Задача №1.

О: $\dot{x} = \theta_1 \sin x + \theta_2 \cos x + u$

Ц: $u: \lim_{t\to\infty} (x(t) - x_м(t)) = 0$

ЭМ: $\dot{x}_м = -\lambda x_м + \lambda g$

Решение: 1) $\exists\ \theta_1, \theta_2$ изв.

$\dot{\varepsilon} = \dot{x}_м - \dot{x} = -\lambda x_м + \lambda g - \theta_1 \sin x - \theta_2 \cos x - u = -\lambda \varepsilon$

$\varepsilon = x_м - x$

$u_н = -\theta_1 \sin x - \theta_2 \cos x - \lambda x + \lambda g$

2) θ_1, θ_2 неизв. $\to \theta_{1,2} \to \hat{\theta}_{1,2}(t)$:

$u_a = -\hat{\theta}_1 \sin x - \hat{\theta}_2 \cos x - \lambda x + \lambda g$

$u_a = \underbrace{-\theta_1 \sin x - \theta_2 \cos x - \lambda x + \lambda g}_{u_н} + (\theta_1 - \hat{\theta}_1)\sin x + (\theta_2 - \hat{\theta}_2)\cos x$

$u_a = u_н + \underbrace{\tilde{\theta}_1 \sin x + \tilde{\theta}_2 \cos x}_{\tilde{\theta}^T \omega}$ $\quad \omega = \begin{bmatrix} \sin x \\ \cos x \end{bmatrix}$

$\dot{\varepsilon} = \underbrace{-\lambda \varepsilon}_{\text{в виду } u_н} - \tilde{\theta}^T \omega$ $\quad \dot{\varepsilon} = \dot{x}_м - \dot{x}$

АА: $\dot{\hat{\theta}} = -\gamma \omega \overset{1}{b^T} \overset{1}{P} \varepsilon$ $\quad P: A^T P + PA = -Q \Rightarrow P = 1$, $A = -\lambda$, $Q = 2\lambda$

$\varepsilon = \underbrace{\frac{-1}{s+\lambda}}_{\text{СПЗ}} [\tilde{\theta}^T \omega] \Rightarrow \dot{\hat{\theta}} = -\gamma \omega \varepsilon$

Задача №1 модиф.

$\dot{x} = \theta u \qquad \theta \geq \underline{\theta} > 0$ – изв.

$\dot{\varepsilon} = \dot{x}_м - \dot{x} = -\lambda x_м + \lambda g - \theta u = -\lambda \varepsilon$, $\varepsilon = x_м - x$

$u_н = \frac{1}{\theta}(-\lambda x + \lambda g)$

$\theta \to \hat{\theta}(t)$:

$u_a = \frac{1}{\hat{\theta}}(-\lambda x + \lambda g)$

Способ 1 $\quad u_a = \frac{\theta \pm \hat{\theta}}{\theta \hat{\theta}}(-\lambda x + \lambda g) = \frac{\tilde{\theta}}{\theta \hat{\theta}}\underbrace{(-\lambda x + \lambda g)}_{u_a} + \frac{1}{\theta}(-\lambda x + \lambda g)$

$u_a = \frac{1}{\theta}\tilde{\theta} u_a + \underbrace{\frac{1}{\theta}(-\lambda x + g)}_{u_н}$

$\dot{\varepsilon} = \dot{x}_м - \dot{x} = \dot{x}_м - \theta u_a = \underbrace{-\lambda \varepsilon}_{\text{в виду } u_н} - \tilde{\theta} u_a$

Способ 2 $\quad \dot{\varepsilon} = \dot{x}_м - \dot{x} = -\lambda x_м + \lambda g - \theta u$

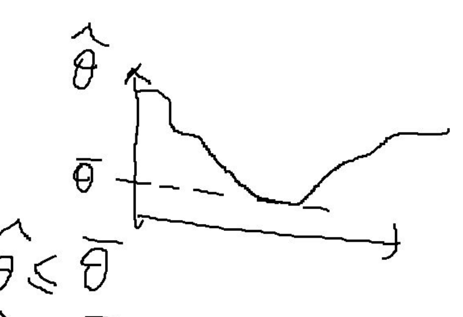
$u_a = \frac{1}{\hat{\theta}}(-\lambda x + \lambda g)$

