Control of Inverted Pendulum and supplementary swing up controller

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1. Introduction

Control systems engineering is a field that deals with controlling and regulating of engineering systems. It is involved in many technological advancements happening in the world today. Many of the control systems are nonlinear, multistate and unstable engineering systems. In robotics, underactuated control systems are also becoming very common. An inverted pendulum consists of all those properties and hence qualifies to be a test bed for control systems design. Apart form the fact that Inverted Pendulums emulate most control systems, its specific design can be applied on real systems as well. The following are examples of Inverted pendulum control.

Handle Robot Boston Dynamics

Falcon 9 Rocket SpaceX

Weather Satellite NASA

2. Aim of this research

The main aim of this research is to swing up and balance an Inverted Pendulum System using classical control methods learned. The chosen methods are the Linear State Space Feedback controller and the Linear quadratic controller. The 2 methods are compared to each other. A nonlinear controller is also designed for swinging from the stable position of the Pendulum to the unstable position

3. Methodology

The Inverted pendulum system is modelled mathematically using Lagrangian mechanics. The model is also obtained quantitatively by giving a known input and observing the output. System validation is done on the transfer function obtained. Feasibility study is conducted before the controller design. Specifications on controller design are given

The Pendulum System Model

$$\ddot{x} = -\frac{\alpha}{M+m}\dot{x} - \frac{ml}{M+m}\ddot{\theta} + \frac{F}{M+m}$$

$$\ddot{\theta} = -\frac{3}{4l}\ddot{x} - \frac{3\beta}{4ml^2}\dot{\theta} + \frac{3g}{4ml^2}\theta$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -33 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 136.125 & 225 & -0.3\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ 16\\ 0\\ -66\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix}$$

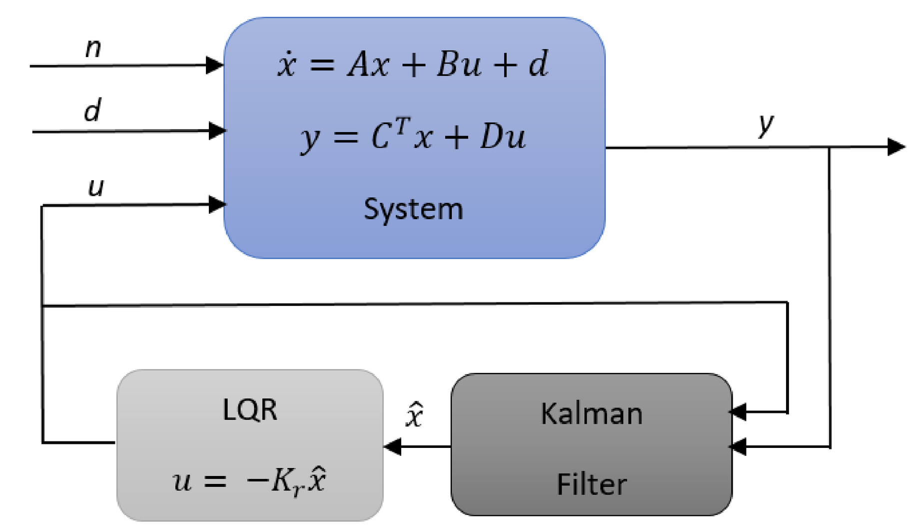
Specifications

- Swing up should be achieved under 10s
- The cart must not hit the end of the track
- The pendulum angle must settle in less than 3s
- Over and undershoot of the pendulum angle should be less the 20%
- The pendulum angle must achieve zero steady state error and capable of disturbance rejection

Controller Design

The Linear State Space Feedback and the Linear Quadratic controllers are designed. To reduce noise and measure all states which are the angle, angular velocity, cart position and velocity, an observer and a Kalman Filter are designed for the respective controllers.

