Heuristic Search

Outline

- Best-first search
- Greedy best-first search
- A^* search
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search
- Genetic algorithms

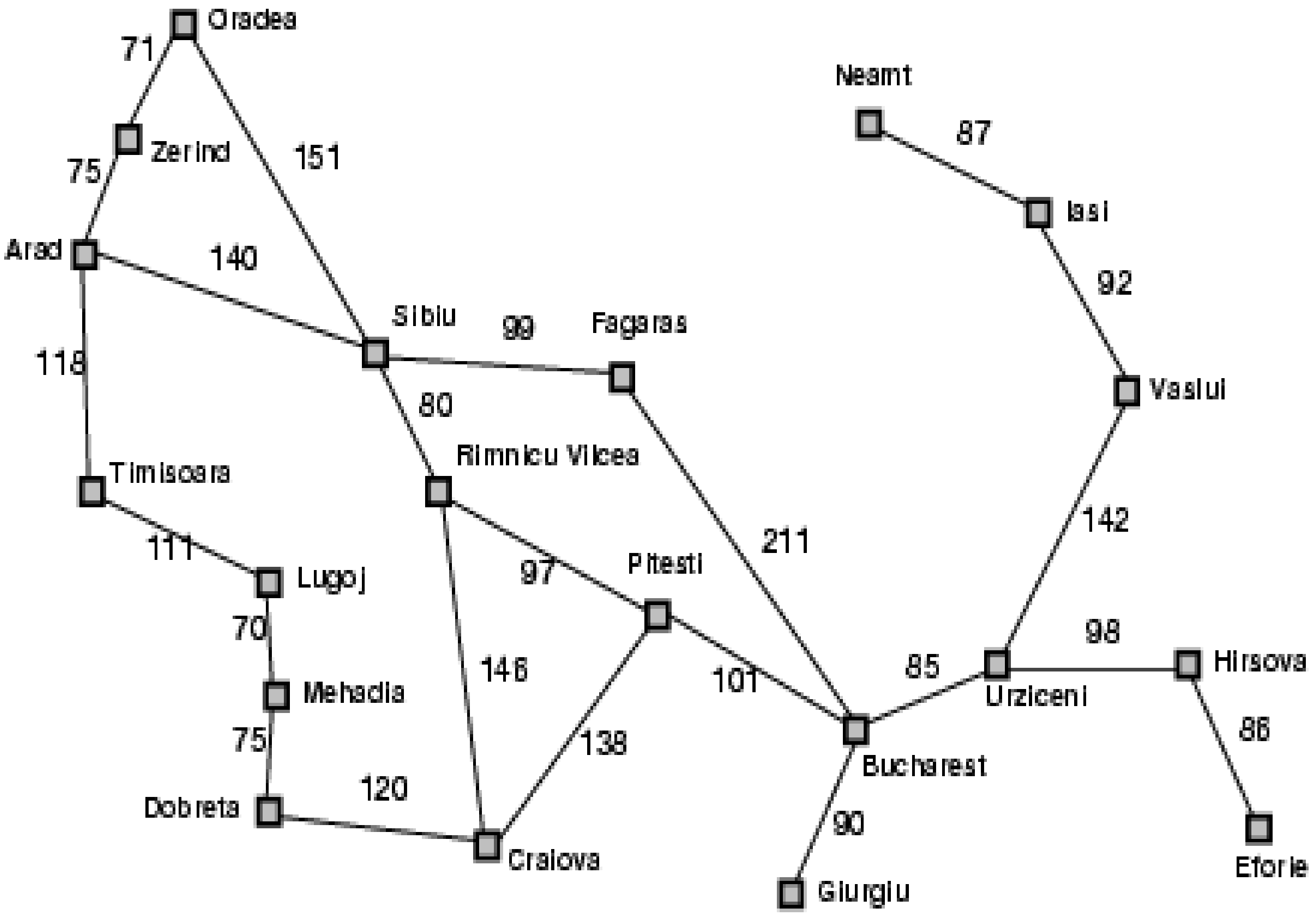
Review: Tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Best-first search

- Idea: use an evaluation function f(n) for each node
  - estimate of "desirability"
  - expand most desirable unexpanded node
- Implementation:
  - Order the nodes in fringe in decreasing order of desirability
- Special cases:
  - greedy best-first search
  - A^* search

Romania with step costs in km

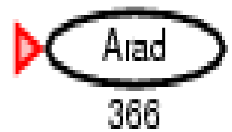


Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

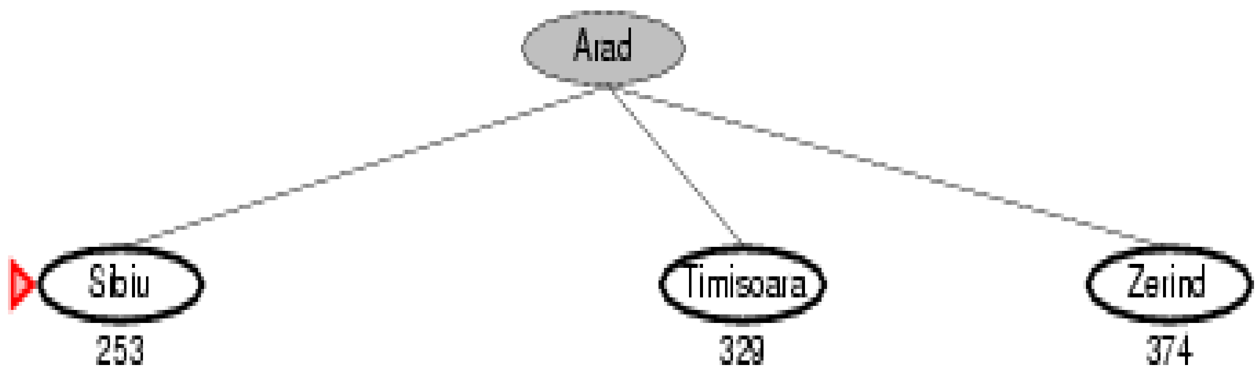
Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)
  - estimate of cost from n state to goal state
  - e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal state

