第9讲 非线性电阻电路的小信号分析方法

1 存在小扰动的非线性电阻电路分析

2 小信号法

3 电路元件的小信号模型

4 MOSFET小信号放大器电路分析

本讲练习题需要用到计算器
还要用点纸笔

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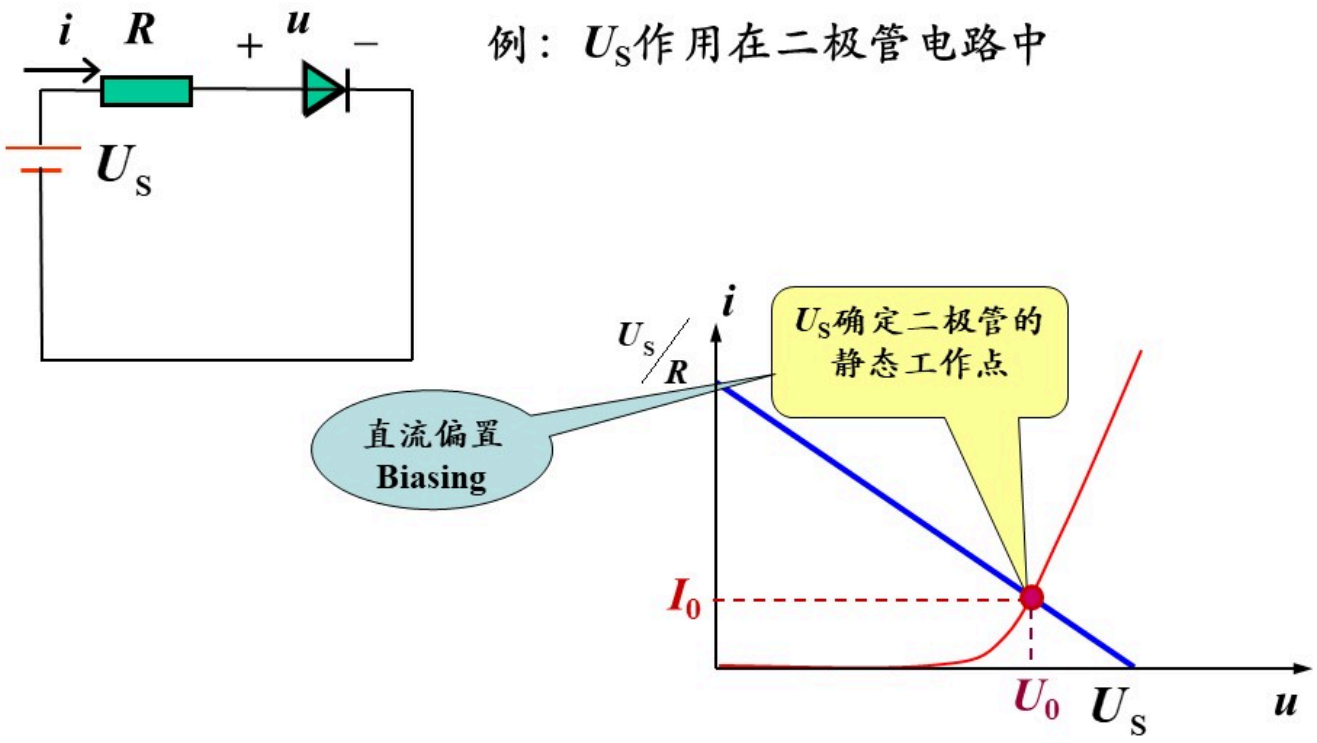
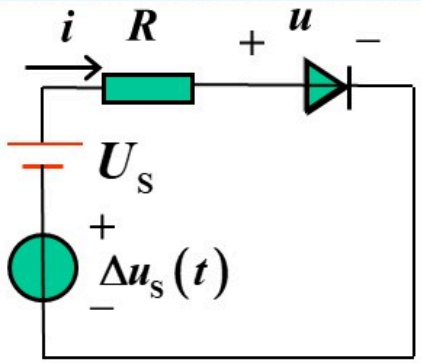
本讲（L9+A2）重难点

- 元件的小信号模型
- MOSFET小信号放大器电路分析

0 前情回顾


i
R
+
u
–
U_S
例：U_S作用在二极管电路中
U_S/R
i
U_S确定二极管的
静态工作点
直流偏置
Biasing
I_0
U_0
U_S
u

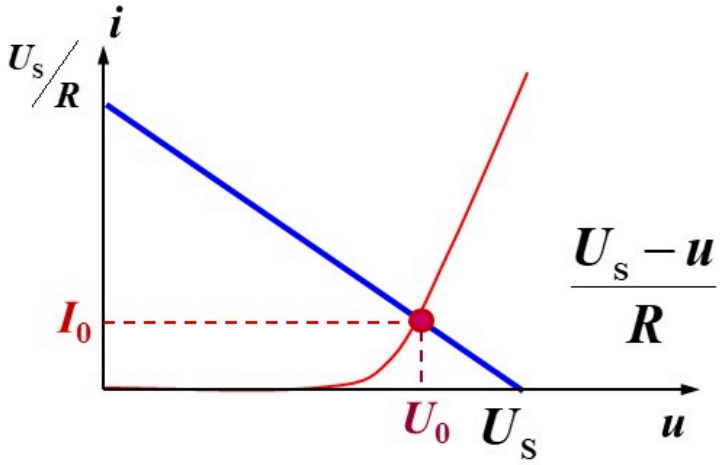
1 存在小扰动的非线性电阻电路分析



电路出现了小扰动电压源$\Delta u_S(t)$

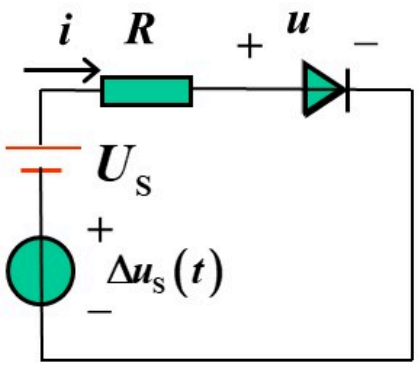
$$\frac{U_S+\Delta u_S(t)-u}{R}=I_S\left(e^{u/U_{TH}}-1\right)$$

如何简化分析?

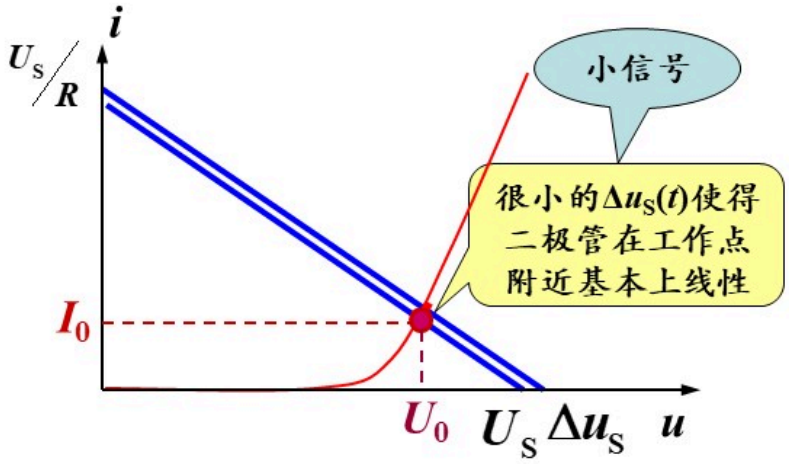


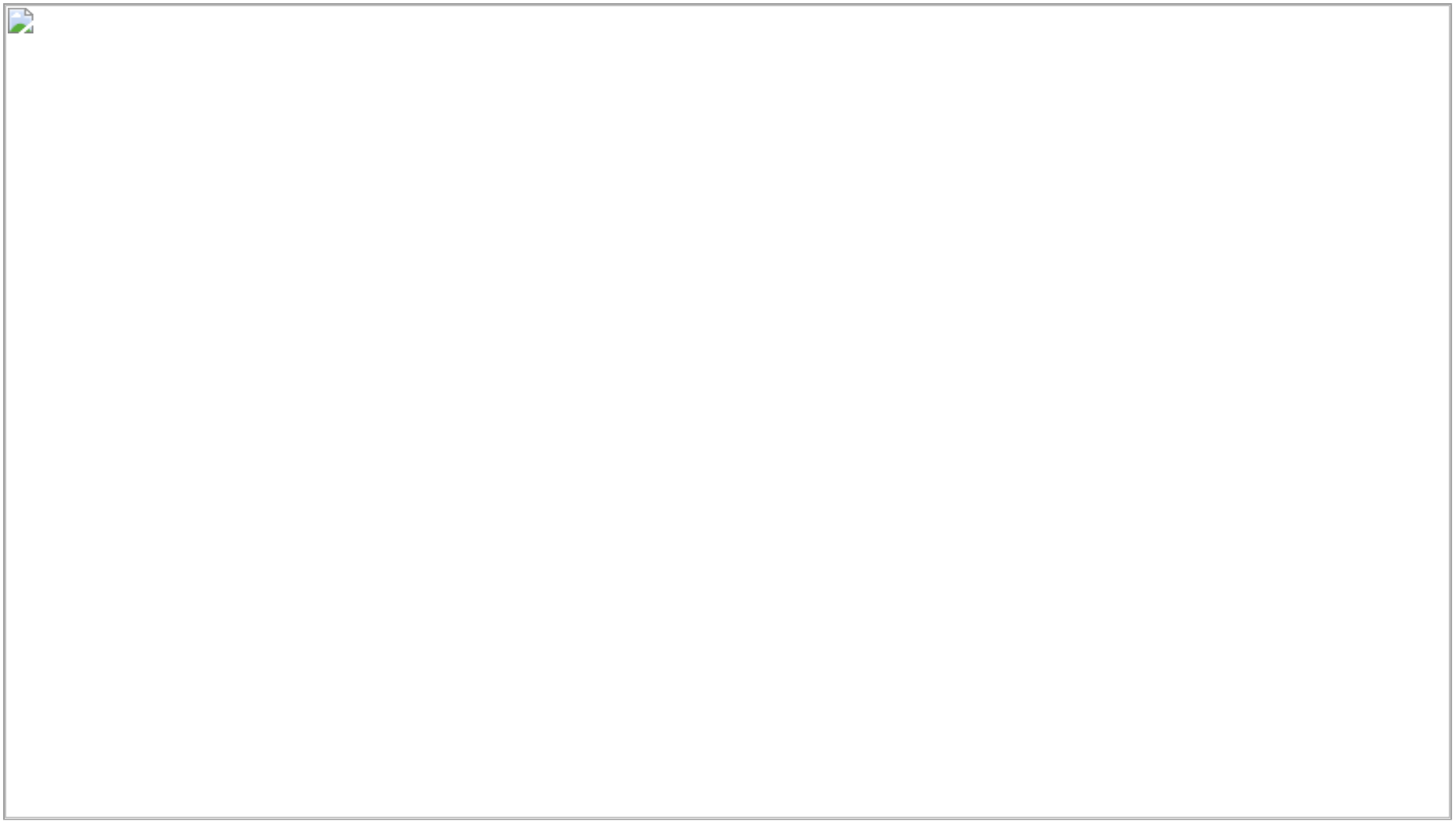
$$\frac{U_S-u}{R}=I_S\left(e^{u/U_{TH}}-1\right)$$

