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2-5　平行板电容器有两层电介质，厚度分别为 d_1 和 d_2，电导率为 σ_1 和 σ_2，介电常数为 ε_1 和 ε_2。电容器上加电压为 U，参见图题2-5。

(1) 若 $\sigma_1=\sigma_2=\sigma$，$\varepsilon_1>\varepsilon_2$；

(2) 若 $\varepsilon_1=\varepsilon_2=\varepsilon$，$\sigma_1>\sigma_2$；

(3) 若 $\sigma_1\neq\sigma_2$，$\varepsilon_1\neq\varepsilon_2$；

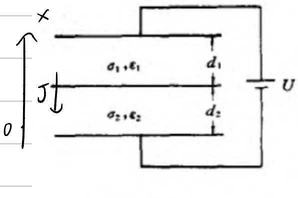

图　题 2-5

(1) 记电流密度为 $\vec{J_1}$，$\vec{J_2}$

由于 $J_{1n}=J_{2n}$，且 $\vec{J_1}$，$\vec{J_2}$ 沿法向

故可记 $\vec{J_1}=\vec{J_2}=\vec{J}$

$$U=\frac{J}{\sigma}\cdot(d_1+d_2) \iff J=\frac{U\sigma}{d_1+d_2}$$

$$\rho_s=D_{2n}-D_{1n}=\varepsilon_2E_2-\varepsilon_1E_1=\frac{U}{d_1+d_2}\cdot(\varepsilon_2-\varepsilon_1)$$

综上

$$\begin{cases}\vec{E_1}=\vec{E_2}=\dfrac{U}{d_1+d_2}\cdot\vec{n}\\ \vec{J_1}=\vec{J_2}=\dfrac{U\sigma}{d_1+d_2}\cdot\vec{n}\\ \varphi(x)=\dfrac{Ux}{d_1+d_2}\\ \rho_s=\dfrac{U}{d_1+d_2}(\varepsilon_2-\varepsilon_1)\end{cases}$$

$\vec{n}$ 为从上极板指向下极板的单位矢量

(2) 同理有 $\vec{J_1}=\vec{J_2}=\vec{J}$

$$\therefore U=E_1d_1+E_2d_2=J\left(\frac{\varepsilon_1}{\sigma_1}+\frac{\varepsilon_2}{\sigma_2}\right)$$

$$\therefore J=\frac{U}{\frac{d_1}{\sigma_1}+\frac{d_2}{\sigma_2}}=\frac{U\sigma_1\sigma_2}{d_1\sigma_2+d_2\sigma_1}$$

$$E_1=\frac{U\sigma_2}{d_1\sigma_2+d_2\sigma_1},\quad E_2=\frac{U\sigma_1}{d_1\sigma_2+d_2\sigma_1}$$

$$\rho_s=D_{2n}-D_{1n}=\frac{U(\sigma_1-\sigma_2)\varepsilon}{d_1\sigma_2+d_2\sigma_1}$$

综上

$$\begin{cases}\vec{E_1}=\dfrac{U\sigma_2}{d_1\sigma_2+d_2\sigma_1}\vec{n}\\ \vec{E_2}=\dfrac{U\sigma_1}{d_1\sigma_2+d_2\sigma_1}\vec{n}\\ \vec{J_1}=\vec{J_2}=\dfrac{U\sigma_1\sigma_2}{d_1\sigma_2+d_2\sigma_2}\cdot\vec{n}\\ \varphi=\begin{cases}\dfrac{U\sigma_1}{d_1\sigma_2+d_2\sigma_1}x, & 0\le x\le d_1\\ \dfrac{U[\sigma_1d_2+\sigma_2(x-d_1)]}{d_1\sigma_2+d_2\sigma_1}, & d_2<x\le d_1+d_2\end{cases}\\ \rho_s=\dfrac{U(\sigma_1-\sigma_2)\varepsilon}{d_1\sigma_2+d_2\sigma_1}\end{cases}$$

(3) 同理 $\vec{J_1}=\vec{J_2}=\vec{J}$

$$U=E_1d_1+E_2d_2=J\cdot\left(\frac{d_1}{\sigma_1}+\frac{d_2}{\sigma_2}\right)$$

$$J=\frac{U\sigma_1\sigma_2}{d_1\sigma_2+d_2\sigma_1}$$

$$\rho_s=P_{2n}-P_{1n}=\frac{U(\sigma_1\varepsilon_2-\sigma_2\varepsilon_1)}{d_1\sigma_2+d_2\sigma_1}$$

$\Downarrow$

$$
\begin{cases}
\vec{E}_1 = \dfrac{U\sigma_2}{d_1\sigma_2 + d_2\sigma_1}\,\vec{n} \\[2ex]
\vec{E}_2 = \dfrac{U\sigma_1}{d_1\sigma_2 + d_2\sigma_1}\,\vec{n} \\[2ex]
\vec{J}_1 = \vec{J}_2 = \dfrac{U\sigma_1\sigma_2}{d_1\sigma_2 + d_2\sigma_2}\cdot\vec{n} \\[2ex]
\varphi = \begin{cases} \dfrac{U\sigma_1}{d_1\sigma_2 + d_2\sigma_1}\,x, & 0 \le x \le d_1 \\[2ex] \dfrac{U[\sigma_1 d_2 + \sigma_2 (x - d_1)]}{d_1\sigma_2 + d_2\sigma_1}, & d_2 < x \le d_1 + d_2 \end{cases} \\[2ex]
\rho_s = \dfrac{U(\sigma_1\varepsilon_2 - \sigma_2\varepsilon_1)}{d_1\sigma_2 + d_2\sigma_1}
\end{cases}
$$