Transformers

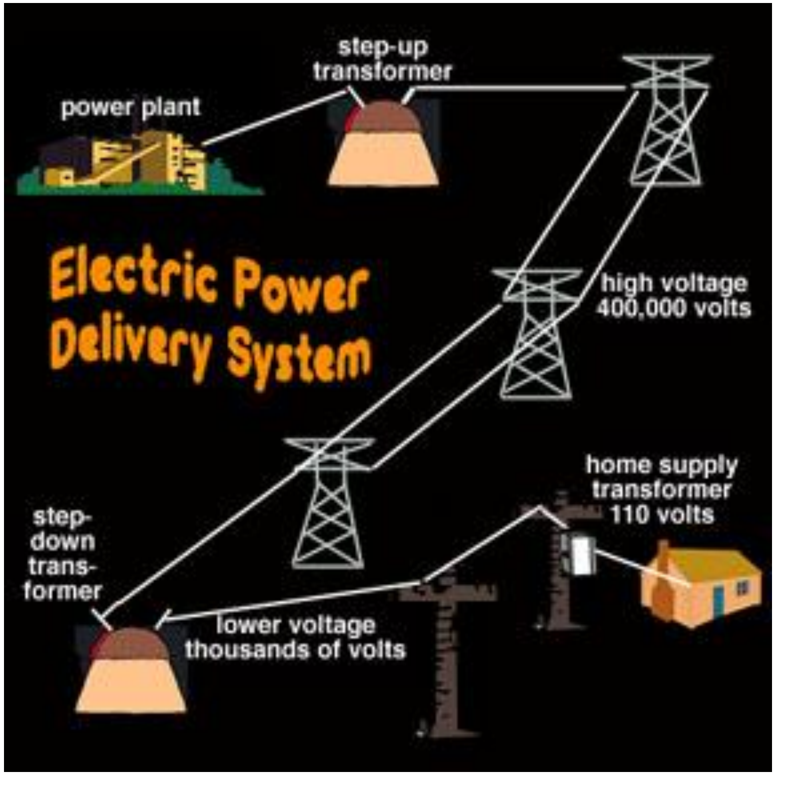
Pinjia Zhang

Chapter 2 Transformers

- Types and construction of transformers
- The ideal transformer
- Theory of operation of real single-phase transformers
- Equivalent circuit of a transformer
- Transformer voltage regulation and efficiency
- The autotransformer
- Three-phase transformer
- Instrument transformers

What is Transformer?

- An Electrical device which changes given an alternating emf into larger or smaller alternating emf.
- It is like a power converter that transfers electrical energy from one circuit to another through inductive coupled conductors. i.e transformer's coils.
- Transformers are used in our homes to keep voltage up to 220 volt.
- Transformer helping to form a safe Electric Power system is shown in Figure:



TRANSFORMER

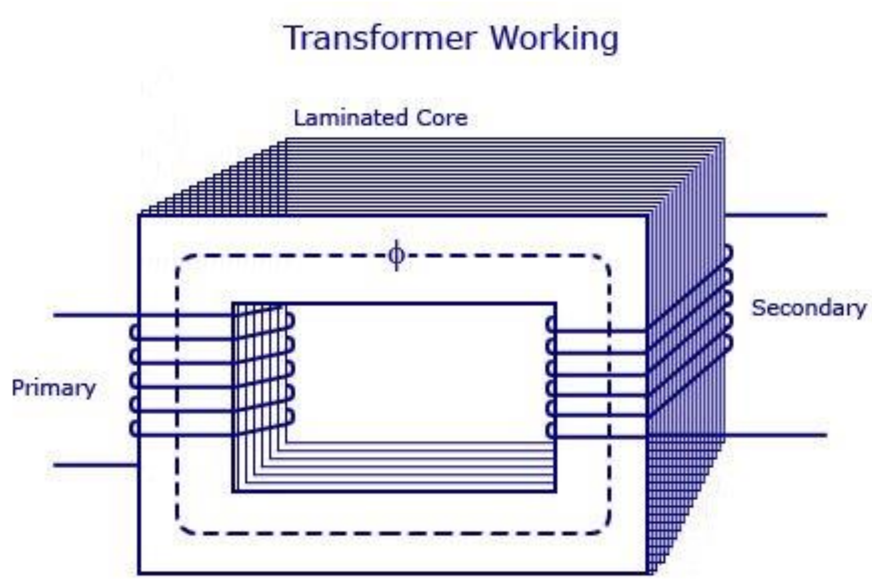
- A transformer is a static device.
- The word ‘transformer’ comes form the word ‘transform’.
- **Transformer** is not an energy conversion device, but it is device that changes AC electrical power at one voltage level into AC electrical power at another voltage level through the action of magnetic field but with a proportional increase or decrease in the current ratings., without a change in frequency.
- It can be either to step-up or step down.

Why transformers are important to modern life

- The transformer ideally changes one ac voltage level to another voltage level without affecting the actual power supplied.
- The transformer can be used in distribution system for efficiency issues.
  - The step-up transformer decreases the line current and decreases the power loss on power line.
  - The transmission/distribution system with transformer can keep high efficiency

WORKING

The main principle of operation of a transformer is mutual inductance between two circuits which is linked by a common magnetic flux. A basic transformer consists of two coils that are electrically separate and inductive, but are magnetically linked through a path of reluctance. The working principle of the transformer can be understood from the figure below



- In short, a transformer carries the operations shown below:

- Transfer of electric power from one circuit to another.
- Transfer of electric power without any change in frequency.
- Transfer with the principle of electromagnetic induction.
- The two electrical circuits are linked by mutual induction.

Transformer Construction

- video

Classification of transformer

- As per phase

1. single phase
2. Three phase

- As per core

1. Core type
2. Shell type

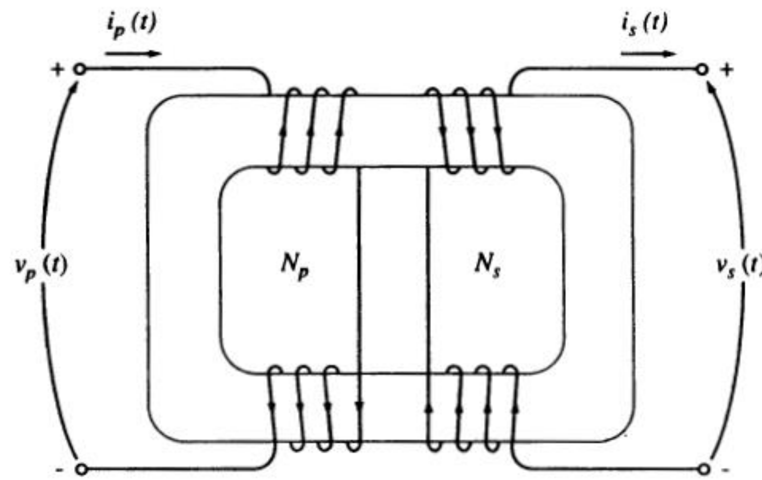
- As per cooling system

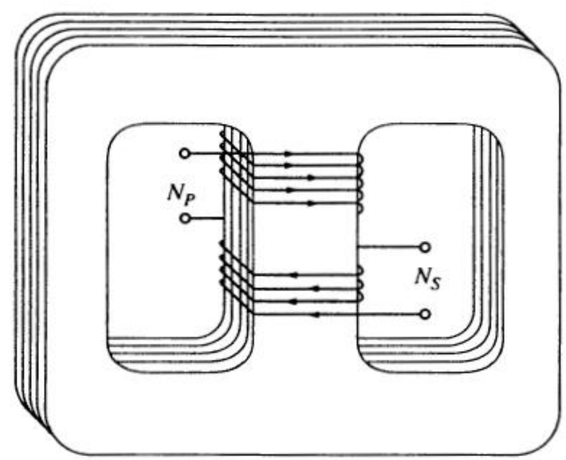
1. Self-cooled
2. Air cooled
3. Oil cooled

Types and construction of transformers

- **Core-form**: consists a simple rectangular laminated piece of steel with the transformer winding wrapped around two sides of the rectangle

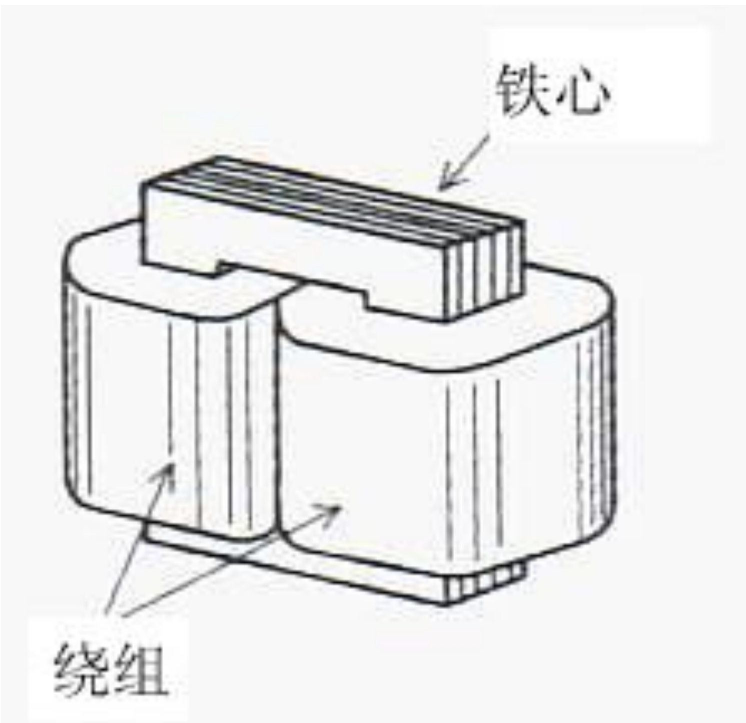
- **Shell-form**: consists three legs laminated core with winding wrapped around the center leg

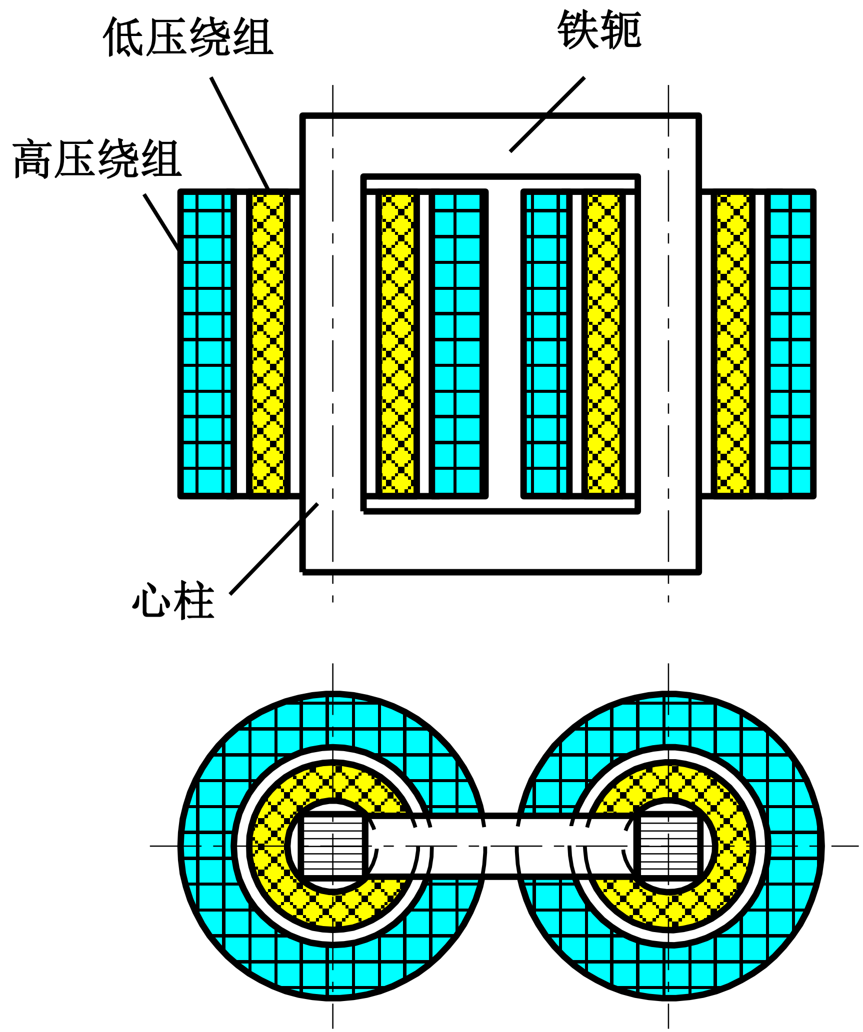




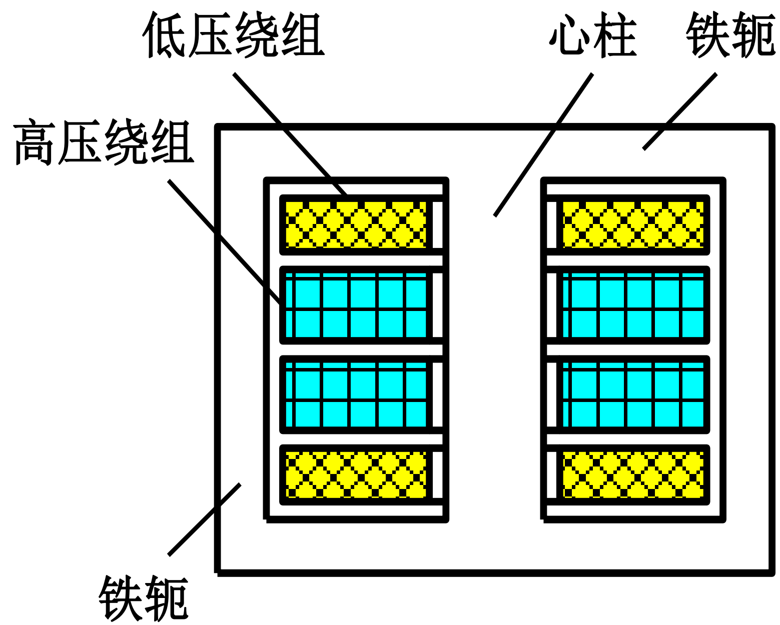
(a)

Core-form transformers

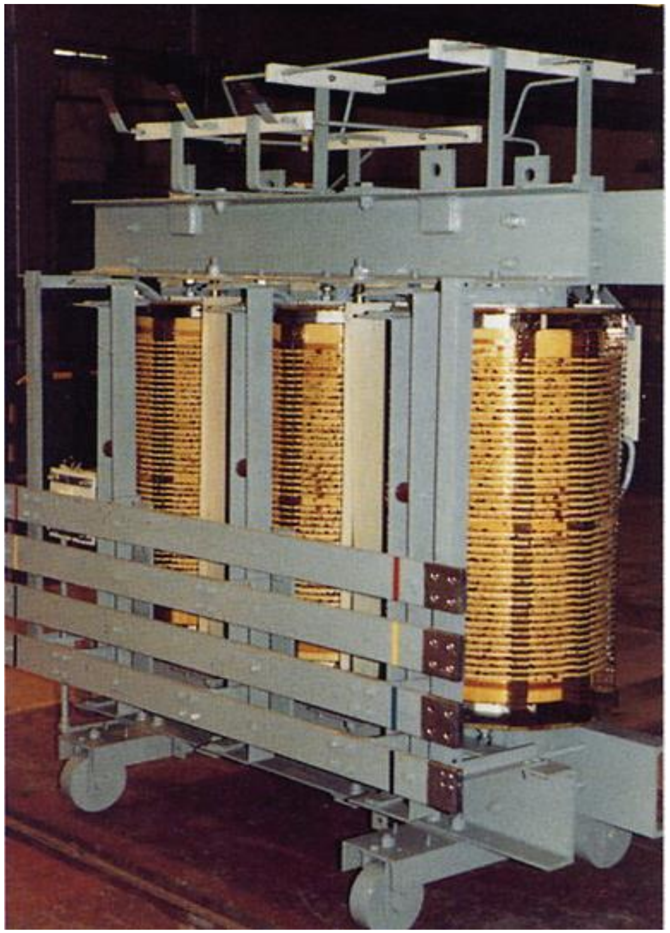




Shell-form transformers

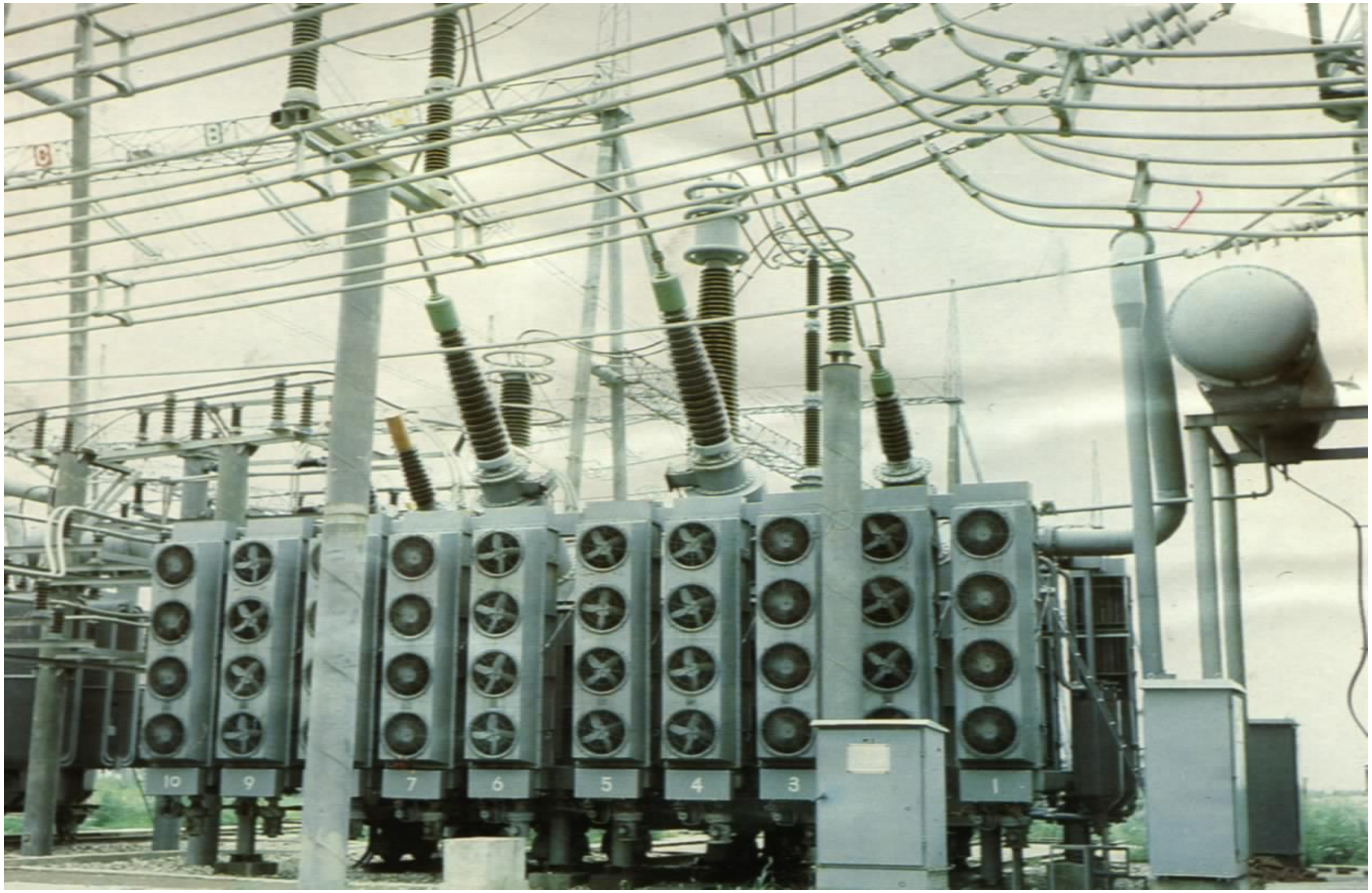


Oil immersed Distribution Transformers



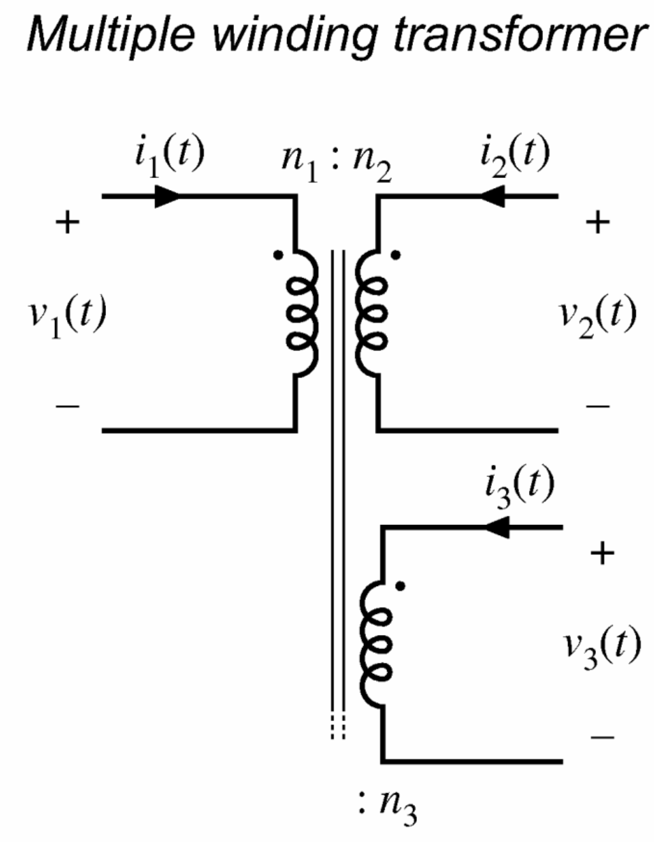
Dry Type Distribution Transformers

Large Power Transformers



Transformers

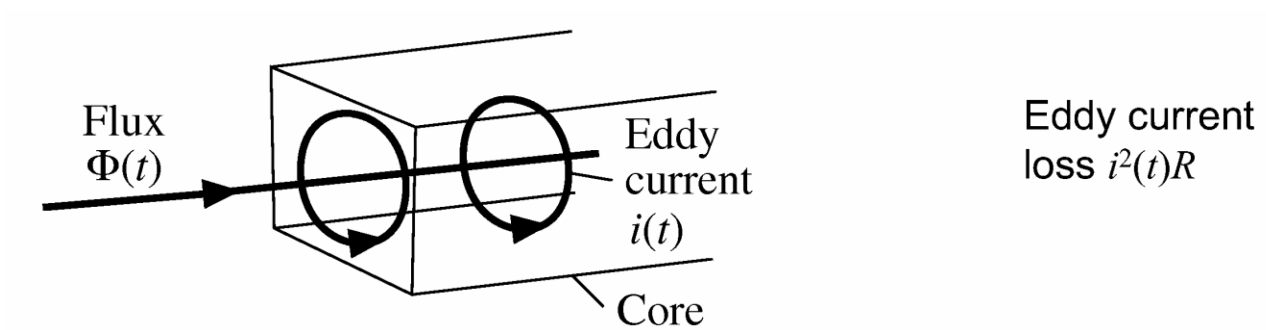
- The transformer winding connected to the power source is called the ***primary winding or input winding***
- The winding connected to the loads is called the ***secondary winding or output winding***



Core material and eddy current

- The core is constructed of ***thin laminations*** electrically isolated from each other in order to minimize the ***eddy currents***.

- Eddy current:



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Lamination video

Special purpose transformers

- ***Unit transformer***: used for voltage up from generator to transmission system.
- ***Substation transformer***: used for voltage down from transmission to distribution
- ***Distribution transformer***: used for voltage down from distribution to actual used levels
- ***Potential transformer*** (PT): 220V at secondary side
- ***Current transformer*** (CT): 5A at secondary side

Ideal Transformer

Assumptions

Neglect

- Leakage flux
- Copper losses
- Hysteresis losses
- Eddy-current losses

The ideal transformer characteristics

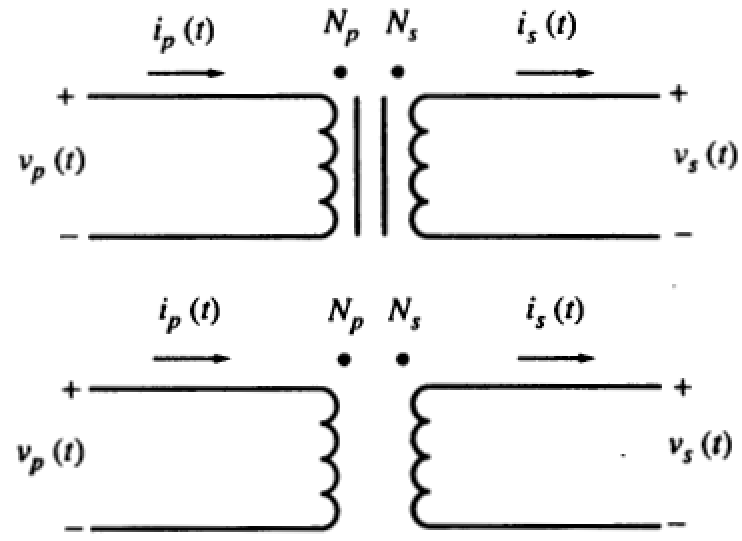
$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a$$

- Where $\boldsymbol{a}=N_p/N_s$ is the turns ratio
- Energy balance relation

$$N_P i_P(t) = N_S i_S(t) \quad \frac{i_P(t)}{i_S(t)} = \frac{1}{a}$$

- Phasor relation

$$\frac{\mathbf{V}_P}{\mathbf{V}_S} = a \quad \frac{\mathbf{I}_P}{\mathbf{I}_S} = \frac{1}{a}$$



(b)

- The turns ratio ***a*** only effects the magnitude not the angle

Dot convention in ideal transformer

- If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also. Voltage polarities are the same with respect to the dots on each side of the core.
- If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding

Power in an ideal transformer

- The power supplied to the transformer by the primary circuit is given by the equation

$$P_{\text{in}} = V_P I_P \cos\theta_P$$

where θ*p* is the angle between the primary voltage and the primary current. The power supplied by the transformer secondary circuit to its loads is given by the equation

$$P_{\text{out}} = V_S I_S \cos\theta_S$$

where θ*s* is the angle between the secondary voltage and the secondary current. Since voltage and current angles are unaffected by an ideal transformer, θ*p* - θ*s* = θ. The primary and secondary windings of an ideal transformer have the *same power factor*.

Power in an ideal transformer

$$P_{\text{out}} = V_S I_S \cos\theta$$

Applying the turns-ratio equations gives $V_s = V_p/a$ and $I_s = aI_p$, so

$$P_{\text{out}} = \frac{V_P}{a}\left(aI_P\right)\cos\theta$$

$$P_{\text{out}} = V_P I_P \cos\theta = P_{\text{in}}$$

Thus, the output power of an ideal transformer is equal to its input power.

The same relationship applies to reactive power Q and apparent power S:

$$Q_{\text{in}} = V_P I_P \sin\theta = V_S I_S \sin\theta = Q_{\text{out}}$$

$$S_{\text{in}} = V_P I_P = V_S I_S = S_{\text{out}}$$

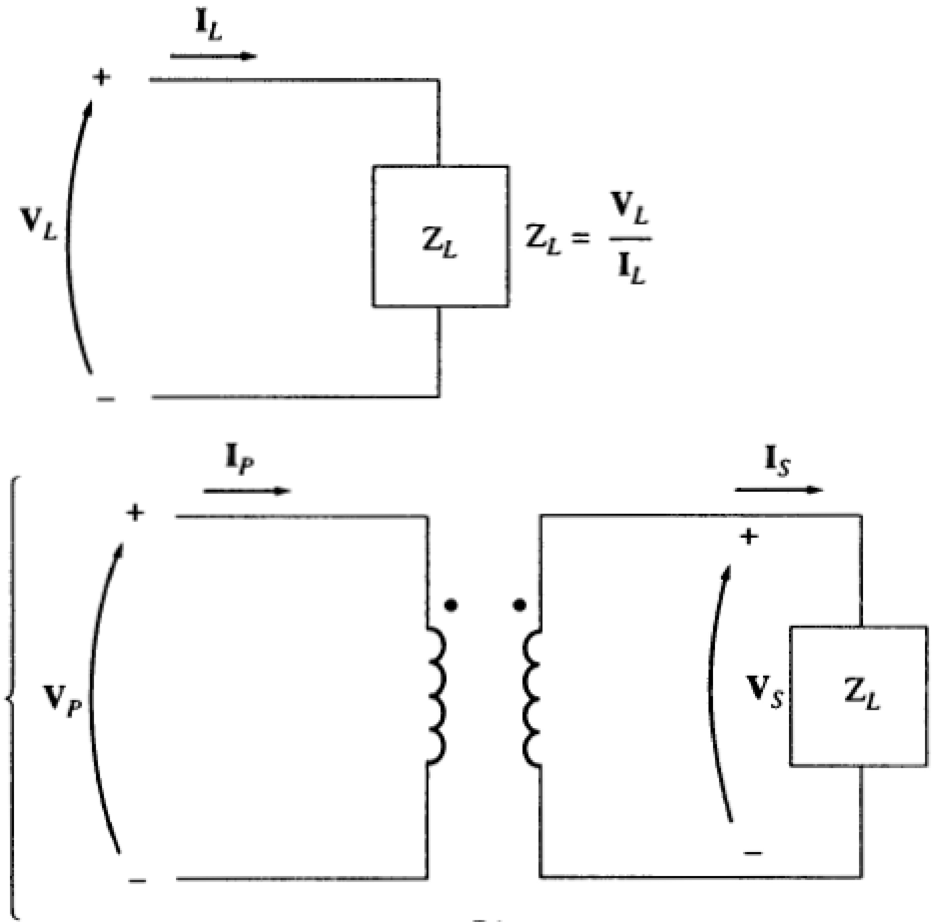
Impedance transformation through a transformer

$$\mathbf{V}_P = a\mathbf{V}_S \qquad \mathbf{I}_P = \frac{\mathbf{I}_S}{a}$$

$$Z_L' = \frac{\mathbf{V}_P}{\mathbf{I}_P} = \frac{a\mathbf{V}_S}{\mathbf{I}_S / a} = a^2 \frac{\mathbf{V}_S}{\mathbf{I}_S}$$

$$Z_L' = a^2 Z_L$$

$$Z_L' = \frac{\mathbf{V}_P}{\mathbf{I}_P}$$



(b)

Analysis of circuits containing ideal transformers

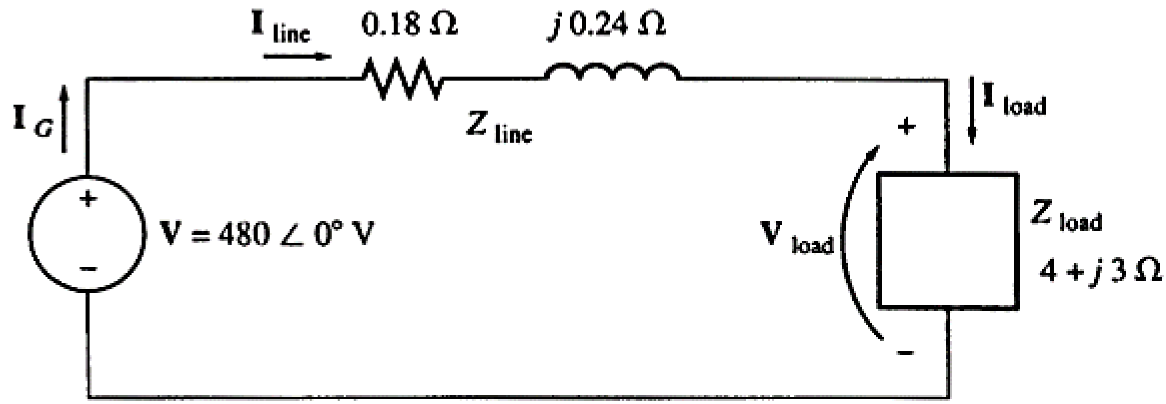
- All the current and voltage are all referred to one side (primary side)
- Note the dot convention for current direction
- Impedance transformation

Example 2-1

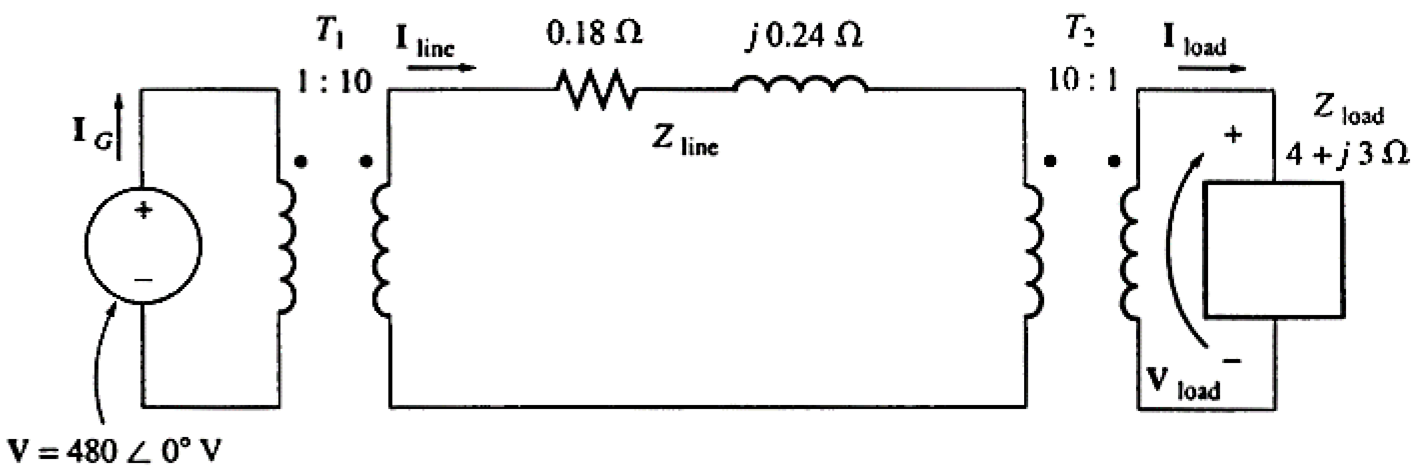
A single-phase power system consists of a 480-V 60-Hz generator supplying a load $Z_{load} = 4 + j3\Omega$ through a transmission line of impedance $Z_{line} = 0.18 + j0.24\Omega$. Answer the following questions about this system.

