SEIR Covid-19 Model

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May 4, 2021

1 Abstract

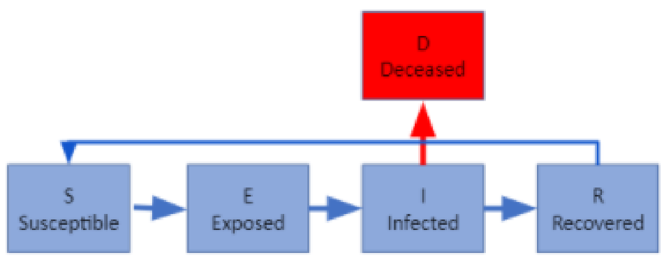
Covid-19 is a wicked problem, which has caused a public health and economic crisis worldwide. In this paper, we employ SEIR(D) model based on the idea in the 20th century by Ross and Hudson (in 1916 and 1917), and Kermack and McKendrick in 1927. With the development of vaccines, we have incorporated a vaccination model, we derive it into vaccination model to see the efficacy of vaccination, and add age-structured compartment to see the impact of Covid-19 vaccine prioritization strategies on cummulative incidence, mortality, and years of life lost.

2 Introduction

1. SEIR(D) Model Equations and rates

An SEIR model is a mathematical model that shows how infectious diseases play out in a specific population. The SEIR model is a compartmental model. The compartments are the susceptible population, S, exposed population, E, infected population, I, and the recovered population, R. Each compartment has their own equations that coupled with the other compartments. We did an SEIR model for covid-19 in Summit county. We also included a death compartment, D, since covid-19 does have a significant fatality rate.

The figure below shows the basic SEIR(D) model for Covid-19. If one person starts off in the susceptible compartment. Some of these people will go into the exposed compartment after being exposed to Covid-19. Those people will then become infected and go into the infected compartment. Infected people have two routes they may go. Either people who got infected go to the deceased compartment or they recover and go to the recovered compartment. After 90 days of a person being recovered they are considered susceptible again because they are able to contract Covid-19 due to the antibodies not being in their system anymore.

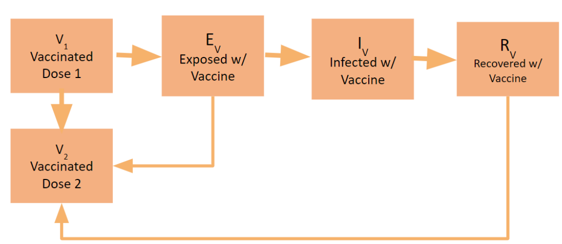


$$\frac{dS}{dt} = -\beta r S I + \nu R$$
$$\frac{dE}{dt} = \beta r S I - \alpha E$$
$$\frac{dI}{dt} = \alpha E - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \nu R$$
$$\frac{dD}{dt} = \mu I$$

The equations above are used to calculate and show the change of the portion of the population that is in each compartment over a 100-day period. The susceptible compartment is represented by $\frac{dS}{dt}$. The equations remove people from this compartment and places them in the exposed compartment based on our fixed infection rate, β, and differing contact rates, r.

2. VEIR Model

This model is an SEIR model but only for the population who has received the vaccine. This model has different compartments since you can still get Covid-19 while the person is waiting to be given their second dose. There are five compartments for this model; the population that has been vaccinated with their first dose, V_1, people received their first dose and then are exposed to someone with Covid-19, E_v, people who received their first dose and then become infected with Covid-19, I_v, those who have recovered from Covid-19 after receiving their first dose, R_v, and the last compartment is the population of people fully vaccinated, V_2

