Chapter 6

Relational Calculus

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Content

- Introduction
- Tuple Relational Calculus (TRC)
- Domain Relational Calculus (DRC)

Introduction

- Is the formal query language
- Introduced by Codd in 1972, "Data Base Systems", Prentice Hall, p33-98

- Properties
  - Nonprocedural language – declarative language
    - Calculus expression specifies *what is to be retrieved* rather than *how to retrieve*
  - *One declarative expression to specify a retrieval request*
    - *There is no description of how to evaluate query*
  - *A calculus expression may be written in different way*
    - *The way it is written has no bearing on how a query should be evaluated*

Introduction

- Categories
  - **Tuple relational calculus – TRC**
    - SQL
  - Domain relational calculus – DRC
    - QBE (Query By Example)
    - DataLog (Database Logic)

Content

- Introduction
- **Tuple relational calculus**
- Domain relational calculus

Tuple relational calculus – TRC

- A simple tuple calculus query is of the form

$$\boxed{\{\ t.A \mid P(t)\ \}}$$

- *t* is a tuple variable
  - Its value is any individual tuple from a relation
  - *t.A* is a value of a tuple *t* at an attribute *A*

- | *(vertical bar) is used to divide the query into two parts:*
  - *P* is a conditional expression involving *t*
    - P(t) has the TRUE or FALSE value depending on *t*
  - The result is the set of all tuples *t* that satisfy *P(t)*

Example 1

- Find employees whose salary is larger than 30000

$$\{ t \mid \underbrace{t \in EMPLOYEE}_{P(t)} \wedge \underbrace{t.SALARY > 30000}_{P(t)} \}$$

  - $t \in$ EMPLOYEE : TRUE
    - If *t* is an instance of relation EMPLOYEE
  - t.SALARY > 30000 : TRUE
    - If the attribute SALARY of tuple *t* has a value being larger than 30000
- The result is all tuples t which satisfy:
  - $t \in$ EMPLOYEE and t.SALARY > 30000

Example 2

- Retrieve the SSN and first name of employees whose salary is larger than 30000

$$\{ t.SSN, t.FNAME \mid t \in EMPLOYEE \wedge t.SALARY > 30000 \}$$

  - The set of SSNs and first names of employees of tuples *t* such that *t* are instances of EMPLOYEE and their values are larger than 30000 at the attribute SALARY

Example 3

- Find employees (SSN) who work for the department 'Nghien cuu'

$$\textbf{t}.\text{SSN} \mid \textbf{t} \in \text{EMPLOYEE}$$

$$\textbf{s} \in \text{DEPARTMENT} \wedge \textbf{s}.\text{DNAME} = \text{'Nghien cuu'}$$

- Select tuples ***t*** that belong to relation *EMPLOYEE*
- Compare ***t*** to a certain tuple ***s*** to find employees working for the department *'Nghien cuu'*
- Use the existential quantifier

$$\boxed{\exists t \in R\ (Q(t))}$$

Existing a tuple t of the relation R such that the expression Q(t) is TRUE → the result of the existential quantifier is TRUE

Example 3

- Find employees (SSN) who work for the department 'Nghien cuu'

$$\{\ t.SSN \mid t \in EMPLOYEE \wedge \exists s \in DEPARTMENT\ (\ \underbrace{s.DNAME = \text{'Nghien cuu'} \wedge s.DNUMBER = t.DNO}_{Q(s)}\ )\ \}$$

Example 4

- Find employees (FNAME) who work on projects or who have dependents

$$\{ t.FNAME \mid t \in EMPLOYEE \wedge ($$

$$\exists s \in WORKS\_ON\ (t.SSN = s.ESSN) \vee$$

$$\exists u \in DEPENDENT\ (t.SSN = u.ESSN)) \}$$

Example 5

- Retrieve the FNAME of employees who participate in projects and have dependents

$$\{\ t.FNAME \mid t \in EMPLOYEE \wedge (\ \exists s \in WORKS\_ON\ (t.SSN = s.ESSN) \wedge \exists u \in DEPENDENT\ (t.SSN = u.ESSN))\ \}$$

