Estimating Molecular Cloud Mass Using Carbon Monoxide Emission Lines

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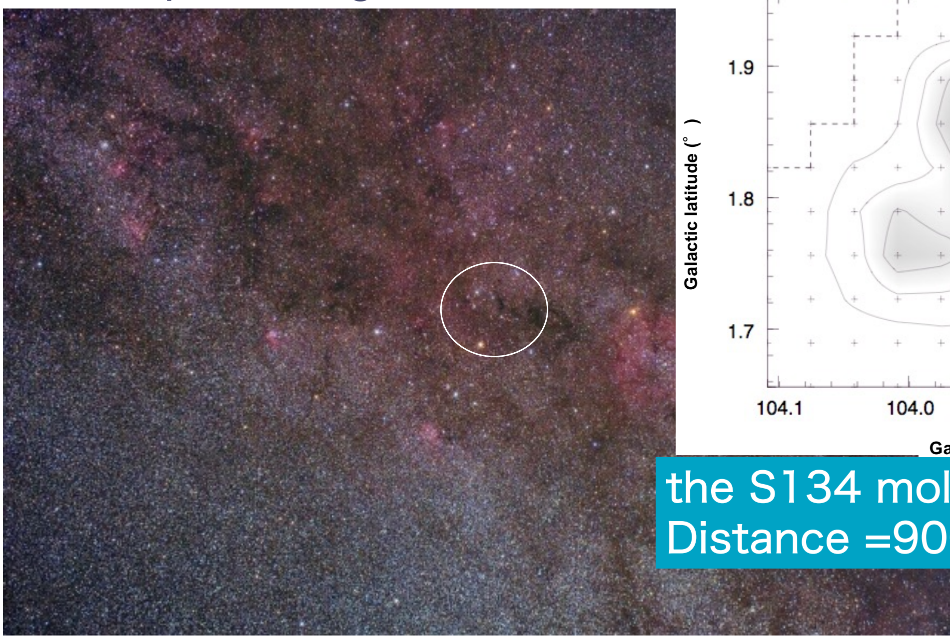
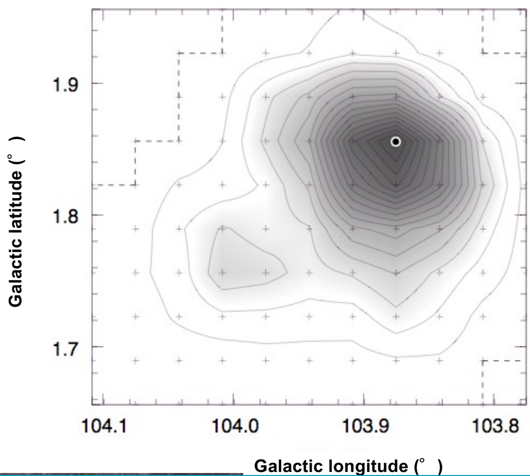
S134
the Cepheus region





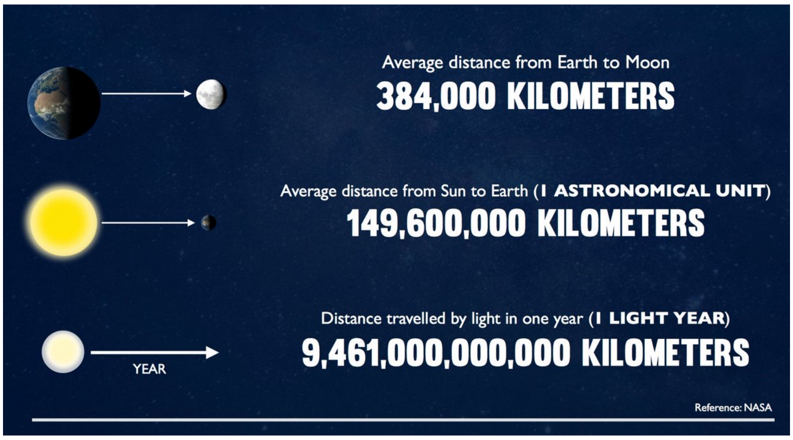
the S134 molecular cloud
Distance =900 parsec

https://orio.blog/

How far is a light-year?

- A light-year is the distance that light travels in one year.
- One light-year is approximately 9.46 trillion kilometers.

9.46×10^{12} km

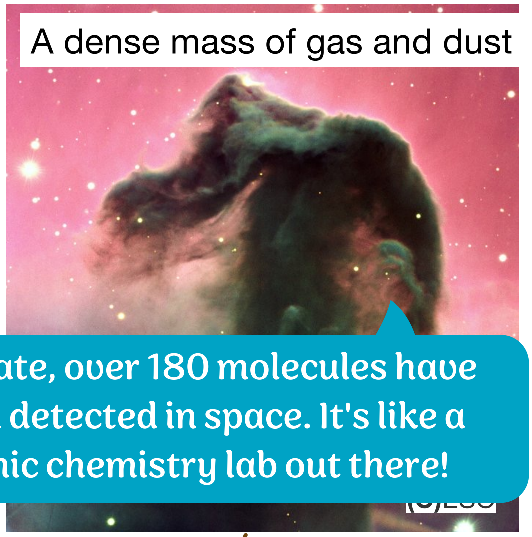


Molecular Cloud

very cold!