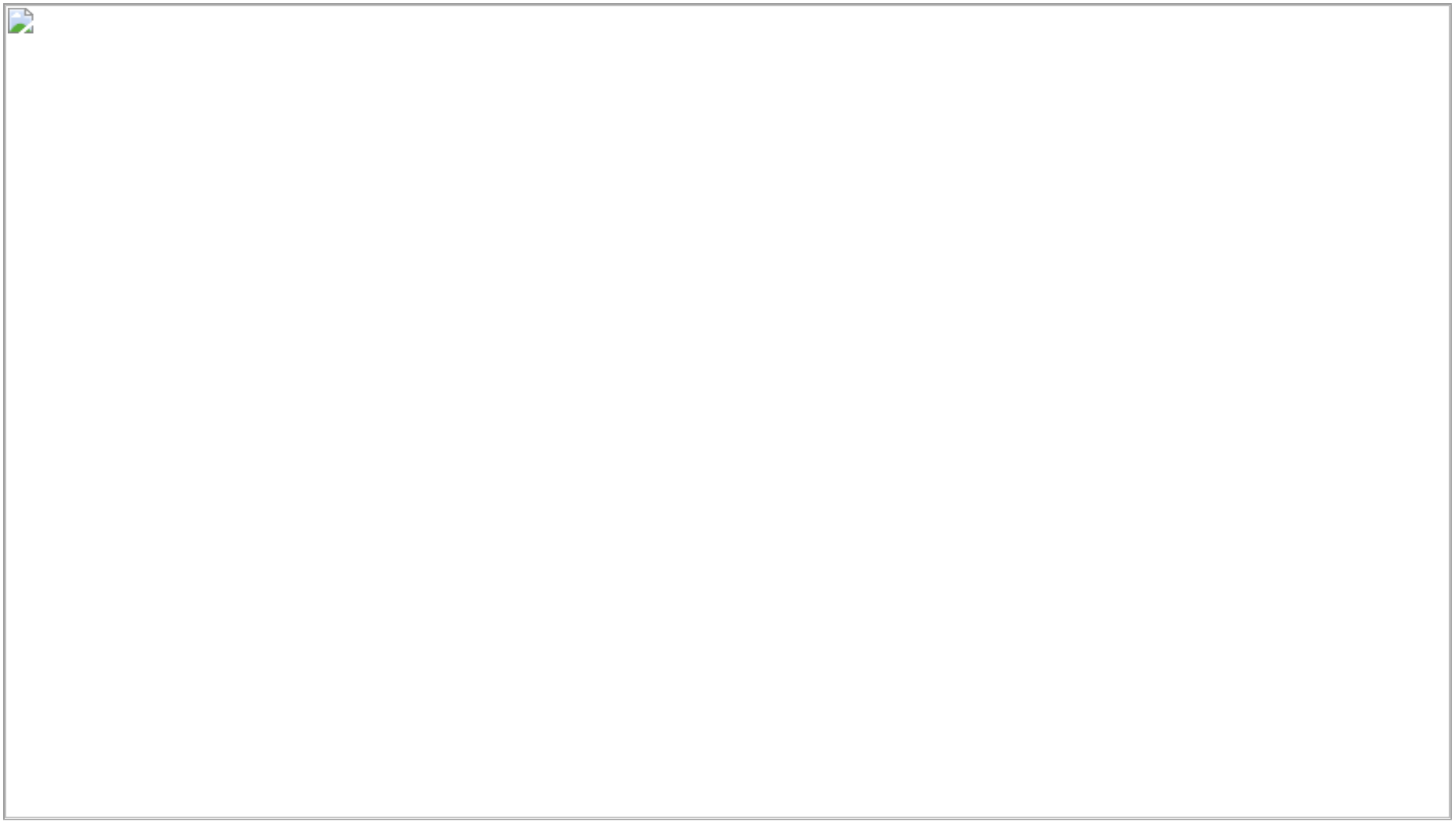
1 存在小扰动的非线性电阻电路分析



电路出现了小扰动电压源$\Delta u_S(t)$







单选题 1分

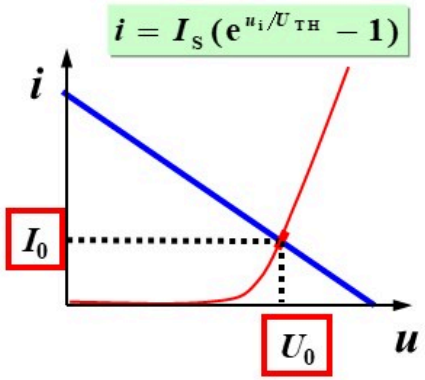
已知U_{TH}=0.025V，I_S=10^{-12}A，U_0=0.7V，则二极管的偏置电流为I_0=_____A

A 0

B 57.85

C 1.446

D 0.026



$$I_0 = I_S\left(e^{U_0/U_{TH}} - 1\right)$$

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单选题 1分

已知U_{TH}=0.025V，I_S=10^{-12}A，
U_0=0.7V，则在二极管直流偏置附近的小信号关系为$\Delta i(t)$=_____*$\Delta u(t)$

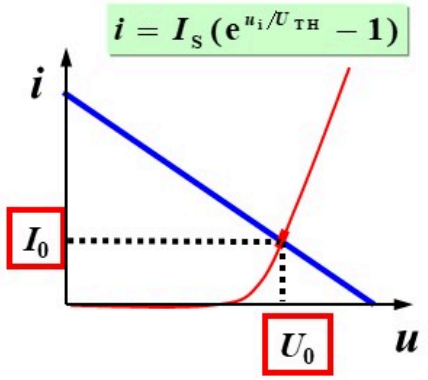
红包

A 0

B 57.85

C 1.446

D 0.026



$$\Delta i(t) = \left.\frac{\mathrm{d}i}{\mathrm{d}u}\right|_{U_0} \Delta u(t)$$

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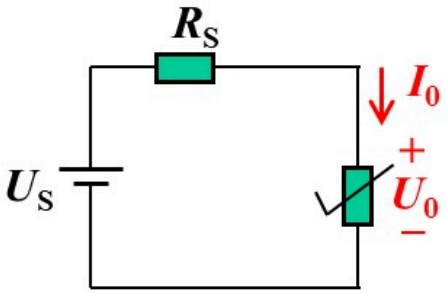
现在关注更为一般的非线性电阻电路(电源/线性/非线性电阻各1个)



U_S 为直流电源

R_S 为线性电阻

非线性电阻 $i = g(u)$

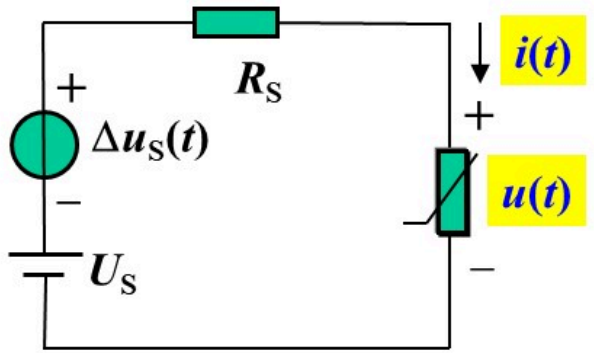
$$\begin{cases} U_S = R_S I_0 + U_0 \\ I_0 = g(U_0) \end{cases} \Rightarrow \begin{cases} U_0 \\ I_0 \end{cases}$$

解析法、分段线性法、图形法

所有支路量均可求解出来

非线性电阻工作点电压电流之间是非线性关系！

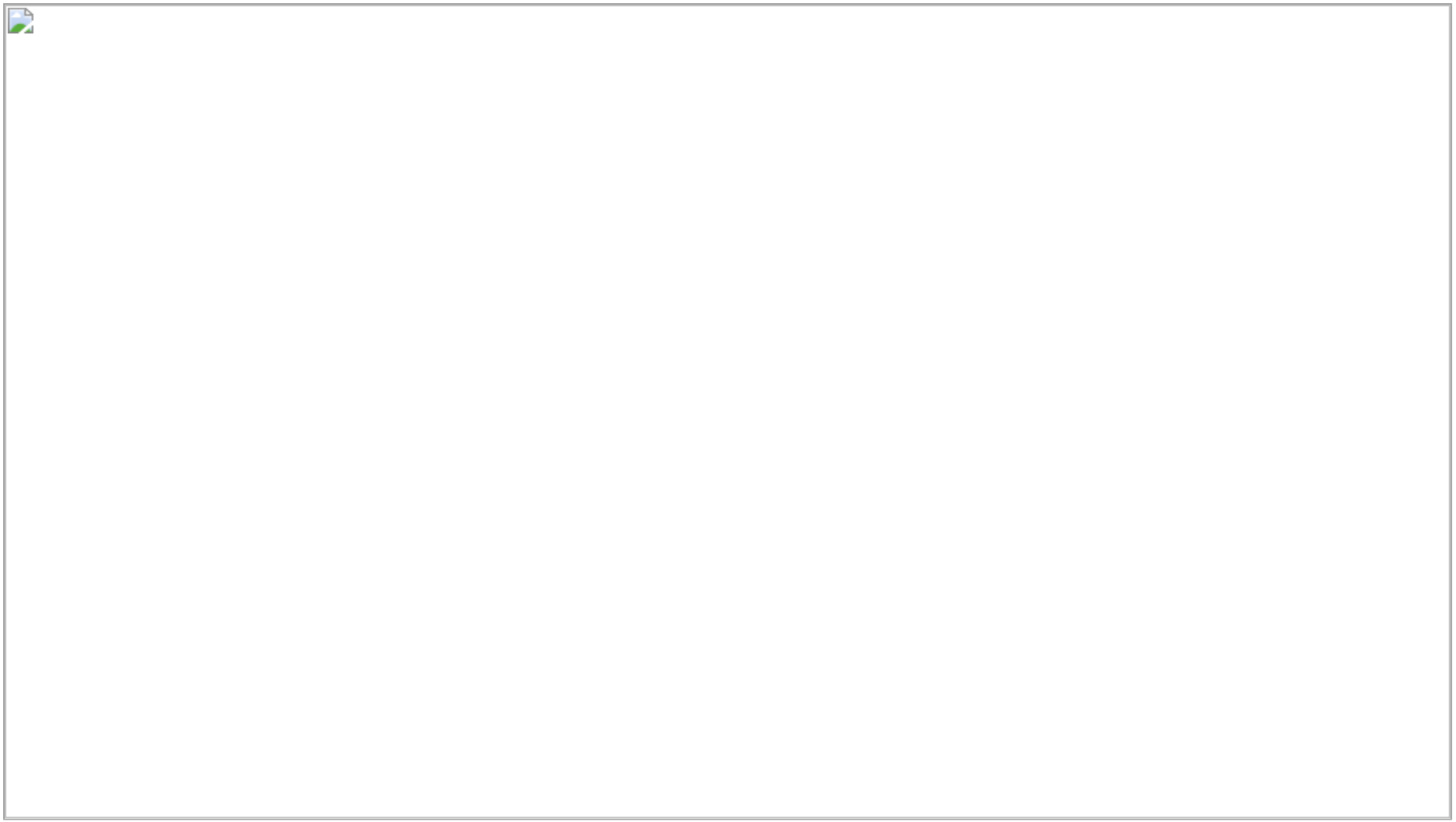
假设激励在U_S基础上，有小扰动电压源$\Delta u_S(t)$，求解支路量

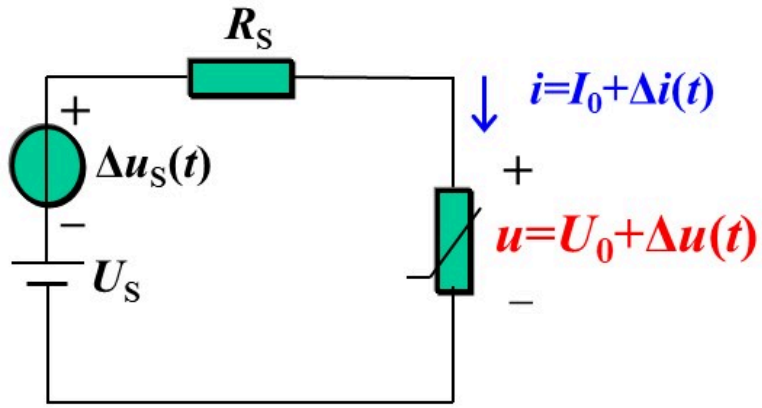


$$\begin{cases} U_S + \Delta u_S(t) = R_S\, i(t) + u(t) \\ i(t) = g(u(t)) \end{cases}$$