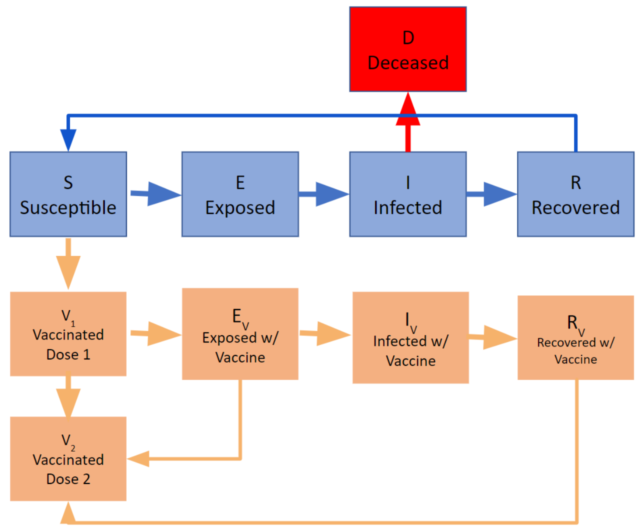


$$
\begin{aligned}
\frac{dV_1}{dt} &= -\eta V_1 - \beta r V_1(1-\xi)(I+I_v) \\
\frac{dE_v}{dt} &= \beta r V_1(1-\xi)(I+I_v) - \alpha E_v - \eta E_v \\
\frac{dI_v}{dt} &= \alpha E_v - \gamma_v I_v \\
\frac{dR_v}{dt} &= \gamma_v I_v - \eta R_v \\
\frac{dV_2}{dt} &= \eta V_1 + \eta E_v + \eta R_v
\end{aligned}
$$

When vaccines begin to rollout, any person who is able to receive their vaccine will be placed into V1. Ohio set up a system where healthcare and frontline workers first received the vaccine, then they allowed older people and people with certain health conditions to receive their vaccine next, and then the younger age groups were allowed to receive the vaccine. We later on implement this into our model by adding age groups and each age group has a different vaccination rate. This current model and set of equations has a constant vaccination rate, 0.01, for the susceptible population. We note that only people who are considered susceptible are receiving the vaccine. We will later combine these equations with the SEIR(D) equations, this will make more sense.

3. Combined SEIR and VEIR Model

As a result, if we combine SEIR and VEIR models, we can clearly see how the result changes since we have the vaccines involved. The following figure shows the complete SEIR with vaccination model and how these parameters related to each other



How did we approach to combine these two models above? We know that there is a chance people in susceptible category will get the vaccine, meanwhile in the latent period, people can also get vaccine. In order to protect ourselves from getting covid-19 again people from recovery category can get vaccine. However, for this model we assume there s minimum 20% of population in Summit County that hesitates vaccines. If the total number of people who can get vaccinated in the susceptible, exposed, recovered compartments are less than 20% of population in Summit County, we set the vaccination rate is 0. Otherwise, we set it to an array which is set up as below.

As a result, the complete model will be:

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta r S(I+I_v) - \nu R - -v_0 \frac{S}{S+E+R} \\
\frac{dE}{dt} &= \beta r S(I+I_v) - \alpha E - -v_0 \frac{E}{S+E+R} \\
\frac{dI}{dt} &= \alpha E - \gamma I - \mu I \\
\frac{dR}{dt} &= \gamma I - v_0 \frac{R}{S+E+R} - \nu R \\
\frac{dD}{dt} &= \mu I \\
\frac{dV1}{dt} &= v_0 - \eta V_1 - \beta r V_1(1-\xi)(I+I_v) \\
\frac{dE_v}{dt} &= \beta r V_1(1-\xi)(I+I_v) - \alpha E_v - \eta E_v \\
\frac{dI_v}{dt} &= \alpha E_v - \gamma_v I_v \\
\frac{dR_v}{dt} &= \gamma_v I_v - \eta R_v \\
\frac{dV_2}{dt} &= \eta V_1 + \eta E_v + \eta R_v
\end{aligned}
$$

The equations above are the combination and interaction of the SEIR(D) model and the VEIR model. The dS/dt equation represents the susceptible compartment. This compartment starts off with a set number where the majority of the population is considered susceptible. The number changes based on a few variables. The number decreases in this compartment based on the amount of people who become exposed and the people who receive their first dose of the vaccine. When people in the susceptible group begin receiving their vaccine they move to V1, which changes based on dV1/dt.

