\theta}\right)$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\lambda\varepsilon^2 - \delta\varepsilon + \frac{\sigma}{\gamma}\tilde{\theta}\hat{\theta}$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\lambda\varepsilon^2 - \delta\varepsilon - \frac{\sigma}{\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta$$

$$\tilde{\theta} = \theta - \hat{\theta}$$

Solution #2

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta$$

Solution #2

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon \pm \frac{1}{2\lambda}\delta^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta \pm \frac{\sigma}{2\gamma}\theta^2$$

Solution #2

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon \pm \frac{1}{2\lambda}\delta^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{\sigma}{\gamma}\tilde{\theta}\theta \pm \frac{\sigma}{2\gamma}\theta^2$$

$$\dot{V}(\varepsilon,\tilde{\theta}) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \frac{\sigma}{2\gamma}\left(\tilde{\theta} - \theta\right)^2 + \frac{\sigma}{2\gamma}\theta^2$$

Solution #2

$$\dot V(\varepsilon,\tilde\theta) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon - \frac{\sigma}{2\gamma}\tilde\theta^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 + \frac{\sigma}{\gamma}\tilde\theta\theta$$

$$\dot V(\varepsilon,\tilde\theta) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 - \frac{\lambda}{2}\varepsilon^2 - \delta\varepsilon \pm \frac{1}{2\lambda}\delta^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 + \frac{\sigma}{\gamma}\tilde\theta\theta \pm \frac{\sigma}{2\gamma}\theta^2$$

$$\dot V(\varepsilon,\tilde\theta) = -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 - \left(\sqrt{\frac{\lambda}{2}}\varepsilon + \sqrt{\frac{1}{2\lambda}}\delta\right)^2 + \frac{1}{2\lambda}\delta^2 - \frac{\sigma}{2\gamma}\left(\tilde\theta - \theta\right)^2 + \frac{\sigma}{2\gamma}\theta^2$$

$$\dot V(\varepsilon,\tilde\theta) \le -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 + \frac{1}{2\lambda}\delta^2 + \frac{\sigma}{2\gamma}\theta^2$$

$$|\delta(t)| \le \bar\delta$$

$$\dot V(\varepsilon,\tilde\theta) \le -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde\theta^2 + \frac{1}{2\lambda}\bar\delta^2 + \frac{\sigma}{2\gamma}\theta^2$$

Solution #2

$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\sigma}{2\gamma}\theta^2$$

$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\frac{\lambda}{2}\varepsilon^2 - \frac{\sigma}{2\gamma}\tilde{\theta}^2 + \bar{\Delta}$$

$$\bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\sigma}{2\gamma}\theta^2$$

$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\kappa V(\varepsilon,\tilde{\theta}) + \bar{\Delta} \qquad \kappa = \min\{\lambda,\sigma\} \qquad (4.13)$$

Solution #2

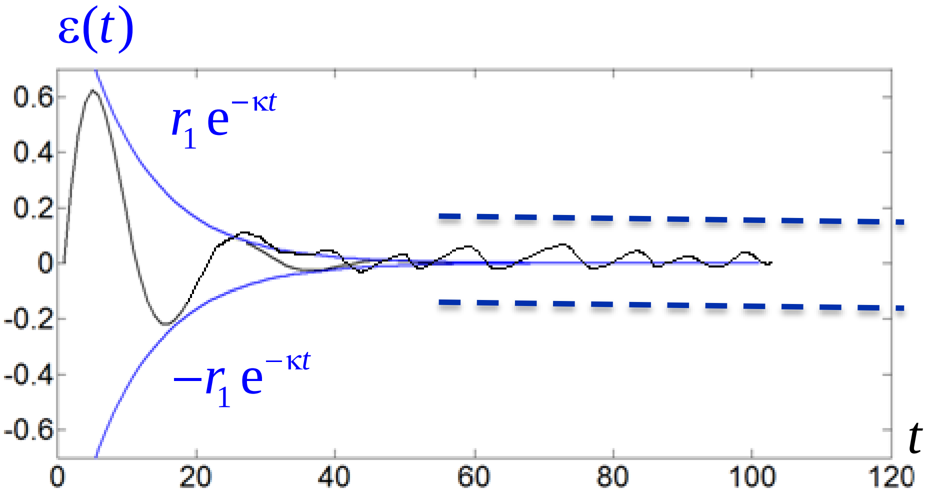
$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\kappa V(\varepsilon,\tilde{\theta}) + \bar{\Delta} \qquad \Rightarrow \qquad V(t) \le \mathrm{e}^{-\kappa t} V(0)\left(1 - \frac{\bar{\Delta}}{\kappa}\right) + \frac{\bar{\Delta}}{\kappa} V(0)$$