Example 6

- Find the FNAME of employees who work on projects and have no dependents

$$\{ t.FNAME \mid t \in EMPLOYEE \wedge$$
$$\exists s \in WORKS\_ON\ (t.SSN = s.ESSN) \wedge$$
$$\neg \exists u \in DEPENDENT\ (t.SSN = u.ESSN) \}$$

Example 7

- For each project in 'TP HCM', find the project number, the department number that controls the project and the FNAME of the manager

$$\{ s.PNUMBER, s.DNUM, t.FNAME \mid s \in PROJECT \land t \in EMPLOYEE \land s.PLOCATION = \text{'TP HCM'} \land \exists u \in DEPARTMENT\ (u.DNUMBER = s.DNUM \land u.MGRSSN = t.SSN) \}$$

Example 8

- Find employees (SSN) who work on <u>all</u> projects

  - Use the universal quantifier

$$\boxed{\forall t \in R\ (Q(t))}$$

If Q is TRUE with all tuples t of relation R, the universal quantifier is TRUE; otherwise FALSE.

Example 8a

- Find employees whose salary is highest.

$$\{\ t.SSN,\ t.LNAME,\ t.FNAME \mid t \in EMPLOYEE \land$$
$$\forall e \in EMPLOYEE\ (t.Salary >= e.Salary)\ \}$$

Example 8

- Find employees (SSN, FNAME, LNAME) who work on all projects

$$\{\, t.SSN, t.LNAME, t.FNAME \mid t \in EMPLOYEE \wedge$$
$$\forall s \in PROJECT\ (\ \exists u \in WORKS\_ON\ ($$
$$u.PNO = s.PNUMBER \wedge$$
$$u.ESSN = t.SSN\ ))\ \}$$

Example 9

- Find employees (SSN, LNAME, FNAME) who work on all projects controlled by the department 4

$$
\{\, t.SSN,\ t.LNAME,\ t.FNAME \mid t \in EMPLOYEE \land \\
\forall s \in PROJECT\ (\\
s.DNUM = 4 \land (\ \exists u \in WORKS\_ON\ (\\
u.PNO = s.PNUMBER \land \\
u.ESSN = t.SSN\ )))\ \}
$$

Example 9

- Find employees (SSN, LNAME, FNAME) who work on all projects controlled by the department 4
  - Use the “implies” operator

$$\boxed{P \Rightarrow Q}$$

If P then Q

Example 9

- Find employees (SSN, LNAME, FNAME) who work on all projects controlled by the department 4

$$
\begin{aligned}
\{\ & t.SSN, t.LNAME, t.FNAME \mid t \in EMPLOYEE \land \\
& \quad \forall s \in PROJECT\ (\\
& \qquad s.DNUM = 4 \Rightarrow (\ \exists u \in WORKS\_ON\ (\\
& \qquad\quad u.PNO = s.PNUMBER \land \\
& \qquad\quad u.ESSN = t.SSN\ )))\ \}
\end{aligned}
$$

Example 9 – Solution 2

- Find employees (SSN, LNAME, FNAME) who work on all projects controlled by the department 4

$$\{ t.SSN, t.LNAME, t.FNAME \mid t \in EMPLOYEE \wedge$$
$$\forall s \in PROJECT\ ($$
$$s.DNUM \neq 4 \vee (\ \exists u \in WORKS\_ON\ ($$
$$u.PNO = s.PNUMBER \wedge$$
$$u.ESSN = t.SSN\ )))\ \}$$

Example 10

- a. Find employees whose salary is larger than at least one employee of department 4.

- b. Find employees whose salary is larger than all employees of department 4.