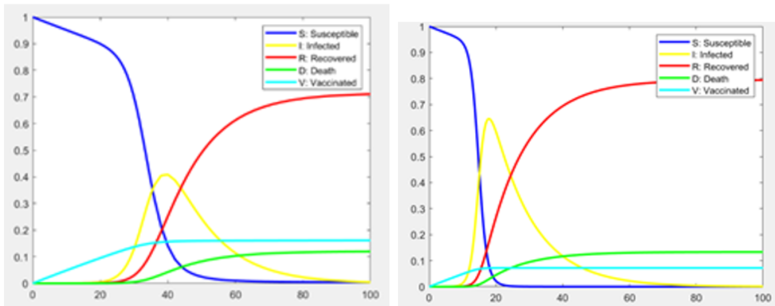


Figure 1: The two figures above show the outcome of two different contact rates where both graphs have a vaccination rate set to 0.5% of the susceptible population being vaccinated per day. The graph on the left has a contact rate of 50 contacts per person per day. The graph on the right has a contact rate of 100 contacts per person per day. When looking at the graphs we see that each line represents a different compartment, susceptible, infected, recovered, dead, and vaccinated. If we focus on the yellow line, the infected compartment, we see that in the right graph, 100 contacts a day, the peak of the proportion of the population who will become infected is over 0.6, 60%. When we compare this to the left graph, 50 contacts a day, we see that the peak of the population that becomes infected is around 0.4, 40%. From this we can see that the proportion of the population that will die is higher in the right graph, the graph with more contacts, than the left graph. From this we can say that when the population of susceptible people is receiving vaccines at a rate of 0.5% per day, we should still be social distancing. Now, let's look at what happens when we double the vaccination rate to 1%.

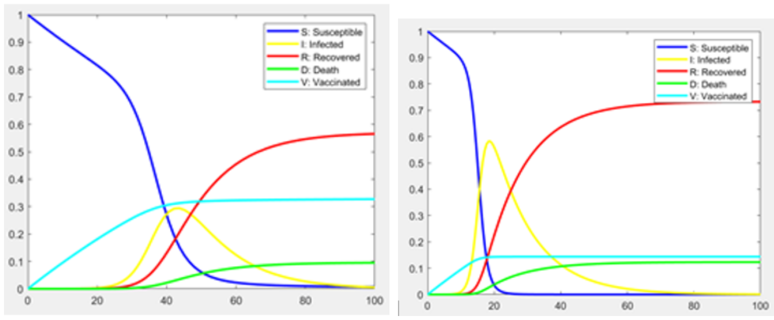
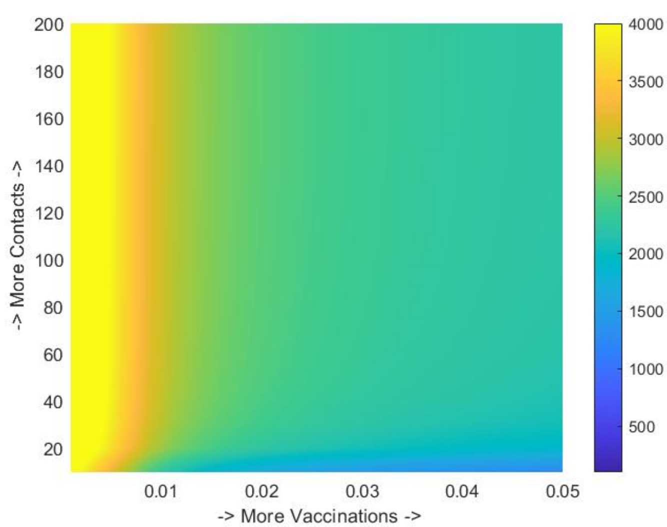


Figure 2: These graphs are similar to the two above them but the difference is we changed the vaccination rate from 0.5% being vaccinated per day to 1% being vaccinated per day. The left graph still has a contact rate of 50 contacts per day and the right graph has a contact rate of 100 per day. First, let's look at the left graph. We see that when people come into contact with 50 people a day when the susceptible population is being vaccinated at a rate of 1% a day, a larger portion of the population is vaccinated than the portion of people who become infected. This is important because people are not dying from the vaccines, but people are dying from covid-19. Even though people become immune to covid-19 for about 90 days after going into the recovered compartment, it is safer for people to be immune because of the vaccine and we want more people vaccinated than we want people becoming infected.