该方程并不好求解

在扰动比较小(且g函数性质比较好)的时候，
存在简单且误差可接受的分析方法

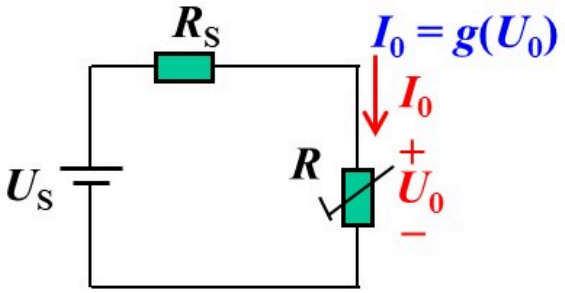




$$\Delta i(t) = g'(U_0)\Delta u(t)$$

$$U_S + \Delta u_S(t) = R_S\,[I_0 + \Delta\, i(t)\,] + U_0 + \Delta\, u(t)$$

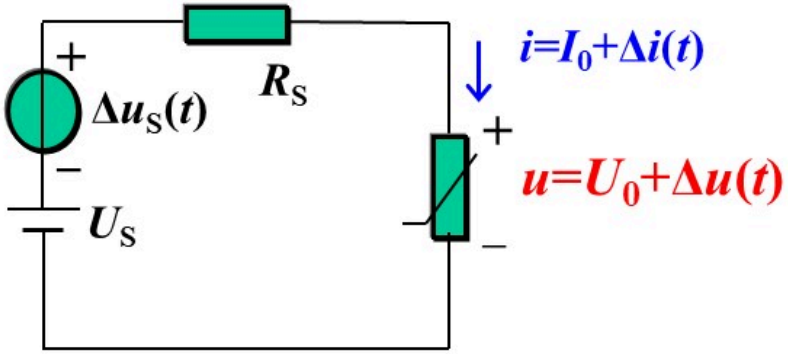
扰动前电路



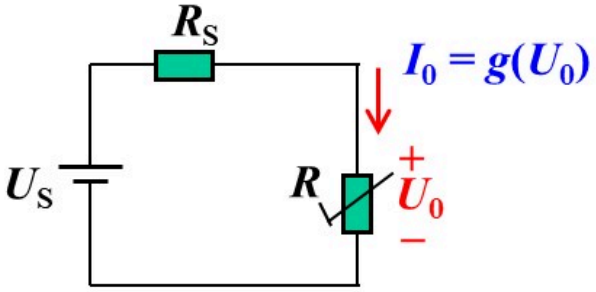
$$U_S = R_S I_0 + U_0$$

观察上式和左图，
关于扰动项$\Delta u(t)$和$\Delta i(t)$的方程是？

关于扰动项$\Delta u(t)$和$\Delta i(t)$的方程
能构成怎样的电路？
（投稿）

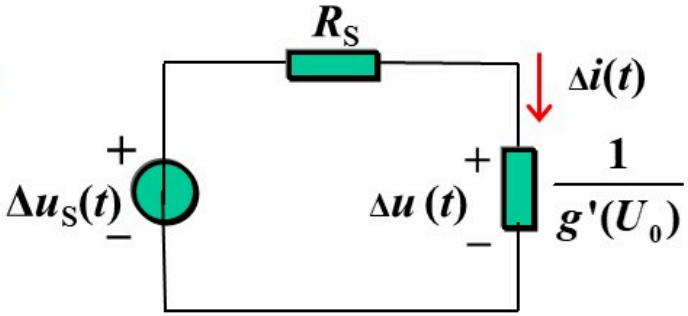


工作点电路：
原拓扑，无扰动，非线性

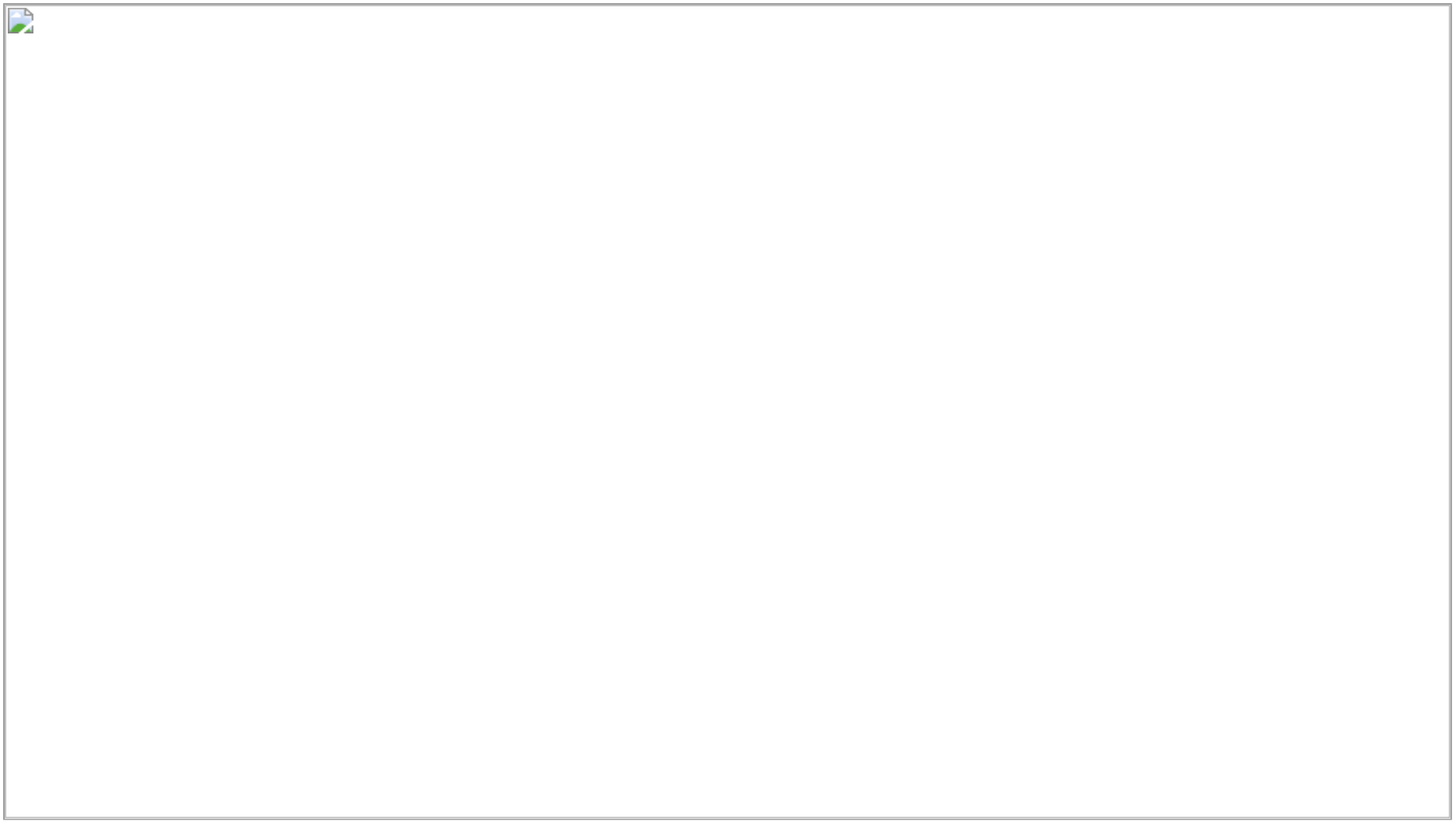


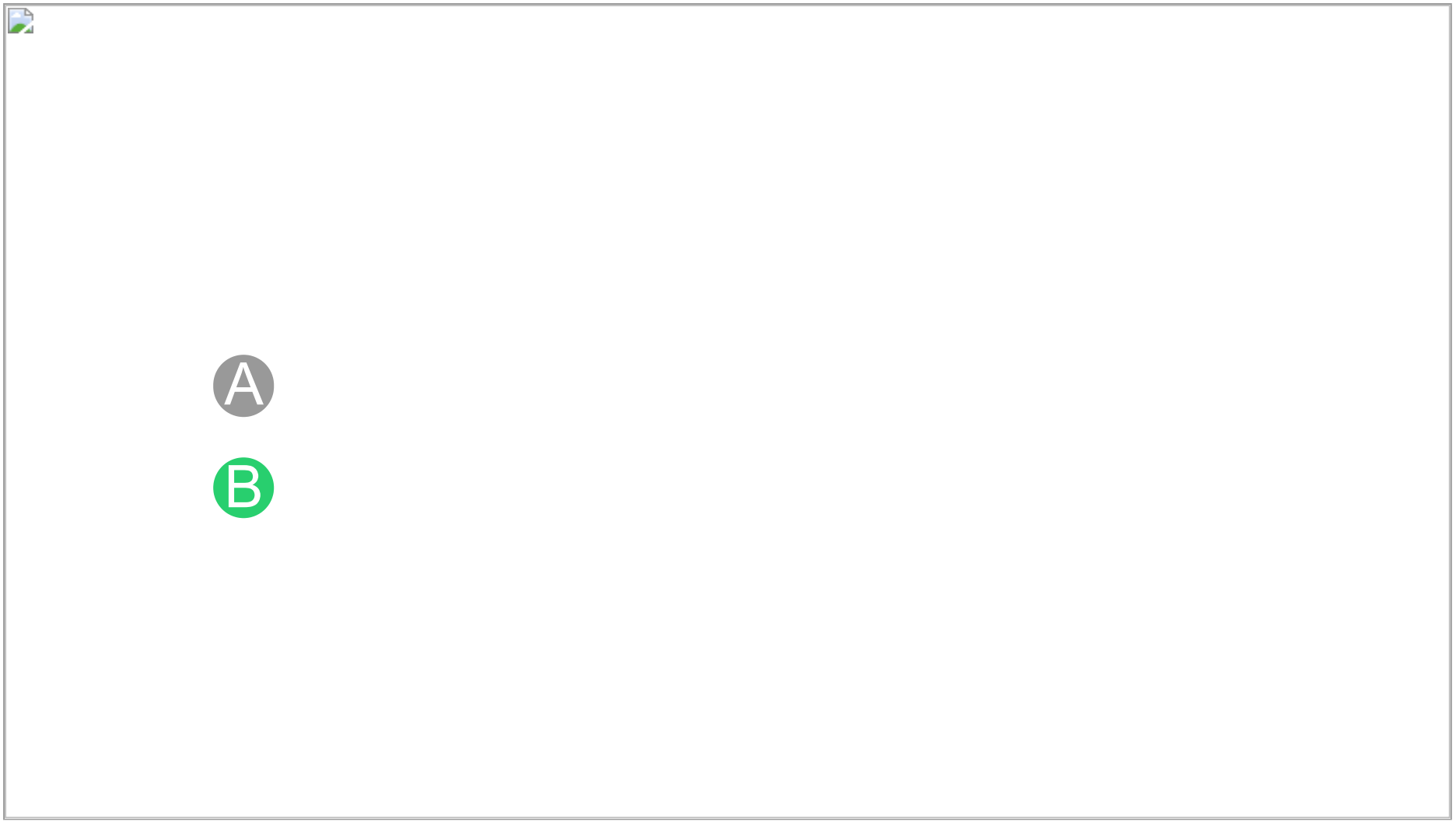
$U_S = R_S I_0 + U_0$

小信号电路：
原拓扑，无直流，线性



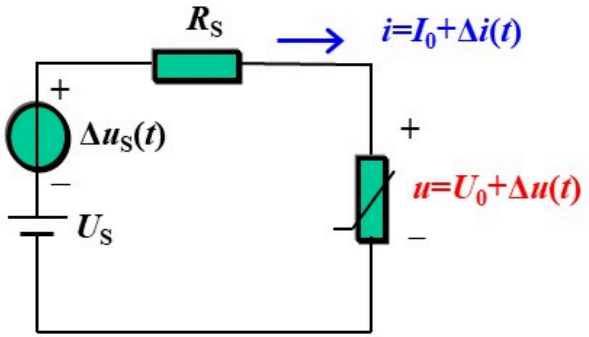
$$\Delta u_S(t) = R_S \Delta i(t) + \Delta u(t)$$




