Logic and Knowledge Representation
Exercise 1

Question 1. Formalize the following in terms of atomic propositions *r*, *b*, and *w*, first making clear how they correspond to the English text.

a. Berries are ripe along the path, but rabbits have not been seen in the area.
b. Rabbits have not been seen in the area, and walking on the path is safe, but berries are ripe along the path.
c. If berries are ripe along the path, then walking is safe if and only if rabbits have not been seen in the area.
d. It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.
e. For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
f. Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.

Question 2. Formalize these statements and determine (with truth tables or otherwise) whether they are consistent, *i.e.*, if there are some assumptions on the atomic propositions that make it true.

The system is in a multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. Either the kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

Question 3. For each of the following propositions, state whether the proposition is *valid*, *satisfiability*, *un-satisfiability*.

a. $p \rightarrow (\neg q \vee r)$
b. $\neg p \rightarrow (q \rightarrow r)$
c. $(p \rightarrow q) \vee (\neg p \rightarrow r)$
d. $(p \rightarrow q) \wedge (\neg p \rightarrow r)$
e. $(p \leftrightarrow q) \wedge (\neg q \leftrightarrow r)$

Question 4. Formalize the following arguments in logical annotation, then determine whether they are *valid*.

a. Either John isn't stupid and he is lazy, or he's stupid. John is stupid. Therefore, John isn't lazy.
b. The butler and the cook are not both innocent. Either the butler is lying or the cook is innocent. Therefore, the butler is either lying or guilty.

Question 5. Determine which of the following are equivalent to each other.

a. $(p \wedge q) \vee (\neg p \wedge \neg q)$
b. $\neg p \vee q$
c. $(p \vee \neg q) \wedge (q \vee \neg p)$
d. $\neg(p \vee q)$
e. $(q \wedge p) \vee \neg p$

Question 6. Formulate the following statements in predicate logic and/or first-order logic, making clear what your atomic predicate symbols stand for.

a. Anyone who has forgiven at least one person is a saint.
b. Nobody in the calculus class is smarter than everybody in the discrete mathematic class.
c. Anyone who has bought a Rolls Royce with cash must have a rich father.
d. If anyone in the college has the measles, then everyone who has a friend in the college must be quarantined.
e. Everyone likes Mary, except Mary herself.
f. Jane saw a bear, and Roger saw one too.
g. All purple mushrooms are poisonous.
h. No student likes every lecture.
i. There are at least two apples in a barrel.
j. There are at least two apples in every barrel.
k. Jane ate a mushroom that she had picked herself.
l. No yellow frogs are edible.
m. All students that had missed a lecture answered at least one question incorrectly.
n. Young creatures who go up in balloons are liable to giddiness.
o. Every bag contains at least one coin.
p. The father of a mother or father is a grandfather.
q. Tom's sister knows Mary's brother.
r. A careless soldier killed himself.

s. If a cake is from France then it has more sugar, if it is made with chocolate then it is made with cream, but if a cake is from Italy then it has more sugar, if it is made with cream then if it is made of chocolate.

t. Mary gave an apple to Tom.

u. One of the apples that Mary gave to Tom is Rotten.

v. The exam was hard but not very hard.

w. You only live twice.

x. By this time tomorrow I shall have finished my exams.