(a) If the power system is exactly as described above, what will the voltage at the load be? What will the transmission line losses be?
(b) Suppose a 1:10 step-up transformer is placed at the generator end of the trans-mission line and a 10:1 step-down transformer is placed at the load end of the line . What will the load voltage be now? What will the transmission line losses be now?

Example 2-1



(a)



(b)

(a) The previous figure(a) shows the power system without transformers. Here $\mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load}$. The line current in this system is given by

$$\begin{aligned}
\mathbf{I}_{line} &= \frac{\mathbf{V}}{Z_{line} + Z_{load}} \\
&= \frac{480\angle 0^\circ \text{ V}}{(0.18\Omega + j0.24\Omega) + (4\Omega + j3\Omega)} \\
&= \frac{480\angle 0^\circ}{4.18 + j3.24} = \frac{480\angle 0^\circ}{5.29\angle 37.8^\circ} \\
&= 90.8\angle -37.8^\circ \text{ A}
\end{aligned}$$

Therefore the load voltage is

$$\begin{aligned}
\mathbf{V}_{load} &= \mathbf{I}_{line} Z_{load} \\
&= \left(90.8\angle -37.8^\circ \text{ A}\right)(4\Omega + j3\Omega) \\
&= \left(90.8\angle -37.8^\circ \text{ A}\right)\left(5\angle 36.9^\circ \Omega\right) \\
&= 454\angle -0.9^\circ \text{ V}
\end{aligned}$$

and the line losses are

$$\begin{aligned}
P_{loss} &= \left(I_{line}\right)^2 R_{line} \\
&= (90.8\text{A})^2 (0.18\Omega) = 1484\text{W}
\end{aligned}$$

(b) The previous figure(b) shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in twosteps:

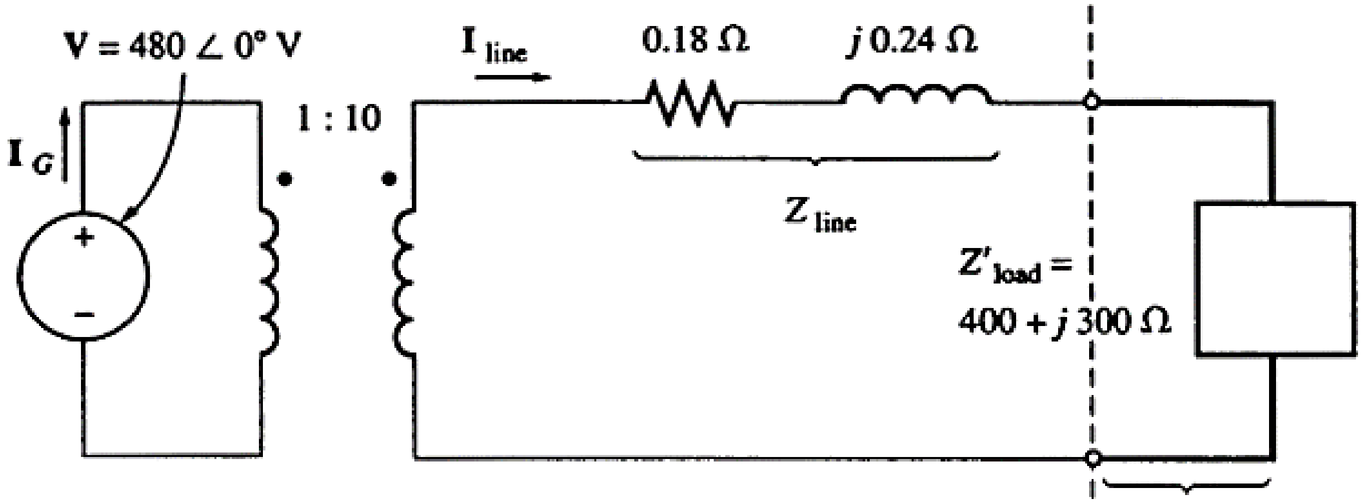
1. Eliminate transformer T, by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T, by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$
\begin{aligned}
Z'_{\mathrm{load}} &= a^2 Z_{\mathrm{load}} \\
&= \left(\frac{10}{1}\right)^2 (4\Omega + j3\Omega) \\
&= 400\Omega + j300\Omega
\end{aligned}
$$

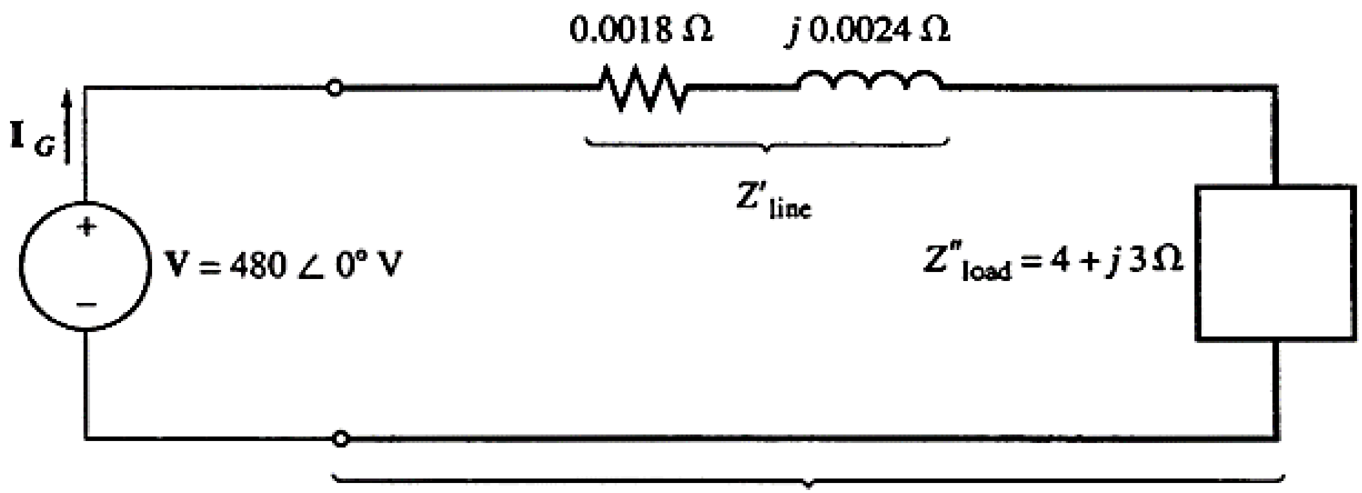
The total impedance at the transmission line level is now

$$
\begin{aligned}
Z_{\mathrm{eq}} &= Z_{\mathrm{line}} + Z_{\mathrm{load}} \\
&= 400.18 + j300.24\Omega = 500.3\angle 36.88^{\circ}\,\Omega
\end{aligned}
$$



(a)

Equivalent circuit



Equivalent circuit

(b)

This equivalent circuit is shown in previous figure(a). The total impedance at the transmission line level (Z_{line} + Z_{load}) is now reflected across T, to the source's voltage level:

$$\begin{aligned} Z'_{eq} &= a^2 Z_{eq} \\ &= a^2\left(Z_{line} + Z'_{load}\right) \\ &= \left(\frac{1}{10}\right)^2 (0.18\Omega + j0.24\Omega + 400\Omega + j300\Omega) \\ &= (0.0018\Omega + j0.0024\Omega + 4\Omega + j3\Omega) \\ &= 5.003\angle 36.88^\circ \Omega \end{aligned}$$

Notice that $Z''_{load} = 4 + j3\Omega$ and $Z'_{line} = 0.0018 + j0.0024\Omega$. The resulting equivalent circuit is shown in the previous figure(b). The generator's current is

$$I_G = \frac{480\angle 0^\circ \text{V}}{5.003\angle 36.88^\circ \Omega} = 95.94\angle -36.88^\circ \text{A}$$

Knowing the current $\mathbf{I}_G$, we can now work back and find $\mathbf{I}_{line}$ and $\mathbf{I}_{load}$. Working back through T_1, we get

$$
\begin{aligned}
N_{P1}\mathbf{I}_G &= N_{S1}\mathbf{I}_{\text{line}} \\
\mathbf{I}_{\text{line}} &= \frac{N_{P1}}{N_{S1}}\mathbf{I}_G \\
&= \frac{1}{10}\left(95.94\angle -36.88^{\circ}\ \text{A}\right) = 9.594\angle -36.88^{\circ}\ \text{A}
\end{aligned}
$$

Working back through T_2 gives

$$
\begin{aligned}
N_{P2}\mathbf{I}_{\text{line}} &= N_{S2}\mathbf{I}_{\text{load}} \\
\mathbf{I}_{\text{load}} &= \frac{N_{P2}}{N_{S2}}\mathbf{I}_{\text{line}} \\
&= \frac{10}{1}\left(9.594\angle -36.88^{\circ}\ \text{A}\right) = 95.94\angle -36.88^{\circ}\ \text{A}
\end{aligned}
$$

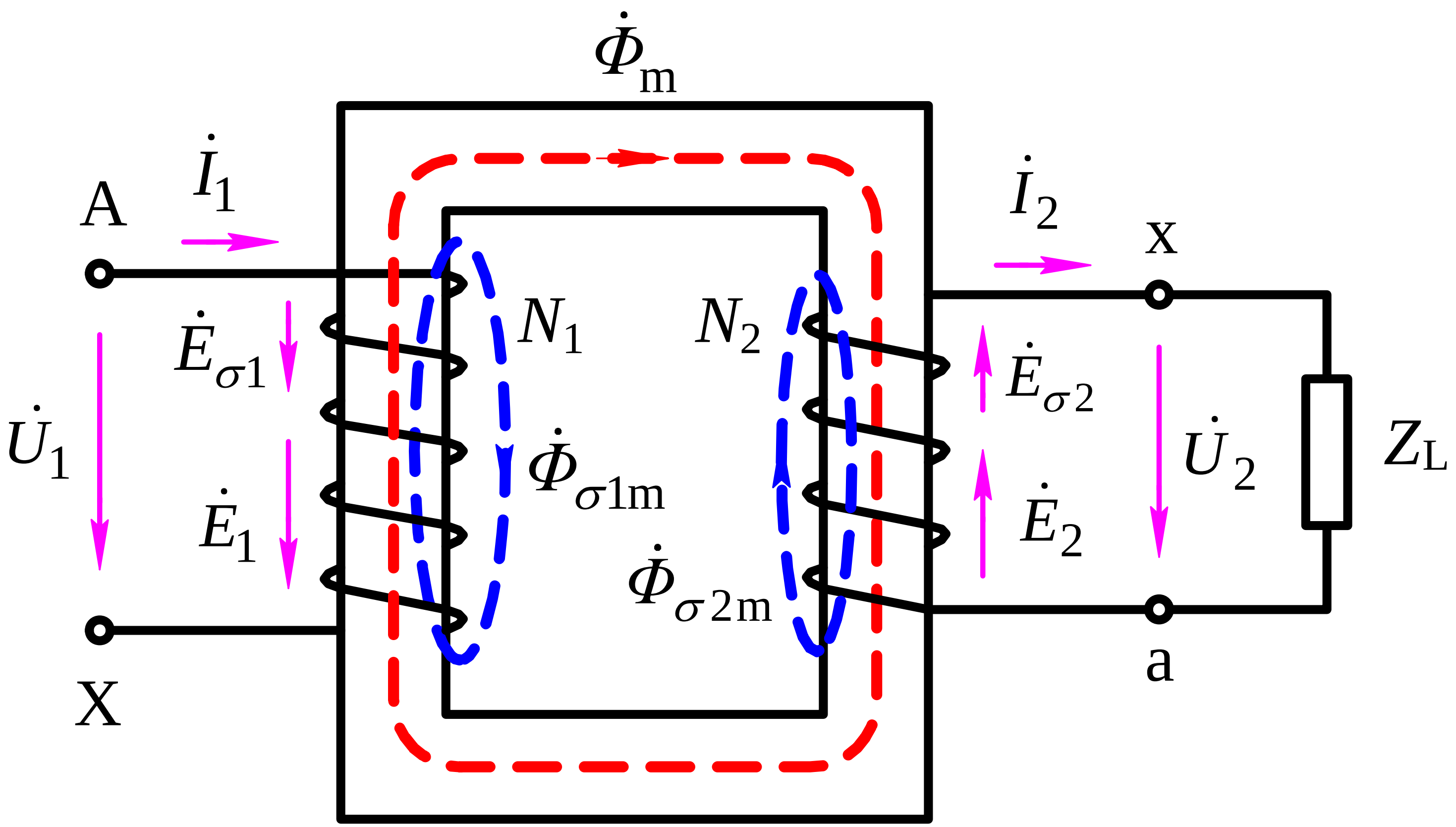
It is now' possible to answer the questions originally asked. The load voltage is given by

$$
\begin{aligned}
\mathbf{V}_{\text{load}} &= \mathbf{I}_{\text{load}}\, Z_{\text{load}} \\
&= \left(95.94\angle -36.88^{\circ}\ \text{A}\right)\left(5\angle 36.87^{\circ}\ \Omega\right) \\
&= 479.7\angle -0.01^{\circ}\ \text{V}
\end{aligned}
$$

and the line losses are given by

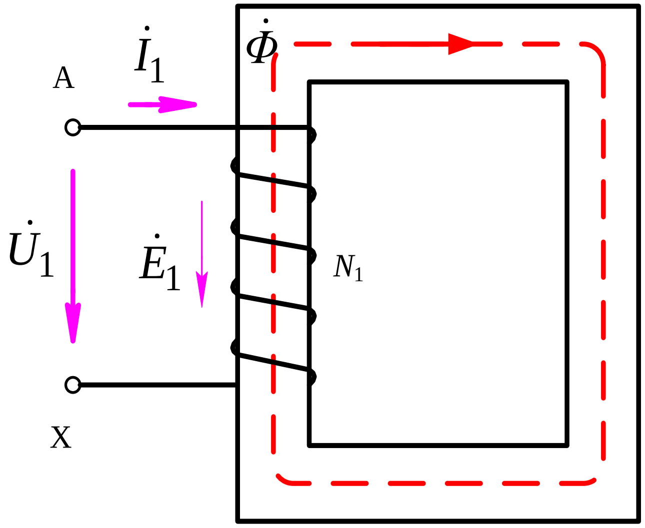
$$
\begin{aligned}
P_{\text{loss}} &= \left(I_{\text{line}}\right)^2 R_{\text{line}} \\
&= (9.594\text{A})^2(0.18\Omega) = 16.7\text{W}
\end{aligned}
$$

Transformer Quantities



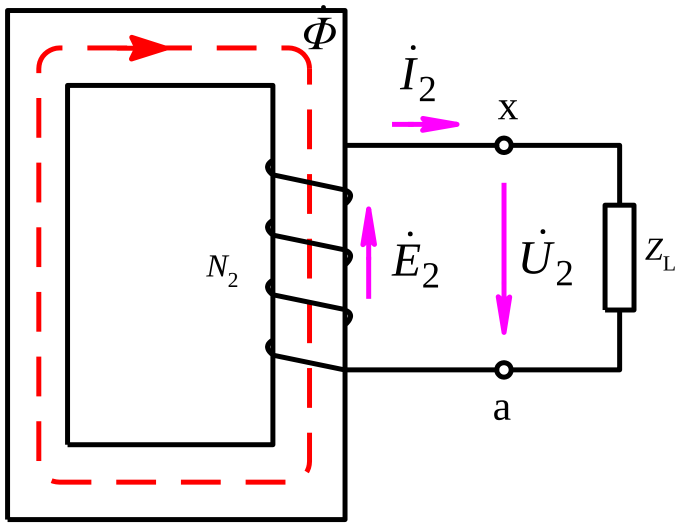
Reference Direction

Current, Voltage, Flux and EMF



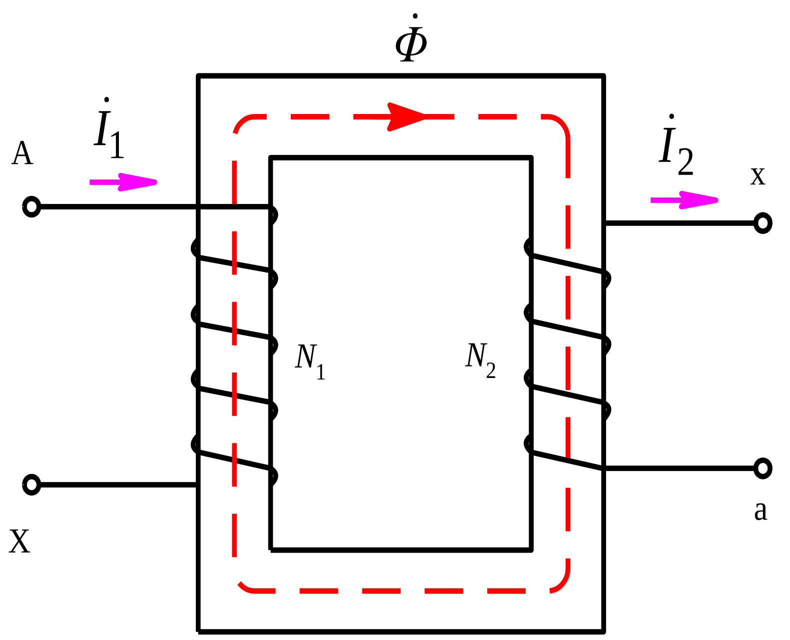
Reference Direction

Current, Voltage, Flux and EMF



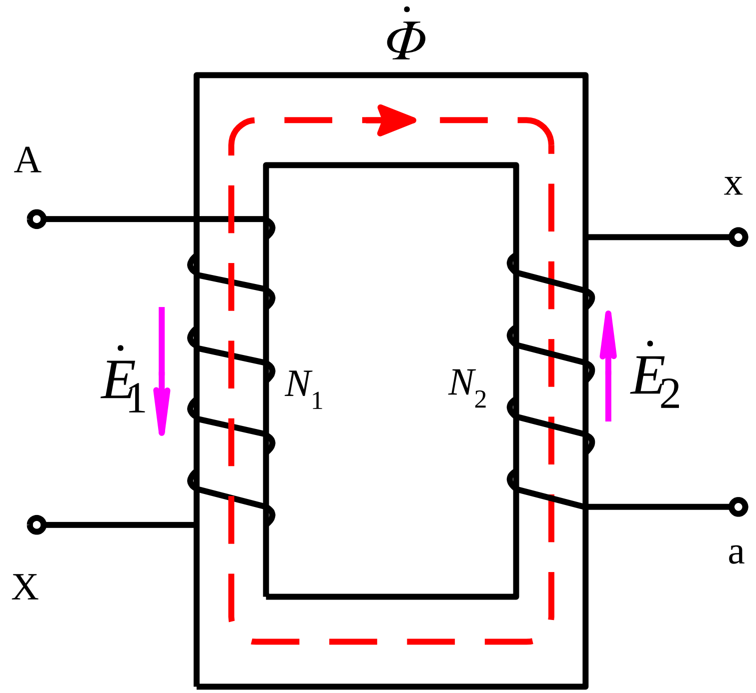
Reference Direction

Current, Flux

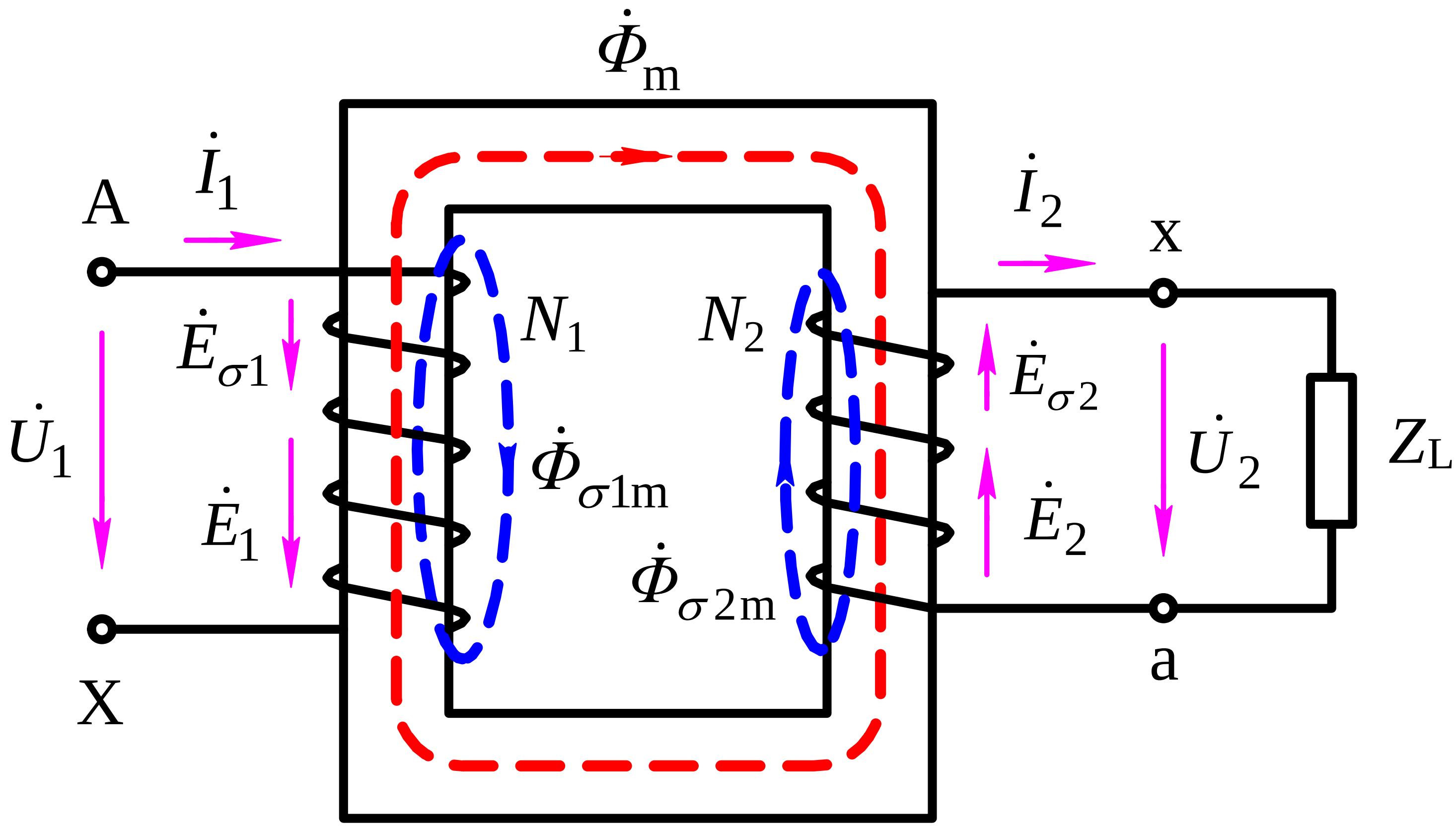


Reference Direction

EMF, Mutual Flux



Reference Direction



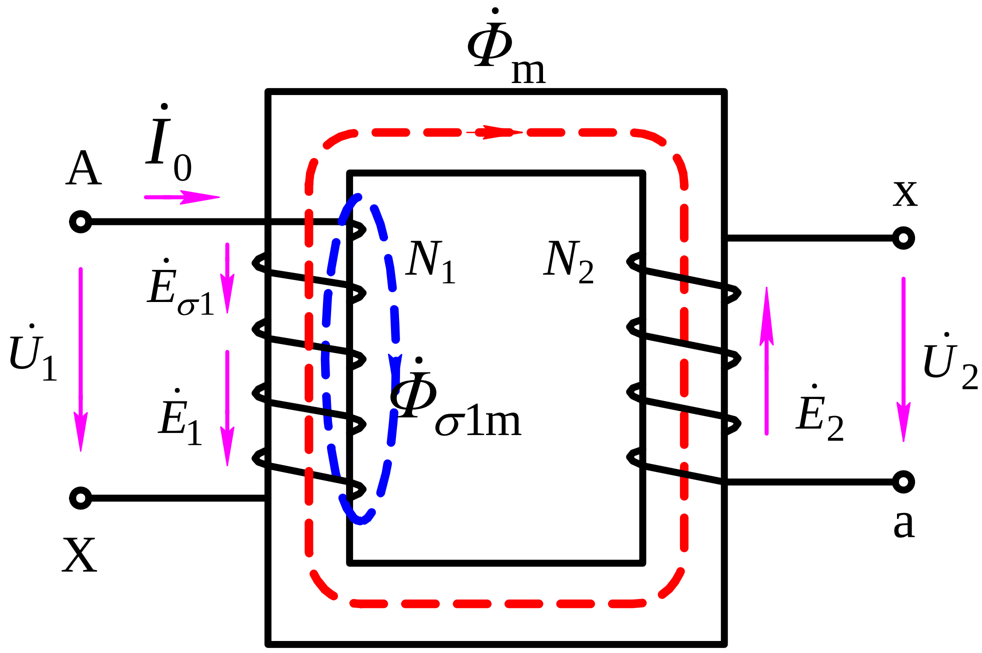
Principles of transformer operation

No load operation of transformers

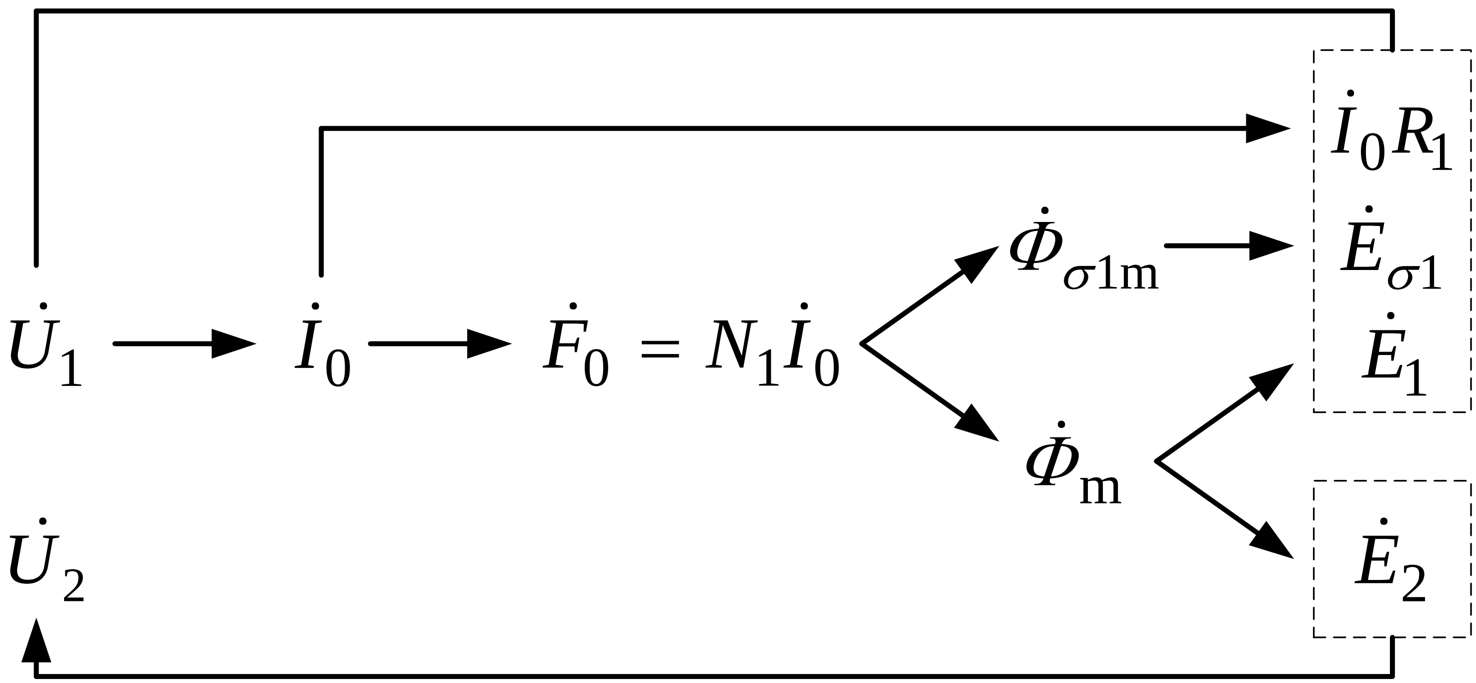
No load condition

(1) primary winding excited with current flowing

(2) secondary winding open circuit with no current flowing

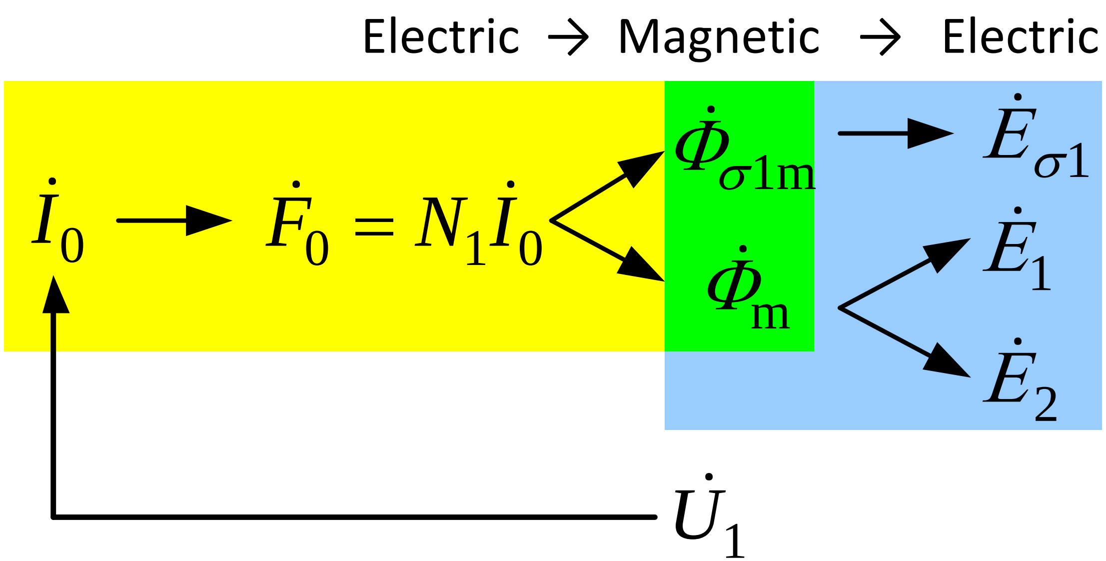


No load operation of transformers



No load operation of transformers

Electromagnetic conversion



No load operation of transformers

Mutual Magnetic Flux

$$\phi = \Phi_{\mathrm{m}} \sin \omega t$$

Φ_{m} (Wb)

