① $F(x) = \int_{-\infty}^{x} f(t)\,dt$

If $x < -1 \Rightarrow F(x) = \int_{-\infty}^{x} 0\,dt = 0$

If $-1 \le x < 2$.

$\Rightarrow F(x) = \int_{-1}^{x} \frac{t^2}{3}\,dt = \frac{t^3}{9}\Big|_{-1}^{x} = \frac{x^3+1}{9}$

If $x \ge 2 \Rightarrow F(x) = \int_{2}^{x} f(t)\,dt + F(2)$

$= 0 + 1 = 1$

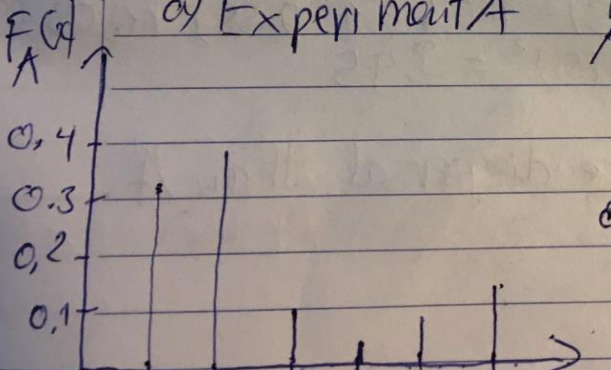
$\Rightarrow F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^3+1}{9} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$.

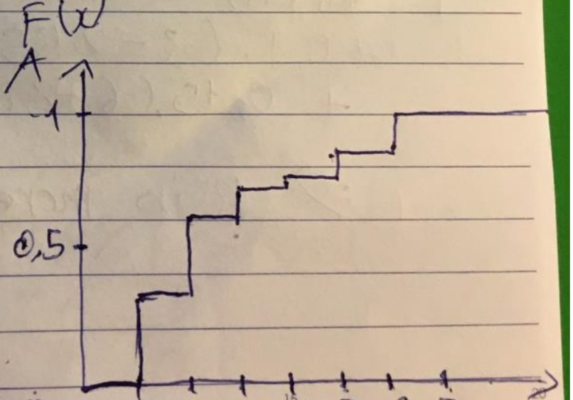
b) $P(0 < x \le 1) = \int_{0}^{1} f(x)\,dx$.

$= \int_{0}^{1} \frac{x^2}{3}\,dx = \frac{1}{9}$

$P(0 \le x \le 1) = P(1) - P(0) = \frac{1^2+1}{9} - \frac{0^2+1}{9} = \frac{1}{9}$

② a) Experiment A





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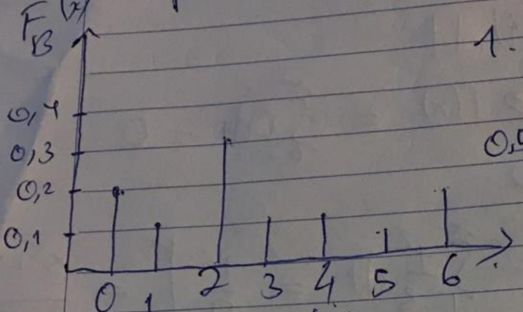
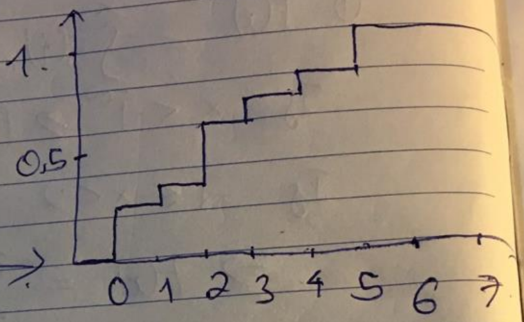
Experiment B.





b/ Experiment A

$E[A] = \text{mean} = 0.31 + 2.0{,}38 + 3.0{,}1 + 4.0{,}01 + 5.0{,}09$
$+ 6.0{,}11 = 2{,}51$

$Var(A) = 0.31(1-2{,}51)^2 + 0{,}38(2-2{,}51)^2 + 0{,}1(3-2{,}51)^2$
$+ 0{,}01(4-2{,}51)^2 + 0{,}09(5-2{,}51)^2 + 0{,}11(6-2{,}51)^2$
$= 2{,}72$

Experiment B.