A
B

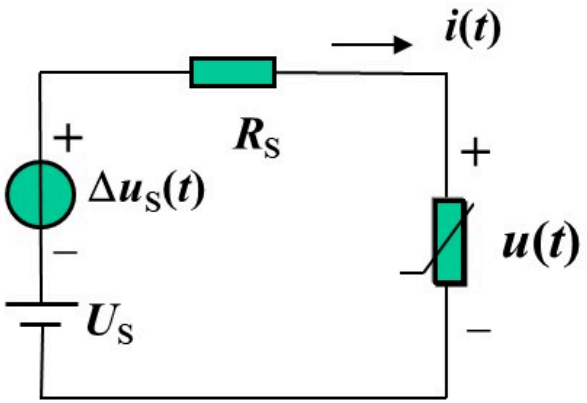
刚才讨论的是电源/线性/非线性电阻/小扰动各1个

- 多个非线性电阻怎么办？
- 多个直流电源怎么办？
  - 电源的作用效果能叠加吗？
- 多个扰动怎么办？
  - 电源和扰动效果的合成不是叠加
  - 多个扰动的作用效果能叠加吗？
- 为什么老强调小扰动/小信号？



此处可以有弹幕

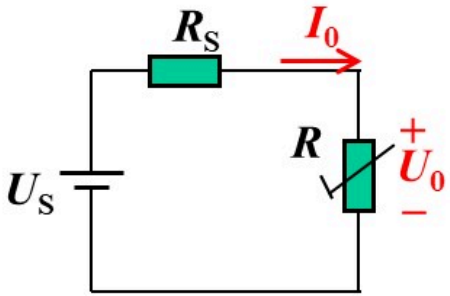
支路量表示方法小结

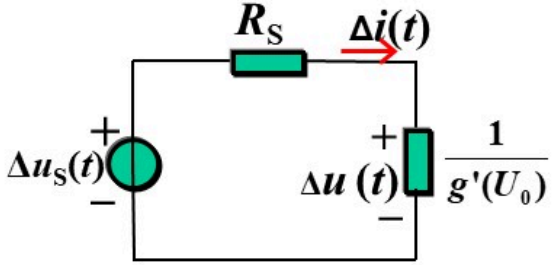


u, i → 可能随时间而变化的量

U, I { 恒定直流量；小信号分析中的工作点

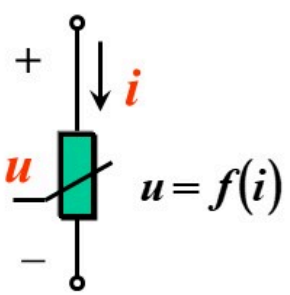
$\Delta u, \Delta i$ → 小信号



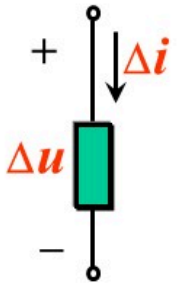


3 电路元件的小信号模型

非线性电阻



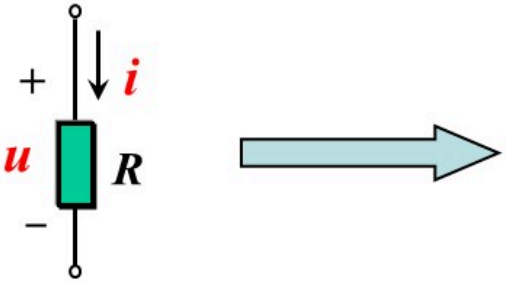
工作点处

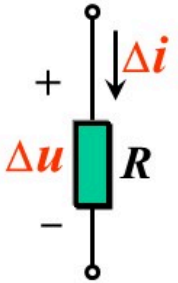


$$\Delta u = \left.\frac{\mathrm{d}f(i)}{\mathrm{d}i}\right|_{i=I_0} \Delta i$$

线性电阻

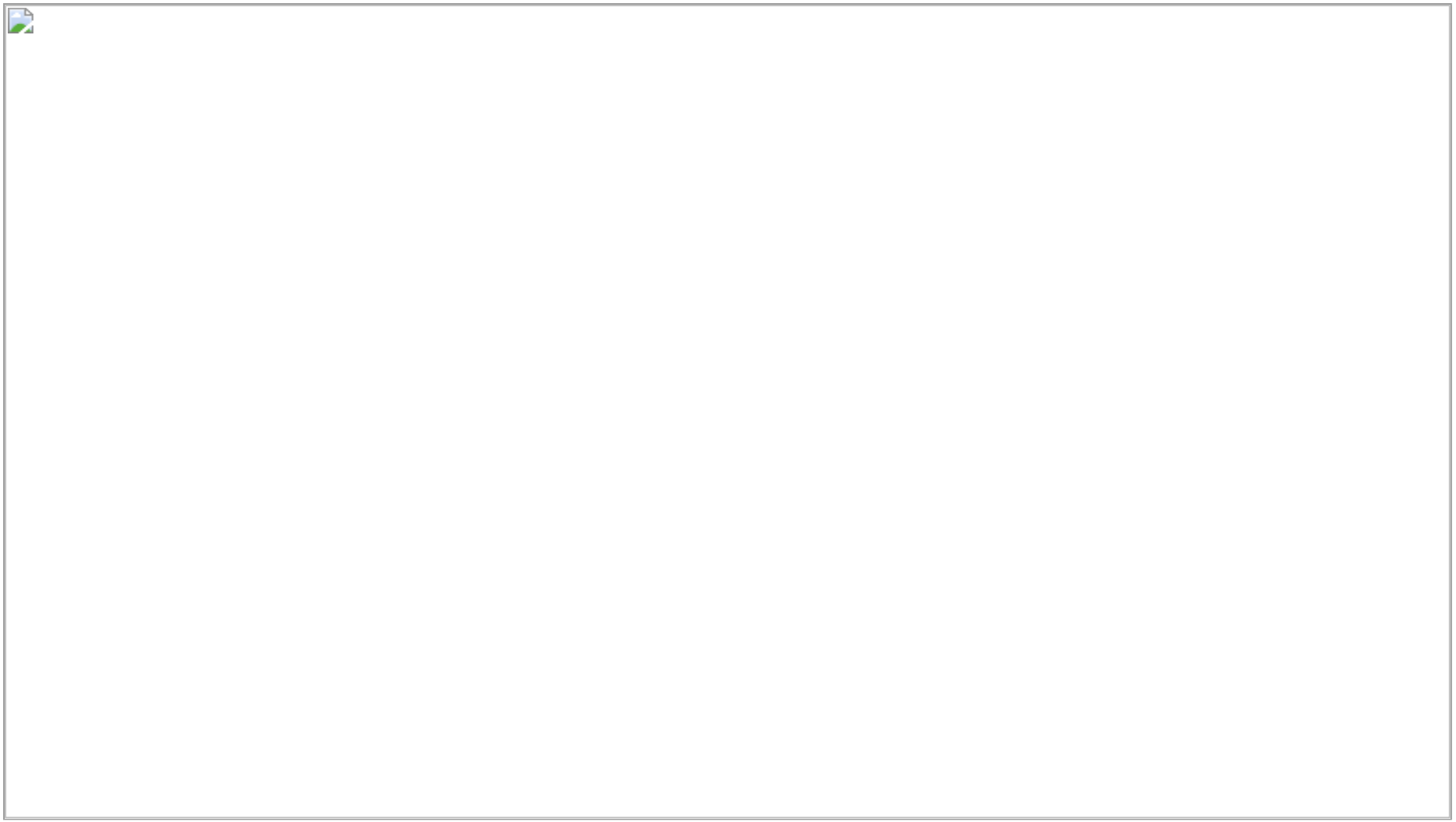
线性电阻





$$\left.\frac{\mathrm{d}(Ri)}{\mathrm{d}i}\right|_{i=I_0} = R$$

线性电阻



单选题 1分

某非线性电阻的u-i关系为 $u = i^3 - 5i$

其工作点I_0=1A时，动态电阻R_d=____Ω

A 1

B 2

C −1

D −2

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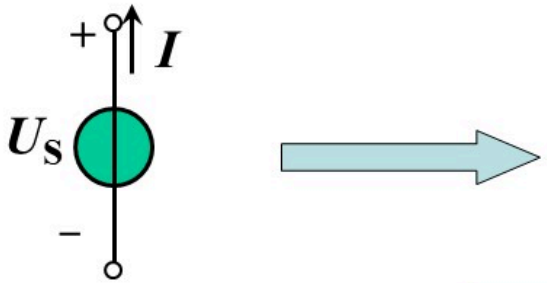
独立电压源



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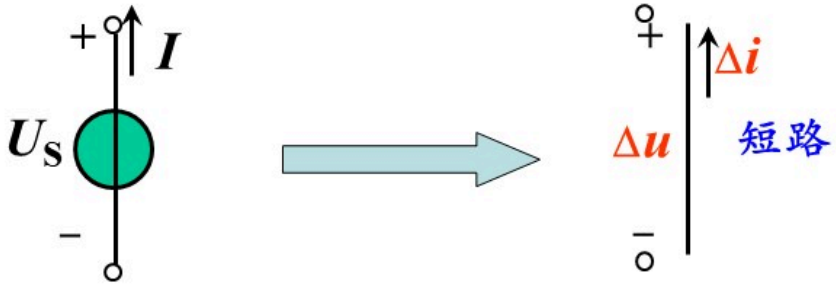
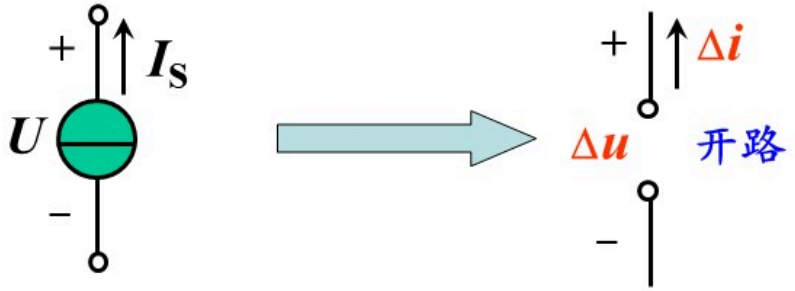
建立工作点
*u-i*特性求导
对Δ*u*-Δ*i*特性的理解

