African Institute for Mathematical Sciences
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Topology-Functional Analysis

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Ex. 1 Let X be the set $(\mathbb{R} \setminus \mathbb{N}) \cup \{1\}$. Define the function $f : \mathbb{R} \to X$ by

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{R} \setminus \mathbb{N} \\ 1, & \text{if } x \in \mathbb{N}. \end{cases}$$

and let $\mathcal{T}$ be defined as follows

$$\mathcal{T} = \{U \subset X \mid f^{-1}(U) \text{ is open in } \mathbb{R}\}.$$

(a) Show that $\mathcal{T}$ is a topology on X.

(b) Show that f is continuous.

(c) Show that $\mathcal{T}$ is Hausdorff.

Ex. 2 Let X be a compact Hausdorff topological space.

(a) Let $x \in X$ and K be a closed subset of X such that $x \notin K$. Show that there exists an open set U containing x, an open set V containing K such that $U \cap V = \emptyset$.

(b) Let K_1 and K_2 be two disjoint closed subsets of X. Show that there exists an open set U containing K_1, un open set V containing K_2 such that $U \cap V = \emptyset$.

(c) Let K be a compact subet of X and Ω be an open subset of X such that $K \subset \Omega$. Show that there exists a compact set K_1 such that

$$K \subset \mathring{K}_1 \subset K_1 \subset \Omega.$$

[Hint : `You may use` (b) `with appropriate sets`.]

(d) Explain briefely, how we can get a sequence $\{K_n\}$ of compact subsets of X such that

$$K \subset \mathring{K}_{n+1} \subset K_{n+1} \subset \mathring{K}_n \subset K_n \subset \Omega.$$