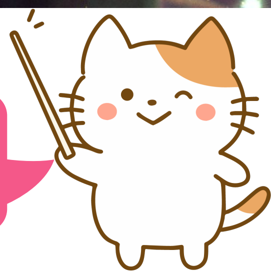
- Temperatures:
  10 K (-263°C)
- Principal components: H_2, He and other molecules
- Densities: $n > 10^{2\text{-}3}$ cm^{-3}
- Size: ~30 light-years

A dense mass of gas and dust



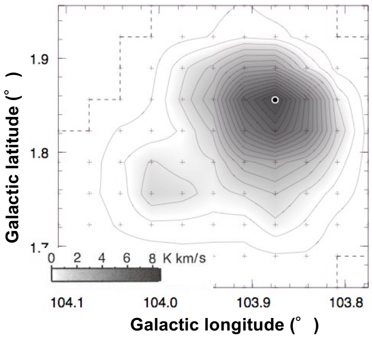
To date, over 180 molecules have been detected in space. It's like a cosmic chemistry lab out there!

The gas is in the molecular state, hence the term 'molecular cloud'



this cloud weighs…

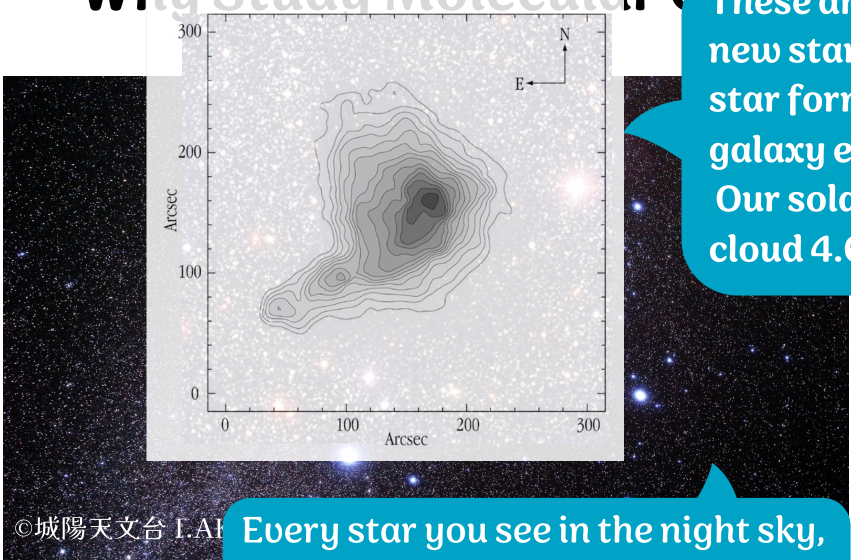
- Less than our Sun
- About the same as our Sun
- 10 times our Sun?
- 100 times our Sun
- More than 1000 times our Sun



Reference points:
Earth mass = 6×10^{24} kg
Sun mass = 2×10^{30} kg = 1 $M_\odot$
Typical molecular cloud: 10-1000 $M_\odot$

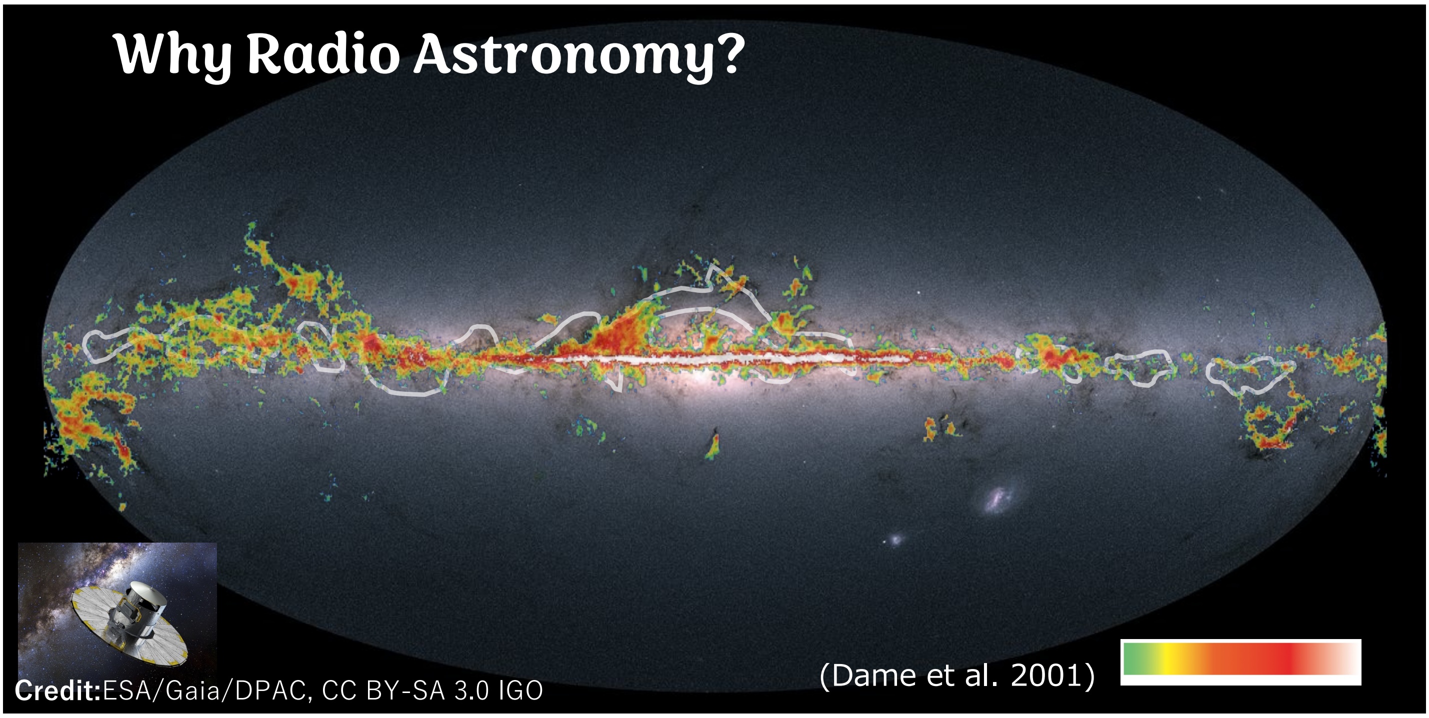
The ⊙ symbol represents the Sun

Why Study Molecular C



These are stellar nurseries - where new stars are born - Understanding star formation helps us understand galaxy evolution
Our solar system formed in such a cloud 4.6 billion years ago

Every star you see in the night sky, including our Sun, was born inside a cloud just like S134.