独立电压源



独立电流源



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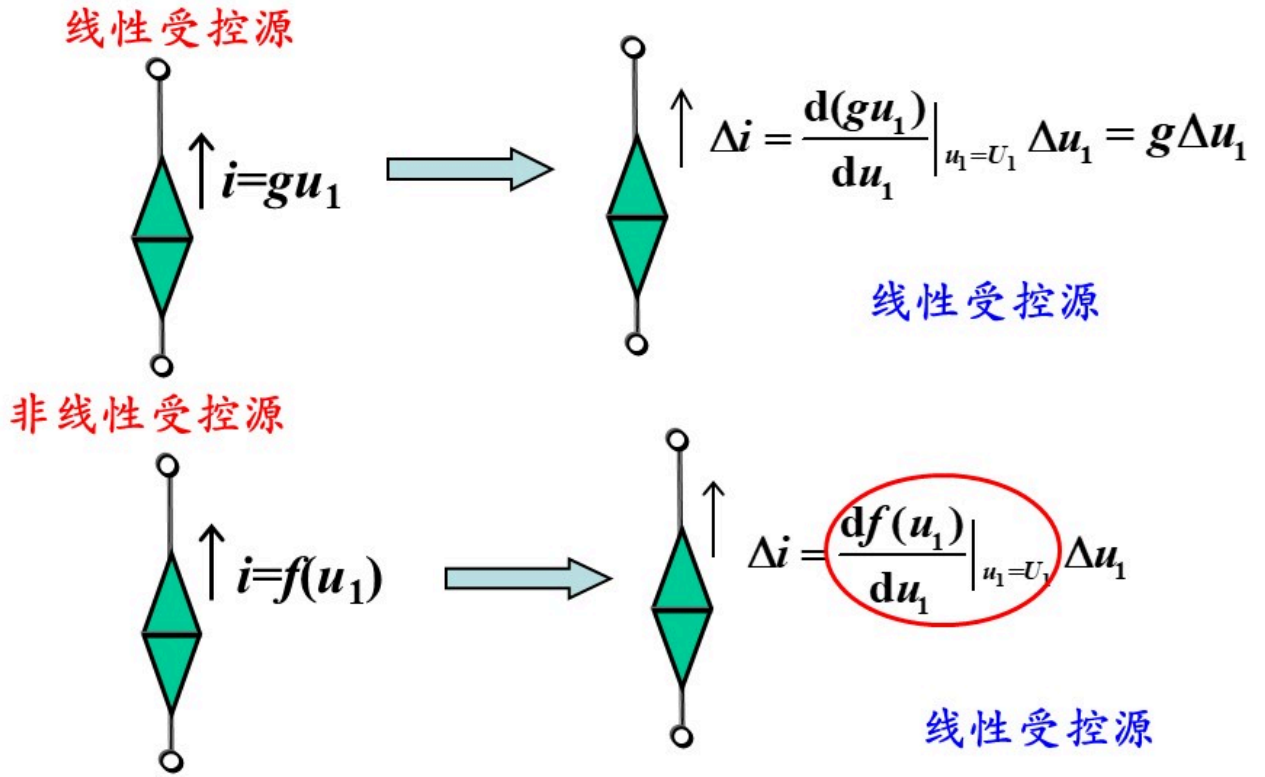
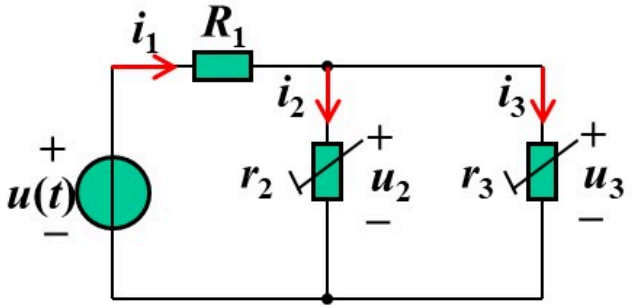

线性受控源
$i=gu_1$
$\Delta i = \left.\frac{\mathrm{d}(gu_1)}{\mathrm{d}u_1}\right|_{u_1=U_1} \Delta u_1 = g\Delta u_1$
线性受控源
非线性受控源
$i=f(u_1)$
$\Delta i = \left.\frac{\mathrm{d}f(u_1)}{\mathrm{d}u_1}\right|_{u_1=U_1} \Delta u_1$
线性受控源

例 已知 $u(t)=7+U_m\sin\omega t$ V，$\omega=100$rad/s，$U_m<<7$V，$R_1=2\Omega$。

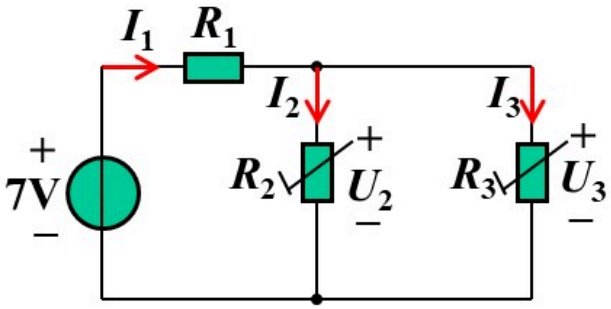


r_2： $u_2=i_2+2\,i_2^3$

r_3： $u_3=2i_3+\,i_3^3$

求电压u_2和电流i_1、i_2、i_3。

第1步：直流电压单独作用，求解静态工作电压，电流。



$$\begin{cases} 2\,I_1+U_2=7 \\ U_2=U_3 \\ I_1=I_2+I_3 \\ U_2=I_2+2\,I_2^3 \\ U_3=2I_3+\,I_3^3 \end{cases}$$

$$\begin{cases} I_1=2\text{A} \\ I_2=1\text{A} \\ I_3=1\text{A} \\ U_2=3\text{V} \\ U_3=3\text{V} \end{cases}$$

第2步：求两个非线性电阻的小信号模型

$$R_{3\mathrm{d}} = \left.\frac{\mathrm{d}u_3}{\mathrm{d}i_3}\right|_{I_3=1\mathrm{A}} = \left.2 + 3i_3^2\right|_{I_3=1\mathrm{A}} = 5\Omega$$

r_2：　$u_2=i_2+2\,i_2^3$

r_3：　$u_3=2i_3+\,i_3^3$

$I_2=I_3=1\mathrm{A}$　$I_1=2\mathrm{A}$

单选题 1分

计算R_2的动态电阻：

R_{2d}=_____Ω

A 1

B 3

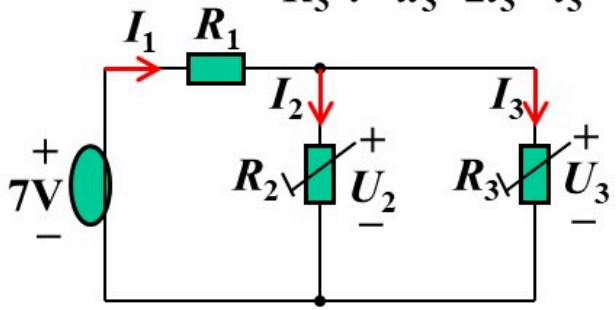
C 5

D 7

$R_1=2\Omega$

R_2： $u_2=i_2+2\,i_2^3$

R_3： $u_3=2i_3+\,i_3^3$



$I_1=2A$
$I_2=1A$
$I_3=1A$

$U_2=3V$
$U_3=3V$

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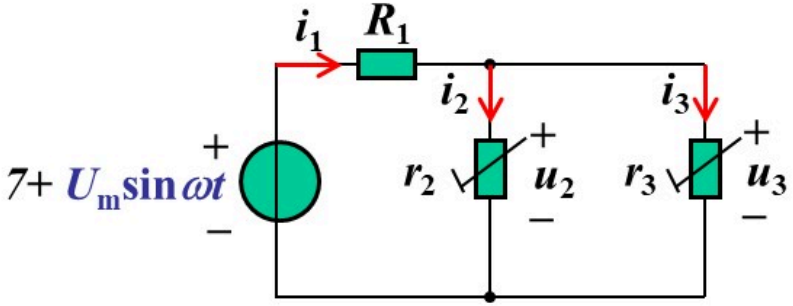
第2步：求两个非线性电阻的小信号模型

$$R_{2\mathrm{d}}=\left.\frac{\mathrm{d}u_2}{\mathrm{d}i_2}\right|_{I_2=1\mathrm{A}}=\left.1+6i_2^2\right|_{I_2=1\mathrm{A}}=7\Omega$$

$$R_{3\mathrm{d}}=\left.\frac{\mathrm{d}u_3}{\mathrm{d}i_3}\right|_{I_3=1\mathrm{A}}=\left.2+3i_3^2\right|_{I_3=1\mathrm{A}}=5\Omega$$