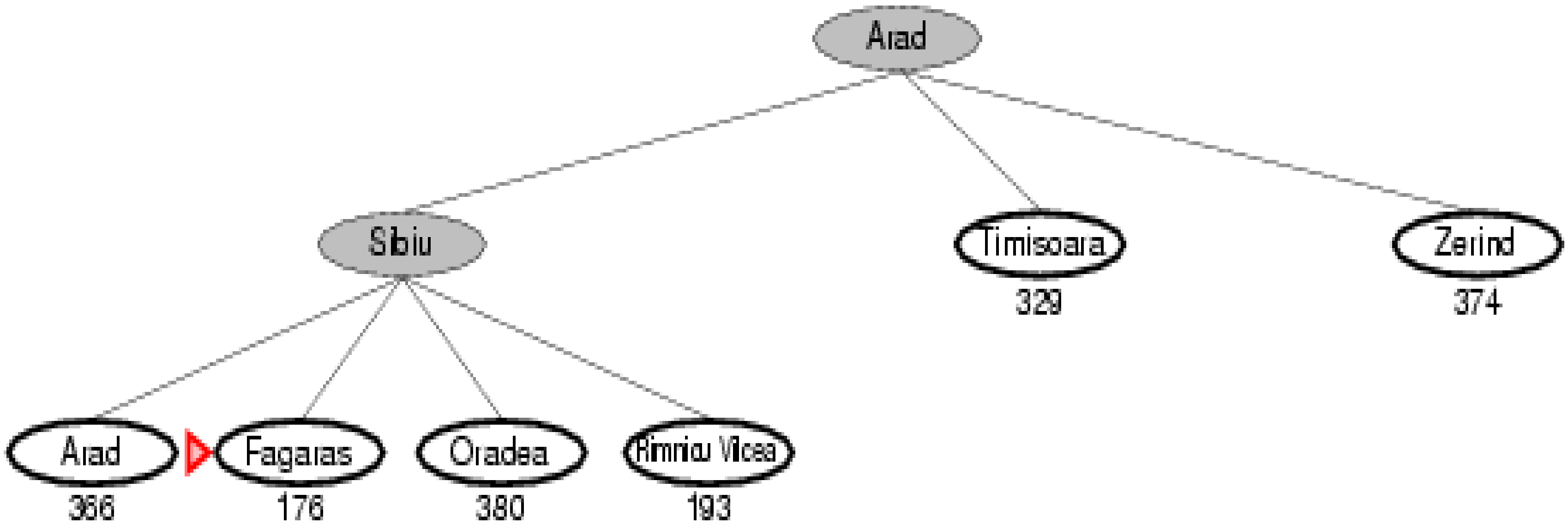
Greedy best-first search example



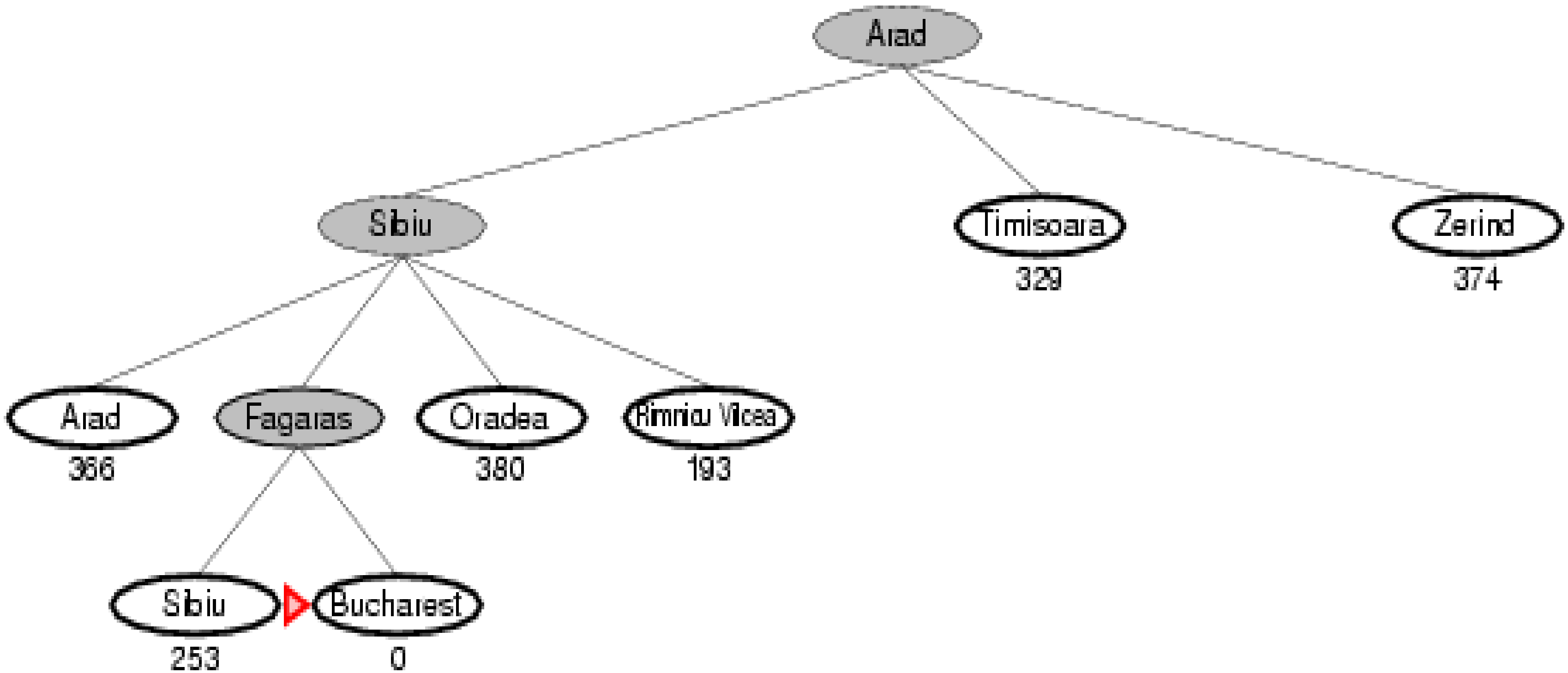
Greedy best-first search example



Greedy best-first search example



Greedy best-first search example



Properties of greedy best-first search

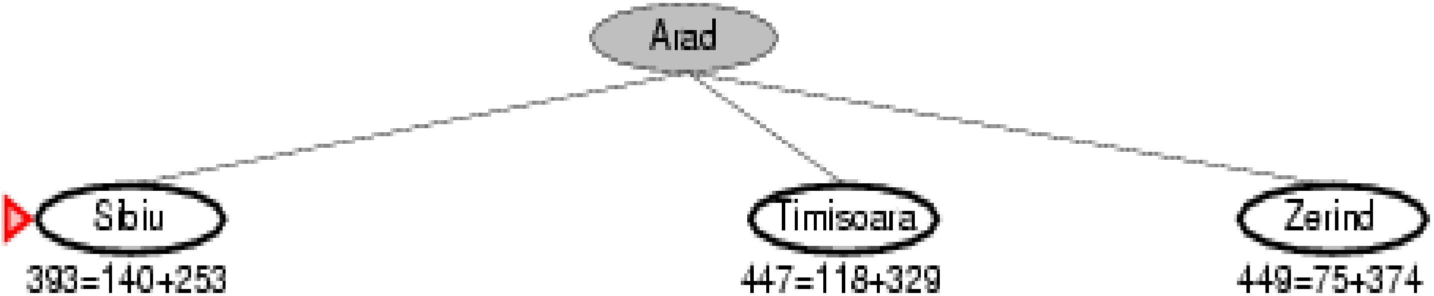