Hydrogen is the most abundant molecule in molecular clouds.

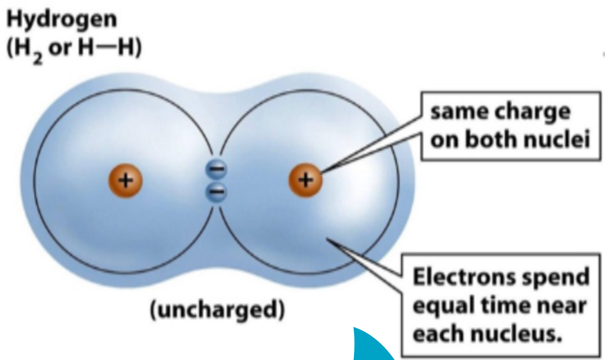
The H_2 Problem

- Most of the cloud is H_2 molecules (99%). But H_2 is "invisible" to radio telescopes
- Why? H_2 is perfectly symmetrical

↓

No radio emission we can detect

Like a perfectly balanced spinning top



A dipole moment occurs when there's an uneven distribution of charge in a molecule, which isn't the case for H_2.

https://www.chemtopper.com/myblog/wp-content/uploads/2019/07/1-2.png

Carbon monoxide Molecule

CO is the second-most abundant molecule in molecular clouds after H_2.

✓CO tends to be present wherever H_2 is found and is often used to trace molecular hydrogen.

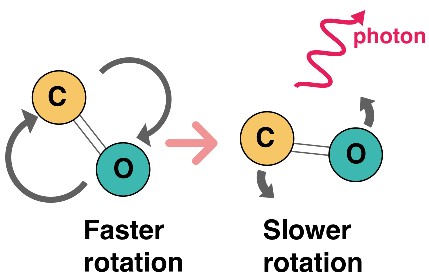
Carbon monoxide Molecule

✓CO has a permanent dipole moment, which allows for strong radio emission.

It means the molecule has a slight electrical imbalance

✓CO gets excited easily, even at cold temperatures

✓CO is chemically stable - survives in harsh space



CO molecules emit photons at millimeter wavelengths when they change rotational states.

The Two-CO Strategy
^{12}CO vs ^{13}CO Explained

^{12}C and ^{13}C are isotopes of carbon

- ^{12}CO - that's regular carbon combined with oxygen
- ^{13}CO - that's heavier carbon (with an extra neutron) combined with oxygen
- $^{13}C/^{12}C$ ratio is 1:90

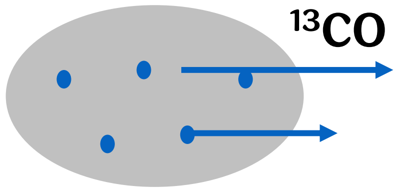
Abundance:
This ratio means that for every 91 carbon atoms in a typical molecular cloud, about 90 will be ^{12}C and only 1 will be ^{13}C.

Optical Depth τ

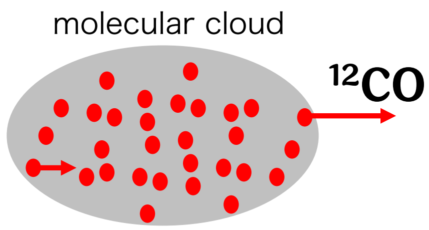
Optical depth measures how opaque or transparent a medium is to radiation.

- The fog represents the medium that radiation is passing through
- The car's headlights are our radiation source
- The slight dimming we see is the effect of optical depth

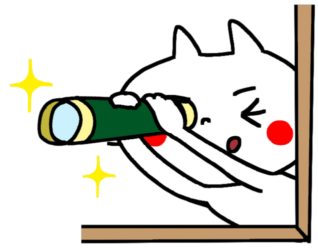
^{12}CO is typically optically thick, ^{13}CO is optically thin



from inside the molecular clouds

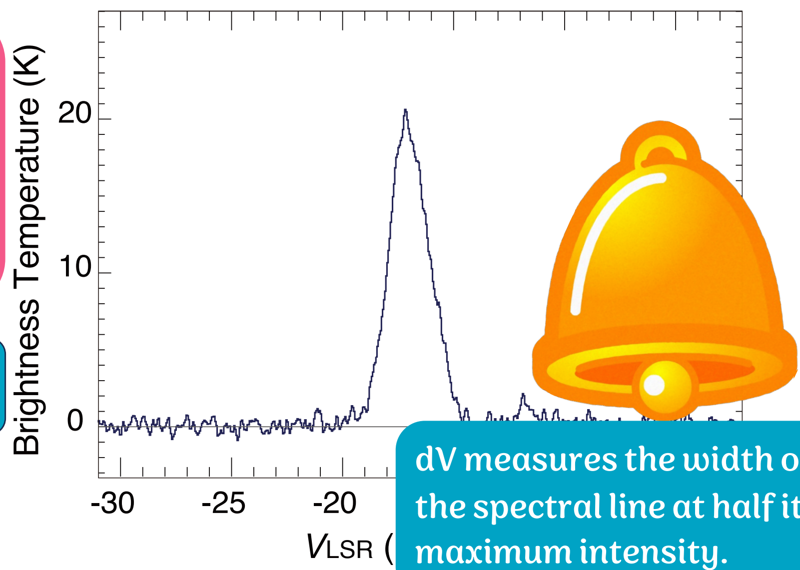


from the outer regions of molecular clouds



The radio intensity is proportional to the temperature of the gas in the molecular cloud.

the Rayleigh-Jeans law



dV measures the width of the spectral line at half its maximum intensity.

