第六章习题解答

9. 原题有误。应加上条件$a, b, c, d \in R$, 否则结论不对。例如$w = z = \frac{iz}{i}$, 则w是恒同映射，但$ad - bc = i^2 - 0 = -1 < 0$.

10. 此题也有误。应改$|w| < 1$为$|w| = 1$. 当$z \to \infty$时，有$|w| \to 1$, 故有$|a| = |c|$, $ad - bc \neq 0$.

12. 令$w^1 = w - 1$，则有$|z| < 1 \to |w^1| < 1$, 因而有

$$w^1 = w - 1 = e^{i\theta}\frac{z - z_0}{1 - \overline{z_0}z},$$

即

$$w = 1 + e^{i\theta}\frac{z - z_0}{1 - \overline{z_0}z}, \theta \in R, \ |z_0| < 1.$$

17. 令$w = \frac{(z-i)}{cz+d}$, 分别将$z_1 = 1, \ z_2 = -i$代入，分别得$w_1 = 1, \ w_2 = -1$, 从而得到c, d的值。再将$z = 0$代入，如果$Im(w(0)) = \frac{-i}{d} > 0$, 则映为上复平面，否则为下复平面。

19(3).

$$z_1 = z^4, \ \ z_2 = \frac{z_1 + 2^4}{z_1 - 2^4}, \ \ w = z_2^2 = \left(\frac{z^4 + 2^4}{z^4 - 2^4}\right)^2.$$

19(8).

$$z_1 = \frac{z+2}{2-z}, (ad - bc > 0), \ z_2 = z_1\pi, \ \ z_3 = z_2 i, \ w = e^{z_3} = e^{\pi i\left(\frac{z+2}{2-z}\right)}.$$

19(9).

$$z_1 = \frac{z-a}{b-a}\pi, \ z_2 = z_1 i, \ \ w = e^{z_2} = e^{\frac{z-a}{b-a}\pi i}.$$