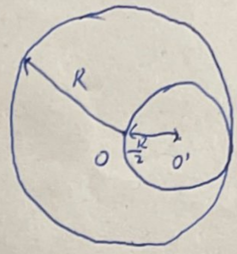
第五章

5.10 已知：半径R，圆洞中心距离原薄板 $\frac{R}{2}$，所剩质量为m.

求：J.



解：$J_1 = \frac{1}{2}m_1(\frac{R}{2})^2 + m_1(\frac{R}{2})^2$

$= \frac{3}{2}m_1(\frac{R}{2})^2$ ／ $m_1 = \pi(\frac{R}{2})^2 a\rho$ a—厚度 ρ—密度

$= \frac{3}{2}\pi(\frac{R}{2})^2 a\rho(\frac{R}{2})^2$

$= \frac{3}{32}\pi a\rho R^4$

$J_2 = \frac{1}{2}m_2R^2$ ／ $m_2 = \pi R^2 a\rho$

$= \frac{1}{2}\pi a\rho R^4$

$J = J_2 - J_1 = \frac{1}{2}\pi a\rho R^4 - \frac{3}{32}\pi a\rho R^4$

$= \frac{13}{32}\pi a\rho R^4$

因为 $m = [\pi R^2 - \pi(\frac{R}{2})^2]a\rho$

$m = \pi R^2 a\rho - \frac{\pi R^2}{4}a\rho$

$m = \frac{3\pi R^2 a\rho}{4}$

$\pi R^2 a\rho = \frac{4m}{3}$

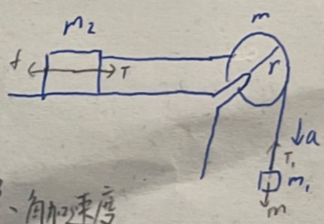
代入 $J = \frac{13}{32}\pi a\rho R^4$

$= \frac{13}{32}\cdot\frac{4m}{3}R^2$

$= \frac{13}{24}mR^2$

5.11. 已知：质量为 m_1, m_2, m，半径为r. m_2与桌面间摩擦系数为 μ_k，求

求：a, T_1, T_2



解：对m_1受力分析：

$m_1g - T_1 = m_1a$

对m_2受力分析

$T_2 - \mu_k m_2 g = m_2 a$

对滑轮

根据转动定理：

$(T_1 - T_2)r = \frac{1}{2}mr^2\beta$ β、角加速度

因为 $a = r\beta$

联立上式：

$$\begin{cases} T_1 = m_1g - m_1a \\ T_2 = m_2a + \mu_k m_2 g \\ (T_1 - T_2)r = \frac{1}{2}mr^2\beta \\ \beta = \frac{a}{r} \end{cases} \Rightarrow$$

$m_1g - m_1a - m_2a - \mu_k m_2 g = \frac{1}{2}mr\frac{a}{r}$

$-m_1a - m_2a - \frac{1}{2}ma = \mu_k m_2 g - m_1 g$

$a = \frac{m_1 - \mu_k m_2}{m_1 + m_2 + \frac{m}{2}}g$

$T_1 = m_1g - m_1a$

$T_1 = m_1(g - \frac{m_1 - \mu_k m_2}{m_1 + m_2 + \frac{m}{2}}g)$

$T_1 = m_1g\frac{(1+\mu_k)m_2 + \frac{m}{2}}{m_1 + m_2 + \frac{m}{2}}$

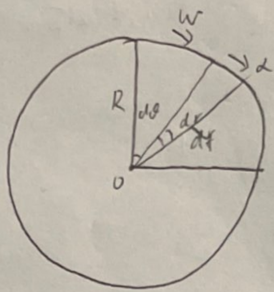
$T_2 = m_2a + \mu_k m_2 g$

$= m_2(\frac{m_1 - \mu_k m_2}{m_1 + m_2 + \frac{m}{2}}g + \mu_k g)$

$= m_2g\frac{(1+\mu_k)m_1 + \frac{m}{2}}{m_1 + m_2 + \frac{m}{2}}$

5.14. 已知半径为R，质量为m，滑动摩擦系数为μ_k，角速度为ω.

求：M, t, A, E_k



解. 设元面积为$ds = rd\theta dr$.

