Example Find Q for reflection in the plane V with equation $x+2y+3z=0$ in $\mathbb{R}^3$.

Perpendicular vector: $\begin{bmatrix}1\\2\\3\end{bmatrix} \rightsquigarrow \vec{u} = \frac{1}{\sqrt{1+4+9}}\begin{bmatrix}1\\2\\3\end{bmatrix} = \frac{1}{\sqrt{14}}\begin{bmatrix}1\\2\\3\end{bmatrix}$

So $Q = I - 2\vec{u}\vec{u}^T = I - \frac{2}{14}\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}$

$$= \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \frac{1}{7}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix} = \frac{1}{7}\begin{bmatrix}6 & -2 & -3\\-2 & 3 & -6\\-3 & -6 & -2\end{bmatrix}$$

If Q is square ($m=n$), we call Q and <u>orthogonal matrix</u>

For an orthogonal matrix Q, $Q^{-1} = Q^T$ since $Q^TQ = I$.

So orthogonal matrices are invertible.

<u>Last time</u>: Orthonormal set of vectors $\{\vec{q}_1, \vec{q}_2, \dots, \vec{q}_n\}$

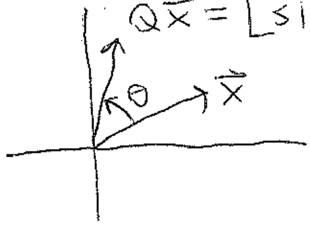
(unit vectors, all perpendicular to each other, so $\vec{q}_i \cdot \vec{q}_j = \begin{cases}1 & \text{if } i=j\\0 & \text{if } i\neq j\end{cases}$

<u>Orthogonal matrix</u> $Q = \begin{bmatrix}\vec{q}_1 & \vec{q}_2 & \cdots & \vec{q}_n\end{bmatrix}$ ← $n\times n$ matrix with orthonormal columns

invertible: $Q^{-1} = Q^T$, or $\boxed{Q^TQ = I}$

<u>1×1 orthogonal matrices</u>: $[1]$ and $[-1]$

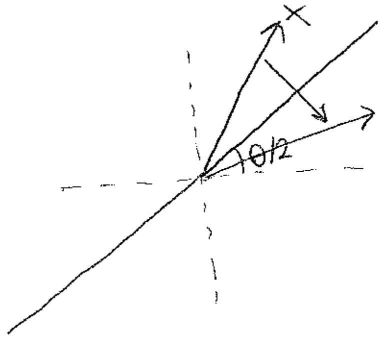
<u>2×2 orthogonal matrices</u>: Rotations



$Q\vec{x} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$

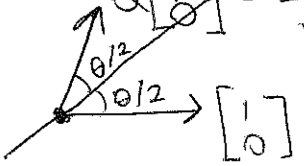
This matrix rotates counterclockwise by θ

Reflections:

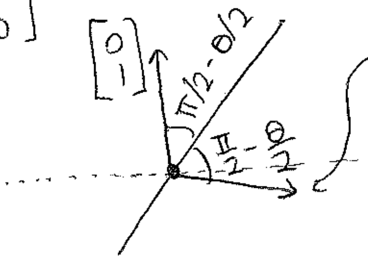


$Q\vec{x} = \begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$

This matrix reflects vectors across $\theta/2$ line



$Q\begin{bmatrix}1\\0\end{bmatrix} =$ ~~clockwise~~ counterclockwise rotation by θ $= \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$ as before



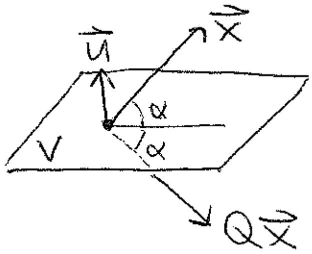
$Q\begin{bmatrix}0\\1\end{bmatrix} =$ clockwise rotation by $\pi - \theta$

$=$ counterclockwise rotation by $\theta - \pi$

$$= \begin{bmatrix}-\sin(\theta-\pi)\\ \cos(\theta-\pi)\end{bmatrix} = \begin{bmatrix}+\sin\theta\\ -\cos\theta\end{bmatrix}$$

So $Q = \begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}$ reflects $\vec{x}$ across the $\theta/2$ line.

Reflection in higher dimensions ($\mathbb{R}^n$):



$V =$ "hyperplane" in $\mathbb{R}^n$ ($(n-1)$-dimensional subspace)

$\vec{u} =$ unit vector $\perp$ to V (basis for $V^\perp$)

$Q =$ matrix that reflects $\vec{x}$ across the hyperplane.

Claim: $Q = I - 2\vec{u}\vec{u}^T$

Why does this work? Check that Q does what we expect to $\vec{u}$ and to vectors in V

① It sends $\vec{u}$ to $-\vec{u}$:

$$Q\vec{u} = (I - 2\vec{u}\vec{u}^T)\vec{u} = I\vec{u} - 2\vec{u}\underbrace{(\vec{u}^T\vec{u})}_{1} = \vec{u} - 2\vec{u} = -\vec{u} \checkmark$$

② It doesn't change vectors in V:

$$Q\vec{v} = (I - 2\vec{u}\vec{u}^T)\vec{v} = I\vec{v} - 2\vec{u}\underbrace{(\vec{u}^T\vec{v})}_{0 \text{ because } \vec{u} \text{ is } \perp \text{ to } V} = \vec{v} \checkmark$$

Is this Q really an orthogonal matrix?

Yes! $Q^TQ = (I - 2\vec{u}\vec{u}^T)^T(I - 2\vec{u}\vec{u}^T) = (I - 2\vec{u}\vec{u}^T)(I - 2\vec{u}\vec{u}^T)$

$$= I - 2\vec{u}\vec{u}^T - 2\vec{u}\vec{u}^T + 4\vec{u}\underbrace{\vec{u}^T\vec{u}}_{1 \text{ because } \|\vec{u}\| = 1}\vec{u}^T$$

$$= I \checkmark$$

Now: What's great about orthogonal matrices (besides having easy inverses)?

They don't change the geometry of vectors:

① $Q\vec{x}$ has the same length as $\vec{x}$.

② The angle between $Q\vec{x}$ and $Q\vec{y}$ is the same as between $\vec{x}$ and $\vec{y}$.

Why? Check lengths: $\|Q\vec{x}\| = \sqrt{(Q\vec{x})^T(Q\vec{x})} = \sqrt{\vec{x}^T \underbrace{Q^T Q}_{I} \vec{x}}$

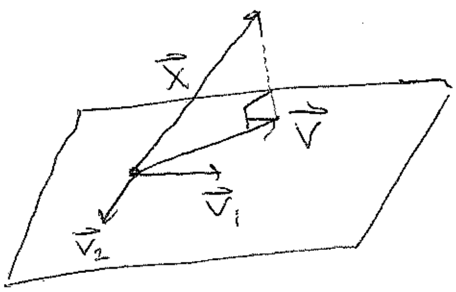
$$= \sqrt{\vec{x}^T\vec{x}} = \|\vec{x}\| \quad \checkmark$$

Same with angles because $(Q\vec{x})^T Q\vec{y} = \vec{x}^T \underbrace{Q^T Q}_{I} \vec{y} = \vec{x}^T\vec{y}$

$(Q\vec{x})\cdot(Q\vec{y}) = \vec{x}\cdot\vec{y}$, i.e., Q "preserves dot products.

Note: Rotations and reflections indeed don't change lengths or angles between vectors, so that's why they are orthogonal matrices.

Another nice feature of orthonormal vectors: they make projections easy: Remember how to project $\vec{x}$ onto V:



$$\vec{v} = P\vec{x} \qquad A(A^TA)^{-1}A^T$$

$$A = \underbrace{\begin{bmatrix} | & | & & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_m \\ | & | & & | \end{bmatrix}}_{\text{Basis for } V}$$

P is a little complicated...

But what if $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is an orthonormal basis for V, i.e. the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are orthogonal unit vectors?

Then we should call A, Q instead: $Q = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \end{bmatrix}$.

And P is much simpler: $P = Q(\underbrace{Q^TQ}_{I})^{-1}Q^T = \boxed{QQ^T}$

<u>Important special case</u>: What if $V =$ all of $\mathbb{R}^n$?

Then proj. of $\vec{x}$ onto V is just $\vec{x}$ itself, and $P = QQ^T$ is just I.
(Q is an orthogonal matrix.)