ω： rad/s

f： Hz

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No load operation of transformers

Induced EMF by Mutual Flux

$$e_1 = -N_1 \frac{\mathrm{d}\phi}{\mathrm{d}t}, \qquad e_2 = -N_2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

Emf on Primary winding:

$$\begin{aligned} e_1 &= -\omega N_1 \Phi_{\mathrm{m}} \cos \omega t \\ &= E_{1\mathrm{m}} \sin\left(\omega t - 90^\circ\right) \end{aligned}$$

$$E_{1\mathrm{m}} = \omega N_1 \Phi_{\mathrm{m}}$$

Emf on Secondary winding:

$$\begin{aligned} e_2 &= -\omega N_2 \Phi_m \cos \omega t \\ &= E_{2m} \sin\left(\omega t - 90^\circ\right) \end{aligned}$$

$$E_{2\mathrm{m}} = \omega N_2 \Phi_{\mathrm{m}}$$

No load operation of transformers

$$\begin{aligned}\dot{E}_1 &= -\mathrm{j}\frac{1}{\sqrt{2}}\omega N_1\dot{\Phi}_{\mathrm{m}} \\ &= -\mathrm{j}\sqrt{2}\pi fN_1\dot{\Phi}_{\mathrm{m}} = -\mathrm{j}4.44\,fN_1\dot{\Phi}_{\mathrm{m}} \\ \dot{E}_2 &= -\mathrm{j}4.44\,fN_2\dot{\Phi}_{\mathrm{m}}\end{aligned}$$

In an ideal transformer $U_1/U_2=N_1/N_2$

No load operation of transformers

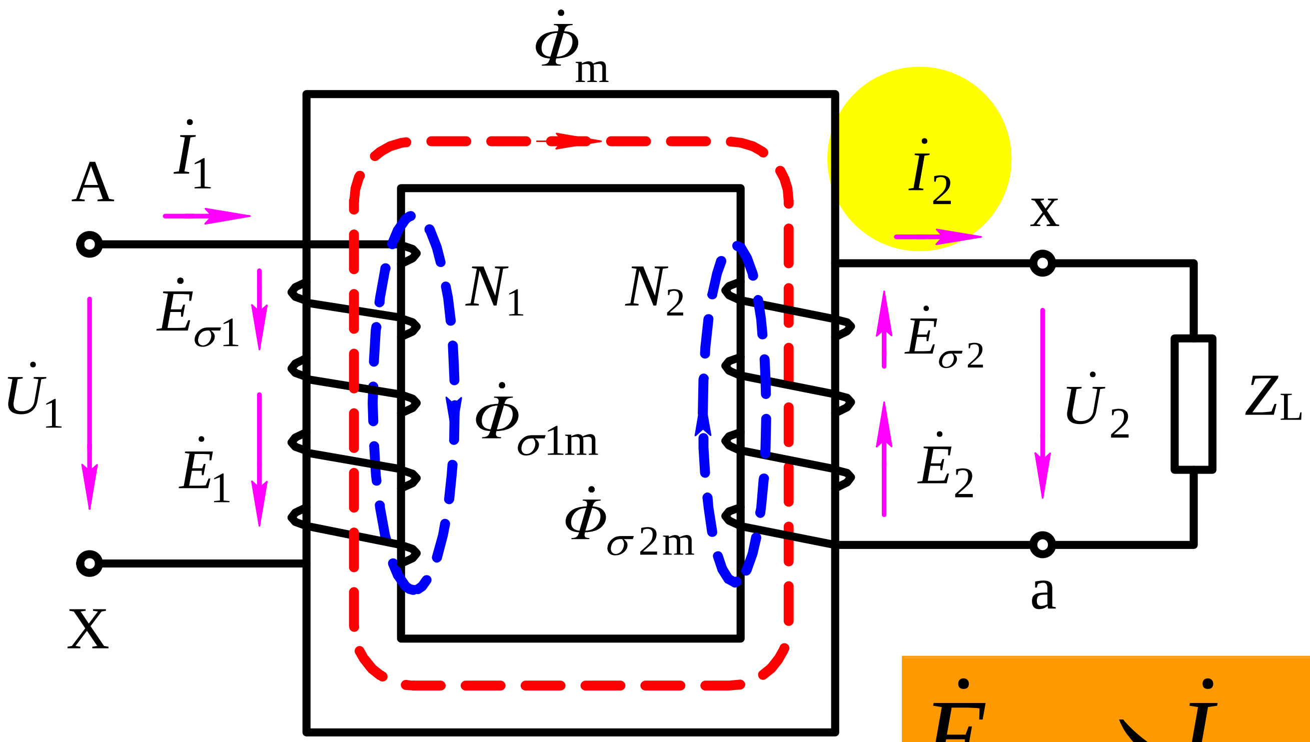
- Neglecting leakage effect and losses

$$\dot{U}_1 = -\dot{E}_1 = \mathrm{j}\frac{1}{\sqrt{2}}\omega N_1 \dot{\Phi}_{\mathrm{m}}$$

$$= \mathrm{j}4.44 f N_1 \dot{\Phi}_{\mathrm{m}}$$

$$= Z_m \cdot \dot{I}_0$$

Loaded operation of transformer



$$\dot{E}_2 \rightarrow \dot{I}_2 \rightarrow \dot{F}_2$$

Loaded operation of transformer

$$\dot{E}_1 = -\mathrm{j}\frac{1}{\sqrt{2}}\omega N_1 \dot{\Phi}_{\mathrm{m}}$$

$$= -\mathrm{j}\sqrt{2}\pi f N_1 \dot{\Phi}_{\mathrm{m}} = -\mathrm{j}4.44 f N_1 \dot{\Phi}_{\mathrm{m}}$$

$$\dot{E}_2 = -\mathrm{j}4.44 f N_2 \dot{\Phi}_{\mathrm{m}}$$

$$\frac{\dot{E}_1}{\dot{E}_2} = \frac{N_1}{N_2} = k$$

The same as no load operation!

What has changed?

- No load operation:

$$\boxed{\dot{F}_0 = N_1 \dot{I}_0}$$

- Loaded operation:

$$\dot{F}_1 = N_1 \dot{I}_1 \qquad \dot{F}_2 = N_2 \dot{I}_2$$

$$\boxed{\dot{F}_m = \dot{F}_1 + \dot{F}_2}$$

Loaded operation of transformer

- MMF under loaded condition is similar to the MMF under no load condition

$$\dot{F}_m \approx \dot{F}_0$$

Why?

$$\dot{F}_0 = \dot{F}_1 + \dot{F}_2$$

$$\dot{F}_1 = \dot{F}_0 + (-\dot{F}_2)$$

$$N_1\dot{I}_1 = N_1\dot{I}_0 + (-N_2\dot{I}_2)$$

Loaded operation of transformer

$$\dot{I}_1 = \dot{I}_0 + (-\frac{N_2}{N_1}\dot{I}_2) = \dot{I}_0 + (-\frac{\dot{I}_2}{k})$$

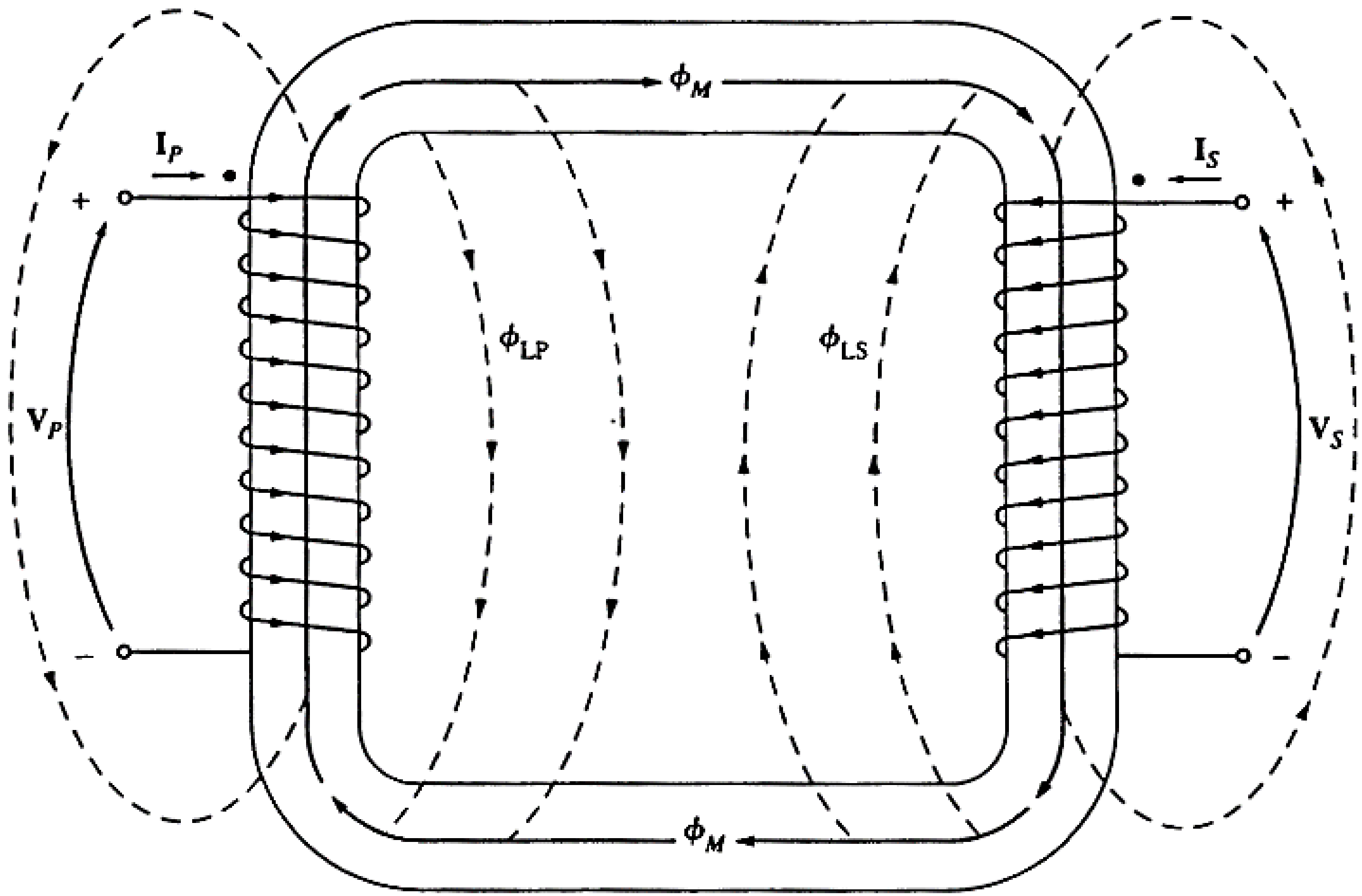
Magnetization current

Load current

Real transformers vs ideal transformers

- Leakage flux
- Copper losses
- Hysteresis losses
- Eddy-current losses

Voltage relation between primary and secondary derived from Faraday's law



Voltage relation

1. Induced voltage on each side

$$\bar{\phi} = \frac{1}{N_P}\int v_P(t)dt$$

2. Primary side flux

$$\bar{\phi}_P = \phi_M + \phi_{\mathrm{LP}}$$

$\bar{\phi}_P$ = total average primary flux

ϕ_M = flux component linking both primary and secondary coils

ϕ_{LP} = primary leakage flux

3. Secondary side flux

$$\bar{\phi}_S = \phi_M + \phi_{LS}$$

$\bar{\phi}_S$ = total average secondary flux

ϕ_M = flux component linking both primary and secondary coils

ϕ_{LS} = secondary leakage flux

Voltage relation

1. Induced voltage on primary side

$$
\begin{aligned}
v_P(t) &= N_P \frac{d\bar{\phi}_P}{dt} \\
&= N_P \frac{d\phi_M}{dt} + N_P \frac{d\phi_{LP}}{dt} \\
v_P(t) &= e_P(t) + e_{LP}(t)
\end{aligned}
$$

2. Induced voltage on secondary side

$$
\begin{aligned}
v_S(t) &= N_S \frac{d\bar{\phi}_S}{dt} \\
&= N_S \frac{d\phi_M}{dt} + N_S \frac{d\phi_{LS}}{dt} \\
&= e_S(t) + e_{LS}(t)
\end{aligned}
$$

Induced voltage relation - Induced by mutual flux

The primary voltage due to the mutual flux is given by

$$e_P(t) = N_P \frac{d\phi_M}{dt}$$

and the secondary voltage due to the mutual flux is given by

$$e_S(t) = N_S \frac{d\phi_M}{dt}$$

Notice from these two relationships that

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

Therefore,

$$\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = a$$

Terminal voltage relation - Neglecting the leakage flux

This equation means that the ratio of the primary voltage caused by the mutual flux to the secondary voltage caused by the mutual flux is equal to the turns ratio of the transformer. Since in a well-designed transformer $\phi_m >> \phi_{LP}$ and $\phi_m >> \phi_{LS}$, the ratio of the total voltage on the primary of a transformer to the total voltage on the secondary of a transformer is approximately

$$\frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a$$

The smaller the leakage fluxes of the transformer are, the closer the total transformer voltage ratio approximates that of the ideal transformer discussed previously.

Modeling the leakage flux by leakage inductance

As explained previously, the leakage flux in the primary windings ϕ_{LP} produces a voltage e_{LP} given by

$$e_{\mathrm{LP}}(t) = N_P \frac{d\phi_{\mathrm{LP}}}{dt}$$

and the leakage flux in the secondary windings ϕ_{LS} produces a voltage e_{LS} given by

$$e_{\mathrm{LS}}(t) = N_S \frac{d\phi_{\mathrm{LS}}}{dt}$$

$$\phi_{\mathrm{LP}} = \left(\mathrm{P}\, N_P\right) i_P$$

$$\phi_{\mathrm{LS}} = \left(\mathrm{P}\, N_S\right) i_S$$

where P = permeance of flux path

N_p = number of turns on primary coil

N_s = number of turns on secondary coil

Modeling the leakage flux by leakage inductance

After substitution, the result is

$$e_{\mathrm{LP}}(t) = N_P \frac{d}{dt}\left(\mathrm{P}\, N_P\right) i_P = N_P^2 \mathrm{P}\, \frac{di_P}{dt}$$

$$e_{\mathrm{LS}}(t) = N_S \frac{d}{dt}\left(\mathrm{P}\, N_S\right) i_S = N_S^2 \mathrm{P}\, \frac{di_S}{dt}$$

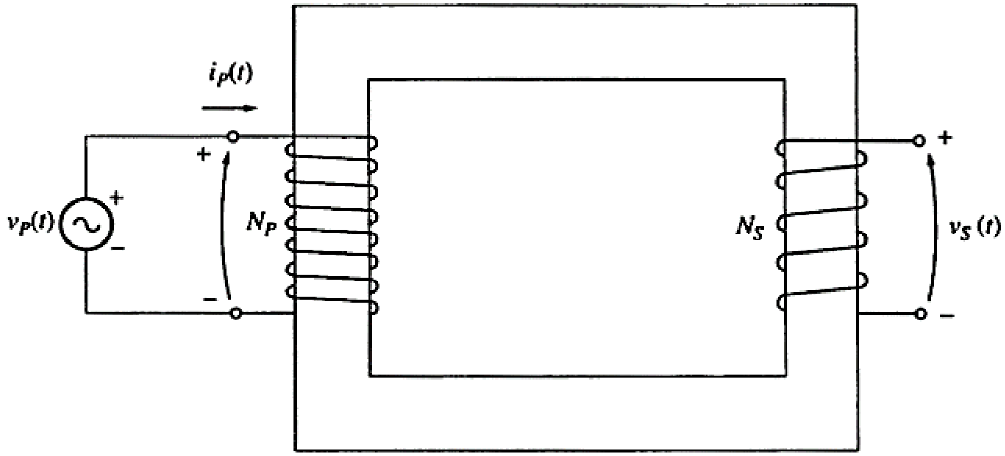
The constants in these equations can be lumped together. Then

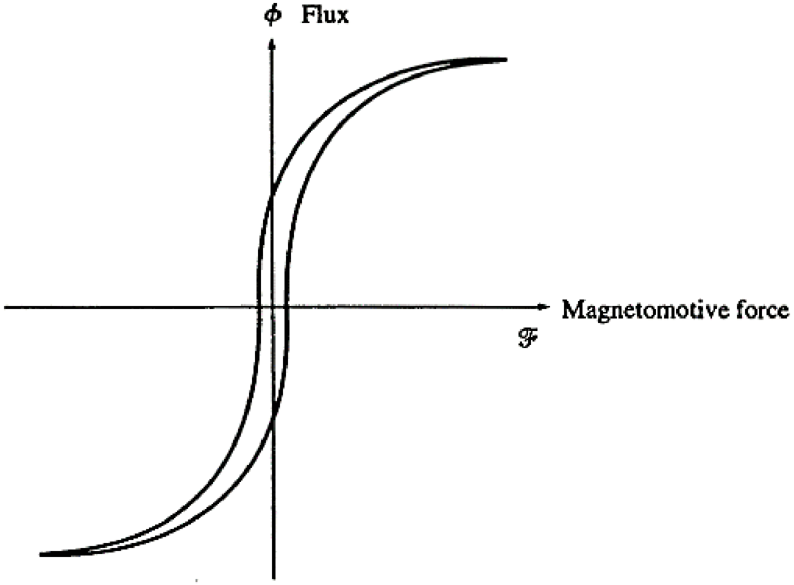
$$e_{\mathrm{LP}}(t) = L_P \frac{di_P}{dt}$$

$$e_{\mathrm{LS}}(t) = L_S \frac{di_S}{dt}$$

Theory of operation of real single-phase transformers – secondary side open

- Secondary side is open circuit
- Input voltage and current to measure hysteresis curve
- Flux is proportional to v_p and magnetomotive force is proportional to i_p
- $i_p(t)$ = 0 for ideal transformer

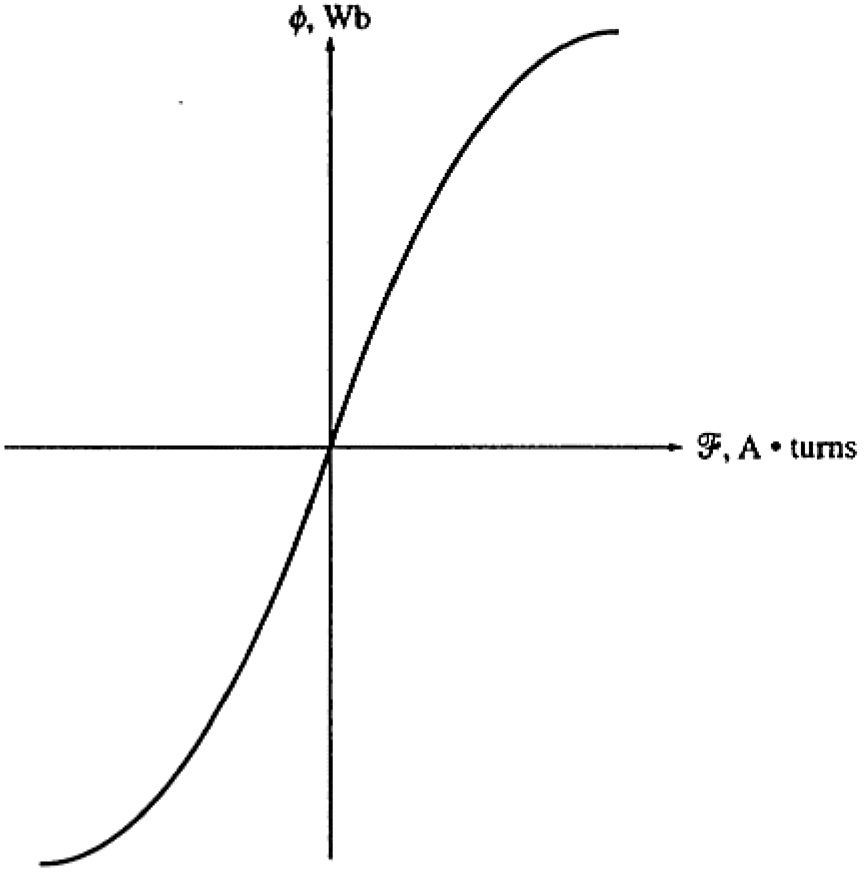




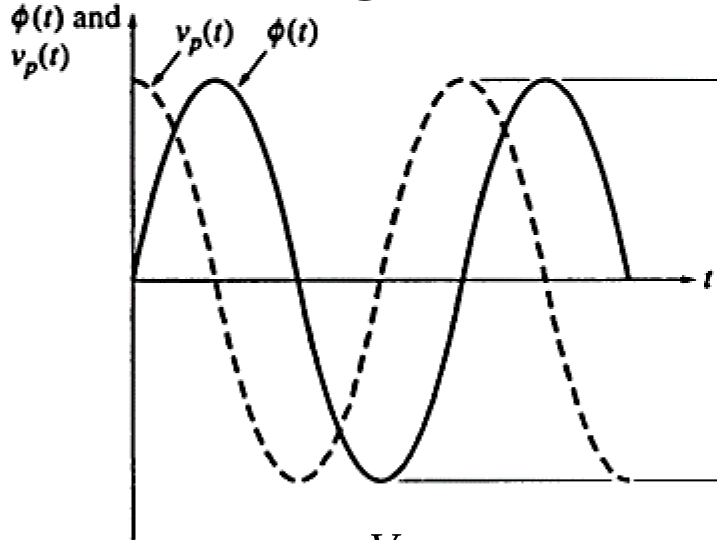
Magnetization current in real transformer

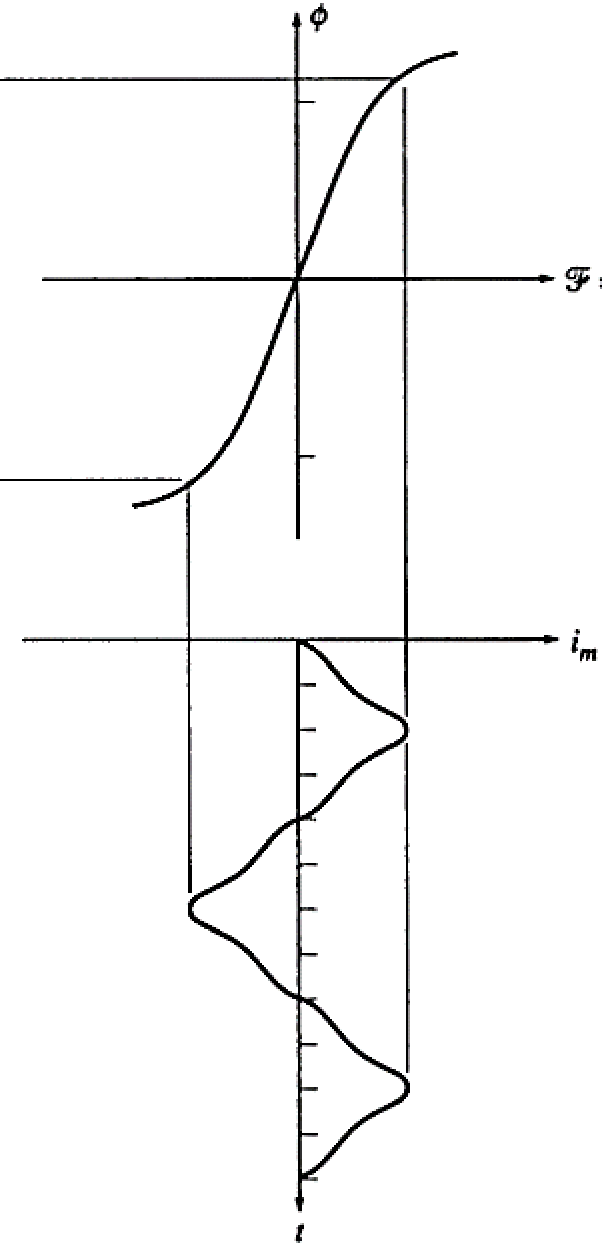
1. The magnetization current i_M is used to generate mutual flux ϕ_M
2. While secondary side is opened, the current measured at primary side contains two parts and is called the excitation current i_{ex}
    1. Magnetization current i_M: to generate mutual flux
    2. Core loss current i_{h+e} : hysteresis and eddy currents

Magnetization curve



Magnetization current





$$\phi(t) = \frac{V_M}{\omega N_p} \sin \omega t$$

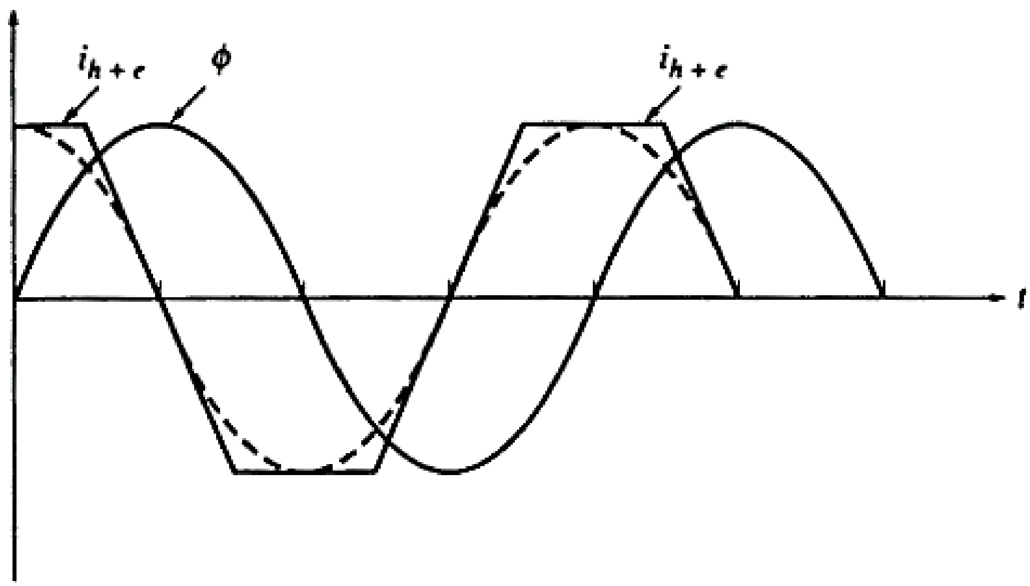
$$v_P(t) = V_M \cos \omega t V$$

(b)

Magnetization current

1. The magnetization current in the transformer is not sinusoidal. The higher-frequency components in the magnetization current are due to magnetic saturation in the transformer core.
2. Once the peak flux reaches the saturation point in the core, a small increase in peak flux requires a very large increase in the peak magnetization current.
3. The fundamental component of the magnetization current lags the voltage applied to the core by 90°
4. The higher-frequency components in the magnetization current can be quite large compared to the fundamental component. In general, the further a trans-former core is driven into saturation, the larger the harmonic components will become.

