第4次应用介绍课

——正弦稳态电路的应用

本节课需要用复数计算器

1 频率特性

2 滤波器

3 互感的应用：变压器

空心变压器

理想变压器

本讲重难点

- 频率特性
- 滤波器
  - 滤波器的截止频率
- 空心变压器的引入阻抗
- 理想变压器的*u-i*关系
  - 双绕组
  - 三绕组（习题课题目布置）
- 含理想变压器电路的计算

1 频率特性

课前预习

电路的频率响应 ⟷ 电路的频率特性

改变电路激励的频率（维持其幅值不变）对电路造成的影响。

- (端口)阻抗
  - 阻抗频率特性
  - 电抗频率特性
- 支路电压电流
  - 电压频率特性
  - 电流频率特性
- 输入输出关系
  - 网络函数/传递函数

$$Z = R + \mathrm{j}\omega L = |Z| \angle \tan^{-1}\frac{\omega L}{R}$$

$$\dot{I} = \frac{U_S}{\sqrt{R^2 + (1/\omega C)^2}} \angle(\varphi + \arctan 1/\omega RC)$$

$$H = \frac{\text{响应}}{\text{激励}}$$

$$H = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\arctan(\omega RC)$$

单选题 1分

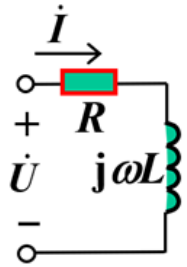
对于正弦激励下的纯电阻电路，只改变电路激励的频率，不会影响电路中某支路量的幅值和初相角

A 正确

B 错误

4

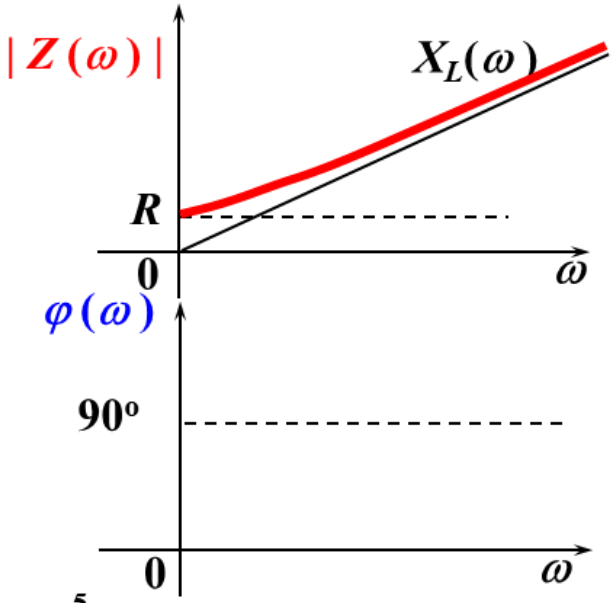
例1 画图示电路入端阻抗频率特性。



$$Z = R + \mathrm{j}\omega L$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

$$\varphi = \arctan\frac{\omega L}{R}$$



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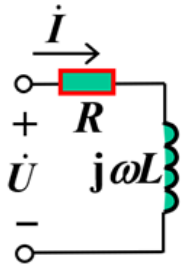

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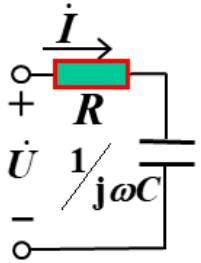
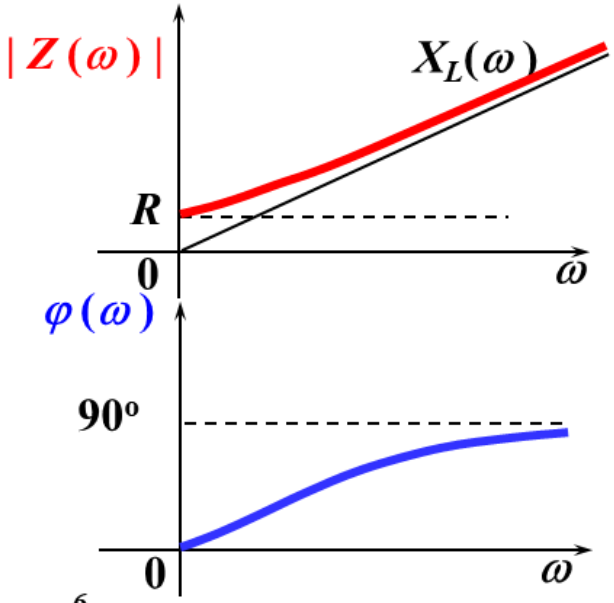
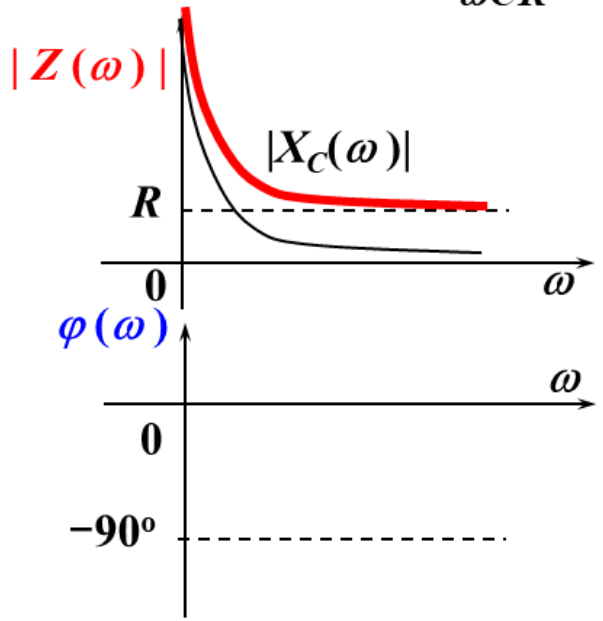


$$Z = R - \mathrm{j}\frac{1}{\omega C}$$

$$|Z| = \sqrt{R^2 + (\frac{1}{\omega C})^2}$$

$$\varphi = -\arctan\frac{1}{\omega CR}$$





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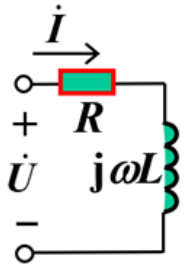

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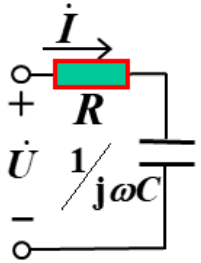

