Problem Discussion

(a) Explain the excitation branch in the exact equivalent circuit of a transformer by describing the reason for all currents flowing in it.

(b) A single-phase, 10-kVA, 200:400-V, 60-Hz transformer has the following test results:

Open-circuit test (HV side open): 200 V, 3.2 A, 450 W
Short-circuit test (LV side shorted): 38 V, 25 A, 600 W

i. Draw the equivalent circuit of the transformer referred to the high-voltage side. Label impedances numerically in ohms and in per unit.

ii. Determine the voltage regulation at rated primary current with 0.6 power factor leading when the load is supplied with rated voltage.

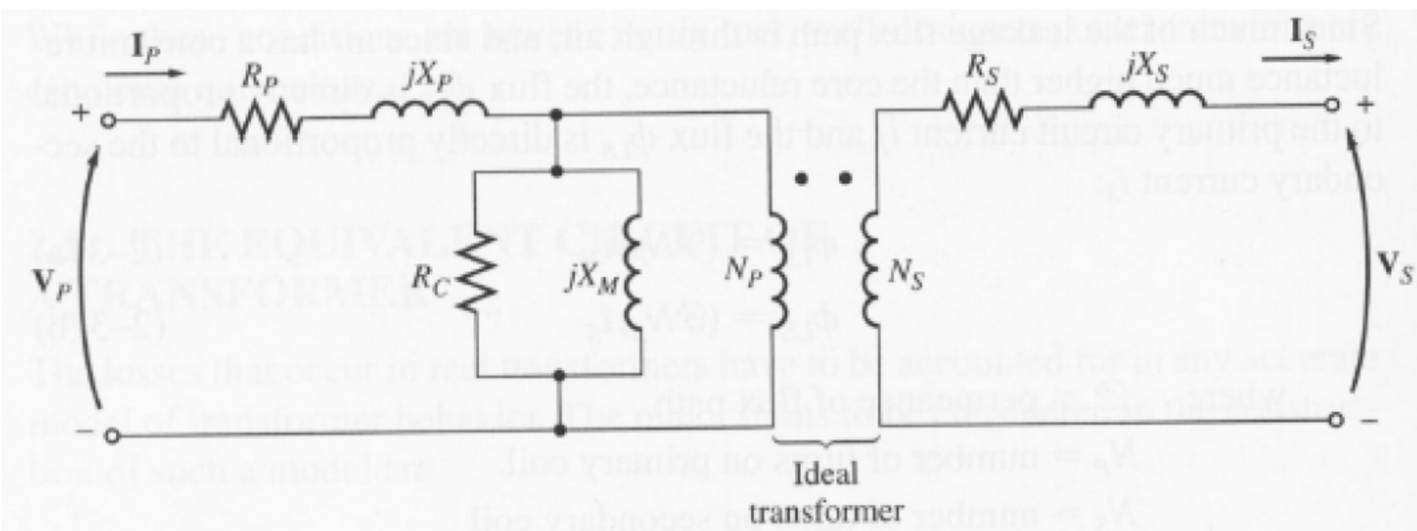
iii. Determine the efficiency of the transformer when the secondary current is at its rated value, load power factor is 0.8 lagging and the primary is supplied with rated voltage.

Part (a)

When an ac power source is connected to the primary of a transformer, a current flows in its primary circuit, even when there is no current in the secondary. The transformer is said to be on no-load. If the secondary current is zero, the primary current should be zero too. However, when the transformer is on no-load, excitation current flows in the primary because of the core losses and the finite permeability of the core.

Modeling the core excitation: I_m is proportional to the voltage applied to the core and lags the applied voltage by 90°. It is modeled by X_M.

Modeling the core loss current: I_{h+e} is proportional to the voltage applied to the core and in phase with the applied voltage. It is modeled by R_C.