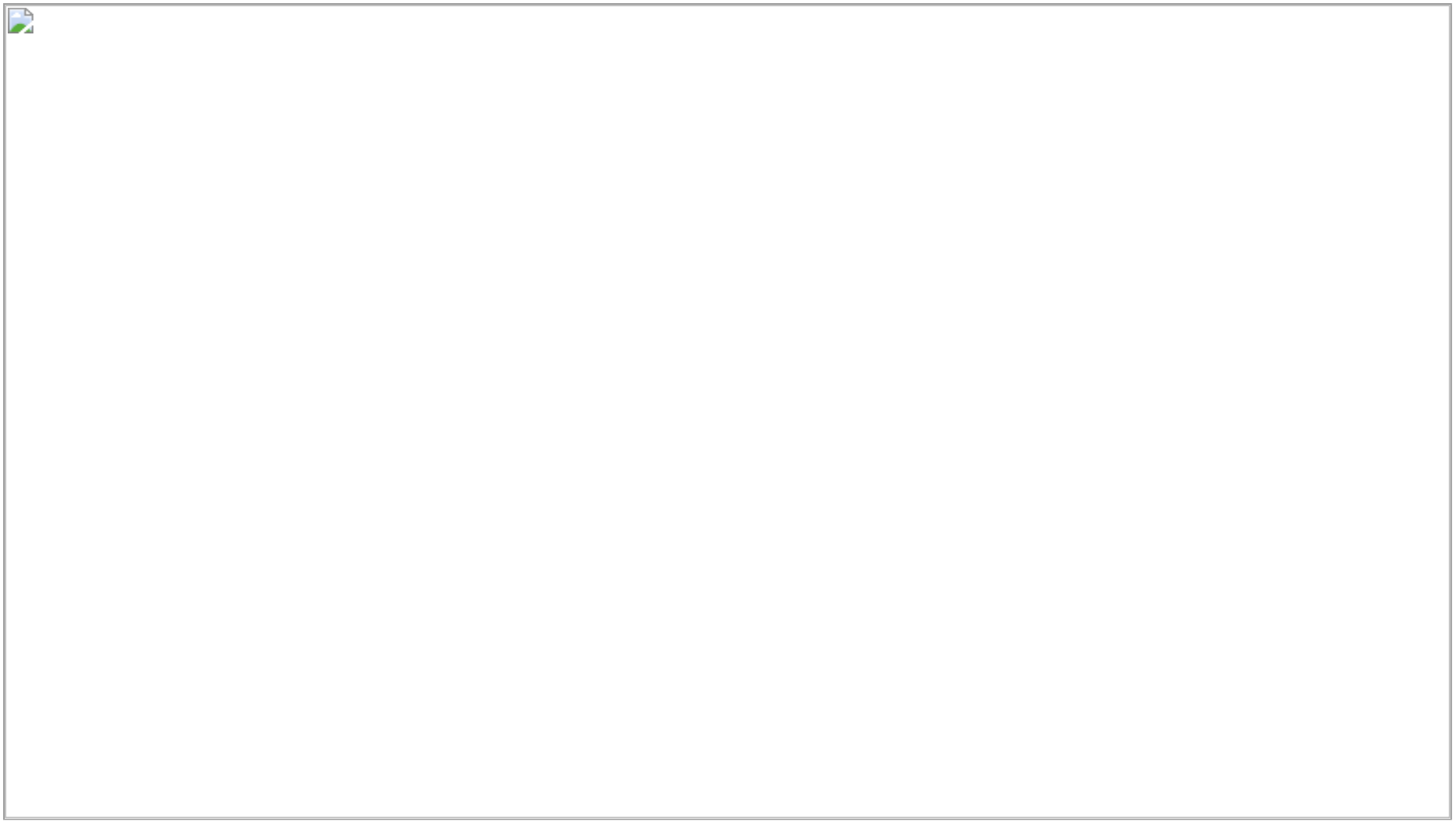
画出小信号工作等效电路，求Δu，Δi。

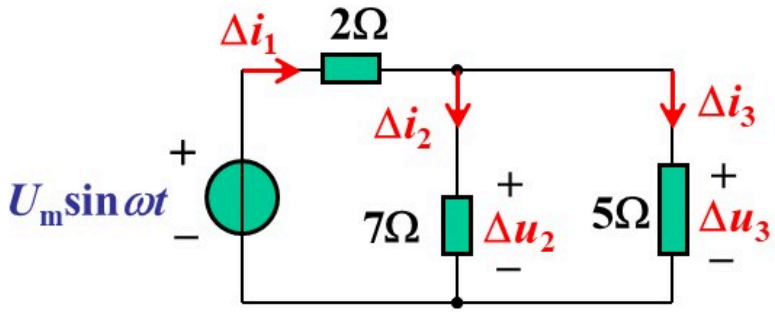


r_2：$u_2=i_2+2\,i_2^3$

r_3：$u_3=2i_3+i_3^3$

$I_2=I_3=1\mathrm{A}$　$I_1=2\mathrm{A}$





$\Delta i_1 = U_m \sin \omega t \,/(2+5//7) = 0.2033\ U_m \sin \omega t$

$\Delta i_2 = \Delta i_1 \times 5/12 = 0.0847\ U_m \sin \omega t$

$\Delta i_3 = \Delta i_1 \times 7/12 = 0.1186\ U_m \sin \omega t$

$\Delta u_2 = 7 \times \Delta i_2 = 0.593\ U_m \sin \omega t$

直流偏置：

$I_2 = I_3 = 1\text{A}$　$I_1 = 2\text{A}$

$U_2 = U_3 = 3\text{V}$

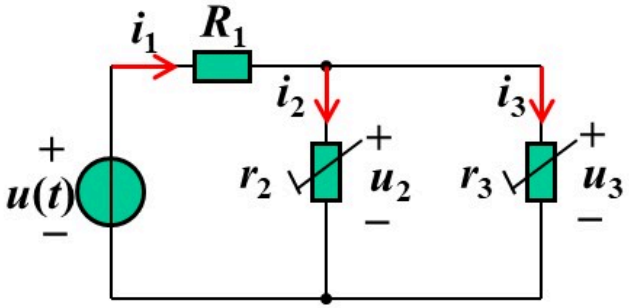
小信号响应：

$\Delta i_1 = 0.2033\, U_m \sin \omega t$

$\Delta i_2 = 0.0847\, U_m \sin \omega t$

$\Delta i_3 = 0.1186\, U_m \sin \omega t$

$\Delta u_2 = 0.593\, U_m \sin \omega t$



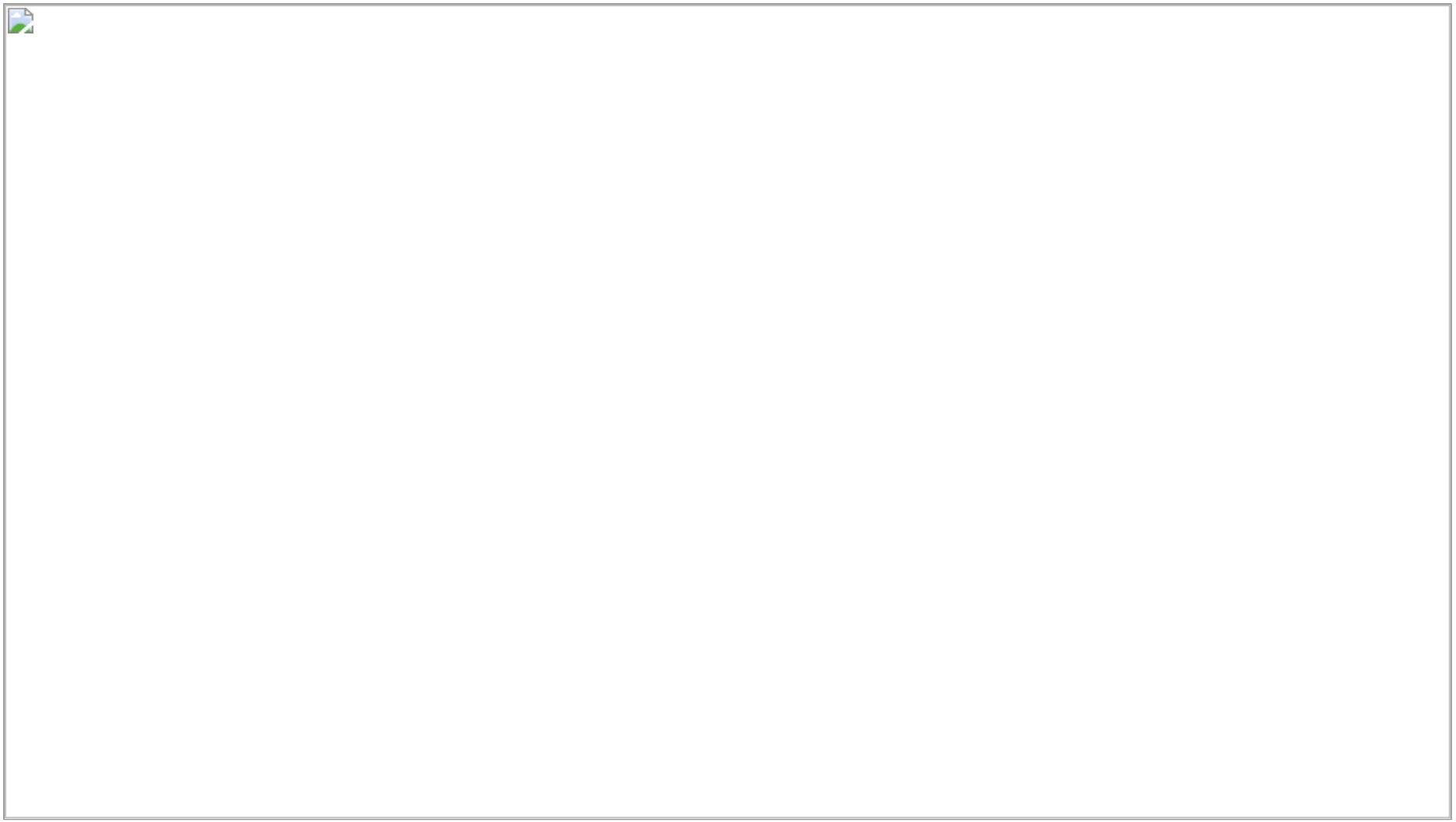
第3步：合成

$i_1 = 2 + 0.2033\, U_m \sin \omega t$ A

$i_2 = 1 + 0.0847\, U_m \sin \omega t$ A

$i_3 = 1 + 0.1186\, U_m \sin \omega t$ A

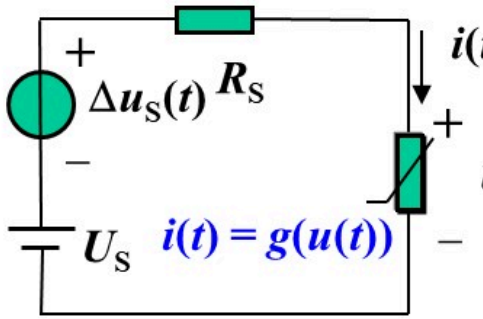
$u_2 = 3 + 0.5932\, U_m \sin \omega t$ V



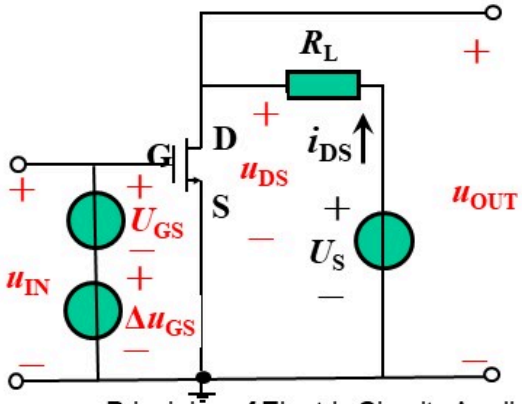
第2次应用介绍课：非线性电阻电路的应用

----(MOSFET构成模拟放大器)

小扰动



小扰动→(小)待放大信号

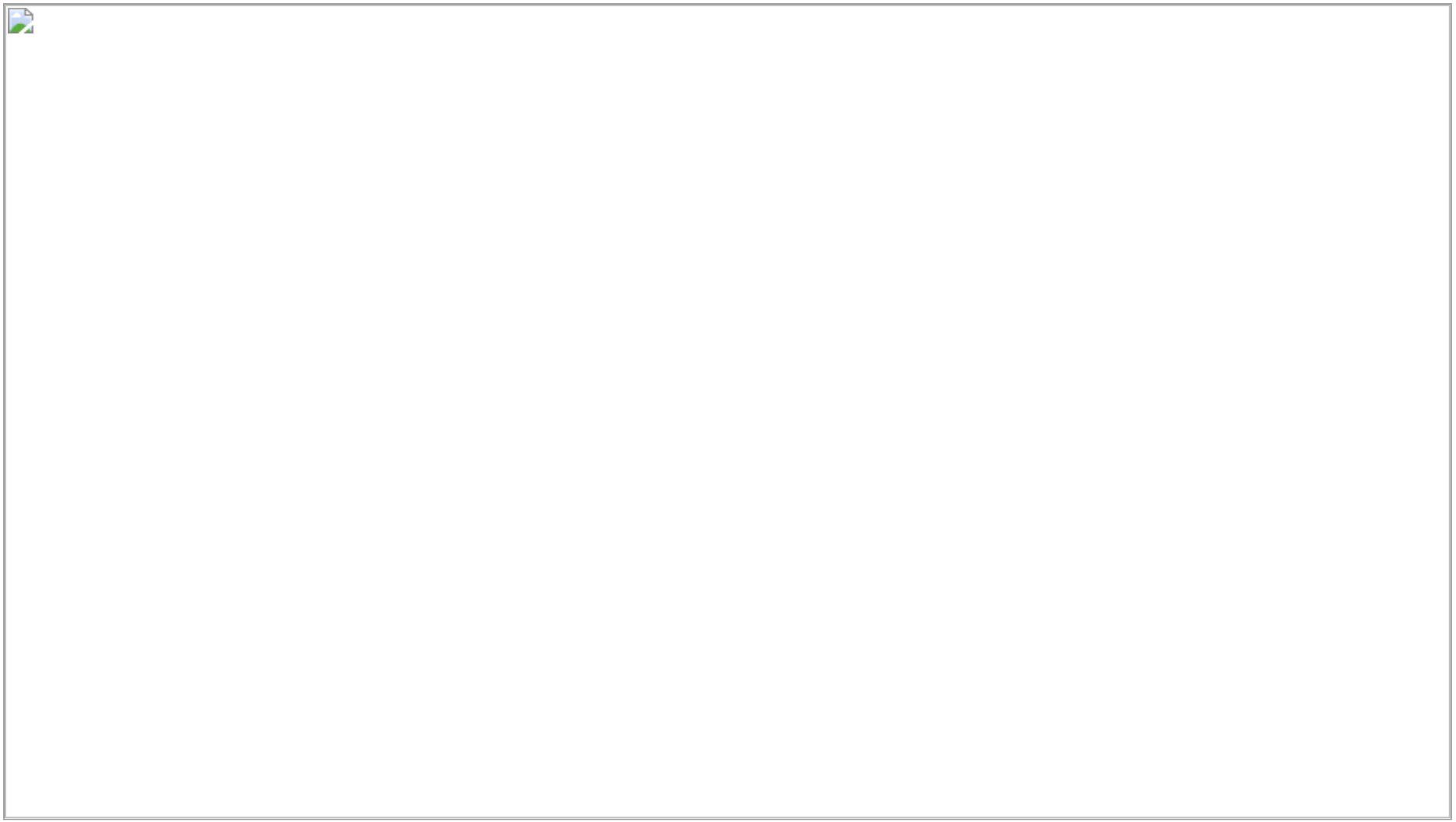


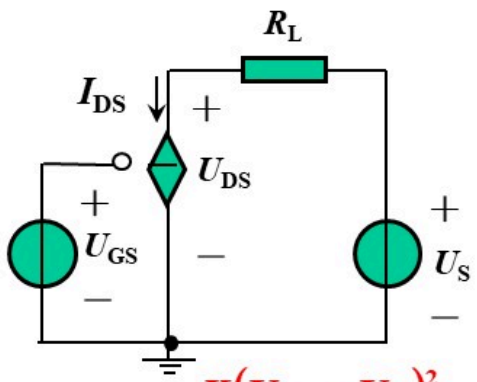
全信号　　直流偏置　(小)待放大信号

$$u_{IN}=u_{GS}=U_{GS}+\Delta u_{GS}$$

$$u_{OUT}=u_{DS}=U_{DS}+\Delta u_{DS}$$

全信号　　直流偏置　(小)放大后信号





$$I_{DS}=\frac{K(U_{GS}-U_T)^2}{2}$$

（1）求直流工作点（解析法）

设MOSFET工作在饱和区

$U_S=10\text{V}$，$U_{GS}=2.5\text{V}$，$K=0.5\text{mA/V}^2$，$U_T=1\text{V}$，$R_L=10\text{k}\Omega$