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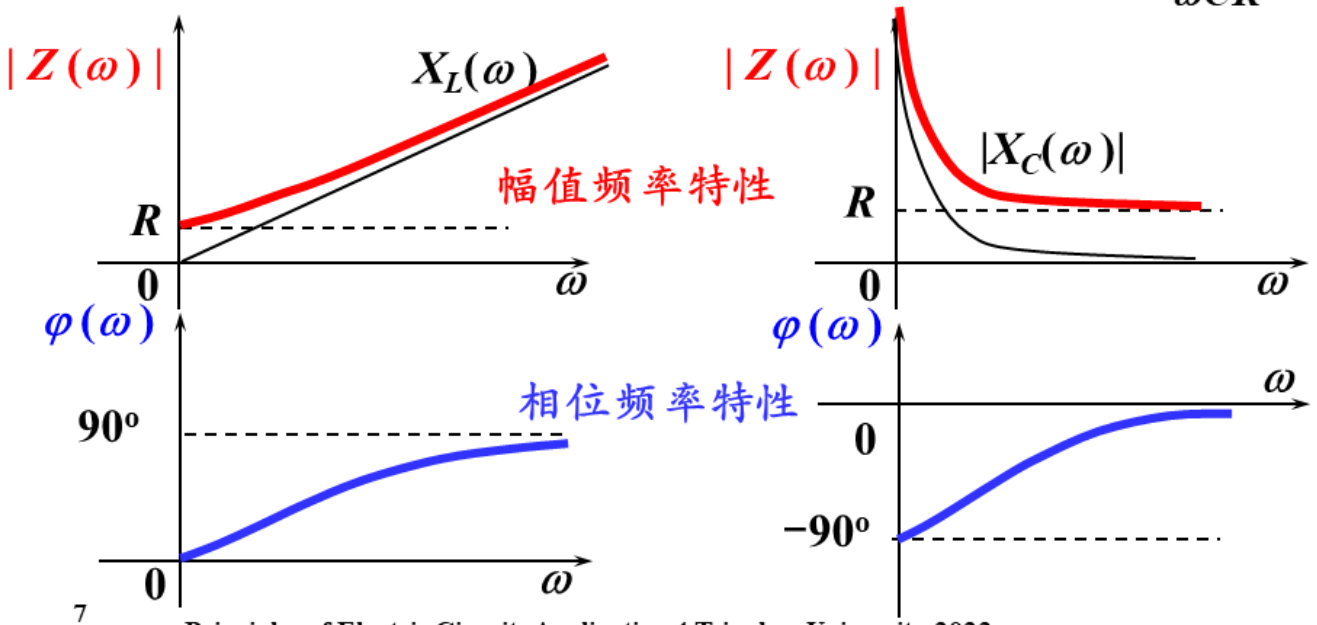
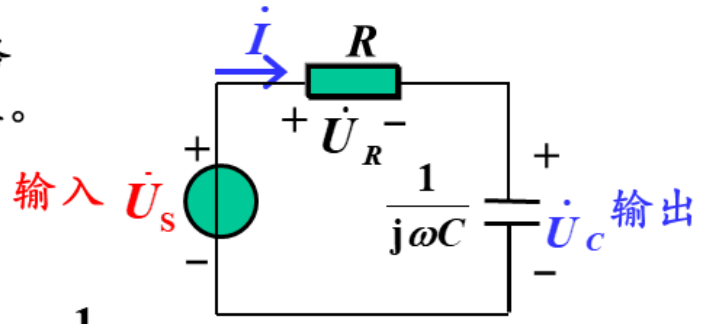
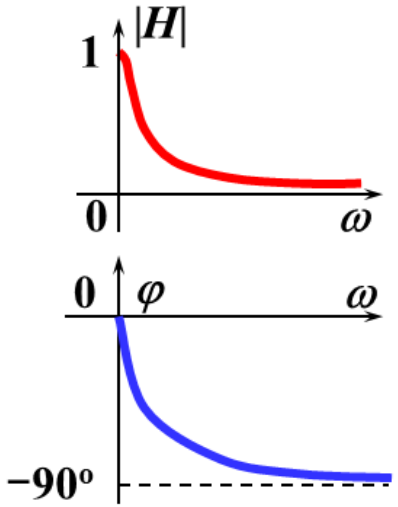


例2　画图示电路网络函数的频率特性。



$$H(\omega)=|H|\angle\varphi=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan(\omega CR)$$



见仿真

设 $RC=1$

ω /(rad/s)	$\lvert H\rvert$	φ (°)
0	1	0
1	0.707	−45
5	0.196	−78.7
10	0.1	−84.3
20	0.05	−87.1
100	0.01	−89.4

实际应用中，经常会出现

- 待考察量的变化非常大
- 频率范围非常宽

阻抗、电压电流、传递函数

$10^{-4}<|H|<10^{+3}$　　1mHz～1GHz

见仿真

幅值/相位频率特性的波形特点被掩盖

怎么办？

此处可以有弹幕

实际应用中，经常会出现

- 待考察量的变化非常大
- 频率范围非常宽

阻抗、电压电流、传递函数

$10^{-4}<|H|<10^{+3}$　　1mHz～1GHz

见仿真

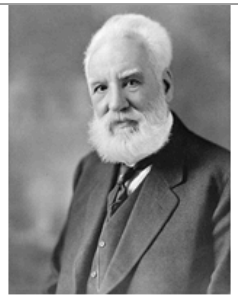
幅值/相位频率特性的波形特点被掩盖

$-4<\lg|H|<+3$　　用对数来表示　　$-3\sim9$

见仿真

分贝(decibel)　　$H_{\mathrm{dB}}=20\lg|H|$

dB的由来和好处



Bell

- 人们在研究声音有多响亮的过程中，提出两种用log方式描述比例大小的手段
- 声音响亮的程度与功率成正比
- 声音的功率与声压平方成正比
- 好处
  - 从容地表示大比例
  - 对>1和<1区分明显

$$\eta = \lg\frac{A}{B}(\text{Bel})$$

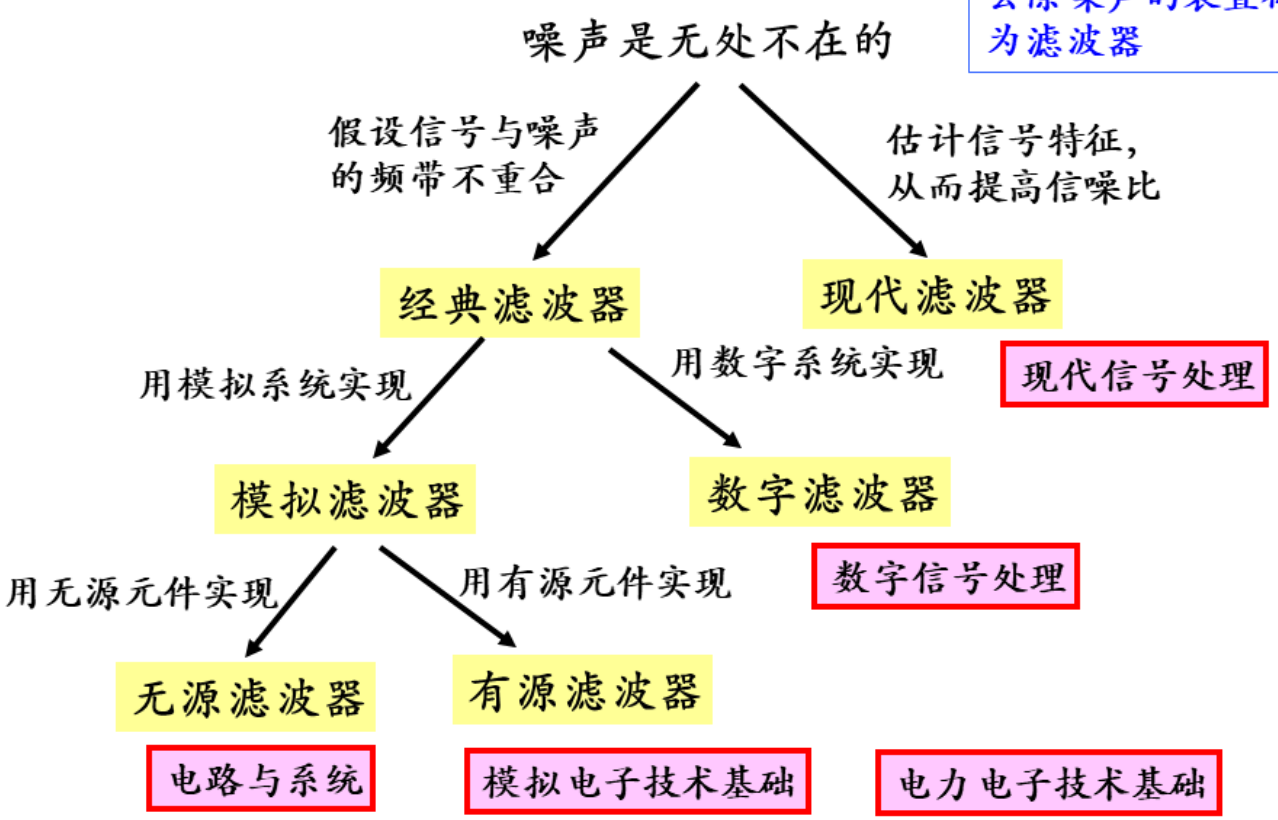
$$\text{dB} = 10\lg\frac{A}{B}$$

$$\text{dB} = 10\lg\frac{P_2}{P_1}$$

$$\text{dB} = 10\lg\frac{U_2^2}{U_1^2} = 20\lg\frac{U_2}{U_1}$$

2 滤波器(Filter)

