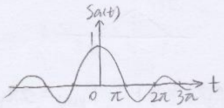
信号抽样 $f_d(n)=f_a(nT_s)$

$e^{j\omega t}=\cos\omega t+j\sin\omega t$

$\cos\omega t=\frac{1}{2}(e^{j\omega}+e^{-j\omega})$

$\sin\omega t=\frac{1}{2}(e^{j\omega}-e^{-j\omega})$

抽样信号 $Sa(t)=\frac{\sin t}{t}$



$\int_{-\infty}^{\infty}Sa(t)\,dt=\pi$

单位冲激 $f(t)\delta(t-t_0)=f(t_0)\delta(t-t_0)$

$\int_{-\infty}^{\infty}f(t)\delta(t-t_0)\,dt=f(t_0)$

$\delta(at)=\frac{1}{|a|}\delta(t)$

单位冲激偶 $\int_{-\infty}^{\infty}\delta'(t-t_0)f(t)\,dt=-f'(t_0)$

$f(t)\delta'(t)=f(0)\delta'(t)-f'(0)\delta(t)$

冲激脉冲抽样仍为连续信号

微分方程齐次解和特征根

λ单实根	$Ae^{\lambda t}$
λ k阶	$A_1e^{\lambda t}+A_2te^{\lambda t}+\cdots+A_kt^{k-1}e^{\lambda t}$
$\lambda_{1,2}=\sigma\pm j\omega$	$Ae^{(\sigma+j\omega)t}+Be^{(\sigma-j\omega)t}$ 或 $Ce^{\sigma t}\cos\omega t+De^{\sigma t}\sin\omega t$
$\lambda_{1,2}=\sigma\pm j\omega$ k阶	$A_1e^{(\sigma+j\omega)t}+A_2te^{(\sigma+j\omega)t}+\cdots+A_kt^{k-1}e^{(\sigma+j\omega)t}$ $+B_1e^{(\sigma-j\omega)t}+B_2te^{(\sigma-j\omega)t}+\cdots+B_kt^{k-1}e^{(\sigma-j\omega)t}$

微分方程有初始状态跳变，差分方程没有

激励函数对应特解

E	A
t^m	$A_mt^m+A_{m-1}t^{m-1}+\cdots+A_1t+A_0$
e^{pt}，p不是特征根	Ae^{pt}
e^{pt}，p k阶特征根	$A_0e^{pt}+A_1te^{pt}+\cdots+A_kt^ke^{pt}$
$\cos\omega t/\sin\omega t$，$\pm j\omega$不是	$A\cos\omega t+B\sin\omega t$
$\cos\omega t/\sin\omega t$，$\pm j\omega$ k阶	$A_0\cos\omega t+A_1t\cos\omega t+\cdots+A_kt^k\cos\omega t$ $+B_0\sin\omega t+B_1t\sin\omega t+\cdots+B_kt^k\sin\omega t$

卷积 $r(t)=e(t)*h(t)=\int_{-\infty}^{\infty}e(\tau)h(t-\tau)\,d\tau$

反褶、平移t、求函数积、净面积

$f(t)*\delta(t-t_0)=f(t-t_0)$ $f(t)*\delta^{(k)}(t-t_0)=f^{(k)}(t-t_0)$

差分方程 $\frac{dr_a(t)}{dt}=\frac{r_d(n)-r_d(n-1)}{T_s}$

$\frac{d^2r_a(t)}{dt^2}=\frac{r_d(n)-2r_d(n-1)+r_d(n-2)}{T_s^2}$

$r_d(0)=r_a(0)$

$r_d(-1)\approx r_a(0)-T_s r_a'(0)$

λ单实根	$A\lambda^n$
λ k阶	$A_1\lambda^n+A_2n\lambda^n+\cdots+A_kn^{k-1}\lambda^n$
$\lambda_{1,2}=\rho e^{\pm j\theta}$	$A\rho^ne^{j\theta n}+B\rho^ne^{-j\theta n}$ 或 $C\rho^n\cos\theta n+D\rho^n\sin\theta n$
$\lambda_{1,2}=\rho e^{\pm j\theta}$ k阶	$(A_1+A_2n+\cdots+A_kn^{k-1})\rho^ne^{j\theta n}+(B_1+B_2n+\cdots+B_kn^{k-1})\rho^ne^{-j\theta n}$

