A. 1. Let $\vec{v_1} = (2,4,5)-(1,5,7) = (1,-1,-2)$

$\vec{v_2} = (1,5,7)-(-1,6,8) = (2,-1,-1)$

$\vec{v_1} \times \vec{v_2} = (-1,-3,1)$

$-1(x-2)-3(y-4)+(z-5)=0$

$\Rightarrow x+3y-z-9=0$

2. $\vec{v} = \vec{OP} = (1,-2,1)$

$(x-1)-2(y+2)+(z-1)=0$

$\Rightarrow x-1-2y-4+z-1=0$

$\Rightarrow x-2y+z-6=0$

3. $\vec{v_1} = (-1,\sqrt{2},1)$ is a vector perpendicular to the plane $z+\sqrt{2}y-x=0$

$\vec{v_2} = (1,0,-1)$ is a vector perpendicular to the plane $z=x$.

$\cos\langle \vec{v_1}, \vec{v_2} \rangle = \dfrac{\vec{v_1}\cdot\vec{v_2}}{\|\vec{v_1}\|\cdot\|\vec{v_2}\|} = -\dfrac{\sqrt{2}}{2}$

$\Rightarrow$ The angle between $\vec{v_1}$ and $\vec{v_2}$ is $\frac{3\pi}{4}$.

So the angle between the planes is $\frac{\pi}{4}$.

B. 1. $\underline{v}\cdot\underline{u_1} = (a\underline{u_1}+b\underline{u_2})\cdot\underline{u_1} = a(\underline{u_1}\cdot\underline{u_1}) + b(\underline{u_2}\cdot\underline{u_1}) = a$.

2. No. $\underline{u}\cdot\underline{v_1} = \underline{u}\cdot\underline{v_2} \Leftrightarrow \underline{u}\cdot(\underline{v_1}-\underline{v_2})=0$. It implies that $\underline{v_1}-\underline{v_2}$ is perpendicular to $\underline{u}$, which is not necessarily 0. For example, let $\underline{u}=(1,0)$, $\underline{v_1}=(1,2)$, $\underline{v_2}=(1,1)$. Then $\underline{u}\cdot\underline{v_1} = \underline{u}\cdot\underline{v_2}$, $\underline{u}\neq 0$ and $\underline{v_1}\neq\underline{v_2}$.

3. $(\underline{w_1}+\underline{w_2})\cdot(\underline{w_1}-\underline{w_2}) = \|\underline{w_1}\| - \|\underline{w_2}\| + (\underline{w_2}\cdot\underline{w_1} - \underline{w_1}\cdot\underline{w_2}) = \|\underline{w_1}\| - \|\underline{w_2}\|$.

So $\underline{w_1}+\underline{w_2}$ and $\underline{w_1}-\underline{w_2}$ are orthogonal iff $\|w_1\|=\|w_2\|$ and $w_1 \neq \pm w_2$.

C. $\underline{u}\times\underline{v} = (1,4,3)$

$(\underline{u}\times\underline{v})\cdot\underline{w} = (1,4,3)\cdot(-1,2,-1) = 4$

$\underline{v}\times\underline{w} = (3,4,5)$

$(\underline{v}\times\underline{w})\cdot\underline{u} = (3,4,5)\cdot(1,-1,1) = 4$

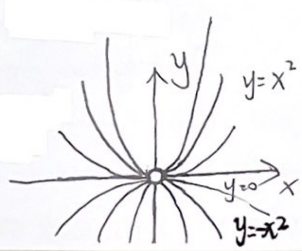
$\Rightarrow (\underline{u}\times\underline{v})\cdot\underline{w} = (\underline{v}\times\underline{w})\cdot\underline{u}$

D. 1. a. $\mathbb{R}^2 \setminus \{(0,y) \mid y\in\mathbb{R}\}$, i.e. $\mathbb{R}^2\setminus$ y-axis.

b. $\mathbb{R}$

c. open, unbounded

d. $\frac{y}{x^2} = c \Leftrightarrow y = cx^2$, $x\neq 0$. $\forall$ fixed $c\in\mathbb{R}$.

