Part 1

1. C

2. A

3. C

4. C

5. A

Part 2a.

$$A = \int_{\sqrt{2}}^{1} 2\pi f(x)$$

$$A = \int_{\sqrt{2}}^{1} 2\pi \frac{1}{\sqrt{(4-x^2)^3}}\, dx.$$

$$A = 2\pi \int_{\sqrt{2}}^{1} \frac{1}{\sqrt{(4-x^2)^3}}\, dx.$$

Part 2b

① $f(x) = x(8-x)^{\frac{1}{3}}$

② $f'(x) = (8-x)^{\frac{1}{3}} + x\,\frac{1}{3}(8-x)^{-\frac{2}{3}}\cdot(-1)$

$\Leftrightarrow\ f'(x) = (8-x)^{\frac{1}{3}} - \dfrac{x}{3\sqrt[3]{(8-x)^2}}$

$\Leftrightarrow\ f'(x) = (8-x)^{\frac{1}{3}} \cdot \dfrac{3\sqrt[3]{(8-x)^2}}{3\sqrt[3]{(8-x)^2}} - \dfrac{x}{3\sqrt[3]{(8-x)^2}}$

$\Leftrightarrow\ f'(x) = \dfrac{24-3x}{3\sqrt[3]{(8-x)^2}} - \dfrac{x}{3\sqrt[3]{(8-x)^2}}$

$\Leftrightarrow\ f'(x) = \dfrac{24-4x}{3\sqrt[3]{(8-x)^2}}$

Let $24-4x=0$ and $3\sqrt[3]{(8-x)^2}=0$.

$\Leftrightarrow\ 24 = 4x \qquad 8-x=0$

$x = 6. \qquad x = 8.$

Therefore, the critical point of $f(x)$ are $x=6$ and $x=8$.

③ $f''(x) = \dfrac{-4(3\sqrt[3]{(8-x)^2}) - (24-x)\cdot(3\frac{2}{3}(8-x)^{-\frac{1}{3}})}{9\sqrt[3]{(8-x)^4}}$

$\Leftrightarrow\ f''(x) = \dfrac{4x-48}{9\sqrt[3]{(8-x)^5}}$

$f(6) = 6\times(8-6)^{\frac{1}{3}}$ $\qquad f(8) = 8\times(8-8)^{\frac{1}{3}}$ $\qquad f(0) = 0\times(8-0)^{\frac{1}{3}}$

$= 6 \qquad = 0. \qquad = 0.$

x	$x<6$	$6<x<8$	$x>8$
$f'(x)$	$+$	$-$	$-$
$f''(x)$	$-$	$-$	$+$

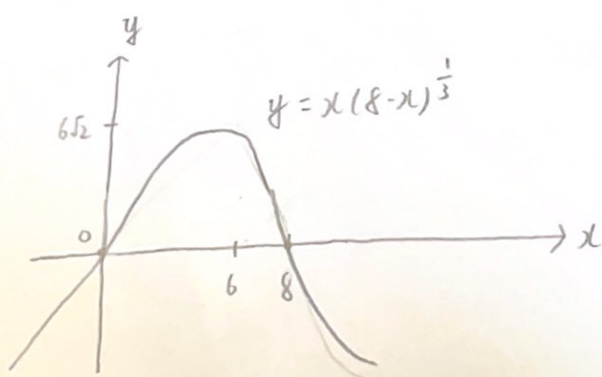
Local extrema.

$$f(6) = 6\times(8-6)^{\frac{1}{3}}$$
$$= 6\sqrt[3]{2}$$

Inflection points. :

$x = 6$.

Convex. $[8, +\infty]$. Concave : $[-\infty, 8]$



Part 2c.

$$\int_0^{\frac{\pi}{2}} \frac{\sin x(\cos x-1)}{(\cos x)^2-5\cos x+6}\,dx.$$

Let $u=\cos x.$ $\qquad\qquad\qquad\qquad \cos\frac{\pi}{2}=0$

$du=-\sin x\,dx.$ $\Leftarrow$ $dx=\frac{du}{-\sin x}.$ $\qquad \cos 0=1$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x(\cos x-1)}{(\cos x)^2-5\cos x+6}\,dx$$

$$=\int_1^0 \frac{\sin x\,(u-1)}{u^2-5u+6}\cdot\frac{du}{-\sin x}$$

$$=\int_1^0 \frac{1-u}{u^2-5u+6}\,du$$

$$=\int_1^0 \frac{1-u}{(u-3)(u-2)}\,du.$$

Let $\frac{1-u}{(u-3)(u-2)}=\frac{A}{u-3}+\frac{B}{u-2}$

$$\frac{A}{u-3}+\frac{B}{u-2}=\frac{Au-2A+Bu-3B}{(u-3)(u-2)}=\frac{(-2A-3B)+(A+B)u}{(u-3)(u-2)}$$

So. $1-u=(-2A-3B)+(A+B)u.$

$$\Rightarrow\begin{cases}1=(-2A-3B)\\-1=(A+B).\end{cases}\Rightarrow\begin{cases}A=-2\\B=1\end{cases}$$

$$\int_1^0 \frac{1-u}{(u-3)(u-2)}$$

$$=\int_1^0 \frac{-2}{u-3}\,du+\int_0^{\frac{\pi}{2}} \frac{1}{u-2}\,du$$

$$=\left(-2\ln|u-3|+\ln|u-2|\right)\Big|_1^0$$

$\longrightarrow$

$= (-2\ln 3 + \ln 2) - ((-2)\ln 2 + \ln 1)$

$= -2\ln 3 + 3\ln 2 - \ln 1$

$= \ln 3^{-2} + \ln 2^{3} + \ln 1^{-1}$

$= \ln(3^{-2} \cdot 2^{3} \cdot 1^{-1})$

$= \ln \frac{8}{9}$