A 16.5 $x^2+y^2-z=0$, $z=2$.

1. $\vec{p}=\vec{k}$, $\nabla f=2x\vec{i}+2y\vec{j}-\vec{k}$ $|\nabla f\cdot\vec{p}|=1$

$|\nabla f|=\sqrt{4x^2+4y^2+1}$

$z=2$ $\Downarrow$ $x^2+y^2=2$

$$S=\iint_R \sqrt{4x^2+4y^2+1}\,dxdy$$
$$=\int_0^{2\pi}\int_0^{\sqrt{2}}\sqrt{4r^2+1}\,r\,dr\,d\theta.$$
$$=\int_0^{2\pi}\left[\frac{1}{12}(4r^2+1)^{\frac{3}{2}}\right]_0^{\sqrt{2}}d\theta \quad =\int_0^{2\pi}\frac{13}{6}d\theta=\frac{13}{3}\pi$$

2. $x^2+y^2-z=0$, $z=2$, $z=6$.

$\vec{p}=\vec{k}$ $\nabla f=2x\vec{i}+2y\vec{j}-\vec{k}$ $|\nabla f\cdot\vec{p}|=1$

$|\nabla f|=\sqrt{4x^2+4y^2+1}$ $2\le x^2+y^2\le 6$.

$$S=\iint_R\sqrt{4x^2+4y^2+1}\,dxdy$$
$$=\int_0^{2\pi}\int_{\sqrt{2}}^{\sqrt{6}}\sqrt{4r^2+1}\,r\,dr\,d\theta=\frac{49}{3}\pi$$

B

15. $g(x,y,z)=xyz$, $x=a$, $y=b$, $z=c$

① $x=a$

$f(x,y,z)=x=a$, $g(x,y,z)=g(a,y,z)=ayz$

$\vec{p}=\vec{i}$, $\nabla f=\vec{i}$, $|\nabla f|=1$

$|\nabla f\cdot\vec{p}|=1$ $d\sigma=dydz$

$$\iint_S g\,d\sigma=\int_0^c\int_0^b ayz\,dydz$$
$$=\frac{ab^2c^2}{4}$$

② $y=b$

by the same method, we can get

$$\iint_S g\,d\sigma=\frac{a^2bc^2}{4}$$

③ $z=c$.

by the same method we can get

$$\iint_S g\,d\sigma=\frac{a^2b^2c}{4}.$$

Therefore: $\iint_S g(x,y,z)\,d\sigma=\frac{abc(ab+ac+bc)}{4}$

16. $g(x,y,z)=xyz$, $x=\pm a$, $y=\pm b$, $z=\pm c$.

① $x=a$:

$f(x,y,z)=x=a$, $g(x,y,z)=ayz$

$\vec{p}=\vec{i}$, $\nabla f=\vec{i}$, $|\nabla f|=1$

$|\nabla f\cdot\vec{p}|=1$, $d\sigma=dzdz$.

$$\iint_S g\,d\sigma=\int_{-b}^{b}\int_{-c}^{c}ayz\,dzdy$$
$$=0$$

② $y=b$

by the same method we can get.

$$\iint_S g\,d\sigma=0$$

③ $z=c$.

by the same method we can get

$$\iint_S g\,d\sigma=0.$$

Therefore $\iint_S g\,d\sigma=0$.

C

19. $\vec{F}(x,y,z)=-\vec{x}+2\vec{j}+3\vec{k}$, $z=0$, $0\le x\le 2$, $0\le y\le 3$.

$g(x,y,z)=z$, $\vec{p}=\vec{k}$

$\nabla g=\vec{k}$, $|\nabla g|=1$, $|\nabla g\cdot\vec{p}|=1$.

$$\text{Flux}=\iint_S \vec{F}\cdot\vec{n}\,d\sigma$$
$$=\iint_R(\vec{F}\cdot\vec{k})\,dA$$
$$=\int_0^2\int_0^3 3\,dy\,dx.$$
$$=18$$

29. $g(x,y,z)=y-e^x=0$

$\nabla g=-e^x\vec{i}+\vec{j}$, $|\nabla g|=\sqrt{e^{2x}+1}$ $\Rightarrow \vec{n}=\dfrac{e^x\vec{i}-\vec{j}}{\sqrt{e^{2x}+1}}$ $\quad \vec{F}\cdot\vec{n}=\dfrac{-2e^x-2y}{\sqrt{e^{2x}+1}}$ $\quad \vec{p}=\vec{i}$

$|\nabla g\cdot\vec{p}|=e^x \Rightarrow d\sigma=\dfrac{\sqrt{e^{2x}+1}}{e^x}dA$

$$\text{Flux}=\iint_R\left(\frac{-2e^x-2y}{\sqrt{e^{2x}+1}}\right)\left(\frac{\sqrt{e^{2x}+1}}{e^x}\right)dA=\iint_R -4\,dA=\int_0^1\int_1^2 -4\,dy\,dz=-4$$

D

36 $f(x,y,z)=4x^2+4y^2-z^2=0$.