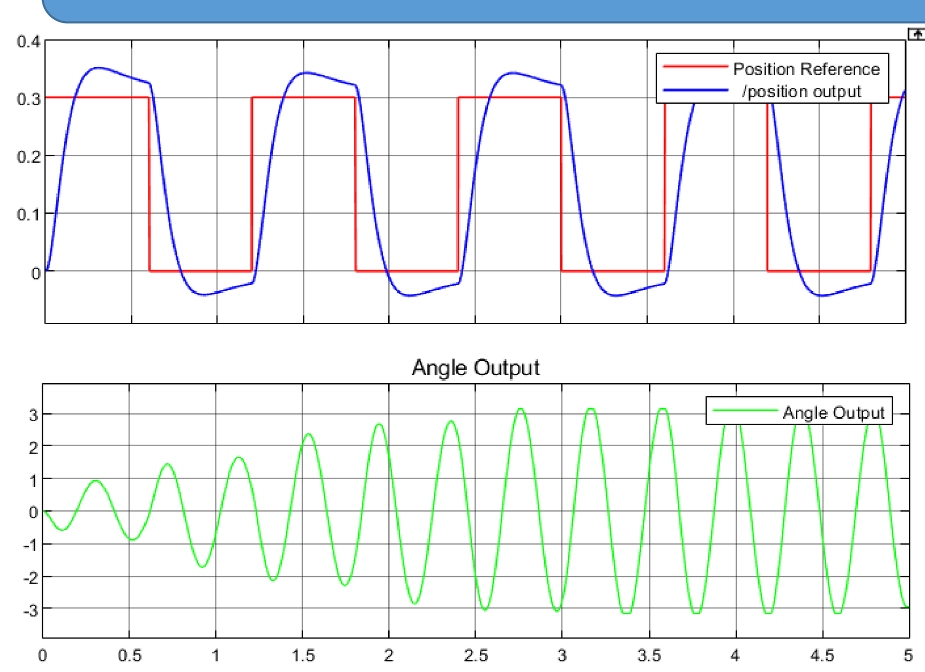
Block Diagram For Pendulum control



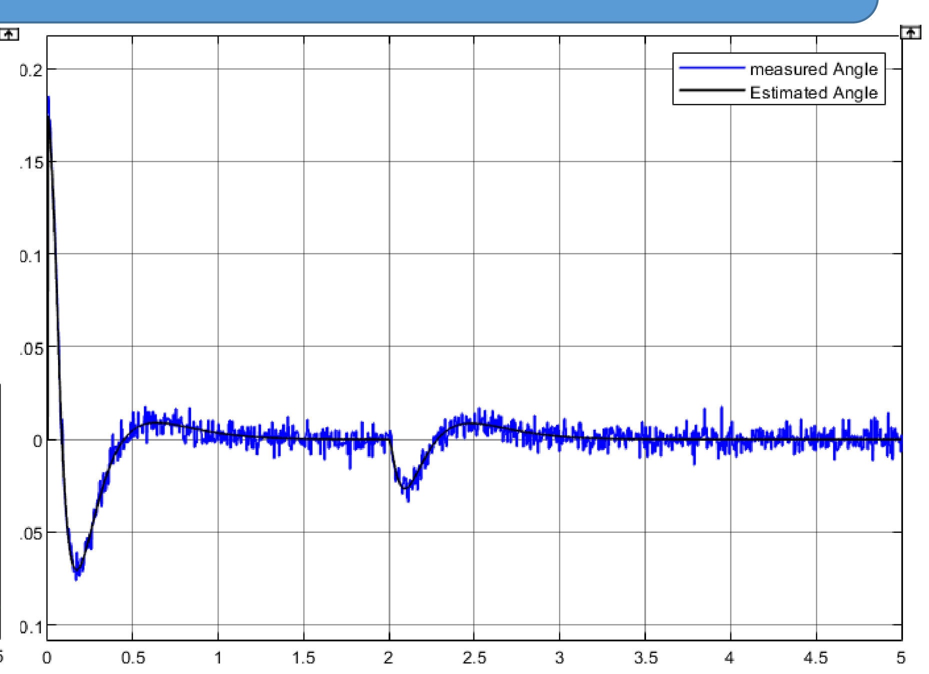
Linear Quadratic Gaussian

The pendulum system is represented by A,B,C and D. There is noise n and disturbance d affecting the system. The Kalman filter improves noise and disturbance rejection

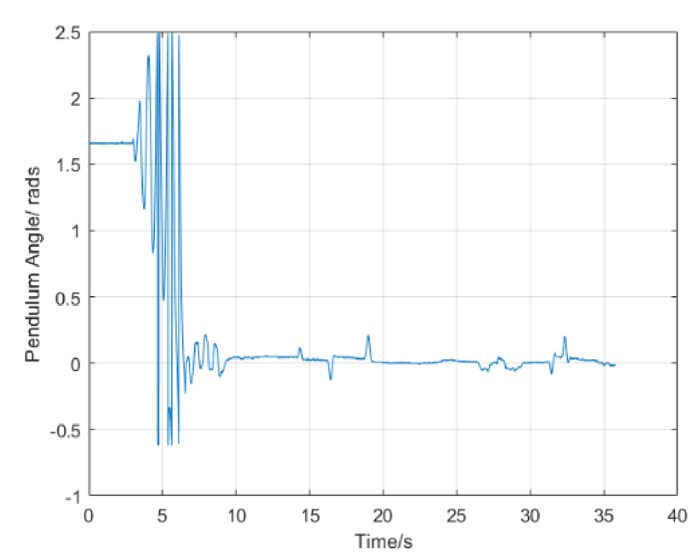
4. Results



Swing up Controller simulation



Balancing Controller simulation



Swing up and Balancing Controller in real time

Controller Type	Overshoot %	Settling Time /s	ISU	ISE
State Space feedback	12%	1s	1.136	0.002911
LQR	10%	1s	0.7933	0.002146

Results Table

5. Conclusions

The LQG controller performed better than the State Space Feedback controller. The Kalman filter is great at filtering noise and estimating states improving the results. LQG is optimized using the cost function hence better results