Lecture 2
Linear Regression and Gradient Descent

LÊ ANH CƯỜNG

Recap from the previous lecture

- Role of machine learning and artificial intelligence.
- A general diagram of AI systems.
- Concepts: knowledge, reasoning, inference, model, learning.
- Issues in machine learning.
- Understanding machine learning concepts through examples.

Outline

- Supervised learning for classification and regression problems
- Linear Regression (LR) model
- Training LR by Gradient Descent Algorithm
- Implementation LR
- Exercises
- Summary

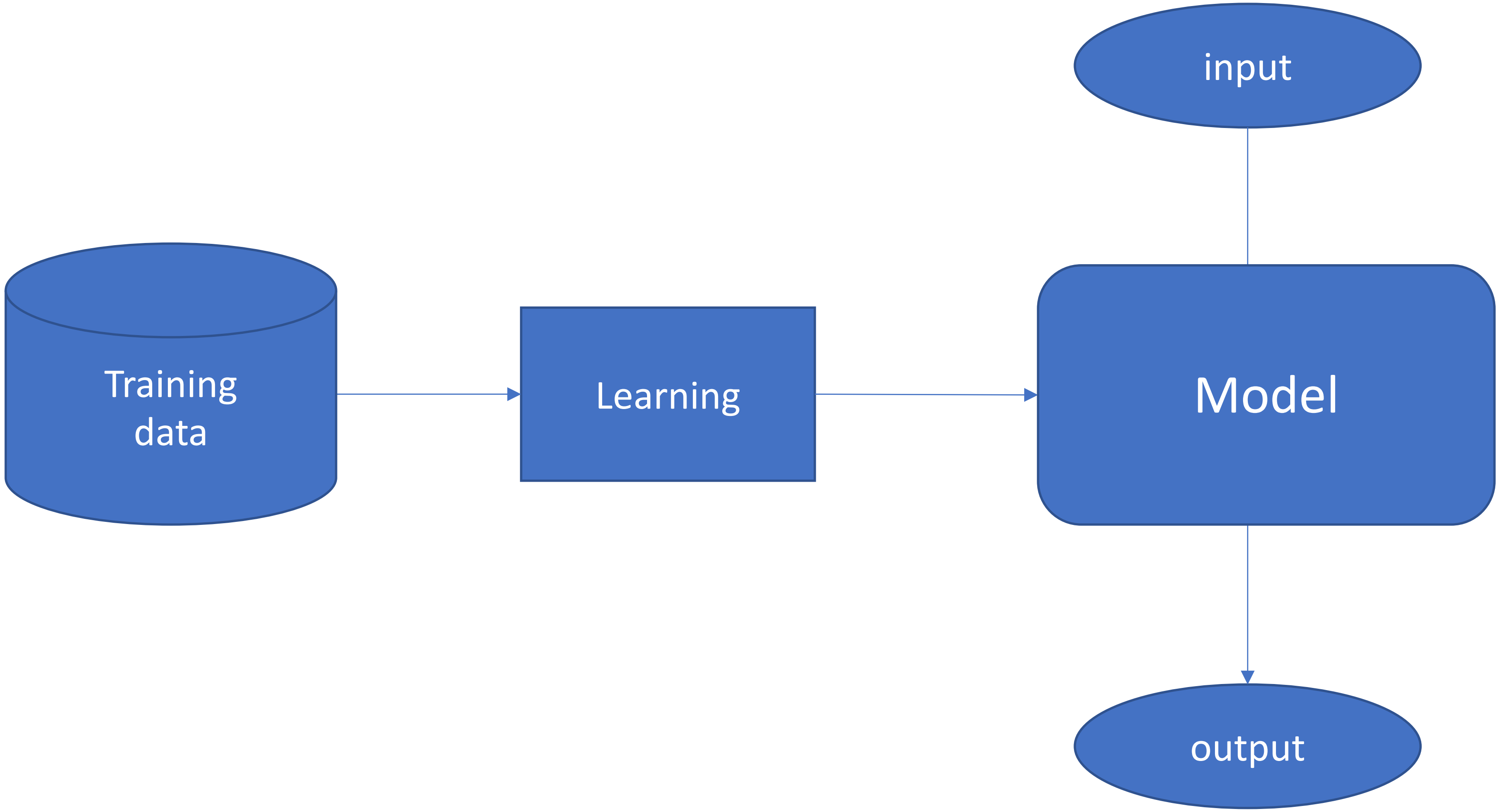
Supervised learning vs Unsupervised learning

- Supervised learning uses training data examples which are labeled as the objective of the task.
- Unsupervised learning doesn’t use labled data, so it aims for different types from supervised learning.

Types of supervised learning problems?