去除噪声的装置称为滤波器



模拟滤波器

从功能上分类

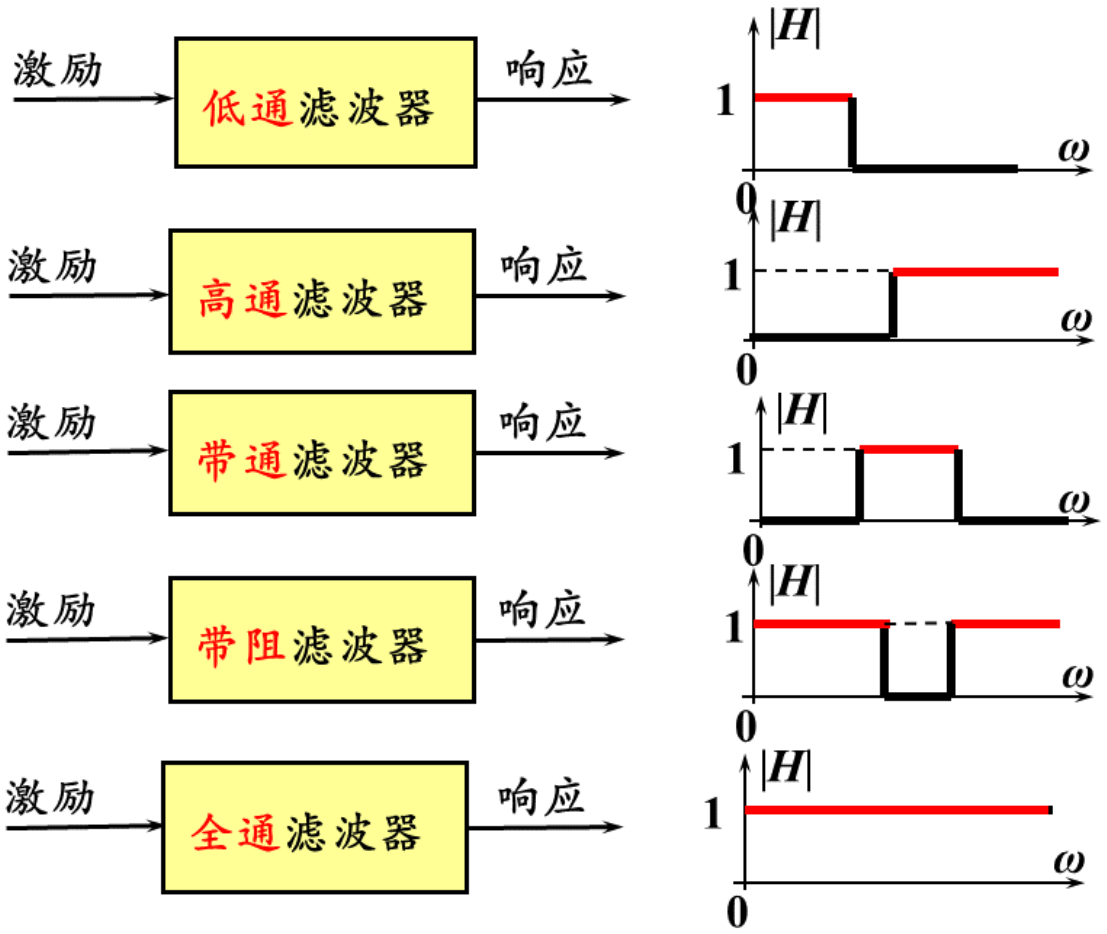
低通(LP)　高通(HP)　带通(BP)　带阻(BS, Notch)　全通(FP)

从实现方式上分类

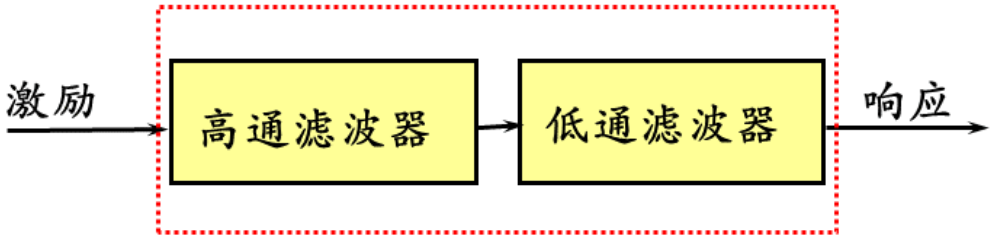
无源滤波器　　有源滤波器

RC　LC　　Op Amp　电力电子器件

用无源元件实现　　用有源元件实现


激励
低通滤波器
响应
|H|
1
0
ω
激励
高通滤波器
响应
|H|
1
0
ω
激励
带通滤波器
响应
|H|
1
0
ω
激励
带阻滤波器
响应
|H|
1
0
ω
激励
全通滤波器
响应
|H|
1
0
ω

单选题 1分



如设计得当，图示滤波器的组合可用作：

A 带通滤波器

B 全通滤波器

C 带阻滤波器

D 以上均有可能

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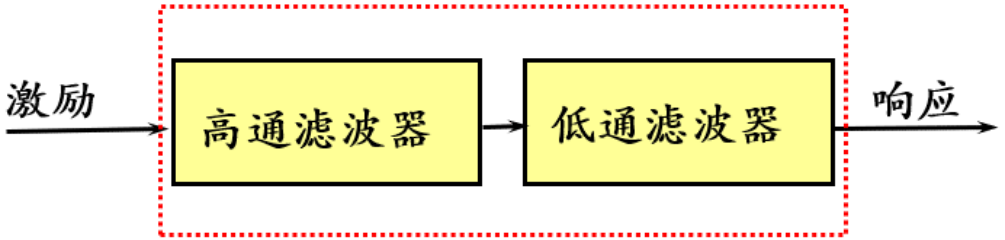