$\nabla f=8x\vec{i}+8y\vec{j}-2z\vec{k} \Rightarrow |\nabla f|=2\sqrt{16x^2+16y^2+z^2}=2\sqrt{4z^2+z^2}=2\sqrt{5}\,z$

$\vec{p}=\vec{k}\Rightarrow|\nabla f\cdot\vec{p}|=2z$.

$d\sigma=\dfrac{2\sqrt{5}z}{2z}dA=\sqrt{5}A$.

$$I_z=\iint_S(x^2+y^2)\delta\,d\sigma$$
$$=\delta\sqrt{5}\iint_R(x^2+y^2)dxy.$$
$$=\delta\sqrt{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta}r^3\,dr\,d\theta=\frac{3\sqrt{5}\pi\delta}{2}$$

E. 16.6.

53. (a) C is $(x,z)=(R+r\cos u, r\sin u) \Rightarrow (x,y,z)$ is on the torus with $\begin{cases}x=(R+r\cos u)\cos v\\ y=(R+r\cos u)\sin v\\ z=r\sin u\end{cases}$ $\quad 0\le u\le 2\pi,\ 0\le v\le 2\pi$

(b) $r_u=(-r\sin u\cos v)\vec{i}-(r\sin u\sin v)\vec{j}+(r\cos u)\vec{k}$

$r_v=(-(R+r\cos u)\sin v)\vec{i}+((R+r\cos u)\cos v)\vec{j}$

$$\vec{r}_u\times\vec{r}_v=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ -r\sin u\cos v&-r\sin u\sin v&r\cos u\\ -(R+r\cos u)\sin v&(R+r\cos u)\cos v&0\end{vmatrix}$$

$|r_u\times r_v|^2=r(R+r\cos u)$, $A=\int_0^{2\pi}\int_0^{2\pi}(rR+r^2\cos u)\,du\,dv$

$=4\pi^2 rR$.

F 16.7

1. $\vec{F}=x^2\vec{i}+2x\vec{j}+z^2\vec{k}$ $\quad C: 4x^2+y^2=4$.

$$\vec{F}=\nabla\times\vec{F}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ x^2&2x&z^2\end{vmatrix}=0\vec{i}+0\vec{j}+(2-0)\vec{k}=2\vec{k},\ \vec{n}=\vec{k}\quad d\sigma=dxdy$$

$\vec{F}\cdot\vec{n}=2$

$\Rightarrow \oint_C\vec{F}\,d\vec{r}=\iint_R 2\,dA=2.$

$=4\pi$

4. $\vec{F}=(y^2+z^2)\vec{i}+(x^2+z^2)\vec{j}+(x^2+y^2)\vec{k}$ $\quad C: x+y+z=1$

$$\vec{F}=\nabla\times\vec{F}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\ \frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\ y^2+z^2&x^2+z^2&x^2+y^2\end{vmatrix}=(2y-2z)\vec{i}+(2z-2x)\vec{j}+(2x-2y)\vec{k},\ \vec{n}=\frac{\vec{i}+\vec{j}+\vec{k}}{\sqrt{3}}$$

$\vec{F}\cdot\vec{n}=0.$

$\oint_C F\cdot dr=\iint_S 0\,d\sigma=0.$

(G) 7. $x=3\cos t$ and $y=2\sin t$

$\vec{F}=(2\sin t)\vec{i}+(9\cos^2 t)\vec{j}+(9\cos^2 t+16\sin^4 t)\sin e^{\sqrt{(6\sin t\cos t(\cdot))}}$ at the shell.

$\vec{r}=(3\cos t)\vec{i}+(2\sin t)\vec{j}$ $\quad \vec{F}\cdot\frac{d\vec{r}}{dt}=-6\sin^2 t+18\cos^3 t.$

$d\vec{r}=(-3\sin t)\vec{i}+(2\cos t)\vec{j}$

$$\iint_S \nabla\times\vec{F}\cdot\vec{n}\,d\sigma=\int_0^{2\pi}(-6\sin^2 t+18\cos^3 t)\,dt=\left[-3t+\frac{3}{2}\sin 2t+6(\sin t)(\cos^2 t+2)\right]_0^{2\pi}=-6\pi$$

(H) 16.8.