- Complete?: No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time?: $O(b^m)$, but a good heuristic can give dramatic improvement
- Space?: $O(b^m)$ → keeps all nodes in memory
- Optimal?: No

A* search

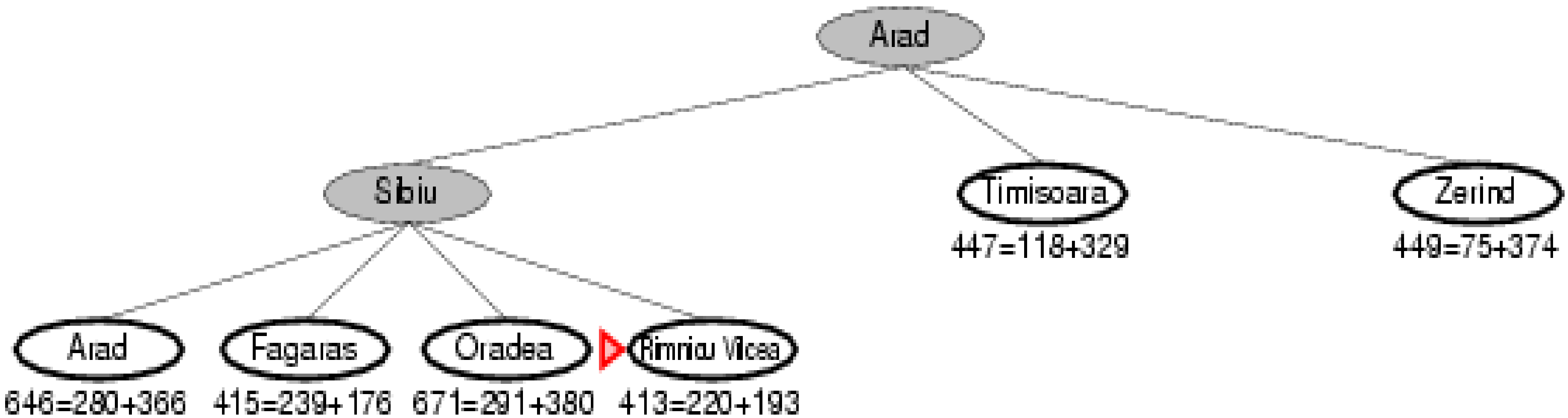
- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal

A* search example

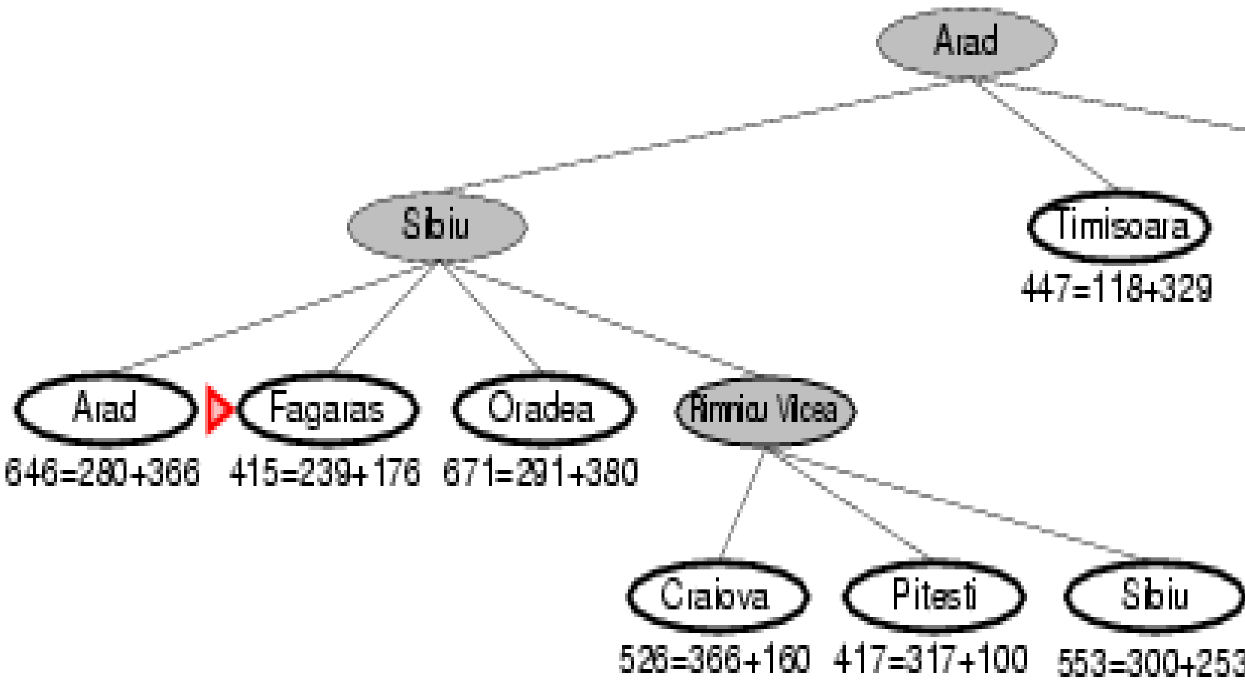
A* search example



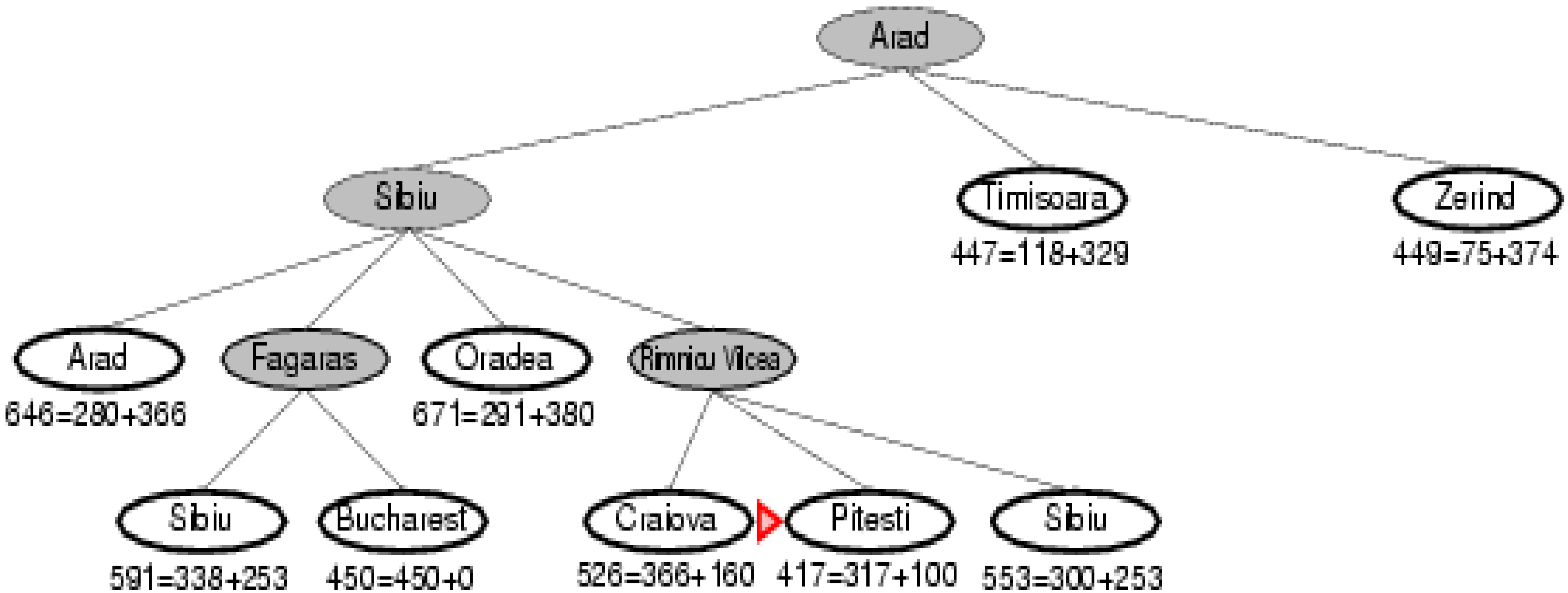
A* search example



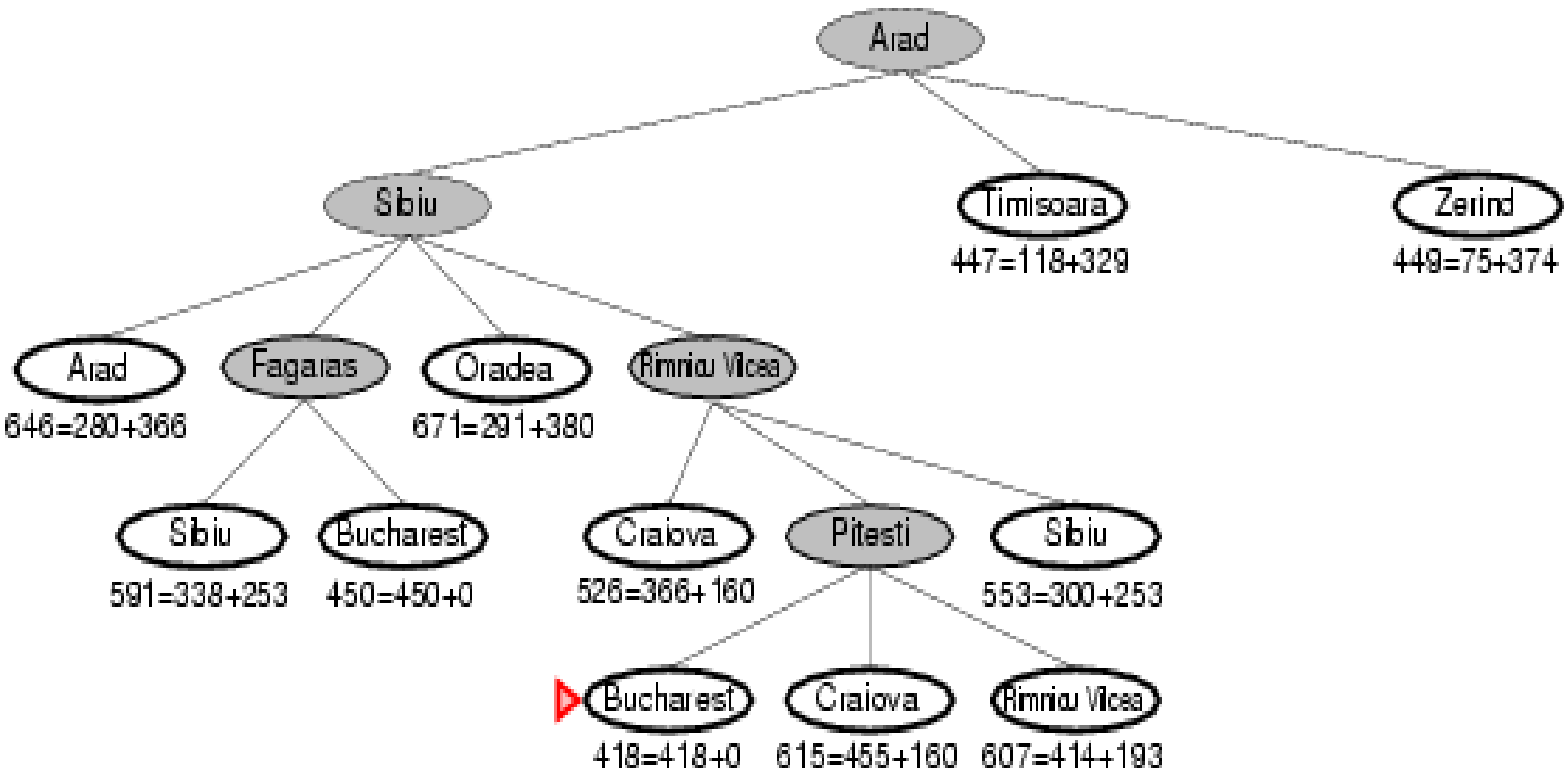
A* search example



A* search example



A* search example

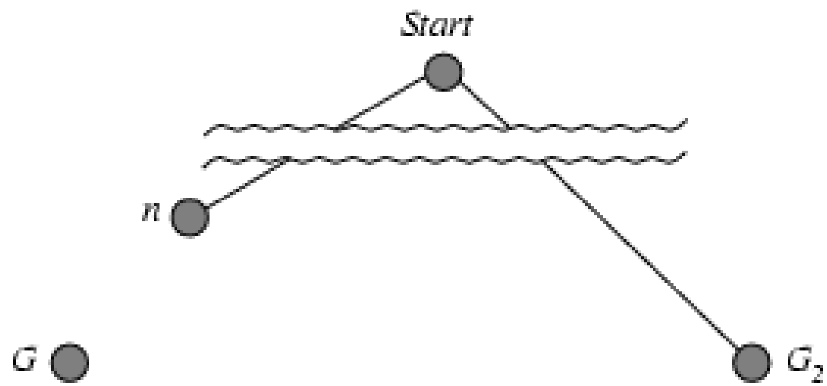


Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
  - Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



  - $f(G_2) = g(G_2)$ since $h(G_2) = 0$
  - $g(G_2) > g(G)$ since G_2 is suboptimal
  - $f(G) = g(G)$ since $h(G) = 0$
  - $f(G_2) > f(G)$ from above