Let's compare these two graphs now. We know that the right graph has double the amount of contacts per day than the left graph, we see that the peak of the portion of the population that becomes infected is about 30% of the population in the left graph and about 60%. The portion of the population that will become infected doubles. Now, let's compare the graphs that both have a contact rate of 100 contacts per day, or the right graphs.

The top right graph has a vaccination rate of 0.5% per day and the bottom right graph has a vaccination rate of 1% per day. These graphs are almost identical. The peak of the portion of the population that will become infected is around 60% in the graph with a vaccination rate of 0.5% a day and around 60% for the graph with a vaccination rate of 1%. This means that even though we doubled the portion of the susceptible population that are being vaccinated a day, the amount of infected people almost stays the same. Thus, the portion of the population that will die will be nearly the same. Now, when we compare the two graphs that are set to 50 contacts per day, we see that the peak of the portion of the population that will become infected goes from around 40% for a vaccination rate of 0.5% per day to about 30% for a vaccination rate of 1% a day. We can say that when we are social distancing and increasing the vaccination rate, less people will be infected. We know that only so many people can be vaccinated a day, so from that statement, we can say that until a large enough portion of the population is immune to covid-19, social distancing still needs to be enforced.

Using the idea of changing contact rates and vaccination rates we can look at this model we can see how many people in Summit County would get deceased after 100 days:



In here we try all possible contact rates from 10 to 200, incubation period is 8 days, vaccine rates from 0.001 to 0.05 with efficacy is 90 percent, 10 percent of getting reinfected, and we assume that recovery rate after vaccination is 1/7.

This result shows that if less than 1% of the population is vaccinated, then no matter how many contacts we make per day, there will be possibly more than 3000 people will get infected and die. However, this result changes as more people are vaccinated. If people social distance and have 20 or less contacts per day, the amount of people dead will lower down to approximately 1500-2000 deaths. We also see that even when the portion of people vaccinated increases, if people are not social distancing and coming into contact with more than 20 people per day, the amount of people that will die stays the same as the portion of the population that gets vaccinated increases.

4. Age structured Idea

We know from the news that age plays a major role in how severe covid-19 side effects can be. We see that the majority of deaths and hospitalizations of people with covid-19 are older people and people who have compromised immune systems. We also know the amount of contacts a person comes into contact with has a significant impact on infection rates. We seperated people into 9 age groups: 0-9, 10-19, 20-29, 30-39, 40-49, 50-59, 60-69, 70-79, and 80+. The contact parameter, r, was a 9x9 matrix, consisting of parameters r for the contact each age group has with another on a given time interval. The matrix r for this age stratified SEIR(D) model is shown below.

