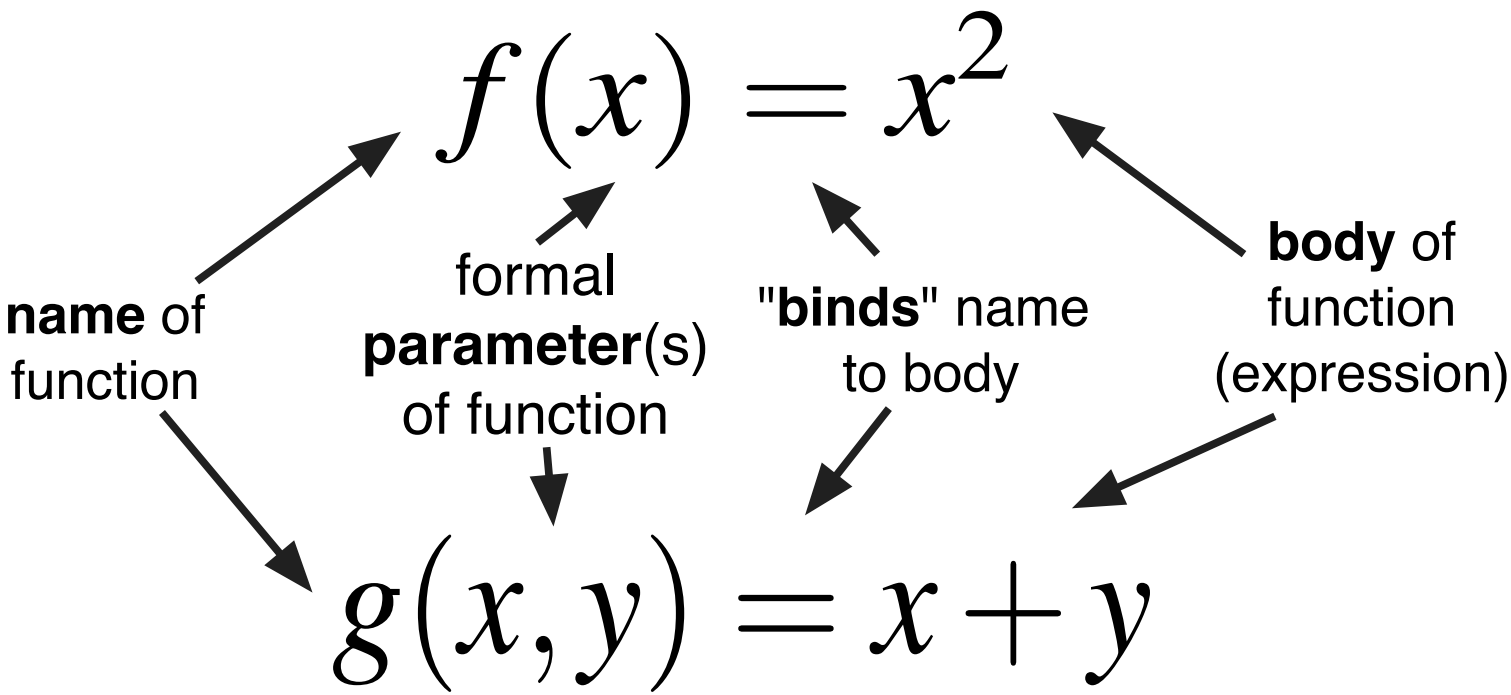
1: Introduction to Racket

Defining Functions in Mathematics



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Although different orderings are possible, we will evaluate these functions left-to-right, choosing to evaluate the innermost function application first.

This will be enforced!

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We insist that we only substitute, in a function application, when all of its arguments are values (not more general expressions).

Function application in Racket

The **syntax** of function application in Racket is different from that of mathematics.

In Racket, the name of the function follows the open parenthesis, and arguments are separated by spaces.

$g(1,3)$ becomes, in Racket, `(g 1 3)`.

$g(g(1,3),f(2))$ becomes `(g (g 1 3) (f 2))`.

Operators in mathematics

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This is what is done in Racket!

Expressions in Racket

Operators in mathematics become functions in Racket.

Parentheses are used only to associate arguments with operations. Ordering of operations is handled by nesting.

- $3 - 2 + 4/5$ becomes `(+ (- 3 2) (/ 4 5))`
- $(6 - 4)(3 + 2)$ becomes `(* (- 6 4) (+ 3 2))`

Any arithmetic expression can be converted into a Racket expression in this way.

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Extra parentheses are harmless in arithmetic expressions, but they are harmful in Racket.

Only use parentheses when necessary (to signal a function application or some other Racket syntax).

The DrRacket environment

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- Interactions window: a read-evaluate-print loop (REPL)
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Numbers in DrRacket

- Integers in Racket are represented exactly (unbounded).
- Rational numbers are represented exactly.
- Evaluating some expressions produces an **inexact** value.
  Example: `(sqrt 2)` evaluates to `#i1.414213562370951`.
  We will not use inexact numbers much (if at all).

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- `(/ 25 0)`

Defining Functions in Racket

$f(x) = x^2$ becomes `(define (f x) (* x x))`.

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`define` is a **form**
(looks like a Racket function application, but treated differently).

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`define` **binds** a name to an expression
(which uses the parameters that follow the name).

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- a list of parameters,
- a single “body” expression.

The body expression typically uses the parameters together with other built-in and user-defined functions.

An application of a user-defined Racket function substitutes argument values for the corresponding parameters in the definition’s body expression.

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This mirrors how we work with mathematical expressions.

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The same process works with user-defined Racket functions, using substitution for function application as we described earlier.

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⇒ (g 4 (* 2 2))
⇒ (g 4 4)
⇒ (+ 4 4)
⇒ 8
```

Parameters in Racket

Each parameter name has meaning only within the body of its function.

```
(define (f x y)
  (+ x y))

(define (g z x)
  (* x z))
```

The two uses of `x` are independent.

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Advantages of constants

- Makes programs easier to understand
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They are sometimes called variables, but their values cannot be changed.

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- Provides a Stepper to let one evaluate expressions step-by-step
- Features: subexpression highlighting, syntax checking, error highlighting, automatic indentation

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Further expressions using the definitions may be evaluated in the Interactions window.

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3. You should be able to define and use functions that do simple arithmetic computations.
4. You should understand the purposes and uses of the Definitions and Interactions windows in DrRacket.