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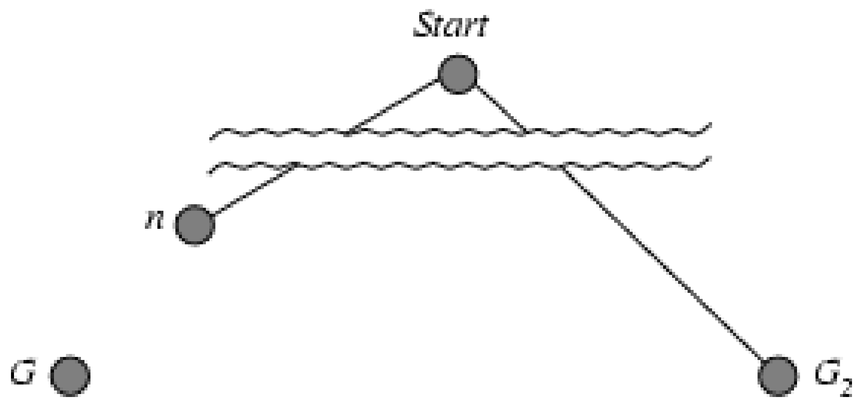
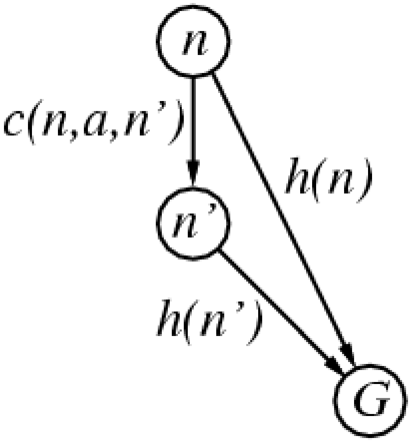


  - $f(G_2) > f(G)$ from above
  - $h(n) \le h^*(n)$ since h is admissible
  - $g(n) + h(n) \le g(n) + h^*(n)$
  - $f(n) \le f(G)$, hence $f(G_2) > f(n)$, and A^* will never select G_2 for expansion

Consistent heuristics

- A heuristic is consistent if for every node n, every successor n' of n generated by any action a,

  h(n) ≤ c(n,a,n') + h(n')



- If h is consistent, we have

  f(n') = g(n') + h(n')

  = g(n) + c(n,a,n') + h(n')

  ≥ g(n) + h(n)