$E[B] = 0{,}02 + 1.0{,}1 + 2.0{,}3 + 3.0{,}1 + 4.0{,}1$
$+ 5.0{,}05 + 6.0{,}15 = 2.55$

$Var(B) = 0.2(0-2{,}55)^2 + 0{,}1(1-2{,}55)^2 + 0{,}3(2-2{,}55)^2$
$+ 0{,}1.(3-2{,}55)^2 + 0{,}1.(4-2{,}55)^2 + 0{,}05(5-2{,}55)^2$
$+ 0{,}15.(6-2{,}25)^2 = 3{,}95$

$\Rightarrow$ B is more dispersal than A.

Thứ ngày

③ Denote:

A: event select a product of line 1

B: event select a product of line 2

C: - - - - - - - - 3

X: - - - - - defective product

a) $P(X) = P(X \cap A) + P(X \cap B) + P(X \cap C)$
$= P(A)\,P(X|A) + P(B)\,P(X|B)$
$+ P(C)\,P(X|C) = 0{,}0245$

b) $P(A|X) = \frac{P(A)\,P(X|A)}{P(X)} = 0{,}245$

similarly: $P(B|X) = 0.551$. $P(C|X) = 0{,}204$

$\Rightarrow$ come from line B

④ $X = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \\ 4 \\ 3 \end{bmatrix}$; $Y = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 3 \\ 3 \end{bmatrix}$; $Z = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 5 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow E(X) = \frac{7}{3}$; $Var(X) = \frac{11}{9}$

$E(Y) = \frac{11}{6}$; $Var(Y) = \frac{29}{36}$

$E(Z) = \frac{7}{2}$; $Var(Z) = \frac{11}{12}$

$Cov(X, Y) = E[XY] - E[X][Y]$
$= \frac{29}{6} - \frac{77}{18} = \frac{5}{9}$

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$$\operatorname{cov}(Y;Z) = \frac{35}{6} - \frac{77}{12} = \frac{-7}{12}$$

$$\operatorname{cov}(Z,X) = \frac{47}{6} - \frac{49}{6} = \frac{-1}{3}$$

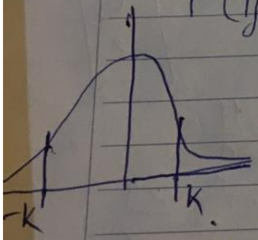
$\Rightarrow$ covariance matrix:
$$\begin{bmatrix} 11/9 & \frac{5}{9} & \frac{-1}{3} \\ 5/9 & \frac{29}{36} & \frac{-7}{12} \\ \frac{-1}{3} & \frac{-7}{12} & \frac{11}{12} \end{bmatrix}$$

5) $U(x-c) = \begin{cases} 1 & \text{if } x \geq c \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow F(x) = \begin{cases} (1-e^{-\alpha x}) & \text{if } x \geq c \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow f(x) = \begin{cases} \alpha e^{\alpha x} & \text{if } x \geq c \\ 0 & \text{otherwise} \end{cases}$

6) $P(\eta - k\sigma < x < \eta + k\sigma) = P\left(\left|\frac{x-\eta}{\sigma}\right| < k\right)$



$= P(|Z| < k)$ where $Z \sim N(0,1)$

$= 2P(Z<k) - 1$

$\Rightarrow P_1 = 2 \cdot 0{,}84134 - 1 = 0{,}68268$

$P_2 = 0{,}9544$

$P_3 = 0{,}9974$

7) $f(x \mid (x-10)^2 < 4) = \begin{cases} \dfrac{f_X(x)}{P((X-10)^2<4)} & \text{if } (x-10)^2 < 4 \\ 0 & \text{otherwise} \end{cases}$

$P((x-10)^2<4)$
whe
$= 0{,}954$
$\Rightarrow f(x \mid ($
6) b)
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Thứ ngày

$P((x-10)^2 < 4) = P(8 < X < 12) = P(-2 < Z < 2)$ where $Z \sim N(0,1)$

$= 0,9544$.

