



ITMO UNIVERSITY

Saint Petersburg, Russia

Augmented error based adaptive control with improved parametric convergence.

Tutorial

Dmitry N. Gerasimov and Vladimir O. Nikiforov

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1. Motivation: MRAC and the Monopoli's scheme of augmented error

1.1. Problem statement and plant parameterization

Plant:

$$y = \frac{b(s)}{a(s)}[u], \quad (1)$$

where $a(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_0$,

$$b(s) = k s^m + b_{m-1}s^{m-1} + \dots + b_0,$$

a_i, b_j are unknown constant coefficients, $n > m$, m are the known orders.

1.1. Problem statement and plant parameterization

Objective:

$$\lim_{t \rightarrow \infty} \varepsilon(t) = \lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0, \quad (2)$$

where y_m is the output of the reference model

$$y_m = \frac{a_m(0)}{a_m(s)}[g] \quad (3)$$

with Hurwitz polynomial $a_m(s)$ of degree $n-m$ with bounded PWC reference $g(t)$.

Assumptions:

1. The polynomial $b(s) = k s^m + b_{m-1} s^{m-1} + \dots + b_0$ is Hurwitz;
2. The sign of the high frequency gain k is known (w.l.g. $k > 0$).

1.1. Problem statement and plant parameterization

Lemma (Monopoli): There exists a constant vector $\psi \in \mathbb{R}^{2n}$

$$\varepsilon \stackrel{\Delta}{=} y - y_m = \frac{k}{a_m(s)} [\psi^T \phi - u] + \sigma, \quad (4)$$

where σ (here and hereafter) is the exponentially decaying term,
 $\phi = [y, v_1, v_2, g]^T$ is the regressor with the vectors $v_1, v_2 \in \mathbb{R}^{n-1}$ generated by
the filters

$$\dot{v}_1 = \Lambda v_1 + \varsigma_{n-1} u,$$

$$\dot{v}_2 = \Lambda v_2 + \varsigma_{n-1} y,$$

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 0 & 1 \\ -\lambda_0 & -\lambda_1 & \dots & \dots & -\lambda_{n-2} \end{bmatrix}, \quad \varsigma_{n-1} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

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Certainty equivalent controller:

$$u = \hat{\psi}^T \phi, \quad (5)$$

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where $\hat{\psi}$ is the vector of adjustable parameters.

Error model:

$$\varepsilon = \frac{k}{a_m(s)} [\tilde{\psi}^T \phi] + \sigma, \quad (6)$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

1.2. The concept of augmented error

Error model:

$$\varepsilon = kW(s) \left[\tilde{\psi}^T \phi \right], \quad (7)$$

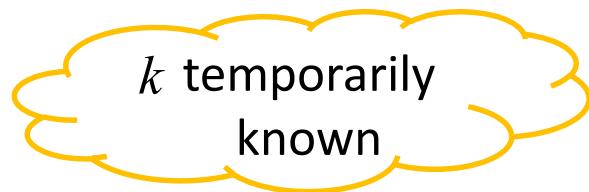
where $\varepsilon \in \mathbb{R}$ is the output error, $\tilde{\psi} = \psi - \hat{\psi} \in \mathbb{R}^q$ is vector of parametric errors, $\phi \in \mathbb{R}^q$ is the vector of measurable functions (regressor), $\hat{\psi}$ is the vector of adjustable parameters (or estimate), $k \in \mathbb{R}$ is a positive control gain, $W(s)$ is the asymptotically stable transfer function.

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1.2. The concept of augmented error

TRANSFER FUNCTION SWAPPING

Augmentation signal:

$$\varsigma_k = W(s) [\hat{\psi}^T \phi] - \hat{\psi}^T \phi_k, \quad (8)$$

where $\phi_k = kW(s)[\phi]$ is the filtered regressor.

Augmented error:

$$\tilde{\varepsilon}_k = \varepsilon + \varsigma_k. \quad (9)$$

Result of swapping:

$$\varepsilon = kW(s) [\tilde{\psi}^T \phi] \longrightarrow \tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (10)$$

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where $\phi_k = kW(s)[\phi]$ is the filtered regressor.

Augmented error:

Result of swapping:

$$\tilde{\varepsilon}_k = \varepsilon + \varsigma_k \cdot \int_0^t \phi^T(\tau) \tilde{\psi}(\tau) d\tau = \int_0^t \phi^T(\tau) d\tau \tilde{\psi} - \int_0^t \int_0^{\tau_2} \phi^T(\tau_1) d\tau_1 \dot{\tilde{\psi}}(\tau_2) d\tau_2 \quad (9)$$

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (10)$$

Integration by parts?

1.2. The concept of augmented error

Integration by parts!

Corollary of the results

Augmentation signal:

$$\zeta_k = -kW_C(s) \left[W_B(s) \left[\phi^T \right] \dot{\hat{\psi}} \right], \quad (11)$$

where $W_C(s) = c^T (sI - A)^{-1}$, $W_B(s) = (sI - A)^{-1} b$ are the transfer function vectors.

Augmented error:

$$\tilde{\varepsilon}_k = \varepsilon - kW_C(s) \left[W_B(s) \left[\phi^T \right] \dot{\hat{\psi}} \right]. \quad (12)$$

Augmented
error

Control
error

Proportional to the time
derivative $\dot{\hat{\psi}}$

1.2. The concept of augmented error

Main idea of the tutorial

is to combine the augmented error approach with modifications of Lion's (DRE) and Kreisselmeier's (MRE) adaptation algorithms ^{1,2,3} with fast parameters tuning to draw the augmentation term to zero AFAP

$$\tilde{\varepsilon}_k = \varepsilon - k W_C(s) \left[W_B(s) \left[\phi^T \right] \dot{\psi} \right]$$

Main contributions:

1. Improvement of output (tracking) error convergence for problems of adaptive control using augmented error: AE+DRE, AE+MRE;
2. Extension of the idea to systems with unknown steady-state coefficients (control gains) k : AE+DRE, AE+MRE;
3. *Ad hoc extensions* of augmented error concept with DRE and MRE to unstable LTI systems, delayed and MIMO LTI systems, nonlinear systems.

1. Lion P. , Rapid identification of linear and nonlinear systems. AIAA J., 5(10), 1835-1842, 1967.

2. Kreisselmeier G. Adaptive observers with exponential rate of convergence. TAC, 22(1), 2-8, 1977.

3. Ortega R., Nikiforov V., and Gerasimov D. On modified parameter estimators for identification and adaptive control: a unified framework and some new schemes, ARC, 50, 278-293., 2020.

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2. Augmented error and improved parametric convergence

TRANSFER FUNCTION SWAPPING

2.1. Schemes for known control gain k

Augmented error:

$$\tilde{\varepsilon}_k = \varepsilon + \zeta_k. \quad (13)$$

Result of swapping:

$$\varepsilon = kW(s) \left[\tilde{\psi}^T \phi \right] \longrightarrow \tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (14)$$

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma \phi_k \tilde{\varepsilon}_k,$$

where $\gamma > 0$ is the adaptation gain.

**Parametric error
model:**

$$\dot{\tilde{\psi}} = -\gamma \phi_k \phi_k^T \tilde{\psi} - \gamma \phi_k \sigma_k \quad (16)$$

Properties:

1. If $\phi_k \in PC$ and defined for all $t \geq 0$, then $\dot{\hat{\psi}} \in L_\infty$ and $\dot{\tilde{\varepsilon}}_k \in L_2$;
2. If $\phi_k, \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \|\tilde{\varepsilon}_k(t)\|, \|\varepsilon(t)\| \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_k \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast and there exists an optimal γ , for which the rate of parametric convergence is maximum.

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Improved adaptation algorithm with DRE:

Dynamically extended error:

$$E_H = H(s) \left[\tilde{\varepsilon}_k + \phi_k^T \hat{\psi} \right] - \Phi_H^T \hat{\psi}, \quad (17)$$

where $H(s) = [H_1(s), H_2(s), \dots, H_{q-1}(s)]$ is the transfer function vector,

$\Phi_H^T = H(s) [\phi_k^T]$ is the dynamically extended regressor.

Dynamically extended error model:

$$E_H = \Phi_H^T \tilde{\psi} + \sigma_H. \quad (18)$$

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TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H, \quad (19)$$

$$\text{where } \Phi_H^T = H(s) \begin{bmatrix} \phi_k^T \end{bmatrix}, \quad E_H = H(s) \begin{bmatrix} \tilde{\varepsilon}_k + \phi_k^T \hat{\psi} \end{bmatrix} - \Phi_H^T \hat{\psi}.$$

Properties:

1. If $\phi_k \in PC$ and defined for all $t \geq 0$, then $\hat{\psi} \in L_\infty$ and $\tilde{\varepsilon}_k \in L_2$;
2. If $\dot{\phi}_k, \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \|\tilde{\varepsilon}_k(t)\|, \|\varepsilon(t)\| \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_k \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ ;
4. If $\lambda_H(t) \notin L_1$, where λ_H is the minimum eigenvalue of $\Phi_H \Phi_H^T$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ asymptotically, and the rate of convergence can be increased by increasing γ .

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Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H, \quad \dot{\tilde{\psi}} = -\gamma \Phi_H \Phi_H^T \tilde{\psi} - \gamma \Phi_H \sigma_H \quad (19)$$

where $\Phi_H^T = H(s) [\phi_k^T]$, $E_H = H(s) [\tilde{\varepsilon}_k + \phi_k^T \hat{\psi}] - \Phi_H^T \hat{\psi}$.

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2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Improved adaptation algorithm with MRE:

Memory extended error:

$$E_L = L(s) \left[\phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi} \right] - \Phi_L \hat{\psi}, \quad (20)$$

where $\Phi_L = L(s) \left[\phi_k \phi_k^T \right] \geq 0$ is the memory extended regressor,

$$L(s) = \prod_{i=1}^p \frac{1}{s + \beta_i} = \frac{1}{d(s)}, \quad \beta_i > 0. \quad (21)$$

Memory extended error model:

$$E_L = \Phi_L^T \tilde{\psi} + \sigma_L. \quad (22)$$

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Error model:

$$\tilde{\varepsilon}_k = \tilde{\psi}^T \phi_k + \sigma_k \quad (15)$$

Improved adaptation algorithm with MRE:

$$\dot{\hat{\psi}} = \gamma E_L, \quad (23)$$

where $\Phi_L = L(s) \begin{bmatrix} \phi_k \phi_k^T \end{bmatrix}$, $E_L = L(s) \begin{bmatrix} \phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi} \end{bmatrix} - \Phi_L \hat{\psi}$.

