H04

A

14.1

Display the values of the functions in Exercises 19–28 in two ways: **(a)** by sketching the surface $z = f(x, y)$ and **(b)** by drawing an assortment of level curves in the function's domain. Label each level curve with its function value.

19. $f(x, y) = y^2$

20. $f(x, y) = 4 - y^2$

21. $f(x, y) = x^2 + y^2$

22. $f(x, y) = \sqrt{x^2 + y^2}$

24. $f(x, y) = 4 - x^2 - y^2$

B

45. The maximum value of a function on a line in space Does the function $f(x, y, z) = xyz$ have a maximum value on the line $x = 20 - t, y = t, z = 20$? If so, what is it? Give reasons for your answer. (*Hint:* Along the line, $w = f(x, y, z)$ is a differentiable function of t.)

46. The minimum value of a function on a line in space Does the function $f(x, y, z) = xy - z$ have a minimum value on the line $x = t - 1, y = t - 2, z = t + 7$? If so, what is it? Give reasons for your answer. (*Hint:* Along the line, $w = f(x, y, z)$ is a differentiable function of t.)

C

14.2

Find the limits in Exercises 1–12.

1. $\displaystyle \lim_{(x, y)\to(0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$

2. $\displaystyle \lim_{(x, y)\to(0,4)} \frac{x}{\sqrt{y}}$

3. $\displaystyle \lim_{(x, y)\to(3,4)} \sqrt{x^2 + y^2 - 1}$

4. $\displaystyle \lim_{(x, y)\to(2,-3)} \left(\frac{1}{x} + \frac{1}{y}\right)^2$

5. $\displaystyle \lim_{(x, y)\to(0,\pi/4)} \sec x \tan y$

D

Find the limits in Exercises 13–20 by rewriting the fractions first.

13. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2 - 2xy + y^2}{x - y}$ **14.** $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2 - y^2}{x - y}$

15. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy - y - 2x + 2}{x - 1}$

16. $\displaystyle\lim_{\substack{(x,y)\to(2,-4)\\ y\neq -4,\, x\neq x^2}} \frac{y + 4}{x^2y - xy + 4x^2 - 4x}$

17. $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$

E

At what points (x, y) in the plane are the functions in Exercises 27–30 continuous?

27. a. $f(x, y) = \sin(x + y)$ **b.** $f(x, y) = \ln(x^2 + y^2)$

28. a. $f(x, y) = \dfrac{x + y}{x - y}$ **b.** $f(x, y) = \dfrac{y}{x^2 + 1}$

At what points (x, y, z) in space are the functions in Exercises 31–34 continuous?

31. a. $f(x, y, z) = x^2 + y^2 - 2z^2$

b. $f(x, y, z) = \sqrt{x^2 + y^2 - 1}$

32. a. $f(x, y, z) = \ln xyz$ **b.** $f(x, y, z) = e^{x+y}\cos z$

50. Continuous extension Define $f(0, 0)$ in a way that extends

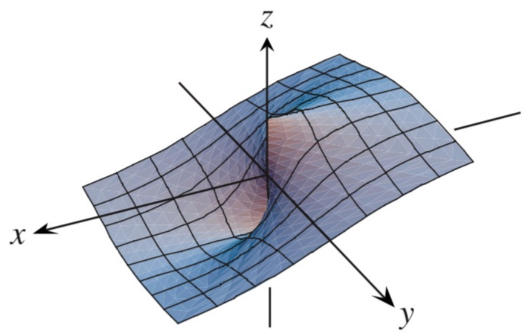
$$f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}$$

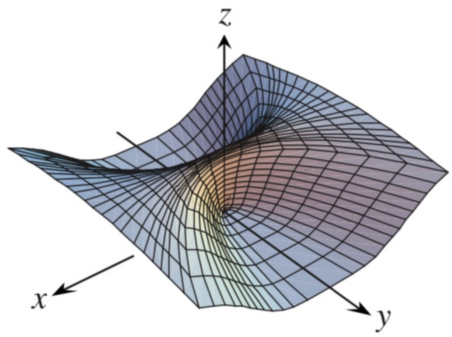
to be continuous at the origin.

F

By considering different paths of approach, show that the functions in Exercises 35–42 have no limit as $(x, y) \to (0, 0)$.

35. $f(x, y) = -\dfrac{x}{\sqrt{x^2 + y^2}}$ **36.** $f(x, y) = \dfrac{x^4}{x^4 + y^2}$





37. $f(x, y) = \dfrac{x^4 - y^2}{x^4 + y^2}$ **38.** $f(x, y) = \dfrac{xy}{|xy|}$

39. $g(x, y) = \dfrac{x - y}{x + y}$

G

14.3

In Exercises 1–22, find $\partial f/\partial x$ and $\partial f/\partial y$.

1. $f(x, y) = 2x^2 - 3y - 4$ **2.** $f(x, y) = x^2 - xy + y^2$

3. $f(x, y) = (x^2 - 1)(y + 2)$

4. $f(x, y) = 5xy - 7x^2 - y^2 + 3x - 6y + 2$

5. $f(x, y) = (xy - 1)^2$ **6.** $f(x, y) = (2x - 3y)^3$

7. $f(x, y) = \sqrt{x^2 + y^2}$ **8.** $f(x, y) = (x^3 + (y/2))^{2/3}$

9. $f(x, y) = 1/(x + y)$ **10.** $f(x, y) = x/(x^2 + y^2)$

H

Find all the second-order partial derivatives of the functions in Exercises 41–46.

41. $f(x, y) = x + y + xy$ **42.** $f(x, y) = \sin xy$

43. $g(x, y) = x^2y + \cos y + y \sin x$

44. $h(x, y) = xe^y + y + 1$ **45.** $r(x, y) = \ln (x + y)$