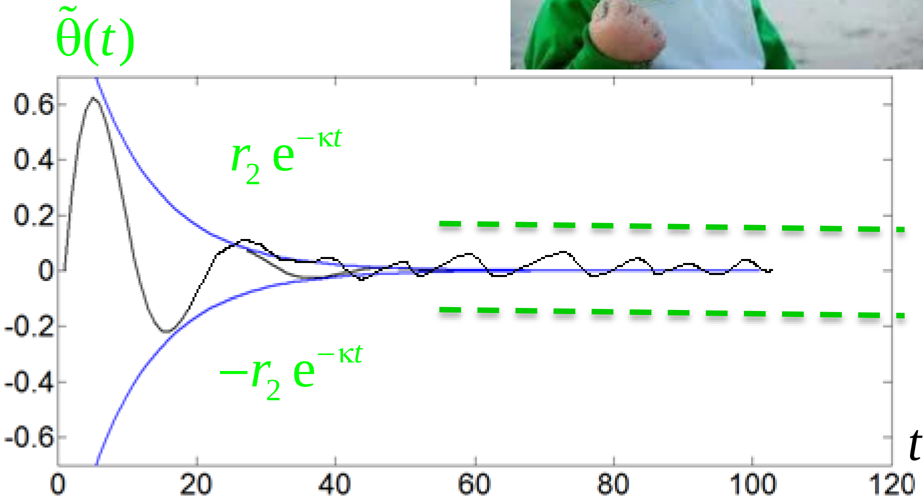
Exponential convergence of ε, $\tilde{\theta}$ to bounded set is proved.

Solution #2

$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\kappa V(\varepsilon,\tilde{\theta}) + \bar{\Delta} \qquad \Rightarrow \qquad V(t) \le e^{-\kappa t} V(0)\left(1-\frac{\bar{\Delta}}{\kappa}\right) + \frac{\bar{\Delta}}{\kappa} V(0)$$

Exponential convergence of ε, $\tilde{\theta}$ to bounded set is proved.





Solution #2

Summary

Adjustable controller:

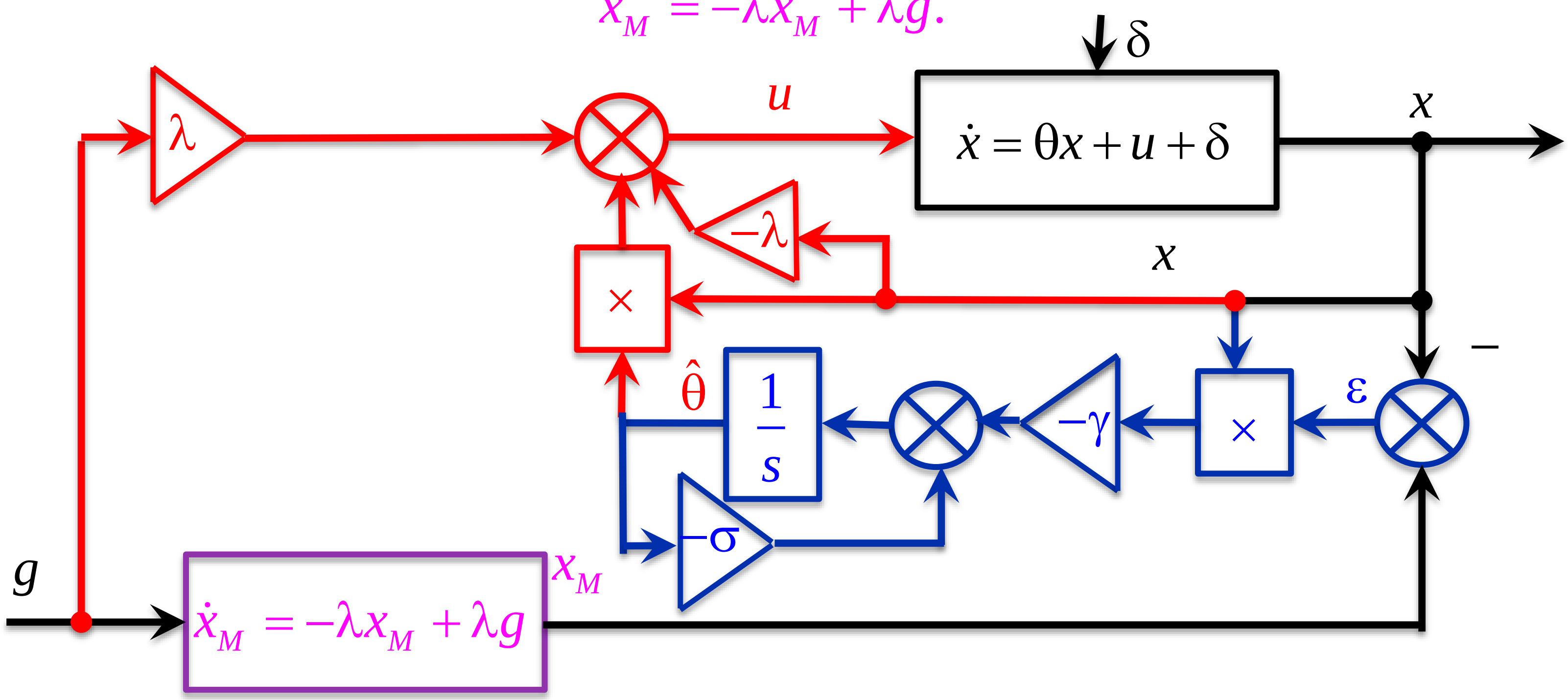
$$u = -\hat{\theta}x - \lambda x + \lambda g \tag{4.8}$$