Doppler effect

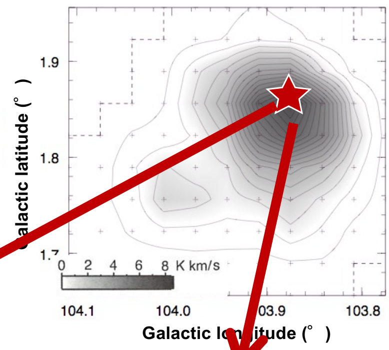
Measurement Challenge

Mission: Find these 3 numbers from your worksheet

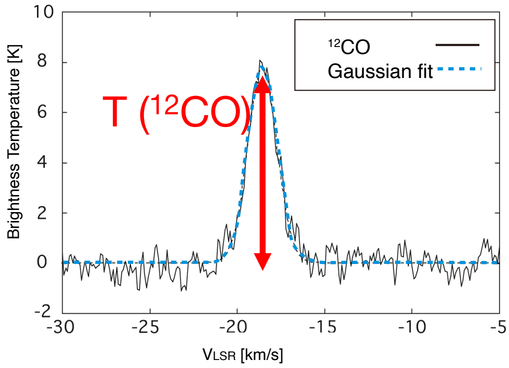
T(^{12}CO) = Peak height of Figure 1

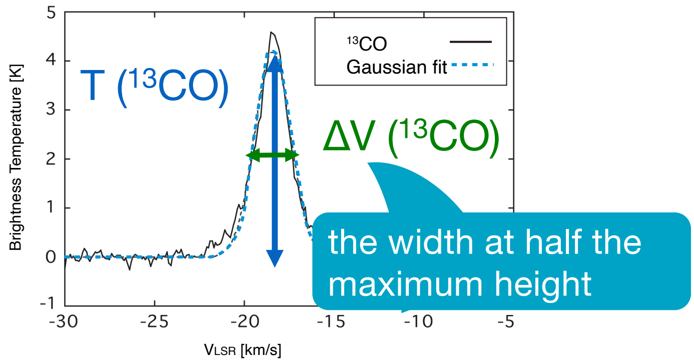
T(^{13}CO) = Peak height of Figure 2

ΔV(^{13}CO) = Width of Figure 2 at half maximum



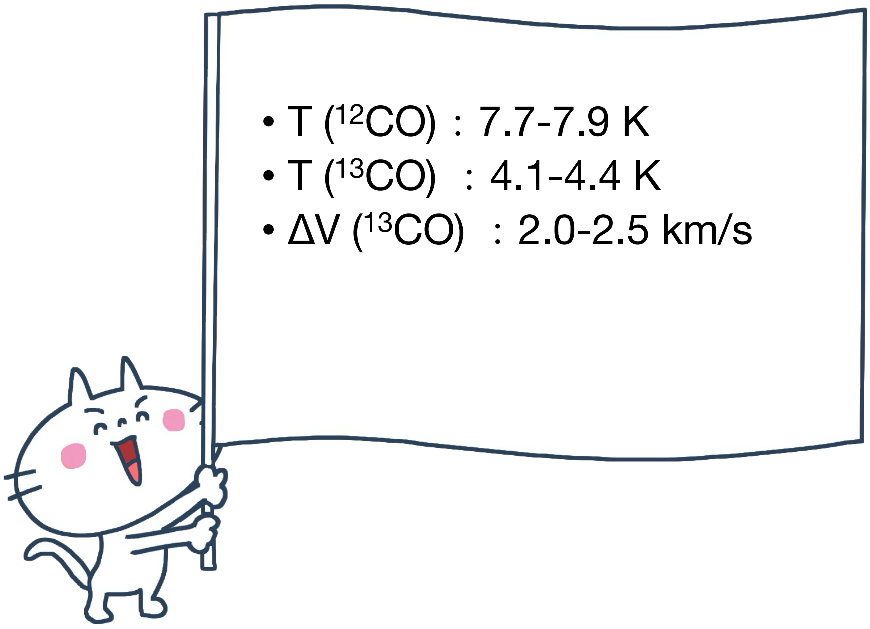
Peak position





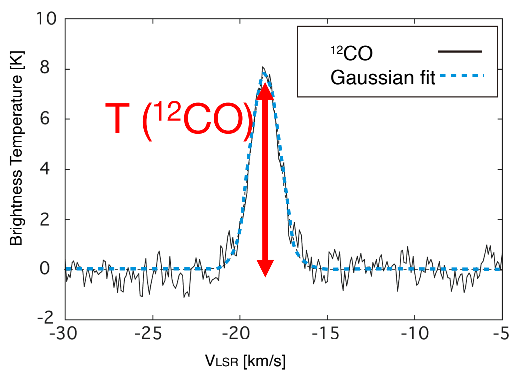
Let's check our measurements!