特解：

E	A
n^m	$A_mn^m+A_{m-1}n^{m-1}+\cdots+A_1n+A_0$
γ^n，γ不是特征根	$A\gamma^n$
γ^n，γ k阶特征根	$A_0\gamma^n+A_1n\gamma^n+\cdots+A_kn^k\gamma^n$
$\cos\theta n/\sin\theta n$，$e^{\pm j\theta}$不是	$A\cos\theta n+B\sin\theta n$
$\cos\theta n/\sin\theta n$，$e^{\pm j\theta}$ k阶	$A_0\cos\theta n+A_1n\cos\theta n+\cdots+A_kn^k\cos\theta n$ $+B_0\sin\theta n+B_1n\sin\theta n+\cdots+B_kn^k\sin\theta n$

零初始状态 $r_d(-1)=r_d(-2)=\cdots=r_d(-N)=0$

特解形式代入差分方程，全解代入初始条件

单位样值响应先迭代，再求零输入

卷积和 $r_d(n)=e_d(n)*h_d(n)=\sum_{k=-\infty}^{n}e_d(k)h_d(n-k)$

$f_d(n-m)*\delta_d(n-k)=f_d(n-m-k)$

内积 $\langle\phi_i(t),\phi_j(t)\rangle=\int_{t_1}^{t_2}\phi_i(t)\phi_j^*(t)\,dt$

$\langle\cos k\omega_1t,\cos k\omega_1t\rangle=\frac{T_1}{2}$

$\langle e^{jk\omega_1t},e^{jk\omega_1t}\rangle=T_1$

傅里叶级数 FS $a_0=\frac{1}{T_1}\int_{t_0}^{t_0+T_1}f(t)\,dt$

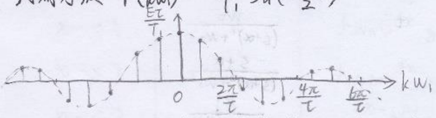
$a_k=\frac{2}{T_1}\int_{t_0}^{t_0+T_1}f(t)\cos k\omega_1t\,dt$

$\phi_k=\arctan(-\frac{b_k}{a_k})$

$F(k\omega_1)=\frac{1}{T_1}\int_{t_0}^{t_0+T_1}f(t)e^{-jk\omega_1t}\,dt$

$F(k\omega_1)=\frac{1}{2}(a_k-jb_k)$，$\phi(k\omega_1)=\phi_k$

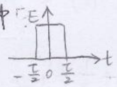
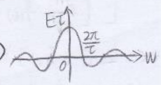
周期方波 $F(k\omega_1)=\frac{E\tau}{T_1}Sa(\frac{k\omega_1\tau}{2})$



时域压缩频域扩展，时域扩展频域压缩

傅里叶变换 FT $F(\omega)=\mathcal{F}[f(t)]=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}\,dt$

(幅值密度) $f(t)=\mathcal{F}^{-1}[F(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega$

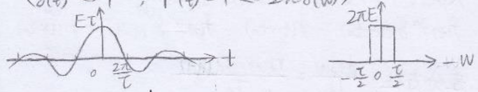
$f(t)$	$F(\omega)$
矩形脉冲 	$E\tau Sa(\frac{\omega\tau}{2})$ 
$\delta(t)$	1
E	$2\pi E\delta(\omega)$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$Ee^{-\alpha t}u(t)$	$\frac{E}{\alpha+j\omega}$
$\begin{cases}Ee^{-\alpha t}, & t>0\\ -Ee^{\alpha t}, & t<0\end{cases}$	$-j\frac{2E\omega}{\alpha^2+\omega^2}$
$sgn(t)$	$\frac{2}{j\omega}$
$\cos\omega_0t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
t	$j2\pi\delta'(\omega)$

(1) $R(\omega)=\int_{-\infty}^{\infty}f(t)\cos\omega t\,dt$，$I(\omega)=-\int_{-\infty}^{\infty}f(t)\sin\omega t\,dt$

实偶函数 $F(\omega)$实，实奇函数 $F(\omega)$纯虚

(2) $\mathcal{F}[F(t)]=2\pi f(-\omega)$

($\delta(t)\Leftrightarrow1$，$F(t)=1\Leftrightarrow2\pi\delta(\omega)$)