Robust modification of adaptation algorithm:

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \sigma \hat{\theta} \tag{4.9}$$

with $\varepsilon = x_M - x$ and reference model

$$\dot{x}_M = -\lambda x_M + \lambda g. \tag{4.3}$$



Solution #2

Summary

Properties of the closed-loop robust system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ together with parametric error $\tilde{\theta}$ exponentially tends to the neighborhood of zero;
3. The radius of neighborhood can be arbitrary reduced by

How?

$$\dot{V}(\varepsilon,\tilde{\theta}) \le -\kappa V(\varepsilon,\tilde{\theta}) + \bar{\Delta} \quad \text{where} \quad \bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\sigma}{2\gamma}\theta^2$$

Solution #2

Summary

Properties of the closed-loop robust system:

1. All the signals in the system are bounded;
2. Control error $\varepsilon = x_M - x$ together with parametric error $\tilde{\theta}$ exponentially tends to the neighborhood of zero;
3. The radius of neighborhood can be arbitrary reduced by

λ  *or* γ  *or* $\sigma \downarrow$

$$\dot{V}(\varepsilon,\tilde{\theta}) \leq -\kappa V(\varepsilon,\tilde{\theta}) + \bar{\Delta} \quad \text{where} \quad \bar{\Delta} = \frac{1}{2\lambda}\bar{\delta}^2 + \frac{\sigma}{2\gamma}\theta^2$$

4. Algorithm provides the compensation of uncertainty.

   If the plant is not disturbed ($\delta = \bar{\delta} = 0$), the error $\varepsilon = x_M - x$ can go to zero, if $\sigma = 0$.

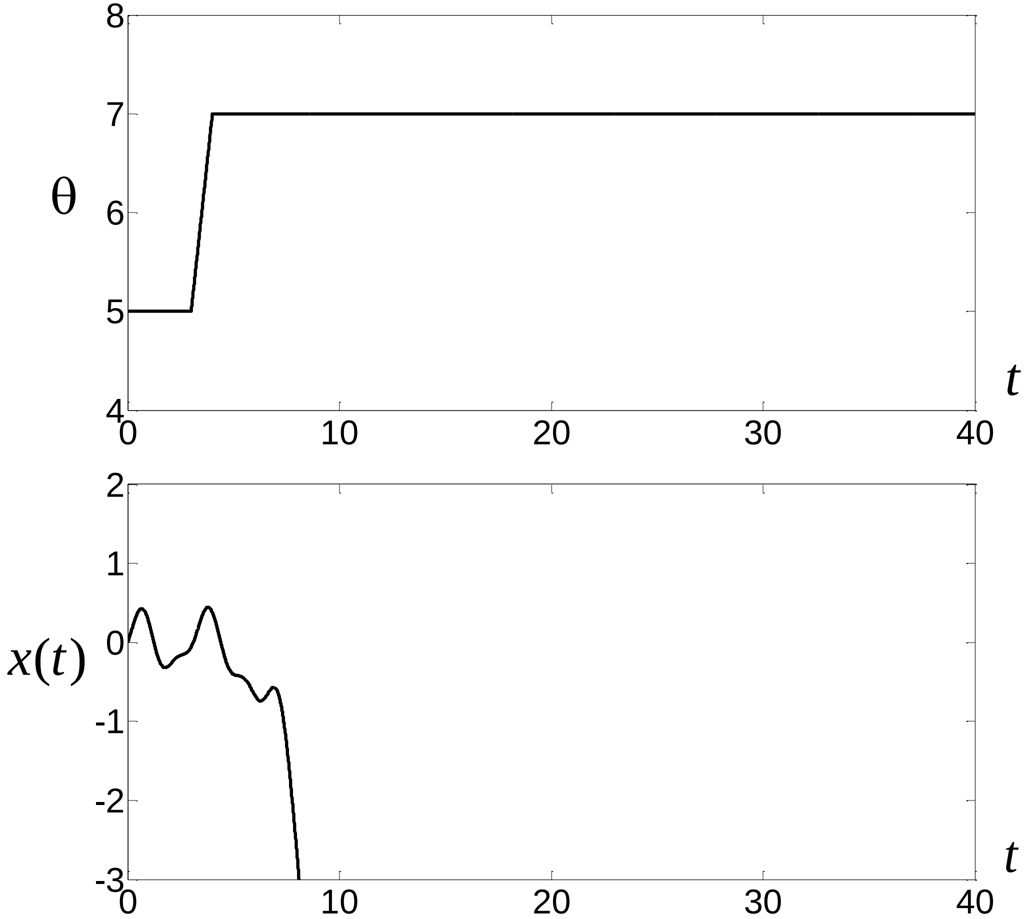
Example: Classical stabilizing control for unstable plant