- T (^{12}CO) : 7.7-7.9 K
- T (^{13}CO) : 4.1-4.4 K
- ΔV (^{13}CO) : 2.0-2.5 km/s



• T(^{12}CO)
→this will give us the real gas temperature
• T(^{13}CO) and ΔV(^{13}CO)
→ these will help us measure optical depth and figure out how many molecules are actually there

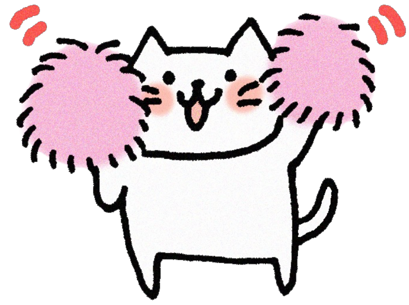
excitation temperature

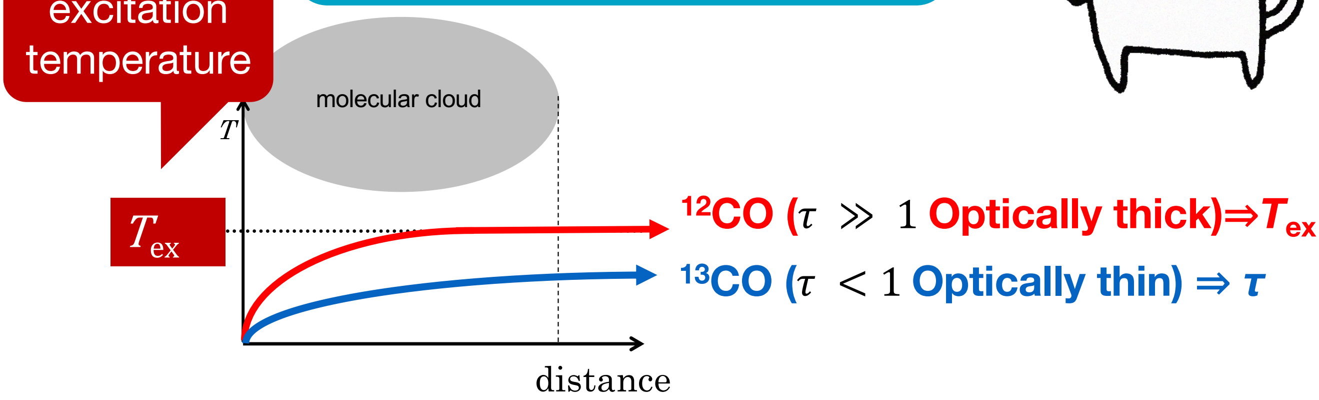


The temperature we measured - T(^{12}CO) - is what we call 'brightness temperature.'

It's related to the real gas temperature, but they're not the same thing.

Excitation temperature is a measure that helps us understand how energetic the molecules are, rather than their physical temperature.





Calculate the excitation temperature Tex

This formula inverts the radiative transfer equation

$$T_{ex} = \frac{5.53}{\ln\left[1 + \frac{5.53}{T(^{12}\mathrm{CO}) + 0.819}\right]}$$

"ln" stands for "natural logarithm" or "Naperian logarithm".

5.53 K = h ν /k for CO J=1→0 transition

0.819 K = cosmic microwave background correction

- ✓ T_{ex} is the excitation temperature we're solving for
- ✓ $T(^{12}\mathrm{CO})$ is the brightness temperature of ^{12}CO that we measured
- ✓ 5.5 and 0.82 are constants related to the properties of the CO molecule and the cosmic microwave background

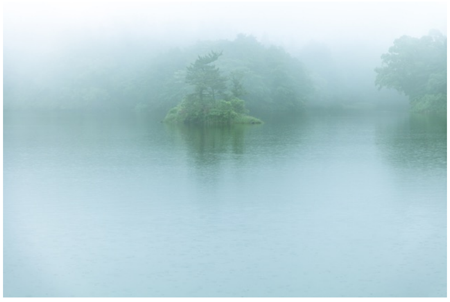
Example

- $T(^{12}\mathrm{CO})$ =7.7 K
- $T_{\mathrm{ex}} = \dfrac{5.53}{\ln\left[1+\frac{5.53}{8+0.819}\right]}$ ≈ 11.1 K (11.0-11.2)

Result: 11K = -262°C! That's 20 times colder than Antarctica!
Molecular clouds are among the coldest places in the universe.

If I shine a flashlight through this cloud, how much light gets through?

- τ = 0.1: very transparent (90% passes through)
- τ = 1: moderately thick (37% passes through)
- τ = 3: very thick (5% passes through)



※ τ is a dimensionless quantity.

How transparent is our cloud to ^{13}CO radiation?

Calculate the optical depth of the ^{13}CO.

$$\tau(^{13}\mathrm{CO}) = -\ln\left\{1 - \frac{T(^{13}\mathrm{CO})}{J(T_{\mathrm{ex}}) - 0.868}\right\}$$

Fundamental concept in radiative transfer
$\tau = \int \alpha \, dl$ (integral of absorption coefficient along path)
Physically: how many 'absorption lengths' we look through"

