AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

# Assignment

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## EXERCICE 1

Let $f(x) = \dfrac{x-1}{2x+1}$ and $X_1 = -1; X_2 = 0; X_3 = 1$

1. Construct the polynome $P$ of Lagrange interpolation of at the points $X_1, X_2$ and $X_3$.

2. Construct the polynome $H$ of Hermite interpolation of at the points $X_1, X_2$ and $X_3$.

3. Construct the polynome $N$ of Newton interpolation of at the points $X_1, X_2$ and $X_3$.

4. Give the regression line at the sense of least squares of the points $\big(X_1, f(X_1)\big); \big(X_2, f(X_2)\big)$ et $\big(X_3, f(X_3)\big)$

## EXERCICE 2

On donne Consider $I = \displaystyle\int_1^4 f(x)dx$ where $f(x) = x^3 - 3x^2 + 2$

1. Give the exact value of $J$.

2. By subdividing the interval $[1,4]$ into 3 intervals calculate $I$
   a) Using the Trapeze method.
   b) Using the Simpson method.

3. Compare these results with the exact value obtained in 1.

4. Program the 2 methods (Trapeze and Simson) then for $N : 100$ calculate then represent the error.

## EXERCICE 3

Write a program which test if for a given matrix $A$ Jacobi method converge.
If yes, so for a vector $X_0$ and a precision epsilon given, solve the system $Ax = b$ where $b$ is also given.
Rappels:

1. The Jacobi method converge if the matrix $A$ is strictly diagonally dominant or if the spectral radius of the Jacobi matrix iteration $J$ is strictly inferior at 1.
2. By given a precision $\varepsilon$ and $X_0$ a initial vector, we have a convergence if $\|X_{k+1} - X_k\| < \varepsilon$ and then the solution is $X_{k+1}$.

Indications: $X_0$ and epsilon given, calculate $X$

$$X(i) = \frac{1}{A(i,i)} * \left(b(i) - \sum_{j=1, j\neq i}^{n} A(i,j) * X_0(j)\right)$$

while $\|X - X_0\| > epsilon$
$X_0 = X$ and

$$X(i) = \frac{1}{A(i,i)} * \left(b(i) - \sum_{j=1, j\neq i}^{n} A(i,j) * X_0(j)\right)$$

Solution of $Ax = b$ by Jacobi and Gauss-Seidel for $X_0$ given.
Exple: $X_0 = (X_0^1, X_0^2, X_0^3)$.

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad b = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad ; X = \begin{bmatrix} X^1 \\ X^2 \\ X^3 \end{bmatrix} \tag{0.1}$$

Solve $Ax = b$ equates to

$$\left[\begin{array}{lcl} X^1 & = & \frac{1}{a_1}(d_1 - b_1 X^2 - c_1 X^3) \\ X^2 & = & \frac{1}{b_2}(d_2 - a_2 X^1 - c_2 X^3) \\ X^3 & = & \frac{1}{c_3}(d_3 - a_3 X^1 - b_3 X^2) \end{array}\right.$$

Jacobi and Gauss-Seidel are the iteratives methods, we give $X_0$ and we calculate $X_1$, etc.
Jacobi

$$\left[\begin{array}{lcl} X_1^1 & = & \frac{1}{a_1}(d_1 - b_1 X_0^2 - c_1 X_0^3) \\ X_1^2 & = & \frac{1}{b_2}(d_2 - a_2 X_0^1 - c_2 X_0^3) \\ X_1^3 & = & \frac{1}{c_3}(d_3 - a_3 X_0^1 - b_3 X_0^2) \end{array}\right.$$

Gauss-Seidel

$$\left[\begin{array}{lcl} X_1^1 & = & \frac{1}{a_1}(d_1 - b_1 X_0^2 - c_1 X_0^3) \\ X_1^2 & = & \frac{1}{b_2}(d_2 - a_2 X_1^1 - c_2 X_0^3) \\ X_1^3 & = & \frac{1}{c_3}(d_3 - a_3 X_1^1 - b_3 X_1^2) \end{array}\right.$$

## Exercice 4

We give

$$A = \begin{bmatrix} 4 & 2 & 1 \\ -1 & 2 & 0 \\ 2 & 1 & 4 \end{bmatrix}; \quad b = \begin{bmatrix} 4 \\ 2 \\ 9 \end{bmatrix}; \quad \text{and} \quad X_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{0.2}$$

Solve using Jacobi, then Gauss-Seidel method $Ax = b$ and by calculating $X_1, X_2, X_3$ and $X_4$

Show that the solution converge to $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

## Exercice 5

We consider the following problem

$$(P_1) \begin{cases} -u'' + u' + u & = & x^2 - x - 4 \ \text{ in } ]0,2[ \\ u(0) & = & 1 \\ u(2) & = & 1 \end{cases}$$

Solve $(P_1)$ by using the finite difference method, taking a centered scheme for $u'$.