1. Complete the objects part of each concept

From the table, the objects (rows) and the attributes (columns) are given. The attributes are **c, e, o, p, s***, and the rows represent specific numbers (objects 0–9).*

The first step is to determine which objects correspond to each attribute combination. Using the table:

Attributes	Objects
[c, e, f, o, p, s]	*{0}*
[c, e, s]	*{0, 4, 6}*
[c, o, s]	*{9}*
[c, e, f]	*{0}*
[f, o, s]	*{9}*
[e, f, p]	*{5}*
[c, s]	*{4, 6, 9}*
[c, e]	*{2, 4, 6}*
[o, s]	*{9}*
[e, f]	*{0, 5}*
[o, p]	*{7}*
[c]	*{2, 4, 6, 8, 9}*
[s]	*{0, 4, 6, 9}*
[e]	*{0, 2, 4, 6, 8}*
[o]	*{1, 3, 7, 9}*
[f]	*{0, 1, 4, 9}*
[p]	*{2, 3, 5, 7}*

Attributes	**Objects**
[]	*{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}*

This gives a clear mapping of objects (numbers) to each concept.

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2. Compute α(β([f, p]))

To find the concept containing **[f, p]***:*

1. **β([f, p])** *gives us the set of objects containing attributes* **f** *and* **p**.
   - *From the table, objects with* **f** *= {0, 1, 4, 9} and objects with* **p** *= {2, 3, 5, 7}.*
   - *The intersection of these sets is empty; no object has both* **f** *and* **p**.
2. *Since* **β([f, p])** *is empty, applying* **α** *(the closure operator) to the empty set yields the full attribute set* **[]***, corresponding to all objects.*

Result: *α(β([f, p])) =* **[]**.

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3. Compute α(β([e, f]))

To find the concept containing **[e, f]***:*

1. **β([e, f])** *gives us the set of objects containing attributes* **e** *and* **f**.
   - *From the table, objects with* **e** *= {0, 2, 4, 6, 8} and objects with* **f** *= {0, 1, 4, 9}.*
   - *The intersection of these sets is {0, 4}.*
2. *Now, apply* **α** *to the set {0, 4}. The closure operator* **α** *expands this set to include all attributes shared by objects {0, 4}:*
   - *Both objects share the attributes* **e, f**.

Result: *α(β([e, f])) =* **[e, f]**.

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4. Immediate successors of [e, f] using Bordat's theorem

Bordat's theorem states:
The immediate successors of a closed set FFF are the inclusion-minimal sets in the family:

{α(β(x+F)):x∉F}.\{ \alpha(\beta(x + F)) : x \notin F \}.{α(β(x+F)):x∈/F}.

Here, F=[e,f]F = [e, f]F=[e,f]. To find the successors:

1. *Identify all attributes xxx not in FFF (not in [e, f]).*
   The attributes are **c, o, p, s**, *so x∈{c,o,p,s}x \in \{ c, o, p, s \}x∈{c,o,p,s}.*
2. *For each xxx, compute α(β(x+[e,f]))\alpha(\beta(x + [e, f]))α(β(x+[e,f])):*
   - *x=cx = cx=c:*
     Add ccc to [e,f][e, f][e,f] → β([e,f,c])\beta([e, f, c])β([e,f,c]). Objects = intersection of **e, f** *({0, 4}) and* **c** *({0, 2, 4, 6, 8}).*
     Result: {0, 4} → **α({0, 4}) = [e, f]** *(no change).*
   - *x=ox = ox=o:*
     Add ooo → β([e,f,o])\beta([e, f, o])β([e,f,o]). Intersection of **e, f** *({0, 4}) and* **o** *({1, 3, 7, 9}) is empty.*
     Result: Empty → closure is **[]**.
   - *x=px = px=p:*
     Add ppp → β([e,f,p])\beta([e, f, p])β([e,f,p]). Intersection of **e, f** *({0, 4}) and* **p** *({2, 3, 5, 7}) is empty.*
     Result: Empty → closure is **[]**.
   - *x=sx = sx=s:*
     Add sss → β([e,f,s])\beta([e, f, s])β([e,f,s]). Intersection of **e, f** *({0, 4}) and* **s** *({0, 4, 6, 9}) = {0, 4}.*
     Result: **α({0, 4}) = [e, f]** *(no change).*
3. *Inclusion-minimal sets: None of the additions expand beyond* **[e, f]**, *so there are* **no immediate successors**.

Result: *[e, f] has no immediate successors.*

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[c] = {0, 4, 6, 8, 9} - compound numbers

[s] = {0, 1, 4, 9} - square numbers

[e] = {0, 2, 4, 6, 8} - even numbers

[o] = {1, 3, 5, 7, 9} - odd numbers

[f] = {0, 1, 4} - square numbers

[p] = {2, 3, 5, 7} - prime numbers

Computing α(β([f,p])): When looking for numbers that are both square (f) and prime (p), there are no numbers in our dataset that satisfy both conditions. Therefore: α(β([f,p])) = {∅}

Computing α(β([e,f])): For numbers that are both even (e) and square (f):

Even numbers: {0, 2, 4, 6, 8}

Square numbers: {0, 1, 4, 9}

Numbers that satisfy both conditions: {0, 4} Therefore: α(β([e,f])) = {0, 4}

Computing immediate successors of [e,f] using Bordat's theorem: Starting with concept [e,f] = {0, 4}, the immediate successors are:

[e] = {0, 2, 4, 6, 8}

[f] = {0, 1, 4} - These are the minimal concepts that properly contain [e,f] in the concept lattice.