质量为$dm = \frac{mrd\theta dr}{\pi R^2}$

$$dM = rdf = r\mu_k dmg = \frac{mg\mu_k r^2 d\theta dr}{\pi R^2}$$

① $M = \int dM = \frac{\mu_k mg}{\pi R^2}\int_0^{2\pi} d\theta \int_0^R r^2 dr = \frac{\mu_k mg}{\pi R^2}\cdot 2\pi \cdot \frac{1}{3}R^3 = \frac{2}{3}\mu_k mgR$

② $t = \frac{\omega}{\alpha} = \frac{\omega}{\frac{M}{\frac{1}{2}mR^2}} = \frac{\omega}{\frac{\frac{2}{3}\mu_k mgR}{\frac{1}{2}mR^2}} = \frac{3R\omega}{4\mu_k g}$

$M = J\alpha$　　转动惯量

③ $A = M\cdot\Delta\theta = M\cdot\omega t = \frac{2}{3}\mu_k mgR\cdot\omega\cdot\frac{3R\omega}{4\mu_k g} = \frac{1}{2}mR^2\omega^2$

④ $E_k = \frac{1}{2}J\omega^2 = \frac{1}{2}(\frac{1}{2}mR^2)\omega = \frac{1}{4}mR^2\omega^2$

5.16. 已知$L = 0.4m$, $M = 1\ kg$, $m = 8g$, $v_0 = 200\ m/s.$, $d = \frac{3L}{4}$

求：ω, θ

解. 根据角动量守恒 $L = r\times p$, $L = J\omega$

$$mv_0\times\frac{3}{4}L = \left[\frac{1}{3}ML^2 + (\frac{3L}{4})^2\right]\omega$$

$$\omega = \frac{3mv_0L}{4(\frac{1}{3}ML^2+\frac{9}{16}L^2)}$$

$$\omega = \frac{3\times0.008\times200\times0.4}{4\times(\frac{1}{3}\times1\times0.4^2+\frac{9}{16}\times0.008\times0.4^2)}$$

$$\omega = 8.88(rad/s)$$

根据机械能守恒：

$$\frac{1}{2}(\frac{1}{3}ML^2+\frac{9}{16}mL^2)\omega^2 = (Mg\frac{L}{2}+mg\frac{3}{4}L)(1-\cos\theta)$$

$$\theta = \arccos\left(1-\frac{\frac{1}{2}(\frac{1}{3}ML^2+\frac{9}{16}mL^2)\omega^2}{Mg\frac{L}{2}+mg\frac{3}{4}L}\right)$$

$$\theta = 94°18'$$

5.19. 已知：$R=2.5m$, $J=3\times10^5 kg\,m^2$, $m=70kg$, $\theta=30°$.

求：v, w, n.

解.

$$mg=\frac{mv^2}{r}$$

$$v=\sqrt{gr}$$

$$v=\sqrt{9.8\times2.5}$$

$$v=4.95(m/s)$$

根据角动量守恒

$$3mvR=Jw$$

$$w=\frac{3mvR}{J}$$

$$w=\frac{3\times70\times4.95\times2.5}{3\times10^5}$$

$$w=8.66\times10^{-3}(rad/s)$$

时间 $t=\frac{\pi}{6w}$，角速度 $=w+\frac{v}{R}$

$$n=\frac{(w+\frac{v}{R})\cdot t}{2\pi}$$

$$n=\frac{(8.66\times10^{-3}+\frac{4.95}{2.5})\cdot\frac{\pi}{6\times8.66\times10^{-3}}}{2\pi}$$

$$n=19.14(圈)$$

5.21 已知：$T=0.033$, $dT=1.26\times10^{-5}\,s/a$

求：α, $\frac{dE}{dt}$, t.

$$\alpha=\frac{dw}{dt}=\frac{d}{dt}\frac{2\pi}{T}=-\frac{2\pi}{T^2}\frac{dT}{dt}=-\frac{2\pi}{0.033^2}\times\frac{1.26\times10^5}{3.15\times10^7}=-2.3\times10^{-9}(rad/s^2)$$

$$\frac{dE}{dt}=\frac{d}{dt}(\frac{1}{2}Jw^2)=\frac{1}{2}J\times2w\frac{dw}{dt}=\frac{2}{5}mR^2w\frac{dw}{dt}$$

$$=\frac{2}{5}\times1.5\times10^{30}\times(10^4)^2\times\frac{2\pi}{0.033}\times(-2.3\times10^{-9})$$

$$=-2.6\times10^{31}(J/s)$$

$$t=\frac{E}{|\frac{dE}{dt}|}=\frac{\frac{1}{2}Jw^2}{|\frac{dE}{dt}|}=\frac{2\times1.5\times10^{30}\times(10^4)^2(2\pi)^2}{2\times5\times0.033^2\times2.6\times10^{31}}$$

$$=4.18\times10^{10}(s)$$

$$=1300(a)$$

5.27. 已知：$T=26000a$，$J=8.05\times10^{37}kg\cdot m^2$，$\theta=23.5°$

求：M，$\left|\frac{dL}{dt}\right|$

解：$\left|\frac{dL}{dt}\right| = L\sin\theta\frac{d\theta}{dt} = J\omega\sin\theta\frac{d\theta}{dt}$

$$= 8.05\times10^{37}\times\frac{2\pi}{86400}\times\sin23.5°\times\frac{2\pi}{26000\times3.15\times10^{7}}$$

$$= 1.79\times10^{22}\,(kg\cdot m/s^2)$$

$M = \left|\frac{dL}{dt}\right| = 1.79\times10^{22}\,(N\cdot m)$