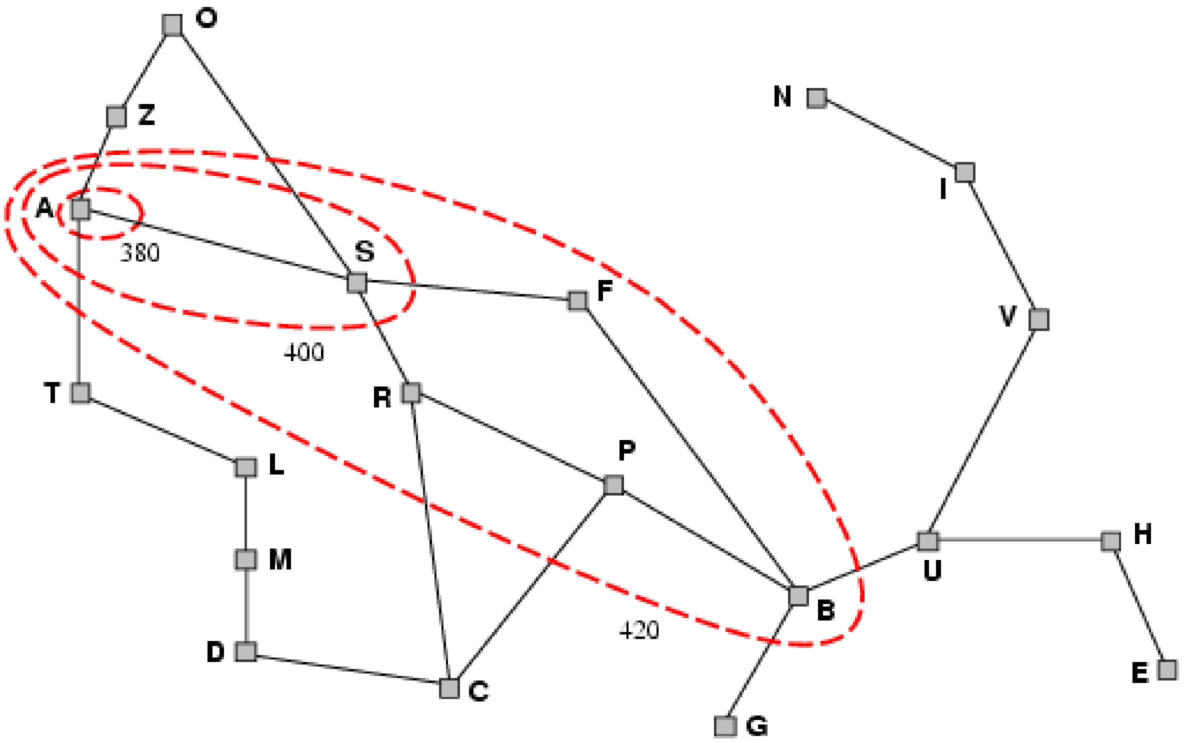
  = f(n)

  – i.e., f(n) is non-decreasing along any path.

- Theorem: If h(n) is consistent, A* using GRAPH-SEARCH is optimal

Optimality of A*

- A* expands nodes in order of increasing f value
- Gradually adds f-contours of nodes
- Contour i has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

- Complete? Yes (unless there are infinitely many nodes with $f \leq f(G)$)

- Time? Exponential

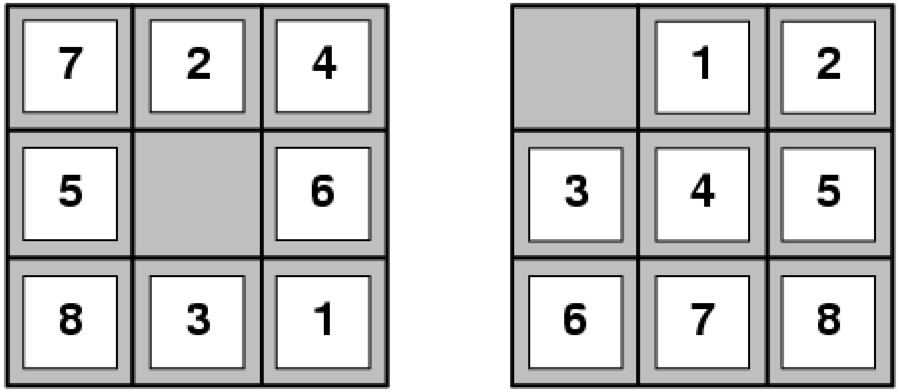
- Space? Keeps all nodes in memory

- Optimal? Yes

Admissible heuristics: 8-puzzle

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State Goal State

- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics: 8-puzzle

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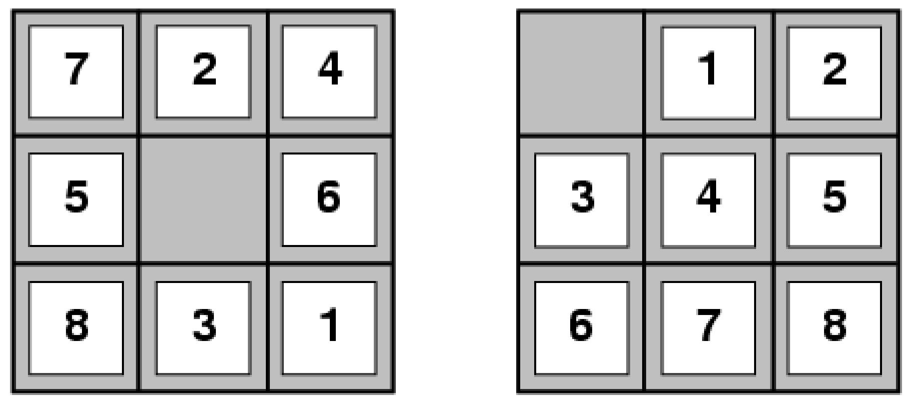


Start State Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1, h_2 is better for search
- Typical search costs (average number of nodes expanded):
  - d=12

    IDS = 3,644,035 nodes
    $A^*(h_1)$ = 227 nodes
    $A^*(h_2)$ = 73 nodes
  - d=24

    IDS = too many nodes
    $A^*(h_1)$ = 39,135 nodes
    $A^*(h_2)$ = 1,641 nodes

Relaxed problems

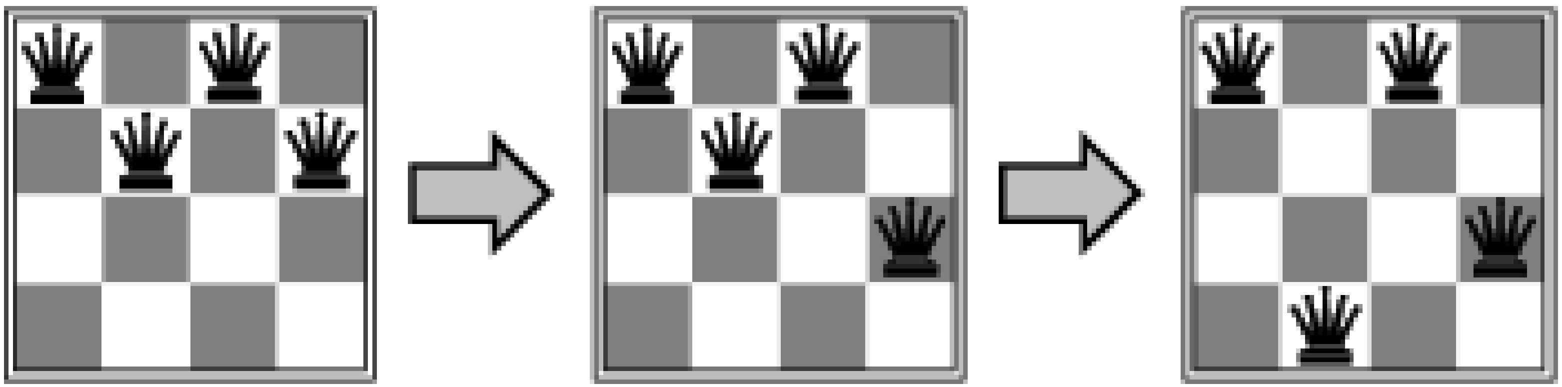
- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Local search algorithms

- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution
- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use local search algorithms keep a single "current" state, try to improve it

Example: *n*-queens

- Put n queens on an n × n board with no two queens on the same row, column, or diagonal



Hill-climbing

- Searching for a goal state = Climbing to the top of a hill
- Heuristic function to estimate how close a given state is to a goal state.

Hill-climbing

- "Like climbing Everest in thick fog with amnesia"

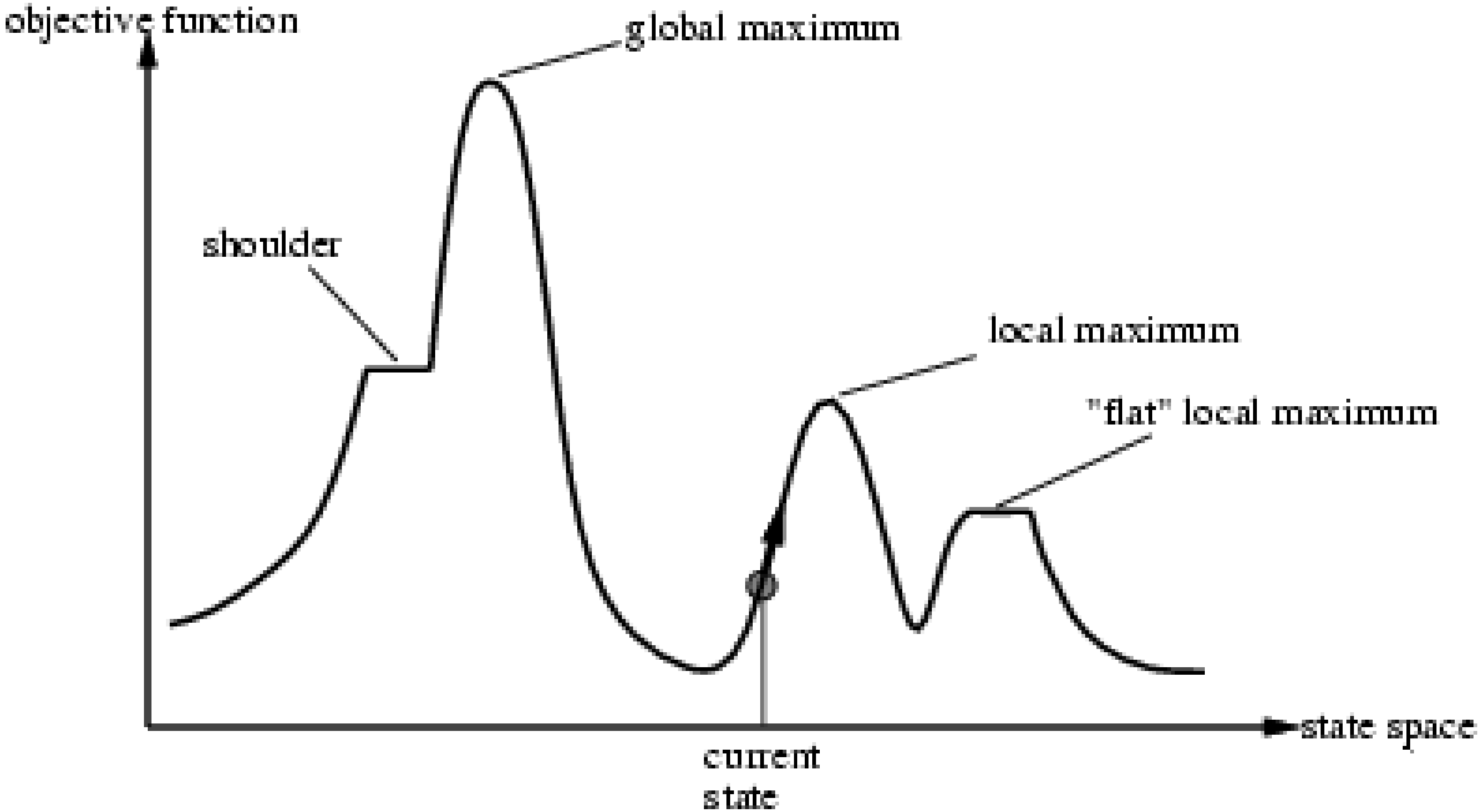
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