This tells us how to write $\vec{x}$ as a linear combination of orthonormal basis vectors.

$$\vec{x} = I\vec{x} = \underbrace{Q}_{\begin{bmatrix} \vec{q}_1 & \vec{q}_2 & \cdots & \vec{q}_n \end{bmatrix}} \underbrace{(Q^T\vec{x})}_{\begin{bmatrix} \vec{q}_1^T\vec{x} \\ \vec{q}_2^T\vec{x} \\ \vdots \\ \vec{q}_n^T\vec{x} \end{bmatrix}}$$

$$= \underbrace{(\vec{q}_1 \cdot \vec{x})}_{c_1} q_1 + \underbrace{(\vec{q}_2 \cdot \vec{x})}_{c_2} \vec{q}_2 + \cdots + \underbrace{(\vec{q}_n \cdot \vec{x})}_{c_n} \vec{q}_n$$

These coefficients are easy to find!

<u>Essential point</u>: It's easy to find how to write <u>any</u> $\vec{x}$ in $\mathbb{R}^n$ as a linear combination of <u>orthonormal</u> basis vectors.

<u>Example</u> $\left\{ \vec{q}_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}, \vec{q}_2 = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \right\}$ is an orthonormal bas for $\mathbb{R}^2$. How do we write $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as a linear combination?

Answer: $\vec{x} = (\vec{q}_1 \cdot \vec{x})\vec{q}_1 + (\vec{q}_2 \cdot \vec{x})\vec{q}_2$

$$= \left(\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \vec{q}_1 + \left(\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) \vec{q}_2$$

$$= (\cos\theta + \sin\theta) \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + (\cos\theta - \sin\theta) \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

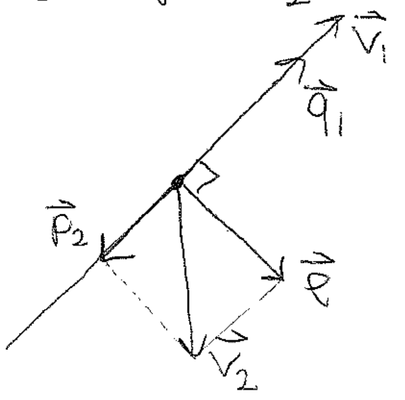
Orthonormal bases are nice, but how do we find them?

<u>Gram-Schmidt Process</u>: Algorithm for turning <u>any</u> basis into an orthonormal one: $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \longrightarrow \{\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_n\}$.

Given a basis $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ for a subspace V:

① "Normalize" $\vec{v}_1$: $\vec{q}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1$ (Now write $Q_1 = \begin{bmatrix} | \\ \vec{q}_1 \\ | \end{bmatrix}$.)

② Project $\vec{v}_2$ onto span$(\vec{q}_1)$ and take $\vec{q}_2$ to be the "normalized error vector":



$$\vec{e} = \vec{v}_2 - \vec{p}_2 = \vec{v}_2 - Q_1Q_1^T\vec{v}_2$$

matrix for projection onto span $(\vec{q}_1)$

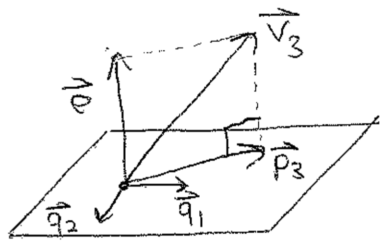
same as $\frac{\vec{v}_1\vec{v}_1^T}{\vec{v}_1^T\vec{v}_1}$

Then $\vec{q}_2 = \frac{1}{\|\vec{e}\|}\vec{e}$

$$= \frac{\vec{v}_2 - Q_1Q_1^T\vec{v}_2}{\|\vec{v}_2 - Q_1Q_1^T\vec{v}_2\|}$$

(Now write $Q_2 = \begin{bmatrix} | & | \\ \vec{q}_1 & \vec{q}_2 \\ | & | \end{bmatrix}$.)

③ Continue: Project $\vec{v}_3$ onto span$(\vec{q}_1, \vec{q}_2)$, take $\vec{q}_3$ to be the normalized error vector.



$$\vec{q}_3 = \frac{1}{\|\vec{e}\|}\vec{e}$$

$$\vec{v}_3 - \underbrace{Q_2Q_2^T\vec{v}_3}_{(\vec{q}_1^T\vec{v}_3)\vec{q}_1 + (\vec{q}_2^T\vec{v}_3)\vec{q}_2}$$

④ Continue in the same way for remaining vectors.

<u>Example</u> Find an orthonormal basis for the plane $x+2y+3z=0$ in $\mathbb{R}^3$.