Properties:

1. If $\phi_k \in PC$ and defined for all $t \geq 0$, then $\hat{\psi} \in L_\infty$ and $\tilde{\varepsilon}_k \in L_2$;
2. If $\dot{\phi}_k, \dot{\phi}_k \in L_\infty$, then $\tilde{\varepsilon}_k, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \|\tilde{\varepsilon}_k(t)\|, \|\varepsilon(t)\| \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_k \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ ;
4. If $\lambda_L(t) \notin L_1$, where λ_L is the minimum eigenvalue of Φ_L , then $\|\tilde{\psi}(t)\| \rightarrow 0$ asymptotically, and the rate of convergence can be increased by increasing γ .

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

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Properties:

5. High order tuner (Bonus Property)

$$\begin{aligned} \hat{\psi}^{(i+1)} &= \gamma \left(Y_L^{(i)} - \sum_{j=0}^{p-1} C_j^i \Phi_L^{(i-j)} \hat{\psi}^{(j)} \right), \quad i = 1, 2, \dots, p, \\ Y_L^{(i)} &= \frac{s^i}{d(s)} \begin{bmatrix} \phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi} \end{bmatrix}, \quad \Phi_L^{(i-j)} = \frac{s^{i-j}}{d(s)} \begin{bmatrix} \phi_k \phi_k^T \end{bmatrix}. \end{aligned} \quad (24a)$$

2.1. Schemes for known control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

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Improved adaptation algorithm with MRE:

$$\dot{\hat{\psi}} = \gamma E_L, \quad (23)$$

where $\Phi_L = L(s) \begin{bmatrix} \phi_k \phi_k^T \end{bmatrix}$, $E_L = L(s) \begin{bmatrix} \phi_k \tilde{\varepsilon}_k + \phi_k \phi_k^T \hat{\psi} \end{bmatrix} - \Phi_L \hat{\psi}$.

Properties:

5. High order tuner (Bonus Property)

$$\hat{\psi}^{(p+1)} + \left(d_{p-1} I_q + \gamma \Phi_L \right) \hat{\psi}^{(p)} + \left(d_{p-2} I_q + \gamma d_{p-1} \Phi_L + \gamma d_p C_{p-1}^p \Phi_L \right) \hat{\psi}^{(p-1)} \quad (24b)$$

$$+ \dots + \left(d_0 I_q + \gamma \sum_{j=1}^p d_j C_1^j \Phi_L^{(j-1)} \right) \dot{\hat{\psi}} = \gamma \phi_k \tilde{\varepsilon}_k$$

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Error model (state-space):

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi, \quad e(0), \quad (24)$$

$$\varepsilon = c^T e, \quad (25)$$

where $e \in \mathbb{R}^n$ is the state error, (A, b, c) is the minimal realization of the transfer function, $W(s) = c^T (sI - A)^{-1} b$, the matrix $A \in \mathbb{R}^{n \times n}$ is Hurwitz.

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Error model (state-space):

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \quad (26)$$

Filters:

$$\dot{\Omega} = A\Omega + kb\phi^T \quad (27)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi} \quad (28)$$

Augmented error:

$$\tilde{e}_k = e + \omega \quad (29)$$

Result of swapping:

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \quad \longrightarrow \quad \tilde{e}_k = \Omega\tilde{\psi} + \sigma \quad (30)$$

2.1. Schemes for known control gain k

STATE-SPACE SWAPPING

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma \Omega^T \tilde{e}_k \quad (31)$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H, \quad (32)$$

where $\Phi_H^T = H(s)[\Omega]$, $E_H = H(s)[\tilde{e}_k + \Omega \hat{\psi}] - \Phi_H^T \hat{\psi}$.

Improved adaptation algorithm with MRE:

$$\dot{\hat{\psi}} = \gamma E_L, \quad (33)$$

where $\Phi_L = L(s)[\Omega^T \Omega]$, $E_L = L(s)[\Omega^T \tilde{e}_k + \Omega^T \Omega \hat{\psi}] - \Phi_L \hat{\psi}$.

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING

Augmentation signal:

$$\zeta_u = W(s) [\hat{\psi}^T \phi] - \hat{\psi}^T \phi_u, \quad (34)$$

where $\phi_u = W(s)[\phi]$ is the filtered regressor.

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k} \zeta_u, \quad (35)$$

where \hat{k} is the additional adjustable parameter.

Result of swapping:

$$\varepsilon = k W(s) [\tilde{\psi}^T \phi] \xrightarrow{\hspace{1cm}} \tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (36)$$

where $\tilde{k} = k - \hat{k}$ is the additional parametric error.

2.2. Schemes for unknown control gain k

Corollary of the results

Augmentation signal:

$$\zeta_u = -W_C(s) \left[W_B(s) \begin{bmatrix} \phi^T \\ \dot{\psi} \end{bmatrix} \right], \quad (37)$$

where $W_C(s) = c^T (sI - A)^{-1}$, $W_B(s) = (sI - A)^{-1} b$ are the transfer function vectors.

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon - \hat{k} W_C(s) \left[W_B(s) \begin{bmatrix} \phi^T \\ \dot{\psi} \end{bmatrix} \right].$$



 Augmented error Control error Proportional to the time derivative $\dot{\psi}$ and \hat{k}

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (38)$$

Gradient adaptation algorithm:

$$\begin{aligned} \dot{\tilde{\psi}} &= \gamma_1 \phi_u \tilde{\varepsilon}_u \begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{k}} \end{bmatrix} = -\gamma \begin{bmatrix} \phi_u \phi_u^T & -\phi_u \zeta_u \\ -\zeta_u \phi_u^T & \zeta_u^2 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \begin{bmatrix} \phi_u \\ \zeta_u \end{bmatrix} \sigma_u \\ \dot{\tilde{k}} &= -\gamma_2 \zeta_u \tilde{\varepsilon}_u, \end{aligned} \quad (39)$$

Parametric error model:

$$\gamma = \text{diag}\{\gamma_1, \gamma_2\} \quad (40)$$

where $\gamma_1, \gamma_2 > 0$ are the adaptation gains.

Properties:

1. If $\phi_u \in PC$ and defined for all $t \geq 0$, then $\hat{\psi}, \hat{k} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\phi_u, \dot{\phi}_u \in L_\infty$, then $\tilde{\varepsilon}_u, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \|\tilde{\varepsilon}_u(t)\|, \|\varepsilon(t)\| \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_u \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast and there exists an optimal γ , for which the rate of parametric convergence is maximum.

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with DRE:

Dynamically extended error:

$$\bar{E}_H = \bar{H}(s) \left[\tilde{\varepsilon}_u - \hat{k}\zeta_u \right] + \hat{k}Z_H, \quad (41)$$

where $\bar{H}(s) = [H_1(s), H_2(s), \dots, H_q(s)]$ is the transfer function vector,

$$\zeta_u = W(s) [\hat{\psi}^T \phi] - \hat{\psi}^T \phi_u,$$

$$Z_H = \bar{H}(s) \left[W(s) [\hat{\psi}^T \phi] \right] - \Phi_H^T \hat{\psi}, \quad \Phi_H^T = \bar{H}(s) [\phi_u].$$

Dynamically extended error model:

$$\bar{E}_H = k\Phi_H^T \tilde{\psi} - \tilde{k}Z_H + \bar{\sigma}_H. \quad (42)$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \quad \gamma_1 > 0 \quad (43)$$

$$\dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \quad \gamma_2 > 0 \quad (44)$$

Parametric error model:

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = -\gamma \Omega_H \begin{bmatrix} kI_q & O_q \\ O_{1 \times q} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \begin{bmatrix} \Phi_H \\ Z_H^T \end{bmatrix} \bar{\sigma}_H,$$

$$\Omega_H = \begin{bmatrix} \Phi_H \Phi_H^T & -\Phi_H Z_H \\ -Z_H^T \Phi_H^T & Z_H^T Z_H \end{bmatrix}, \quad \gamma = \text{diag}\{\gamma_1, \gamma_2\}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \quad \gamma_1 > 0 \quad (43)$$

$$\dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \quad \gamma_2 > 0 \quad (44)$$

Properties:

1. If $\phi_u \in PC$ and defined for all $t \geq 0$, then $\hat{\psi}, \hat{k} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\dot{\phi}_u, \dot{\phi}_u \in L_\infty$, then $\dot{\tilde{\varepsilon}}_u, \dot{\hat{\psi}}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \dot{\tilde{\varepsilon}}_u(t), \varepsilon(t) \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_u \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ_1, γ_2 ;
4. If $\bar{\lambda}_H(t) \notin L_1$, where $\bar{\lambda}_H$ is the minimum eigenvalue of Ω_H then $\tilde{\psi}(t), \tilde{k}(t) \rightarrow 0$ asymptotically, and the rate of convergence can be increased by increasing γ_1, γ_2 .