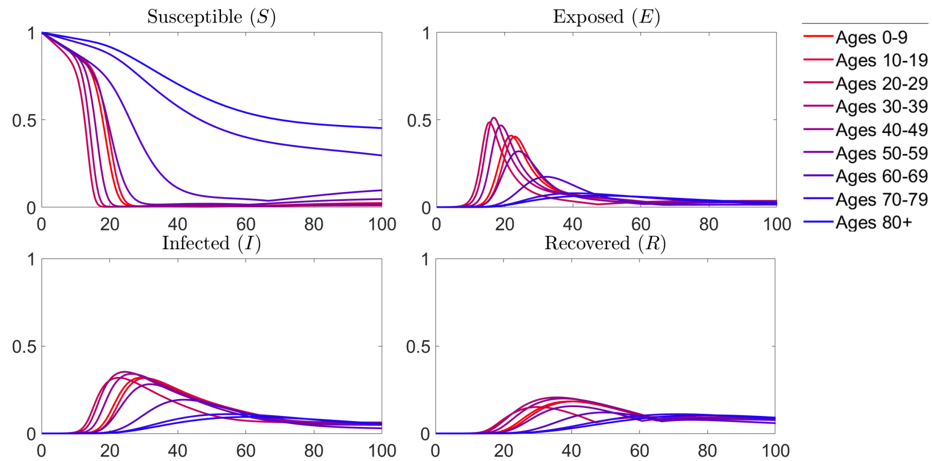
$$
r = \begin{bmatrix}
40 & 20 & 3 & 15 & 10 & 5 & 3 & 2 & 1 \\
20 & 40 & 5 & 5 & 10 & 15 & 5 & 3 & 1 \\
3 & 5 & 60 & 30 & 10 & 3 & 2 & 1 & 1 \\
15 & 5 & 30 & 50 & 30 & 10 & 3 & 2 & 1 \\
10 & 10 & 10 & 30 & 40 & 15 & 2 & 2 & 3 \\
5 & 15 & 3 & 10 & 15 & 20 & 10 & 3 & 2 \\
3 & 5 & 2 & 3 & 2 & 10 & 15 & 2 & 1 \\
2 & 3 & 1 & 2 & 2 & 3 & 2 & 10 & 10 \\
1 & 1 & 1 & 1 & 3 & 2 & 1 & 10 & 10
\end{bmatrix}
$$

Here, i-th column/row represents interaction between each age group, for example, first column/row shows the daily contact that people in the same age group from 0-9 have with each others, and first column with second column means the daily contact that people in age group from 0-9 have with people in age group from 10-19.

a. Adding Age-structured idea to SEIR model

$$
\begin{aligned}
\frac{dS_i}{dt} &= -\beta(r_{i1}\frac{I_1}{N_1} + r_{i2}\frac{I_2}{N_2} + ... + r_{i9}\frac{I_9}{N_9})S_i + \nu R_i \\
\frac{dE_i}{dt} &= \beta(r_{i1}\frac{I_1}{N_1} + r_{i2}\frac{I_2}{N_2} + ... + r_{i9}\frac{I_9}{N_9})S_i - \alpha E_i \\
\frac{dI_i}{dt} &= \alpha E_i - \gamma I_i - \mu I_i \\
\frac{dR_i}{dt} &= \gamma I_i - \nu R_i \\
\frac{dD_i}{dt} &= \mu I_i
\end{aligned}
$$

In this equation, if we change index of i from 1 to 9 we can get the SEIR(D) equation correspoding to each age group. After changing the index i, we will get the result as following:

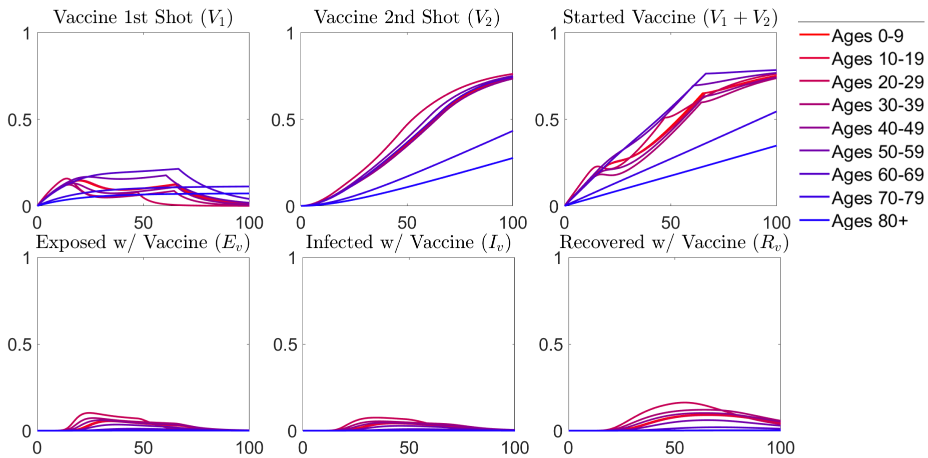


This model can be adjusted so that populations strictly adhere to social distancing guidelines, populations that somewhat follow these guidelines, and populations where only certain age groups follow directives. A large problem across the country, many younger people are ignoring the calls for social distancing and neglecting stay-at-home orders, so a model can be developed such that this is applied.