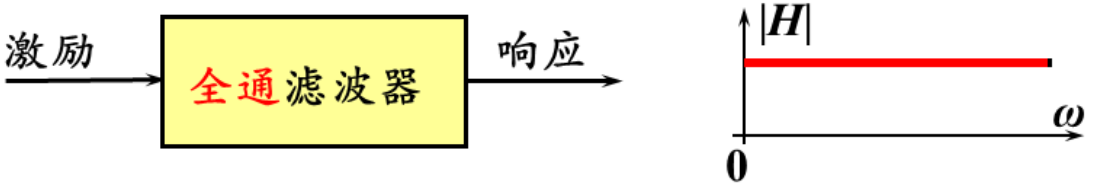
如设计得当，图示滤波器的组合可用作：

带通滤波器

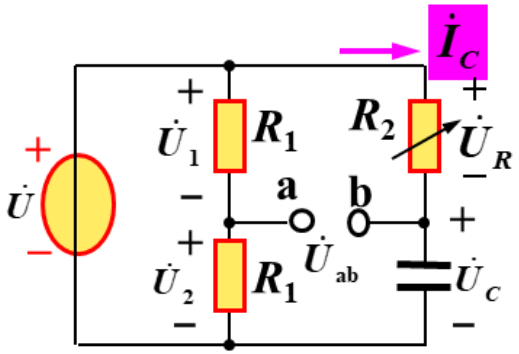
该怎么做带阻滤波器？

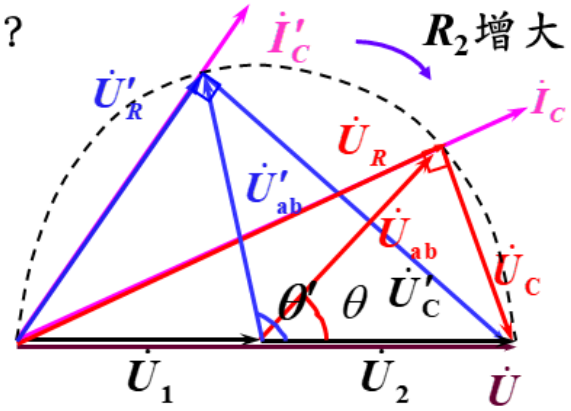
投稿

全通滤波器是个啥？

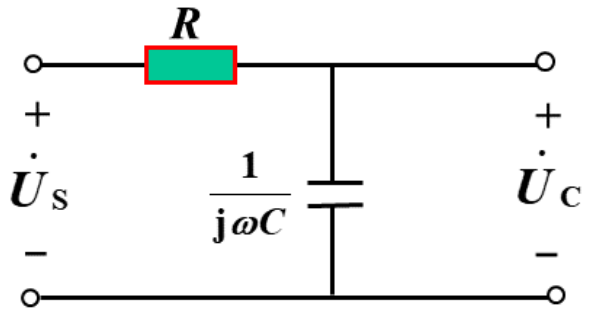


例3　当R_2由0→∞时，$\dot{U}_{ab}$如何变化？

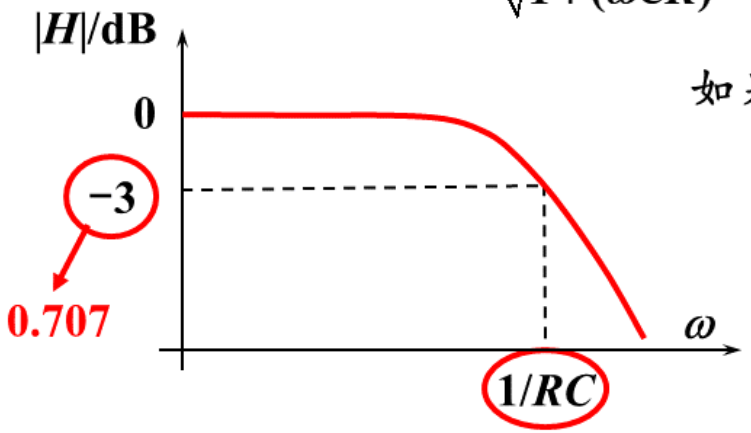




例1 RC低通滤波器



$$H(\omega)=|H|\angle\varphi=\frac{1}{\sqrt{1+(\omega CR)^2}}\angle-\arctan(\omega CR)$$

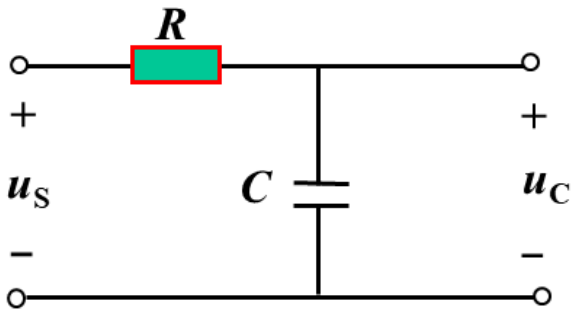


如果 $RC>>1$ 截止频率低

下面我们来讨论
这个(截止频率低的)低通滤波器
在时域是个什么

低通滤波器

截止频率 半功率频率



低通滤波器

已知

$u_C(0^-)=0$，$u_S=10\text{V}$

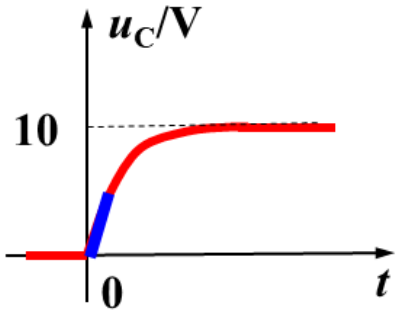
时域

$$RC\frac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=u_S \longrightarrow u_C(t)=10(1-\mathrm{e}^{-\frac{t}{RC}})$$