$\Rightarrow f(x \mid (x-10)^2 < 4) = \begin{cases} \dfrac{\varphi(x)}{0,9544} & \text{if } 8 < x < 12 \\ 0 & \text{otherwise} \end{cases}$

⑥ b) $P_k = 2P(Z < k) - 1 = 0,99$

$\Leftrightarrow P(Z < k) = 0,995$

$\Leftrightarrow k = 2,57$.

Similarly $\Rightarrow$ for case 0.99: $k = 3.29$

for case $0,999$, $k = 3,86$

for case 0.9 : $k = 1,65$

c. let z_γ be the value of Z which determines the right area $= \gamma$

$(Z \sim N(0,1))$

$\Rightarrow P(\eta - z_v \sigma < x < \eta + z_v \sigma)$

$= P\left(\left|\dfrac{x-\eta}{\sigma}\right| < z_v\right)$.

$\Rightarrow P(|Z| < z_v) = \gamma \Leftrightarrow 2P(Z < z_v) - 1 = \gamma$

$\Leftrightarrow \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{z_0} e^{\frac{-x^2}{2}} dx = \dfrac{1+\gamma}{2}$.

⑧ X : random variable measuring resistance

$P(X > 43) = P(Z > 1,5) = 1 - P(Z < 1,5)$

$= 0,06681$

$)^2 < 4$.

Thứ ngày

⑨ $P(1 \le X \le 2) = P(\frac{1}{2} < Z < 1)$

$= \Phi(1) - \Phi(\frac{1}{2}) = 0{,}1992$

$P(1 \le X \le 2 | X \ge 1) = \dfrac{P(1 \le X \le 2)}{P(X \ge 1)}$

$= \dfrac{P(1 \le X \le 2)}{1 - P(X \le 1)} = 0{,}944$

⑩ $P(X < 1024) = P(Z < 1.2) = 0{,}8849$

$P(X < 1024 | X > 961) = P(Z < 1.2 | Z > -1.95)$

$= \dfrac{P(-1{,}95 < Z < 1{,}2)}{P(Z > 1{,}95)} = 0{,}8819$

$P(31 < \bar{X} < 32) = \dfrac{P(-1{,}95 < Z < 1{,}2)}{P(Z \ge -50} = 0{,}8593$

⑪ $P(\frac{X-79}{7} > a) = 0{,}12 \Leftrightarrow P(Z > a) = 0{,}19$

$\Leftrightarrow a = 1.17 \Leftrightarrow X > 82{,}19$

$\Rightarrow$ Smallest score forget A is 83

⑫ Exact value : $P(\mu - 3\sigma < X < \mu + 3\sigma)$

$= P(|Z| < 3) = 2P(Z < 3) - 1$

$= 0{,}9973$

⑬

Thứ ngày

(13) Binomial distribution: $g(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$

$\Rightarrow P(25 \leq x \leq 30) = \sum_{i=25}^{30} g(i)$

$= \sum_{i=25}^{30} \binom{80}{i} \frac{1}{4^i} \cdot \left(\frac{3}{4}\right)^{80-i} = 0,119$

AS $np > 5$: $np > 5 \Rightarrow$ normal approp.