b. Adding Age-structured idea to VEIR model

$$\begin{aligned}
\frac{dV_{1_i}}{dt} &= v_0 - \eta V_{1_i} - \beta(r_{i1}\frac{I_{v_1}}{N_1} + r_{i2}\frac{I_{v_2}}{N_2} + ... + r_{i9}\frac{I_{v_9}}{N_9})V_i(1-\xi) \\
\frac{dE_{v_i}}{dt} &= \beta(r_{i1}\frac{I_{v_1}}{N_1} + r_{i2}\frac{I_{v_2}}{N_2} + ... + r_{i9}\frac{I_{v_9}}{N_9})V_i(1-\xi) - \alpha E_{v_i} - \eta E_{v_i} \\
\frac{dI_{v_i}}{dt} &= \alpha E_{v_i} - \gamma_v I_{v_i} \\
\frac{dR_{v_i}}{dt} &= \gamma_v I_{v_i} - \eta R_{v_i} \\
\frac{dV_{2_i}}{dt} &= \eta V_{1_i} + \eta E_{v_i} + \eta R_{v_i}
\end{aligned}$$

In this equation, if we change index of i from 1 to 9 we can see how important vaccines are to each age groups and to answer the question "After getting vaccinated, do we need to follow the social distancing guidelines anymore?". After changig the index i, we will get the result as following:



For the first three graphs, we can see older people more likely to only have half vaccination because they follow the social distancing guidelines accordingly, and they have less years to lose which we will talk later in this paper. However, if we look at younger age groups in the bottom three graphs, since they were ignoring the calls for social distancing and neglecting stay-at-home orders, so even though they are fully vaccinated, there is still a small possibility where they can get reinfected.

b. Complete Age-structure model

$$
\begin{aligned}
\frac{dS_i}{dt} &= \beta(r_{i1}\frac{I_1}{N_1} + r_{i2}\frac{I_2}{N_2} + ... + r_{i9}\frac{I_9}{N_9})S_i + \nu R_i - v_0 N_i \frac{S_i}{S_i + E_i + R_i} \\
\frac{dE_i}{dt} &= \beta(r_{i1}\frac{I_1}{N_1} + r_{i2}\frac{I_2}{N_2} + ... + r_{i9}\frac{I_9}{N_9})S_i - \alpha E_i - v_0 N_i \frac{E_i}{S_i + E_i + R_i} \\
\frac{dI_i}{dt} &= \alpha E_i - \gamma I_i - \mu I_i \\
\frac{dR_i}{dt} &= \gamma I_i - \nu R_i - v_0 N_i \frac{R_i}{S_i + E_i + R_i} \\
\frac{dD_i}{dt} &= \mu I_i \\
\frac{dV_{1_i}}{dt} &= v_0 - \eta V_{1_i} - \beta(r_{i1}\frac{I_{v_1}}{N_1} + r_{i2}\frac{I_{v_2}}{N_2} + ... + r_{i9}\frac{I_{v_9}}{N_9})V_i(1-\xi) \\
\frac{dE_{v_i}}{dt} &= \beta(r_{i1}\frac{I_{v_1}}{N_1} + r_{i2}\frac{I_{v_2}}{N_2} + ... + r_{i9}\frac{I_{v_9}}{N_9})V_i(1-\xi) - \alpha E_{v_i} - \eta E_{v_i} \\
\frac{dI_{v_i}}{dt} &= \alpha E_{v_i} - \gamma_v I_{v_i} \\
\frac{dR_{v_i}}{dt} &= \gamma_v I_{v_i} - \eta R_{v_i} \\
\frac{dV_{2_i}}{dt} &= \eta V_{1_i} + \eta E_{v_i} + \eta R_{v_i}
\end{aligned}
$$

These equations are used to show the changes for each compartment with respect to each age group. These equations account for the SEIR model with vaccinations and show the difference in each age group. We use the contact matrix to differentiate each age group. These age groups are defined with the equations with i, where i ranges from 1 to 9. The graphs above show how the different age groups change throughout each compartment. From the graphs above we see that as the older age groups are being vaccinated the portion of people who are getting infected are mostly from the younger age groups. Since we know that covid-19 symptoms are more severe for the older age groups, it is more important for the older age groups to get vaccinated and avoid infection. This being said, we can safely say that as the older age groups are getting vaccinated at the same rate as the other age groups, their infection rate goes down much faster than the other age groups as you can see from the infected graph above. If we prioritize the older age groups to receive their vaccines, we will be able to decrease the mortality rate significantly.