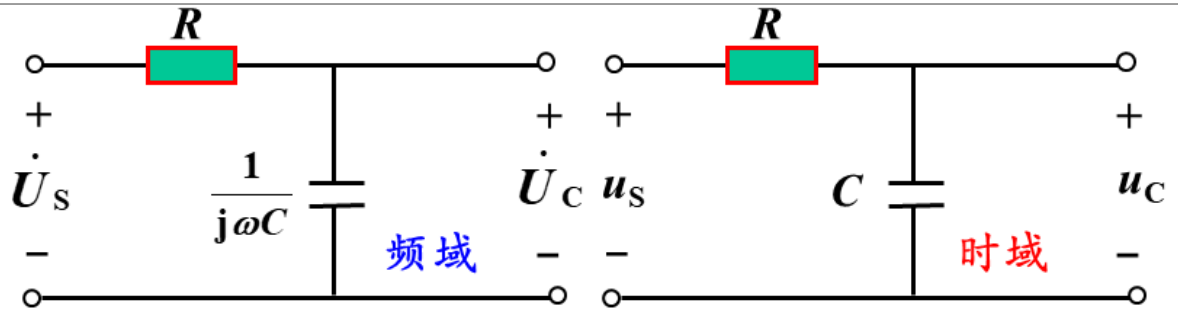
$RC>>1$

t很小

$$u_C\approx\frac{1}{RC}\int u_S\mathrm{d}t \qquad u_C\approx 10\frac{t}{RC}$$



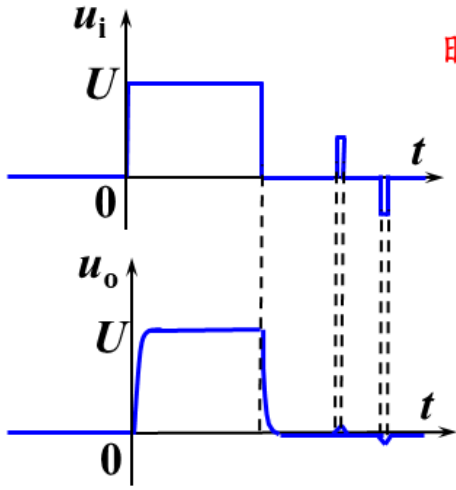
近似积分器



（截止频率比较低的）低通滤波器 ⟷ $RC \gg 1$ ⟷ （积分速度比较慢的）积分器

频域

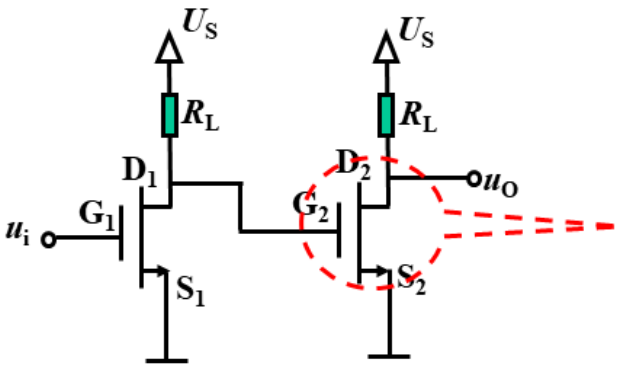
毛刺变化快 → 高频成分多 → 高频成分被滤除 → 毛刺被抑制



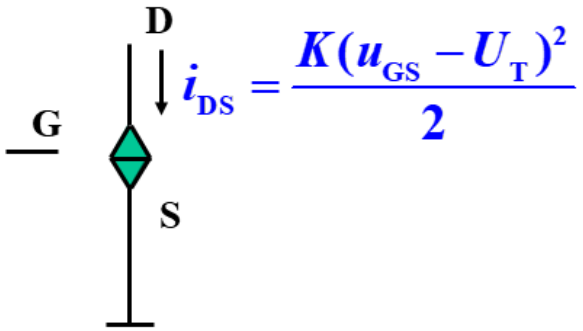
时域

毛刺时间短 → 积分量小 → 毛刺被抑制

例2：用频率特性来分析
——MOSFET小信号放大器的增益

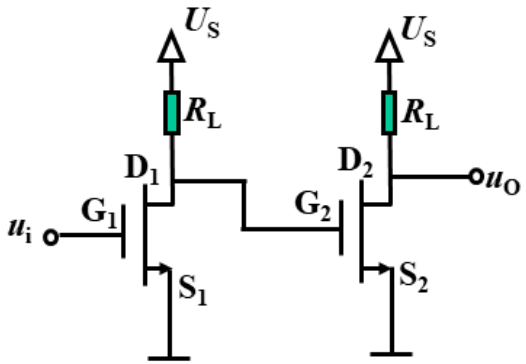


电流源模型



$$i_{DS} = \frac{K(u_{GS} - U_T)^2}{2}$$

导通状态

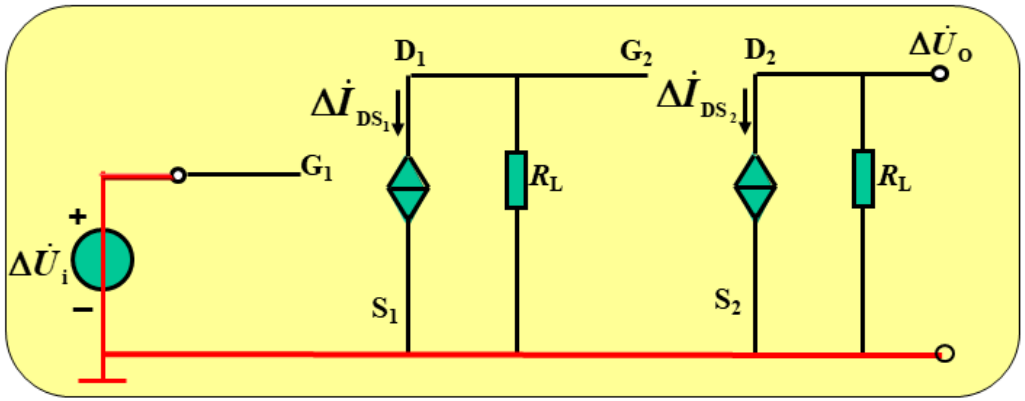


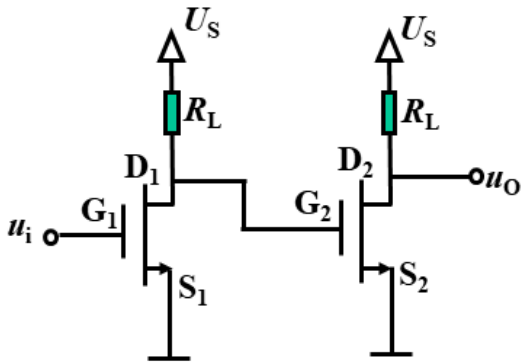
$$\Delta\dot{I}_{DS_1} = g_m \Delta\dot{U}_{G_1S_1}$$

$$\Delta\dot{I}_{DS_2} = g_m \Delta\dot{U}_{G_2S_2}$$

MOSFET放大器的小信号相量电路模型

求输入$\Delta\dot{U}_O$





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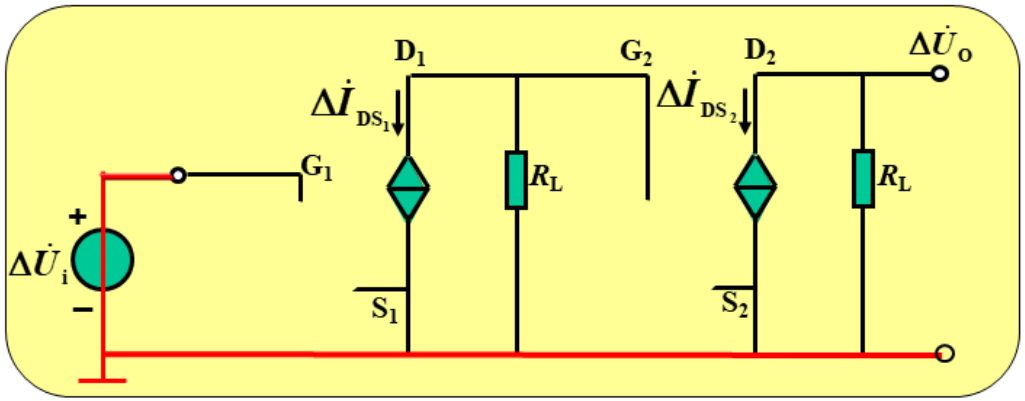
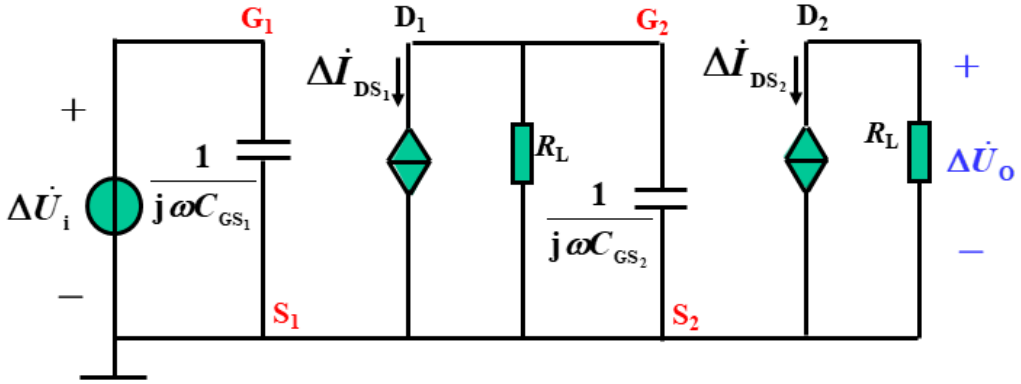