- Classification
  - Single label
  - Multi-labels
- Regression
  - Real Values
- Sequence generation
  - Sequence of real values
  - Sequence of labels

Supervised Learning: a general model



Regression vs Classification

- Classification is about predicting a label and Regression is about predicting a quantity.
- Classification is the problem of predicting a discrete class label output for an example. The predicted label is from a set of predefined labels.
  - For example: predict an email is spam or ham
- Regression is the problem of predicting a continuous quantity output for an example.
  - For example: predict house price from its features (i.e. information).

Regression (Hồi qui)

- From Height, predict Weight?

Height(cm)	Weight(kg)
147	49
150	50
153	51
155	52
158	54
160	56
163	58
165	59

Training data

(x1, y1)

(x2, y2)

.

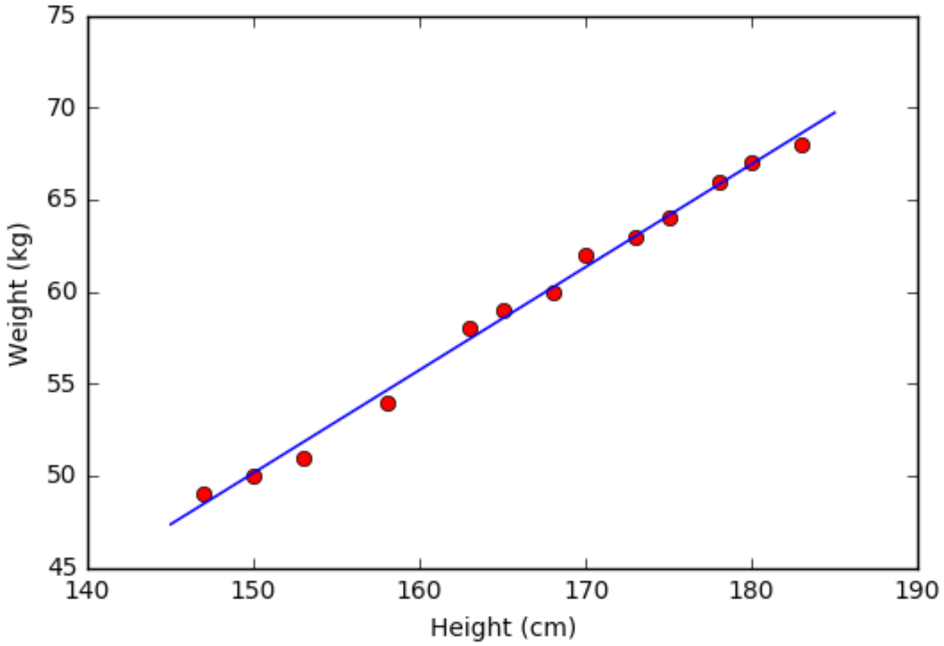
.

.

(xn, yn)