★Background radiation correction

$$J(T_{\mathrm{ex}}) = \frac{h\nu/k}{\exp(h\nu/kT_{\mathrm{ex}}) - 1}$$

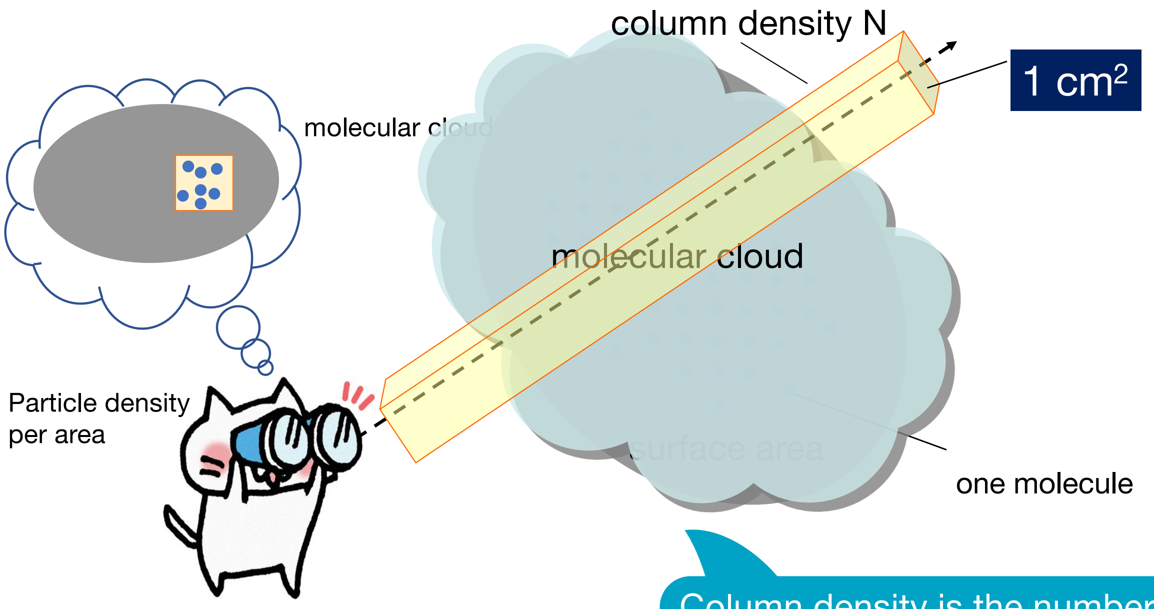
J(T), is a concept used in radio astronomy to describe the intensity of radiation in terms of temperature.

h (Planck's constant) = 6.626×10^{-34} J·s, k (Boltzmann constant) = 1.380649×10^{-23} J/K,
ν (frequency of ^{13}CO J=1-0 transition) ≈ 1.1020137×10^{11} Hz

Example

- T_{ex} = 11.1 K and $T(^{13}\mathrm{CO})$ =4.2 K
- $J(T_{\mathrm{ex}})$ = hν/k / [exp(hν/kTex) - 1]
  = 5.29 / [exp(5.29/11.1) - 1] ≈ 8.67 K
- $\tau(^{13}\mathrm{CO}) \approx 0.77$ (0.74-0.84)

τ = 0.77 means 'moderately transparent'
When light travels through material, it gets weaker at a rate proportional to how bright it currently is.
With τ = 0.77, the transmittance is e^(-0.77) ≈ 0.46
We see through about 46% of the cloud thickness!



Column density is the number of molecules per unit area along our line of sight through the cloud.

Calculate the column density of ^{13}CO, $N(13\mathrm{CO})$

2.52×10^{14}: Conversion factor from radiative transfer theory

$$\boxed{N(^{13}\mathrm{CO}) = \frac{2.52\times10^{14}\,\tau(^{13}\mathrm{CO})\,\Delta V(^{13}\mathrm{CO})\,T_{\mathrm{ex}}}{1-\exp(-5.29/T_{\mathrm{ex}})}}$$

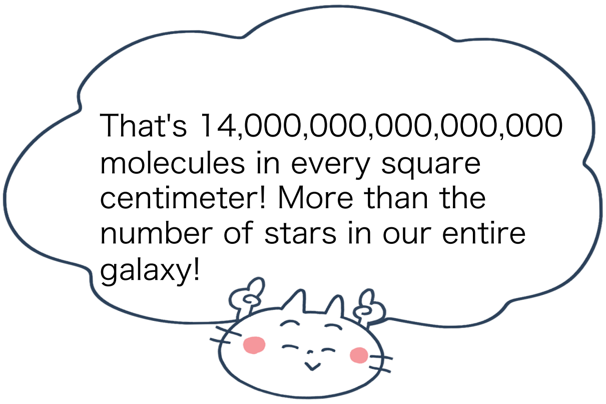
[cm^{-2}]

- ✓ N(^{13}CO) is the column density we're solving for
- ✓ τ is the optical depth of ^{13}CO
- ✓ ΔV :Velocity range of ^{13}CO in km/s
- ✓ T_{ex} is the excitation temperature in Kelvin

Example

- T_{ex} = 11.1 K, $\tau(^{13}\mathrm{CO})$ =0.77, and $\Delta V(^{13}\mathrm{CO})$ = 2.4 km/s
- $N(^{13}\mathrm{CO})$

$$= 2.52\times10^{14}\times0.77\times2.4\times11.1/\ [1-\exp(-5.29/11.1)]$$
$$\approx 1.4\times\ 10^{16}\mathrm{cm}^{-2}$$



Convert to hydrogen molecules

$$\boxed{N(\mathrm{H_2}) = 5.0\times10^{5}N(^{13}\mathrm{CO})}\quad [\mathrm{cm}^{-2}]$$

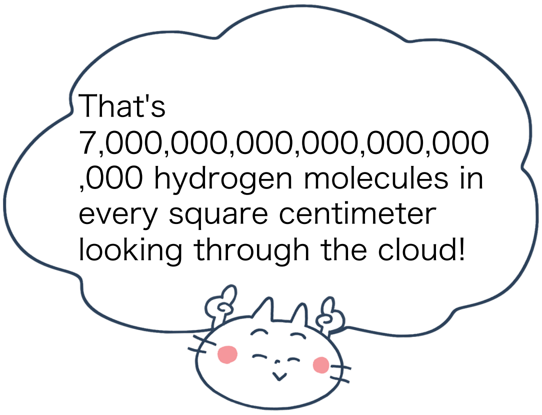
Dickman (1978)
Bohlin et al.(1978)

✓This N(H_2) represents the total hydrogen both molecular and atomic, converted to equivalent H_2 molecules.
✓This equation is based on statistical studies of many molecular clouds.
✓It provides a reliable estimate but remember it's an average relationship.