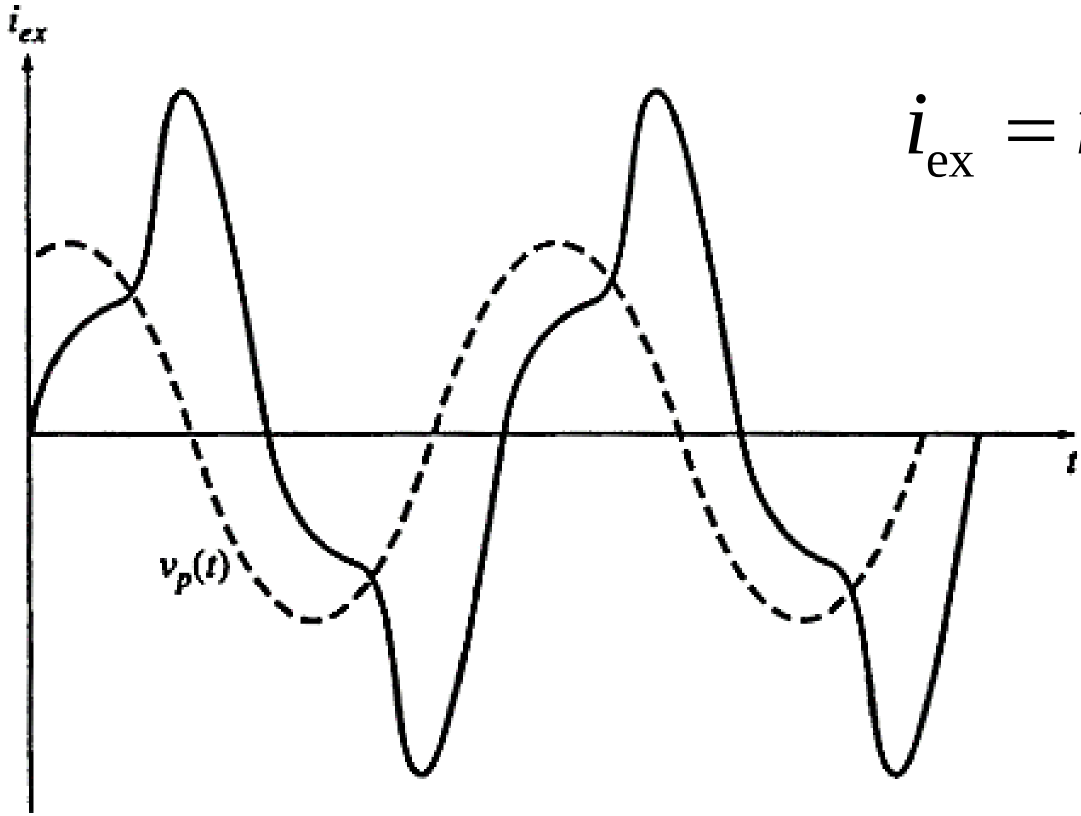
Core loss current



1, The core-loss current is nonlinear because of the nonlinear effects of hysteresis.

2. The fundamental component of the core-loss current is in phase with the voltage applied to the core.

Excitation current i_{ex}



$$i_{\mathrm{ex}} = i_m + i_{h+e}$$

The assumptions from real to ideal transformer

1. The core must have no hysteresis or eddy currents.

2. The magnetization curve must have the shape shown previously. Notice that for an unsaturated core the net magnetomotive force $\mathcal{F}_{net} = 0$, implying that $N_p i_p = N_s i_s$

3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.

4.The resistance of the transformer windings must be zero.

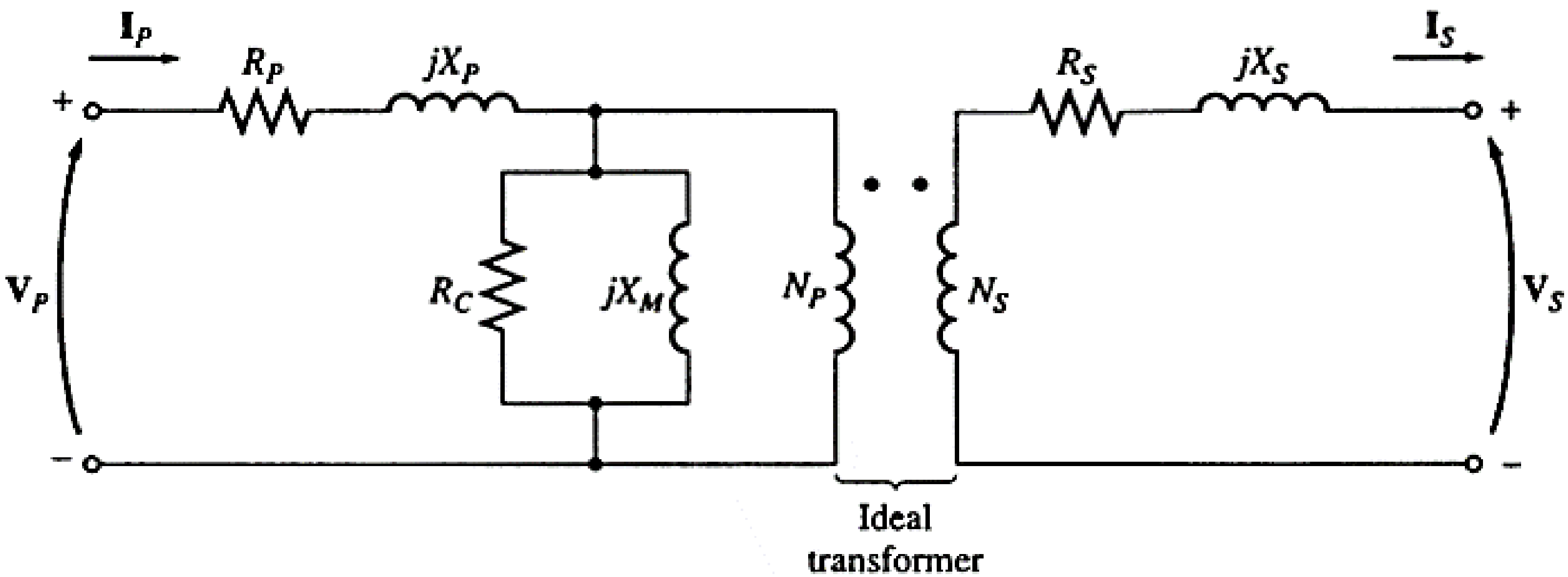
The equivalent circuit of a transformer – to model the non-ideal characteristics

1. *Copper (I^2R) losses*. Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.
2. *Eddy current losses*. Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
3. *Hysteresis losses*. Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle, as explained previously. They are a complex, nonlinear function of the voltage applied to the transformer.
4. *Leakage flux*. The fluxes ϕ_{LP} and ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be accounted for.

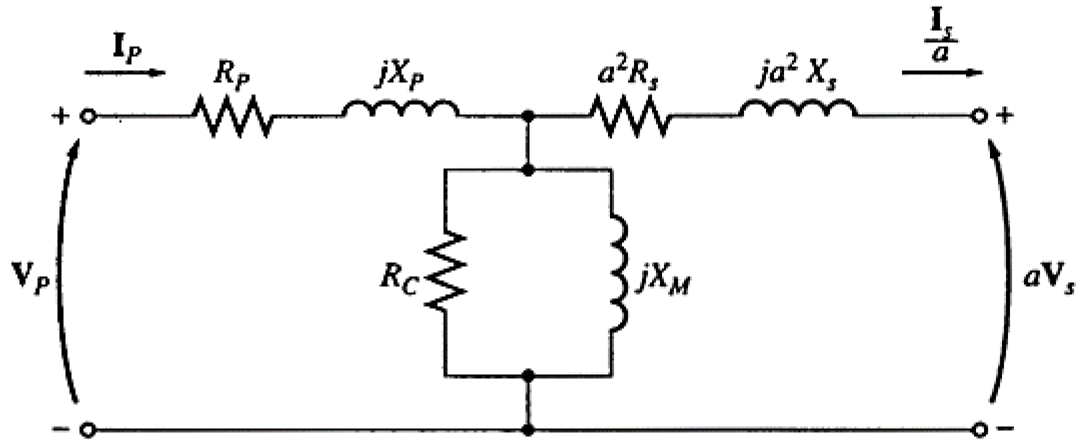
Modeling excitation current and copper loss

- The hysteresis and eddy currents is in-phase with input voltage (modeled as a shunt resistor R_c)
- The magnetization current is lagging input voltage by 90 degrees (modeled as a shunt inductor X_m)
- The copper loss can be modeled as the series resistors R_p and R_s

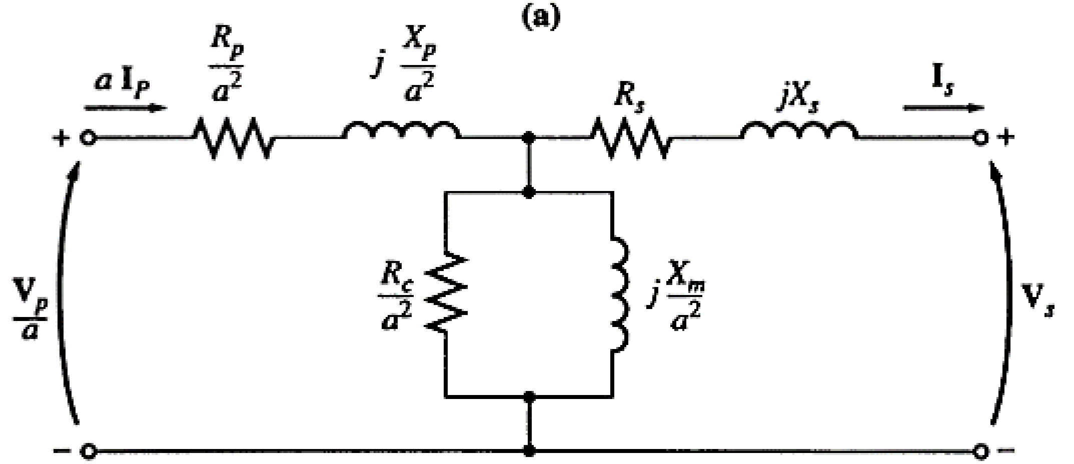
The resulting equivalent circuit



Impedance transform to primary or secondary side



(a)



(b)

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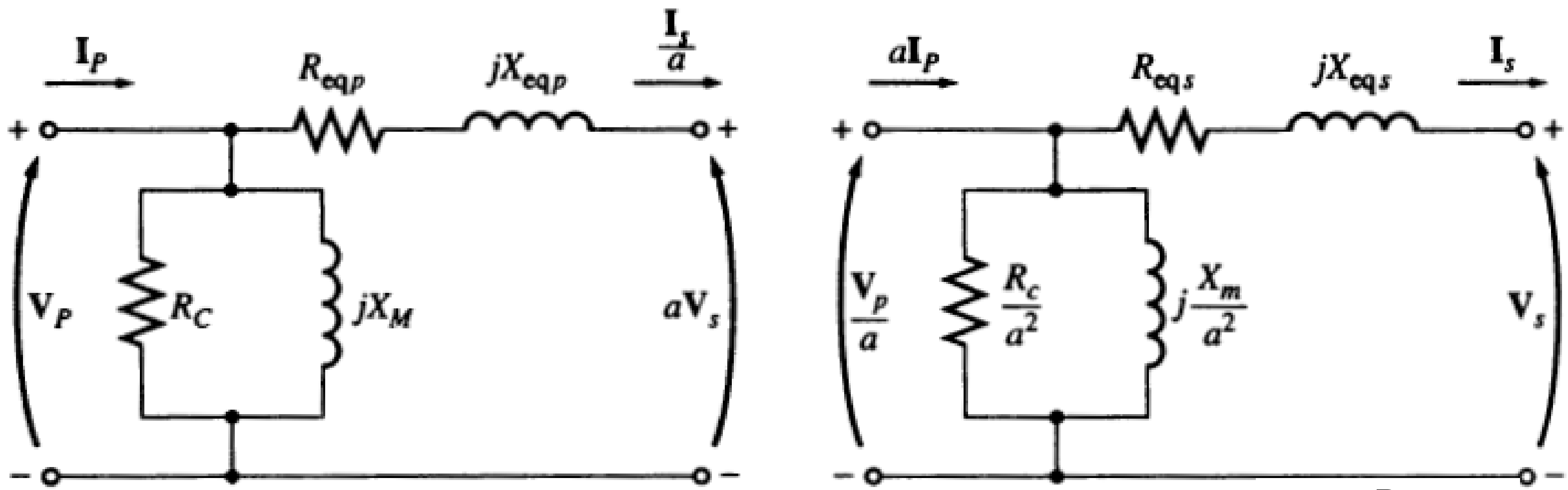
Approximate equivalent circuit



(a)

$$R_{\text{eq}\,p} = R_p + a^2 R_s$$

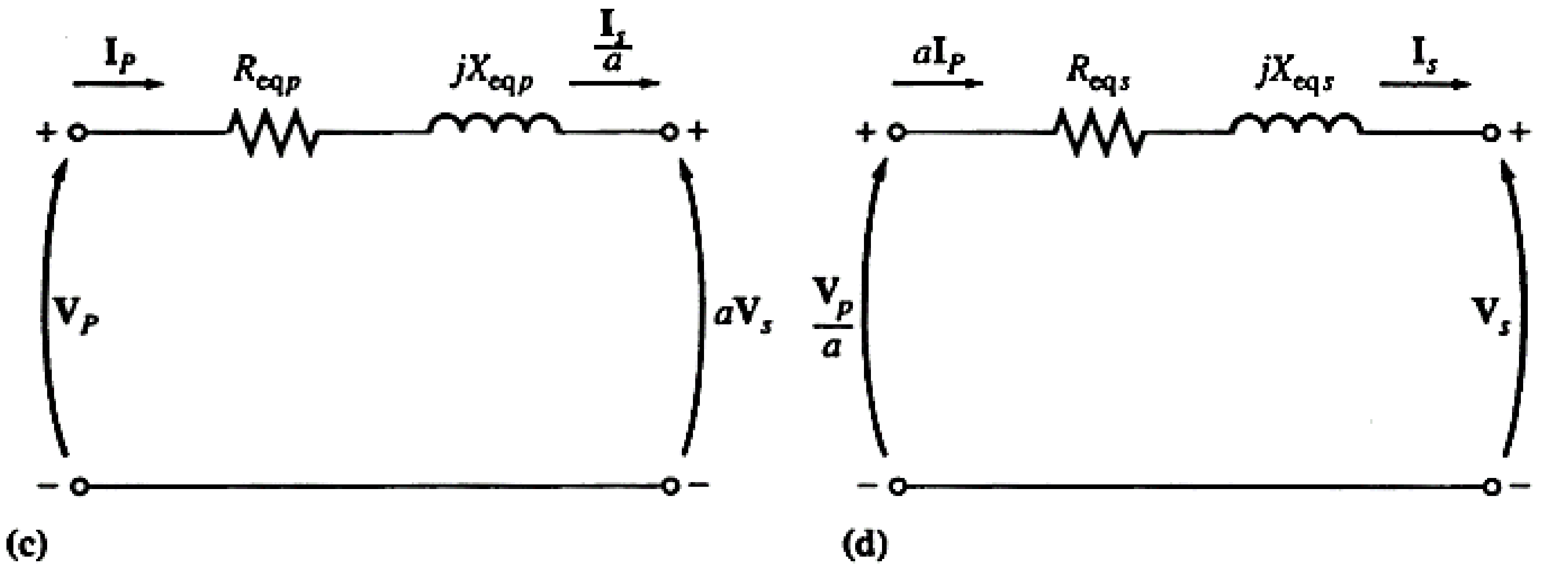
$$X_{\text{eq}\,p} = X_p + a^2 X_s$$

(b)

$$R_{\text{eq}\,s} = \frac{R_p}{a^2} + R_s$$

$$X_{\text{eq}\,s} = \frac{X_p}{a^2} + X_s$$

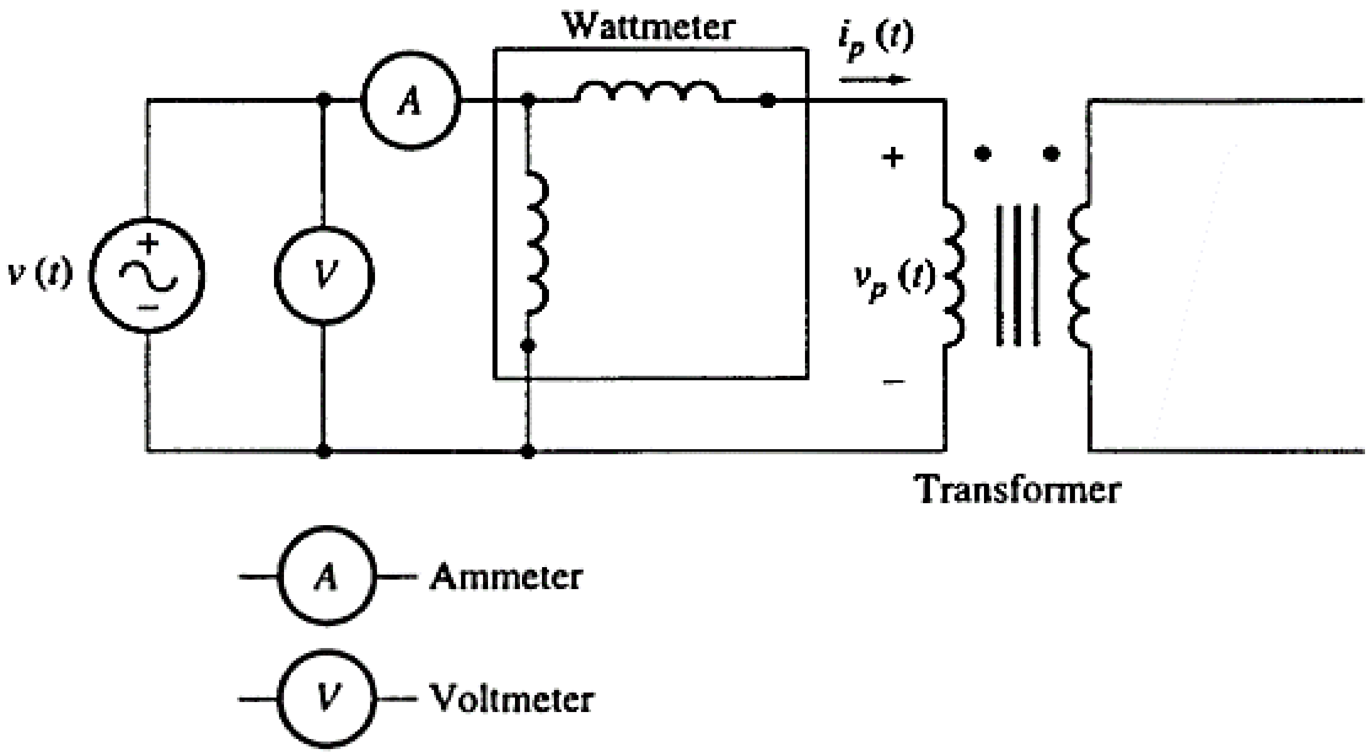
Neglecting excitation current



Measure the equivalent circuit parameters

- There are two types of measurements used for determination the equivalent circuit parameters
- ***Open circuit test*** – used to measure excitation branch
- ***Short circuit test*** – used to measure series branch

Open circuit test



Open circuit test

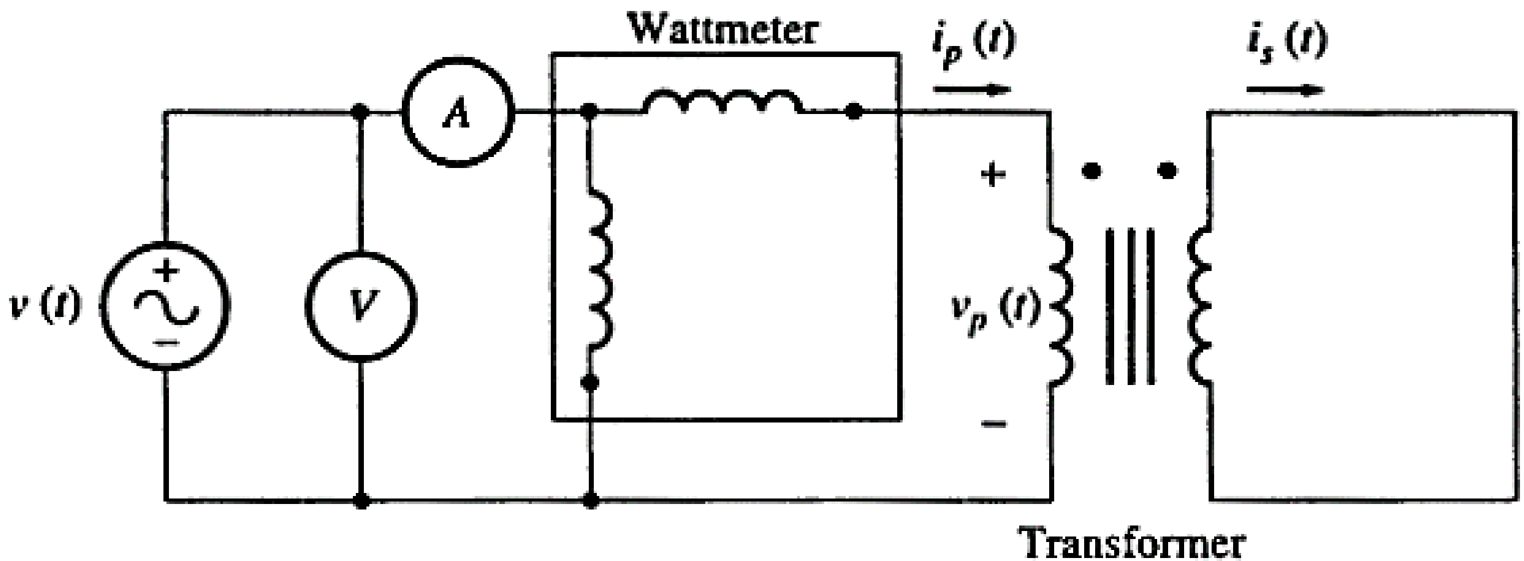
- Under the open circuit condition, all the input current flows through the excitation branch (The measurement is normally done on the *low-voltage* side, since lower voltages are easier to work with.)

$$
\begin{aligned}
Y_E &= G_C - jB_M \\
&= \frac{1}{R_C} - j\frac{1}{X_M} \\
\theta &= \cos^{-1}\frac{P_{\mathrm{OC}}}{V_{\mathrm{OC}} I_{\mathrm{OC}}} \\
\mathrm{PF} &= \cos\theta = \frac{P_{\mathrm{OC}}}{V_{\mathrm{OC}} I_{\mathrm{OC}}}
\end{aligned}
$$

$$
\left|Y_E\right| = \frac{I_{\mathrm{OC}}}{V_{\mathrm{OC}}}
$$

$$
\begin{aligned}
Y_E &= \frac{I_{\mathrm{OC}}}{V_{\mathrm{OC}}}\angle -\theta \\
&= \frac{I_{\mathrm{OC}}}{V_{\mathrm{OC}}}\angle -\cos^{-1}\mathrm{PF}
\end{aligned}
$$

Short circuit test



Short circuit test

- At secondary side short circuit condition, the input voltage must be a very low value to prevent input large short circuit current (The measurement is normally done on the *high-voltage* side, since lower currents are easier to work with)

$$\left|Z_{\mathrm{SE}}\right| = \frac{V_{\mathrm{SC}}}{I_{\mathrm{SC}}}$$

$$\mathrm{PF} = \cos\theta = \frac{P_{\mathrm{SC}}}{V_{\mathrm{SC}} I_{\mathrm{SC}}}$$

$$\mathrm{Z}_{\mathrm{SE}} = \frac{V_{\mathrm{SC}}\angle 0^{\circ}}{I_{\mathrm{SC}}\angle -\theta^{\circ}} = \frac{V_{\mathrm{SC}}}{I_{\mathrm{SC}}}\angle\theta^{\circ}$$

$$\theta = \cos^{-1}\frac{P_{\mathrm{SC}}}{V_{\mathrm{SC}} I_{\mathrm{SC}}}$$

$$Z_{\mathrm{SE}} = R_{\mathrm{eq}} + jX_{\mathrm{eq}}$$

$$= \left(R_P + a^2 R_S\right) + j\left(X_P + a^2 X_S\right)$$

Example 2-2

The equivalent circuit impedances of a 20-kVA, 8000/240-V, 60-Hz transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken:

Open-circuit test (on primary)	Short-circuit test (on primary)
$V_{oc} = 8000$ V	$V_{sc} = 489$ V
$I_{oc} = 0.214$ A	$I_{sc} = 2.5$ A
$P_{oc} = 400$ W	$P_{sc} = 240$ W

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

The power factor during the open-circuit test is

$$\begin{aligned}\mathrm{PF} &= \cos\theta = \frac{P_{\mathrm{OC}}}{V_{\mathrm{OC}} I_{\mathrm{OC}}} \\ &= \cos\theta = \frac{400\mathrm{W}}{(8000\mathrm{V})(0.214\mathrm{A})} \\ &= 0.234 \text{ lagging}\end{aligned}$$

The excitation admittance is given by

$$\begin{aligned}Y_E &= \frac{I_{\mathrm{OC}}}{V_{\mathrm{OC}}} \angle -\cos^{-1}\mathrm{PF} \\ &= \frac{0.214\mathrm{A}}{8000\mathrm{V}} \angle -\cos^{-1} 0.234 \\ &= 0.0000268 \angle -76.5^{\circ}\,\Omega \\ &= 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}\end{aligned}$$

Therefore,

$$R_C = \frac{1}{0.0000063} = 159\text{k}\Omega$$

$$X_M = \frac{1}{0.0000261} = 38.4\text{k}\Omega$$

The power factor during the short-circuit test is

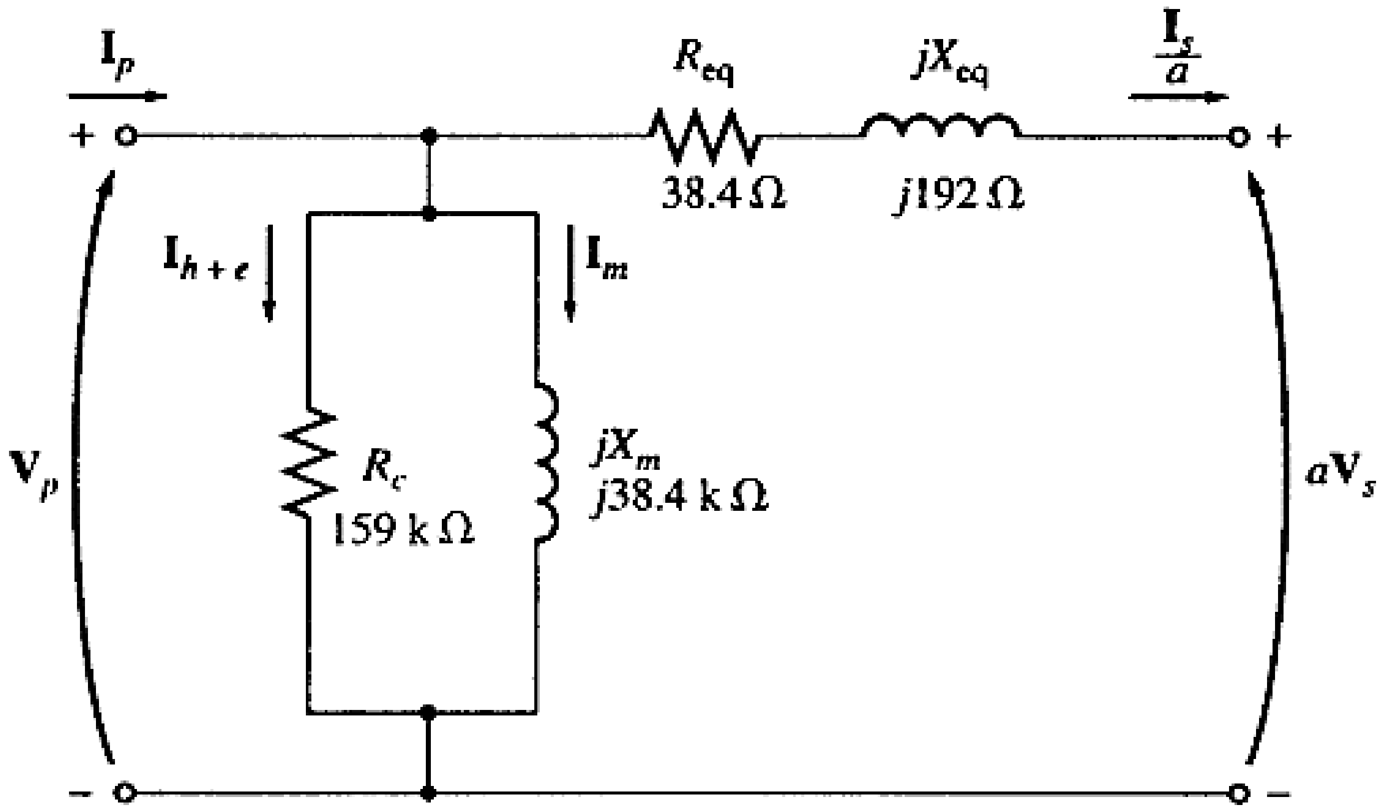
$$\begin{aligned}\text{PF} &= \cos\theta = \frac{P_{\text{SC}}}{V_{\text{SC}} I_{\text{SC}}} \\ &= \cos\theta = \frac{240\text{W}}{(489\text{V})(2.5\text{A})} = 0.196 \text{ lagging}\end{aligned}$$

The series impedance is given by

$$\begin{aligned}Z_{\text{SE}} &= \frac{V_{\text{SC}}}{I_{\text{SC}}} \angle -\cos^{-1}\text{PF} \\ &= \frac{489\text{V}}{2.5\text{A}} \angle 78.7^{\circ} \\ &= 195.6\angle 78.7^{\circ} = 38.4 + j192\Omega\end{aligned}$$

Therefore, the equivalent resistance and reactance are

$$R_{\text{eq}} = 38.4\Omega \quad X_{\text{eq}} = 192\Omega$$


$\mathbf{I}_p$
R_{eq}
jX_{eq}
$\frac{\mathbf{I}_s}{a}$
+
+
38.4 Ω
j192 Ω
$\mathbf{I}_{h+e}$
$\mathbf{I}_m$
$\mathbf{V}_p$
R_c
159 k Ω
jX_m
j38.4 k Ω
$a\mathbf{V}_s$
-
-

Transformer voltage regulation and efficiency

Because a real transformer has series impedances within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *voltage regulation* (VR). *Full-load voltage regulation* is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load. It is defined by the equation

$$\mathrm{VR} = \frac{V_{S,\mathrm{nl}} - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%$$

Since at no load, $V_s = V_p/a$, the voltage regulation can also be expressed as

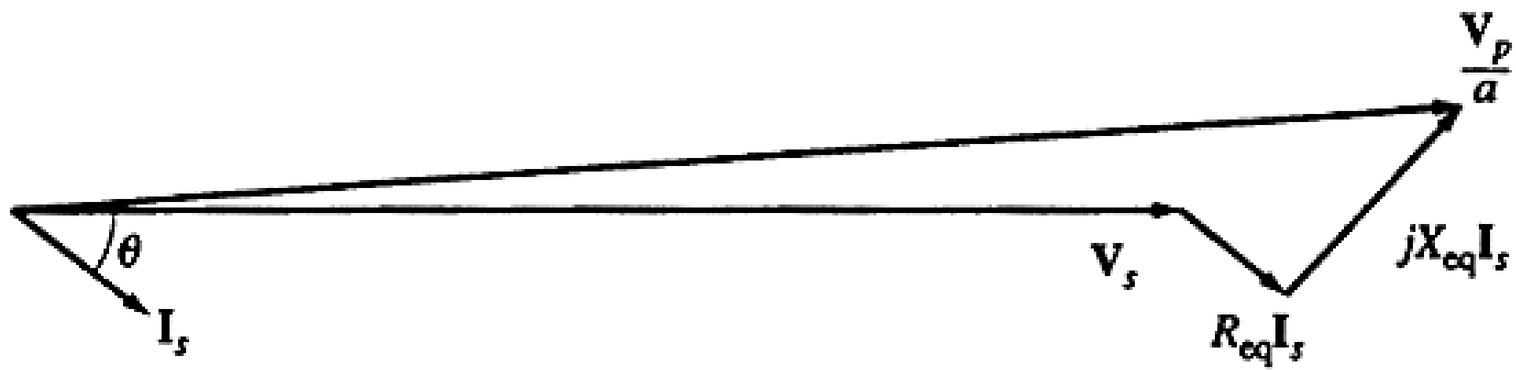
$$\mathrm{VR} = \frac{V_P / a - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%$$