First find <u>any</u> basis: $V = \text{all} \begin{bmatrix} -2y-3z \\ y \\ z \end{bmatrix} = \text{all } y\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$\vec{v}_1$ $\vec{v}_2$

Now apply Gram-Schmidt:

① $\vec{q}_1 = \frac{1}{\|\vec{v}_1\|}\vec{v}_1 = \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

② "Error" vector: $\vec{v}_2 - Q_1Q_1^T\vec{v}_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} -2 & 1 & 0 \end{bmatrix}\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} - \frac{6}{5}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}.$$ Now normalize: $\vec{q}_2 = \frac{1}{\|\vec{e}\|}\vec{e}$

$$\vec{q}_2 = \frac{1}{\sqrt{9+36+25}}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix} = \frac{1}{\sqrt{70}}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}$$

So <u>orthonormal basis</u> for the plane is: $\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{70}}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix} \right\}$

What if we wanted to turn $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ into an orthonormal basis for $\mathbb{R}^3$?

We would get $\vec{q}_1 = \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{q}_2 = \frac{1}{\sqrt{70}}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}$ again. What about $\vec{q}_3$?

"Error" vector: $\vec{e} = \vec{v}_3 - Q_2Q_2^T\vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -2/\sqrt{5} & -3/\sqrt{70} \\ 1/\sqrt{5} & -6/\sqrt{70} \\ 0 & 5/\sqrt{70} \end{bmatrix} \cdot \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -3/\sqrt{70} & -6/\sqrt{70} & 5/\sqrt{70} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -2/\sqrt{5} & -3/\sqrt{70} \\ 1/\sqrt{5} & -6/\sqrt{70} \\ 0 & 5/\sqrt{70} \end{bmatrix}\begin{bmatrix} -2/\sqrt{5} \\ -3/\sqrt{70} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 4/5 + 9/70 \\ -2/5 + 18/70 \\ -15/70 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 65/70 \\ -10/70 \\ -15/70 \end{bmatrix} = \frac{1}{70}\begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Finally, normalize: $\vec{q}_3 = \frac{1}{\sqrt{9+4+1}}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{\sqrt{14}}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Orthonormal basis for $\mathbb{R}^3$:

$$\left\{ \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{70}}\begin{bmatrix} -3 \\ -6 \\ 5 \end{bmatrix}, \frac{1}{\sqrt{14}}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$$

Maybe not as "easy" as $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, but Gram-Schmidt is useful because it gives you orthonormal basis for <u>any</u> subspace.

QR Factorization

$\{\vec v_1, \vec v_2, \dots, \vec v_n\} \xrightarrow{\text{Gram-Schmidt}} \{\vec q_1, \vec q_2, \dots, \vec q_n\}$

Any basis for V — Orthonormal basis

$A = \begin{bmatrix} \vec v_1 & \vec v_2 & \cdots & \vec v_n \end{bmatrix}$ $\xleftrightarrow[\text{matrices related?}]{\text{How are these}}$ $Q = \begin{bmatrix} \vec q_1 & \vec q_2 & \cdots & \vec q_n \end{bmatrix}$

Write $\vec v$'s as linear combinations of orthonormal $\vec q$'s:

$$\vec v_1 = (\vec q_1 \cdot \vec v_1)\vec q_1 + (\vec q_2 \cdot \vec v_2)\vec q_2 + \cdots + (\vec q_n \cdot \vec v_n)\vec q_n$$

$$= \begin{bmatrix} \vec q_1 & \vec q_2 & \cdots & \vec q_n \end{bmatrix} \begin{bmatrix} \vec q_1 \cdot \vec v_1 \\ \vec q_2 \cdot \vec v_1 \\ \vdots \\ \vec q_n \cdot \vec v_1 \end{bmatrix} = Q \begin{bmatrix} \vec q_1 \cdot \vec v_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

These are all 0 since $\vec v_1 \perp \vec q_i$ for $i > 1$.

1st column of a matrix, R

2nd column: $\vec v_2 = (\vec q_1 \cdot \vec v_2)\vec q_1 + (\vec q_2 \cdot \vec v_2)\vec q_2 + \cdots + (\vec q_n \cdot \vec v_2)\vec q_n$

$$= Q \begin{bmatrix} \vec q_1 \cdot \vec v_2 \\ \vec q_2 \cdot \vec v_2 \\ \vdots \\ \vec q_n \cdot \vec v_n \end{bmatrix} = Q \begin{bmatrix} \vec q_1 \cdot \vec v_2 \\ \vec q_2 \cdot \vec v_2 \\ \vdots \\ 0 \end{bmatrix}$$

These are all 0, since $\vec q_i \perp \vec v_2$ if $i > 2$

2nd column of R.