$$\boxed{\dot{x} = \theta x + u + \delta}$$

$$u = -6x$$

$$\boxed{\theta = 5}$$

$$\boxed{\delta(t) = 0,5\sin(4t) + 0,75\cos(2t)}$$



Example: Adaptive robust stabilizing control for the plant $\boxed{\dot{x} = \theta x + u + \delta}$

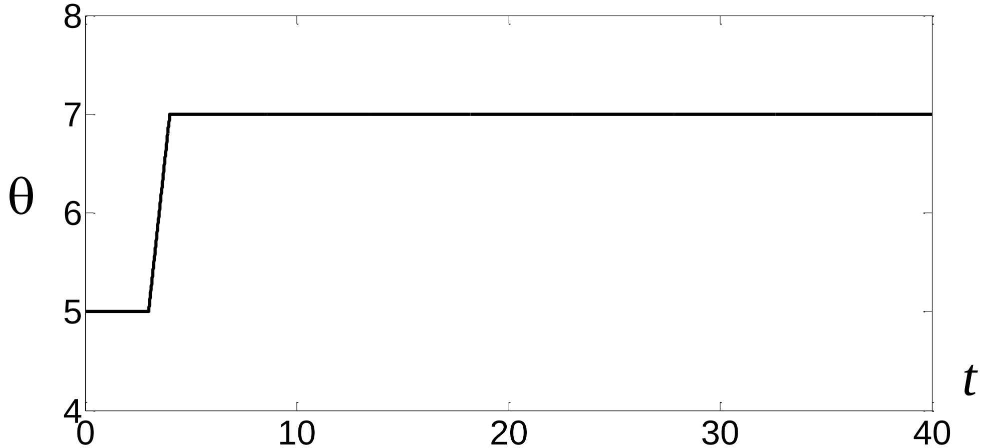
$$u = -6x - \hat{\theta}x,$$

$\boxed{\theta = 5}$

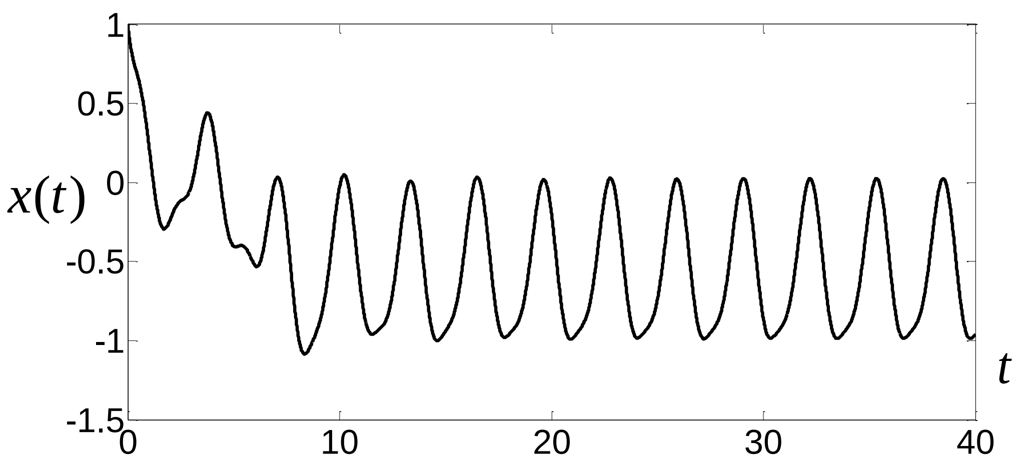
$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \hat{\theta}, \qquad \varepsilon = x_M - x,$$

