Logic and Knowledge Representation
Exercise 2

1. Conjunctive Normal Form (CNF)

A proposition sentence ϕ is said to be in *Conjunctive Normal Form* if ϕ is true, false, or a conjunction of α_i:

$$\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n \qquad \textbf{(1)}$$

In **(1)**, each α_i is a clause (a disjunction of β_j):

$$\alpha = (\beta_1 \vee \beta_2 \vee \dots \vee \beta_n) \qquad \textbf{(2)}$$

Each β in **(2)** is called a *literal*, which is either a variable or the negation of a variable, $\beta = p$, or $\beta = \neg p$.

2. Rule-Based Inference Systems

An *argument* in propositional logic is a sequence of propositions. All but the final proposition is called *premises*. The last statement is the *conclusion*. The argument is valid if the premises imply the conclusion.

A tautology contains two parts, the premises, p_i, and the conclusion, q, as presented in **(3)**.

$$(p_1 \wedge p_2 \wedge \dots \wedge p_n) \to q \qquad \textbf{(3)}$$

An inference system in Propositional Logic can also be specified as a set $\mathcal{R}$ of inference, or derivation rules. Each rule is a *pattern* premises/conclusion.

No.	*Rules of Inference*	*Tautology*
1	Modus Ponens	$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$ $((p \to q) \wedge p) \to q$
2	Modus Tollens	$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$ $((p \to q) \wedge \neg q) \to \neg p$

No.	*Rules of Inference*	*Tautology*
3	Hypothetical Syllogism	$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$ $\big((p \to q) \land (q \to r)\big) \to (p \to r)$
4	Disjunctive Syllogism	$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$ $\big((p \lor q) \land (\neg p)\big) \to q$
5	Addition	$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$ $p \to (p \lor q)$
6	Simplification	$\begin{array}{l} p \land q \\ \hline \therefore q \end{array}$ $(p \land q) \to p$
7	Conjunction	$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$ $\big((p) \land (q)\big) \to (p \land q)$
8	Resolution *	$\begin{array}{l} \neg p \lor r \\ p \lor q \\ \hline \therefore q \lor r \end{array}$ $\big((\neg p \lor r) \land (p \lor q)\big) \to (q \lor r)$

3. Proof by Contradiction

The following steps illustrate how-to prove a proposition Q from P.

- Translate Q and P to logic.
- Let $P' = P \cup \{\neg Q\}$
- Transform all formula in P' to implicative normal form.
- Derive contradiction.

For example, assume we have the following facts in knowledge base:

- allergies(X) → sneeze(X)
- cat(Y) ∧ allergicToCats(X) → allergies(X)
- cat(felix)
- allergicToCats(mary)

The question is can we derivative that sneeze(mary).

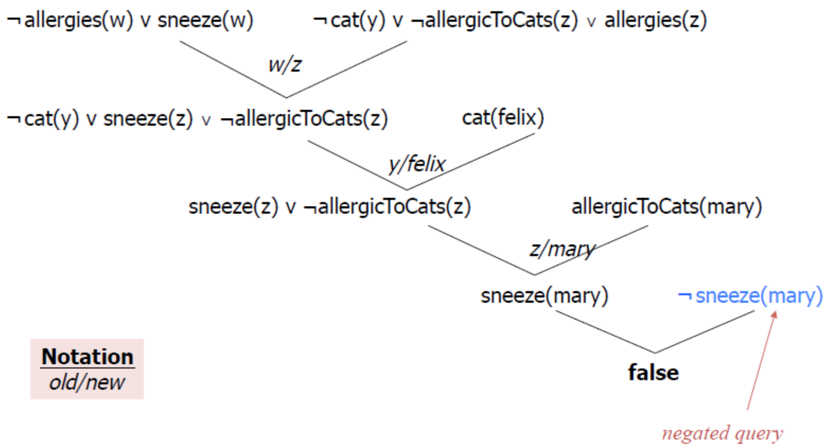

Figure 1 Proof by contradiction

4. Question

Question 1. Convert the given propositional sentences to CNF.

a. $\big((A \rightarrow B) \land \neg B\big) \rightarrow \neg A$

b. $(\neg A \rightarrow B) \rightarrow \big((B \rightarrow A) \land \neg A\big)$

c. $(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$

d. $(A \land B) \rightarrow (\neg A \leftrightarrow B)$

Question 2. Prove the following propositional clause using *contradiction*.

a. $P \land Q \vdash P \lor Q$

b. $\{P \lor Q; Q \rightarrow (R \land S); (P \land R) \rightarrow U\} \vdash U$

c. $\{P \lor Q; Q \lor R; \neg P \lor \neg Q; \neg P \lor \neg R; \neg Q \lor \neg R\} \vdash Q$

d. If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.

e. If a^2 is even, $a \in \mathbb{Z}$, then a is even.

f. If k is odd, $k \in \mathbb{Z}$, then k^2 is odd.

g. If k^2 is odd, $k \in \mathbb{Z}$, then k is odd.

h. There exist no integers a and b for which $21a + 30b = 1$.

i. There exist no integers a and b for which $18a + 6b = 1$.

Question 3. Given the following facts, how can you conclude *"It's good to walk"* by using *contradiction* method?

- It is raining, it is snowing or it is dry.
- It is warm.
- It is not raining.
- It is not snowing.
- If the weather is nice, then it is good to walk.
- If the weather is dry and warm, the weather is nice.

Question 4. Write the truth table for the following propositional sentences.

a. $\big((P \land Q) \lor R\big) \rightarrow \big((R \lor \neg U) \leftrightarrow P\big)$
b. $\neg\big(P \leftrightarrow (Q \land \neg R)\big) \leftrightarrow (U \rightarrow P)$

Question 5. Given the following facts, how can you conclude *"Gianni climbs mount Everest"* by using *contradiction* method?

- If Gianni is a climber and he is fit, he climbs mount Everest.
- If Gianni is not lucky and he is not fit, he does not climb mount Everest.
- Gianni is fit.

Question 6. Formulate the following statements in predicate logic and/or first-order logic, making clear what your atomic predicate symbols stand for.

a. Anyone who has forgiven at least one person is a saint.
b. Nobody in the calculus class is smarter than everybody in the discrete mathematic class.
c. Anyone who has bought a Rolls Royce with cash must have a rich father.
d. If anyone in the college has the measles, then everyone who has a friend in the college must be quarantined.
e. Everyone likes Mary, except Mary herself.
f. Jane saw a bear, and Roger saw one too.
g. All purple mushrooms are poisonous.
h. No student likes every lecture.
i. There are at least two apples in a barrel.
j. There are at least two apples in every barrel.
k. Jane ate a mushroom that she had picked herself.
l. No yellow frogs are edible.
m. All students that had missed a lecture answered at least one question incorrectly.
n. Young creatures who go up in balloons are liable to giddiness.

o. Every bag contains at least one coin.

p. The father of a mother or father is a grandfather.

q. Tom's sister knows Mary’s brother.

r. A careless soldier killed himself.

s. If a cake is from France then it has more sugar, if it is made with chocolate then it is made with cream, but if a cake is from Italy then it has more sugar, if it is made with cream then if it is made of chocolate.

t. Mary gave an apple to Tom.

u. One of the apples that Mary gave to Tom is Rotten.

v. The exam was hard but not very hard.

w. You only live twice.

x. By this time tomorrow I shall have finished my exams.