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

Filter:

$$L(s) = \prod_{i=1}^p \frac{1}{s + \beta_i} = \frac{1}{d(s)}, \quad \beta_i > 0. \quad (45)$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \hat{k}\zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

Memory extended error:

$$\bar{E}_L = \begin{bmatrix} L(s) \left[\phi_u \left(\tilde{\varepsilon}_u - \hat{k}\zeta_u \right) \right] \\ L(s) \left[W(s) \left[\hat{\psi}^T \phi \right] \left(\tilde{\varepsilon}_u - \hat{k}\zeta_u \right) \right] \end{bmatrix} + \hat{k} Z_L, \quad (46)$$

where $\zeta_u = W(s) \left[\hat{\psi}^T \phi \right] - \hat{\psi}^T \phi_u$,

$$Z_L = \begin{bmatrix} L(s) \left[\phi_u W(s) \left[\hat{\psi}^T \phi \right] \right] \\ L(s) \left[W(s) \left[\hat{\psi}^T \phi \right]^2 \right] \end{bmatrix} - \begin{bmatrix} L(s) \left[\phi_u \phi_u^T \right] \\ L(s) \left[W(s) \left[\hat{\psi}^T \phi \right] \phi_u^T \right] \end{bmatrix} \hat{\psi}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

Memory extended error model:

$$\bar{E}_L = \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} + \bar{\sigma}_L, \quad (47)$$

where

$$\Omega_L = \begin{bmatrix} L(s)[\phi_u \phi_u^T] & L(s)[\phi_u W(s)[\hat{\psi}^T \phi]] \\ L(s)[W(s)[\hat{\psi}^T \phi] \phi_u^T] & L(s)[W(s)[\hat{\psi}^T \phi]^2] \end{bmatrix}, \quad \Psi = \begin{bmatrix} kI_q & \hat{\psi} \\ O_{1 \times q} & -1 \end{bmatrix}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k\tilde{\psi}^T \phi_u - \tilde{k}\zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

Memory extended error model:

$$\bar{E}_L = \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} + \bar{\sigma}_L, \quad (47)$$

where

$$\Omega_L = \begin{bmatrix} L(s)[\phi_u \phi_u^T] & L(s)[\phi_u W(s)[\hat{\psi}^T \phi]] \\ L(s)[W(s)[\hat{\psi}^T \phi] \phi_u^T] & L(s)[W(s)[\hat{\psi}^T \phi]^2] \end{bmatrix} \geq 0, \quad \Psi = \begin{bmatrix} kI_q \geq 0 & \hat{\psi} \\ O_{1 \times q} & -1 \end{bmatrix}.$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \hat{\Psi}^T \bar{E}_L, \quad \gamma = \text{diag}\{\gamma_1, \gamma_2\} > 0, \quad (48)$$

Parametric error model:

$$\begin{bmatrix} \dot{\tilde{\psi}} \\ \dot{\tilde{k}} \end{bmatrix} = -\gamma \hat{\Psi}^T \Omega_L \Psi \begin{bmatrix} \tilde{\psi} \\ \tilde{k} \end{bmatrix} - \gamma \hat{\Psi}^T \bar{\sigma}_L,$$

$$\hat{\Psi} = \begin{bmatrix} I_q & \hat{\psi} \\ O_{1 \times q} & -1 \end{bmatrix}, \quad \Psi = \begin{bmatrix} kI_q & \hat{\psi} \\ O_{1 \times q} & -1 \end{bmatrix}$$

2.2. Schemes for unknown control gain k

TRANSFER FUNCTION SWAPPING ADAPTATION ALGORITHMS

Error model:

$$\tilde{\varepsilon}_u = k \tilde{\psi}^T \phi_u - \tilde{k} \zeta_u + \sigma_u \quad (38)$$

Improved adaptation algorithm with MRE:

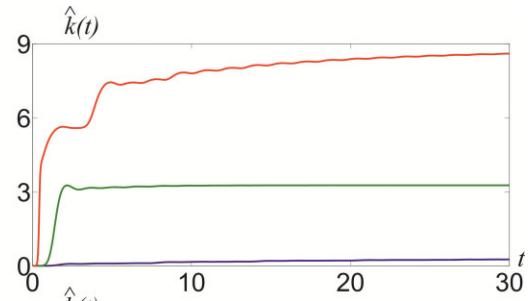
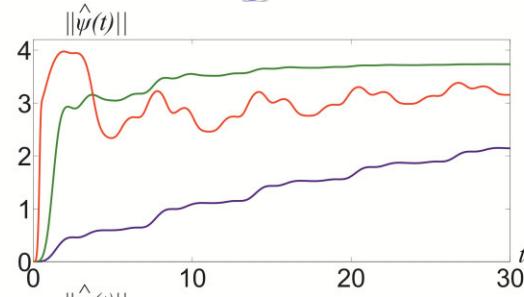
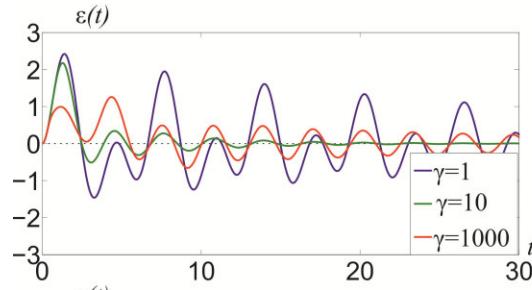
$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \hat{\Psi}^T \bar{E}_L, \quad \gamma = \text{diag}\{\gamma_1, \gamma_2\} > 0, \quad (48)$$

Properties:

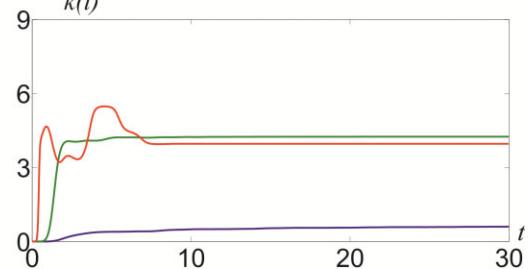
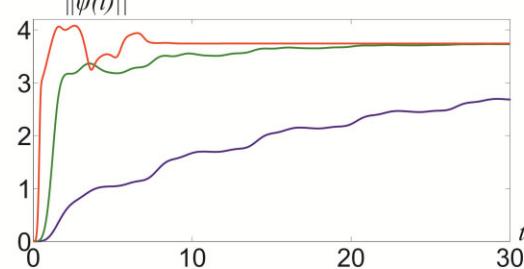
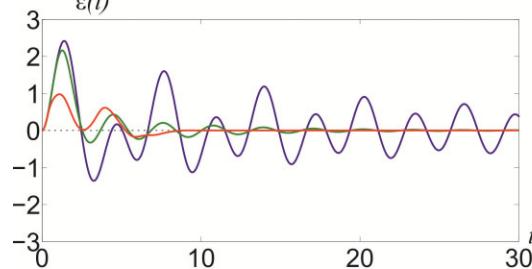
1. If $\phi_u \in PC$ and defined for all $t \geq 0$, then $\dot{\hat{\psi}}, \dot{\hat{k}} \in L_\infty$ and $\tilde{\varepsilon}_u \in L_2$;
2. If $\phi_u, \dot{\phi}_u \in L_\infty$, then $\tilde{\varepsilon}_u, \hat{\psi}, \dot{\hat{\psi}} \in L_\infty$ and $\|\dot{\hat{\psi}}(t)\|, \|\tilde{\varepsilon}_u(t)\|, \|\varepsilon(t)\| \rightarrow 0$ as $t \rightarrow \infty$;
3. If $\phi_u \in PE$, then $\|\tilde{\psi}(t)\| \rightarrow 0$ exponentially fast, and the rate of parametric convergence can be increased by increasing γ_1, γ_2 ;
4. If $\bar{\lambda}_\Omega(t) \notin L_1$, where $\bar{\lambda}_\Omega$ is the minimum eigenvalue of $\hat{\Psi}^T \Omega_L \hat{\Psi}$ then $\tilde{\psi}(t), \tilde{k}(t) \rightarrow 0$ asymptotically, and the rate of convergence can be increased by increasing γ_1, γ_2 .

2.2. Schemes for unknown control gain k

Gradient
AA:



AA with
DRE:



AA with
MRE:

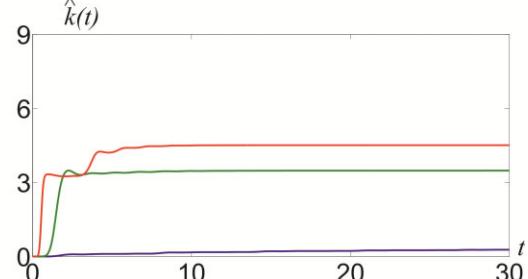
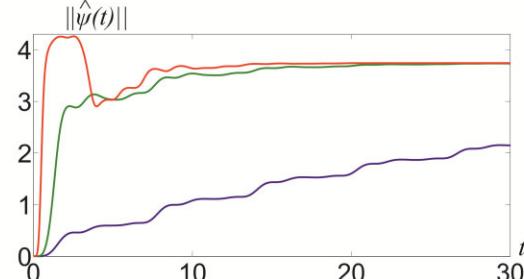
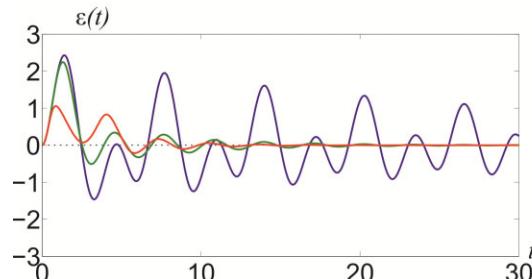
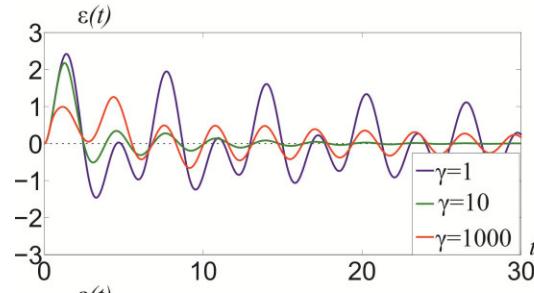


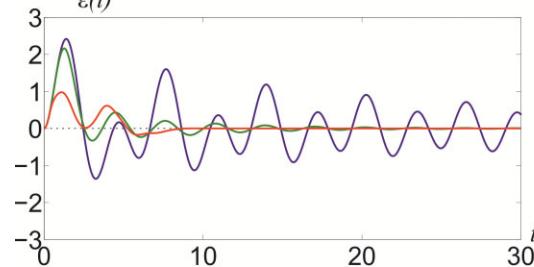
Fig. 1. Identification results in different adaptation algorithms with different gains $\gamma = \gamma_1 = \gamma_2$

2.2. Schemes for unknown control gain k

Gradient
AA:



AA with
DRE:



AA with
MRE:

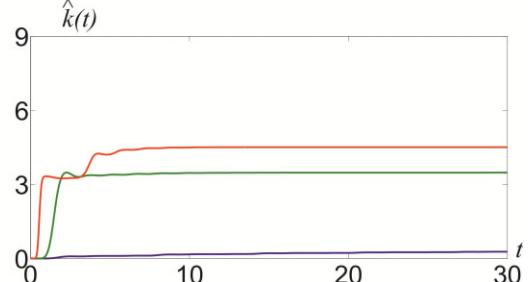
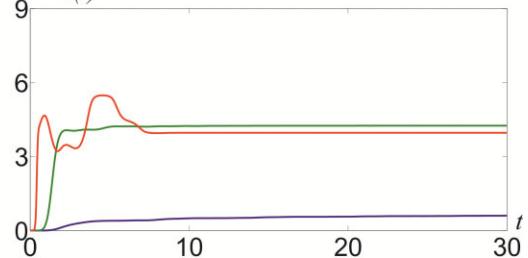
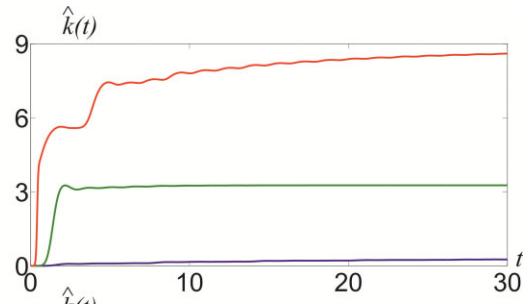
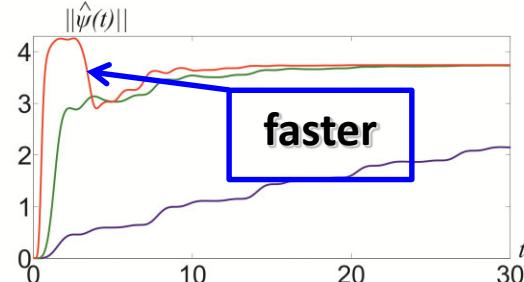
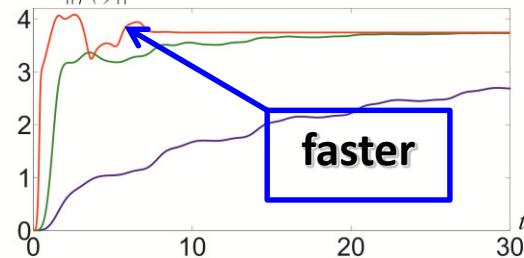
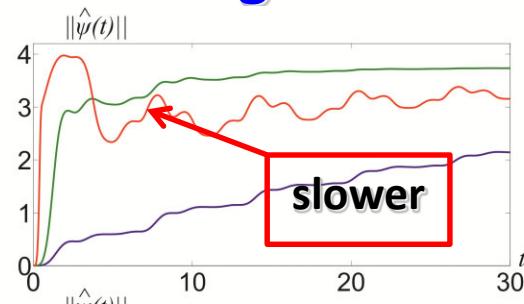
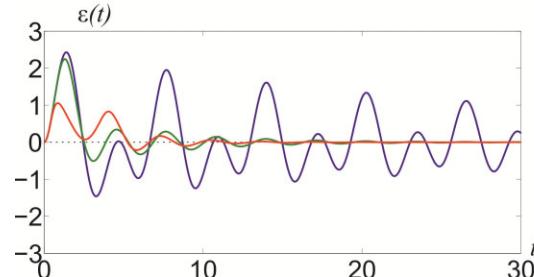


Fig. 1. Identification results in different adaptation algorithms with different gains $\gamma = \gamma_1 = \gamma_2$

2.2. Schemes for unknown control gain k

STATE-SPACE SWAPPING

Error model:

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \quad (49)$$

Filters:

$$\dot{\Omega} = A\Omega + b\phi^T \hat{\psi} \quad \dot{\bar{\Omega}} = A\bar{\Omega} + b\phi^T, \quad (50)$$

$$\dot{w} = Aw - \hat{k}\bar{\Omega}\dot{\hat{\psi}} + \dot{\hat{k}}\bar{w}, \quad \dot{\bar{w}} = A\bar{w} - \bar{\Omega}\dot{\hat{\psi}} \quad (51)$$

Augmented error:

$$\tilde{e}_u = e + w \quad (52)$$

Result of swapping:

$$\dot{e} = Ae + kb\tilde{\psi}^T \phi \quad \xrightarrow{\hspace{1cm}} \quad \tilde{e}_u = k\bar{\Omega}\tilde{\psi} - \tilde{k}\bar{w} + \bar{\sigma}, \quad (53)$$

where $\tilde{k} = k - \hat{k}$ is the parametric error.

2.2. Schemes for unknown control gain k

STATE-SPACE SWAPPING

Gradient adaptation algorithm:

$$\dot{\hat{\psi}} = \gamma_1 \bar{\Omega}^T \tilde{e}_u, \quad \dot{\hat{k}} = -\gamma_2 \bar{w} \tilde{e}_u \quad (54)$$

Improved adaptation algorithm with DRE:

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \quad \dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \quad (55)$$

where $\bar{E}_H = \bar{H}(s) [\tilde{e}_u - \hat{k} \bar{w}] + \hat{k} Z_H$, $Z_H = \bar{H}(s) [\Omega] - \Phi_H^T \hat{\psi}$, $\Phi_H^T = \bar{H}(s) [\bar{\Omega}^T]$,
 $\bar{H}(s) = [H_1(s), H_2(s), \dots, H_q(s)]$

2.2. Schemes for unknown control gain k

STATE-SPACE SWAPPING

Improved adaptation algorithm with MRE:

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \hat{\Psi}^T \bar{E}_L, \quad (56)$$

where $\gamma = \text{diag}\{\gamma_1, \gamma_2\}$, $\hat{\Psi} = \begin{bmatrix} I_q & \hat{\psi} \\ O_{1 \times q} & -1 \end{bmatrix}$,

$$\bar{E}_L = \begin{bmatrix} L(s) \left[\bar{\Omega}^T (\tilde{e}_u - \hat{k} \bar{w}) \right] \\ L(s) \left[\Omega^T (\tilde{e}_u - \hat{k} \bar{w}) \right] \end{bmatrix} + \hat{k} Z_L, \quad Z_L = \begin{bmatrix} L(s) \left[\bar{\Omega}^T \Omega \right] \\ L(s) \left[\Omega^T \Omega \right] \end{bmatrix} - \begin{bmatrix} L(s) \left[\bar{\Omega}^T \bar{\Omega} \right] \\ L(s) \left[\Omega^T \bar{\Omega} \right] \end{bmatrix} \hat{\psi}.$$

2.2. Schemes for unknown control gain (example)

Example 1. Model reference adaptive control

Plant:

$$y = \frac{k}{s^2 + a_1 s + a_0} [u],$$

where $k = 5$, $a_0 = 1$, $a_1 = 2$ are the unknown parameters,

Reference model:

$$y_m = \frac{20}{s^2 + 9s + 20} [g],$$

where $g(t) = \sin(t) + 2$.

Objective:

$$\lim_{t \rightarrow \infty} \varepsilon(t) = \lim_{t \rightarrow \infty} (y_m(t) - y(t)) = 0.$$

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{v}_1 = -5v_1 + u, \quad \dot{v}_2 = -5v_2 + y$$

forming $\phi = [y, v_1, v_2, g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k} \varsigma_u, \quad \varsigma_u = W(s) [\hat{\psi}^T \phi] - \hat{\psi}^T \phi_u, \quad \phi_u = W(s) [\phi]$$

Algorithm of adaptation (gradient):

$$\dot{\hat{\psi}} = \gamma_1 \frac{\phi_u}{r^2} \tilde{\varepsilon}_u, \quad \dot{\hat{k}} = -\gamma_2 \frac{\varsigma_u}{r^2} \tilde{\varepsilon}_u, \quad \gamma_1, \gamma_2 > 0,$$

where $r = \sqrt{1 + \phi_u^T \phi_u}$ is the normalizing divider.

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{v}_1 = -5v_1 + u, \quad \dot{v}_2 = -5v_2 + y$$

forming $\phi = [y, v_1, v_2, g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\varepsilon}_u = \varepsilon + \hat{k}\varsigma_u, \quad \varsigma_u = W(s)[\hat{\psi}^T \phi] - \hat{\psi}^T \phi_u, \quad \phi_u = W(s)[\phi]$$

Algorithm of adaptation (improved with DRE):

$$\dot{\hat{\psi}} = \gamma_1 \Phi_H \bar{E}_H, \quad \dot{\hat{k}} = -\gamma_2 Z_H^T \bar{E}_H, \quad \gamma_1, \gamma_2 > 0,$$

$$\bar{E}_H = \bar{H}(s) \left[\frac{\tilde{\varepsilon}_u - \hat{k}\varsigma_u}{r} \right] + \hat{k} Z_H, \quad Z_H = \bar{H}(s) \left[\frac{W(s)[\hat{\psi}^T \phi]}{r} \right] - \Phi_H^T \hat{\psi}, \quad \Phi_H^T = \bar{H}(s) \left[\frac{\phi_u}{r} \right].$$

2.2. Schemes for unknown control gain (example)

Filters:

$$\dot{v}_1 = -5v_1 + u, \quad \dot{v}_2 = -5v_2 + y$$

forming $\phi = [y, v_1, v_2, g]^T$.