$\boxed{\delta(t) = 0,5\sin(4t) + 0,75\cos(2t)}$

$$\dot{x}_M = -6x_M$$



$\gamma = 2$



Example: Adaptive robust stabilizing control for the plant $\boxed{\dot{x} = \theta x + u + \delta}$

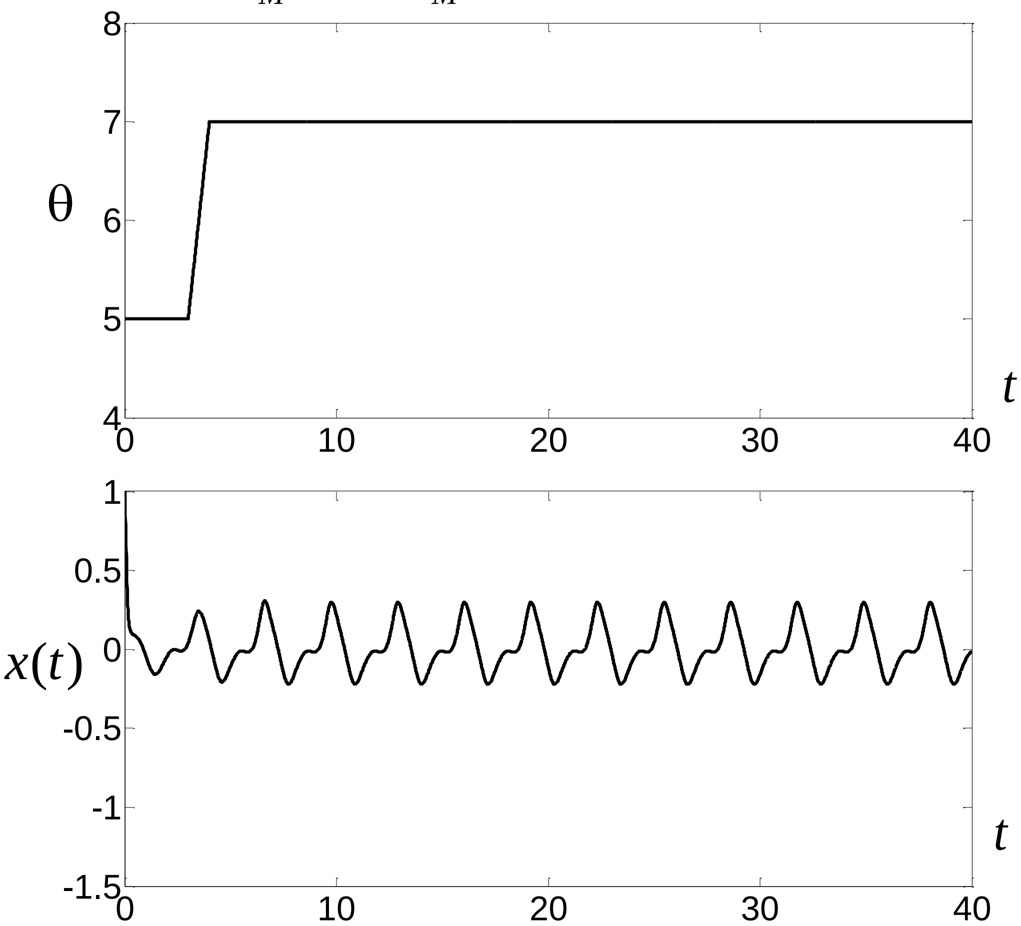
$$u = -6x - \hat{\theta}x,$$

$$\boxed{\theta = 5}$$

$$\dot{\hat{\theta}} = -\gamma x \varepsilon - \hat{\theta}, \qquad \varepsilon = x_M - x, \qquad \boxed{\delta(t) = 0{,}5\sin(4t) + 0{,}75\cos(2t)}$$

$$\dot{x}_M = -6x_M$$

$\gamma = 200$



Example: Adaptive robust tracking control for the plant $\boxed{\dot{x} = \theta x + u + \delta}$