How to calculate the voltage regulation – transformer phasor diagram

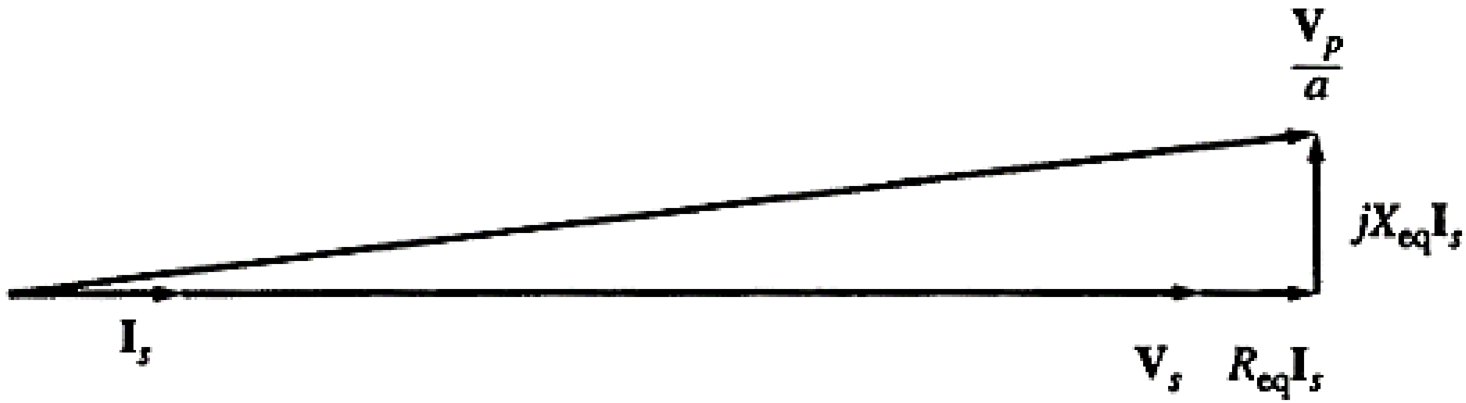
- Use the phasor relation to obtain the voltage regulation

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{\text{eq}}\mathbf{I}_S + jX_{\text{eq}}\mathbf{I}_S$$

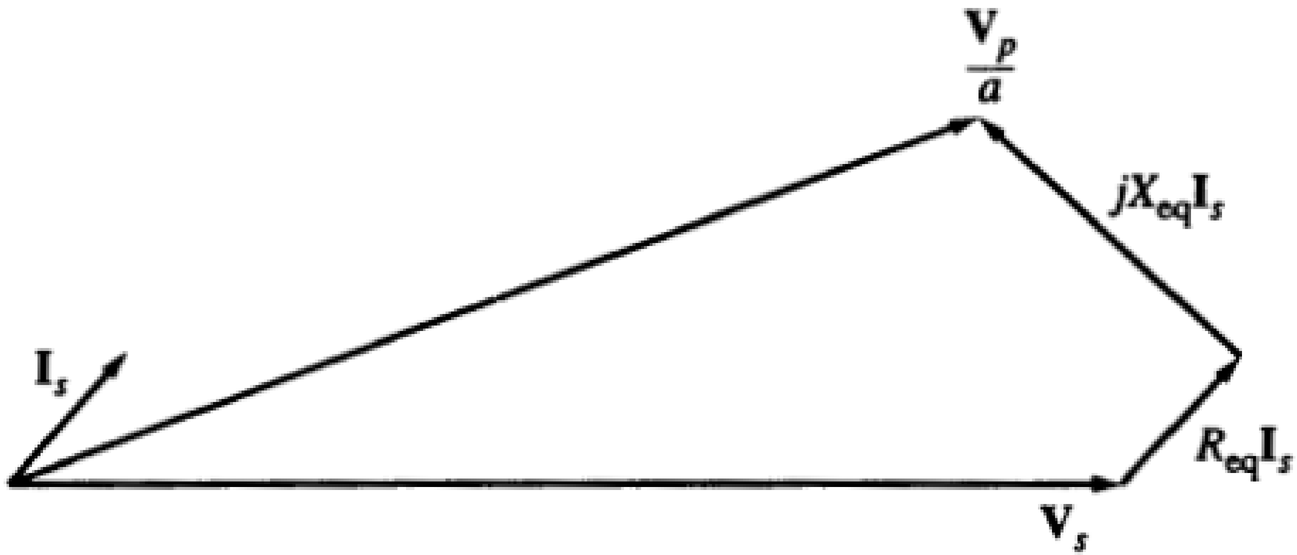
Phasor diagram - lagging



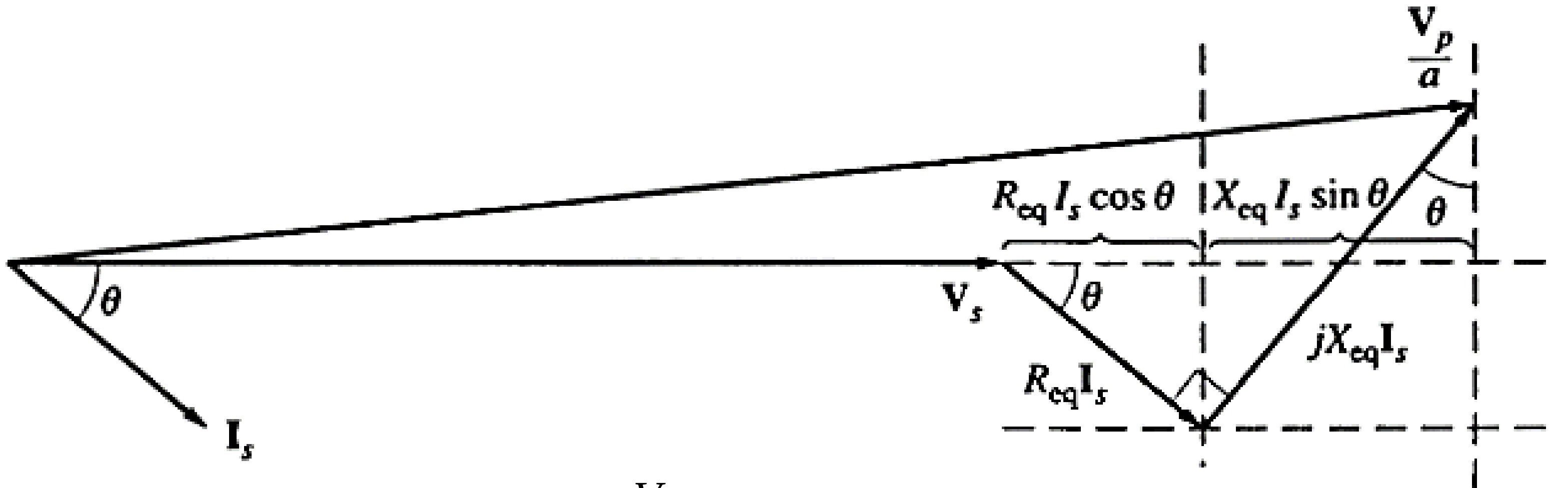
Phasor diagram - unit



Phasor diagram - leading



Approximation



$$\frac{V_p}{a} \approx \mathrm{V}_s + R_{\mathrm{eq}} I_s \cos\theta + X_{\mathrm{eq}} I_s \sin\theta$$

Voltage Regulation Characteristics

Lagging P.F.	$V_P / a > V_S$	V.R. > 0
Unity P.F.	$V_P / a > V_S$	V.R. >0 (smaller)
Leading P.F.	$V_S > V_P / a$	V.R. < 0

Transformer efficiency

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} \times 100\%$$

1. *Copper (I^2R) losses*. These losses are accounted for by the series resistance in the equivalent circuit.
2. *Hysteresis losses*. These losses were explained previously and are accounted for by resistor R_C.
3. *Eddy current losses*. These losses were explained previously and are accounted for by resistor R_C.

Transformer efficiency

To calculate the efficiency of a transformer at a given load, just add the losses from each resistor and apply the previous equation. Since the output power is given by

$$P_{\text{out}} = V_S I_S \cos\theta_{\text{S}}$$

the efficiency of the transformer can be expressed by

$$\eta = \frac{V_S I_S \cos\theta}{P_{\text{Cu}} + P_{\text{core}} + V_S I_S \cos\theta} \times 100\%$$

When is the efficiency maximized?

$$\eta = \frac{P_2}{P_1} = \frac{P_1 - \Sigma p}{P_1} = (1 - \frac{\Sigma p}{P_2 + \Sigma p})$$

$$P_2 = U_2 I_2 \cos\varphi_2 \approx U_{2N} I_2 \cos\varphi_2$$

$$= \beta U_{2N} I_{2N} \cos\varphi_2 = \beta S_N \cos\varphi_2$$

$$p_{Fe} \approx p_0, \qquad p_{Cu} = p_k = \frac{I_2^2}{I_{2N}^2} p_{kN} = \beta^2 p_{kN}$$

$$\eta = \left(1 - \frac{p_0 + \beta^2 p_{kN}}{\beta S_N \cos\varphi_2 + p_0 + \beta^2 p_{kN}}\right) \times 100\%$$

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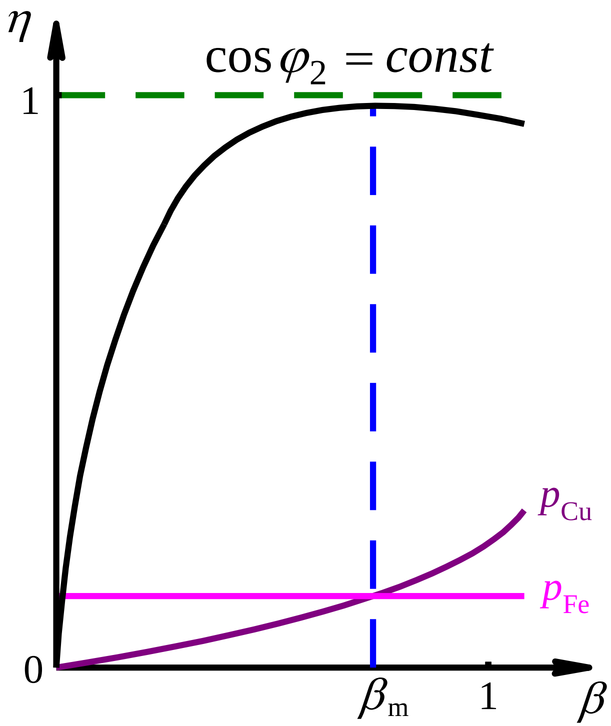
For constant power factor

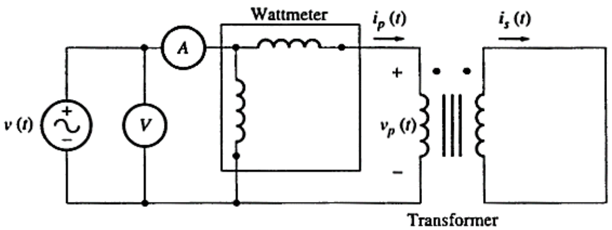


$$p_0 = \beta_m^2 p_{kN}$$

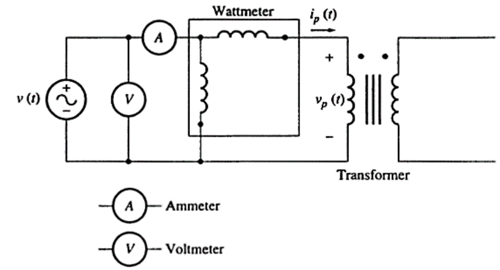
$$\beta_m = \sqrt{\frac{p_0}{p_{kN}}}$$

$$\beta = \frac{I_1}{I_{1N}} = \frac{I_2}{I_{2N}}$$

Efficiency is optimized when copper loss = core loss



Example 2-5



A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the primary side of the transformer:

Open-circuit test	Short-circuit test
V_{OC}=2300V	V_{SC}=47V
I_{OC}=0.21A	I_{SC}=6.0A
P_{OC}=50W	P_{SC}=160W

The data have been taken by using the connections shown previously.

(a) Find the equivalent circuit of this transformer referred to the high-voltage side.
(b) Find the equivalent circuit of this transformer referred to the low-voltage side.
(c) Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor.
(d) Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
(e) What is the efficiency of the transformer at full load with a power factor of 0.8lagging?

(a) The excitation branch values of the transformer equivalent circuit can be calculated from the open-circuit test data, and the series elements can be calculated from the short-circuit test data, From the open-circuit test data, the open-circuit impedance angle is

$$\begin{aligned}\theta_{\mathrm{OC}} &= \cos^{-1}\frac{P_{\mathrm{OC}}}{V_{\mathrm{OC}}I_{\mathrm{OC}}} \\ &= \cos^{-1}\frac{50\mathrm{W}}{(2300\mathrm{V})(0.21\mathrm{A})} = 84^{\circ}\end{aligned}$$

The excitation admittance is thus

$$\begin{aligned}Y_E &= \frac{I_{\mathrm{OC}}}{V_{\mathrm{OC}}}\angle -84^{\circ} \\ &= \frac{0.21\mathrm{A}}{2300\mathrm{V}}\angle -84^{\circ} \\ &= 9.13\times 10^{-5}\angle -84^{\circ}\,\Omega = 0.0000095 - j0.0000908\Omega\end{aligned}$$

The elements of the excitation branch referred to the primary are

$$R_C = \frac{1}{0.0000095} = 105\mathrm{k}\Omega$$

$$X_M = \frac{1}{0.0000908} = 11\mathrm{k}\Omega$$

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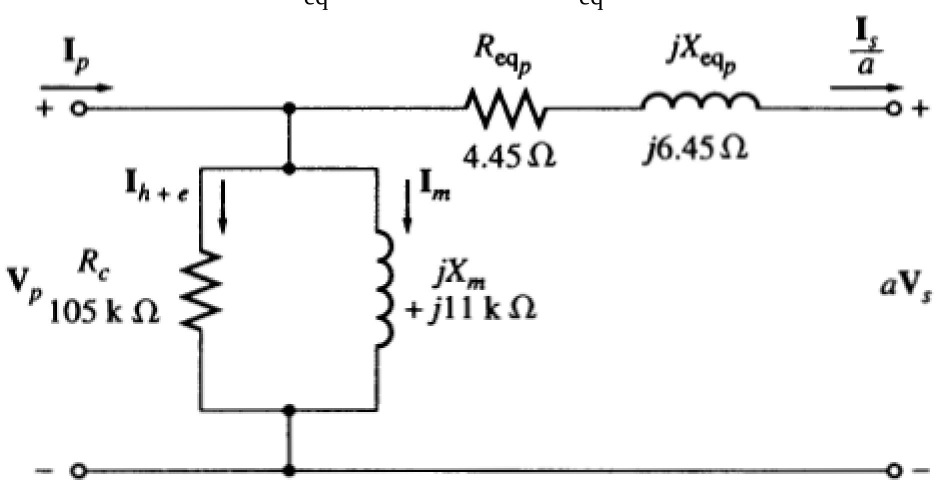
$$
\begin{aligned}
\theta_{SC} &= \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} \\
&= \cos^{-1}\frac{160\mathrm{W}}{(47\mathrm{V})(6\mathrm{A})} = 55.4^{\circ}
\end{aligned}
$$

The equivalent series impedance is thus

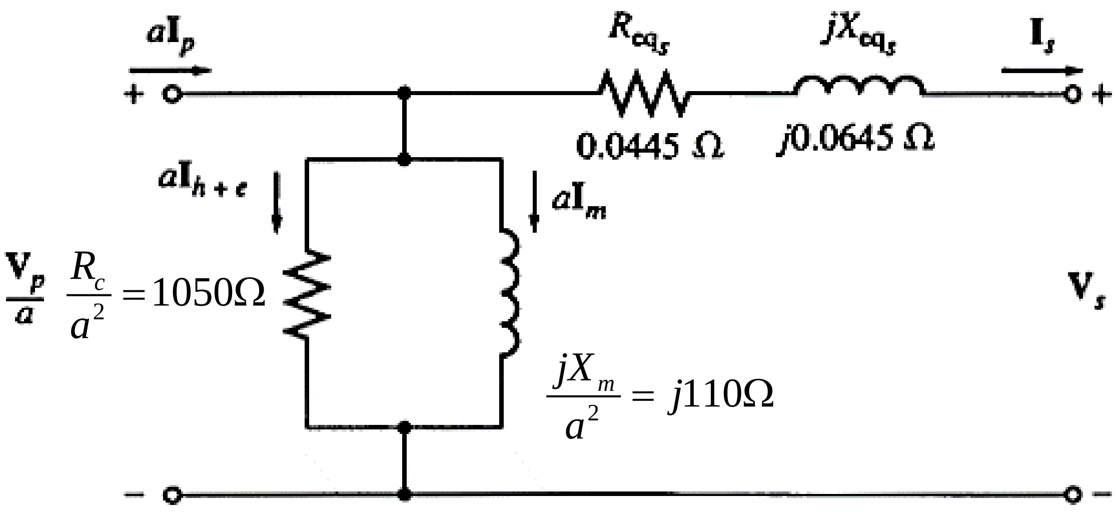
$$
\begin{aligned}
Z_{SE} &= \frac{V_{SC}}{I_{SC}}\angle\theta_{SC} \\
&= \frac{47\mathrm{V}}{6\mathrm{A}}\angle 55.4^{\circ}\,\Omega \\
&= 7.833\angle 55.4^{\circ} = 4.45 + j6.45
\end{aligned}
$$

The series elements referred to the primary are

$$
R_{eq} = 4.45\Omega \quad X_{eq} = 6.45\Omega
$$



(a)



(b)

$$R_C = 1050\Omega \quad R_{\text{eq}} = 0.0445\Omega$$

$$X_M = 110\Omega \quad X_{\text{eq}} = 0.0645\Omega$$

(c) The full-load current on the secondary side of this transformer is

$$I_{S,\text{ rated}} = \frac{S_{\text{rated}}}{V_{S,\text{ rated}}} = \frac{15{,}000\text{VA}}{230\text{V}} = 65.2\text{A}$$

To calculate V_p/a, we use

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{\text{eq}}\mathbf{I}_S + jX_{\text{eq}}\mathbf{I}_S$$

At PF = 0.8 lagging, current $\mathbf{I}_S$ = 65.2∠-36.9° A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\text{ V} + (0.0445\Omega)\left(65.2\angle -36.9^\circ\text{ A}\right) + j(0.0645\Omega)\left(65.2\angle -36.9^\circ\text{ A}\right)\\ &= 230\angle 0^\circ\text{ V} + 2.90\angle -36.9^\circ\text{ V} + 4.21\angle 53.1^\circ\text{ V}\\ &= 230 + 2.32 - j1.74 + 2.52 + j3.36\\ &= 234.84 + j1.62 = 234.85\angle 0.40^\circ\text{ V}\end{aligned}$$

The resulting voltage regulation is

$$\begin{aligned}\text{VR} &= \frac{V_P/a - V_{S,\text{f}}}{V_{S,\text{n}}}\times 100\%\\ &= \frac{234.85\text{V} - 230\text{V}}{230\text{V}}\times 100\% = 2.1\%\end{aligned}$$

At PF = 1.0, current $\mathbf{I}_S$ = 65.2∠0° A. Therefore.

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\,\mathrm{V} + (0.0445\Omega)\left(65.2\angle 0^\circ\,\mathrm{A}\right) + j(0.0645\Omega)\left(65.2\angle 0^\circ\,\mathrm{A}\right) \\ &= 230\angle 0^\circ\,\mathrm{V} + 2.90\angle 0^\circ\,\mathrm{V} + 4.21\angle 90^\circ\,\mathrm{V} \\ &= 230 + 2.90 + j4.21 \\ &= 232.9 + j4.21 = 232.94\angle 1.04^\circ\,\mathrm{V}\end{aligned}$$

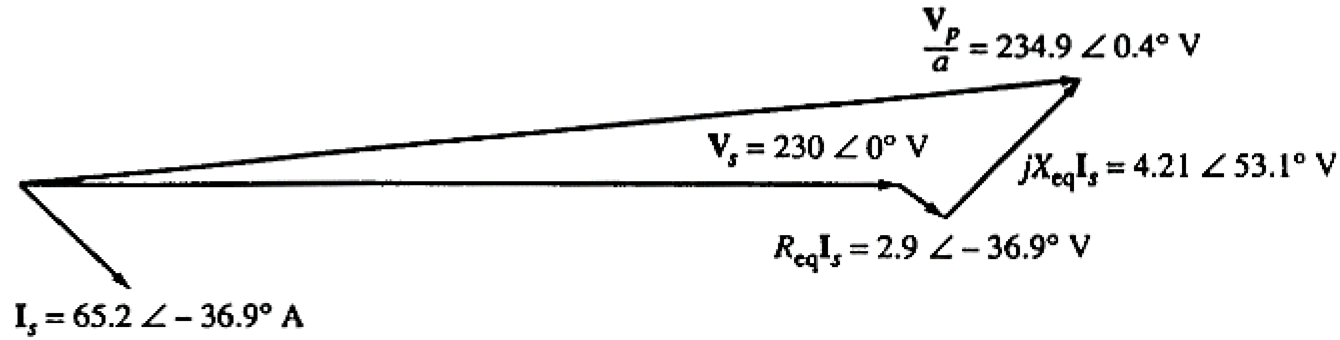
The resulting voltage regulation is

$$\mathrm{VR} = \frac{232.94\mathrm{V} - 230\mathrm{V}}{230\mathrm{V}} \times 100\% = 1.28\%$$

At PF = 0.8 leading, current $\mathbf{I}_S$ = 65.2∠36.9° A. Therefore,

$$\begin{aligned}\frac{\mathbf{V}_P}{a} &= 230\angle 0^\circ\,\mathrm{V} + (0.0445\Omega)\left(65.2\angle 36.9^\circ\,\mathrm{A}\right) + j(0.0645\Omega)\left(65.2\angle 36.9^\circ\,\mathrm{A}\right) \\ &= 230\angle 0^\circ\,\mathrm{V} + 2.90\angle 36.9^\circ\,\mathrm{V} + 4.21\angle 126.9^\circ\,\mathrm{V} \\ &= 230 + 2.32 + j1.74 - 2.52 + j3.36 \\ &= 229.80 + j5.10 = 229.85\angle 1.27^\circ\,\mathrm{V}\end{aligned}$$

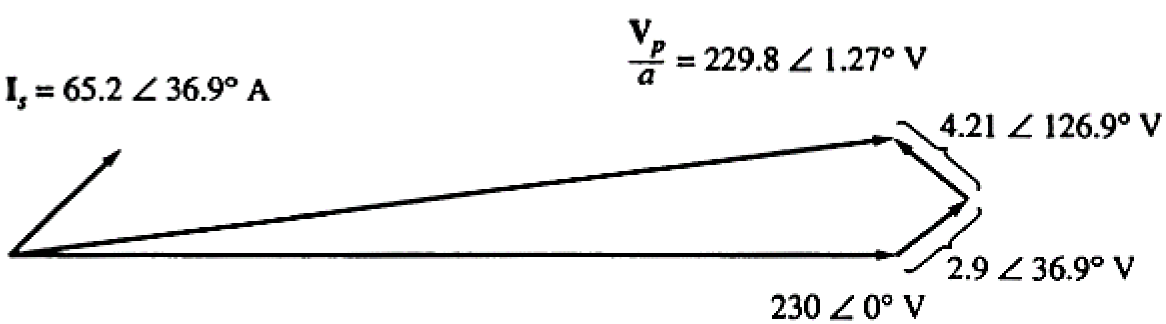
$$VR = \frac{229.85\mathrm{V} - 230\mathrm{V}}{230\mathrm{V}} \times 100\% = -0.062\%$$


$\frac{\mathbf{V}_p}{a} = 234.9 \angle 0.4° \text{ V}$
$\mathbf{V}_s = 230 \angle 0° \text{ V}$
$jX_{eq}\mathbf{I}_s = 4.21 \angle 53.1° \text{ V}$
$R_{eq}\mathbf{I}_s = 2.9 \angle -36.9° \text{ V}$
$\mathbf{I}_s = 65.2 \angle -36.9° \text{ A}$

(a)


$\frac{\mathbf{V}_p}{a} = 232.9 \angle 1.04° \text{ V}$
$4.21 \angle 90° \text{ V}$
$\mathbf{I}_s = 65.2 \angle 0° \text{ A}$
$230 \angle 0° \text{ V}$
$2.9 \angle 0° \text{ V}$

(b)


$\frac{\mathbf{V}_p}{a} = 229.8 \angle 1.27° \text{ V}$
$\mathbf{I}_s = 65.2 \angle 36.9° \text{ A}$
$4.21 \angle 126.9° \text{ V}$
$2.9 \angle 36.9° \text{ V}$
$230 \angle 0° \text{ V}$

(c)

(e) To find the efficiency of the transformer, first calculate its losses. The copper losses

$$P_{\text{Cu}} = \left(I_S\right)^2 R_{\text{eq}} = (65.2\text{A})^2(0.0445\Omega) = 189\text{W}$$

The core losses are given by

$$P_{\text{core}} = \frac{\left(V_P / a\right)^2}{R_C} = \frac{(234.85\text{V})^2}{1050\Omega} = 52.5\text{W}$$

The output power of the transformer at this power factor is

$$\begin{aligned} P_{\text{out}} &= V_S I_S \cos\theta \\ &= (230\text{V})(65.2\text{A})\cos 36.9^\circ = 12,000\text{W} \end{aligned}$$

Therefore, the efficiency of the transformer at this condition is

$$\begin{aligned} \eta &= \frac{V_S I_S \cos\theta}{P_{\text{Cu}} + P_{\text{core}} + V_S I_S \cos\theta} \times 100\% \\ &= \frac{12,000\text{W}}{189\text{W} + 52.5\text{W} + 12,000\text{W}} \times 100\% \\ &= 98.03\% \end{aligned}$$

Transformer taps and voltage regulation

- The taps of transformer is used to change the effective turns ratio of transformer

A 500-kVA，13,200/480-V distribution transformer has four 2.5 percent taps on its primary winding. What are the voltage ratios of this transformer at each tap setting?