5. Cube $\vec{F}=(y-x)\vec{i}+(z-y)\vec{j}+(y-x)\vec{k}$ $\quad D: x=\pm1, y=\pm1, z=\pm1.$

$\frac{\partial(y-x)}{\partial x}=1,\ \frac{\partial(z-y)}{\partial y}=-1,\ \frac{\partial(y-x)}{\partial z}=0$ $\quad \nabla\cdot\vec{F}=-2.$ $\quad \text{Flux}=\int_{-1}^1\int_{-1}^1\int_{-1}^1 -2\,dx\,dy\,dz=-16.$

8. $\vec{F}=x^2\vec{i}+xz\vec{j}+3z\vec{k}$ $\quad D: x^2+y^2+z^2\le4$

$\frac{\partial(x^2)}{\partial x}=2x,\ \frac{\partial(xz)}{\partial y}=0,\ \frac{\partial(3z)}{\partial z}=3$ $\quad \nabla\cdot\vec{F}=2x+3.$ $\quad \text{Flux}=\iiint_D(2x+3)\,dV=\int_0^{2\pi}\int_0^{\pi}\int_0^2(2\rho\sin\phi\cos\theta+3)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta$

$=32\pi.$

(I) 26.

$\vec{F}=\vec{C}\Rightarrow\nabla\cdot\vec{F}=0\Rightarrow\text{Flux}=\iint_S\vec{F}\cdot\vec{n}\,d\sigma=\iiint_D\nabla\cdot\vec{F}\,dV=\iiint_D 0\,dV=0.$

27. (a) $\iint_S\nabla f\cdot\vec{n}\,d\sigma=\iiint_D\nabla\cdot\nabla f\,dV=\iiint_D\nabla^2 f\,dV=\iiint_D 0\,dV=0$ $\quad$ since f is harmonic.

(b) $f\nabla f=(f\frac{\partial f}{\partial x})\vec{i}+(f\frac{\partial f}{\partial y})\vec{j}+(f\frac{\partial f}{\partial z})\vec{k}\Rightarrow\nabla\cdot f\nabla f=f\nabla^2 f+|\nabla f|^2=0.$ $\quad \iint_S f\nabla f\cdot\vec{n}\,d\sigma=\iiint_D|\nabla f|^2\,dV$

(J) 16.7

26. $\frac{\partial P}{\partial y}=0,\ \frac{\partial N}{\partial z}=0,\ \frac{\partial M}{\partial z}=0,\ \frac{\partial P}{\partial x}=0,\ \frac{\partial N}{\partial x}=\frac{y^2-x^2}{(x^2+y^2)^2},\ \frac{\partial M}{\partial y}=\frac{y^2-x^2}{(x^2+y^2)^2}\Rightarrow\vec{F}=\left[\frac{y^2-x^2}{(x^2+y^2)^2}-\frac{y^2-x^2}{(x^2+y^2)^2}\right]\vec{k}=\vec{0}$

However, $x^2+y^2=1$, $\vec{r}=(\cos t)\vec{i}+(\sin t)\vec{j}\Rightarrow\frac{d\vec{r}}{dt}=(-\sin t)\vec{i}+(\cos t)\vec{j}$

$\vec{F}=(-\sin t)\vec{i}+(\cos t)\vec{j}=\vec{F}\cdot\frac{d\vec{r}}{dt}=\sin^2 t+\cos^2 t=1.\Rightarrow\oint_C\vec{F}\cdot d\vec{r}=\oint_0^{2\pi}1\,dt=2\pi$

11. Let S_1 and S_2 be oriented surfaces that span C and that induce the same positive direction C. Then

$\iint_{S_1}\nabla\times\vec{F}\cdot\vec{n}_1\,d\sigma_1=\oint_C\vec{F}\cdot d\vec{r}=\iint_{S_2}\nabla\times\vec{F}\cdot\vec{n}_2\,d\sigma_2$

12. $\iint_S\nabla\times\vec{F}\cdot\vec{n}\,d\sigma=\iint_{S_1}\nabla\times\vec{F}\cdot\vec{n}\,d\sigma+\iint_{S_2}\nabla\times\vec{F}\cdot\vec{n}\,d\sigma$ and since S_1 and S_2 are joined by the simple closed curve C, each of the above integrals will be equal to a circulation integral on C. But for one surface the circulati will be counterclockwise, and for the other surface the circulat will be clockwise. Since the integrands are the same, the sum will be $0\Rightarrow\iint_S\nabla\times\vec{F}\cdot\vec{n}\,d\sigma=0.$