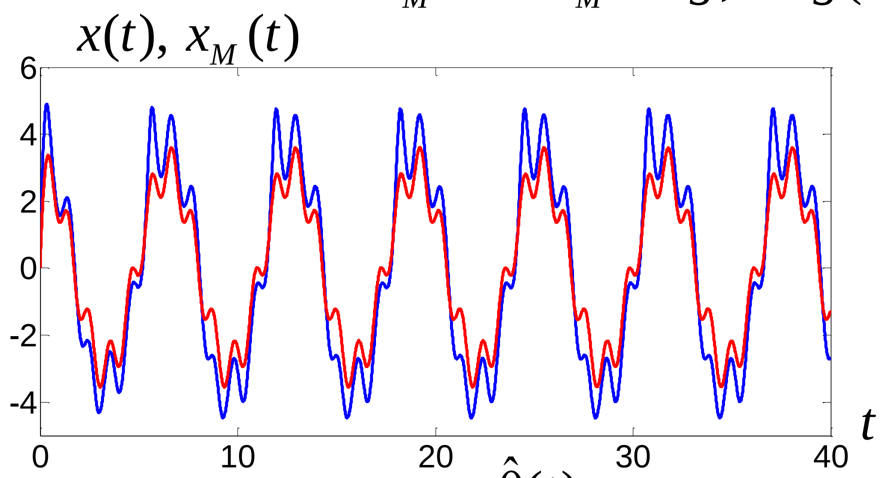
$$u = -6x - \hat{\theta}x + 6g,$$

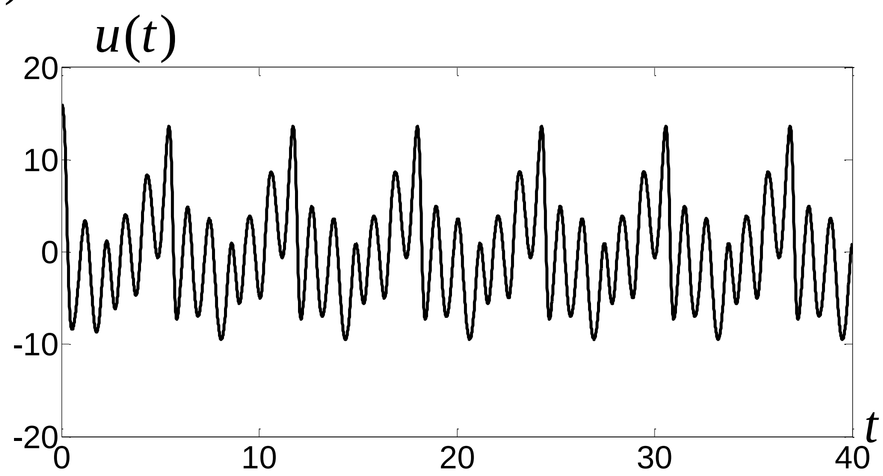
$$\boxed{\theta = 5}$$

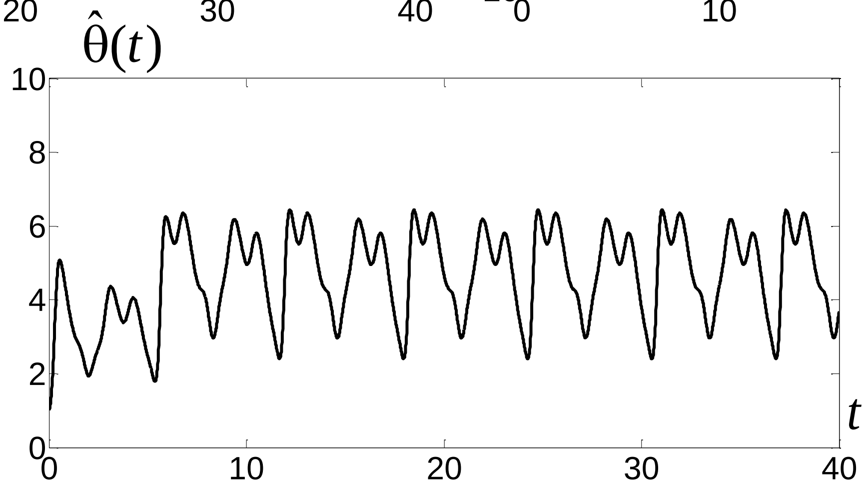
$$\dot{\hat{\theta}} = -2x\varepsilon - \hat{\theta}, \qquad \varepsilon = x_M - x,$$

$$\boxed{\delta(t) = 0{,}5\sin(4t) + 0{,}75\cos(2t)}$$

$$\dot{x}_M = -6x_M + 6g, \qquad g(t) = \sin 6t + 3\cos t$$







5. Generalized Algorithm of Adaptive and Robust Controller Design

How to design an adaptive control?