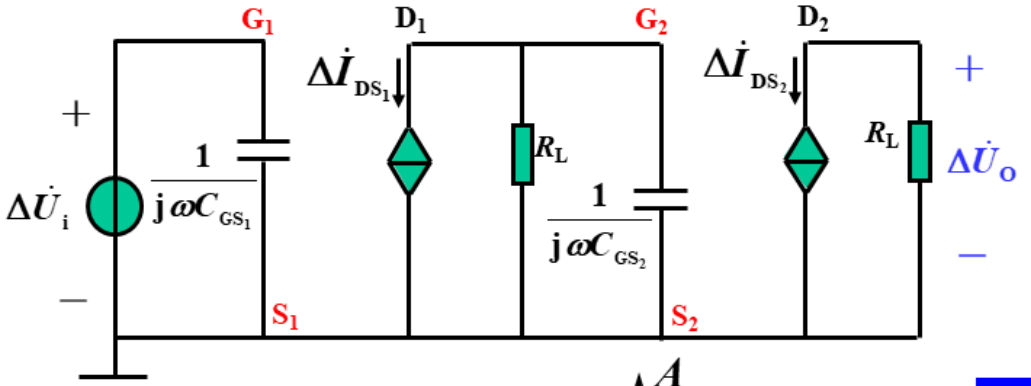
$$\Delta\dot{I}_{DS_1} = g_m \Delta\dot{U}_{G_1S_1} \qquad \Delta\dot{I}_{DS_2} = g_m \Delta\dot{U}_{G_2S_2}$$



$$\Delta\dot{U}_O = -\Delta\dot{I}_{DS_2} R_L = -g_m \Delta\dot{U}_{G_2S_2} R_L$$

$$\Delta\dot{U}_{G_2S_2} = -\frac{R_L \times \frac{1}{j\omega C_{GS_2}}}{R_L + \frac{1}{j\omega C_{GS_2}}} \times \Delta\dot{I}_{DS_1} = -\frac{R_L}{1 + j\omega C_{GS_2} R_L} \times \Delta\dot{I}_{DS_1}$$

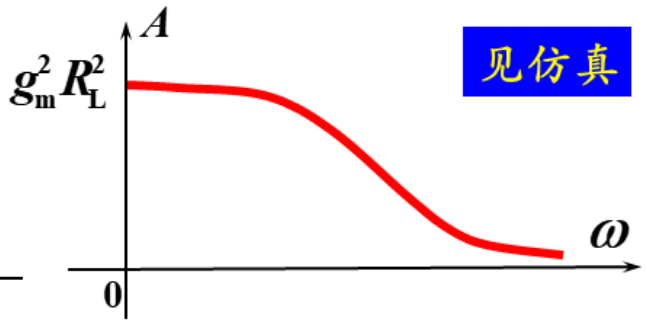
$$\Delta\dot{U}_O = g_m R_L \times \frac{R_L}{1 + j\omega C_{GS_2} R_L} \times \Delta\dot{I}_{DS_1} = \frac{g_m^2 R_L^2}{1 + j\omega C_{GS_2} R_L} \times \Delta\dot{U}_{G_1S_1}$$



$$\Delta\dot{U}_{\mathrm{O}}=\frac{g_{\mathrm{m}}^{2}R_{\mathrm{L}}^{2}}{1+\mathrm{j}\omega C_{\mathrm{GS_2}}R_{\mathrm{L}}}\times\Delta\dot{U}_{\mathrm{G_1S_1}}$$

$$\frac{\Delta\dot{U}_{\mathrm{O}}}{\Delta\dot{U}_{\mathrm{i}}}=\frac{\Delta\dot{U}_{\mathrm{O}}}{\Delta\dot{U}_{\mathrm{G_1S_1}}}=\frac{g_{\mathrm{m}}^{2}R_{\mathrm{L}}^{2}}{1+\mathrm{j}\omega C_{\mathrm{GS_2}}R_{\mathrm{L}}}$$

$$A=\left|\frac{\Delta\dot{U}_{\mathrm{O}}}{\Delta\dot{U}_{\mathrm{i}}}\right|=\frac{g_{\mathrm{m}}^{2}R_{\mathrm{L}}^{2}}{\sqrt{1+\left(\omega C_{\mathrm{GS_2}}R_{\mathrm{L}}\right)^{2}}}$$



见仿真

结论：

由于存在寄生电容，MOSFET小信号放大器的增益随着频率增加而减小。

单选题 1分

前述考虑了C_{GS}的MOSFET小信号放大器，可视作哪种滤波器？

A 高通

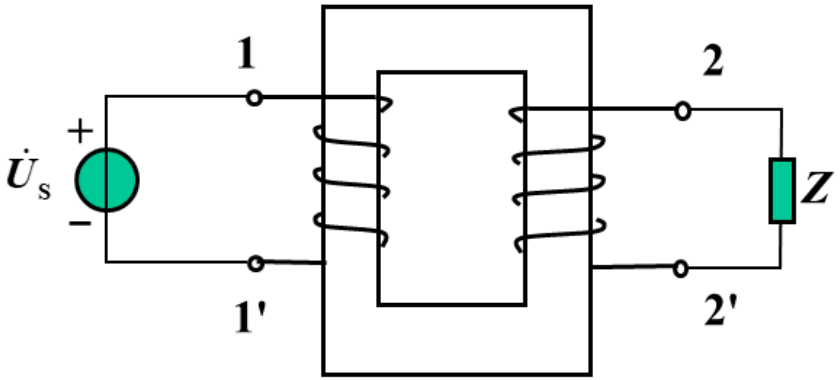
B 低通

C 带通

D 带阻

$$A=\left|\frac{\Delta\dot{U}_{O}}{\Delta\dot{U}_{i}}\right|=\frac{g_{m}^{2}R_{L}^{2}}{\sqrt{1+\left(\omega C_{GS_2}R_{L}\right)^{2}}}$$

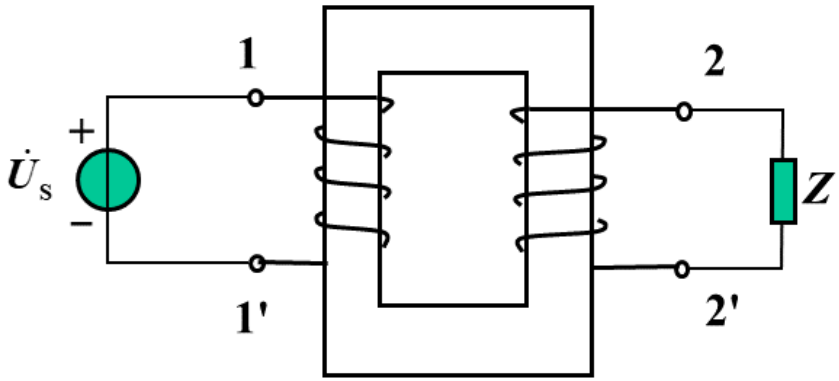
3 变压器 (Transformer)