(3) $\mathcal{F}[f(at)]=\frac{1}{|a|}F(\frac{\omega}{a})$

$F(0)=f(0)\cdot\tau$，$f(0)=\frac{1}{2\pi}F(0)\cdot B_\omega$

(4) $\mathcal{F}[f(t\pm t_0)]=F(\omega)e^{\pm j\omega t_0}$

$\mathcal{F}[f(t)e^{\pm j\omega_0t}]=F(\omega\mp\omega_0)$

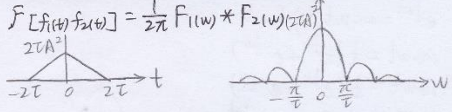
(5) $\mathcal{F}[\frac{d^nf(t)}{dt^n}]=(j\omega)^nF(\omega)$

$\mathcal{F}[t^nf(t)]=j^n\frac{d^nF(\omega)}{d\omega^n}$

(6) $\mathcal{F}[\int_{-\infty}^{t}f(\tau)d\tau]=\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$

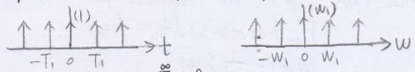
(7) $\mathcal{F}[f_1(t)*f_2(t)]=F_1(\omega)F_2(\omega)$

$\mathcal{F}[f_1(t)f_2(t)]=\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$



周期信号FT：$F_p(\omega)=2\pi\sum_{k=-\infty}^{\infty}F_{gp}(k\omega_1)\delta(\omega-k\omega_1)$

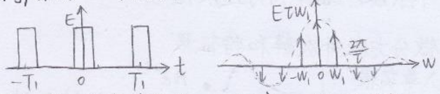
$\delta_p(t)=\sum_{i=-\infty}^{\infty}\delta(t-iT_1)\rightarrow\Delta_p(\omega)=\omega_1\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_1)$



延拓周期信号：$f_p(t)=\sum_{i=-\infty}^{\infty}f(t-iT_1)$

$F_p(\omega)=\omega_1\sum_{k=-\infty}^{\infty}F(\omega)|_{\omega=k\omega_1}\delta(\omega-k\omega_1)$ （$F(\omega)$的冲激抽样）

$2\pi F_{gp}(k\omega_1)=\omega_1F(\omega)|_{\omega=k\omega_1}$ （$F(\omega)$的数值抽样）



抽样定理：频率有限$(-\omega_m,\omega_m)\rightarrow\omega_s\geq2\omega_m$

拉普拉斯 $F(s)=L[f(t)]=\int_{0_-}^{\infty}f(t)e^{-st}\,dt$

$f(t)=L^{-1}[F(s)]=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}F(s)e^{st}\,ds$

$F(\omega)=F(s)|_{s=j\omega}+\pi\sum_{i=1}^{N}k_i\delta(\omega-\omega_i)$ $j\omega_i$是虚轴上单极点

$f(t)$	$F(s)$	收敛域
$\delta(t)$	1	s平面
$u(t)$	$\frac{1}{s}$	$\sigma>0$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$\sigma>-\alpha$
$e^{-\alpha t}\sin\omega_0t$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\sigma>-\alpha$
$e^{-\alpha t}\cos\omega_0t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\sigma>-\alpha$
$t^ne^{-\alpha t}$	$\frac{n!}{(s+\alpha)^{n+1}}$	$\sigma>-\alpha$

(1) $L[f(t-t_0)u(t-t_0)]=e^{-st_0}F(s)$

$L[f(t)e^{-s_0t}]=F(s+s_0)$

(2) $L[\frac{df(t)}{dt}]=sF(s)-f(0_-)$

$L[\frac{d^n}{dt^n}f(t)]=s^nF(s)-s^{n-1}f(0_-)-s^{n-2}f'(0_-)\cdots-f^{n-1}(0_-)$

$L[-tf(t)]=\frac{d}{ds}F(s)$

(3) $L[\int_{-\infty}^{t}f(x)dx]=\frac{1}{s}F(s)+\frac{1}{s}f^{(-1)}(0_-)$

$L[\frac{f(t)}{t}]=\int_s^{\infty}F(\lambda)\,d\lambda$

(4) $L[f_1(t)*f_2(t)]=F_1(s)F_2(s)$

$L[f_1(t)f_2(t)]=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}F_1(p)F_2(s-p)\,dp$

(5) $\lim_{t\to0+}f(t)=\lim_{s\to\infty}sF(s)$，$\lim_{t\to\infty}f(t)=\lim_{s\to0}sF(s)$