5. Generalized Algorithm of Adaptive and Robust Controller Design

1. Problem statement of adaptive control (state feedback):

Plant:

$$\dot{x} = f(\theta, x, u, \delta), \qquad x(0), \tag{5.1}$$

where θ is the vector of unknown parameters (or functions),

$f \in R^n$ is continuous nonlinear mapping, $\delta \in R^m : \|\delta(t)\| \le \bar{\delta}$ is the disturbance.

Objective is to design a control u providing the following inequality:

$$|x_M(t) - x(t)| \le \Delta \text{ for } any\ t \ge T, \tag{5.2}$$

where x_M is the state of the reference model

$$\dot{x}_M = A_M x_M + b_M g, \tag{5.3}$$

g is the reference signal, A_M, b_M are the design matrix parameters.

5. Generalized Algorithm of Adaptive and Robust Controller Design

1. Problem statement of adaptive control (output feedback):

Plant:

$$\begin{aligned} \dot{x} &= f(\theta, x, u, \delta), \qquad x(0), \\ y &= h(\theta, x, u, \delta) \end{aligned} \tag{5.1}$$

where θ is the vector of unknown parameters (or functions),

$f \in R^n$ is continuous nonlinear mapping, $\delta \in R^m : \|\delta(t)\| \leq \bar{\delta}$ is the disturbance.

Objective is to design a control u providing the following inequality:

$$|y_M(t) - y(t)| \leq \Delta \quad \text{for } any\ t \geq T, \tag{5.2}$$

where y_M is the output of the reference model

$$y_M = W_M(s)[g], \tag{5.3}$$

g is the reference signal, $W_M(s)$ is the transfer function.

5. Generalized Algorithm of Adaptive and Robust Controller Design

2. Nonadaptive controller design:

Let the plant parameters (functions) θ be known.

5. Generalized Algorithm of Adaptive and Robust Controller Design

2. Nonadaptive controller design:

Let the plant parameters (functions) θ be known.

Luggage of classical control theory

$\Rightarrow$

Nonadaptive control

$$u = U(\theta, x, e, g), \qquad (5.4)$$

where $e = x_M - x$ is the control error, U is the nonlinear static or dynamical scalar function.

5. Generalized Algorithm of Adaptive and Robust Controller Design

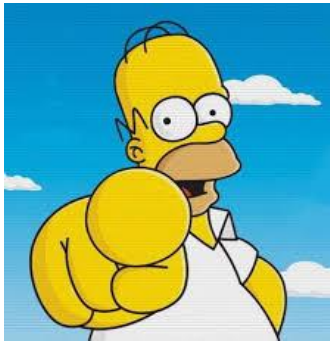
3. Adjustable controller design

Parameters (functions) θ are unknown.

5. Generalized Algorithm of Adaptive and Robust Controller Design

3. Adjustable controller design

Parameters (functions) θ are unknown.



Substitute estimates $\hat{\theta}$ for θ in control (5.4)

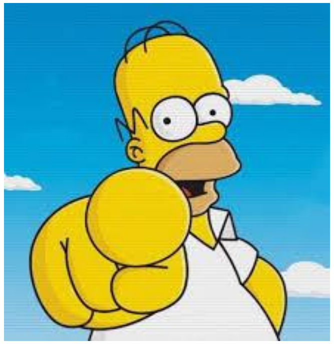
and obtain adjustable controller:

$$\boxed{u = U(\hat{\theta}, x, e, g)} \tag{5.5}$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

3. Adjustable controller design

Parameters (functions) θ are unknown.



Substitute estimates $\hat{\theta}$ for θ in control (5.4)

and obtain adjustable controller:

$$\boxed{u = U(\hat{\theta}, x, e, g)} \tag{5.5}$$

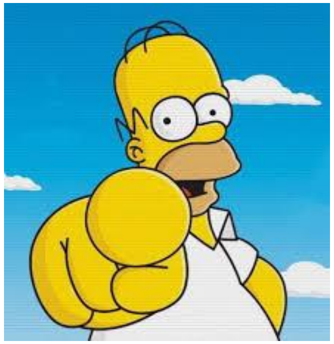
Substitute (38) into the plant $\dot{x} = f(\theta, x, u, \delta)$:

$$\dot{x} = f(\theta, x, U(\hat{\theta}, x, e, g), \delta)$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

3. Adjustable controller design

Parameters (functions) θ are unknown.