利用互感的作用来传递能量

- 交流变压、变流
- 传送功率
- 电隔离
- 阻抗匹配

研究思路



1 考虑线圈内阻，求从原边(副边)看的等效电路

↓ 空芯变压器模型

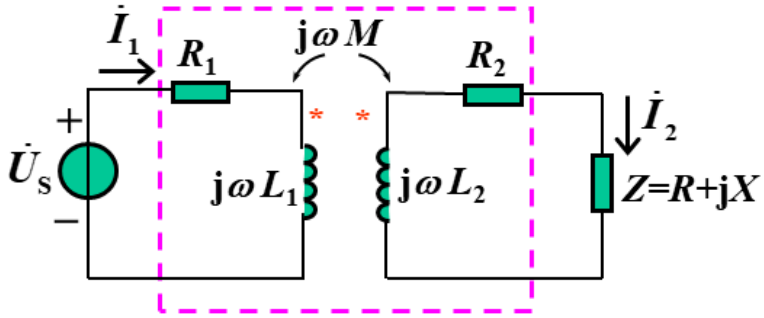
2 忽略考虑线圈内阻，且耦合系数为1

↓ 全耦合变压器模型

3 感值趋向于无穷大

理想变压器模型

(1) 空心变压器



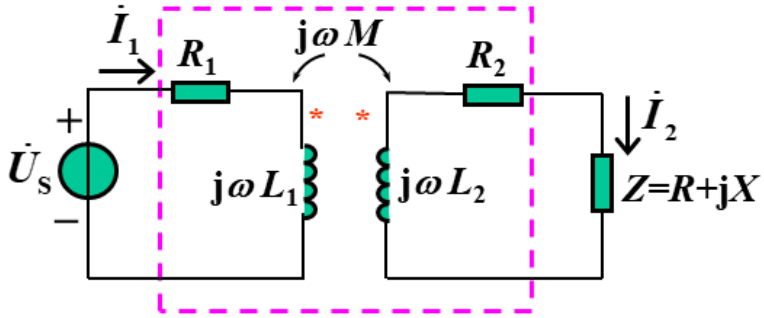
原边回路总抗阻

$$Z_{11}=R_1+j\omega L_1$$

副边回路总阻抗

$$Z_{22}=(R_2+R)+j(\omega L_2+X)$$

(1) 空心变压器



原边回路总抗阻

$Z_{11}=R_1+j\omega L_1$

副边回路总阻抗

$Z_{22}=(R_2+R)+j(\omega L_2+X)$

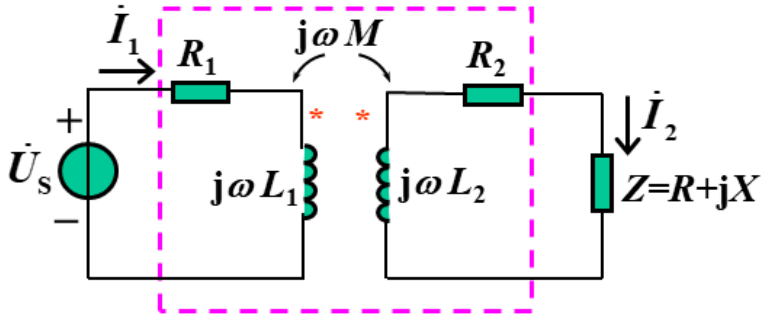
$$\begin{cases} Z_{11}\dot{I}_1 - j\omega M\dot{I}_2 = \dot{U}_S \\ -j\omega M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases}$$

$$\dot{I}_2 = \frac{j\omega M\dot{I}_1}{Z_{22}}$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$

原边(电源侧看过去的)等效电路

(1) 空心变压器



原边回路总抗阻

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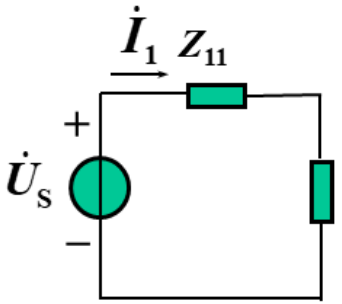
$$Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_{11} + \frac{(\omega M)^2}{Z_{22}}$$

$$\dot{I}_2 = \frac{j\omega M\dot{I}_1}{Z_{22}}$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$



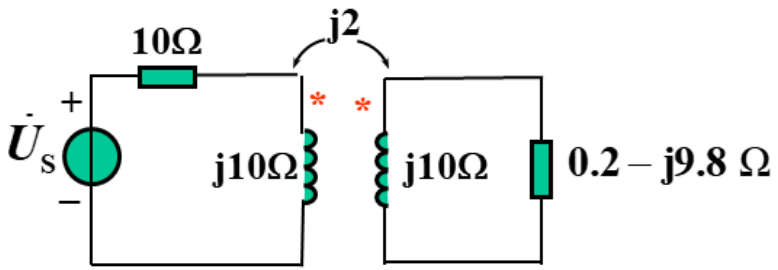
原边等效电路

$$Z_l \overset{\text{def}}{=} \frac{(\omega M)^2}{Z_{22}}$$

引入阻抗

单选题 1分

副边反映在原边回路中的引入阻抗为
（红包）



A $10 + j10\ \Omega$

B $8.32 + j0.41\ \Omega$

C $-\ j0.4\ \Omega$

D $10 - j10\ \Omega$

$Z_{11}=R_1+j\omega L_1$

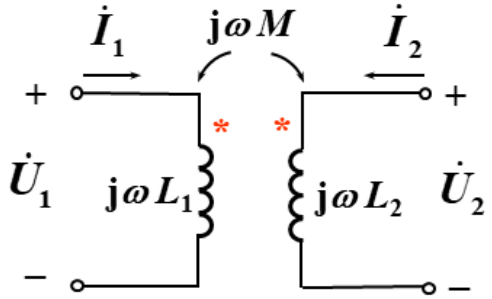
$Z_{22}=(R_2+R)+j(\omega L_2+X)$

$$Z_l \overset{\text{def}}{=} \frac{(\omega M)^2}{Z_{22}}$$

(2) 全耦合变压器 (unity-coupled transformer)

$M=\sqrt{L_1L_2}$

忽略电阻



$$\dot{U}_1 = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2 \qquad \dot{U}_2 = j\omega M\dot{I}_1 + j\omega L_2\dot{I}_2$$

$$= j\omega L_1\dot{I}_1 + j\omega\sqrt{L_1L_2}\,\dot{I}_2 \qquad = j\omega\sqrt{L_1L_2}\,\dot{I}_1 + j\omega L_2\dot{I}_2$$

$$\dot{U}_1\sqrt{L_2} = \dot{U}_2\sqrt{L_1}$$

$$\frac{\dot{U}_1}{\dot{U}_2} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} = n \rightarrow \text{变比}$$

变压器实现变压

单选题 1分

全耦合变压器原边线圈1000匝，副边线圈5000匝，原边电压有效值为_____时，副边电压有效值为1100V。

A 1100V

B 5500V

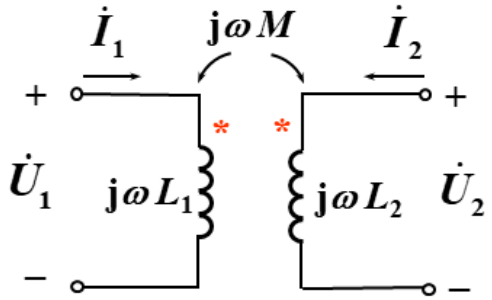
C 220V

D 条件不足，无法计算

全耦合变压器电压、电流关系

$$M=\sqrt{L_1L_2}$$

$$\frac{\dot{U}_1}{\dot{U}_2}=n \qquad \sqrt{\frac{L_1}{L_2}}=\frac{N_1}{N_2}=n$$



$$\dot{U}_1=j\omega L_1\dot{I}_1+j\omega M\dot{I}_2$$

$$\dot{I}_1=\frac{\dot{U}_1}{j\omega L_1}-\frac{M}{L_1}\dot{I}_2=\frac{\dot{U}_1}{j\omega L_1}-\frac{\sqrt{L_1L_2}}{L_1}\dot{I}_2=\frac{\dot{U}_1}{j\omega L_1}-\sqrt{\frac{L_2}{L_1}}\dot{I}_2$$

$$\dot{I}_1=\frac{\dot{U}_1}{j\omega L_1}-\frac{1}{n}\dot{I}_2$$

原边1000匝，副边5000匝的变压器
和原边1匝，副边5匝的变压器

有什么区别?