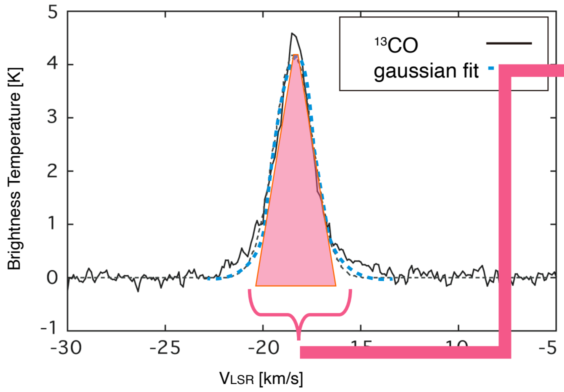
Example

- $N(^{13}\mathrm{CO}) \approx 1.36\times 10^{16}\ \mathrm{cm}^{-2}$
- $N(\mathrm{H_2}) = 5.0\times10^{5}\times1.37\times 10^{16} \approx 7\times10^{21}\ \mathrm{cm}^{-2}$

(~10^{21} — ~10^{22} cm^{-2})

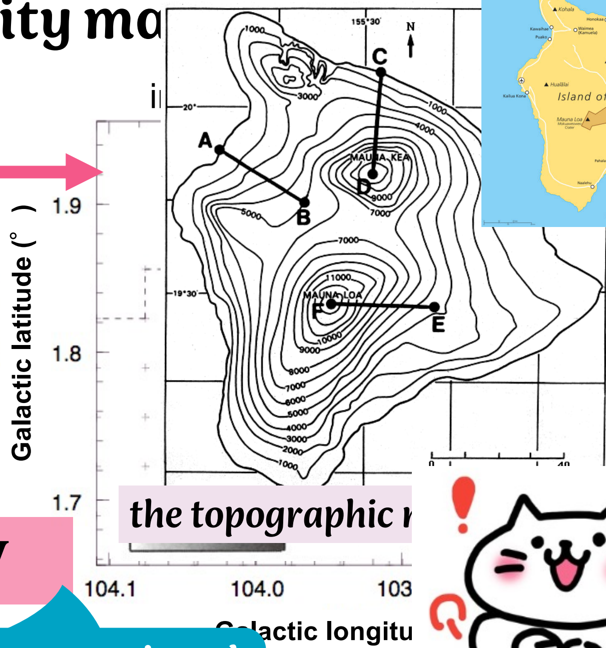


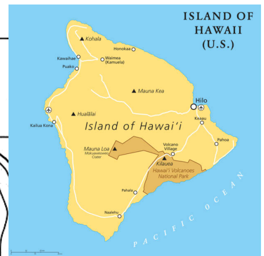
Integrated intensity ma

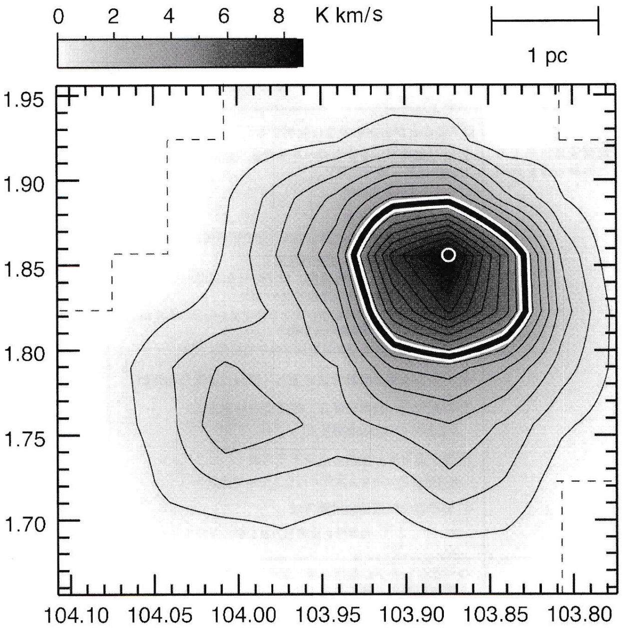


integrated intensity $\int T\mathrm{d}V$

The integrated intensity is proportional to the number of molecules.







Measure the area S of the molecular cloud at half the maximum integrated intensity (within the thick black line) in the ^{13}CO integrated intensity map shown in Figure 3.
Express this measurement in cm^2.

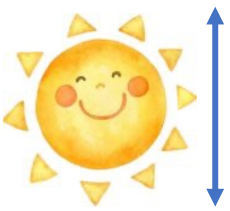
- Contour lines: Each line represents a specific intensity level. The lowest contour line shows 0.5 K km/s. Each next line increases by 0.5 K km/s.
- Black dot: This marks the spot where the intensity is highest. Thick line: This outlines where the intensity is half of the maximum.