Control:

$$u = \hat{\psi}^T \phi$$

Augmented error:

$$\tilde{\epsilon}_u = \epsilon + \hat{k}\zeta_u, \quad \zeta_u = W(s)[\hat{\psi}^T \phi] - \hat{\psi}^T \phi_u, \quad \phi_u = W(s)[\phi]$$

Algorithm of adaptation (improved with MRE):

$$\begin{bmatrix} \dot{\hat{\psi}} \\ \dot{\hat{k}} \end{bmatrix} = \gamma \begin{bmatrix} I_q & O_q \\ \hat{\psi}^T & -1 \end{bmatrix} \bar{E}_L, \quad \gamma = \text{diag}\{\gamma_1, \gamma_2\} > 0,$$

$$\bar{E}_L = \begin{bmatrix} L(s) \left[\phi_u (\tilde{\epsilon}_u - \hat{k}\zeta_u) / r^2 \right] \\ L(s) \left[W(s) [\hat{\psi}^T \phi] (\tilde{\epsilon}_u - \hat{k}\zeta_u) / r^2 \right] \end{bmatrix} + \hat{k} Z_L, \quad Z_L = \begin{bmatrix} L(s) \left[\phi_u W(s) [\hat{\psi}^T \phi] / r^2 \right] \\ L(s) \left[W(s) [\hat{\psi}^T \phi]^2 / r^2 \right] \end{bmatrix} - \begin{bmatrix} L(s) \left[\phi_u \phi_u^T / r^2 \right] \\ L(s) \left[W(s) [\hat{\psi}^T \phi] \phi_u^T / r^2 \right] \end{bmatrix} \hat{\psi}.$$

2.2. Schemes for unknown control gain (example)

Simulation results:

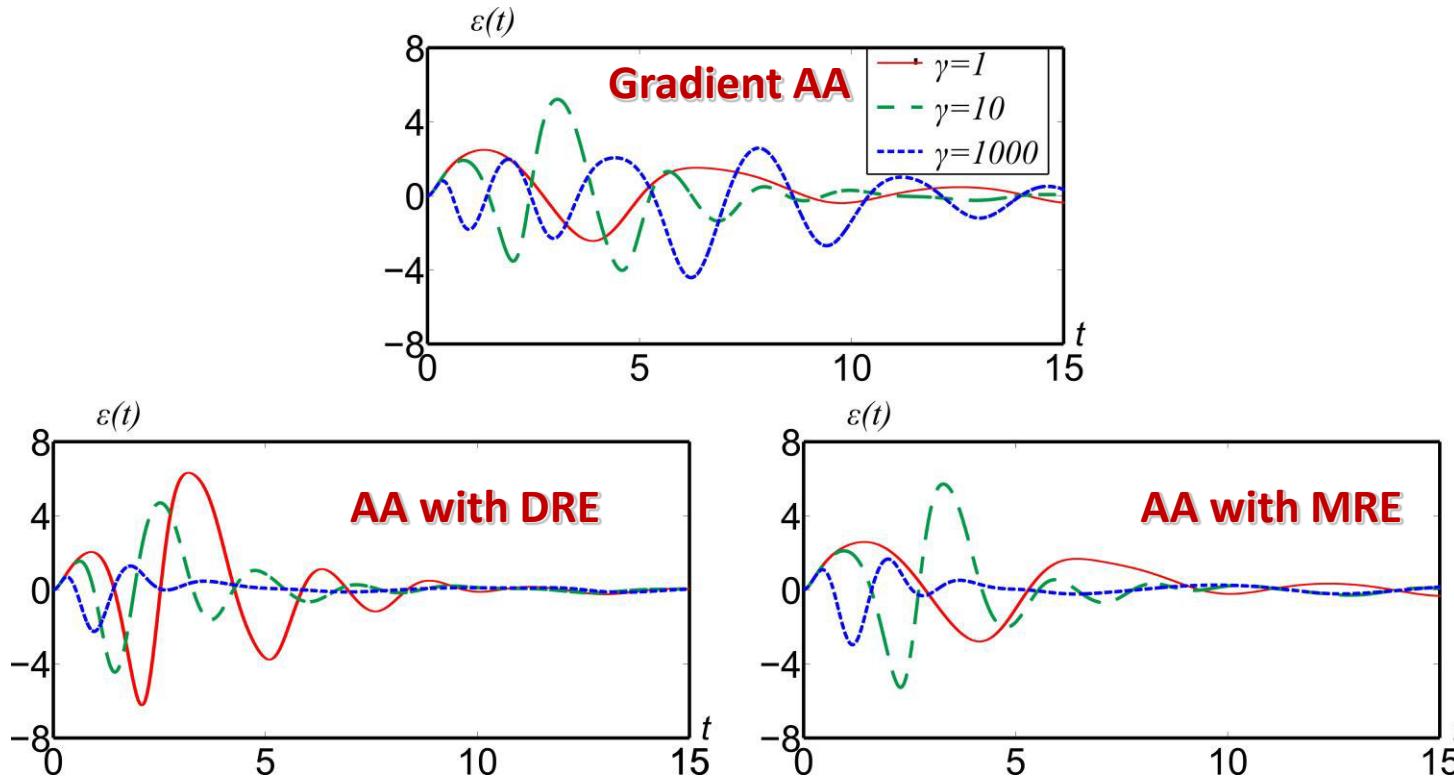
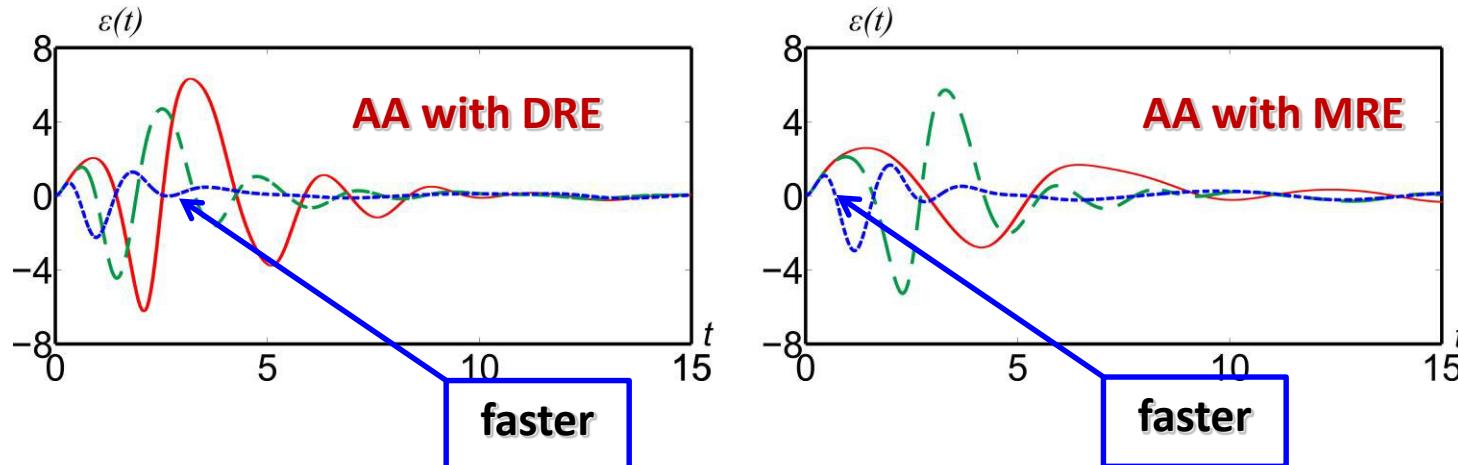
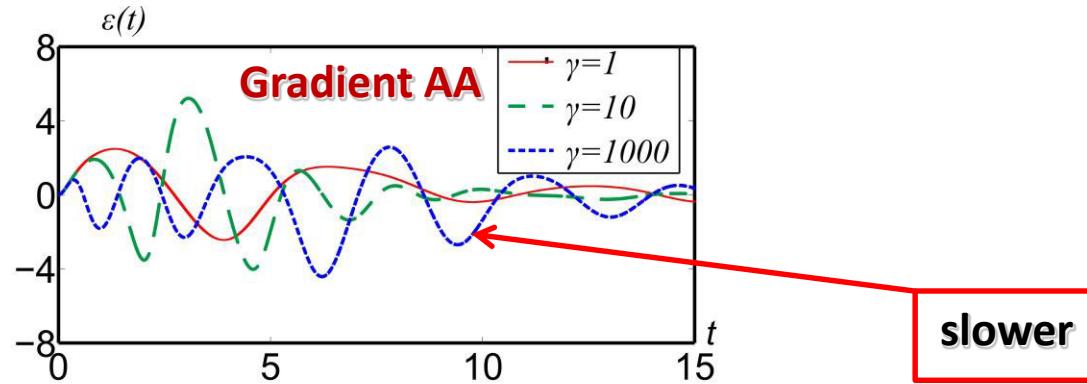


Fig. 2. Evolutions of tracking errors in the systems closed by different adaptation algorithms

2.2. Schemes for unknown control gain (example)

Simulation results:



3. Development of the concept and *adhoc* modifications of augmented error

3.1. Swapping for unstable systems

Error model (state-space):

$$\dot{e} = Ae + b\tilde{\psi}^T \phi, \quad e(0), \quad (57)$$

$$\varepsilon = c^T e, \quad (58)$$

where A is not Hurwitz, A, b , and c are known.

Filters:

$$\dot{\Omega} = A_e \Omega + b \phi^T, \quad (59)$$

$$\dot{\omega} = A_e \omega - \Omega \dot{\psi} - l_e c^T e \quad (60)$$

where the vector of feedback gains l_e is selected so that $A_e = A - l_e c^T$ is Hurwitz.

3.1. Swapping for unstable systems

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + c^T \omega \quad (61)$$

Result of swapping:

$$\dot{e} = Ae + b\tilde{\psi}^T \phi, \quad \longrightarrow \quad \tilde{e} = \Omega \tilde{\psi} + \sigma_e, \quad (62)$$

$$\varepsilon = c^T e \quad \tilde{\varepsilon} = c^T \Omega \tilde{\psi} + c^T \sigma_e \quad (63)$$

3.2. Swapping for systems with input delay

Error model (state-space):

$$\dot{e} = Ae + b\tilde{\psi}^T(t - \tau)\phi, \quad (64)$$

$$\varepsilon = c^T e, \quad (65)$$

where A is Hurwitz, A, b , and c are known, τ is delay.

Filters:

$$\dot{\Omega} = A\Omega + b\phi^T, \quad (66)$$

$$\dot{\omega} = A\omega - \Omega\dot{\hat{\psi}} - b\phi^T(\hat{\psi}(t - \tau) - \hat{\psi}(t)) \quad (67)$$

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + c^T \omega \quad (68)$$

3.2. Swapping for systems with input delay

Result of swapping:

$$\dot{e} = Ae + b\tilde{\psi}^T(t - \tau)\phi, \quad \xrightarrow{\hspace{1cm}} \quad \tilde{e} = \Omega\tilde{\psi}(t) + \sigma_e, \quad (69)$$

$$\varepsilon = c^T e \quad \xrightarrow{\hspace{1cm}} \quad \tilde{\varepsilon} = c^T \Omega \tilde{\psi}(t) + c^T \sigma_\tau \quad (70)$$

3.2. Swapping for systems with input delay (example)

Example 2. Disturbance compensation in delayed systems

Plant:

$$\dot{x} = Ax + b(u(t - \tau) + \delta), \quad (71)$$

$$y = c^T x, \quad (72)$$

where A, b , and c are known, τ is delay. State x is measurable,

$$\delta(t) = a_0 + \sum_{i=1}^p a_i \sin(\omega_i t + \varphi_i)$$

is the inaccessible for measurement disturbance with unknown a_0, a_i, ω_i , and φ_i but known maximum number of harmonics p .