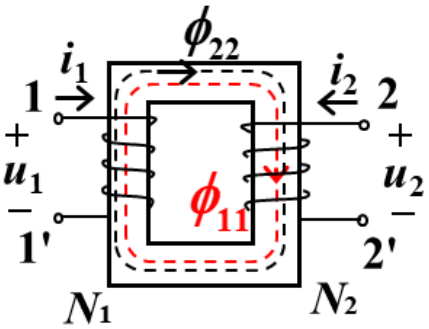
此处可以有弹幕

全耦合变压器电压、电流关系

$$\begin{cases} \dfrac{\dot{U}_1}{\dot{U}_2} = n \\ \dot{I}_1 = \dfrac{\dot{U}_1}{\mathrm{j}\omega L_1} - \dfrac{1}{n}\dot{I}_2 \end{cases}$$

(3) 理想变压器 (ideal transformer)

$$n=\frac{N_1}{N_2}$$



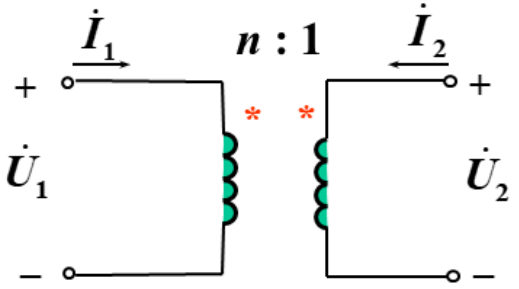
全耦合变压器

$$\begin{cases}\dfrac{\dot{U}_1}{\dot{U}_2}=n\\ \dot{I}_1=\dfrac{\dot{U}_1}{\mathrm{j}\omega L_1}-\dfrac{1}{n}\dot{I}_2\end{cases}$$

若$L_1\to\infty$(原因可以是磁导率$\mu\to\infty$或者匝数$N_1\to\infty$)，同时确保L_1/L_2比值不变($L_2\to\infty$)，且全耦合($M\to\infty$)，有

$$\begin{cases}\dot{U}_1=n\dot{U}_2\\ \dot{I}_1=-\dfrac{1}{n}\dot{I}_2\end{cases}$$

理想变压器的元件特性



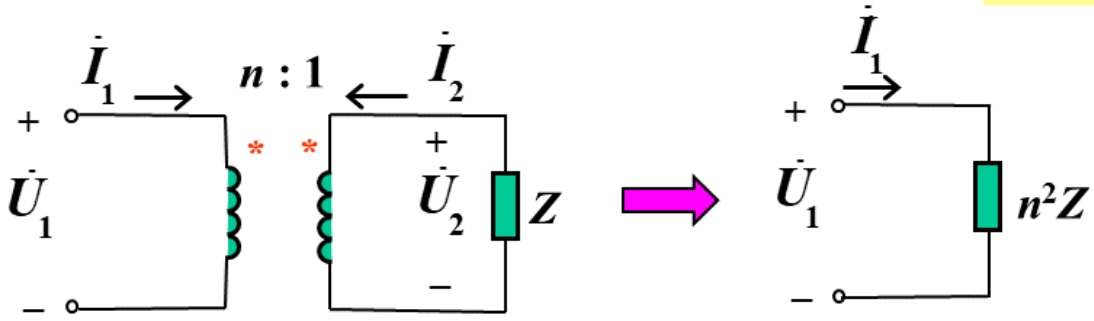
理想变压器的电路模型

理想变压器模型看不出电感!

理想变压器的阻抗变换性质：

$$\dot{U}_1 = n\dot{U}_2$$

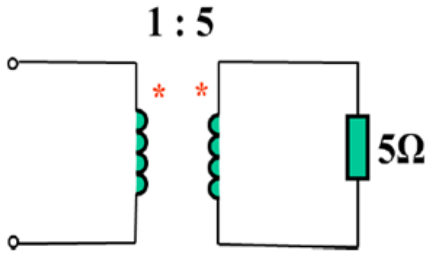
$$\dot{I}_1 = -\frac{1}{n}\dot{I}_2$$



$$\frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-\frac{1}{n}\dot{I}_2} = n^2\left(-\frac{\dot{U}_2}{\dot{I}_2}\right) = n^2 Z$$

单选题 1分

从理想变压器原边看进去，
等效电阻为



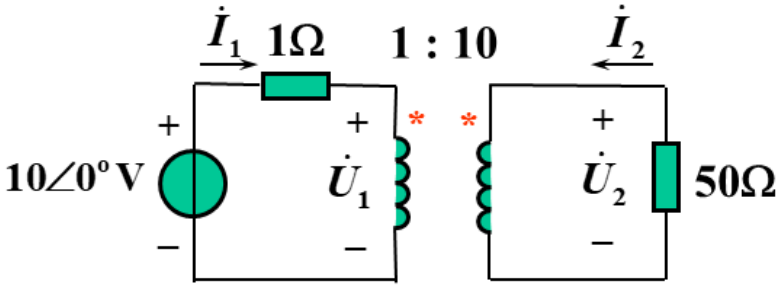
A 0.2 Ω

B 1 Ω

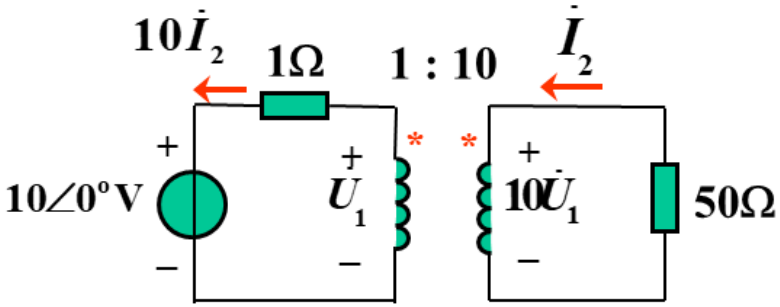
C 25 Ω

D 125 Ω

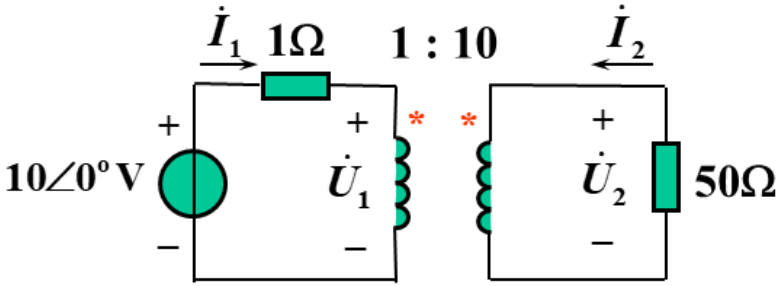
例　求图中电压$\dot{U}_2$。



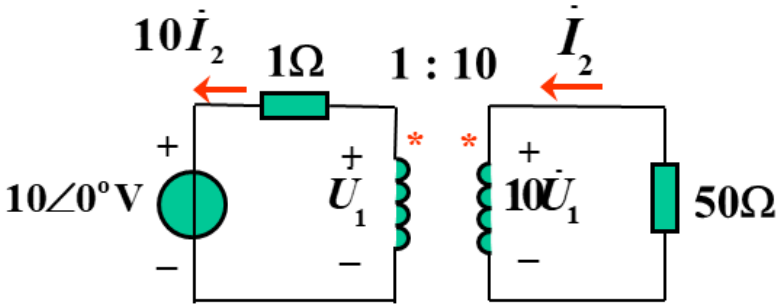
解：



例　求图中电压$\dot{U}_2$。



解：



$$\begin{cases} 10\dot{U}_1 = -50\dot{I}_2 \\ 10\dot{I}_2 + 10 = \dot{U}_1 \end{cases}$$

$$\dot{U}_2 = 33.3\angle 0^\circ \text{ V}$$