

$$\mathbf{x}_i = \begin{pmatrix} X_e \\ Y_e \\ \theta_e \end{pmatrix}^T \quad (1)$$

$$\ell_i = \sqrt{(X_e - X_i + a_i C - b_i S)^2 + (Y_e - Y_i + b_i C + a_i S)^2} \quad (2)$$

$$\dot{\ell}_i = \frac{\ell_i}{\mathbf{x}_e} \dot{\mathbf{x}}_e = \mathbf{J}_i \dot{\mathbf{x}}_e \quad (3)$$

$$\mathbf{J}_i = \begin{pmatrix} \frac{2 X_e - 2 X_i + 2 a_i C - 2 b_i S}{2 \sqrt{(X_e - X_i + a_i C - b_i S)^2 + (Y_e - Y_i + b_i C + a_i S)^2}} \\ \frac{2 Y_e - 2 Y_i + 2 b_i C + 2 a_i S}{2 \sqrt{(X_e - X_i + a_i C - b_i S)^2 + (Y_e - Y_i + b_i C + a_i S)^2}} \\ - \frac{2 (b_i C + a_i S) (X_e - X_i + a_i C - b_i S) - 2 (a_i C - b_i S) (Y_e - Y_i + b_i C + a_i S)}{2 \sqrt{(X_e - X_i + a_i C - b_i S)^2 + (Y_e - Y_i + b_i C + a_i S)^2}} \end{pmatrix}^T \quad (4)$$