Formal definition

- A general expression is of the form

$$\{ t_1.A_i, t_2.A_j, \ldots, t_n.A_m \mid P(t_1, t_2, \ldots, t_n, \ldots, t_{n+m}) \}$$

  - $t_1, t_2, \ldots, t_n$ are tuple variables
  - $A_i, A_j, \ldots, A_m$ are attributes of tuples *t*
  - P is a condition or well-formed formula
    - P is made up of predicate calculus atoms

Tuple variable

- Free variable

$$\{ t \mid t \in EMPLOYEE \wedge t.SALARY > 30000 \}$$

t is a free variable

- Bound variable

$$\{ t \mid t \in EMPLOYEE \wedge \exists s \in DEPARTMENT\ (s.DNUMBER = t.PNO) \}$$

Free variable

Bound variable

Atoms

- (i) $\boxed{t \in R}$
  - t is a tuple variable
  - R is a relation

$t \in$ EMPLOYEE

- (ii) $\boxed{t.A\ \theta\ s.B}$
  - A is an attribute of the tuple variable t
  - B is an attribute of the tuple variable s
  - θ is comparison operators, eg. $<$, $>$, $\leq$, $\geq$, $\neq$, $=$

t.SSN = s.ESSN

- (iii) $\boxed{t.A\ \theta\ c}$
  - C is a constant
  - A is an attribute of the tuple variable t
  - θ is comparison operators, eg. $<$, $>$, $\leq$, $\geq$, $\neq$, $=$

t.SALARY > 30000

Atoms

- Each of atoms evaluates to either TRUE or FALSE for a specific combination of tuples

- Formula (i) $\boxed{t \in R}$
  - TRUE value if *t* is a tuple of the specified relation *R*
  - FALSE value if *t* does not belong to *R*

R	A	B	C
	α	10	1
	α	20	1

$t1 = <\alpha, 10, 1>$ $t1 \in R$ has the TRUE value

$t2 = <\alpha, 20, 2>$ $t2 \in R$ has the FALSE value

Atoms

- Formula (ii) $\boxed{t.A \; \theta \; s.B}$ and (iii) $\boxed{t.A \; \theta \; c}$
  - If the tuple variables are assigned to tuples such that they satisfy the condition, then the atom is TRUE

R	A	B	C
	α	10	1
	α	20	1

If *t* is the tuple <α, 10, 1>

Then *t.B > 5* has the TRUE value (10 > 5)

Rules

- (1) Every atom is formula
- (2) If P is a formula then
  - $\neg P$ is a formula
  - (P) is a formula
- (3) If P_1 and P_2 are formulas then
  - $P_1 \vee P_2$ is a formula
  - $P_1 \wedge P_2$ is a formula
  - $P_1 \Rightarrow P_2$ is a formula

Rules

- (4) If P(t) is a formula then
  - $\forall t \in R$ (P(t)) is a formula
    - TRUE when *P(t)* is TRUE for all tuples in *R*
    - FALSE when there is one tuple that makes *P(t)* FALSE
  - $\exists t \in R$ (P(t)) is a formula
    - TRUE when there exists some tuple that makes *P(t)* TRUE
    - FALSE when *P(t)* is FALSE for all tuples *t* in *R*

Rules

- (5) If P is an atom then
  - Tuple variables *t* in *P* are free variables
- (6) Formulas $P = P_1 \wedge P_2$, $P = P_1 \vee P_2$, $P = P_1 \Rightarrow P_2$
  - A variable *t* in *P* is free or bound variable will depends on its role in P_1 and P_2

Transform

- (i) $P1 \wedge P2 = \neg (\neg P1 \vee \neg P2)$
- (ii) $\forall t \in R\ (P(t)) = \neg \exists t \in R\ (\neg P(t))$
- (iii) $\exists t \in R\ (P(t)) = \neg \forall t \in R\ (\neg P(t))$
- (iv) $P \Rightarrow Q = \neg P \vee Q$

Safe expression

- Examine

$$\{ t \mid \neg(t \in \text{EMPLOYEE}) \}$$

- Unsafe
  - Many tuples in the universe that are not EMPLOYEE tuples
  - Even though they do not exist in the database
  - The result is infinitely numerous

Safe expression

- Safe expression
  - Guarantee to yield *a finite number of tuples*

- A formula P is called safe expression
  - If its resulting values are from <u>the domain of P</u>
    - The domain of a tuple relational calculus expression: DOM(P)
    - The set of all values
      - Either appear as constant values in P
      - Or exist in any tuple in the relation referenced in P