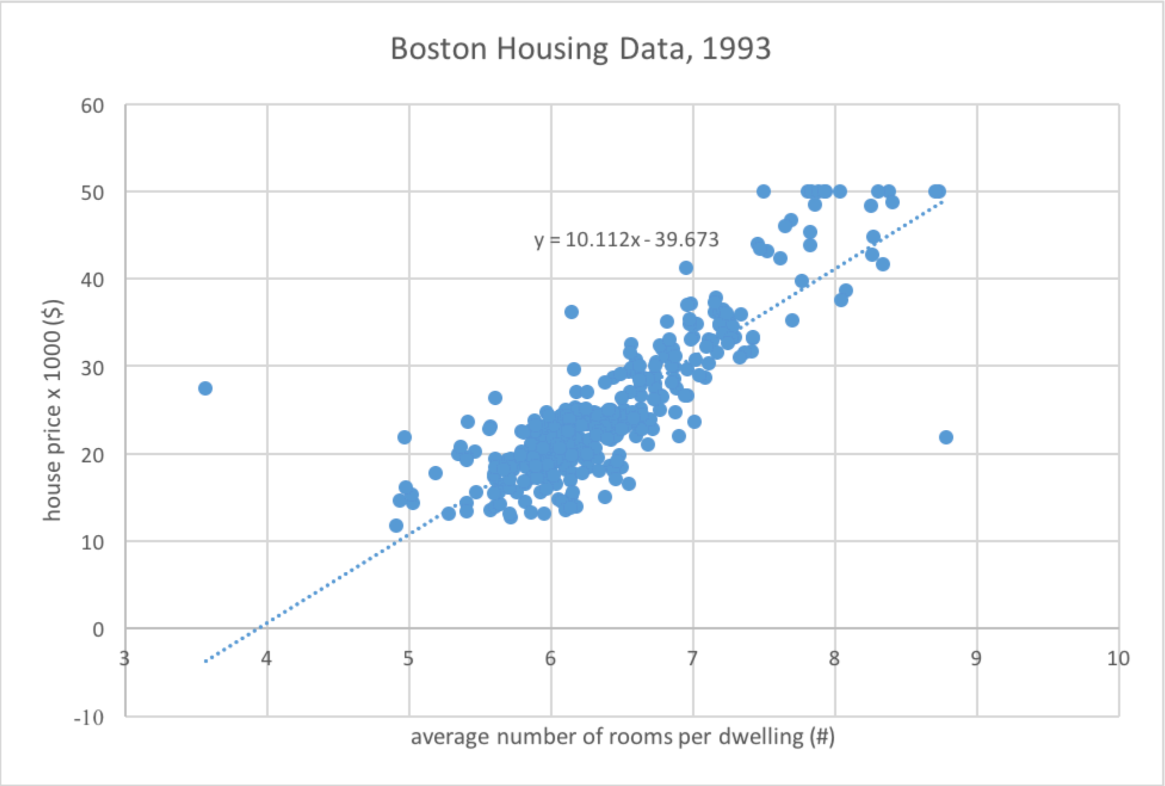
Linear Regression

- How is the relationship between Height and Weight?
- Suppose that Weight linearly depends on Height



Linear Regression

- Example of house pricing



Model Learning

1. Choose a model form?
2. Determine a loss function.
3. Learning: minimize the loss function then obtain parameters' values

Simple Linear Regression: one variable

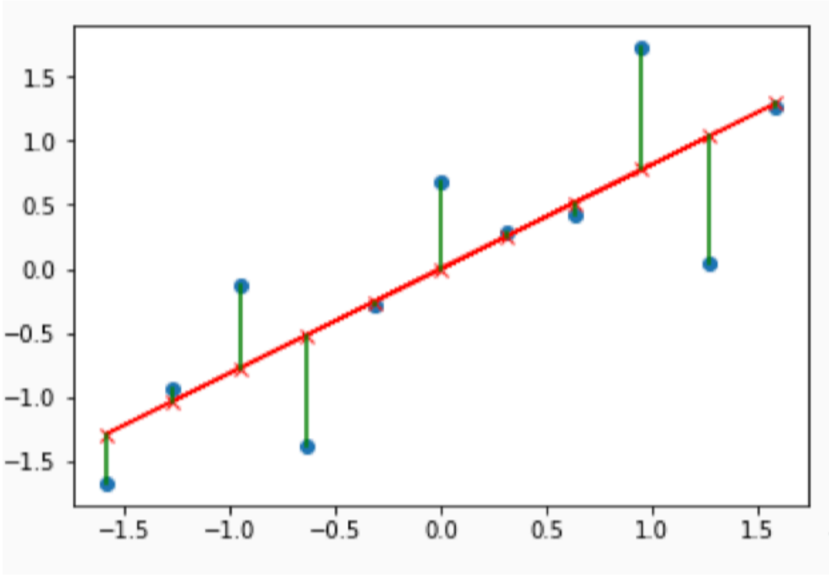
Training data: $(x_1, y_1), (x_2, y_2). . . (x_n, y_n)$