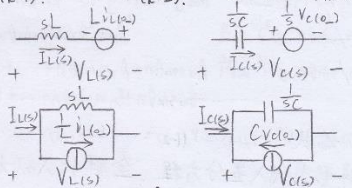
极点p_i单根 $A_i=[(s-p_i)F(s)]_{s=p_i}$

p_i k阶 $A_k=[\frac{1}{(k-1)!}\frac{d^{k-1}}{ds^{k-1}}[(s-p_1)^kF(s)]]_{s=p_1}$

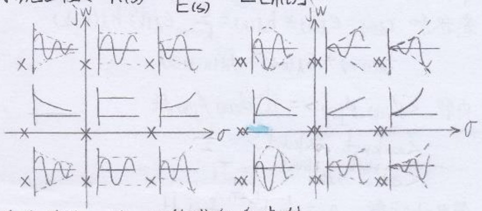
$f(t)=\frac{A_1}{(k-1)!}t^{k-1}e^{p_1t}+\frac{A_2}{(k-2)!}t^{k-2}e^{p_1t}+\cdots+A_ke^{p_1t}$

回路分析

结点分析



系统函数 $H(s)=\frac{R(s)}{E(s)}=L[h(t)]$



系统稳定 = $H(s)$收敛域包含虚轴

频率响应 $H(\omega)=\mathcal{F}[h(t)]=H(s)|_{s=j\omega}$

$<\phi_{di(n)}, \phi_{dj(n)}> = \sum_{n=n_0}^{n_0+N-1} \phi_{di(n)} \phi_{dj(n)}^{*}$

$\theta_1 = \frac{2\pi}{N_1}$ $\quad\quad T_s = \frac{T_1}{N_1}$

离散复指数函数集中，只有N_1个互不相同的元素

$X_{d(k)} = DFS[x_{d(n)}] = \sum_{n=n_0}^{n_0+N-1} x_{d(n)} e^{-j\theta_1 kn}$，周期$N_1$

$x_{d(n)} = IDFS[X_{d(k)}] = \frac{1}{N_1}\sum_{k=0}^{N_1-1} X_{d(k)} e^{j\theta_1 kn}$

$x_{d(n)}$偶，$X_{d(k)}$实

冲激抽样$x_{as(t)}$的FS：$X_{asg}(kw_1) = \frac{1}{T_1} X_{dg}(k)$

$\therefore X_{ag}(kw_1) = \frac{1}{N_1} X_{dg}(k)$

$X_{dg}(k)$是$X_{ag}(kw_1)$的周期延拓

混叠误差：k次谐波和$N_1 m \pm k$次谐波混叠

$X_d(e^{j\theta}) = DTFT[x_{d(n)}] = \sum_{n=-\infty}^{\infty} x_{d(n)} e^{-j\theta n}$，周期$2\pi$

$x_{d(n)} = IDTFT[X_d(e^{j\theta})] = \frac{1}{2\pi}\int_{2\pi} X_d(e^{j\theta}) e^{j\theta n} d\theta$

离散周期信号傅里叶变换不存在

(1) $DTFT[x_d(n-n_0)] = X_d(e^{j\theta}) e^{-j\theta n_0}$

$DTFT[x_{d(n)} e^{j\theta_0 n}] = X_d(e^{j(\theta-\theta_0)})$

(2) $DTFT[n x_{d(n)}] = j\frac{d}{d\theta}[X_d(e^{j\theta})]$

(3) $DTFT[x_{d(-n)}] = X_d(e^{-j\theta})$

(4) $x_{d(n)}$实，$|X_d(e^{j\theta})|$偶，$\phi_{d(\theta)}$奇

$x_{d(n)}$实偶，$X_d(e^{j\theta})$实偶；

$x_{d(n)}$实奇，$X_d(e^{j\theta})$纯虚奇

(5) $DTFT[x_{d(n)} * h_{d(n)}] = X_d(e^{j\theta}) H_d(e^{j\theta})$

$DTFT[x_{d(n)} h_{d(n)}] = \frac{1}{2\pi}[X_d(e^{j\theta}) * H_d(e^{j\theta})]$

(6) $\sum_{n=-\infty}^{\infty} |x_{d(n)}|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X_d(e^{j\theta})|^2 d\theta$