Safe expression

- Example

$$\{ t \mid t \in EMPLOYEE \wedge t.SALARY > 30000 \}$$

- DOM($t \in EMPLOYEE \wedge t.SALARY > 30000$)
- The set of values
  - Lager than 30000 at the attribute SALARY
  - Other values at the remaining attributes that appear in EMPLOYEE
- Safe expression

Content

- Introduction
- Tuple relational calculus
- **Domain relational calculus**

Domain relational calculus

- An expression of the domain calculus is of the form

$$\boxed{\{ x_1, x_2, \ldots, x_n \mid P(x_1, x_2, \ldots, x_n) \}}$$

- $x_1, x_2, \ldots, x_n$ are domain variables
  - Accepting single values from the domain of attributes
- P is a formula of variables $x_1, x_2, \ldots, x_n$
  - P is formed from atoms
- The result
  - The set of values such that when assigned to variables x_i, they make *P* TRUE

Example 1

- Find employees whose salary is larger than 30000

$$\{ r, s \mid \exists x (\langle p, q, r, s, t, u, v, x, y, z \rangle \in EMPLOYEE \land x > 30000) \}$$

Example 3

- Find employees (SSN) who work for the department 'Nghien cuu'

$$
\begin{aligned}
\{\, s \mid \exists z\, (& \\
& \langle p, q, r, s, t, u, v, x, y, z\rangle \in \text{EMPLOYEE} \wedge \\
& \exists a, b\, (\, \langle a, b, c, d\rangle \in \text{DEPARTMENT} \wedge \\
& \quad a = \text{'Nghien cuu'} \wedge b = z\,))\, \}
\end{aligned}
$$

Example 10

- Find employees (SSN, LNAME, FNAME) who have no dependents

$$\{ p, r, s \mid \exists s\ ($$

$$\langle p, q, r, s, t, u, v, x, y, z \rangle \in EMPLOYEE \wedge$$

$$\neg \exists a\ (\langle a, b, c, d, e \rangle \in DEPENDENT \wedge a = s)) \}$$

Atoms

- (i) $\boxed{\langle x_1, x_2, \ldots, x_n \rangle \in R}$
  - x_i is a domain variable
  - R is a relation with *n* attributes

- (ii) $\boxed{x \; \theta \; y}$
  - x, y are domain variables
  - Domains of *x* and *y* are identical
  - θ is comparison operators, eg. $<$, $>$, $\leq$, $\geq$, $\neq$, $=$

- (iii) $\boxed{x \; \theta \; c}$
  - c is a constant
  - x is a domain variable
  - θ is comparison operators, eg. $<$, $>$, $\leq$, $\geq$, $\neq$, $=$

Discussion

- Atoms evaluate to either TRUE or FALSE for a set of values
  - Called the truth values of the atoms

- Rules and transforms are in the similar way to the tuple calculus

Safe expression

- Examine

$$\{ p, r, s \mid \neg (<p, q, r, s, t, u, v, x, y, z> \in EMPLOYEE) \}$$

  - Values in the result do not belong to the domain of the expression
  - Unsafe

Safe expression

- Examine

$$\{\ x \mid \underbrace{\exists y\ (\langle x, y\rangle \in R)}_{\text{Formula 1}} \wedge \underbrace{\exists z\ (\neg \langle x, z\rangle \in R \wedge P(x, z))}_{\text{Formula 2}}\ \}$$

  - *R* is a relation with a finite number of values
  - We also have a finite number of values that does not belong to *R*
  - Formula 1: examine values in *R* only
  - Formula 2: could not validate cause we do not know the finite number of values of variable *z*

Safe expresion

- Expression

$$\{ x_1, x_2, \ldots, x_n \mid P(x_1, x_2, \ldots, x_n) \}$$

is safe if :

- Values that appear in tuples of the expression must belong to the domain of *P*

- ∃ quantifiers: expression ∃x (Q(x)) is TRUE iff
  - Values of *x* belong to DOM(Q) and make *Q(x)* TRUE

- ∀ quantifiers: expression ∀x (Q(x)) is TRUE iff
  - *Q(x)* is TRUE for all values of *x* belonging to DOM(Q)