$$y = f(\mathrm{x}) = w_0 + w_1 x$$

Learn w_0 and w_1

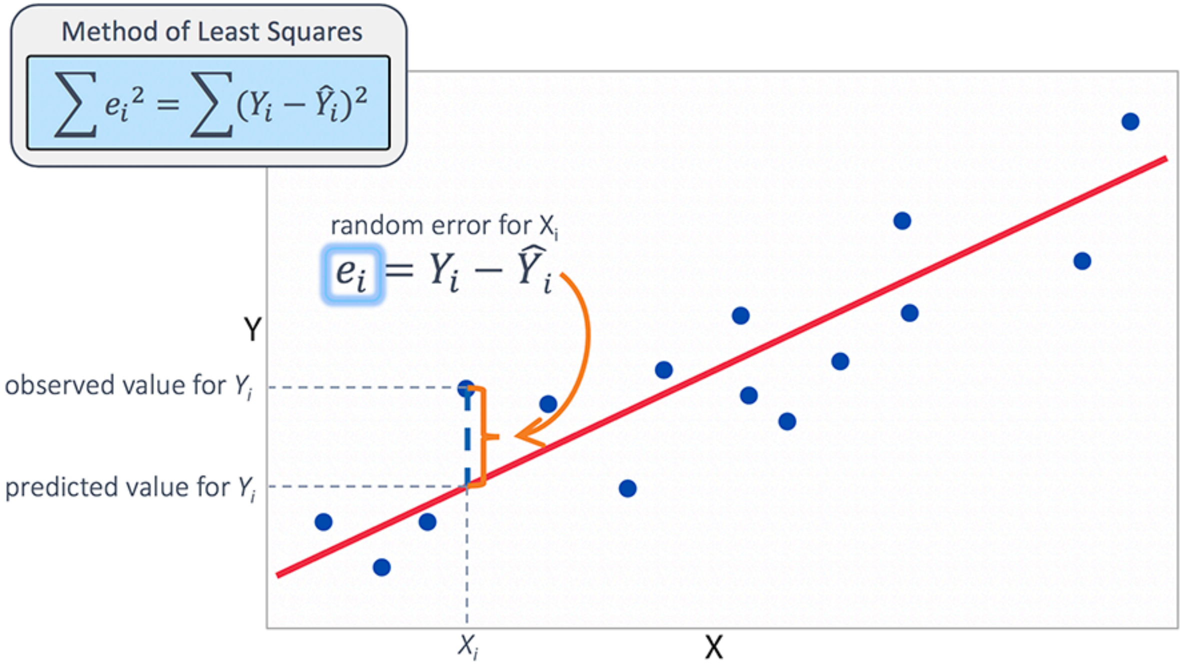
Loss Function

Training data: $(x_1, y_1), (x_2, y_2)... (x_n, y_n)$



$$y = f(\mathrm{x}) = w_0 + w_1 x$$

Loss Function



$$y = f(\mathrm{x}) = w_0 + w_1 x$$

$$\text{Loss} = L = \frac{1}{2N}\sum_{i=1}^{N}(f(x_i) - y_i)^2 = \frac{1}{2N}\sum_{i=1}^{N}((w_0 + w_1 x_i) - y_i)^2$$

Simple Linear Regression: one variable

$y = f(\mathrm{x}) = \ w_0 + \ w_1 x$

$$\text{Loss} = L = \frac{1}{2N}\sum_{i=1}^{N}(f(x_i) - y_i)^2 = \frac{1}{2N}\sum_{i=1}^{N}((w_0 + \ w_1 x_i) - y_i)^2$$

Parameters: w_0, w_1

Goal: minimize Loss function

$$\frac{\partial L}{\partial w_0} = 0, \qquad \frac{\partial L}{\partial w_1} = 0$$

Linear Regression: Learning

Goal: minimize Loss function

$$\frac{\partial L}{\partial w_0} = 0, \qquad \frac{\partial L}{\partial w_1} = 0$$

$$L = \frac{1}{2N}\sum_{i=1}^{N}(f(x_i) - y_i)^2 =$$

$$= \frac{1}{2N}\sum_{i=1}^{N}((w_0 + w_1 x_i) - y_i)^2$$

$$\frac{\partial L}{\partial w_0} = \frac{1}{N}\sum_{i=1}^{N}(f(x_i) - y_i)$$

$$\frac{\partial L}{\partial w_1} = \frac{1}{N}\sum_{i=1}^{N}(f(x_i) - y_i)\, x_i$$

Multiple Linear Regression

$(X_{\mathrm{i}}, \mathrm{y}_{\mathrm{i}})$

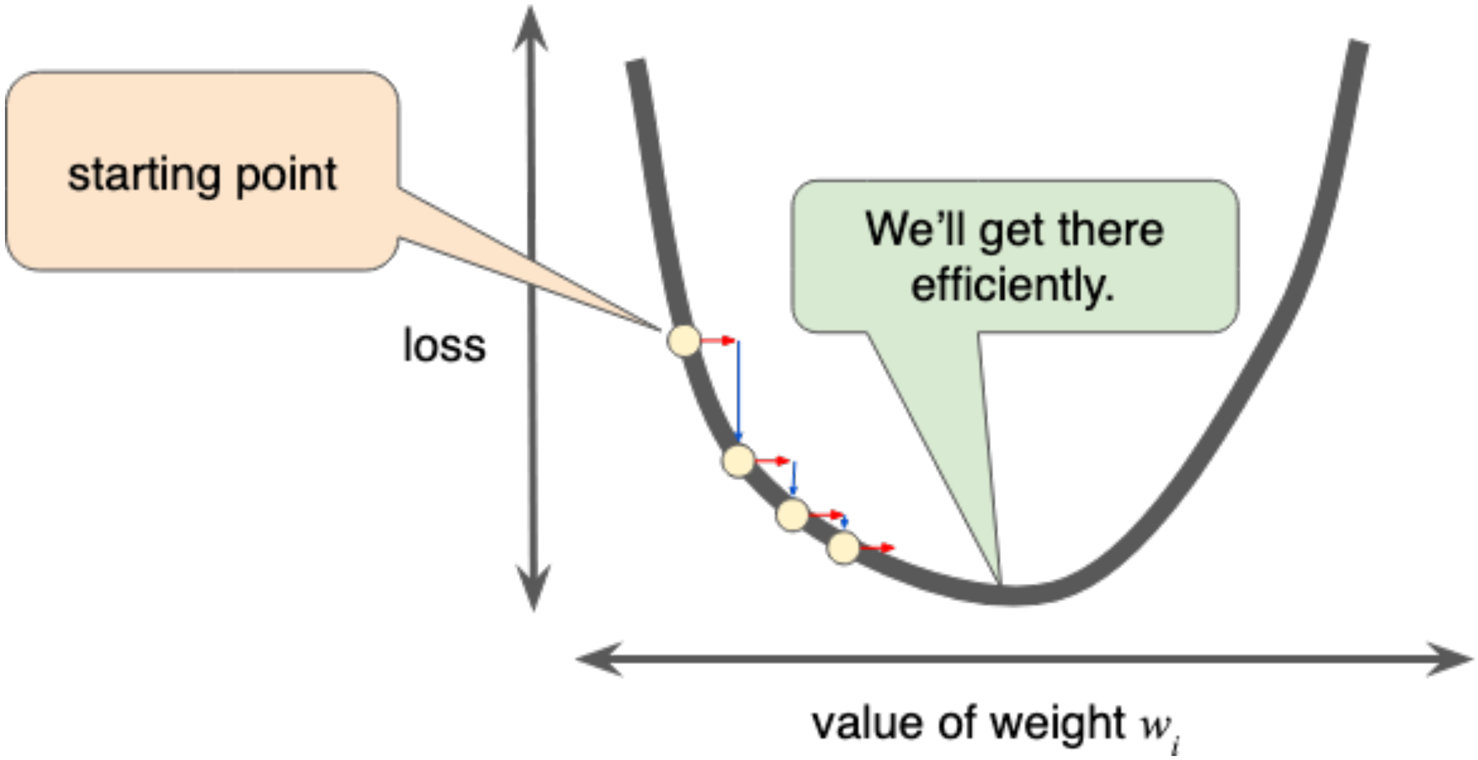
$X_{\mathrm{i}} = (x_{i1}, \ldots, x_{\mathrm{k}})$