<u>Solution</u>

The five possible voltage ratings of this transformer are

+5.0% tap	13860/480V
+2.5% tap	13,530/480 V
Nominal rating	13,200/480 V
-2.5% tap	12,870/480V
-5.0% tap	12,540/480V

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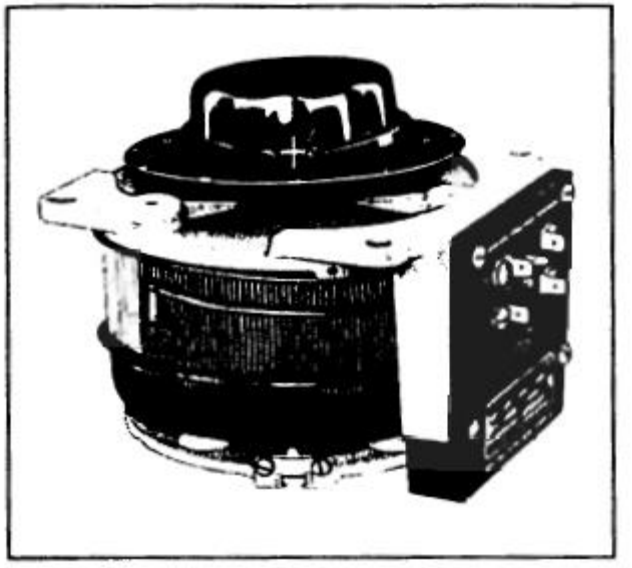
TCUL and voltage regulator

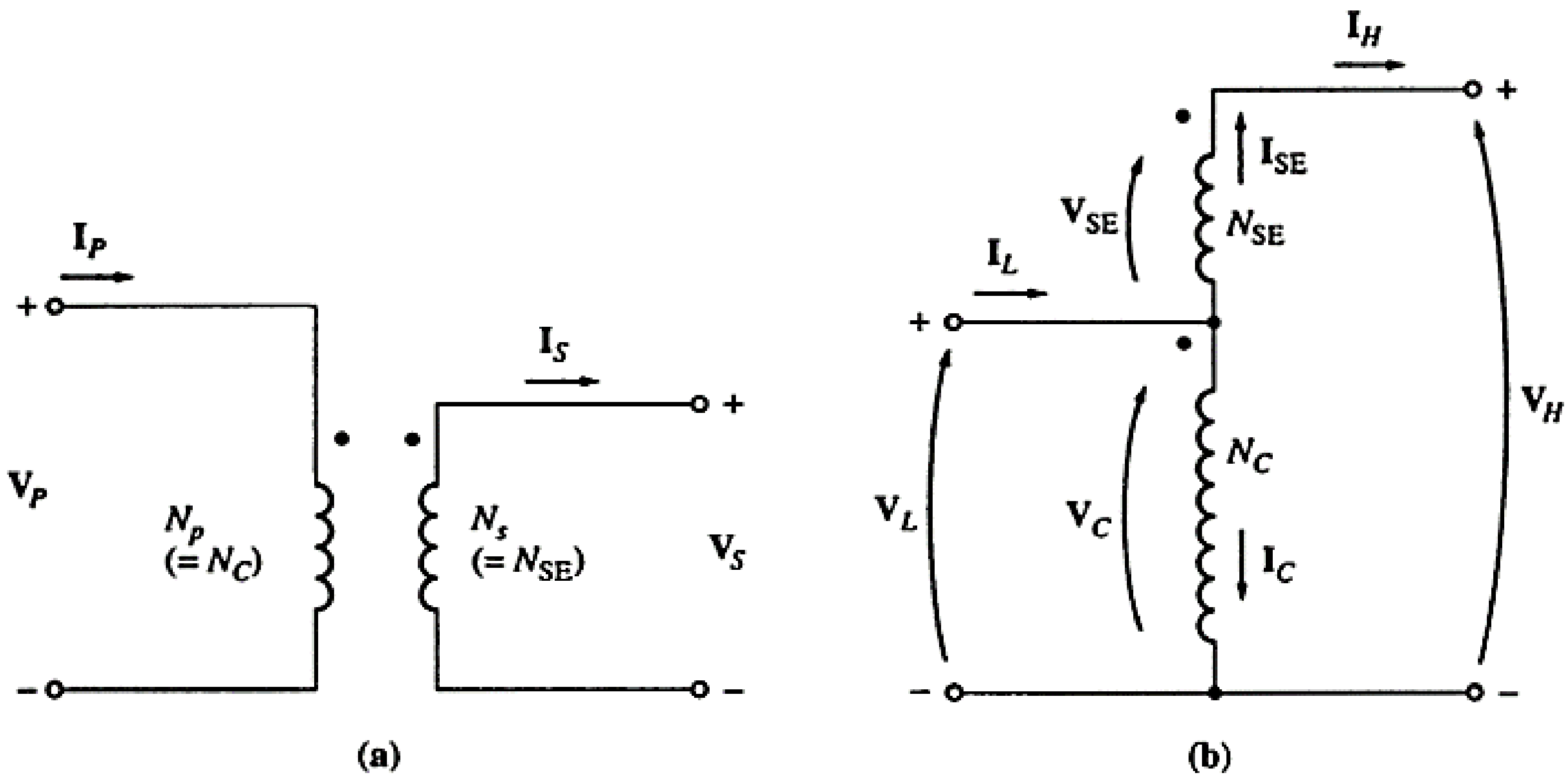
1. The ***tape changing under load*** (TCUL) is a transformer with the ability to change taps while power is connected to it
2. The ***voltage regulator*** is the TCUL with voltage sensing circuitry that automatically change taps to maintain the output voltage level

The autotransformer

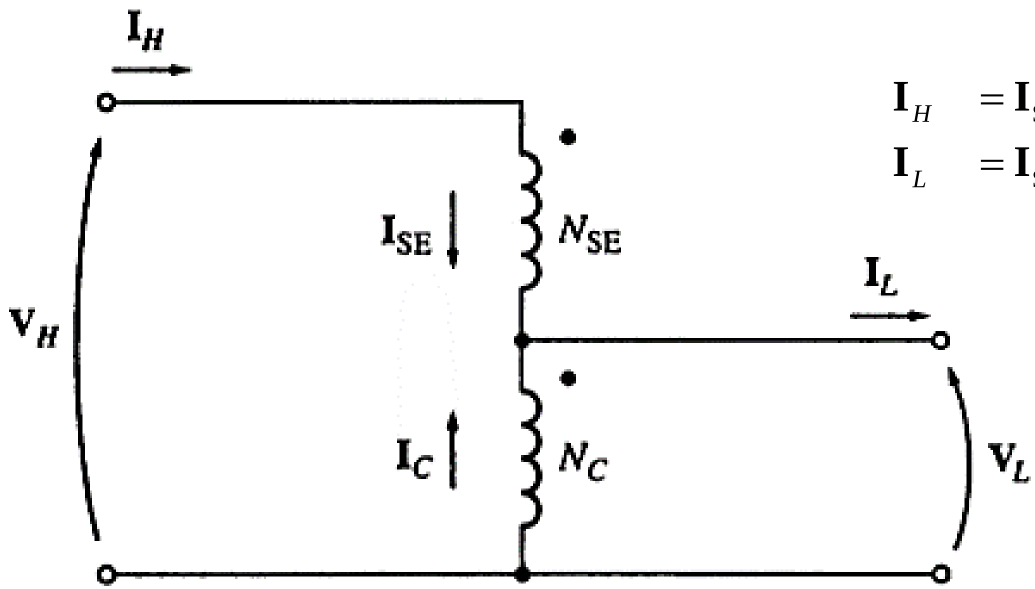
- Continuously tune the output voltage magnitude
- The size of auto transformer is smaller than the size of conventional transformer
- Output terminal is not electrical isolation



The step-up connection of autotransformer from conventional transformer



The step-down connection of autotransformer



$$\mathbf{I}_H = \mathbf{I}_{SE}$$

$$\mathbf{I}_L = \mathbf{I}_{SE} + \mathbf{I}_C$$

Voltage and current in both coils

- ***Coil*** voltage and current in Nc and Nse still follow the voltage and current relation

$$\frac{\mathbf{V}_C}{\mathbf{V}_{\mathrm{SE}}} = \frac{N_C}{N_{\mathrm{SE}}}$$

$$N_C \mathbf{I}_C = N_{\mathrm{SE}} \mathbf{I}_{\mathrm{SE}}$$

- The autotransformer ***terminal*** voltage and current

$$\begin{aligned} \mathbf{V}_L &= \mathbf{V}_C \\ \mathbf{V}_H &= \mathbf{V}_C + \mathbf{V}_{\mathrm{SE}} \\ \mathbf{I}_L &= \mathbf{I}_C + \mathbf{I}_{\mathrm{SE}} \\ \mathbf{I}_H &= \mathbf{I}_{\mathrm{SE}} \end{aligned}$$

Terminal voltage and current relation of autotransformer

$$\mathbf{V}_H = \mathbf{V}_C + \mathbf{V}_{\mathrm{SE}}$$

But $V_C/V_{SE} = N_C/N_{SE}$, so

$$\mathbf{V}_H = \mathbf{V}_C + \frac{N_{\mathrm{SE}}}{N_C}\mathbf{V}_C$$

Finally, noting that $V_L = V_C$, we get

$$\begin{aligned}\mathbf{V}_H &= \mathbf{V}_L + \frac{N_{\mathrm{SE}}}{N_C}\mathbf{V}_L \\ &= \frac{N_{\mathrm{SE}} + N_C}{N_C}\mathbf{V}_L\end{aligned}$$

or

$$\frac{\mathbf{V}_L}{\mathbf{V}_H} = \frac{N_C}{N_{\mathrm{SE}} + N_C}$$

Terminal voltage and current relation of autotransformer

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{\mathrm{SE}}$$

From a previous equation, $I_C = (N_{SE}/N_C)I_{SE}$, so

$$\mathbf{I}_L = \frac{N_{\mathrm{SE}}}{N_C}\mathbf{I}_{\mathrm{SE}} + \mathbf{I}_{\mathrm{SE}}$$

Finally, noting that $I_H = I_{SE}$, we find

$$\mathbf{I}_L = \frac{N_{\mathrm{SE}}}{N_C}\mathbf{I}_H + \mathbf{I}_H$$

$$= \frac{N_{\mathrm{SE}} + N_C}{N_C}\mathbf{I}_H$$

or

$$\frac{\mathbf{I}_L}{\mathbf{I}_H} = \frac{N_{\mathrm{SE}} + N_C}{N_C}$$

Apparatus power rating advantage in autotransformer

- There are two types of rating
  - Power rating on terminals S_{IO}
  - Power rating on windings S_w
- Terminals power rating S_{IO}

$$S_{\text{in}} = V_L I_L \quad S_{\text{out}} = V_H I_H \quad S_{\text{in}} = S_{\text{out}} = S_{\text{IO}}$$

- Winding power rating S_w

$$S_W = V_C I_C = V_{\text{SE}} I_{\text{SE}}$$

Apparatus power rating advantage in autotransformer

- Relation

$$
\begin{aligned}
S_W &= V_C I_C \\
&= V_L \left(I_L - I_H \right) \\
&= V_L I_L - V_L I_H
\end{aligned}
$$

$$
\frac{S_{\text{IO}}}{S_W} = \frac{N_{\text{SE}} + N_C}{N_{\text{SE}}}
$$

$$
\begin{aligned}
S_W &= V_L I_L - V_L I_L \frac{N_C}{N_{\text{SE}} + N_C} \\
&= V_L I_L \frac{\left(N_{\text{SE}} + N_C \right) - N_C}{N_{\text{SE}} + N_C} \\
&= S_{\text{IO}} \frac{N_{\text{SE}}}{N_{\text{SE}} + N_C}
\end{aligned}
$$

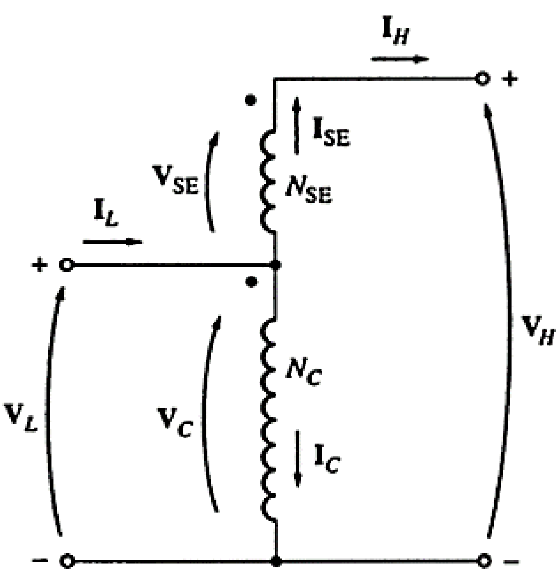
Example 2-7

A 100-VA 120/12-V transformer is to be connected so as to form a step-up autotransformer (see the figure below). A primary voltage of 120 V is applied to the transformer

(a) What is the secondary voltage of the transformer?

(b) What is its maximum voltampere rating in this mode of operation?

(c) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12-V operation.

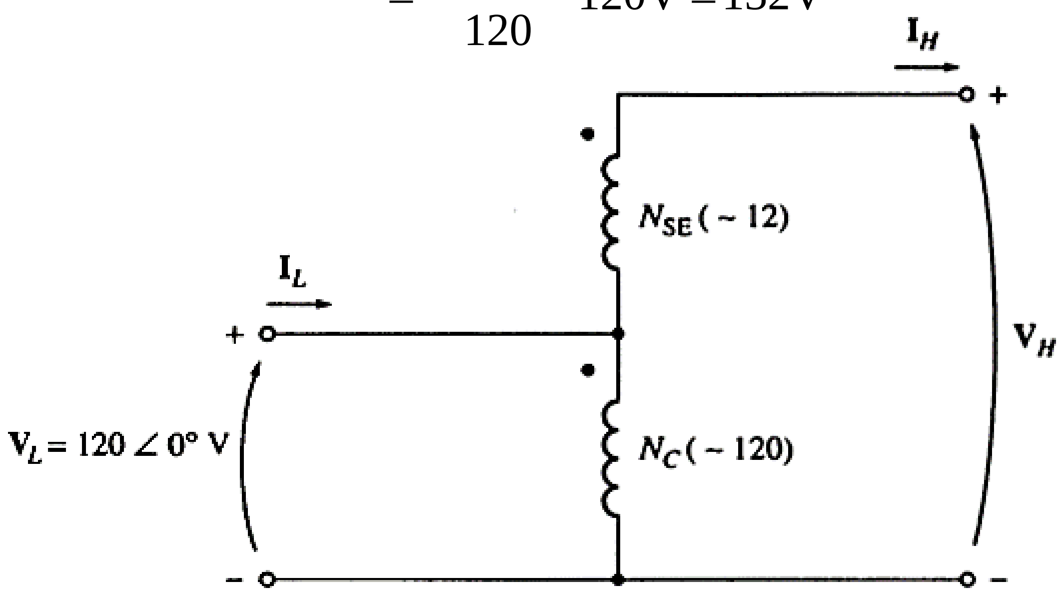


To accomplish a step-up transformation with a 120-V primary, the ratio of the turns on the common winding Nc to the turns on the series winding Nse in this transformer must be 120:12 (or 10:1).

(a) This transformer is being used as a step-up transformer. The secondary voltage is V_H, and from the equation below

$$\mathbf{V}_H = \frac{N_{\mathrm{SE}} + N_C}{N_C}\mathbf{V}_L$$

$$= \frac{12+120}{120}120\mathrm{V} = 132\mathrm{V}$$



(b) The maximum voltampere rating in either winding of this transformer is 100 VA. How much input or output apparent power can this provide? To find out, examine the series winding. The voltage V_{SE} on the winding is 12 V, and the voltampere rating of the winding is 100 VA. Therefore, the maximum series winding current is

$$I_{\mathrm{SE,max}} = \frac{S_{\max}}{V_{\mathrm{SE}}} = \frac{100\mathrm{VA}}{12\mathrm{V}} = 8.33\mathrm{A}$$

Since I_{SE} is equal to the secondary current I_S (or I_H) and since the secondary voltage V_S = V_H = 132 V, the secondary apparent power is

$$\begin{aligned} S_{\mathrm{out}} &= V_S I_S = V_H I_H \\ &= (132\mathrm{V})(8.33\mathrm{A}) = 1100\mathrm{VA} = S_{\mathrm{in}} \end{aligned}$$

(c) The rating advantage can be calculated from part (b) or separately from equations. From part b,

$$\frac{S_{\mathrm{IO}}}{S_W} = \frac{1100\mathrm{VA}}{100\mathrm{VA}} = 11$$

Hence,

$$\begin{aligned} \frac{S_{\mathrm{IO}}}{S_W} &= \frac{N_{\mathrm{SE}} + N_C}{N_{\mathrm{SE}}} \\ &= \frac{12+120}{12} = \frac{132}{12} = 11 \end{aligned}$$

Autotransformer summarize

- When two voltages are fairly close to each other
- The power advantage is very large
- There is a direct physical connection between primary and secondary sides.
- The autotransformer is a convenient and inexpensive way to tie **nearly equal** two voltages together
- The electrical isolation of two sides is lost

Example 2-8

A transformer is rated at 1000 kVA, 12/1.2 kV, 60 Hz when it is operated as a conventional two-winding transformer. Under these conditions, its series resistance and reactance are given as 1 and 8 percent per unit, respectively. This transformer is to be used as a 13.2/12-kV step-down autotransformer in a power distribution system. In the autotransformer connection, **(a)** what is the transformer's rating when used in this manner and **(b)** what is the transformer's series impedance in per-unit?

(a) The N_C/N_{SE} turns ratio must be 12:1.2 or 10:1. The voltage rating of this transformer will be 13.2/12 kV, and the apparent power (voltampere) rating will be

$$
\begin{aligned}
S_{\mathrm{IO}} &= \frac{N_{\mathrm{SE}} + N_C}{N_{\mathrm{SE}}} S_W \\
&= \frac{1+10}{1} 1000\mathrm{kVA} = 11{,}000\mathrm{kVA}
\end{aligned}
$$

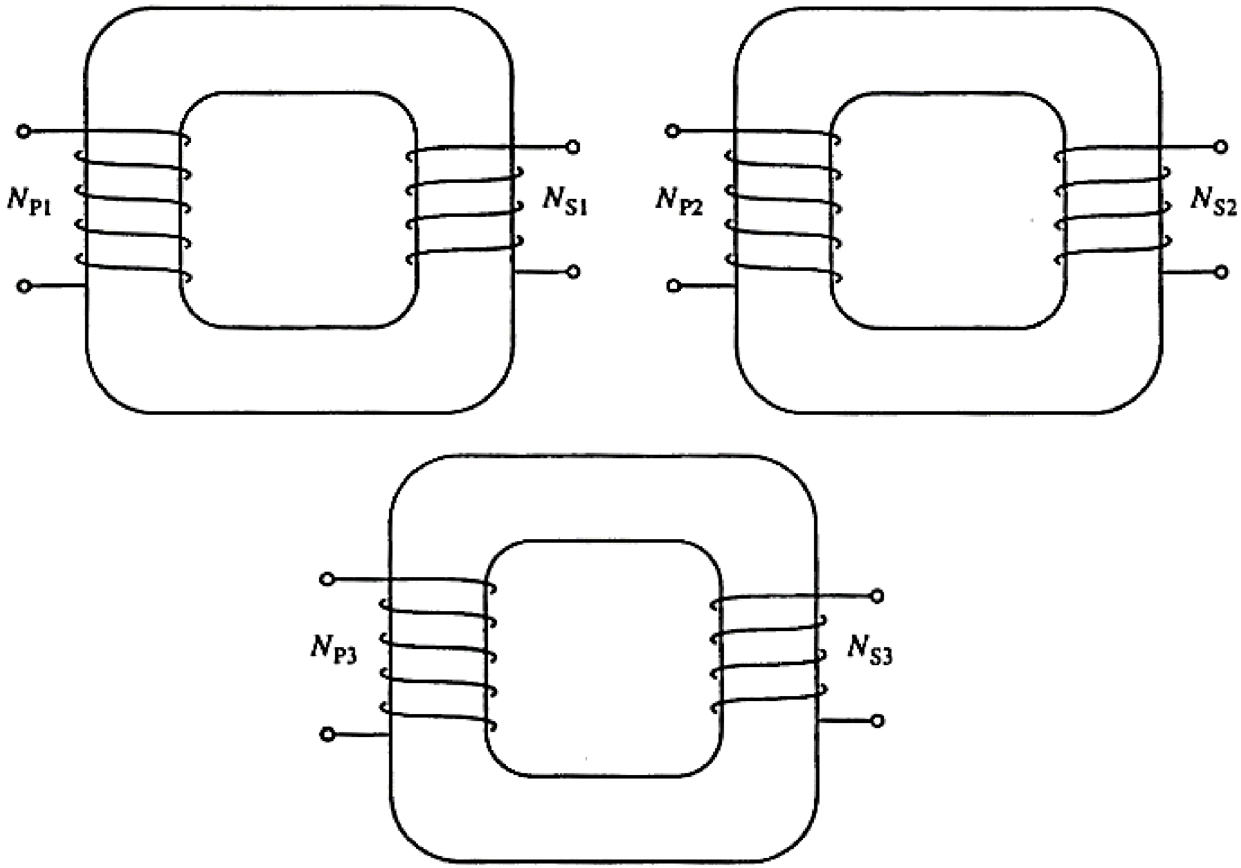
(b) The transformer's impedance in a per-unit system when connected in the conventional manner is

$$Z_{\mathrm{eq}} = 0.01 + j0.08\mathrm{pu} \quad \text{separate windings}$$

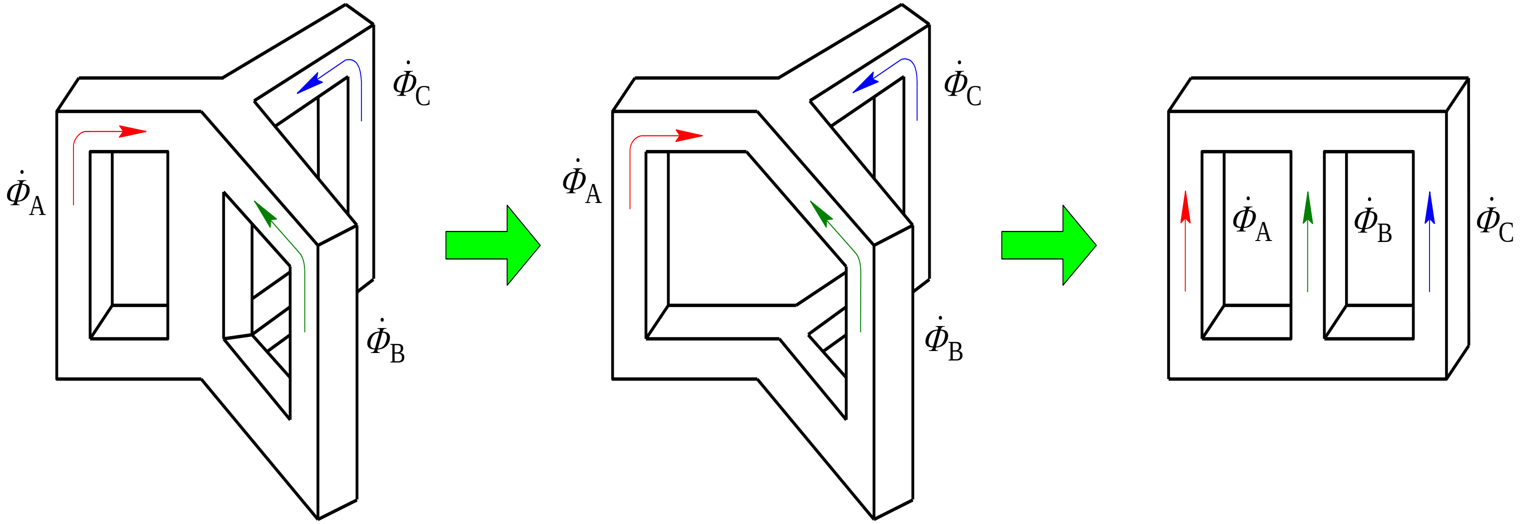
The apparent power advantage of this autotransformer is 11, so the per-unit impedance of the autotransformer connected as described is

$$
\begin{aligned}
Z_{\mathrm{eq}} &= \frac{0.01 + j0.08}{11} \\
&= 0.00091 + j0.00727\mathrm{pu} \quad \text{autotransformer}
\end{aligned}
$$

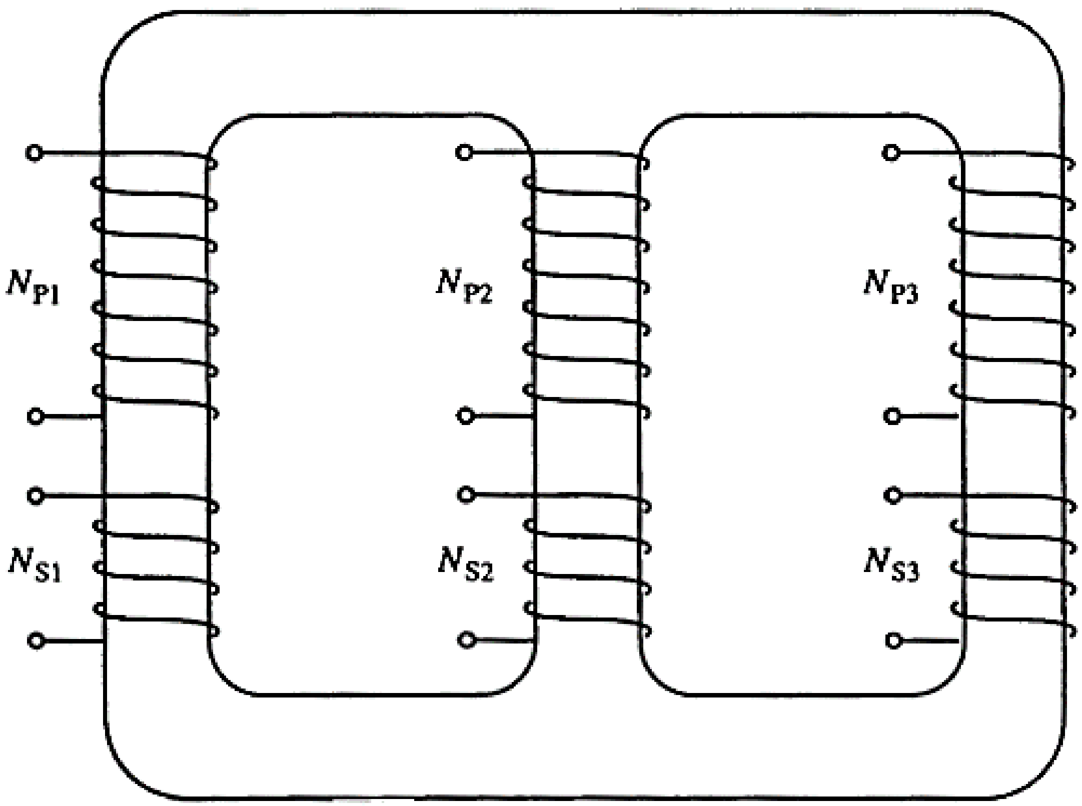
Three-phase transformer – three single-phase transformer banks



Three-phase transformer



Three-phase transformer – one three-lags transformer banks

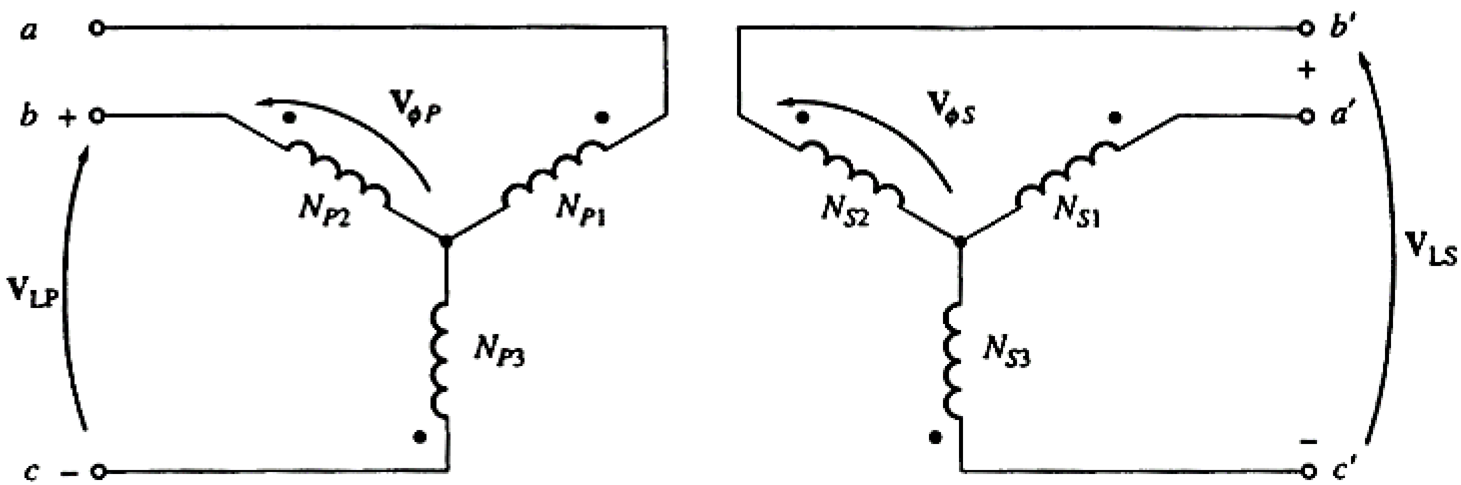


Three types of terminal connection

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any three-phase transformer can be independently connected in either a wye (Y) or a delta (Δ). This gives a total of four possible connections for a three-phase transformer bank:

1. Wye-wye(Y-Y)
2. Wye-delta(Y-Δ)
3. Delta-wye(Δ -Y)
4. Delta-delta (Δ - Δ)

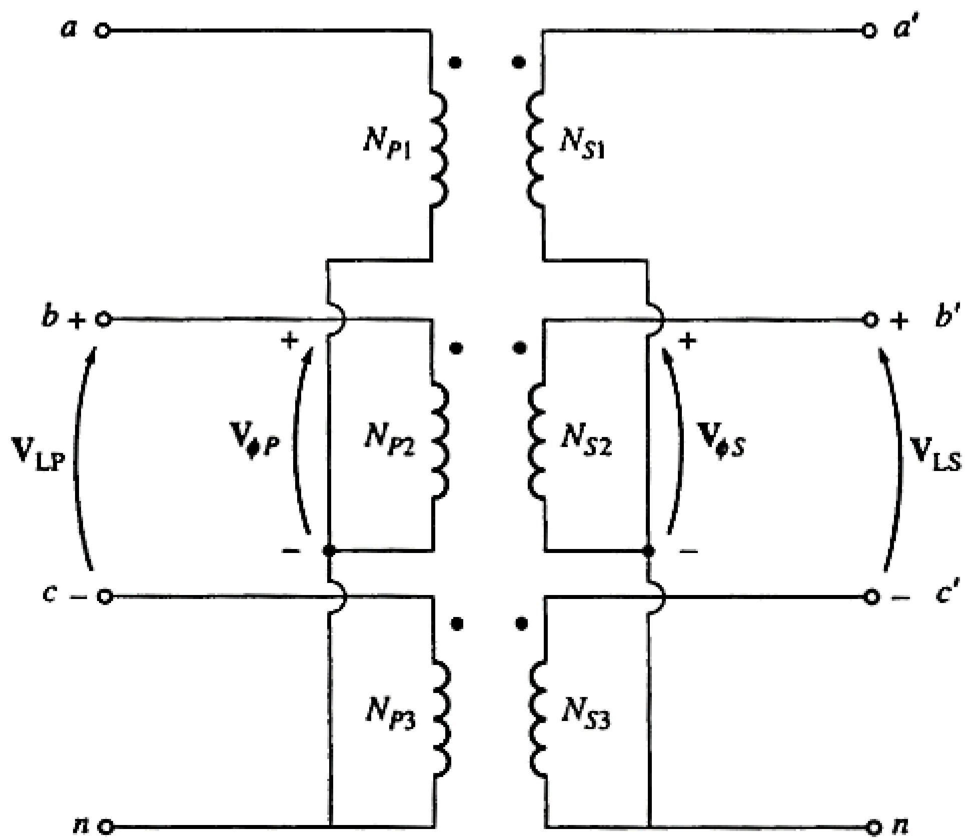
Wye-wye connection



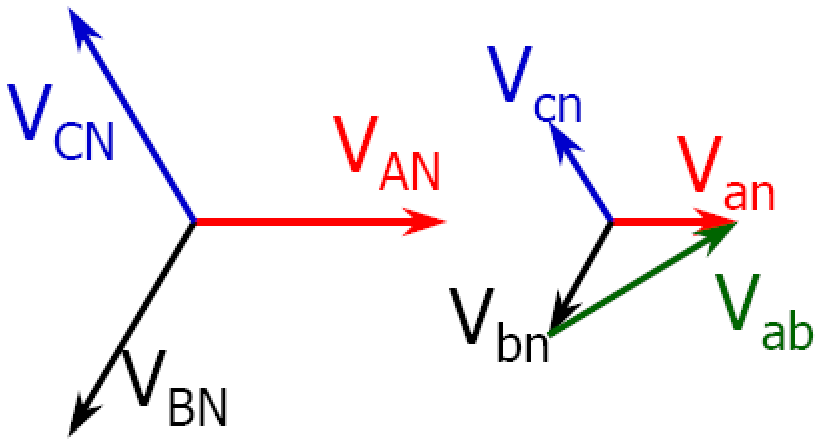
$$\frac{V_{\mathrm{LP}}}{V_{\mathrm{LS}}}=\frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}}=a \quad \mathrm{Y}-\mathrm{Y}$$

1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.