$\dot{\varepsilon} = -\lambda x_м + \lambda g - \frac{\theta \pm \hat{\theta}}{\hat{\theta}}(-\lambda x + \lambda g)$

$\dot{\varepsilon} = -\lambda x_м + \cancel{\lambda g} - \tilde{\theta} u_a + \lambda x - \cancel{\lambda g}$

$\dot{\varepsilon} = -\lambda \varepsilon - \tilde{\theta} u_a$

$\dot{\hat{\theta}} = -\gamma u \varepsilon$



$\dot{\hat{\theta}} = -\begin{cases} 0, & \text{если } \hat{\theta} \leq \underline{\theta} \\ \gamma u \varepsilon, & \text{если } \hat{\theta} > \underline{\theta} \end{cases}$

Задача №2 Робастное или Адаптивное роб. упр. скал. об.

Решается задача АУ:

$u = \mathcal{U}(\hat{\theta}, g, x)$

АА: $\dot{\hat{\theta}} = F(x, \varepsilon)$

Р: $\hat{\theta} = F$

АР: $\dot{\hat{\theta}} = F - \sigma \hat{\theta}$

Задача №3.

О: $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\theta_1 \sin x_2 - \theta_2 \cos x_1 + u \end{cases}$

$\dot{x} = A_0 x + b_0 (u + \theta^T \omega)$

$\Rightarrow A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, b_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\omega = [-\sin x_2 \ -\cos x_1]^T$

ЭМ: $\begin{cases} \dot{x}_{м1} = x_{м2} \\ \dot{x}_{м2} = -a_{м0} x_{м1} - a_{м1} x_{м2} + b_{м0} g \end{cases}$

$\Rightarrow \dot{x}_м = A_м x_м + b_м g$ $\quad A_м = \begin{bmatrix} 0 & 1 \\ -a_{м0} & -a_{м1} \end{bmatrix}$, $b_м = \begin{bmatrix} 0 \\ b_{м0} \end{bmatrix}$

Решение

$u_н = \theta_1 \sin x_2 + \theta_2 \cos x_1 - a_{м0} x_1 - a_{м1} x_2 + b_{м0} g$

$e = x_м - x$ $\quad (e(t) = \exp\{A_м t\} e(0))$

$\dot{e} = \underbrace{A_м x_м + b_м g}_{\dot{x}_м} \underbrace{- A_0 x - b_0 (u + \theta^T \omega)}_{-\dot{x}} \overset{\text{жел.}}{=} A_м e = x_м - x$

$A_м \cancel{x_м} + b_м g - A_0 x - b_0 (u + \theta^T \omega) = \cancel{A_м x_м} - A_м x$

$b_0 (u + \theta^T \omega) = (A_м - A_0) x + b_м g$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} (u + \theta^T \omega) = \underbrace{\begin{bmatrix} 0 & 0 \\ -a_{м0} & -a_{м1} \end{bmatrix}}_{\begin{bmatrix} 0 \\ 1 \end{bmatrix} [-a_{м0} \ -a_{м1}]} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} b_{м0} g$

$u_н = -a_{м0} x_1 - a_{м1} x_2 - \theta^T \omega + b_{м0} g$

$\exists\ \theta$ - неизв $\Rightarrow \theta \to \hat{\theta}(t)$:

$u_a = \underbrace{-a_{м0} x_1 - a_{м1} x_2 - \hat{\theta}^T \omega + b_{м0} g - \theta^T \omega}_{u_н} + \theta^T \omega$

$u_a = u_н + \tilde{\theta}^T \omega$

$\dot{e} = A_м x_м + b_м g - A_0 x - b_0 (u + \theta^T \omega)$

$\dot{e} = \underbrace{A_м e}_{\text{в виду } u_н} - \underbrace{b_0 \tilde{\theta}^T \omega}_{\text{в виду } \tilde{\theta}^T \omega}$

АА: $\dot{\hat{\theta}} = -\gamma \omega b_0^T P e$ $\quad$ Р: $A_м^T P + P A_м = -Q$

Задача №4 Робастное или Адаптивное роб. упр. 2-х м. об.