$$U_{OUT}=U_{DS}=U_S-I_{DS}R_L$$

$$U_{OUT}=U_S-\frac{K(U_{GS}-U_T)^2}{2}R_L$$

$$U_{OUT}=U_{DS}=10-\frac{0.5\times(2.5-1)^2}{2}\times 10=4.375\text{V}$$

恒流区工作条件：$0<(U_{GS}-U_T)<U_{DS}$ 满足

还需验证MOSFET不工作在电阻区(略)

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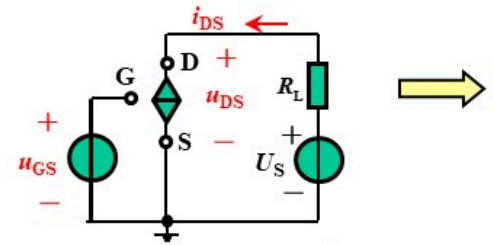
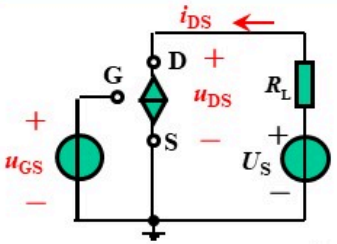
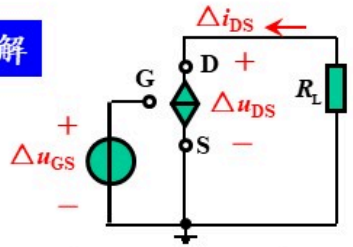
（2）画小信号电路，求小信号解



$$i_{DS}=\frac{K\left(u_{GS}-U_T\right)^2}{2}$$

（2）画小信号电路，求小信号解



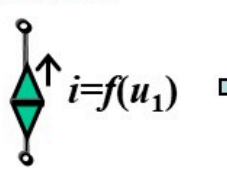


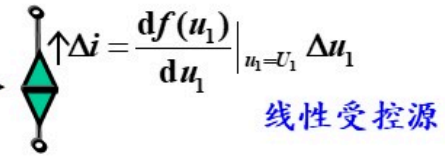
$$i_{DS}=\frac{K\left(u_{GS}-U_T\right)^2}{2}$$

$$\Delta i_{DS}=\left.\frac{\mathrm{d}\left(\frac{K\left(u_{GS}-U_T\right)^2}{2}\right)}{\mathrm{d}u_{GS}}\right|_{u_{GS}=U_{GS}}\Delta u_{GS}$$

$$\boxed{\Delta i_{DS}=K\left(U_{GS}-U_T\right)\Delta u_{GS}}$$

非线性受控源





线性受控源

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单选题 1分

已知MOSFET工作于电流源区，K=0.5mA/V²，U_T=1V，当U_{GS}=1.3V时，其对应的线性受控源关系为Δi_{DS}(A)=____*Δu_{GS}(V)（注意单位）

$$i_{DS}=\frac{K(u_{GS}-U_T)^2}{2}$$

A 0.15 (A/V)

B 0.045 (A/V)

$$\Delta i_{DS}=K(U_{GS}-U_T)\Delta u_{GS}$$

C 0.00015 (A/V)

D 0.000845 (A/V)

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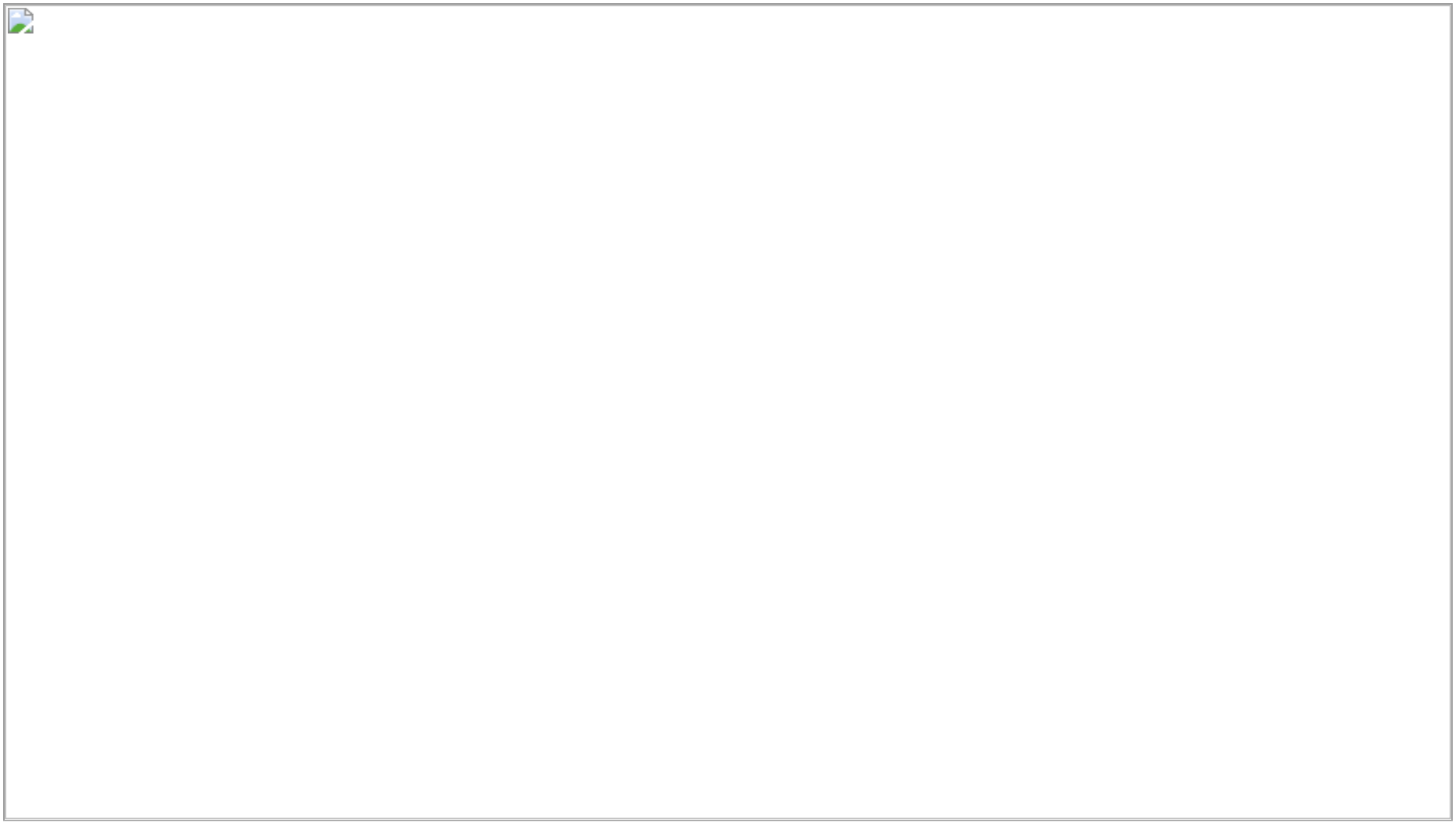
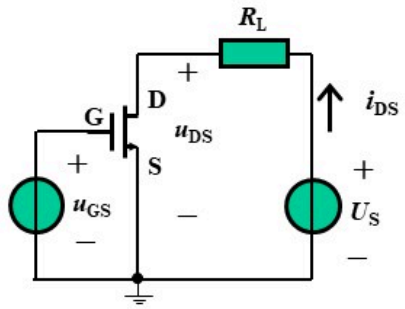


第3步：合成

$$u_{OUT} = U_{OUT} + \Delta u_{OUT}$$

$$U_{OUT} = U_S - \frac{K\left(U_{GS} - U_T\right)^2}{2} R_L$$

$$\Delta u_{OUT} = -K\left(U_{GS} - U_T\right) R_L \Delta u_{IN}$$

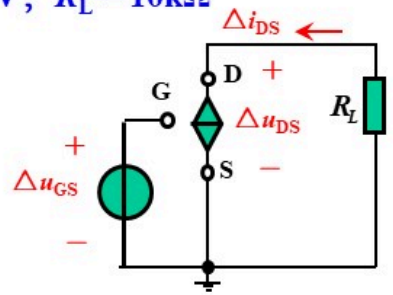


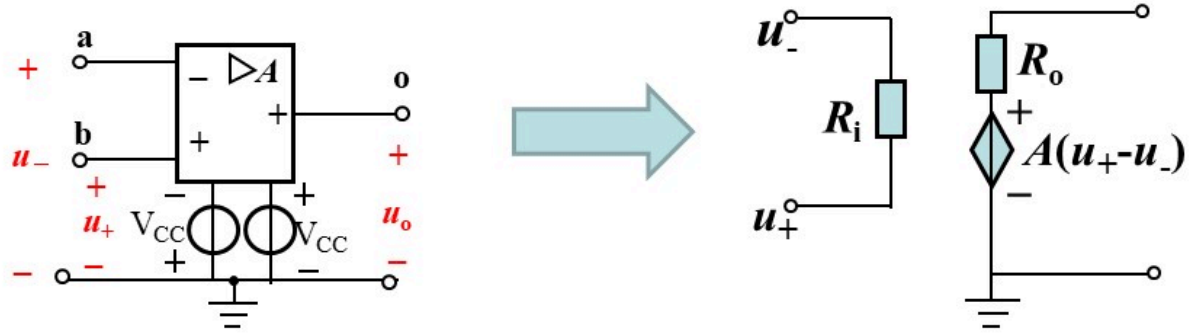
$U_S = 10\text{V}$，$U_{GS} = 2.5\text{V}$，$K = 0.5\text{mA/V}^2$，$U_T = 1\text{V}$，$R_L = 10\text{k}\Omega$