Angular and Distance Units in Astronomy

◇ Angular Units

1°= 60′

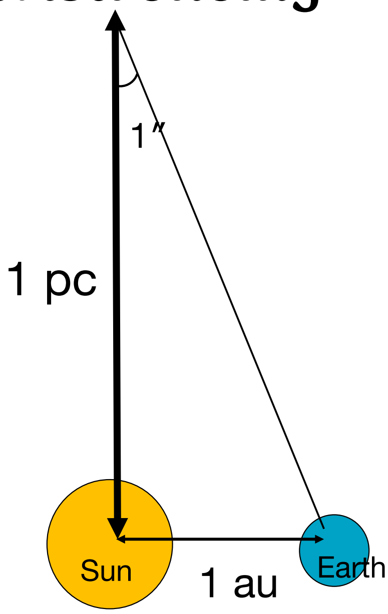
1′= 60″ ※1°=π/180 [rad]



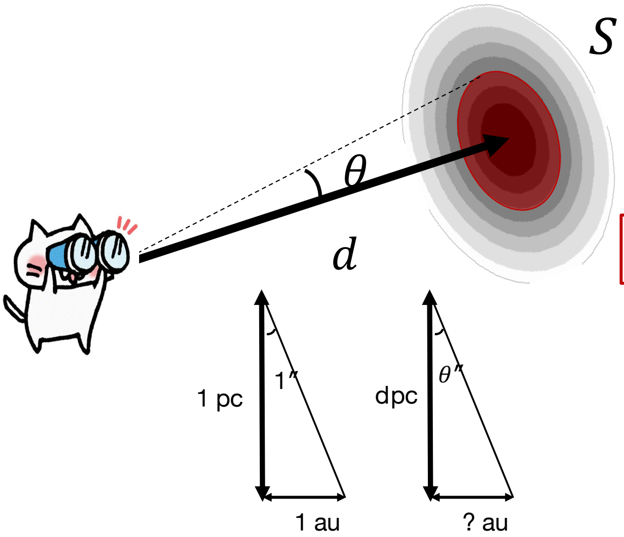
◇ Distance Units

★An **astronomical unit (=au)** is defined as the average distance between the Earth and the Sun. **1 au ～ 1.496×10^{13} [cm]**

★ A **parsec (=pc)** is defined as the distance at which 1 au subtends an angle of 1“. **1 pc ～ 3.08×10^{18} [cm]**



S can be calculated from its angular size and distance.



1 au = 1.496 × 10^{13} cm

When the angular size is very small, the value of the angle in radians is so small that the following relationship can be used.

$$1'' \sim \frac{1\ \mathrm{au}}{1\ \mathrm{pc}}$$

$$\boxed{r = \text{apparent size}(\theta) \times \text{distance(d)}}$$

Example: $\theta\ = 10'', \mathrm{d} =\ 200\ \mathrm{pc}$

$r = 10'' \times (1\ \mathrm{au}/1\ \mathrm{pc}) \times 200\ \mathrm{pc}$

$$\boxed{S \sim \pi(\theta \times d \times \{1\ \mathrm{au}\})^2}$$

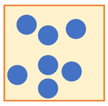
Example

This cloud is 30 light-years across! That's 300,000 times wider than Earth's orbit around the Sun!

To put this in perspective - if the Sun were the size of a grain of sand, this cloud would be about 10 kilometers across!

- Angular size(radius):
  θ ≈ (0.04°=) 144”
- Linear size calculation:
  1" ≈(1 au)/(1 pc) (1" at 1 pc ≈ 1 au)
  144" × (1 au / 1 pc) × 900 pc = 129,600 au
  =129,600 au × (1.496 × 10^{13} cm/1 au) = 1.9388160×10^{18} cm
- Area calculation:
  S = πr^2 = π × (1.939 × 10^{18} cm$)^2$ ≈ 1.2 × 10^{37} cm^2

Understanding the Mass Formula

Mass of a molecular cloud= Area of molecular cloud X  column density (Number of molecules per cm²) X the mass of a single molecule

$$M = \frac{\mu\, \mathrm{m_p} N(\mathrm{H_2}) S}{\ln 2} \frac{1}{M_\odot}$$

So our formula tells us

- Count all the molecules ($N(H_2)$ × S)
- Account for different particle types (μ × mp)
- Correct for the cloud's shape (÷ ln2)
- Express in convenient units (÷ M☉)

$$M = \frac{\mu\, \mathrm{m_p} N(\mathrm{H_2}) S}{\ln 2} \frac{1}{M_\odot}$$

$\ln 2 \approx 0.69314718$

✓μ = 2.4: average molecular weight (includes helium and heavier elements)
✓mp: proton mass (the basic building block)
✓$N(H_2)$: column density we just calculated
✓S: area we just measured
✓1/ln(2): correction for Gaussian distribution
✓M☉: solar mass (convenient unit).

Real clouds aren't uniform - they're densest in the center and thinner at the edges, like a Gaussian bell curve.
The half-maximum contour contains about 50% of the total mass, so we need to multiply by 1/ln(2) ≈ 1.44.

Example

$$\boxed{\begin{aligned} M &= \frac{\mu\, \mathrm{m_p} N(\mathrm{H_2}) S}{\ln 2} \frac{1}{M_\odot} \\ &= \frac{\mu\, \mathrm{m_p} N(\mathrm{H_2}) S}{\ln 2} \frac{1}{1.989\times 10^{33}} \end{aligned}}$$

$\ln 2 \approx 0.69314718$

$M = (2.4 \times 1.6735 \times 10^{-24} \times 6.9 \times 10^{21} \times 1.2 \times 10^{37}) / 0.69314718 / (1.989 \times 10^{33})$

$M \approx 240\ M_\odot$

Great job everyone!

200 $M_\odot$ - 400 $M_\odot$

This cloud contains enough material to make 240 stars like our Sun, or about 80 million Earth-masses!

Instagram・X：
shimoikura_lab