$$y_{\mathrm{i}} = f(\mathrm{X}_{\mathrm{i}}) = w_0 x_{\mathrm{i}0} + w_1 x_{\mathrm{i}1} + \ldots + w_{\mathrm{k}} x_{\mathrm{i}k}$$

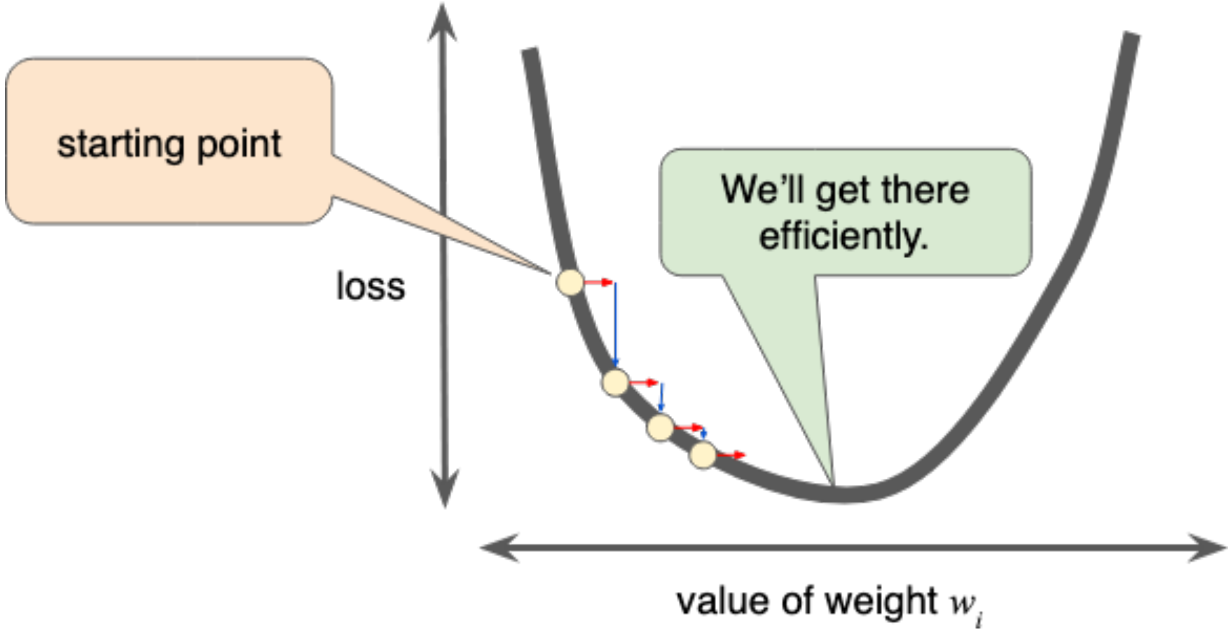
$$\text{Loss} = L = \frac{1}{2N}\sum_{i=1}^{N}(f(\mathrm{X}) - y_i)^2 = \frac{1}{2N}\sum_{i=1}^{N}((w_{\mathrm{i}0} + w_{\mathrm{i}1}x + \cdots + w_{\mathrm{ik}}x) - y_i)^2 =$$

$$\frac{\partial L}{\partial w_{\mathrm{t}}} = \frac{1}{N}\sum_{i=1}^{N}(f(\mathrm{X}_i) - y_i)\, x_{i\mathrm{t}}$$