Objective:

$$\lim_{t \rightarrow \infty} y(t) = 0. \quad (73)$$

3.2. Swapping for systems with input delay (example)

Internal model of the disturbance:

$$\dot{z} = \Gamma z, \quad z(0), \quad (74)$$

$$\delta = h^T z, \quad (75)$$

where $\Gamma \in \mathbb{R}^{m \times m}$ is an unknown matrix with simple eigenvalues with zero real parts, h is an unknown m -dimensional vector, z is the inaccessible for measurement state.

3.2. Swapping for systems with input delay (example)

Internal model of the disturbance:

$$\dot{z} = \Gamma z, \quad z(0), \quad (74)$$

$$\delta = h^T z, \quad (75)$$

where $\Gamma \in \mathbb{R}^{m \times m}$ is an unknown matrix with simple eigenvalues with zero real parts, h is an unknown m -dimensional vector, z is the inaccessible for measurement state.

Assumptions:

- the plant is controllable;
- the pair (Γ, h) is observable;
- the dimension m (corresponding to the maximum number of harmonics) is known.

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \quad (76)$$

$$\delta = \theta^T \xi, \quad (77)$$

where $G \in \mathbb{R}^{m \times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

3.2. Swapping for systems with input delay (example)

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Autonomous model:

$$\dot{\xi} = (G + l\theta^T) \xi \quad (78)$$

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Autonomous model:

$$\dot{\xi} = (G + l\theta^T) \xi \quad (78)$$

$$\xi(t) = e^{(G+l\theta^T)t} \xi(t_0) \Rightarrow \xi(t+\tau) = e^{(G+l\theta^T)\tau} \xi(t)$$

3.2. Swapping for systems with input delay (example)

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$$\xi(t) = e^{(G+l\theta^T)t} \xi(t_0) \Rightarrow \xi(t) = e^{(G+l\theta^T)\tau} \xi(t - \tau)$$

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \quad (76)$$

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Autonomous model:

$$\dot{\xi} = (G + l\theta^T) \xi \quad (78)$$

New parameterization:

$$\delta(t) = \theta^T e^{(G+l\theta^T)\tau} \xi(t-\tau) \quad (79)$$

3.2. Swapping for systems with input delay (example)

Canonical form of disturbance model:

$$\dot{\xi} = G\xi + l\delta, \quad (76)$$

$$\delta = \theta^T \xi, \quad (77)$$

where $G \in \mathbb{R}^{m \times m}$ is a Hurwitz matrix, l is a vector selected so that the pair (G, l) is controllable, θ is a vector of unknown parameters.

Autonomous model:

$$\dot{\xi} = (G + l\theta^T) \xi \quad (78)$$

New parameterization:

$$\delta(t) = \psi^T \xi(t - \tau), \quad (79)$$

where $\psi = e^{(G^T + \theta l^T)\tau} \theta$ is the new vector of unknown parameters.

3.2. Swapping for systems with input delay (example)

Plant with parameterized disturbance:

$$\dot{x} = Ax + b(u(t - \tau) + \psi^T \xi(t - \tau)) \quad (80)$$

Exponentially stable disturbance observer:

$$\hat{\xi} = \eta + Nx, \quad (81)$$

$$\dot{\eta} = G\eta + (GN - NA)x - bu(t - \tau), \quad (82)$$

where $\hat{\xi}$ is the estimate of ξ , $N \in \mathbb{R}^{m \times n}$ is the matrix selected so that
 $Nb = l$. (83)

3.2. Swapping for systems with input delay (example)

Plant with parameterized disturbance:

$$\dot{x} = Ax + b(u(t - \tau) + \psi^T \xi(t - \tau)) \quad (80)$$

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$$\dot{\eta} = G\eta + (GN - NA)x - bu(t - \tau), \quad (82)$$

where $\hat{\xi}$ is the estimate of ξ , $N \in \mathbb{R}^{m \times n}$ is the matrix selected so that
 $Nb = l$. (83)

Plant with parameterized disturbance:

$$\dot{x} = Ax + b(u(t - \tau) + \psi^T \hat{\xi}(t - \tau)) \quad (84)$$

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$u = -\hat{\psi}^T \hat{\xi},$$

$$\dot{x} = Ax + b(u(t-\tau) + \psi^T \hat{\xi}(t-\tau)) \quad (85)$$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$\dot{x} = Ax + b(u(t-\tau) + \psi^T \hat{\xi}(t-\tau)) \quad (85)$$

$u = -\hat{\psi}^T \hat{\xi},$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

Error model:

$$\dot{x} = Ax + b(\tilde{\psi}^T(t-\tau) \hat{\xi}(t-\tau)), \quad (86)$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

3.2. Swapping for systems with input delay (example)

Certainty equivalent control:

$$u = -\hat{\psi}^T \hat{\xi}, \quad (85)$$

where $\hat{\psi} \in R^m$ is the vector of adjustable parameter.

Error model:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right), \quad (86)$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

$$\dot{\tilde{\psi}}(t-\tau) = \gamma \hat{\xi}(t-\tau) b^T P x$$

3.2. Swapping for systems with input delay (example)

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t - \tau), \quad (87)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi} - b\hat{\xi}^T(t - \tau)(\hat{\psi}(t - \tau) - \psi) \quad (88)$$

Augmented error:

$$\tilde{x} = x + \omega \quad (89)$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t - \tau)\hat{\xi}(t - \tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \quad (90)$$

3.2. Swapping for systems with input delay (example)

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$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \quad (87)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi} - b\hat{\xi}^T(t-\tau)(\hat{\psi}(t-\tau) - \psi) \quad (88)$$

Augmented error:

$$\tilde{x} = x + \omega \quad (89)$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \quad (90)$$

Algorithm of adaptation (gradient):

$$\dot{\psi} = \gamma\Omega^T\tilde{x}, \quad \gamma > 0 \quad (91)$$

3.2. Swapping for systems with input delay (example)

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \quad \text{---} \quad \dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \quad (87)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi} - b\hat{\xi}^T(t-\tau)(\hat{\psi}(t-\tau) - \hat{\psi}) \quad (88)$$

Augmented error:

$$\tilde{x} = x + \omega \quad (89)$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \quad (90)$$

Algorithm of adaptation (improved with DRE):

$$\dot{\hat{\psi}} = \gamma \Phi_H E_H, \quad (92)$$

where $\Phi_H^T = H(s)[\Omega]$, $E_H = H(s)[\tilde{x} + \Omega\hat{\psi}] - \Phi_H^T \hat{\psi}$.

3.2. Swapping for systems with input delay (example)

Filters:

$$\dot{\Omega} = A\Omega + b\hat{\xi}^T(t-\tau), \quad (87)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi} - b\hat{\xi}^T(t-\tau)(\hat{\psi}(t-\tau) - \hat{\psi}) \quad (88)$$

Augmented error:

$$\tilde{x} = x + \omega \quad (89)$$

Result of swapping:

$$\dot{x} = Ax + b\left(\tilde{\psi}^T(t-\tau)\hat{\xi}(t-\tau)\right) \longrightarrow \tilde{x} = \Omega\tilde{\psi} + \sigma_e, \quad (90)$$

Algorithm of adaptation (improved with MRE):

$$\dot{\hat{\psi}} = \gamma E_L, \quad (93)$$

where $\Phi_L = L(s)[\Omega^T \Omega]$, $E_L = L(s)[\Omega^T \tilde{x} + \Omega^T \Omega \hat{\psi}] - \Phi_L \hat{\psi}$.

3.2. Swapping for systems with input delay (example)

Simulation results:

$$\begin{aligned}\dot{x} &= Ax + b(u(t-\tau) + \delta), \\ y &= c^T x,\end{aligned} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix},$$

where $\delta(t) = a \sin(\omega t + \varphi)$ with unknown $a = 2$, $\omega = 1$, $\varphi = 1$; $\tau = 1\text{sec}$ is known. For the observer design, the matrices are selected as follows:

$$G = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}, \quad l = \begin{bmatrix} 0 \\ 20 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 20 \end{bmatrix}.$$

3.2. Swapping for systems with input delay (example)

Simulation results:

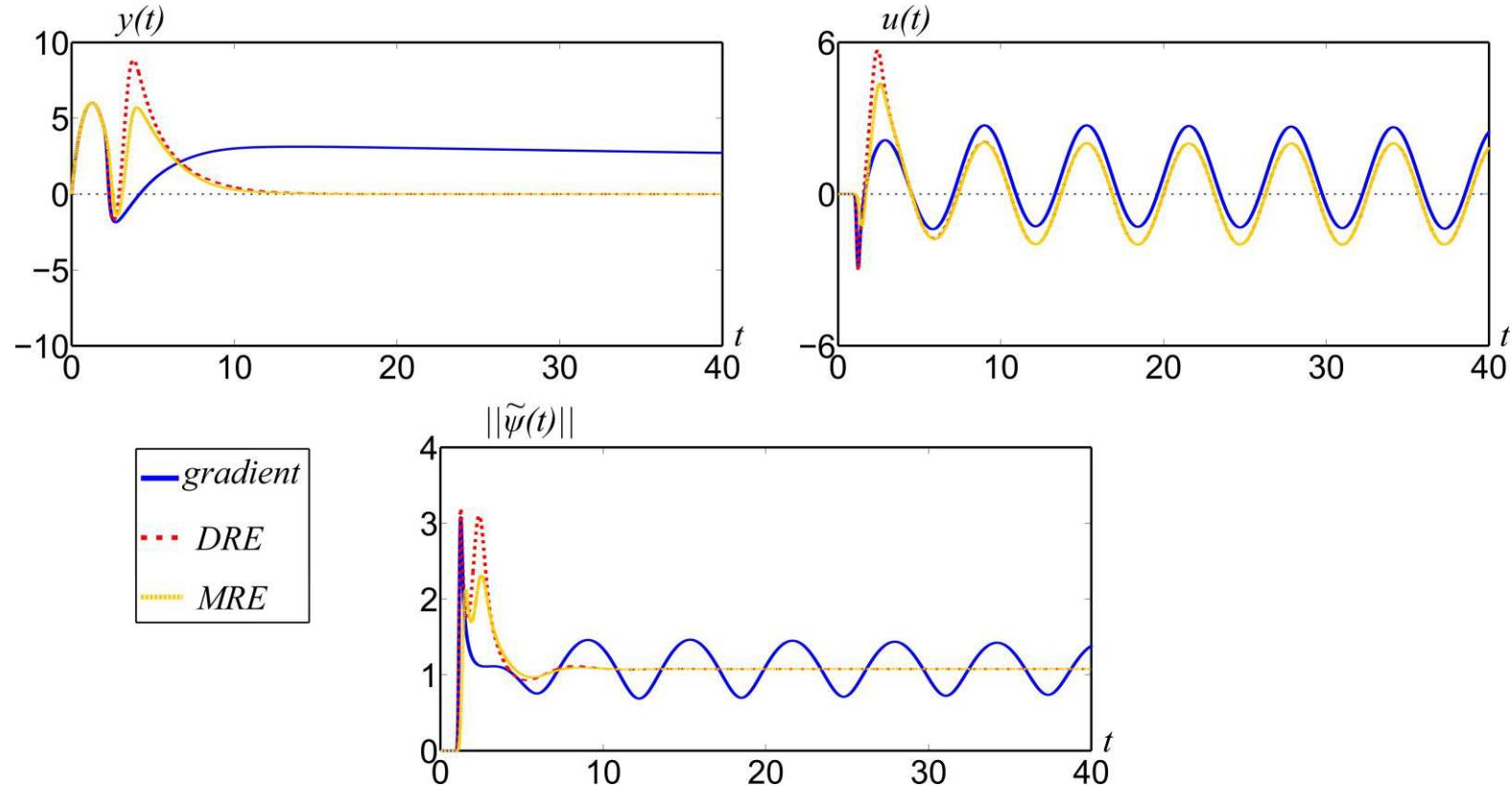


Fig. 3. Adaptive compensation of sinusoidal disturbance in delayed system with $\gamma = 5$.

3.3. Swapping for systems with state and input delays

Error model (state-space):

$$\dot{e} = A_0 e + \sum_{i=1}^r A_i e(t - \tau_{xi}) + \sum_{j=1}^p b_j \tilde{\psi}^T(t - \tau_{uj}) \phi_j, \quad (94)$$

$$\varepsilon = c^T e, \quad (95)$$

where A_0 , A_i , b_j , and c are known, τ_{xi} , τ_{uj} are known delays. The plant is asymptotically stable.

Filters:

$$\dot{\Omega} = A_0 \Omega + \sum_{i=1}^r A_i \Omega(t - \tau_{xi}) + \sum_{j=1}^p b_j \phi_j^T, \quad (96)$$

$$\dot{\omega} = A_0 \omega - \Omega \dot{\hat{\psi}} - \sum_{j=1}^p b_j \phi_j^T (\hat{\psi}(t - \tau_{uj}) - \hat{\psi}) \quad (97)$$

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + c^T \omega \quad (98)$$

3.3. Swapping for systems with state and input delays

Result of swapping:

$$\dot{e} = A_0 e + \sum_{i=1}^r A_i e(t - \tau_{xi}) + \sum_{j=1}^p b_j \tilde{\psi}^T(t - \tau_{uj}) \phi_j, \quad \longrightarrow \quad \tilde{e} = \Omega \tilde{\psi}(t) + \sigma_e, \quad (99)$$

$$\varepsilon = c^T e \quad \longrightarrow \quad \tilde{\varepsilon} = c^T \Omega \tilde{\psi}(t) + c^T \sigma_\tau \quad (100)$$

3.4. Nonlinear swapping

Error model:

$$\dot{e} = A(e, t)e + b(e, t)\tilde{\psi}^T \phi, \quad e(0), \quad (101)$$

$$\varepsilon = c^T(e, t)e, \quad (102)$$

where $A(e, t) \in \mathbb{R}^{n \times n}$, $b(e, t) \in \mathbb{R}^n$, and $c(e, t) \in \mathbb{R}^n$ are the nonlinear mappings, locally Lipschitz in e and bounded in t . The matrix $A(e, t)$ is such that the system

$$\dot{x} = A(e, t)x \quad (103)$$

is exponentially stable.

3.4. Nonlinear swapping

Error model:

$$\dot{e} = A(e, t)e + b(e, t)\tilde{\psi}^T \phi, \quad e(0), \quad (101)$$

$$\varepsilon = c^T(e, t)e, \quad (102)$$

where $A(e, t) \in \mathbb{R}^{n \times n}$, $b(e, t) \in \mathbb{R}^n$, and $c(e, t) \in \mathbb{R}^n$ are the nonlinear mappings, locally Lipschitz in e and bounded in t . The matrix $A(e, t)$ is such that the system

$$\dot{x} = A(e, t)x \quad (103)$$

is exponentially stable.

Filters:

$$\dot{\Omega} = A(e, t)\Omega + b(e, t)\phi^T, \quad (104)$$

$$\dot{\omega} = A(e, t)\omega - \Omega\dot{\hat{\psi}} - b(e, t)\phi^T\hat{\psi} \quad (105)$$

3.4. Nonlinear swapping

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + c^T(e, t)\omega \quad (106)$$

Result of swapping:

$$\dot{e} = A(e, t)e + b(e, t)\tilde{\psi}^T\phi, \quad \xrightarrow{\hspace{1cm}} \quad \tilde{e} = \Omega\tilde{\psi} + \sigma_n, \quad (107)$$

$$\varepsilon = c^T(e, t)e, \quad \tilde{\varepsilon} = c^T(e, t)\Omega\tilde{\psi} + c^T(e, t)\sigma_n \quad (108)$$

where $\sigma_n(t)$ exponentially decays according to $\dot{\sigma}_n = A(e, t)\sigma_n$ with IC $\sigma_n(0) = e(0) + \omega(0) - \Omega(0)\tilde{\psi}(0)$.

3.4. Nonlinear swapping (example)

Example 3. Adaptive backstepping control with improved parameters tuning

Plant:

$$\dot{x}_i = x_{i+1} + \phi_i^T(x_1, \dots, x_i)\psi, \quad i = 1, 2, \dots, n-1, \quad (109)$$

$$\dot{x}_n = u + \phi_n^T(x)\theta, \quad (110)$$

$$y = x_1, \quad (111)$$

where $x \in \mathbb{R}^n$ is the measurable state vector with the elements x_1, x_2, \dots, x_n , ϕ_i, ϕ_n are the sufficiently smooth vector-functions, $\psi \in \mathbb{R}^q$ is the vector of unknown parameters.

Objective:

$$\lim_{t \rightarrow \infty} \varepsilon(t) = \lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0. \quad (112)$$

3.4. Nonlinear swapping (example)

Backstepping procedure (based on modular identifiers):

$$\alpha_1(x_1, \hat{\psi}, y_m) = -(c_1 + s_1)z_1 - \phi_1^T \hat{\psi}, \quad (113)$$

$$\alpha_i(x_1, \dots, x_i, \hat{\psi}, \dots, \hat{\psi}^{(i-1)}, y_m, \dots, y_m^{(i)}) = -z_{i-1} - (c_i + s_i)z_i \quad (114)$$

$$-\left(\phi_i^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j^T \right) \hat{\psi} + \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}^{(j-1)}} \hat{\psi}^{(j)} + \frac{\partial \alpha_{i-1}}{\partial y_m^{(j-1)}} y_m^{(j)} \right),$$

where $i = 2, 3, \dots, n$, $z_1 = x_1 - y_m$, $z_i = x_i - \alpha_{i-1} - y_m^{(i-1)}$ are the new state errors,

$$s_1 = \mu_1 \|\phi_1\|^2, \quad s_i = \mu_i \left\| \phi_i^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j^T \right\|^2$$

are the damping terms, $\hat{\psi}$ is the vector of adjustable parameters, $c_i, \mu_i > 0$ are constant design parameters.

3.4. Nonlinear swapping (example)

Actual control:

$$u = \alpha_n + y_m^{(n)} \quad (115)$$

3.4. Nonlinear swapping (example)

Actual control:

$$u = \alpha_n + y_m^{(n)} \quad (115)$$

Closed-loop error model:

$$\dot{z} = A_z(z, \phi_i, \hat{\psi}, \hat{\psi}^{(j)})z + B_z(z, \phi_i, \hat{\psi}, \hat{\psi}^{(j)})\tilde{\psi}, \quad \varepsilon = c_z z, \quad (116)$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors,

$$A_z = \begin{bmatrix} -c_1 - s_1 & 1 & \cdots & 0 \\ -1 & -c_2 - s_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots 0 & -1 & -c_n - s_n \end{bmatrix}, \quad B_z = \begin{bmatrix} \phi_1^T \\ \phi_2^T - \frac{\partial \alpha_1}{\partial x_1} \phi_1^T \\ \vdots \\ \phi_n^T - \sum_{j=1}^{n-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j^T \end{bmatrix}, \quad c_z = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

3.4. Nonlinear swapping (example)

Swapping filters:

$$\dot{\Omega} = A_z \Omega + B_z, \quad \text{--- } \circ \quad \circ \quad \quad \dot{z} = A_z z + B_z \tilde{\psi} \quad (117)$$

$$\dot{\omega} = A_z \omega - \Omega \dot{\hat{\psi}} - B_z \hat{\psi} \quad (118)$$

Augmented error:

$$\tilde{z} = z + \omega \quad (119)$$

Result of swapping:

$$\dot{z} = A_z z + B_z \tilde{\psi} \quad \longrightarrow \quad \tilde{z} = \Omega \tilde{\psi} + \sigma_n$$

3.4. Nonlinear swapping (example)

Swapping filters:

$$\dot{\Omega} = A_z \Omega + B_z, \quad \text{--- } \circ \quad \circ \quad \quad \dot{z} = A_z z + B_z \tilde{\psi} \quad (117)$$

$$\dot{\omega} = A_z \omega - \Omega \dot{\hat{\psi}} - B_z \hat{\psi} \quad (118)$$

Augmented errors:

$$\tilde{z} = z + \omega \quad (119)$$

Result of swapping:

$$\dot{z} = A_z z + B_z \tilde{\psi} \quad \xrightarrow{\hspace{1cm}} \quad \tilde{z} = \Omega \tilde{\psi} + \sigma_n$$

Algorithm of adaptation (improved with MRE):

$$\dot{\hat{\psi}} = \gamma E_L, \quad (121)$$

where $\Phi_L = L(s) [\Omega^T \Omega]$, $E_L = L(s) [\Omega^T \tilde{z} + \Omega^T \Omega \hat{\psi}] - \Phi_L \hat{\psi}$.

