

# ML tuần 3

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## 1 Problem 1, find parameter w for linear regression

To find parameter  $\theta$  or  $w$  we minimize:

$$P = \frac{\beta}{2} \sum_{n=1}^N \{y(x_n, w) - t_n\}^2$$

We suppose that the linear regression model is as the following:

$$y(x_n, n) = w_1 x_n + w_0$$

$$x = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}; \quad t = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}; \quad w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

Then,

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} w_1 x_1 + w_0 \\ w_2 x_2 + w_0 \\ \vdots \\ w_n x_n + w_0 \end{bmatrix} = x.w$$

$$t - y = \begin{bmatrix} t_1 - y_1 \\ t_2 - y_2 \\ \vdots \\ t_n - y_n \end{bmatrix}$$

$$\begin{aligned} \rightarrow \|t - y\|_2^2 &= (t_1 - y_1)^2 + \cdots + (t_n - y_n)^2 \\ &= \sum (t_i - y_i)^2 = P \end{aligned}$$

$$\rightarrow P = \|t - y\| = \|t - xw\| = (xw - t)^T(xw - t)$$

Take the derivate of P:

$$\frac{\partial(P)}{\partial(w)} = 2x^T(t - xw) = 0$$

$$\leftrightarrow x^t = x^T xw$$

$$\leftrightarrow w = (x^T x)^{-1} x^T . t$$

## 2 Problem 4: Chứng minh $X^T X$ invertible khi $X$ full rank

$X$  full rank  $\Rightarrow$  Ta có  $X$  độc lập tuyến tính

$$\Rightarrow \vec{v}^T X^T X \vec{v} = \vec{v}^T \vec{0} = 0$$

$$\Rightarrow (X \vec{v})^T X \vec{v} = 0$$

$$\Rightarrow (X \vec{v}) \cdot (X \vec{v}) = 0$$

$$\Rightarrow X \vec{v} = \vec{0}$$

Suy ra: Nếu  $\vec{v} \in N(X^T X) \Rightarrow \vec{v} \in N(X)$

$$\Rightarrow \vec{v} \text{ phải bằng } \vec{0}$$

$$\Rightarrow N(X^T X) = N(X) = \{\vec{0}\}$$

$\Rightarrow X^T X$  độc lập tuyến tính và  $X^T X$  là 1 ma trận vuông  $\Rightarrow X^T X$  invertible