Continue, and get

$$A = \begin{bmatrix} \vec v_1 & \vec v_2 & \cdots & \vec v_n \end{bmatrix} = Q \underbrace{\begin{bmatrix} \vec q_1 \cdot \vec v_1 & \vec q_1 \cdot \vec v_2 & \cdots & \vec q_1 \cdot \vec v_n \\ 0 & \vec q_2 \cdot \vec v_2 & \cdots & \vec q_2 \cdot \vec v_n \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \vec q_n \cdot \vec v_n \end{bmatrix}}_{\text{Upper triangular matrix } R.}$$

Example

$A = \begin{bmatrix} -2 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, We saw $Q = \begin{bmatrix} -2/\sqrt{5} & -3/\sqrt{70} & 1/\sqrt{14} \\ 1/\sqrt{5} & -6/\sqrt{70} & 2/\sqrt{14} \\ 0 & 5/\sqrt{70} & 3/\sqrt{14} \end{bmatrix}$

What is R?

$$R = \begin{bmatrix} q_1 \cdot v_1 & q_1 \cdot v_2 & q_1 \cdot v_3 \\ 0 & q_2 \cdot v_2 & q_2 \cdot v_3 \\ 0 & 0 & q_3 \cdot v_3 \end{bmatrix} = \begin{bmatrix} \sqrt{5} & 6/\sqrt{5} & -2/\sqrt{5} \\ 0 & \sqrt{14/5} & -3/\sqrt{70} \\ 0 & 0 & 1/\sqrt{14} \end{bmatrix}$$

$$\text{So } QR = \begin{bmatrix} -2/\sqrt{5} & -3/\sqrt{70} & 1/\sqrt{14} \\ 1/\sqrt{5} & -6/\sqrt{70} & 2/\sqrt{14} \\ 0 & 5/\sqrt{70} & 3/\sqrt{14} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 6/\sqrt{5} & -2/\sqrt{5} \\ 0 & \sqrt{14/5} & -3/\sqrt{70} \\ 0 & 0 & 1/\sqrt{14} \end{bmatrix} = \begin{bmatrix} -2 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = A$$

Why is $A = QR$ useful? It simplifies solving equations and least squares:

$$A\vec{x} = \vec{b} \longleftrightarrow QR\vec{x} = \vec{b} \xrightarrow[Q^T]{\text{Multiply by}} \underbrace{(Q^T Q)}_{I} R\vec{x} = Q^T\vec{b}$$

$$\longrightarrow \underbrace{R\vec{x} = Q^T\vec{b}}$$

upper triangular system (easy once you know Q and R)

Least squares: What is A is $m \times n$, with $m > n$, so $A\vec{x} = \vec{b}$ probably doesn't have a solution?

Approximate solution $\hat{x}$: $A^T A \hat{x} = A^T \vec{b}$ (normal equations)

$$\text{Same as: } (QR)^T QR\hat{x} = (QR)^T\vec{b} \rightsquigarrow R^T \underbrace{Q^T Q}_{I} R\hat{x} = R^T Q^T \vec{b} \longrightarrow$$

$$R^T R\hat{x} = R^T Q^T \vec{b} \xrightarrow[\text{invertible}]{R^T \text{ is}} R\hat{x} = Q^T\vec{b}$$

(same upper triangular system)

Example Approximate solution to

$$\underbrace{\begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{\vec{b}}, \qquad A = QR = \begin{bmatrix} -2/\sqrt{5} & -3/\sqrt{70} \\ 1/\sqrt{5} & -6/\sqrt{70} \\ 0 & 5/\sqrt{70} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 6/\sqrt{5} \\ 0 & \sqrt{14/5} \end{bmatrix}$$

Just need to solve $R\hat{x} = Q^T \vec{b} = \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -3/\sqrt{70} & -6/\sqrt{70} & 5/\sqrt{70} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2/\sqrt{5} \\ -3/\sqrt{70} \end{bmatrix}$

So $\sqrt{5}\, x + \frac{6}{\sqrt{5}} y = -\frac{2}{\sqrt{5}} \rightarrow x = \frac{1}{\sqrt{5}}\left(-\frac{2}{\sqrt{5}} - \frac{6}{\sqrt{5}} \cdot \frac{3}{5}\right) = -\frac{2}{5} - \frac{18}{25} = -\frac{28}{25}$

$\sqrt{\frac{14}{5}}\, y = -\frac{3}{\sqrt{70}} \rightarrow y = -\frac{3\sqrt{5}}{\sqrt{14 \cdot 70}} = -\frac{3}{5}$