Решается задача АУ:

$u = \mathcal{U}(\hat{\theta}, g, x)$

АА: $\dot{\hat{\theta}} = F(x, \varepsilon)$

Р: $\hat{\theta} = F$

АР: $\dot{\hat{\theta}} = F - \sigma \hat{\theta}$

Задача №5

$\dot{\hat{x}} = A\hat{x} + bu + l(y - \hat{y})$

$\hat{y} = c^T \hat{x}$

$\ddot{y} + a_1 \dot{y} + a_0 y = b_0 u \to y = \theta^T \omega$ (л. 6.1, 7)

$\omega = \left[\frac{1}{k(s)}[y], \frac{s}{k(s)}[y], \frac{1}{k(s)}[u]\right]^T$

$\dot{y} = x_2 = \theta^T \dot{\omega}$

$\dot{\omega} = \left[\frac{s}{k(s)}[y], \frac{s^2}{k(s)}[y], \frac{s}{k(s)}[u]\right]^T$

$k(s) = s^2 + k_1 s + k_0$

Альтернативное решение (см. 6.3, пример)

$y = \psi^T \varphi \qquad \varphi = \left[y, \frac{1}{s+k}[y], \frac{1}{s+k}[u]\right]^T$

Расширение на нелин. случай

О: $\ddot{y} + a_1 \dot{y} y^2 + a_0 y = b_1 \dot{u} + b_0 u^3$ $\quad \Big| \ \frac{1}{k(s)} = s^2 + k_1 s + k_0$

цель: $y = \omega_0 + \theta^T \omega$

$\frac{s^2 \pm k_1 s \pm k_0}{k(s)}[y] + a_1 \underbrace{\frac{s}{k(s)}}_{\omega_2}\left[\frac{y^3}{3}\right] + a_0 \underbrace{\frac{1}{k(s)}[y]}_{\omega_1} = b_1 \underbrace{\frac{s}{k(s)}[u]}_{\omega_4} + b_0 \underbrace{\frac{1}{k(s)}[u]}_{\omega_3}$ $\quad \dot{y} y^2$

$y - \underbrace{\frac{k_1 s + k_0}{k(s)}[y]}_{\omega_0} = \theta^T \omega$ $\quad \omega = \begin{bmatrix} -\omega_1 \\ -\omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$ $\theta = \begin{bmatrix} a_0 \\ a_1 \\ b_0 \\ b_1 \end{bmatrix}$

$y = \omega_0 + \theta^T \omega$

Задача №6. Синтезировать Ад. набл. для $\dot{y}$

Уст. О: $\ddot{y} + a_1 \dot{y} y^2 + a_0 y = b_1 \dot{u} + b_0 u^3$ $\quad a_0, a_1, b_0, b_1$ - неизв

Ц: $\hat{\dot{y}}$: $\lim_{t\to\infty} (\hat{\dot{y}}(t) - \dot{y}(t)) = 0$

Решение: 1) Решить з №5

$y = \omega_0 + \theta^T \omega$

2) Парам. $\dot{y}$

$\dot{y} = \dot{\omega}_0 + \theta^T \dot{\omega}$

$\dot{\omega}_1 = \frac{s}{k(s)}[y], \dot{\omega}_2 = \frac{s^2}{k(s)}\left[\frac{y^3}{3}\right]$

$\dot{\omega}_3 = \frac{s}{k(s)}[u], \dot{\omega}_4 = \frac{s^2}{k(s)}[u]$

3) $\varepsilon \overset{\Delta}{=} y - \omega_0 - \hat{\theta}^T \omega$

$\varepsilon = \tilde{\theta}^T \omega$

$\Downarrow$

$\dot{\hat{\theta}} = \gamma \omega \varepsilon$

4) Набл

$\hat{\dot{y}} = \dot{\omega}_0 + \hat{\theta}^T \dot{\omega}$

Задача №0

$V = \frac{1}{2}\varepsilon^2 + \frac{1}{2}\tilde{\theta}^2$

а) $\dot{V} = -\varepsilon^2$ $\quad$ д) $\dot{V} = -\varepsilon^2 - 2\tilde{\theta}^2 \leq -\frac{1}{2}\varepsilon^2 - \frac{1}{2}\tilde{\theta}^2 = -V$

в) $\dot{V} =$