$X_{a(w)} = T_s X_d(e^{j\theta})$

$DFS[x_{dp(n)}] = DTFT[x_{d(n)}]_{\theta=k\theta_1}$

延拓周期信号的DFS是原非周期信号DTFT的抽样

$X_{d(z)} = Z[x_{d(n)}] = \sum_{n=-\infty}^{\infty} x_{d(n)} z^{-n}$

$x_{d(n)} = Z^{-1}[X_{d(z)}] = \frac{1}{j2\pi}\oint_c X_{d(z)} z^{n-1} dz$

$Z[\delta_{d(n)}] = 1$ $\quad , 0<|z|<\infty$

$Z[\delta_d(n-m)] = z^{-m}$ $\quad , 0<|z|$

$Z[\delta_d(n+m)] = z^{m}$ $\quad , |z|<\infty$

$Z[u_{d(n)}] = \frac{z}{z-1}$ $\quad , |z|>1$

$Z[u_d(n-1)] = \frac{1}{z-1}$ $\quad , |z|>1$

$Z[u_d(-n)] = \frac{1}{1-z}$ $\quad , |z|<1$

	$x_{d(n)}$	$X_{d(z)}$	收敛域
	$a^n u_{d(n)}$	$\frac{z}{z-a}$	$|z|>|a|$
右边	$n a^n u_{d(n)}$	$\frac{az}{(z-a)^2}$	$|z|>|a|$
	$\frac{(n+1)\cdots(n+m)a^n u_{d(n)}}{m!}$	$\frac{z^{m+1}}{(z-a)^{m+1}}$	$|z|>|a|$
左边	$-a^n u_{d(-n-1)}$	$\frac{z}{z-a}$	$|z|<|a|$

(1) 双边 $Z[x_{d(n\pm m)}] = z^{\pm m} X_{d(z)}$

右边 $Z[x_d(n+m)] = z^m [X_{d(z)} - \sum_{k=0}^{m-1} x_{d(k)} z^{-k}]$

$Z[x_d(n-m)] = z^{-m} X_{d(z)}$

(2) 两序列线性组合零极点相消，收敛域扩大

(3) $Z[n x_{d(n)}] = -z\frac{d}{dz} X_{d(z)}$

$Z[a^n x_{d(n)}] = X_d(\frac{z}{a})$

(4) $Z[x_{d(n)} * h_{d(n)}] = X_{d(z)} H_{d(z)}$

(5) $x_{d(0)} = \lim_{z\to\infty} X_{d(z)}$

$\lim_{n\to\infty} x_{d(n)} = \lim_{z\to 1}[(z-1) X_{d(z)}]$

DTFT $X_d(e^{j\theta}) = X_{d(z)}|_{z=e^{j\theta}}$ （取$r=1$）

若收敛域不含单位圆，则$DTFT[x_{d(n)}]$不存在

$e^{sT_s} = z$，$s = \frac{\ln z}{T_s}$，$r = e^{\sigma T_s}$，$\theta = wT_s$

$X_{a(s)} = \sum_{i=1}^{M} \frac{A_i}{s-p_i}$

$X_{d(z)} = \sum_{i=1}^{M} \frac{A_i z}{z-e^{p_i T_s}}$

冲激响应不变：

$H_{d(z)} = Z[T_s h_a(nT_s)] = T_s \sum_{i=1}^{n} \frac{A_i z}{z-e^{p_i T_s}}$

$h_{d(n)} = T_s h_a(nT_s)$

z变换解差分方程

$r_{d(n-2)}$ $\quad z^{-2}R_{d(z)} + z^{-1} r_{d(-1)} + r_{d(-2)}$

$r_{d(n+2)}$ $\quad z^{2}R_{d(z)} - z^{2} r_{d(0)} - z\, r_{d(1)}$

系统稳定：收敛域含单位圆

所有极点在单位圆内（因果系统）

$W_N = e^{-j\frac{2\pi}{N}}$

$X_{d(k)} = DFT[x_{d(n)}] = \sum_{n=0}^{N-1} x_{d(n)} e^{-j\frac{2\pi}{N}kn}$

$= \sum_{n=0}^{N-1} x_{d(n)} W_N^{kn}$ $\quad , k=0,1,\cdots,N-1$

$x_{d(n)} = IDFT[X_{d(k)}] = \frac{1}{N}\sum_{k=0}^{N-1} X_{d(k)} e^{j\frac{2\pi}{N}kn}$

$x_{d(n)}$是周期信号抽样时，DFT就是DFS

$x_{d(n)}$是非周期信号抽样时，DFT是DTFT抽样

$x_{d(n)}$ N为周期延拓 $x_{d((n))_N}$

线时移取主值 $x_{d((n\pm m))_N} G_{dN(n)}$

$\sum_{n=0}^{N-1} |x_{d(n)}|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X_{d(k)}|^2$