Linear Regression: Learning by Gradient Descent



Linear Regression: Learning by Gradient Descent

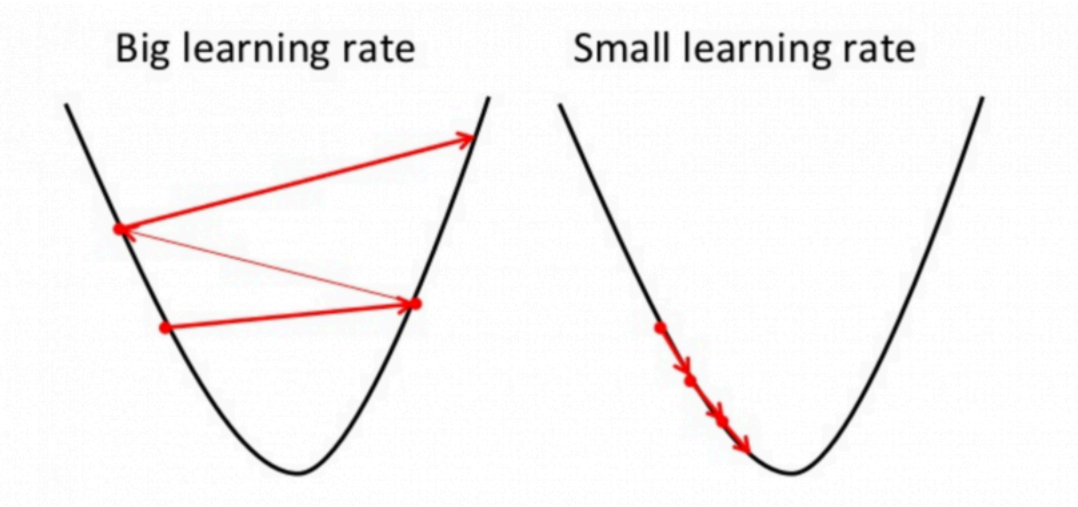


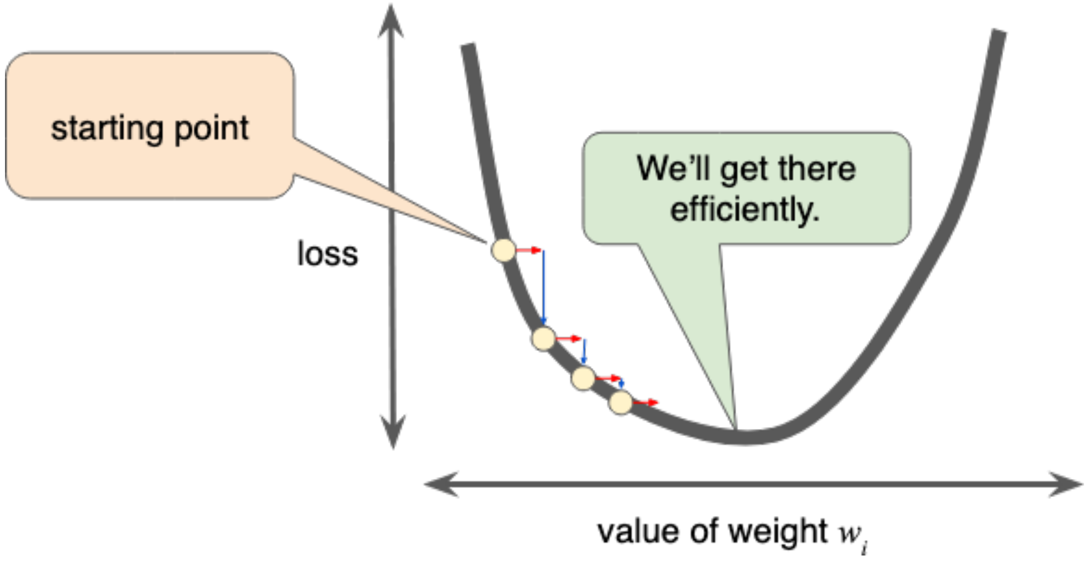
Update parameters:

Learning rate

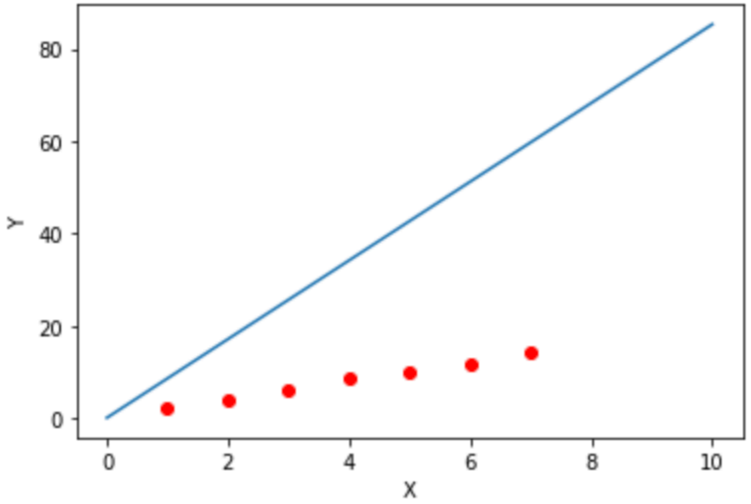
$$w_i = w_i - \mu \frac{\partial L}{\partial w_i}$$