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$

$$\dot{x}_2 = x_3,$$

$$\dot{x}_3 = u,$$

$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

For controller design we use $y_m(t) = \sin(t) + 1$ and derive the control law

$$\alpha_1 = -(c_1 + s_1)z_1 - \phi_1^T \hat{\psi},$$

$$\alpha_2 = -z_1 - (c_2 + s_2)z_2 + \frac{\partial \alpha_1}{\partial x_1} \phi_1^T \hat{\psi} + \frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}} \dot{\hat{\psi}} + \frac{\partial \alpha_1}{\partial y_m} \dot{y}_m,$$

$$u = -z_2 - (c_3 + s_3)z_3 + \frac{\partial \alpha_2}{\partial x_1} \phi_1^T \hat{\psi} + \sum_{j=1}^2 \left(\frac{\partial \alpha_2}{\partial x_j} x_{j+1} + \frac{\partial \alpha_2}{\partial \hat{\theta}^{(j-1)}} \hat{\psi}^{(j)} + \frac{\partial \alpha_2}{\partial y_m^{(j-1)}} y_m^{(j)} \right) + \ddot{y}_m.$$

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$

$$\dot{x}_2 = x_3,$$

$$\dot{x}_3 = u,$$

$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

with $z_1 = x_1 - y_m, z_2 = x_2 - \alpha_1 - \dot{y}_m, z_3 = x_3 - \alpha_2 - \ddot{y}_m$ the damping terms

$$s_1 = 10^{-3} \|\phi_1\|^2, \quad s_2 = 10^{-3} \left\| \frac{\partial \alpha_1}{\partial x_1} \phi_1 \right\|^2, \quad s_3 = 10^{-3} \left\| \frac{\partial \alpha_2}{\partial x_1} \phi_1 \right\|^2$$

the swapping filters

3.4. Nonlinear swapping (example)

Simulation results:

$$\dot{x}_1 = x_2 + \phi_1^T(x_1)\psi,$$

$$\dot{x}_2 = x_3,$$

$$\dot{x}_3 = u,$$

$$y = x_1,$$

$$\phi_1(x_1) = \begin{bmatrix} x_1^2 \\ \cos(x_1) \end{bmatrix}, \quad \psi = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

and the adaptation algorithm

$$\dot{\hat{\psi}} = \gamma E_L,$$

where $\Phi_L = L(s)[\Omega^T \Omega]$, $E_L = L(s)[\Omega^T \tilde{z} + \Omega^T \Omega \hat{\psi}] - \Phi_L \hat{\psi}$, $L(s) = \frac{1}{s+1}$

or

$$\ddot{\hat{\psi}} + (I_2 + \gamma \Phi_L) \dot{\hat{\psi}} = \gamma \Omega^T \tilde{z}. \quad (122)$$

3.4. Nonlinear swapping (example)

Simulation results:

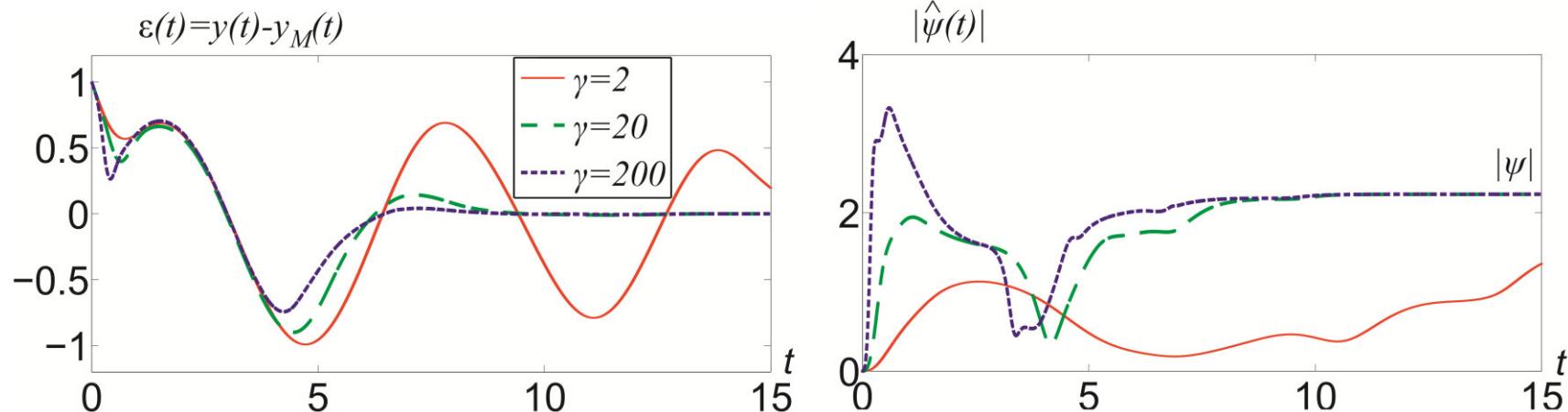


Fig. 4. Transients in the adaptive system closed by backstepping controller with adaptation algorithm with MRE for different γ .

3.5. Swapping for systems with unknown control gain

Error model:

$$\dot{e} = A(e, t)e + \textcolor{blue}{k}b(e, t)\tilde{\psi}^T\phi, \quad e(0), \quad (123)$$

$$\varepsilon = c^T(e, t)e, \quad (124)$$

where $A(e, t) \in \mathbb{R}^{n \times n}$, $b(e, t) \in \mathbb{R}^n$, and $c(e, t) \in \mathbb{R}^n$ are the nonlinear mappings, locally Lipschitz in e and bounded in t . The matrix $A(e, t)$ is such that the system

$$\dot{x} = A(e, t)x$$

is exponentially stable.

Filters:

$$\dot{\Omega} = A(e, t)\Omega + b(e, t)\phi^T\hat{\psi} \quad \dot{\bar{\Omega}} = A(e, t)\bar{\Omega} + b(e, t)\phi^T, \quad (125)$$

$$\dot{w} = A(e, t)w - \hat{k}\bar{\Omega}\dot{\hat{\psi}} + \dot{\hat{k}}\bar{w}, \quad \dot{\bar{w}} = A(e, t)\bar{w} - \bar{\Omega}\dot{\hat{\psi}} \quad (126)$$

3.5. Swapping for systems with unknown control gain

Augmented error:

$$\tilde{e} = e + w \quad (127)$$

Result of swapping:

$$\dot{e} = A(e, t)e + \textcolor{blue}{k} b(e, t) \tilde{\psi}^T \phi \longrightarrow \tilde{e} = k \bar{\Omega} \tilde{\psi} - \tilde{k} \bar{w} + \bar{\sigma}, \quad (128)$$

where $\tilde{k} = k - \hat{k}$ is the parametric error.

3.6. Swapping for MIMO linear systems

Error model:

$$\dot{e} = Ae + B\tilde{\Psi}\phi, \quad e(0), \quad (129)$$

$$\varepsilon = Ce, \quad (130)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $C \in \mathbb{R}^{m \times n}$ are the constant matrices, A is Hurwitz, $\tilde{\Psi} = \Psi - \hat{\Psi} \in \mathbb{R}^{m \times q}$ is the matrix of parametric errors, Ψ is a matrix of unknown parameters, $\hat{\Psi}$ is the matrix of adjustable parameters, $\phi \in \mathbb{R}^q$ is the regressor.

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$$\dot{e} = Ae + B\Phi\tilde{\Psi},$$

$$\Phi = \begin{bmatrix} \phi^T & O_{1 \times q} & \cdots & O_{1 \times q} \\ O_{1 \times q} & \phi^T & \cdots & O_{1 \times q} \\ \vdots & \vdots & \ddots & \vdots \\ O_{1 \times q} & O_{1 \times q} & \cdots & \phi^T \end{bmatrix} \in \mathbb{R}^{m \times q \cdot m}, \quad \tilde{\Psi} = \text{vec}\{\tilde{\Psi}\} = \text{vec}\left\{\begin{bmatrix} \tilde{\Psi}_1^T \\ \tilde{\Psi}_2^T \\ \vdots \\ \tilde{\Psi}_m^T \end{bmatrix}\right\} = \begin{bmatrix} \tilde{\Psi}_1 \\ \tilde{\Psi}_2 \\ \vdots \\ \tilde{\Psi}_m \end{bmatrix} \in \mathbb{R}^{q \cdot m}.$$

3.6. Swapping for MIMO linear systems

Filters:

$$\dot{\Omega} = A\Omega + B\Phi, \quad \Xi = C\Omega, \quad (131)$$

$$\dot{\omega} = A\omega - \Omega\dot{\psi}, \quad \xi = C\omega \quad (132)$$

Augmented errors:

$$\tilde{e} = e + \omega, \quad \tilde{\varepsilon} = \varepsilon + \xi \quad (133)$$

Result of swapping:

$$\dot{e} = Ae + B\tilde{\Psi}\phi, \quad \xrightarrow{\hspace{1cm}} \quad \tilde{e} = \Omega\tilde{\psi} + \sigma, \quad (134)$$

$$\varepsilon = Ce \quad \xrightarrow{\hspace{1cm}} \quad \tilde{\varepsilon} = \Xi\tilde{\psi} + C\sigma \quad (135)$$

Conclusion

1. It is shown in a systematic way that the concept of augmented error can be combined with adaptation algorithms with DRE and MRE used for improvement of transient performance of adaptive systems;
2. Special *ad hoc* extensions of the swapping schemes used in augmented errors are illustrated.

The extensions can be mixed depending on the control problem conditions.

Conclusion

1. It is shown in a systematic way that the concept of augmented error can be combined with adaptation algorithms with DRE and MRE used for improvement of transient performance of adaptive systems;
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The extensions can be mixed depending on the control problem conditions.

Next steps:

1. Application of DREM₁ / MREM₂ for improvement of transient performance;
2. Involving of exponentially decaying terms into regressors;
3. Schemes with finite time convergence of parametric errors.

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