Substitute estimates $\hat{\theta}$ for θ in control (5.4)

and obtain adjustable controller:

$$\boxed{u = U(\hat{\theta}, x, e, g)} \tag{5.5}$$

Substitute (38) into the plant $\dot{x} = f(\theta, x, u, \delta)$:

$$\dot{x} = f(\theta, x, U(\hat{\theta}, x, e, g), \delta)$$

Form the error $e = x_M - x$ and take its derivative:

5. Generalized Algorithm of Adaptive and Robust Controller Design

Form the error $e = x_M - x$ and take its derivative:

$$\dot{e} = \dot{x}_M - \dot{x} = \left(A_M x_M + b_M g\right) - f(\theta, x, U(\hat{\theta}, x, \varepsilon, g), \delta)$$

$$\Downarrow$$

Signal Error Model $$\boxed{\dot{e} = E\left(e, \tilde{\theta}, t\right)} \qquad (5.6)$$

where E is the nonlinear static vector function,

$\tilde{\theta} = \theta - \hat{\theta}$ is the parametric error.

5. Generalized Algorithm of Adaptive and Robust Controller Design

4. Adaptation algorithm design

Form the parametric error model

Parametric Error Model $$\boxed{\dot{\tilde{\theta}} = \Omega(e,t),} \tag{5.7}$$

where Ω is the implementable (measurable) function to be determined.

5. Generalized Algorithm of Adaptive and Robust Controller Design

4. Adaptation algorithm design

Form the parametric error model

Parametric Error Model $$\boxed{\dot{\tilde{\theta}} = \Omega(e,t),} \tag{5.7}$$

where Ω is the implementable (measurable) function to be determined.

Adaptation algorithm:

$$\boxed{\dot{\hat{\theta}} = -\Omega(e,t),} \qquad \dot{\tilde{\theta}} = -\dot{\hat{\theta}} \tag{5.8}$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

5. Determination of Ω

Signal Error Model $\boxed{\dot{e} = E\left(e, \tilde{\theta}, t\right)}$ (5.6)

Parametric Error Model $\boxed{\dot{\tilde{\theta}} = \Omega\left(e, t\right)}$ (5.7)

Choose a Lyapunov function candidate

$$V = V(e, \tilde{\theta}, t).$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

5. Determination of Ω

Signal Error Model

$$\boxed{\dot{e} = E\left(e, \tilde{\theta}, t\right)} \tag{5.6}$$

Parametric Error Model

$$\boxed{\dot{\tilde{\theta}} = \Omega\left(e, t\right),} \tag{5.7}$$

Choose a Lyapunov function candidate

$$V = V(e, \tilde{\theta}, t) \le c_1 \left\| e \right\|^2 + c_2 \left\| \tilde{\theta} \right\|^2 .$$

Take its derivative in view of (5.6) and (5.7).

5. Generalized Algorithm of Adaptive and Robust Controller Design

$$\dot{e} = E\left(e, \tilde{\theta}, t\right) \qquad \dot{\tilde{\theta}} = \Omega\left(e, t\right)$$

$$\dot{V}(e, \tilde{\theta})$$

Condition

$$\dot{V}(e, \tilde{\theta}) < -c_3 \left\|e\right\|^2 \ (e(t) \to 0 \ \ as \ \ t \to \infty)$$

gives

$$\Downarrow$$

$$\Omega\left(e, t\right)$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

$$\boxed{\dot{e} = E\left(e, \tilde{\theta}, t\right)} \qquad \boxed{\dot{\tilde{\theta}} = \Omega\left(e, t\right)}$$

$$\dot{V}(e, \tilde{\theta})$$

Condition

$$\dot{V}(e, \tilde{\theta}) < -c_3 \left\| e \right\|^2 \ (e(t) \to 0 \ \ as \ \ t \to \infty)$$

gives

$$\Downarrow$$

Adaptation algorithm: $\Omega\left(e, t\right)$

$$\boxed{\dot{\hat{\theta}} = -\Omega\left(e, t\right)} \tag{5.8}$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

Summary

Adjustable control

$$u = U(\hat{\theta}, x, \varepsilon, g) \tag{5.5}$$

Adaptation algorithm

$$\dot{\hat{\theta}} = -\Omega(e, t) \tag{5.8}$$

5. Generalized Algorithm of Adaptive and Robust Controller Design

There is no any universal approach of Lyapunov function choice!

5. Generalized Algorithm of Adaptive and Robust Controller Design

There is no any universal approach of Lyapunov function choice!

However, there are standard errors models with preliminary selected Lyapunov functions used for design of adaptation algorithms and oriented to different problems of adaptive control and identification