2-b-1 $S_{base} = 10\,kVA$ $V_{base} = 400V$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{400^2}{10\,k} = 16\,\Omega$$

Open circuit: in primary

$$|Y_{oc}| = \frac{I_{oc}}{V_{oc}} = \frac{3.2}{200} = 0.016$$

$$\angle Y_{oc} = -\cos^{-1}\frac{P_{oc}}{V_{oc} I_{oc}} = -\cos^{-1}\frac{450}{3.2 \cdot 200} = -45.33^{\circ}$$

$$Y_{oc} = 0.016\angle -45.33 = (0.0112 - j0.0113)\,mho$$

$$\Rightarrow R_{Cp} = \frac{1}{0.0112} = 89.28\,\Omega$$ R_{Cp}: $R_{C(primary)}$

$$X_{mp} = \frac{1}{0.0113} = 88.49\,\Omega$$ X_{mp} $X_{m(primary)}$

referred to secondary

$$Z_s = \frac{Z_p}{a^2} \qquad a = \frac{V_p}{V_s} = 0.5$$

$$\Rightarrow Z_s = 4Z_p \Rightarrow R_{C_{secondary}} = 357.12\,\Omega$$

$$X_m = 88.49 \times 4 = 353.96$$

$$R_{C\,pu} = \frac{R_{Cs}}{Z_{base}} = \frac{357.12}{16} = 22.32\ p.u.$$

$$X_{m\,pu} = \frac{X_{ms}}{Z_{base}} = \frac{353.96}{16} = 22.122\ p.u.$$

Short circuit test. Secondary

$$|Z_{SC}| = \frac{V_{SC}}{I_{SC}} = \frac{38}{25} = 1.52$$

$$\angle Z_{SC} = \cos^{-1} \frac{P_{SC}}{V_{SC} I_{SC}} = 50.83^\circ$$

$$Z_{SC} = 1.52 \angle 50.83 = (.96 + j1.178)\ \Omega$$

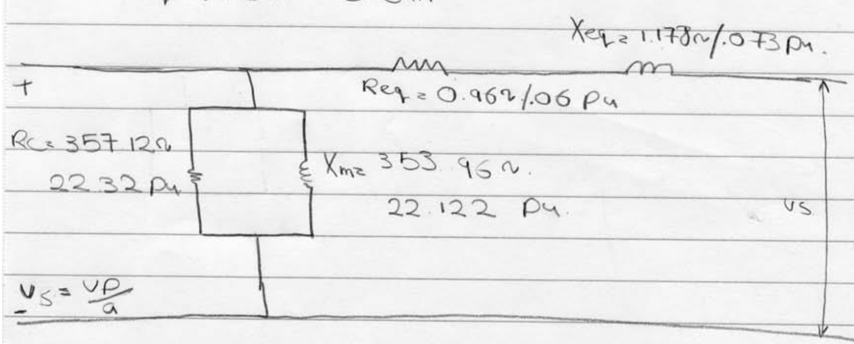
$\Rightarrow$ $R_{eq} = 0.96\ \Omega$

$X_{eq} = 1.178\ \Omega$

$$R_{eq\,pu} = \frac{R_{eq}}{Z_{base}} = \frac{.96}{16} = 0.06\ pu$$

$$X_{eq\,pu} = \frac{X_{eq}}{Z_{base}} = \frac{1.178}{16} = 0.073\ pu$$

The equivalent circuit



When circuit is referred to primary.

$V_{FL} = 200V$

$\cos\theta = 0.6$

$\theta = \cos^{-1}(0.6) = 53.13°$

$|I_2| = \frac{P}{V} = \frac{10000}{200} = 50A$

$I_2 = 50 \angle 53.13° \ A.$

$V_{NL} = 200 + 50\angle 53.13 \left(0.239 + j.295\right)$

$= 196.24 \angle 5.38 \ V.$

Voltage regulation $= \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$

$= \frac{196.24 - 200}{200} \times 100$

$= -1.88\%.$

When circuit is referred to secondary.

$V_{FL} = 400V.$

$\cos\theta = 0.6$

$\theta = \cos^{-1}(0.6) = 53.13$

$|I_2| = \frac{P}{V} = \frac{10000}{400} = 25 \ A.$

$I_2 = 25 \angle 53.13^\circ \ A$

$$V_{NL} = 400 + 25\angle 53.13^\circ (0.957 + 1.18j)$$
$$= 392.48 \angle 5.38^\circ$$

$$\text{Voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$
$$= \frac{392.48 - 400}{400} \times 100$$
$$= -1.88\%.$$

When circuit referred to primary.

$$|I_2| = \frac{10000}{200} = 50\ A$$

$\cos\theta = 0.8$

$\theta = \cos^{-1}(0.8) = -36.87°$ (-ve because lagging).

$I_2 = 50 \angle -36.87°\ A.$

$$I_c = \frac{400}{89.29} = 2.24A.$$

$$I_M = \frac{200}{88.49j} = -2.26j\ A.$$

$$P_{Loss} = I_c^2 R_c + I_2^2 R_{eq}$$
$$= (2.24)^2 (89.29) + (50\angle -36.87°)^2 (0.239)$$
$$= 841.21 \angle -42.99°\ W.$$

$$I_1 = I_2 + I_c + I_M$$
$$= 50\angle -36.87° + 2.24 - 2.26j$$
$$= 53.15 \angle -37.37°$$

$$P_{in} = V |I_1| \cos\theta$$
$$= \cancel{200 \times 53.15 \angle -37.37°}$$
$$= 200 \times 53.15 \times 0.8$$
$$= 8504\ W.$$

$$\eta = \frac{P_{in} - P_{Loss}}{P_{in}} \times 100 = \frac{8504 - 841.21}{8504} \times 100 = 90.1\%$$

When circuit is referred to secondary

$$|I_2| = \frac{10000}{400} = 25A.$$

$\cos\theta = 0.8$

$\theta = \cos^{-1}(0.8) = -36.87°$ (-ve because lagging).

$I_2 = 25 \angle -36.87°$ A.

$$I_c = \frac{400}{357.16} = 1.12A.$$

$$I_M = \frac{400}{353.96j} = -1.3j$$

$$P_{Loss} = I_c^2 R_c + I_2^2 R_{eq}.$$
$$= (1.12)^2 (357.16) + (25 \angle -36.87°)^2 (0.957)$$
$$= 841.75 \angle -43.01° \text{ W}$$

$$I_1 = I_2 + I_c + I_M$$
$$= 25 \angle -36.87° + 1.12 + -1.3j$$
$$= 26.68 \angle -37.66 \text{ A}.$$

$P_{in} = V|I_1| \cos\theta = 400 \times 26.68 \times 0.8 = 8537.6$ W.

$$\eta = \frac{P_{in} - P_{Loss}}{P_{in}} \times 100$$
$$= \frac{8537.6 - 841.75}{8537.6} \times 100$$
$$= 90.1 \%.$$

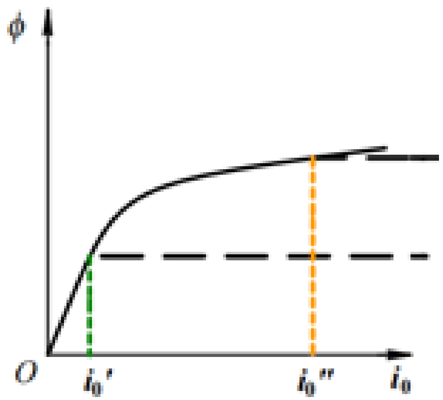
A single-phase transformer is rated at 220V 50Hz. We now have it connected to a 240V 60Hz power supply, how do these parameters change? Primary leakage inductance_________________, mutual inductance_________________, core loss_________________. Assume the original transformer has 1100 turns on the primary side, and we reduce it to 1000 turns before connecting it to the 240V 60Hz power supply, how do these parameters change compared to the 220V/50Hz case? Primary leakage inductance_________________, mutual inductance_________________, core loss_________________.(12 points)

$$p_{core} \propto f^{1.3} B^2$$

$L_{\sigma 1}$ does not change

$$E = 4.44 fN\Phi$$

$$\frac{\Phi'}{\Phi} = \frac{24 \times 5}{22 \times 6} = \frac{10}{11}$$



μ increases

$L_M \propto \mu N^2$, increases

$p_{core} \propto f^{1.3} B^2 = 1.048$, increases

if N=1000,

$L_{\sigma 1}$ decreases

$$\frac{\Phi'}{\Phi} = \frac{24 \times 5 \times 1100}{22 \times 6 \times 1000} = 1,$$ does not change

μ does not change

$L_M \propto \mu N^2$ decreases

$p_{core} \propto f^{1.3} B^2 = 1.2^{1.3}$ increases

For a 4-pole, three-phase, salient-pole-rotor synchronous machine, the rated speed is 1500 RPM. There are a total of 24 slots on the stator.

1. What is the electrical frequency of the machine.

2. If we need to use single layer winding, what is the winding factor?

3. If we decide to use double layer winding, how would you design the winding to reduce both 5^{th} and 7^{th} harmonics?

4. Assume there can be a total of 12 turns per slot, and a total of two parallel connections per phase. Based on your design, what is the flux needed to generated 300 Volts (fundamental component) open-loop voltage per phase at the output?

5. Assume the air gap is evenly distributed and the machine is 5 cm long, the flux density in the airgap is 1 T, what is the required diameter of the rotor?

1. $f = \frac{np}{60} = \frac{1500\times 2}{60} = 50Hz$

2. $\alpha = \frac{2\pi p}{Q} = \frac{\pi}{6}$, $q = \frac{Q}{2p\times 3} = \frac{24}{2\times 3\times 2} = 2$

$$K_b = \frac{\sin q\frac{\alpha}{2}}{q\sin\frac{\alpha}{2}} = 0.9659$$

3. $y = \frac{5}{6}$

4. $N_K = 12,\ a = 2,\ p = 2,\ q = 2,$

$$N_c = \frac{2pqN_k}{a} = 48,\quad K_p = \sin\left(y\frac{\pi}{2}\right) = 0.9659$$

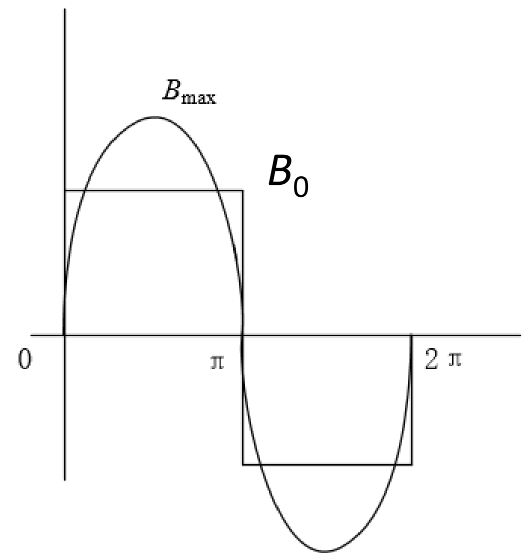
$$\phi = \frac{E}{4.44 f N_c K_b K_p} = \frac{300}{4.44\times 50\times 48\times 0.9659^2} = 0.0302\,wb$$

Correction：
Nk=6

$$\int BdS = \frac{2}{\pi} B_{\max} L \frac{\pi d}{4} = \Phi$$

$$B_{\max} = \frac{4}{\pi} B_0$$

$$d = \frac{\pi\Phi}{2B_0 L}$$



For the same machine with the winding design you chose,

1. Assume the rms of the stator current is 100 Amps, what is the peak value of the mmf generated per phase?

2. What is the magnitude of the fifth space harmonic mmf? And what frequency does it vary at? At what speed does it rotate in the airgap?

3. If the stator current has 100Amps RMS in fundamental frequency and 10 Amps RMS in third harmonic frequency, what is the peak value of the total mmf ?

4. At what speed does the 5^{th} space harmonic mmf caused by the third harmonic current rotate in the air gap?

1. $f_m = \frac{1}{2} q N_k i_m = \frac{1}{2} \times 2 \times 12 \times \sqrt{2} \times 100 = 1697.06\ A \cdot t$

2. $F_5 = \frac{1}{5} \times 1.35 \times \frac{N_k K_{b5} K_{p5} I}{P}$

$K_{b5} = \frac{\sin\left(2 \times \frac{5 \times \frac{\pi}{6}}{2}\right)}{2 \sin\left(\frac{5 \times \frac{\pi}{6}}{2}\right)} = 0.2588$ $\quad K_{p5} = \sin\left(5 \times \frac{5}{6} \times \frac{\pi}{2}\right)$

$= 0.2588$

$\therefore F_5 = 43.40\ A \cdot t$

又 $\because f_5 = F_5 \cos(5\alpha + \omega t)$, $\quad \therefore \alpha = -\frac{\omega t}{5}$

$\therefore f_5 = 50\ Hz$, $\omega_5 = -\frac{\omega}{5} = -\frac{2\pi f}{5} = -20\pi\ rad/s$

3. $f_m = \frac{1}{2} q N_k i_m = \frac{1}{2} \times 2 \times 12 \times \sqrt{2} \times 110$

$= 1866.7\ A \cdot t$

4.

$\cos 3\omega t \cdot \cos 5\alpha$

$= \frac{1}{2} [\cos(3\omega t + 5\alpha) + \cos(3\omega t - 5\alpha)]$

$\omega = \frac{3\omega}{5}$

For a four-pole three-phase synchronous machine, the stator winding is double-layer, the phase band is 60° and the parallel branch number $a = 1$. There are 48 coils in the stator in total. The pitch of the coil can largely reduce the 5th and 7th harmonics of EMF.

(1) The slot number Q= ____________, the slot number per pole per phase of this winding q=__________, the pitch of the coil y_1 = _____________

(2) The slots of the stator are consecutively numbered in a clockwise direction, and the reference direction of the induced EMF of each coil is defined in the same way. When the non-sinusoidally distributed air gap MMF rotates clockwise, the induced EMFs in the upper-layer coil in the No.4 and No.8 slot are e_{K4} and e_{K8}. The phase angle difference between the fundamental component of e_{K4} and e_{K8} is ____________________, The phase angle difference between the phase angle of the 3rd harmonics in the two coils is __________________

(3) If the RMS values of the fundamental component and 5th harmonics of the induced EMF induced by the air-gap MMF in one phase group per pole are 200V and 2V respectively, the ratio of the fundamental component to 5th harmonics of the **flux per pole** is ____________, the ratio of the fundamental component to 5th harmonics of the amplitude of the **flux density** is ___________.

(4) If an 8-pole rotor is installed inside the stator by mistake, and the amplitude of the fundamental flux density is reduced to 1/4 of the original value, the RMS value of the fundamental component in the induced EMF in one phase stator winding changes to ____________ times of the original value.

(1) Q=48， q=48/4/3=4,y=5/6

(2) ek4 leads ek8 π/3 ; ek4 leads ek8 π;

For each slot has electric angle difference of 360/48*2=15°

(3)

$$\frac{\Phi_1}{\Phi_5} = \frac{5E_1 k_{dp5}}{E_5 k_{dp1}} = 28.72 \qquad \frac{B_1}{B_5} = \frac{\Phi_1 \frac{S}{5}}{\Phi_5 S} = 5.745$$

(4) 0