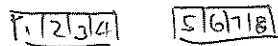


$$\hookrightarrow 2(n-3)! \quad 2 \cdot 5!$$



$$\hookrightarrow 2(n-4)! \quad 2 \cdot 4!$$



$$\hookrightarrow 2$$

in general, for array size  $n$ , there are

$$2(n-2)! + 2(n-3)! + \dots + 2(n-\frac{n}{2})! + 2$$

worst case ( $n-1$  key cmp in 1 merge)      best case ( $\frac{n}{2}$  key cmp in 1 merge)

$$= 2 + \sum_{i=2}^{\frac{n}{2}} 2(n-i)!$$

$$= 2 + 2 \cdot \sum_{i=2}^{\frac{n}{2}} (n-i)!$$

combinations of ordered subarray, each occurring with equal probability in one merge

$$\text{probability of best case: } \frac{2}{2 + 2 \cdot \sum_{i=2}^{\frac{n}{2}} (n-i)!} = \frac{1}{1 + \sum_{i=2}^{\frac{n}{2}} (n-i)!} = P(\frac{n}{2})$$

( $\frac{n}{2}$  key cmp)

$$\text{probability of each case } j = \frac{2(n-j)!}{2 + 2 \cdot \sum_{i=2}^{\frac{n}{2}} (n-i)!} = \frac{(n-j)!}{1 + \sum_{i=2}^{\frac{n}{2}} (n-i)!} = P(\text{num of key cmp})$$

number of key cmp

$$\text{on average, number of key comparisons in one merge} = P(\frac{n}{2}) \times \frac{n}{2} + P(\frac{n}{2}+1) \times (\frac{n}{2}+1) + \dots + P(j) \times j + \dots + P(n-1) \times (n-1)$$