$$u_{OUT} = 4.375 - 7.5\Delta u_{IN}$$

见仿真

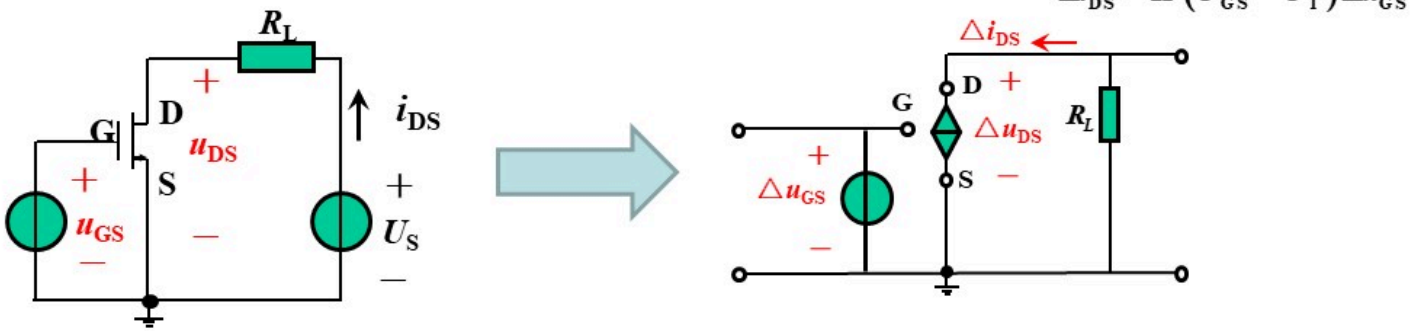
再论no free lunch



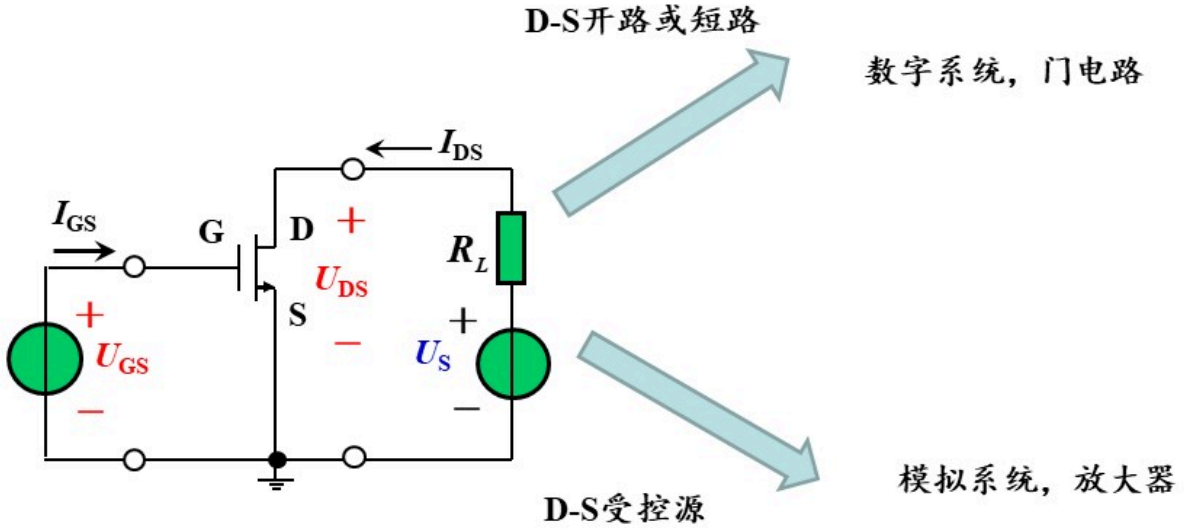


直流电源去哪啦?

$$\Delta i_{DS} = K\left(U_{GS} - U_T\right)\Delta u_{GS}$$

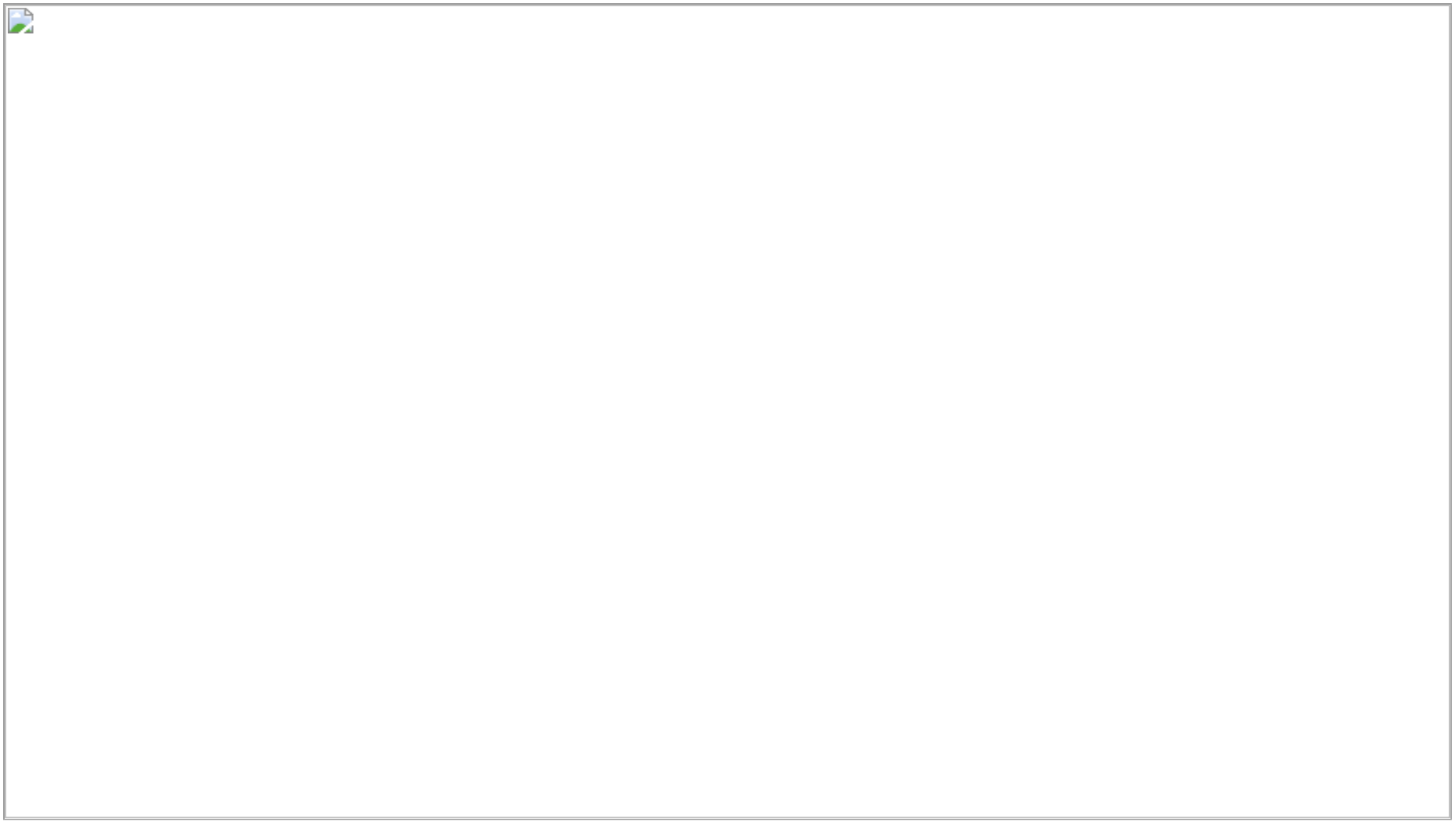
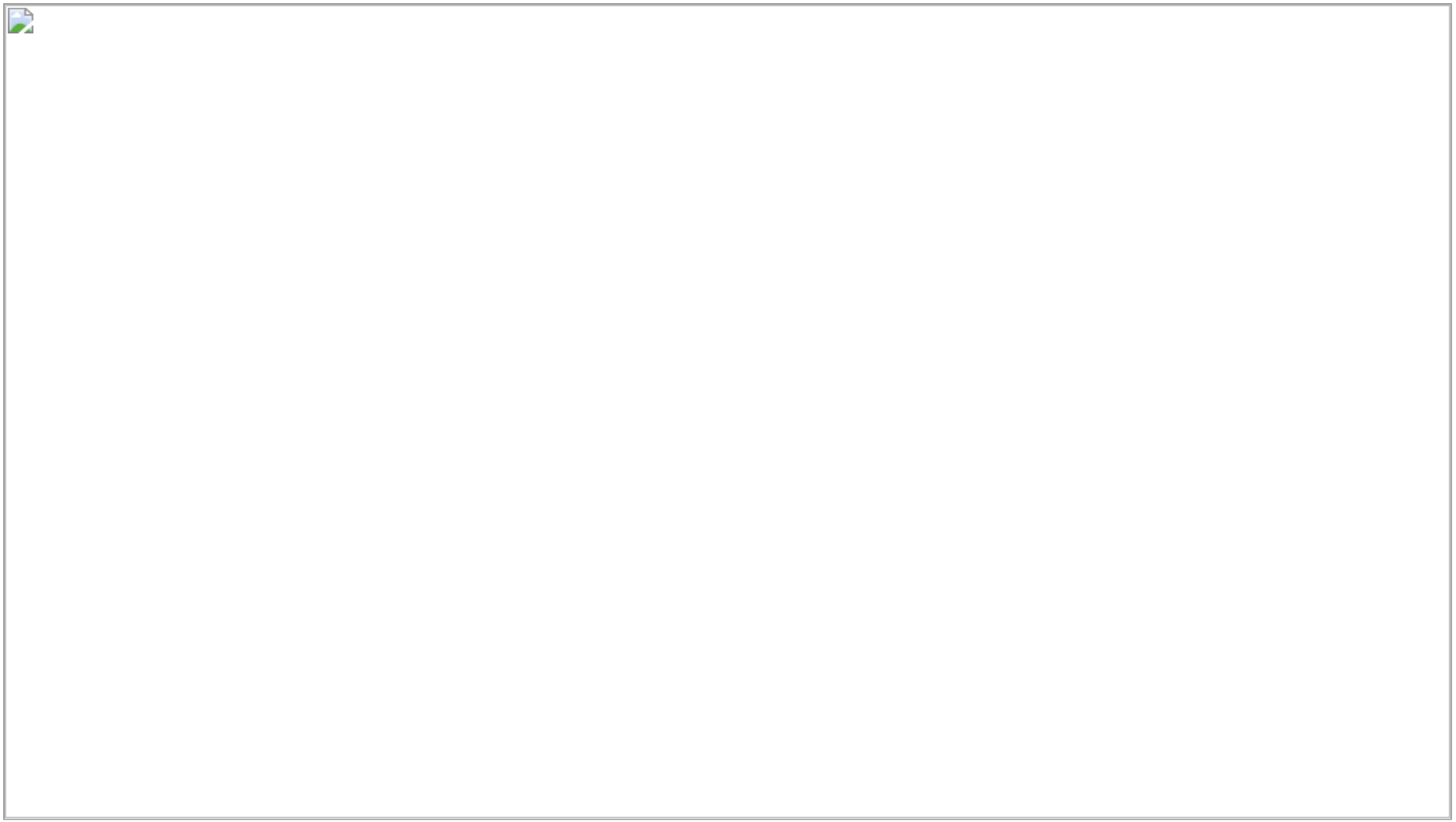


回顾一下MOSFET



让我们来看一看2020年旗舰手机的显示屏

型号	显示屏
iPhone 11	Liquid 视网膜高清显示屏 6.1 英寸 LCD 屏
iPhone 11 Pro Max	超视网膜 XDR 显示屏 5.8 英寸或 6.5 英寸 OLED 屏
Galaxy S20+	动态 AMOLED 2X 动感十足的观看体验 一手掌握
华为Mate 30 Pro	88° 超曲面OLED环幕屏*1 环幕视界， 侧屏触控自在掌握
小米 10 Pro	定制三星 AMOLED 高端双曲面 90Hz刷新率+180Hz采样率 刷新屏幕体验

AMOLED
是一种OLED

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