Wye-wye connection



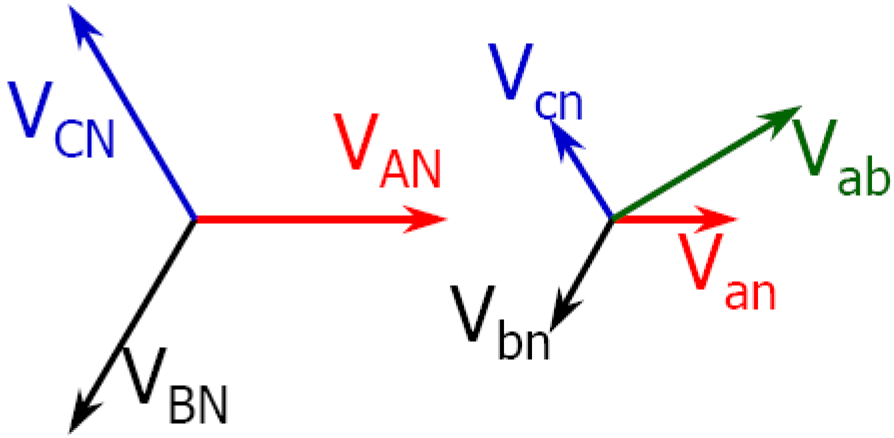
Wye-wye connection



$$V_{ab} = V_{an} - V_{bn}$$
$$V_{bc} = V_{bn} - V_{cn}$$
$$V_{ca} = V_{cn} - V_{an}$$

$$V_{AN} = aV_{an}$$
$$V_{AB} = aV_{ab}$$

Wye-wye connection



$$V_{ab} = V_{an} - V_{bn}$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$V_{ca} = V_{cn} - V_{an}$$

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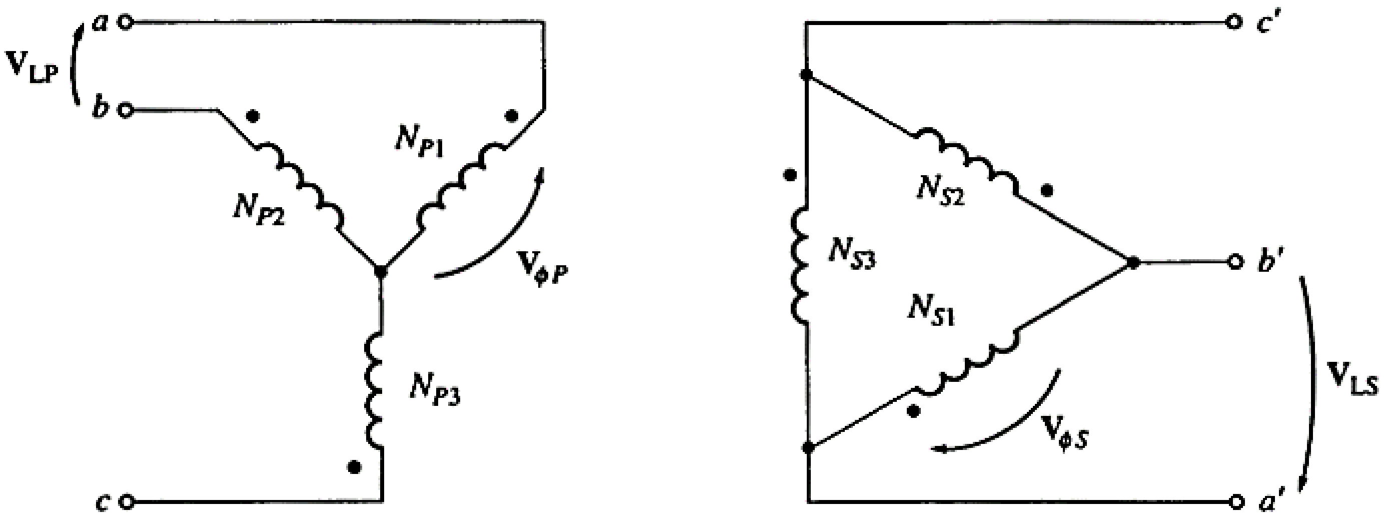
Solving the unbalance and 3rd harmonic problems in wye-wye connection

1. ***Solid ground the neutral of transformers***: solve the unbalance problem and support a return path to the 3rd harmonic component.
2. ***Add a Δ-connected third winding***: Since 3rd harmonic components are in-phase in each branch of Δ-connection, 3rd harmonic components will be limited in Δ-connection as the circulating current.

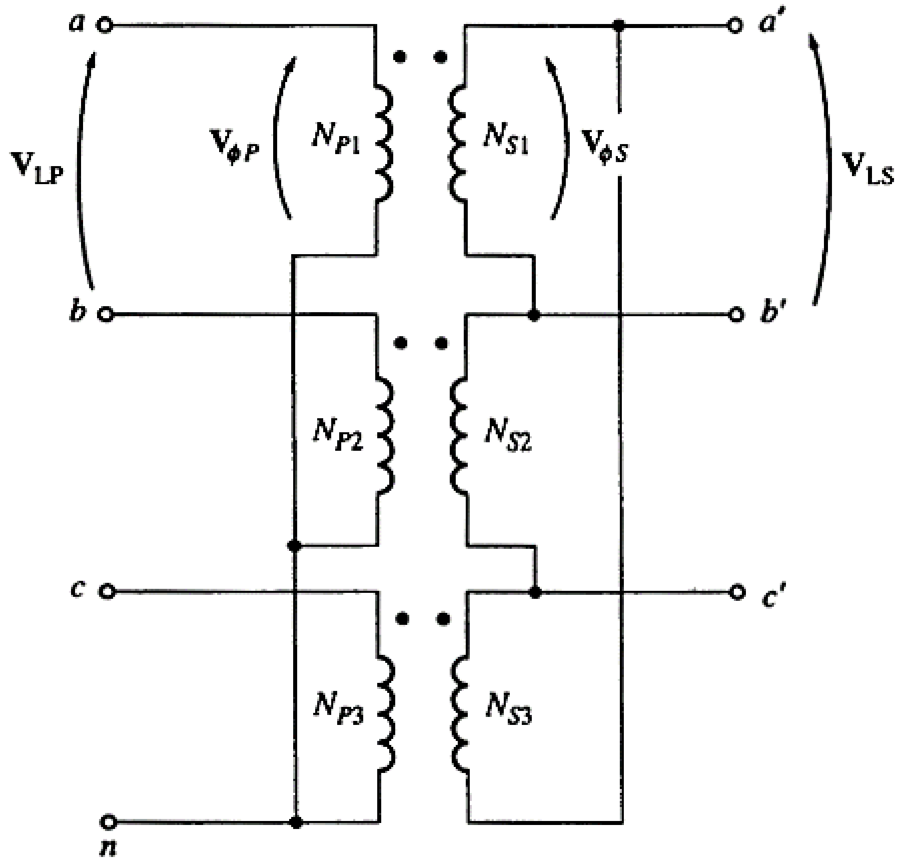
Wye-delta connection



$$\frac{V_{\phi P}}{V_{\phi S}} = a \qquad \frac{V_{\mathrm{LP}}}{V_{\mathrm{LS}}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}}$$

$$\frac{V_{\mathrm{LP}}}{V_{\mathrm{LS}}} = \sqrt{3}a \qquad \mathrm{Y}-\Delta$$

Wye-delta connection



(b)

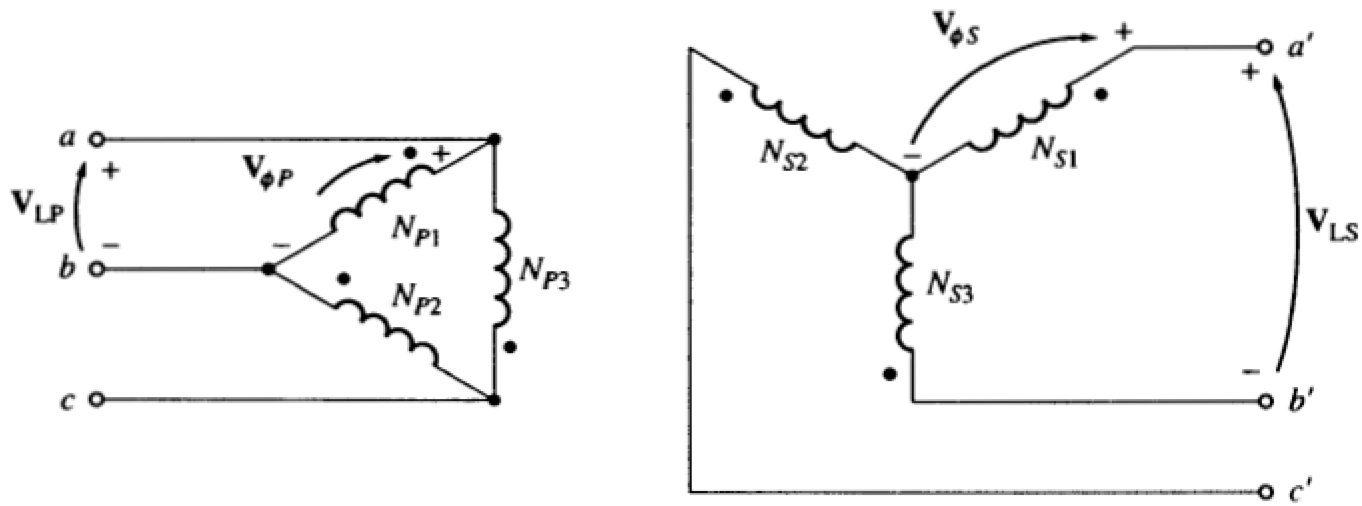
Wye-delta connection summarize

1. No 3rd harmonic component problem:

2. There are phase difference between each sides

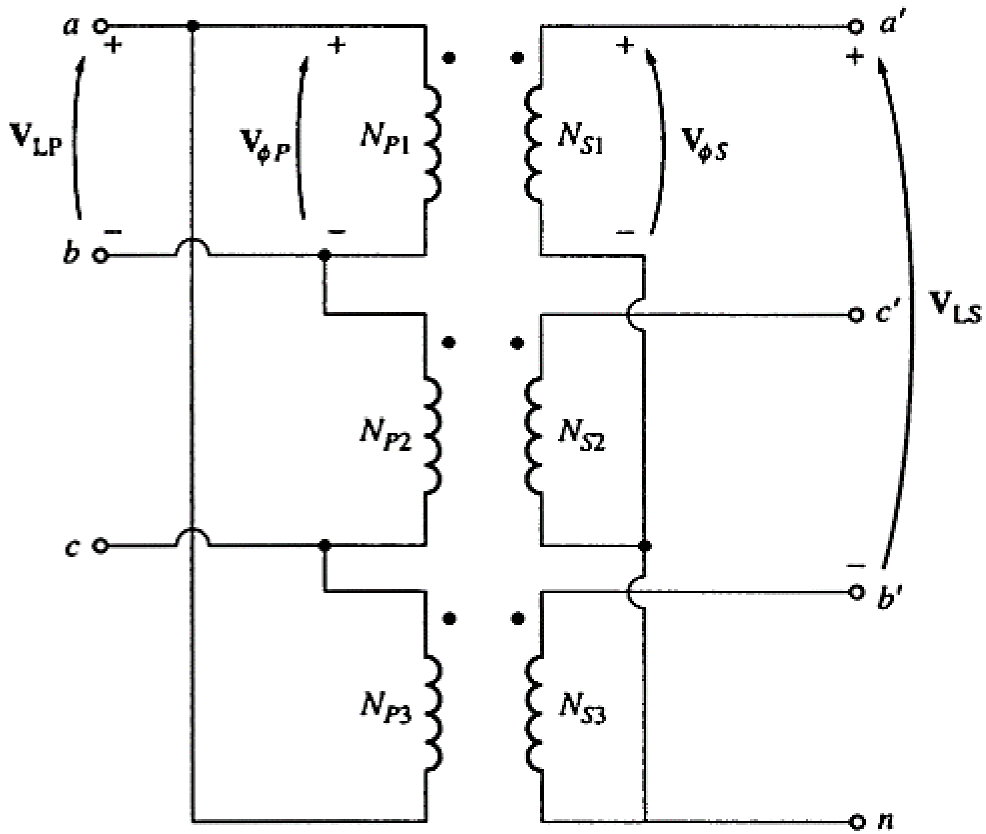
3. In United state, the secondary voltage will lag the primary voltage 30 degrees

Delta-wye connection



$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{V_{\varphi P}}{\sqrt{3}V_{\varphi S}} = \frac{a}{\sqrt{3}}}$$

Delta-wye connection



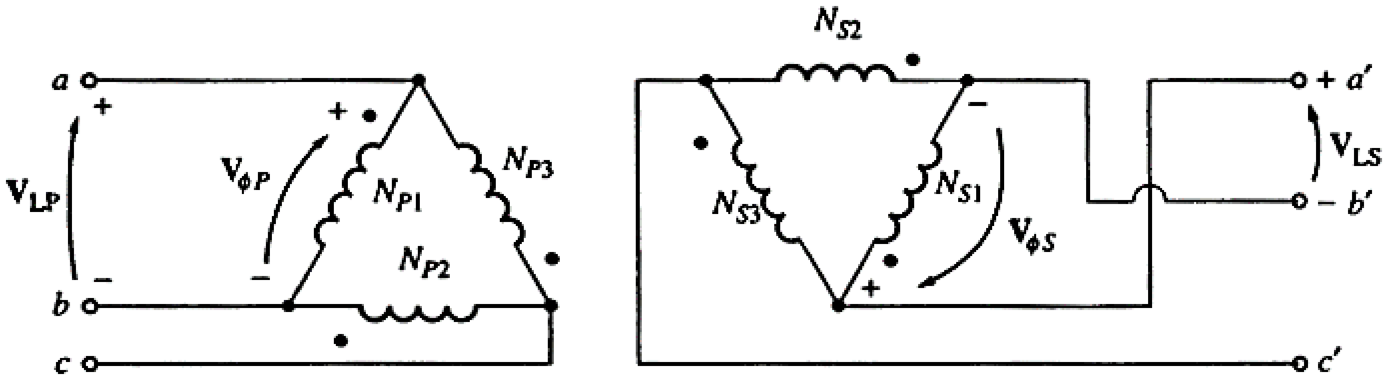
Delta-wye connection

- Common connection:
  - Used on three-wire (delta) to four wire (wye)
  - Used to isolate ground on wye side from source ground on delta side

Delta-wye or wye-delta

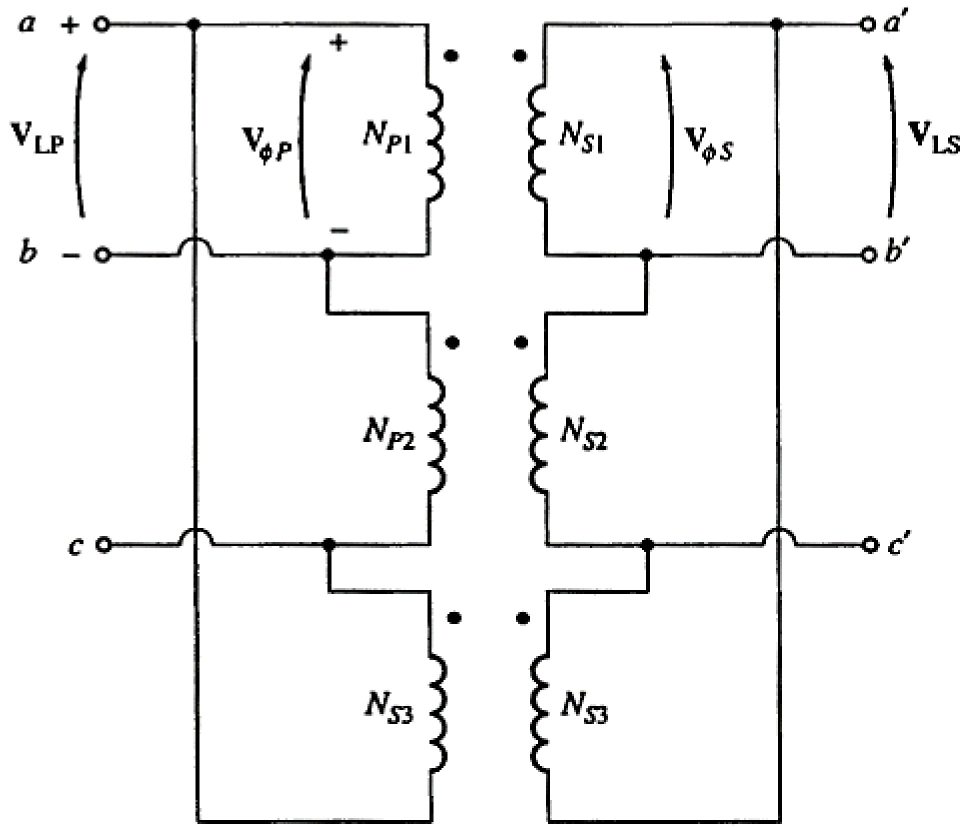
- Common for wye-delta step-up transformer banks in generating plants
- Common for delta-wye step-down banks in substation

Delta-delta connection



$$\frac{V_{\mathrm{LP}}}{V_{\mathrm{LS}}} = \frac{V_{\phi P}}{V_{\phi S}} = a \quad \Delta - \Delta$$

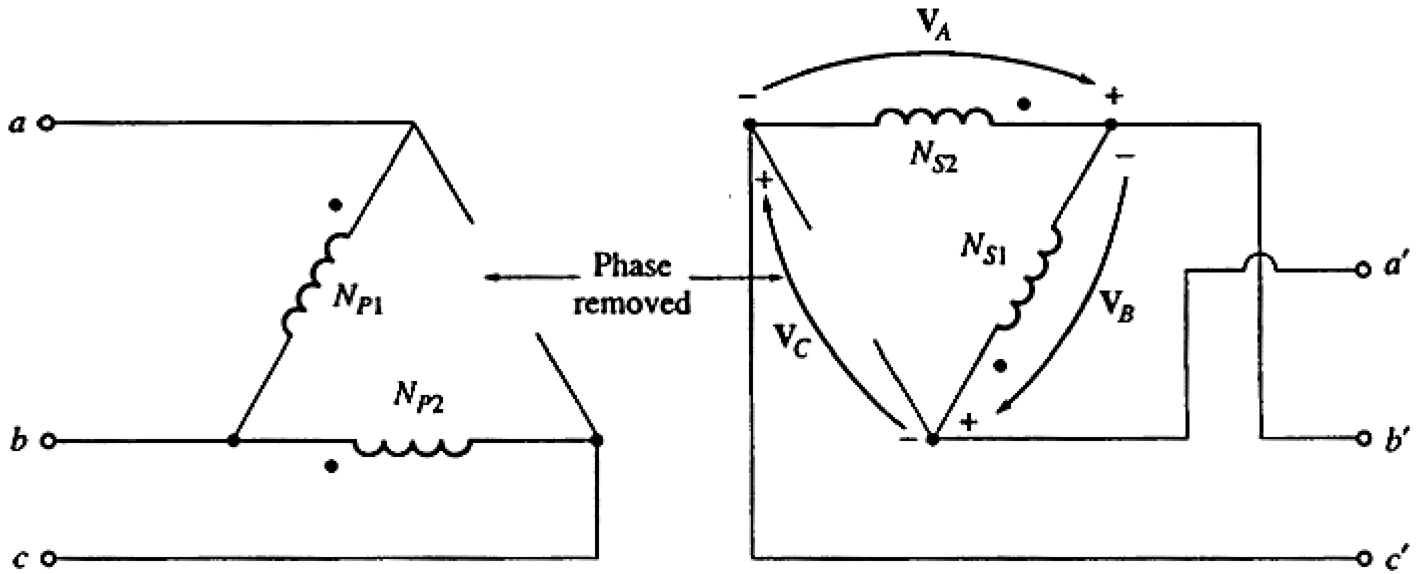
Delta-delta connection



Three-phase transformation using two transformers

1. The open-Δ (or V-V) connection
2. The open-Y-open- Δ connection
3. The Scott-T connection
4. The three-phase T connection

V-V connection

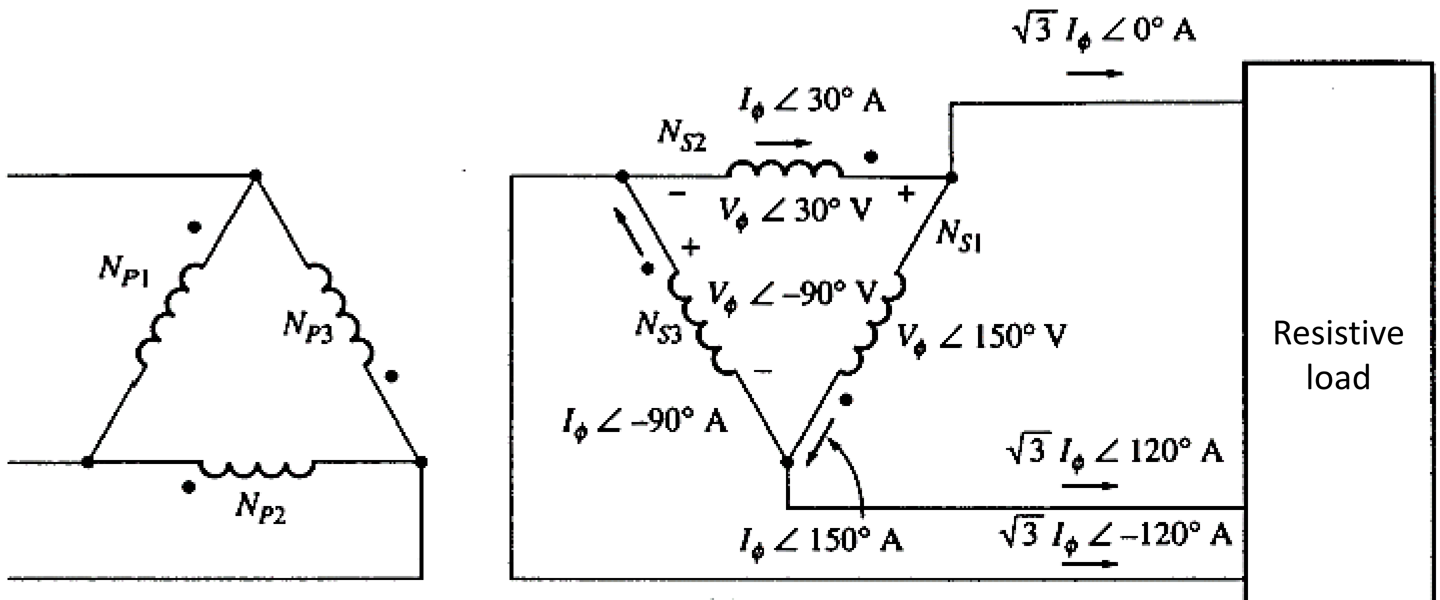


$$\mathbf{V}_A = V\angle 0^\circ \text{ V}$$

$$\mathbf{V}_B = V\angle 120^\circ \text{ V}$$

$$\begin{aligned}
\mathbf{V}_C &= -\mathbf{V}_A - \mathbf{V}_B \\
&= -V\angle 0^\circ - V\angle -120^\circ \\
&= -V - (-0.5V - j0.866V) \\
&= -0.5V + j0.866V \\
&= V\angle 120^\circ \quad \text{V}
\end{aligned}$$

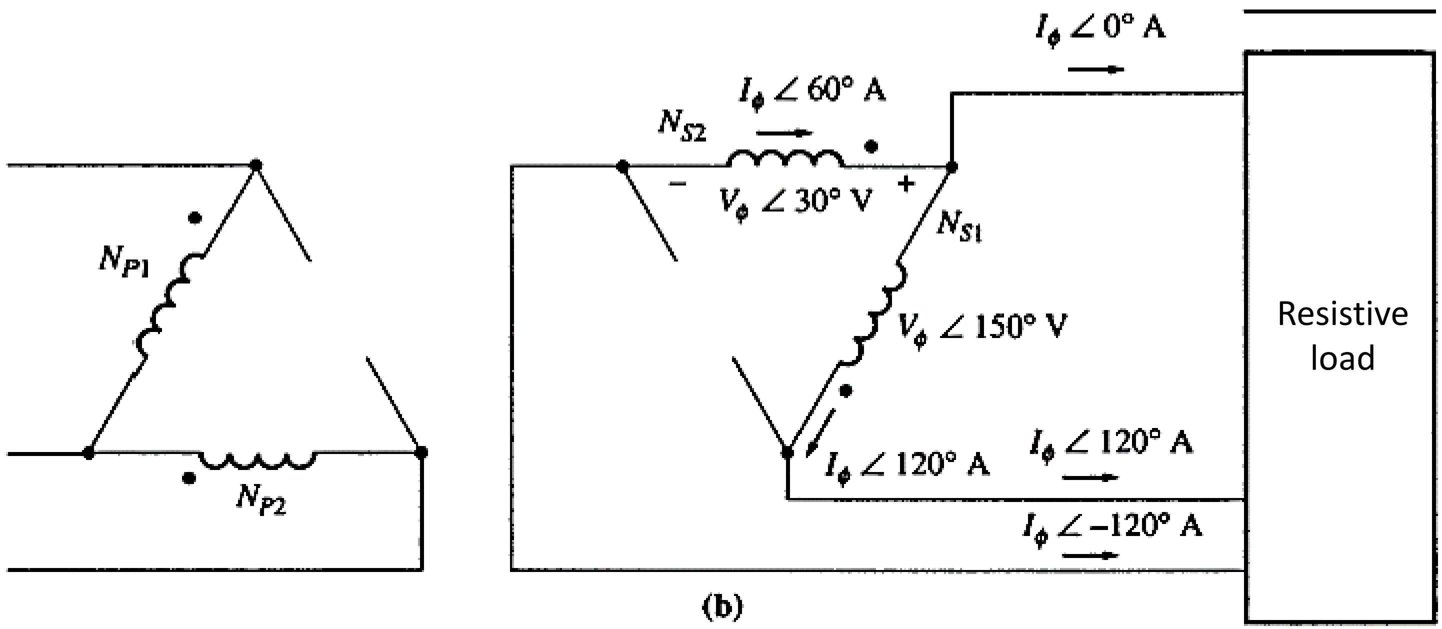
Power rating of Δ-Δ connection



(a)

$$
\begin{aligned}
P &= 3V_\phi I_\phi \cos\theta \\
&= 3V_\phi I_\phi
\end{aligned}
$$

Power rating of V-V connection



(b)

$$
\begin{aligned}
P_1 &= 3V_\phi I_\phi \cos\left(150^\circ - 120^\circ\right) \\
&= 3V_\phi I_\phi \cos 30^\circ \\
&= \frac{\sqrt{3}}{2} V_\phi I_\phi
\end{aligned}
$$

$$
\begin{aligned}
P_2 &= 3V_\phi I_\phi \cos\left(30^\circ - 60^\circ\right) \\
&= 3V_\phi I_\phi \cos\left(-30^\circ\right) \\
&= \frac{\sqrt{3}}{2} V_\phi I_\phi
\end{aligned}
$$

$$P = \sqrt{3} V_\phi I_\phi$$

Comparison of power rating

1. The power rating comparison

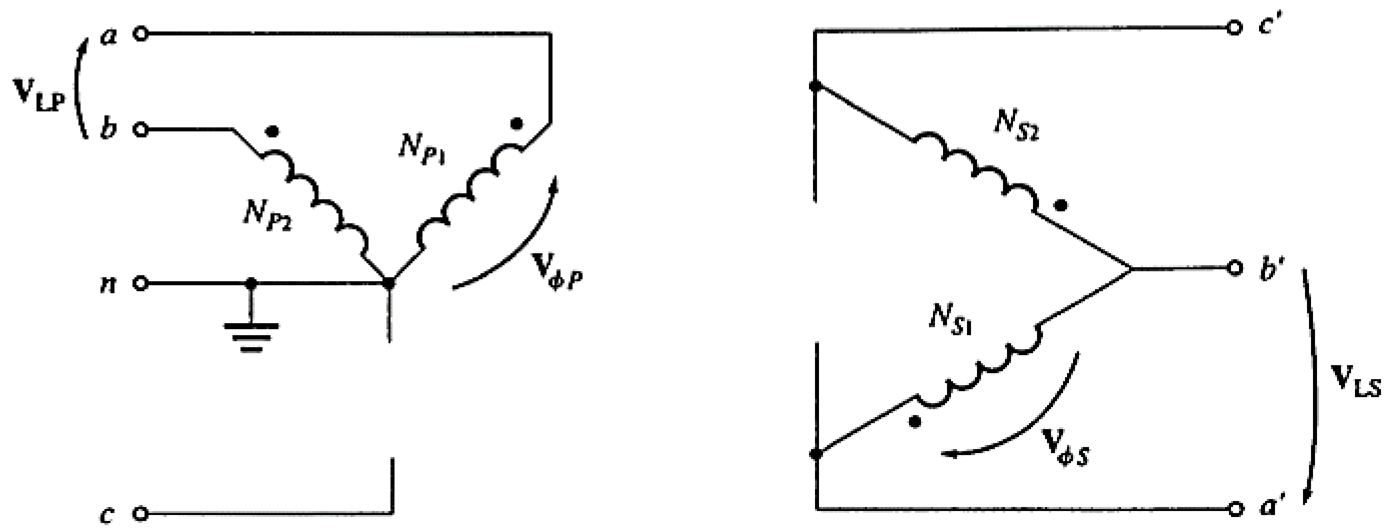
$$\frac{P_{\text{open }\Delta}}{P_{\text{3 phase}}} = \frac{\sqrt{3}V_\phi I_\phi}{3V_\phi I_\phi} = \frac{1}{\sqrt{3}} = 0.577$$

2. Where is the power rating ?
   - The existing reactive power will consume the power rating

$$\begin{aligned} Q_1 &= 3V_\phi I_\phi \sin\left(150^\circ - 120^\circ\right) \\ &= 3V_\phi I_\phi \sin 30^\circ \\ &= 1/2V_\phi I_\phi \end{aligned}$$

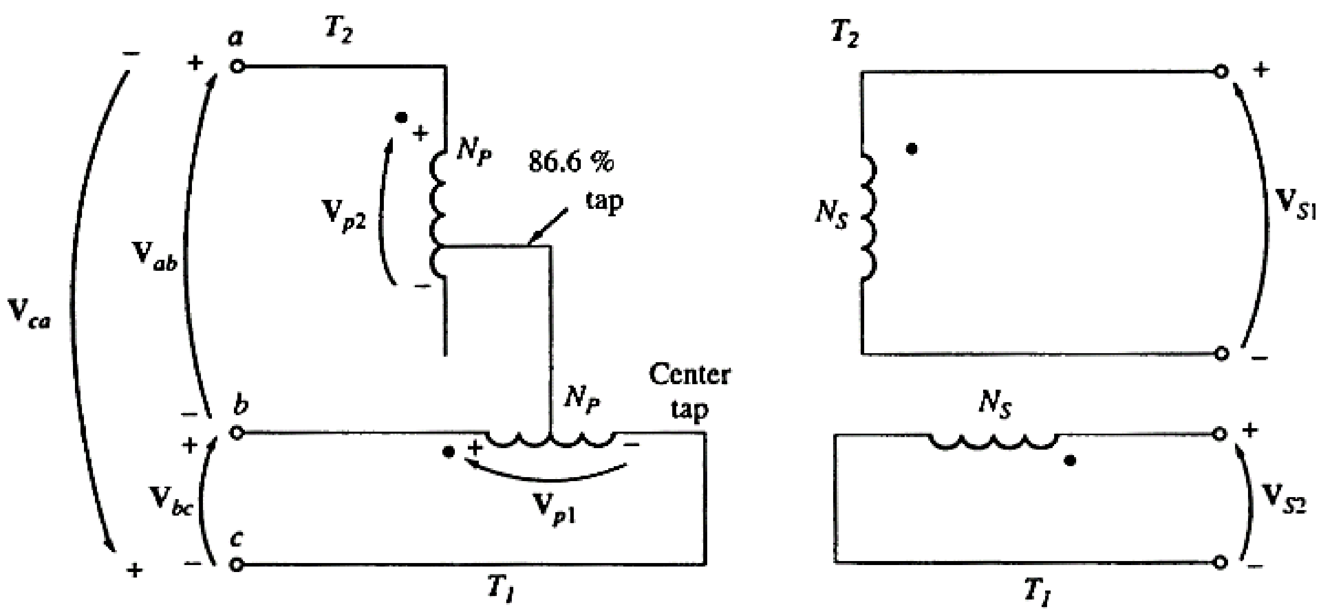
$$\begin{aligned} Q_2 &= 3V_\phi I_\phi \sin\left(30^\circ - 60^\circ\right) \\ &= 3V_\phi I_\phi \sin\left(-30^\circ\right) \\ &= -1/2V_\phi I_\phi \end{aligned}$$

Open Y-open Δ connection



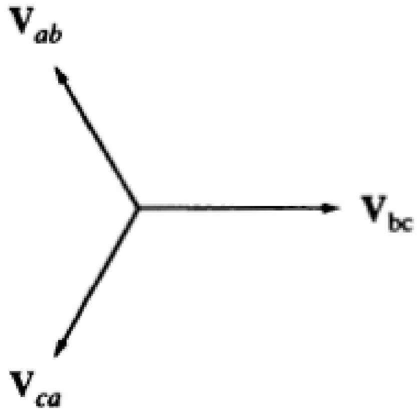
Scott-T connection – railroad applications

1. While in railroad applications, there always need two-phase power system for supporting northbound and southbound rails respectively.

