1.

Solution

(a) If the speed of rotation ω of the shaft is 500 rad/s, then the voltage induced in the rotating loop will be

$$e_{\text{ind}} = 2rlB\omega$$
$$e_{\text{ind}} = 2(0.25\text{ m})(0.5\text{ m})(0.4\text{ T})(500\text{ rad/s}) = 50\text{ V}$$

Since the external battery voltage is only 48 V, this machine is operating as a *generator*, charging the battery.

(b) The current flowing out of the machine is approximately

$$i = \frac{e_{\text{ind}} - V_B}{R} = \left(\frac{50\text{ V} - 48\text{ V}}{0.4\ \Omega}\right) = 5.0\text{ A}$$

(*Note* that this value is the current flowing while the loop is under the pole faces. When the loop goes beyond the pole faces, e_{ind} will momentarily fall to 0 V, and the current flow will momentarily reverse. Therefore, the *average* current flow over a complete cycle will be somewhat less than 5.0 A.)

(c) If the speed of the rotor were increased to 550 rad/s, the induced voltage of the loop would increase to

$$e_{\text{ind}} = 2rlB\omega$$
$$e_{\text{ind}} = 2(0.25\text{ m})(0.5\text{ m})(0.4\text{ T})(550\text{ rad/s}) = 55\text{ V}$$

and the current flow out of the machine will increase to

$$i = \frac{e_{\text{ind}} - V_B}{R} = \left(\frac{55\text{ V} - 48\text{ V}}{0.4\ \Omega}\right) = 17.5\text{ A}$$

(d) If the speed of the rotor were decreased to 450 rad/s, the induced voltage of the loop would fall to

$$e_{\text{ind}} = 2rlB\omega$$
$$e_{\text{ind}} = 2(0.25\text{ m})(0.5\text{ m})(0.4\text{ T})(450\text{ rad/s}) = 45\text{ V}$$

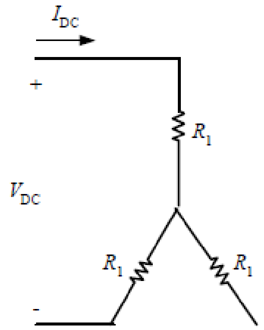
Here, e_{ind} is less than V_B, so current flows into the loop and the machine is acting as a motor. The current flow into the machine would be

$$i = \frac{V_B - e_{\text{ind}}}{R} = \left(\frac{48\text{ V} - 45\text{ V}}{0.4\ \Omega}\right) = 7.5\text{ A}$$

2.

SOLUTION From the DC test,

$$2R_1 = \frac{13.5\text{ V}}{64\text{ A}} \qquad \Rightarrow \qquad R_1 = 0.105\ \Omega$$



In the no-load test, the line voltage is 208 V, so the phase voltage is 120 V. Therefore,

$$X_1 + X_M = \frac{V_\phi}{I_{A,\text{nl}}} = \frac{120\text{ V}}{24.0\text{ A}} = 5.00\ \Omega \quad @ \quad 60\text{ Hz}$$

In the locked-rotor test, the line voltage is 24.6 V, so the phase voltage is 14.2 V. From the locked-rotor test *at 15 Hz*,

$$\left|Z'_{\text{LR}}\right| = \left|R_{\text{LR}} + jX'_{\text{LR}}\right| = \frac{V_\phi}{I_{A,\text{LR}}} = \frac{14.2\text{ V}}{64.5\text{ A}} = 0.220\ \Omega$$

$$\theta'_{\text{LR}} = \cos^{-1}\frac{P_{\text{LR}}}{S_{\text{LR}}} = \cos^{-1}\left[\frac{2200\text{ W}}{\sqrt{3}(24.6\text{ V})(64.5\text{ A})}\right] = 36.82°$$

Therefore,

$$R_{\text{LR}} = \left|Z'_{\text{LR}}\right| \cos\theta_{\text{LR}} = (0.220\ \Omega)\cos(36.82°) = 0.176\ \Omega$$

$$\Rightarrow \qquad R_1 + R_2 = 0.176\ \Omega$$

$$\Rightarrow \qquad R_2 = 0.071\ \Omega$$

$$X'_{\text{LR}} = \left|Z'_{\text{LR}}\right| \sin\theta_{\text{LR}} = (0.2202\ \Omega)\sin(36.82°) = 0.132\ \Omega$$

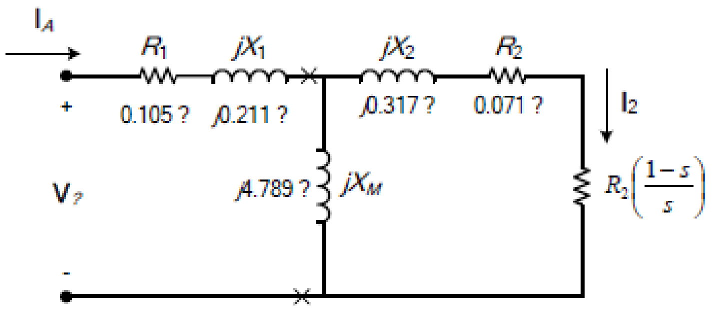
At a frequency of 60 Hz,

$$X_{\text{LR}} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) X'_{\text{LR}} = 0.528\ \Omega$$

For a Design Class B motor, the split is $X_1 = 0.211\ \Omega$ and $X_2 = 0.317\ \Omega$. Therefore,

$$X_M = 5.000\ \Omega - 0.211\ \Omega \ = \ 4.789\ \Omega$$

The resulting equivalent circuit is shown below:



A MATLAB program to calculate the torque-speed characteristic of this motor is shown below:

```
% M-file: prob6_20.m
% M-file create a plot of the torque-speed curve of the
%   induction motor of Problem 6-20.

% First, initialize the values needed in this program.
r1 = 0.105;                   % Stator resistance
x1 = 0.211;                   % Stator reactance
r2 = 0.071;                   % Rotor resistance
x2 = 0.317;                   % Rotor reactance
xm = 4.789;                   % Magnetization branch reactance
v_phase = 208 / sqrt(3);      % Phase voltage
n_sync = 1200;                % Synchronous speed (r/min)
w_sync = 125.7;               % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 6-41a and 6-43.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;             % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;         % Mechanical speed

% Calculate torque versus speed
for ii = 1:51
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
           (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}');
ylabel('\bf\tau_{ind}');
title ('\bfInduction Motor Torque-Speed Characteristic');
```

```
grid on;
```

The resulting plot is shown below: