H06

A

14.6

In Exercises 1–8, find equations for the

(a) tangent plane and **(b)** normal line at the point P_0 on the given surface.

1. $x^2 + y^2 + z^2 = 3, \quad P_0(1, 1, 1)$

2. $x^2 + y^2 - z^2 = 18, \quad P_0(3, 5, -4)$

3. $2z - x^2 = 0, \quad P_0(2, 0, 2)$

In Exercises 13–18, find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

13. Surfaces: $x + y^2 + 2z = 4, \quad x = 1$

Point: $(1, 1, 1)$

14. Surfaces: $xyz = 1, \quad x^2 + 2y^2 + 3z^2 = 6$

Point: $(1, 1, 1)$

B

19. By about how much will

$$f(x, y, z) = \ln\sqrt{x^2 + y^2 + z^2}$$

change if the point $P(x, y, z)$ moves from $P_0(3, 4, 12)$ a distance of $ds = 0.1$ unit in the direction of $3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$?

20. By about how much will

$$f(x, y, z) = e^x \cos yz$$

change as the point $P(x, y, z)$ moves from the origin a distance of $ds = 0.1$ unit in the direction of $2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$?

21. By about how much will

$$g(x, y, z) = x + x\cos z - y\sin z + y$$

change if the point $P(x, y, z)$ moves from $P_0(2, -1, 0)$ a distance of $ds = 0.2$ unit toward the point $P_1(0, 1, 2)$?

24. Changing temperature along a space curve The Celsius temperature in a region in space is given by $T(x, y, z) = 2x^2 - xyz$. A particle is moving in this region and its position at time t is given by $x = 2t^2, y = 3t, z = -t^2$, where time is measured in seconds and distance in meters.

a. How fast is the temperature experienced by the particle changing in degrees Celsius per meter when the particle is at the point $P(8, 6, -4)$?

b. How fast is the temperature experienced by the particle changing in degrees Celsius per second at P?

C

In Exercises 25–30, find the linearization $L(x, y)$ of the function at each point.

25. $f(x, y) = x^2 + y^2 + 1$ at **a.** $(0, 0)$, **b.** $(1, 1)$

26. $f(x, y) = (x + y + 2)^2$ at **a.** $(0, 0)$, **b.** $(1, 2)$

27. $f(x, y) = 3x - 4y + 5$ at **a.** $(0, 0)$, **b.** $(1, 1)$

Find the linearizations $L(x, y, z)$ of the functions in Exercises 37–42 at the given points.

41. $f(x, y, z) = e^x + \cos(y + z)$ at

a. $(0, 0, 0)$ **b.** $\left(0, \frac{\pi}{2}, 0\right)$ **c.** $\left(0, \frac{\pi}{4}, \frac{\pi}{4}\right)$

42. $f(x, y, z) = \tan^{-1}(xyz)$ at

a. $(1, 0, 0)$ **b.** $(1, 1, 0)$ **c.** $(1, 1, 1)$

D

14.7

Find all the local maxima, local minima, and saddle points of the functions in Exercises 1–30.

1. $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$

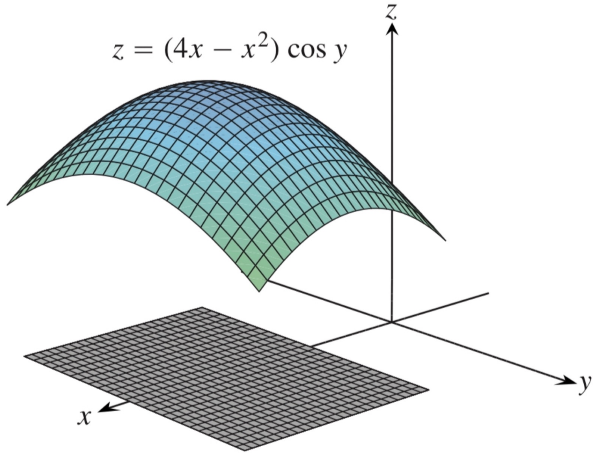
28. $f(x, y) = \frac{1}{x} + xy + \frac{1}{y}$

30. $f(x, y) = e^{2x} \cos y$

In Exercises 31–38, find the absolute maxima and minima of the functions on the given domains.

31. $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0, y = 2, y = 2x$ in the first quadrant

37. $f(x, y) = (4x - x^2) \cos y$ on the rectangular plate $1 \le x \le 3$, $-\pi/4 \le y \le \pi/4$ (see accompanying figure).



E

42. Find the critical point of

$$f(x, y) = xy + 2x - \ln x^2 y$$

in the open first quadrant ($x > 0, y > 0$) and show that f takes on a minimum there (Figure 14.47).

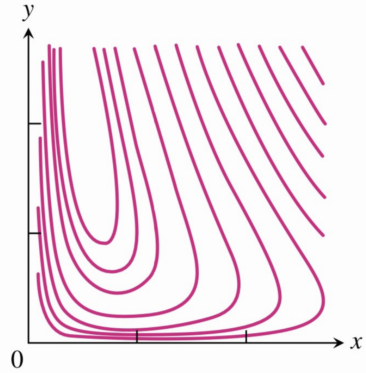


FIGURE 14.47 The function $f(x, y) = xy + 2x - \ln x^2 y$ (selected level curves shown here) takes on a minimum value somewhere in the open first quadrant $x > 0, y > 0$ (Exercise 42).

44. The discriminant $f_{xx} f_{yy} - f_{xy}{}^2$ is zero at the origin for each of the following functions, so the Second Derivative Test fails there. Determine whether the function has a maximum, a minimum, or neither at the origin by imagining what the surface $z = f(x, y)$ looks like. Describe your reasoning in each case.

c. $f(x, y) = xy^2$

e. $f(x, y) = x^3 y^3$

50. Find the point on the graph of $z = x^2 + y^2 + 10$ nearest the plane $x + 2y - z = 0$.

F

15.1

In Exercises 1–10, sketch the region of integration and evaluate the integral.

1. $\displaystyle\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$

2. $\displaystyle\int_0^3 \int_{-2}^0 (x^2 y - 2xy)\, dy\, dx$

5. $\displaystyle\int_0^{\pi} \int_0^{x} x \sin y\, dy\, dx$

6. $\displaystyle\int_0^{\pi} \int_0^{\sin x} y\, dy\, dx$

In Exercises 11–16, integrate f over the given region.

11. Quadrilateral $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x, y = 2x, x = 1, x = 2$

G

60. Converting to a double integral Evaluate the integral

$$\int_0^2 (\tan^{-1}\pi x - \tan^{-1} x)\, dx.$$

(*Hint:* Write the integrand as an integral.)

61. Maximizing a double integral What region R in the xy-plane maximizes the value of

$$\iint_R (4 - x^2 - 2y^2)\, dA?$$

Give reasons for your answer.

62. Minimizing a double integral What region R in the xy-plane minimizes the value of

$$\iint_R (x^2 + y^2 - 9)\, dA?$$

Give reasons for your answer.

H

In Exercises 21–30, sketch the region of integration and write an equivalent double integral with the order of integration reversed.

21. $\displaystyle\int_0^1 \int_2^{4-2x} dy\, dx$

26. $\displaystyle\int_0^{\ln 2} \int_{e^x}^{2} dx\, dy$

29. $\displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\, dx\, dy$

In Exercises 31–40, sketch the region of integration, reverse the order of integration, and evaluate the integral.

39. Square region $\iint_R (y - 2x^2)\, dA$ where R is the region bounded by the square $|x| + |y| = 1$

40. Triangular region $\iint_R xy\, dA$ where R is the region bounded by the lines $y = x$, $y = 2x$, and $x + y = 2$