| | | | 6 | | | | 7 | 8 | / | | | 7 | 8 | | | | 6 |

1 to 5

↳ $2(n-3)!$ $2 \cdot 5!$

| | | 5 | | 6 | 7 | 8 | / | 6 | 7 | 8 | | | | 5 |

1 to 4

↳ $2(n-4)!$ $2 \cdot 4!$

| 1 | 2 | 3 | 4 | | 5 | 6 | 7 | 8 |

↳ 2

in general, for array size n,

there are

worst case ($n-1$ key cmp in 1 merge)

best case ($\frac{n}{2}$ key cmp in 1 merge)

$$2(n-2)! + 2(n-3)! + \dots + 2\left(n-\frac{n}{2}\right)! + 2$$

$$= 2 + \sum_{i=2}^{\frac{n}{2}} 2(n-i)!$$

$$= 2 + 2 \cdot \sum_{i=2}^{\frac{n}{2}} (n-i)!$$

combinations of ordered subarray, each occurring with equal probability in one merge

probability of best case ($\frac{n}{2}$ key cmp):

$$\frac{2}{2+2\cdot\sum_{i=2}^{\frac{n}{2}}(n-i)!} = \frac{1}{1+\sum_{i=2}^{\frac{n}{2}}(n-i)!} = P\left(\frac{n}{2}\right)$$

probability of each case j (except best case) =

j ← number of key cmp

$$\frac{2(n-j)!}{2+2\sum_{i=2}^{\frac{n}{2}}(n-i)!} = \frac{(n-j)!}{1+\sum_{i=2}^{\frac{n}{2}}(n-i)!} = P(\text{num of key cmp})$$

on average, number of key comparisons in one merge

$$= P\left(\frac{n}{2}\right)\times\frac{n}{2} + P\left(\frac{n}{2}+1\right)\times\left(\frac{n}{2}+1\right) + \dots + P(j)\times j + \dots + P(n-1)\times(n-1)$$