Calculus A(2) Midterm

23/04/15

A

3 marks

Find equations for the following planes.

1. The plane through $(2,4,5)$, $(1,5,7)$, and $(-1,6,8)$.
2. The plane through $P=(1,-2,1)$ perpendicular to the vector from the origin to P.

(2 marks)

3. Find the angle between the planes

$$z+\sqrt{2}y-x=0, \quad z=x.$$

(1 marks)

B

3 marks

1. If $\underline{u}_1$ and $\underline{u}_2$ are orthogonal unit vectors and

$$\underline{v}=a\underline{u}_1+b\underline{u}_2,$$

find $\underline{v}.\underline{u}_1$.

2. Does $\underline{u}.\underline{v}_1=\underline{u}.\underline{v}_2$ with $\underline{u}\neq\underline{0}$ imply $\underline{v}_1=\underline{v}_2$? Give a reason.
3. Assume $\underline{w}_1+\underline{w}_2$ and $\underline{w}_1-\underline{w}_2$ are both non-zero. When are they orthogonal?

C

2 marks

For vectors

$$\begin{aligned}\underline{u}&=\underline{i}-\underline{j}+\underline{k}\\ \underline{v}&=2\underline{i}+\underline{j}-2\underline{k}\\ \underline{w}&=-\underline{i}+2\underline{j}-\underline{k}\end{aligned}$$

verify that the following holds.

$$(\underline{u}\times\underline{v}).\underline{w}=(\underline{v}\times\underline{w}).\underline{u}$$

D 6 marks

For the functions $f(x, y)$ given by

1. y/x^2
2. $\sqrt{x+y}$
3. $\tan^{-1}(y/x)$

find

a. the domain,

b. the range,

say if the domain is

c. closed/open/neither, bounded/unbounded,

and

d. sketch some level curves.

E 3 marks

A flat plate has shape $R = \{x^2 + y^2 \le 1\}$. The temperature on the plate is

$$T(x, y) = x^2 + 2y^2 - x.$$

Find the hottest and coldest points on the plate, including the boundary, and the temperatures there.

absolute.

F 3 marks

The Laplace equation for a function $f(x, y)$ is

$$f_{xx} + f_{yy} = 0.$$

Show that the following functions satisfy it.

1. $x^2 - y^2$
2. $\ln \sqrt{x^2 + y^2}$

Show that if $z = g(u, v)$ satisfies $g_{uu} + g_{vv} = 0$ and

$$u = (x^2 - y^2)/2, \quad v = xy$$

then z satisfies $z_{xx} + z_{yy} = 0$.

G 4 marks

Sketch the curve $f(x, y) = c$ together with ∇f and the tangent line at P. Write an equation for the tangent line.

1. $x^2 - y = 1$, $P = (\sqrt{2}, 1)$
2. $xy = -4$, $P = (2, -2)$

H 4 marks

Recall that arc length is given by the formula

$$s = \int_a^b |\underline{v}(t)|\, dt.$$

Find the length of the following curves.

1. $\underline{r}(t) = (e^t \cos t, e^t \sin t, e^t)$ for $-\ln 4 \le t \le 0$
2. $\underline{r}(t) = (1 + 2t, 1 + 3t, 6 - 6t)$ for $-1 \le t \le 0$

I

5 marks

Recall that the centroid of a planar region R is given by

$$(\bar{x}, \bar{y}) = \left(\int_R x\,dA, \int_R y\,dA\right) \Big/ \int_R dA.$$

1. Sketch the following region R and find its centroid.

$$R: \quad \begin{array}{l} 0 \le x \le \pi \\ 0 \le y \le \sin x \end{array}$$

(3 marks)

2. Find the centroid of the tetrahedron with vertices as follows.

$$\begin{array}{ll} (0,0,0) & (1,0,0) \\ (0,1,0) & (0,0,1) \end{array}$$

(2 marks)

J

7 marks

Integrate the following.

1.
$$\int_{-1}^{0}\int_{-1}^{1} x + y + 1\,dx\,dy$$

2.
$$\int_{\pi}^{2\pi}\int_{0}^{\pi} \sin x + \cos y\,dx\,dy$$

3. $f(x, y) = x^2 + y^2$ over the triangle with vertices

$$(0,0), \quad (1,0), \quad (0,1).$$

(3 marks)

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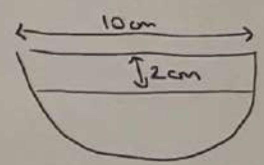
4. A hemispherical bowl, diameter 10cm, is filled with water to 2cm from the top. Find the volume of water in the bowl.

(2 marks)

side view:



5. For the planar region

$$R: \quad \begin{array}{l} a \le x \le b \\ c \le y \le d \end{array}$$

we have

$$\iint_R f(x)g(y)\,dA = \int_a^b f(x)\,dx \int_c^d g(y)\,dy.$$

Why?

(2 marks)