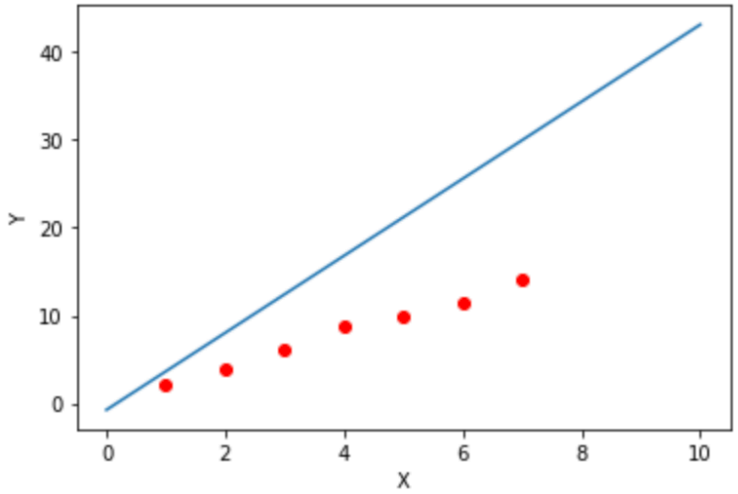
Linear Regression: Learning by Gradient Descent

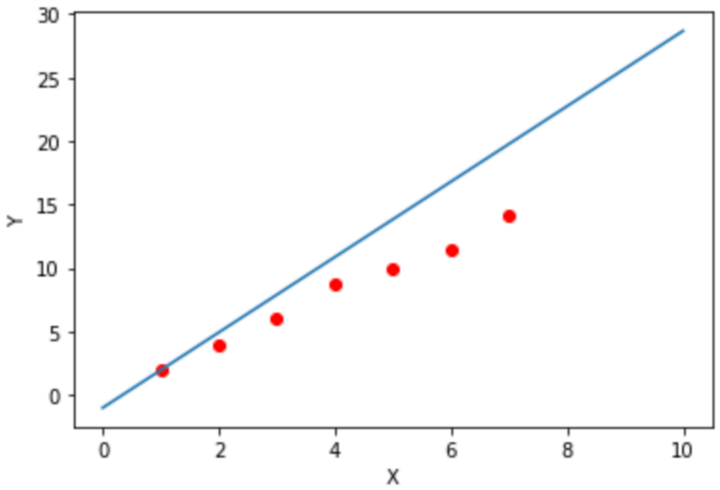


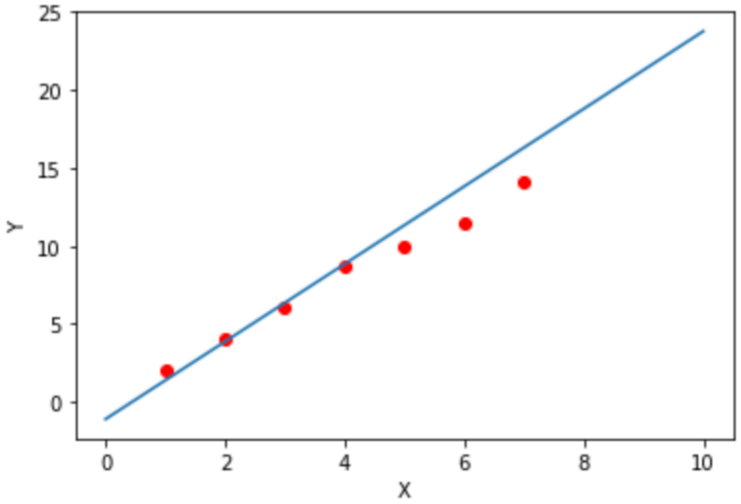


Learning progress

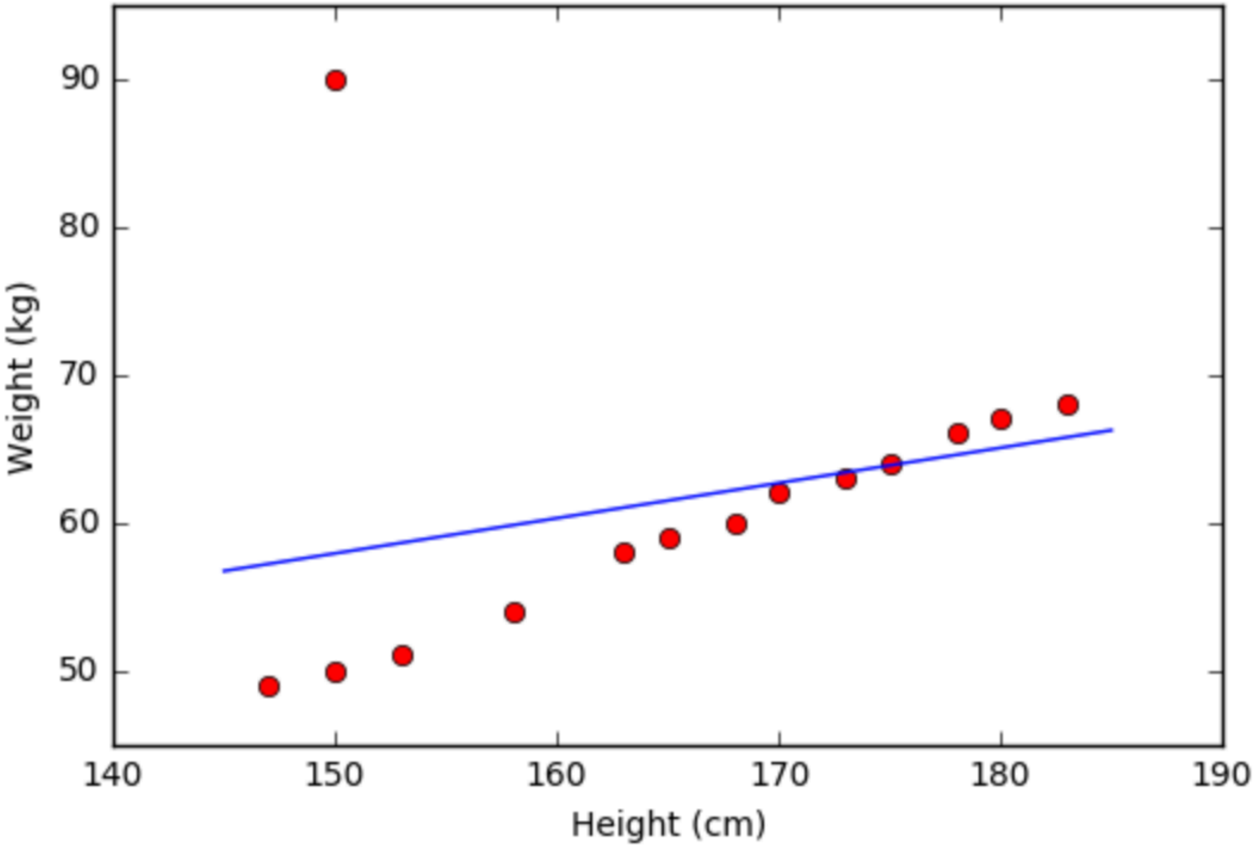








Issues:
Linear Regression is sensitive with outliers



Summary

- Supervised learning vs unsupervised learning.
- Classification vs regression
- Linear regression form.
- Learning Linear Regression model by Gradient Descent algorithm?

Exercise

- Implement Linear Regression for the problem of house pricing with multiple variables:
  - Firstly derive steps and mathematical formulas in the algorithm.
  - Secondly, implementation using python.

Answer the questions:

1. What is the objective of a supervised learning model? General model?
2. What is linear regression model?
3. Formulating the linear regression model with single variable?
   - Function y = f(x)
   - Loss function
4. Learning parameters by Gradient Descent
   - Update parameters?
   - Learning rate?
5. Implementation