FFT：$\frac{N}{2}$个偶数$+\frac{N}{2}$个奇数的DFT，要求$N=2^M$

$W_N^{kn} = W_N^{((kn))_N}$，$W_N^{2} = W_{\frac{N}{2}}$

$W_N^{(kn+\frac{N}{2})} = -W_N^{kn}$

抽样率 $f_s = \frac{1}{T_s}$ $\quad\quad f_s > 2f_m$

抽样长度T_1

频率分辨率 $f_1 = \frac{1}{T_1} = \frac{f_s}{N}$

不失真传输：幅频特性是常数，相频特性是过原点直线

理想滤波器物理不可实现：非因果

巴特沃兹低通滤波 $|H_{a(w)}| = \frac{1}{\sqrt{1+(\frac{w}{w_c})^{2n}}}$

$H_{(s)} = \frac{1}{B_n(s')}\Big|_{s'=\frac{s}{w_c}}$

$-20\lg|H(w_p)| \le \alpha_p$，$-20\lg|H(w_e)| \ge \alpha_e$

对离散信号$\theta=2k\pi$是最低频，$\theta=(2k+1)\pi$是最高频

递归算法 $r_{d(n)} = \sum_{i=0}^{M} b_i e_{d(n-i)} - \sum_{k=1}^{N} a_k r_{d(n-k)}$

$H_{d(z)} = \frac{\sum_{i=0}^{M} b_i z^{-i}}{1+\sum_{k=1}^{N} a_k z^{-k}}$ $\quad\quad$ IIR

非递归算法 $r_{d(n)} = \sum_{i=0}^{M} b_i e_{d(n-i)}$

$H_{d(z)} = \sum_{i=0}^{M} b_i z^{-i}$ $\quad\quad$ FIR

双线性变换 $s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}$

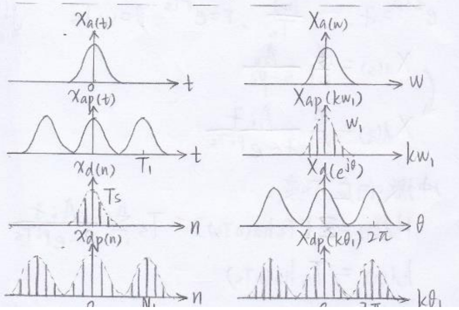
$z = \frac{1+\frac{T_s}{2}s}{1-\frac{T_s}{2}s}$

给定模拟滤波器 (w_{Ap}, α_p) (w_{Ae}, α_e)

$\theta = wT_s$ 得到 (θ_p, α_p) (θ_e, α_e)

$w = \frac{2}{T_s}\tan\frac{\theta}{2}$ 得到 (w_p, α_p) (w_e, α_e)

$H_{d(z)} = H_{a(s)}\Big|_{s=\frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}}$



四种变换的关系：

(1) $x_{ap(t)}$是$x_{a(t)}$ T_1周期延拓，$X_{ap}(kw_1)$是$X_{a(w)}$抽样

$X_{ap}(kw_1) = \frac{1}{T_1} X_{a(w)}|_{w=kw_1}$

(2) $x_{dp(n)}$是$x_{d(n)}$ N_1周期延拓，$X_{dp}(k\theta_1)$是$X_d(e^{j\theta})$抽样

$X_{dp}(k\theta_1) = X_d(e^{j\theta})|_{\theta=k\theta_1}$

(3) $x_{d(n)}$是$x_{a(t)}$ T_s抽样，$X_d(e^{j\theta})$是$X_{a(w)}$ w_s延拓

$X_d(e^{j\theta}) = \frac{1}{T_s}[\sum_{m=-\infty}^{\infty} X_a(w+mw_s)]_{w=\frac{\theta}{T_s}}$

满足抽样定理：$X_d(e^{j\theta})|_{\theta=\frac{w}{T_s}} = \frac{1}{T_s} X_{a(w)}$

(4) $x_{dp(n)}$是$x_{ap(t)}$ T_s抽样，且$T_s N_1 = T_1$

$X_{dp}(k\theta_1)$是$X_{ap}(kw_1)$ $w_s = w_1 N_1$周期延拓

$X_{dp}(k\theta_1) = N_1[\sum_{m=-\infty}^{\infty} X_{ap}(kw_1+mw_s)]_{kw_1=\frac{k\theta_1}{T_s}}$

满足抽样定理：$X_{dp}(k\theta_1) = N_1 X_{ap}(kw_1)|_{kw_1=\frac{k\theta_1}{T_s}}$