The following result shows life lost throughout 100 days to see which age group has priority to get vaccine first:

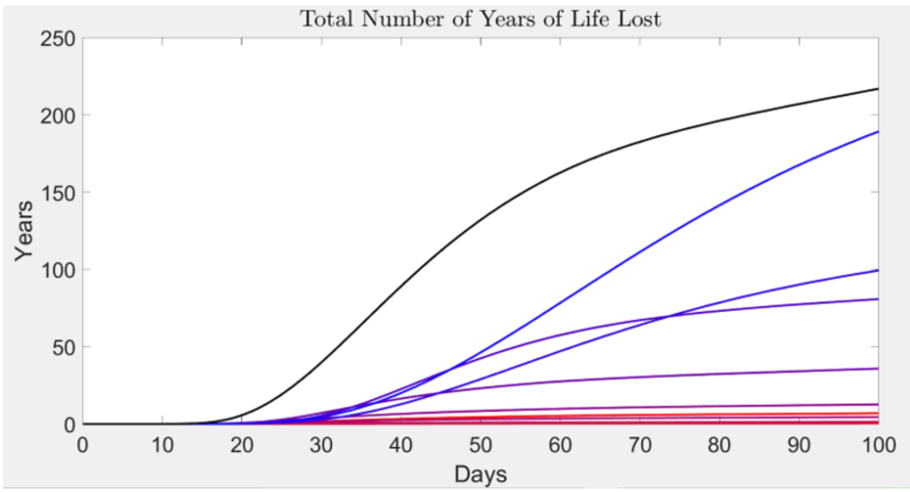


Figure 3: Original Contact Matrix - The figure above shows the number of years of life that will be lost over a 100 day period for each age group. The black line is the combined total number of years lost for all people. To enhance the graph, we divided the number of years by 10. As you can see, from our current contact matrix around 2250 years will be lost in just 100 days of the pandemic.

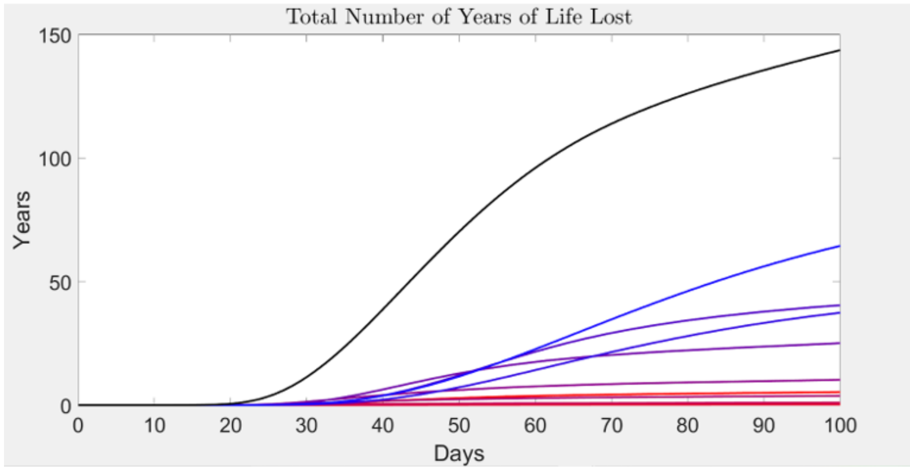


Figure 4: 75% of the contacts - This figure shows how the number of years of life lost when we lower the contact rates in our matrix by 25%. From the figure before, we see that there will be around 2250 deaths after 100 days and this plot shows us that the number drops to less than 1500 years of life lost. If we round this number up to 1500 years and divide that by 2250, we see that the number of years drops by approximately 34% when we drop the contacts by only 25%. Let's see what happens when we lower contacts by 50%.