6. What are limitations of linear regression

```python
# hàm y = f(x)
# w: tham số
def f(x,w):
    return w[0]+w[1]*x

# tính hàm loss
def loss(x,y,w):
    d = 0
    for i in range(len(x)):
        d += (y[i] - (w[0] + w[1]*x[i]))**2
    return d/(2*len(x))

# tính đạo hàm tại điểm w[0] và w[1]
def derivative(x,y,w):
    d0=0
    d1=0
    for i in range(len(x)):
        d1 += x[i]*(f(x[i],w)-y[i])
        d0 += f(x[i],w)-y[i]
    return d0/len(x),d1/len(x)
```

```python
# training
epoch = 10
learning_rate = 0.01
w = [1,1] # y = x + 1
los_old = 0
for i in range(epoch):
    # hiển thị đồ thị
    plt.plot(x,y,'ro')
    plt.xlabel('X')
    plt.ylabel('Y')
    x0 = np.linspace(start=1, stop=10, num=50)
    y0 = w[0]+w[1]*x0
    plt.plot(x0,y0)
    plt.show()

    # cập nhật tham số
    los = loss(x,y,w)
    print('epoch_',i,':')
    print(los,' : ',los_old)
    if los>(los_old-0.0001) and i>0:
        break
    los_old = los

    # cập nhật
    a,b = derivative(x,y,w)# cho w0 và w1
    w[0] = w[0]-a*learning_rate
    w[1] = w[1]-b*learning_rate
```