$\mu = np = 20$

$\sigma = \sqrt{npq} = \sqrt{15}$

1.95)

$\Rightarrow P(25 \leq X \leq 30) \approx P(1.39 < Z < 2.58)$

$= 0,09359$

(14) $P(-4 < X < 20) = P(-12 < X - 8 < 12)$

.8593

$= P(|X-8| < 12)$

$= P(|X-8| \leq 4.3) \geq 1 - \frac{1}{4^2} = \frac{15}{16}$

$P(|X-8| \geq 6) \leq \frac{1}{2^2} = \frac{1}{4}$

(15) $E[X] = \int_0^1 x . 6x(1-x)\,dx = \frac{1}{2}$

$Var(X) = \int_0^1 \left(x - \frac{1}{2}\right)^2 . 6x(1-x)\,dx = \frac{1}{20}$

$F(X) = \begin{cases} 0 & \text{if } x < 0 \\ 3x^2 - 2x^3 & \text{if } 0 \leq x \leq 1 \end{cases}$

$\rightarrow P(\mu - 2\sigma < X < \mu + 2\sigma) = P(0.05 < X < 0.94)$

$= 0,984$

use chebyshev: $P(\ldots) \geq 0,75$

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Thứ ngày

Statistic exercises

① a. Names of candidates
b. Out comes (Head or Tail)
c. Lasting time of pairs
d. Duration of travel recorded

② a. mean: $\frac{5+11+9+5+10+15+6+10+5+10}{10}$
$= 8,6$

b) median : 5 5 5 6 9, 10 10 10 11 15
$\frac{9+10}{2} = 9,5$

c) mode : 5 and 10

③ mean $= \frac{159,5}{60} = 2,6583$

median $= 2,7$

var $= 0,337$

std $= 0,58$

④ For a random sample of $n = 16$

$E[\bar{X}] = \mu = 800$

$var(\bar{X}) = \frac{\sigma^2}{16} = 100$

population is $\approx$ normal distribution

$\Rightarrow$ So its sample $\bar{X} \sim N(800; 100)$

$\Rightarrow P(\bar{X} < 775) = P(Z < -2,5) = 0,0621$

Thứ ngày

⑤ $var(\bar{X}) = 4 = \frac{\sigma^2}{36} \Rightarrow \sigma^2 = 144.$

$\Rightarrow$ new size $= \frac{144}{1.2^2} = 100$

⑥ a sample of size 25 : $E[\bar{X}] = 174{,}5$

$var(\bar{X}) = \frac{6{,}9^2}{25} = 1{,}9$

b) $P(172{,}5 < \bar{X} < 175{,}8) = P(-1{,}45 < Z < 0{,}94)$

$= 0{,}7529$

$\Rightarrow$ # sample mean : 150

c) $P(\bar{X} < 172) = P(Z < -1{,}81) = 0{,}0351$

$\Rightarrow$ # sample mean : 7

⑦ $\bar{x} = 2{,}6$; $\sigma = 0{,}3$

$P\left(\left|\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}\right| < z_{\alpha/2}\right) = 1 - \alpha$

$\Leftrightarrow P\left(\left|\frac{2{,}6 - \mu}{0{,}3/6}\right| < z_{\alpha/2}\right) = 1 - \alpha$

Case 95% : $z_{\alpha/2} = 1{,}96 \Rightarrow$

$\Rightarrow 2{,}505 \le \mu \le 2{,}698$

Case : 99% : $z_{\alpha/2} = 2{,}575$

$\Rightarrow 2{,}471 \le \mu \le 2{,}729$

Thứ ngày

(8) $H_0: \mu = 20000$ $H_1: \mu > 20000$

We can reject H_0 if p-value $\le 0,05$

or if $23500 >$ critical value c

Where $\frac{c-20000}{3900/\sqrt{10}} = z_{0,05}$

$\Leftrightarrow c = 20639$

$\Rightarrow$ reject H_0

(9) $E[(X_{t+k} - X_t)^2] = E[X_{t+k}^2 - 2X_{t+k} \cdot X_t + X_t^2]$

$= E[X_{t+k}^2] + E[X_t^2] - 2E(X_{t+k} \cdot X_t]$

$= var(X_{t+k}) - E^2[X_{t+k}] + var(X_t) - E^2[X_t]$

$- 2(Cov(X_t, X_{t+k}) - E(X_t] \cdot E[X_{t+k}])$

$= 2R_0 - 2R_k$

$\Rightarrow E[X^2] = 2(R_0 - R_3) = 2A(1-e^{-3\lambda})$