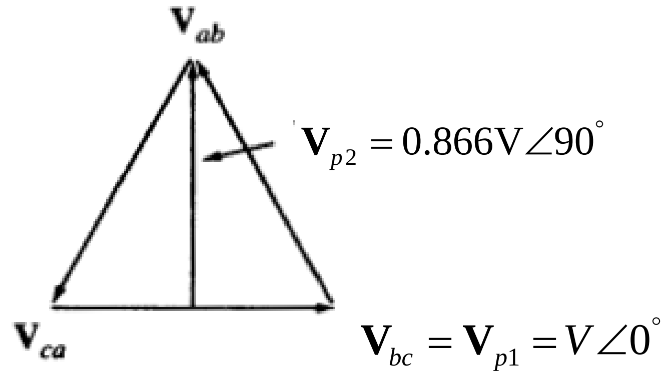


(a)

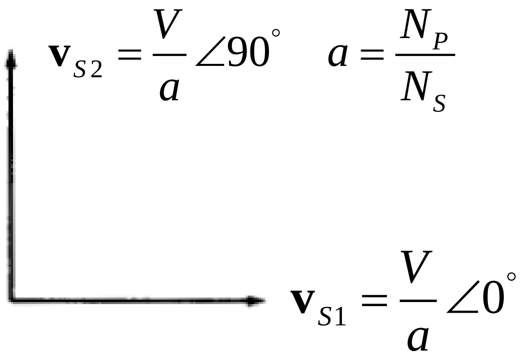
$$\mathbf{V}_{ab} = V\angle 120^\circ$$
$$\mathbf{V}_{bc} = V\angle 0^\circ$$
$$\mathbf{V}_{ca} = V\angle -120^\circ$$



(b)

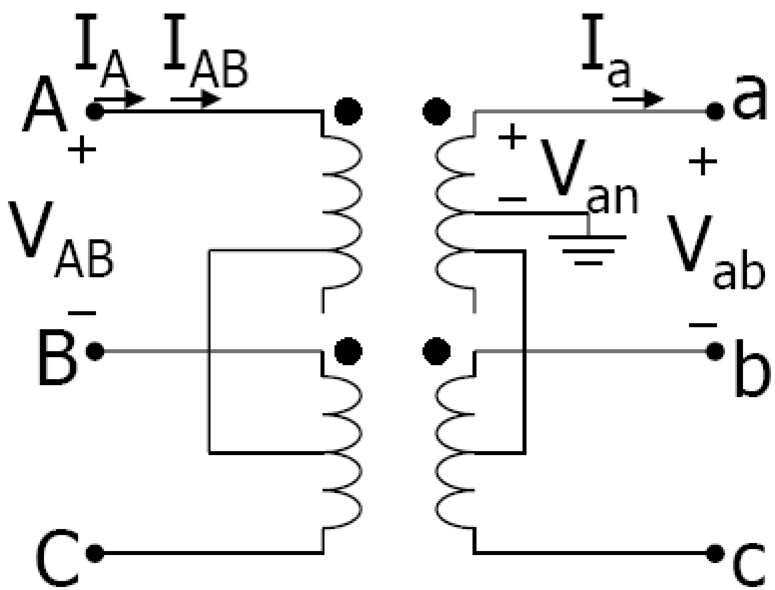


(c)



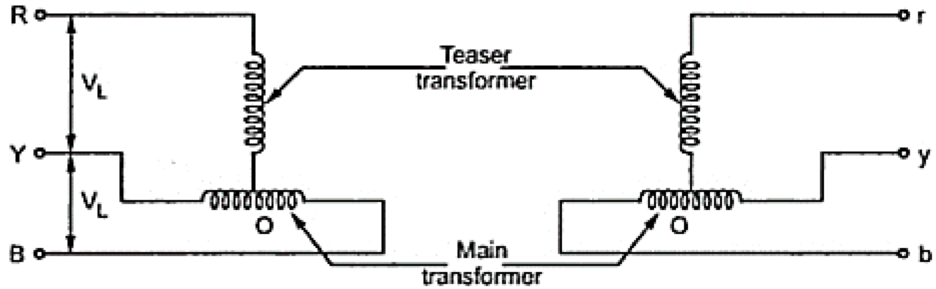
(d)

T-T three phase connection

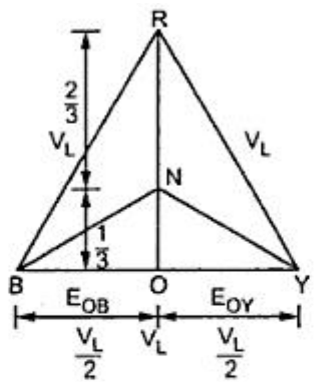


T-T three-phase connection

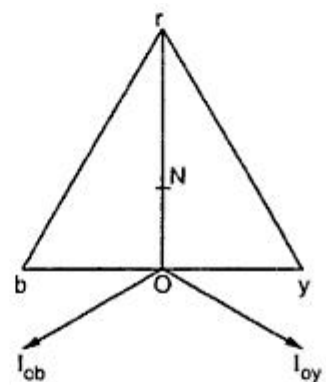
T-T three phase connection



Primary side　　　　Secondary side



Primary side



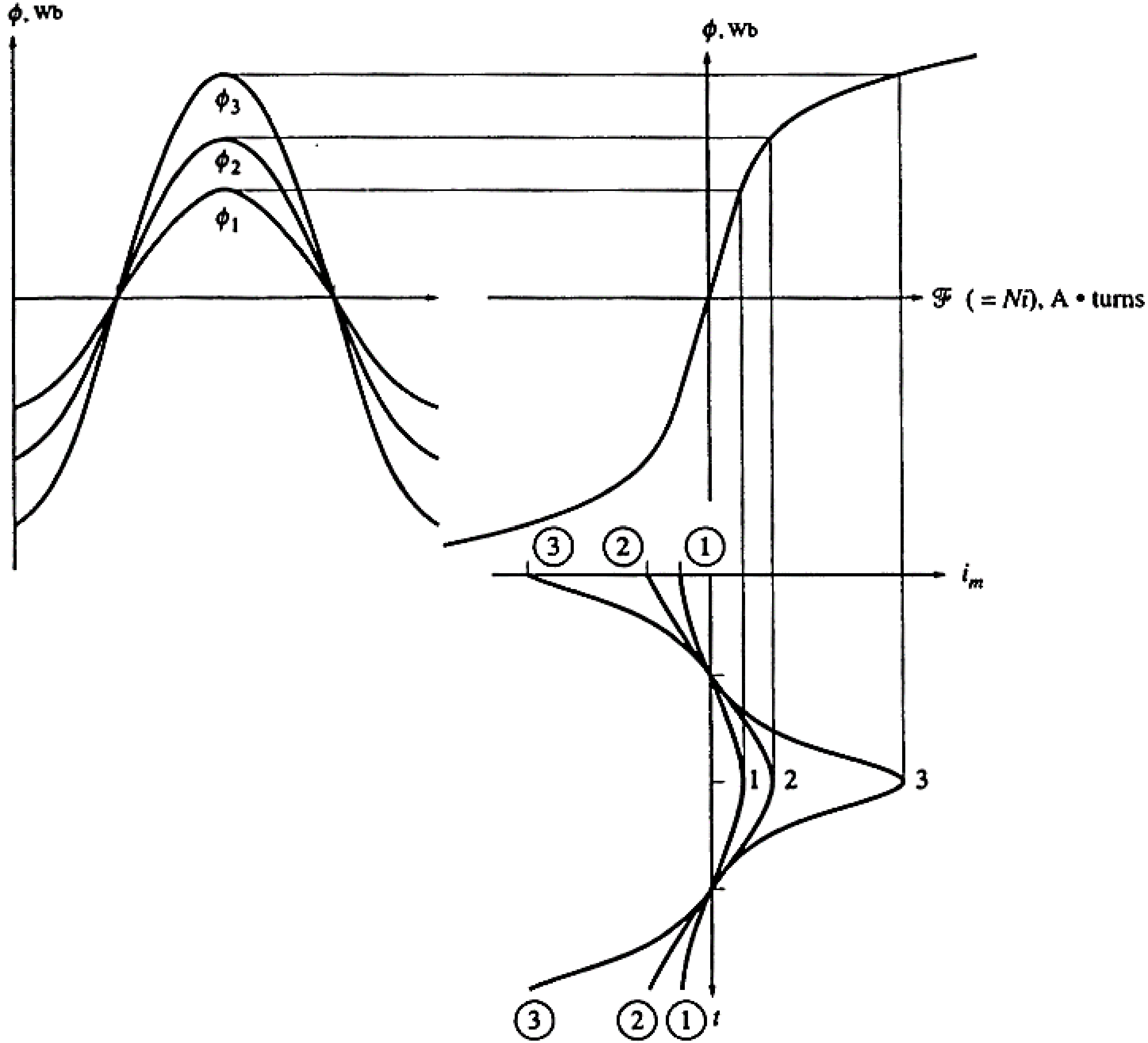
Secondary side

Transformer rating and relative problems

1. Voltage rating
   - Prevent the over-voltage insulation problem
   - Prevent the saturation of magnetization curve

$$
\begin{aligned}
v(t) &= V_M \sin \omega t \quad \mathrm{V} \\
\phi(t) &= \frac{1}{N_P}\int v(t)dt \\
&= \frac{1}{N_P}\int V_M \sin \omega t dt \\
\phi(t) &= -\frac{V_M}{\omega N_P}\cos \omega t
\end{aligned}
$$

$$\phi_{\max} = \frac{V_{\max}}{\omega N_P}$$



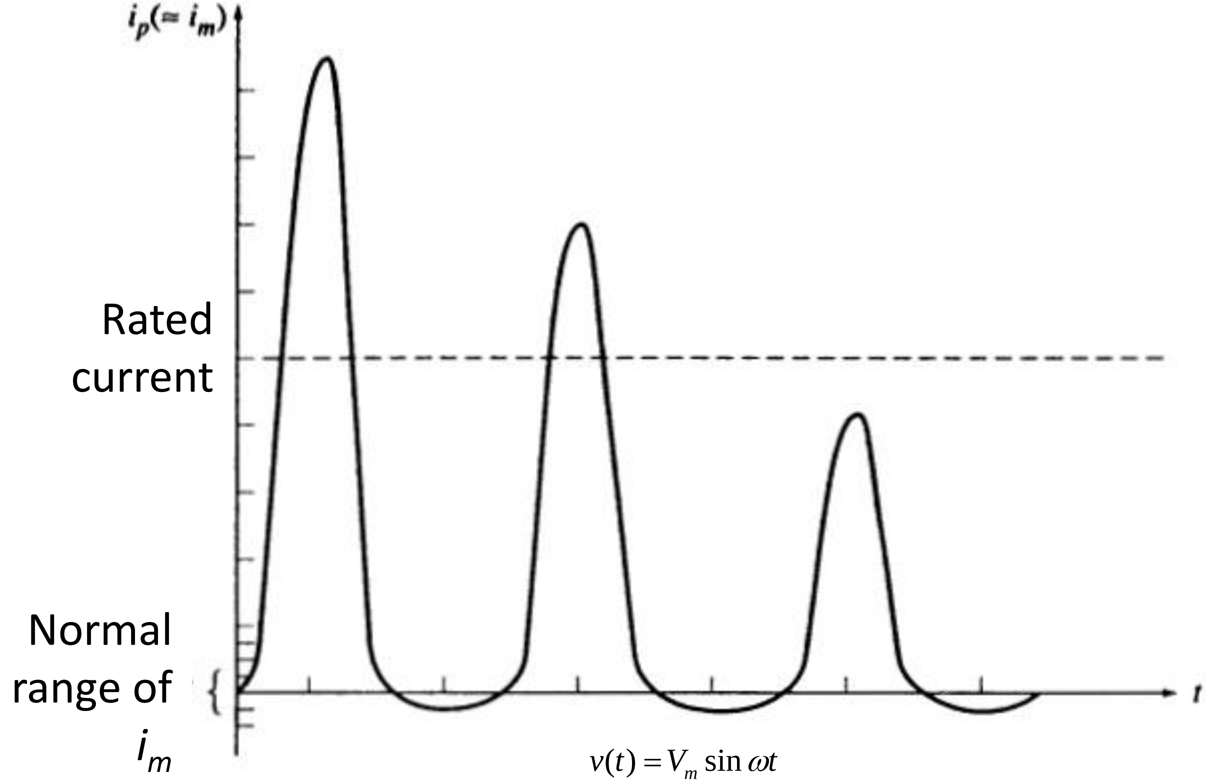
φ, Wb
φ3
φ2
φ1
φ, Wb
ℱ (= Ni), A • turns
i_m
3
2
1
1
2
3
3
2
1
t

Frequency limitation

- If a 60Hz transformer operates on 50Hz, its applied voltage must be reduced by one-sixth
- If a 50Hz transformer operates on 60Hz, its applied voltage may rise 20 percents.

$$\phi_{\max} = \frac{V_{\max}}{\omega N_P}$$

Inrush current problem



$$v(t)=V_M \sin\omega t \quad \mathrm{V}$$

$$\phi(t)=\frac{1}{N_P}\int_0^{\pi/\omega} V_M \sin\omega t dt$$

$$=-\frac{V_M}{\omega N_P}\cos\omega t\Big|_0^{\pi/\omega}$$

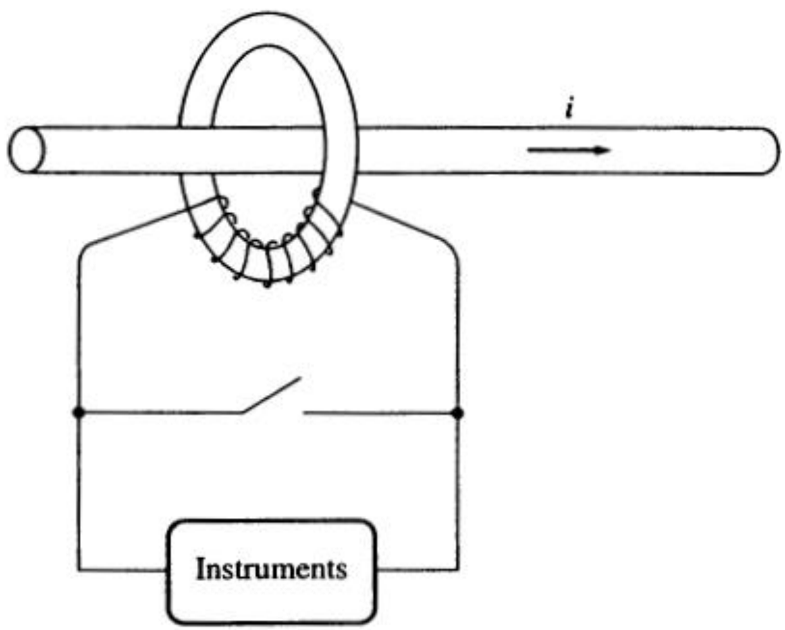
$$=-\frac{V_M}{\omega N_P}[(-1)-(1)]$$

$$\phi_{\max}=\frac{2V_{\max}}{\omega N_P}$$

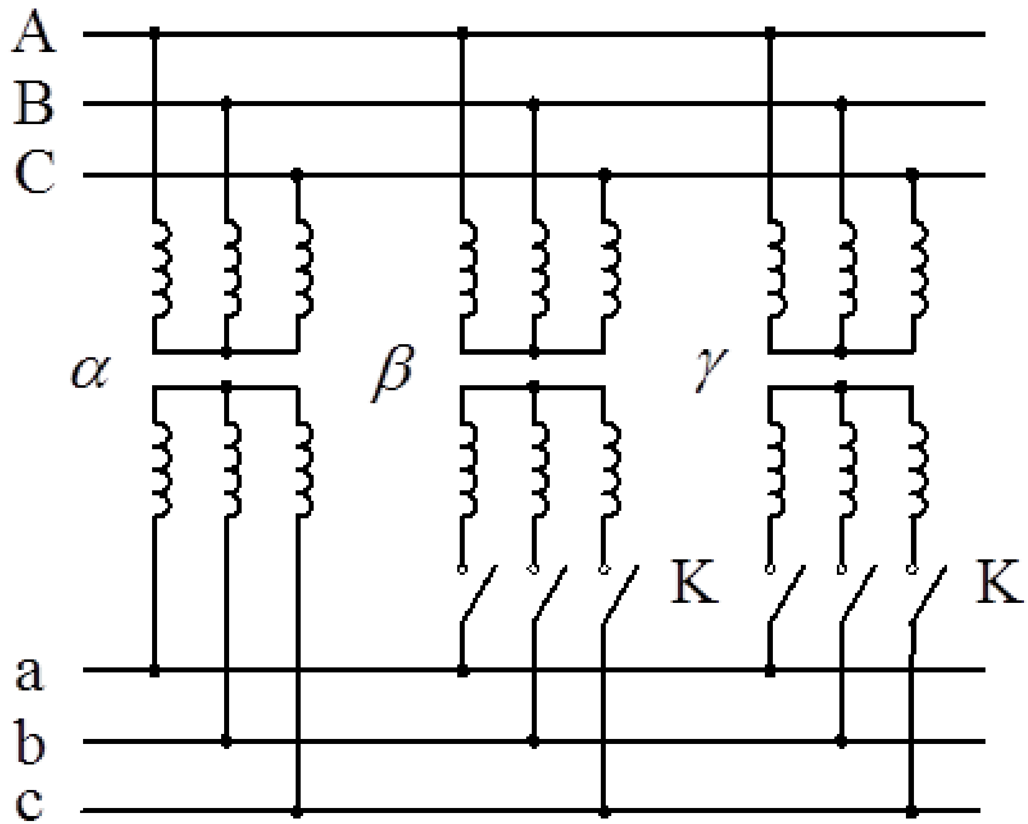
Instrument transformer

- Potential transformer (PT)

- Current transformer (CT)



Parallel operation of transformers



Parallel operation of transformers

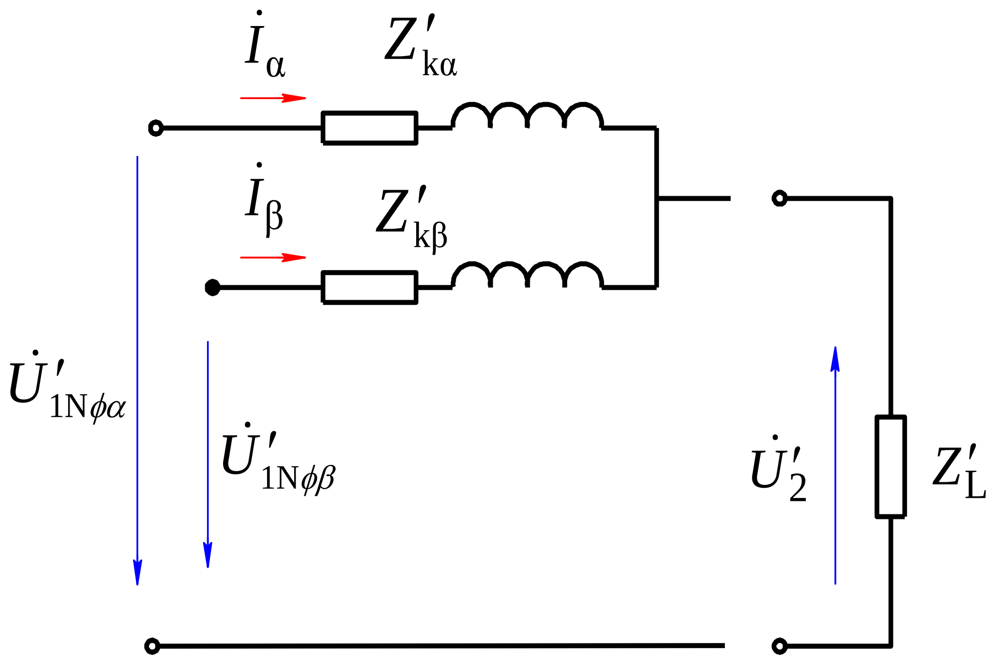
- Advantage
  - Flexible facing load change. Improve overall efficiency
  - Cost effective. Reduce size of backup transformer
  - Easy for maintenance
  - Expandable for planning.

Parallel operation of transformers

- Ideal operation:
  - No circulating current under no load condition
  - Equal loading under loaded condition

- How to achieve that:
  - Exactly the same voltage rating
  - Exactly the same phase angle between primary and secondary windings

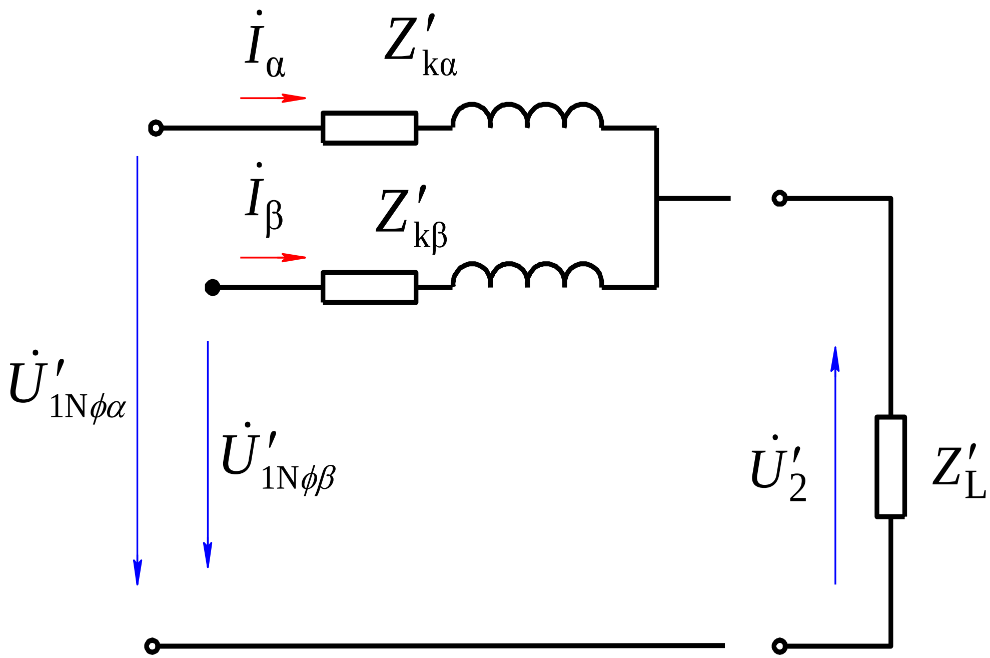
Parallel operation of transformers



$$\dot{I}_{\alpha} = -\dot{I}_{\beta} = \frac{\dot{U}'_{1N\phi\alpha} - \dot{U}'_{1N\phi\beta}}{Z'_{k\alpha} + Z'_{k\beta}}$$

Difference in rated voltage must be smaller than 0.5%

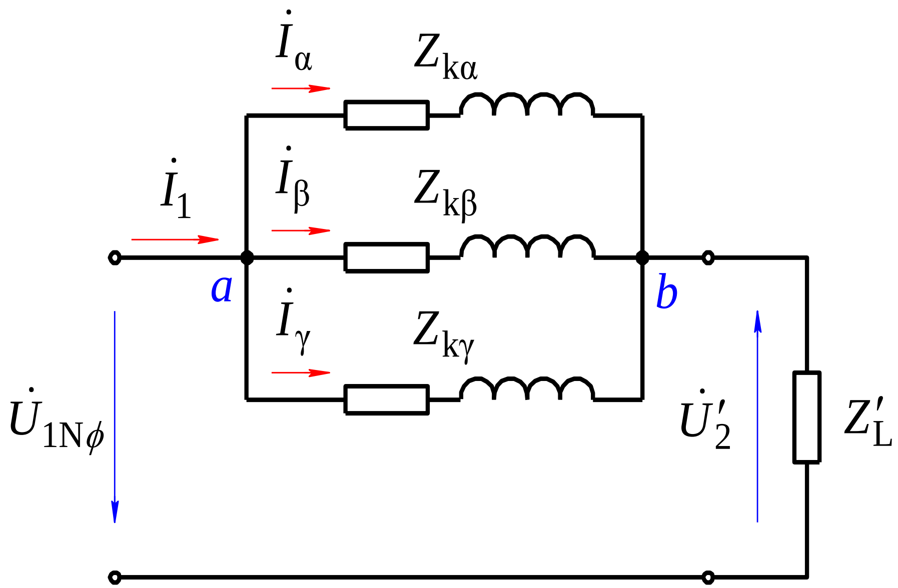
Parallel operation of transformers



$$\dot{I}_{\alpha} = -\dot{I}_{\beta} = \frac{\dot{U}'_{1N\phi\alpha} - \dot{U}'_{1N\phi\beta}}{Z'_{k\alpha} + Z'_{k\beta}}$$

The same type of connection must be used among transformers

Parallel operation of transformers



How are loading current shared?

Parallel operation of transformers

$$\dot{U}_{\mathrm{ab}} = \dot{I}_{\alpha} Z_{\mathrm{k}\alpha} = \dot{I}_{\beta} Z_{\mathrm{k}\beta} = \dot{I}_{\gamma} Z_{\mathrm{k}\gamma}$$

$$\dot{I}_{\alpha} : \dot{I}_{\beta} : \dot{I}_{\gamma} = \frac{1}{Z_{\mathrm{k}\alpha}} : \frac{1}{Z_{\mathrm{k}\beta}} : \frac{1}{Z_{\mathrm{k}\gamma}}$$

$$I_{\alpha} : I_{\beta} : I_{\gamma} = \frac{1}{\left|Z_{\mathrm{k}\alpha}\right|} : \frac{1}{\left|Z_{\mathrm{k}\beta}\right|} : \frac{1}{\left|Z_{\mathrm{k}\gamma}\right|}$$

$$\beta_{\alpha} : \beta_{\beta} : \beta_{\gamma} = \underline{I}_{\alpha} : \underline{I}_{\beta} : \underline{I}_{\gamma} = \frac{1}{\left|\underline{Z}_{\mathrm{k}\alpha}\right|} : \frac{1}{\left|\underline{Z}_{\mathrm{k}\beta}\right|} : \frac{1}{\left|\underline{Z}_{\mathrm{k}\gamma}\right|}$$

All depending on the short circuit impedance!

Parallel operation of transformers

- Conditions for parallel operation of transformers:
  - Difference in rated voltage must be smaller than 0.5%
  - Use the same type of connection
  - Use transformers with similar short circuit impedance for equal load sharing

Per Unit System

- Per unit system, a system of dimensionless parameters, is used for computational convenience and for readily comparing the performance of a set of transformers or a set of electrical machines.

$$\boldsymbol{PU\,Value = \frac{Actual\ Quantity}{Base\ Quantity}}$$

Where ‘actual quantity’ is a value in volts, amperes, ohms, etc.
$[VA]_{base}$ and $[V]_{base}$ are chosen first.

Typically, rated values are used as the base value.

Per Unit System

$$I_{base} = \frac{[VA]_{base}}{[V]_{base}}$$

$$P_{base} = Q_{base} = |S_{base}| = [VA]_{base} = [V]_{base}[I]_{base}$$

$$R_{base} = X_{base} = |Z_{base}| = \frac{[V]_{base}}{[I]_{base}} = \frac{[V]_{base}^2}{S_{base}} = \frac{[V]_{base}^2}{[VA]_{base}}$$

$$Y_{base} = \frac{[I]_{base}}{[V]_{base}}$$

$$|Z|_{PU} = \frac{|Z|_{ohm}}{|Z_{base}|}$$

$$\boldsymbol{\left[[VA]_{base}\right]_{pri} = \left[[VA]_{base}\right]_{sec}}$$

$$\boldsymbol{\frac{\left[[V]_{base}\right]_{pri}}{\left[[V]_{base}\right]_{sec}} = turns\ ratio}$$

Per Unit System

- For transformers
  - The rated VA is used as the base value
  - The rated voltages of the primary winding and the secondary winding are used as the base values, respectively.
  - The rated currents of the primary winding and the secondary winding are used as the base values, respectively.
  - The base impedance if the ratio of base voltage over base current

Per Unit System

$$\underline{R}'_2 = \frac{I_{1N\phi} R'_2}{U_{1N\phi}} = \frac{I_{1N\phi} k^2 R_2}{U_{1N\phi}} = \frac{k I_{1N\phi} R_2}{U_{1N\phi}/k} = \frac{I_{2N\phi} R_2}{U_{2N\phi}} = \underline{R}_2$$

$$\underline{R}_{\mathrm{k}} = \underline{R}_1 + \underline{R}_2$$

$$\underline{X}_{\mathrm{k}} = \underline{X}_{\sigma 1} + \underline{X}_{\sigma 2}$$

$$\underline{Z}_{\mathrm{k}} = \underline{Z}_1 + \underline{Z}_2$$

$$\underline{\dot{I}}_0 = \underline{\dot{I}}_1 + \underline{\dot{I}}_2$$

Example

A 20-kVA, 8000:480-V distribution transformer has the following resistances and reactances:

$R_P = 32$ ohm	$R_S = 0.05$ ohm
$X_P = 45$ ohm	$X_S = 0.06$ ohm
$R_C = 250{,}000$ ohm	$X_M = 30{,}000$ ohm

The excitation branch impedances are referred to the high-voltage side.

a) Find the equivalent circuit of the transformer referred to the high-voltage side.

b) Find the per unit equivalent circuit of this transformer.

c) Assume that the transformer is supplying rated load at 480 V and 0.8 power factor lagging. What is this transformer's input voltage? What is its voltage regulation?

d) What is this transformer's efficiency under the conditions of part (c)?

Recap

1. You must know the magnetization current, core loss current and excitation current
2. You must know the equivalent circuit of real transformer
3. Voltage regulation and efficiency of transformer
4. The differences between four types connection of three-phase transformer
5. V-V connection, Scott T connection