

Object

$$\dot{x} = \theta x + u$$

Reference:

$$\dot{x}_M = -\lambda x_m + \lambda g$$

Error:

$$\varepsilon = x_M - x$$

$$\begin{aligned}\dot{\varepsilon} &= \dot{x}_M - \dot{x} \\ &= -\lambda x_m + \lambda g - \theta x - u\end{aligned}$$

Exponential Stability:

$$\begin{aligned}\dot{\varepsilon} &= -\lambda \varepsilon \\ -\lambda x_m + \lambda g - \theta x - u &= -\lambda \varepsilon \\ u &= -\lambda(\varepsilon - x_M) + \lambda g - \theta x \\ u &= -\lambda x + \lambda g - \theta x\end{aligned}$$

Adaptive:

$$u = -\hat{\theta}x - \lambda x + \lambda g$$

Thay vào Object:

$$\begin{aligned}\dot{x} &= \theta x - \hat{\theta}x - \lambda x + \lambda g \\ &= \tilde{\theta}x - \lambda x + \lambda g\end{aligned}$$

Thay vào Error:

$$\begin{aligned}\dot{\varepsilon} &= \dot{x}_M - \dot{x} \\ &= -\lambda x_m + \lambda g - \tilde{\theta}x + \lambda x - \lambda g \\ &= -\lambda \varepsilon - \tilde{\theta}x\end{aligned}$$

Lyapunov:

$$V = \frac{1}{2}\varepsilon^2 + \frac{1}{2}\tilde{\theta}^2$$