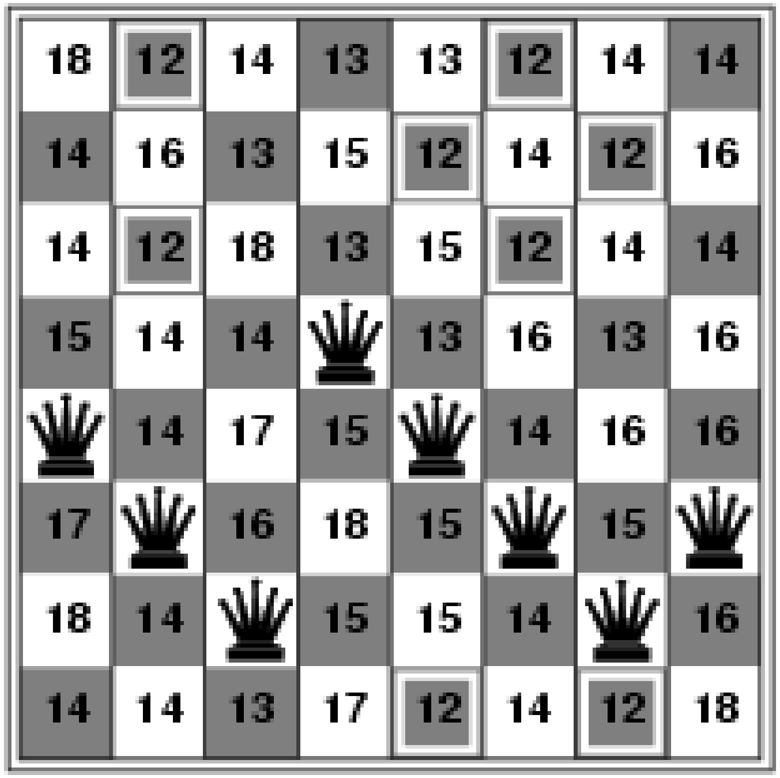
Hill-climbing: Disadvantages

- Local maximum
- Plateau
- Ridge

Hill-climbing: Disadvantages

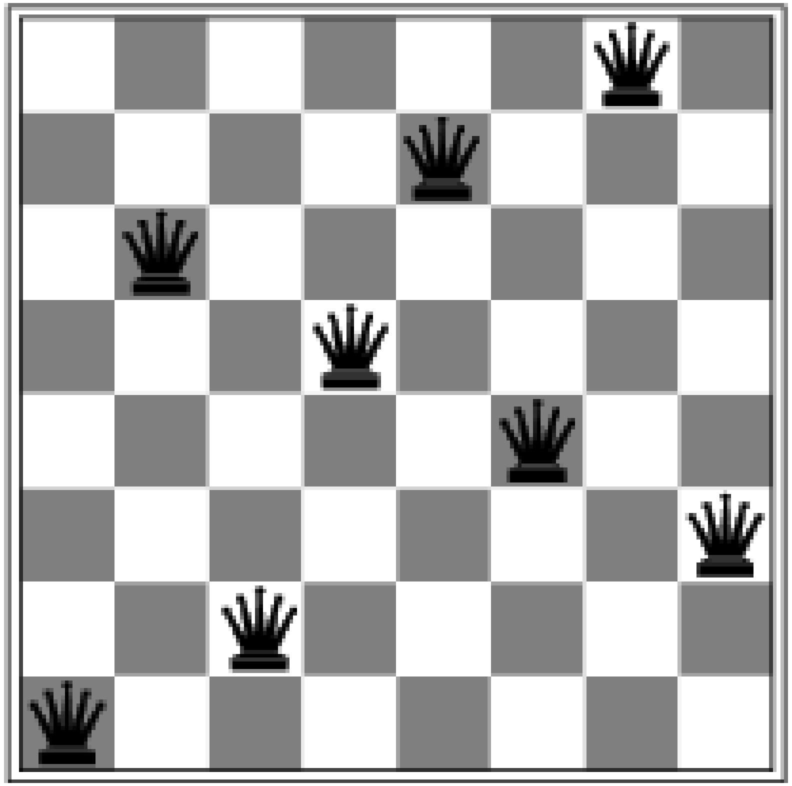


Hill-climbing search: 8-queens problem



- h = number of pairs of queens that are attacking each other, either directly or indirectly (h = 17 for the above state)

Hill-climbing search: 8-queens problem



- A local minimum with h = 1

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing search

- One can prove: If T decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

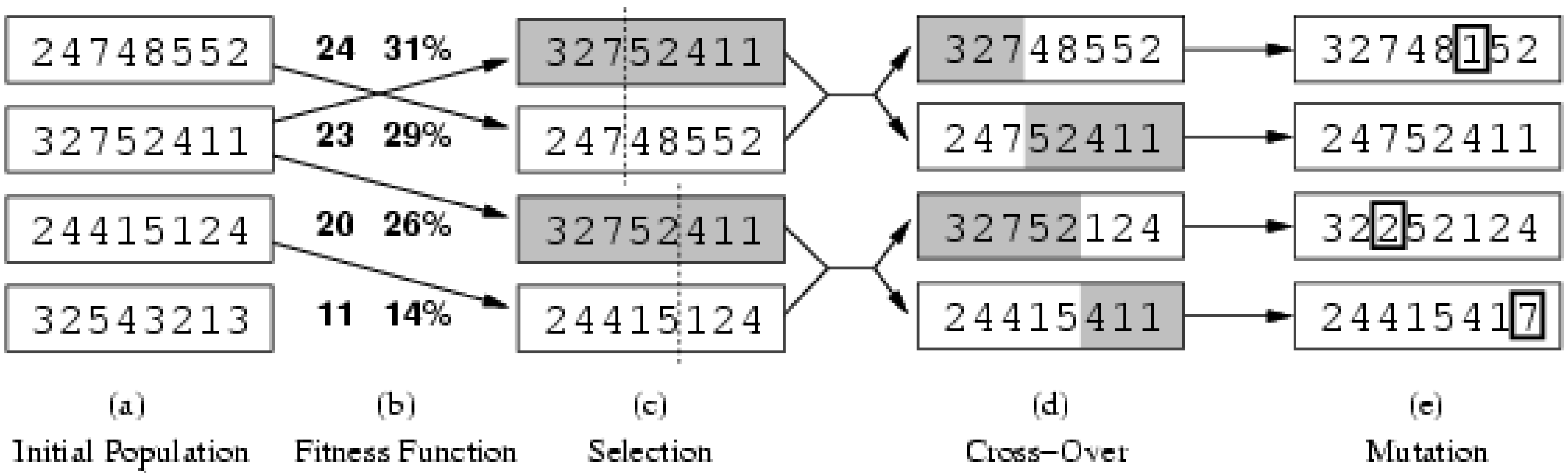
Local beam search

- Keep track of k states rather than just one
- Start with k randomly generated states
- At each iteration, all the successors of all k states are generated
- If any one is a goal state, stop; else select the k best successors from the complete list and repeat.

Genetic algorithms

- A successor state is generated by combining two parent states
- Start with k randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by
  - Selection
  - Crossover
  - Mutation

Genetic algorithms



- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)
- 24/(24+23+20+11) = 31%
- 23/(24+23+20+11) = 29% etc

Genetic algorithms