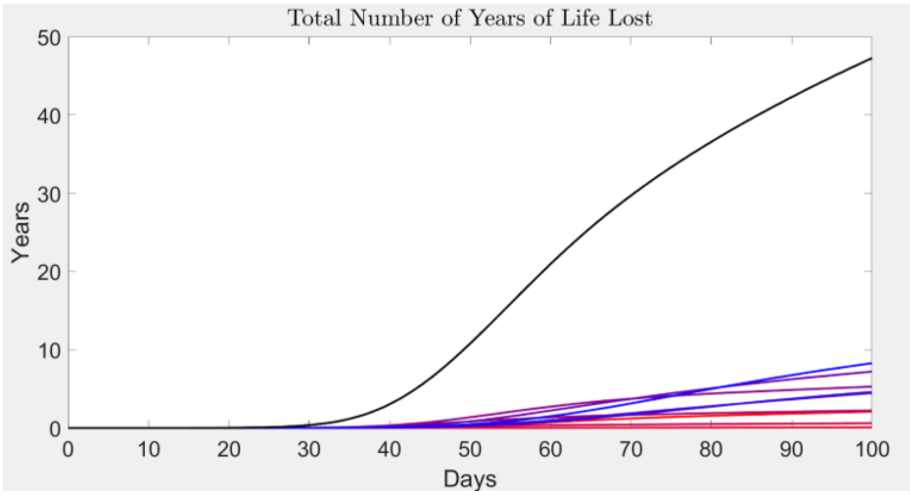


Figure 5: 50% of the contacts - This figure shows the number of years of life lost over 100 days when we lower the contact rates in our matrix by 50%. The number of years is now less than 500. If we round this number up to 500 years and divide that by 2250, we see that the number of years drops by approximately 78%. If we implement strict social distancing and lower contact rates by 50%, we can save at least 1750 years of life over a 100 day period. Imagine how many years that we could save in a whole year, 365 days. Further supporting social distancing during a pandemic.

Therefore, more days pass, there are more deaths for higher age groups. Which means getting vaccines in age-structured shows that the strategy to give out vaccines should prioritize higher age groups first, younger after.

3 Conclusion

In conclusion, we have shown how implementing vaccinations and lowering a person's contacts works in lowering death and infection rates. If we had lowered contact rates by implementing stronger social distancing rules and restrictions during the peak of the pandemic, we would have a much smaller amount of deaths. There is also the issue that people still get infected even when vaccines are rolling out. There are still many people who have not been vaccinated yet. We should keep restrictions until the country has reached some sort of herd immunity.

We note that the older groups of people have a higher mortality rate compared to the younger groups. This proves the idea of allowing the older generation to get their vaccines before the other age groups would be the most beneficial when trying to lower the mortality rate. We can also conclude that social distancing should be implemented the strongest for the older age groups, but from our contact matrix, we see that the older age groups come into contact with all other age groups. The older groups may be spreading to each other but, the younger age groups are also infecting them, further justifying that everyone should be social distancing and getting vaccinated.

Supplemetary Tables

Parameter	Description	Value
ν	re-infectivity rate	0.05
α	incubation period after exposure	0.125
γ	length of recovery	$\frac{1}{14}$
β	infectivity rate	0.01
μ	death rate	[0.00001, 0.00003, 0.0001, 0.0004, 0.0012, 0.004, 0.0136, 0.0455, 0.1524] * $\frac{1}{30}$
v_0	vaccination rate	0.1
η	duration between first vaccine dose and second vaccine dose rate	21 days
γ_v	recovery rate from infection after vaccination	$\frac{1}{7}$
ξ	efficacy of vaccine	0.9
N_i	Number of people in age group i	[24462, 23888, 33574, 24592, 24110, 26129, 23749, 10877, 6933]

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