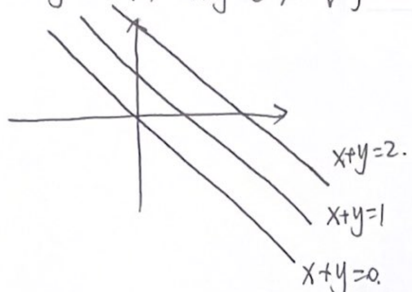


2. a. $\{(x,y)\in\mathbb{R}^2 \mid y \geq -x\}$.

b. $[0,+\infty)$

c. closed, unbounded

d $\sqrt{x+y}=c \Leftrightarrow x+y=c^2$, $\forall$ fixed $c \geq 0$

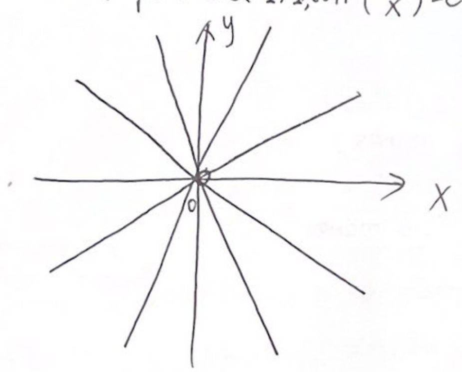


3. a. $\mathbb{R}^2 \setminus \{(0,y) \mid y\in\mathbb{R}\}$

b. $(-\frac{\pi}{2}, \frac{\pi}{2})$

c. open, unbounded

d. $\forall$ fixed $c\in(-\frac{\pi}{2},\frac{\pi}{2})$, $\tan^{-1}(\frac{y}{x})=c \Leftrightarrow \tan(c)=\frac{y}{x} \Leftrightarrow y=\tan(c)\cdot x$, $x\neq 0$.



E. Since $T(x,y)$ is differentiable, the hottest and coldest points on the plate can only be inside the plate where $T_x=T_y=0$ and on the unit circle.

1) interior points: $T_x=2x-1=0$, $T_y=4y=0$ yielding the single point $(x,y)=(\frac{1}{2},0)$.
The temperature there is $T(\frac{1}{2},0)=-\frac{1}{4}$.

2) boundary points: On the unit circle, $x^2+y^2=1$.
$T(x,y)=x^2+2y^2-x=2(x^2+y^2)-x^2-x=-x^2-x+2$
which can be regarded as a function of x defined on the closed interval $[-1,1]$.
Its extreme values may occur at the endpoints
$x=-1$: $T(-1,0)=2$; $x=1$; $T(1,0)=0$.
and at the interior points where $T'(x,y)=-2x-1=0$.
The only two interior points where $T'(x,y)=0$ are $x=-\frac{1}{2}$, $y=\pm\frac{\sqrt{3}}{2}$,
where $T(-\frac{1}{2},\pm\frac{\sqrt{3}}{2})=\frac{9}{4}$.

So the hottest point on the plate are $(-\frac{1}{2},\pm\frac{\sqrt{3}}{2})$, and the temperature there is $\frac{9}{4}$.
The coldest point on the plate is $(\frac{1}{2},0)$, and the temperature there is $-\frac{1}{4}$.

F. 1. $f(x,y)=x^2-y^2$

$f_x=2x,\quad f_{xx}=2$

$f_y=-2y,\quad f_{yy}=-2$

$\Rightarrow f_{xx}+f_{yy}=0.$

2. $f(x,y)=\ln\sqrt{x^2+y^2}$.

$f_x=\dfrac{1}{\sqrt{x^2+y^2}}\cdot\dfrac{1}{2}\cdot\dfrac{2x}{\sqrt{x^2+y^2}}=\dfrac{x}{x^2+y^2},\quad f_{xx}=\dfrac{x^2+y^2-2x^2}{(x^2+y^2)^2}=\dfrac{y^2-x^2}{(x^2+y^2)^2}.$

$f_y=\dfrac{1}{\sqrt{x^2+y^2}}\cdot\dfrac{1}{2}\cdot\dfrac{2y}{\sqrt{x^2+y^2}}=\dfrac{y}{x^2+y^2},\quad f_{yy}=\dfrac{x^2+y^2-2y^2}{(x^2+y^2)^2}=\dfrac{x^2-y^2}{(x^2+y^2)^2}=-\dfrac{y^2-x^2}{(x^2+y^2)^2}$

$\Rightarrow f_{xx}+f_{yy}=0.$

3. $z_x=g_u\cdot u_x+g_v\cdot v_x=x\,g_u+y\,g_v,$

$z_{xx}=g_u+x(g_{uu}\cdot u_x+g_{uv}\cdot v_x)+y(g_{vu}\cdot u_x+g_{vv}\cdot v_x)$

$=g_u+x^2g_{uu}+xy\,g_{uv}+xy\,g_{vu}+y^2g_{vv}.$

$z_y=g_u\cdot u_y+g_v\cdot v_y=-y\,g_u+x\,g_v.$

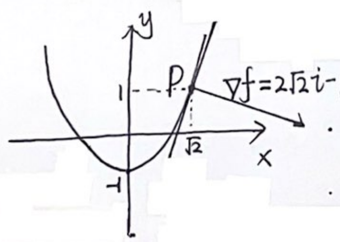
$z_{yy}=-g_u-y(g_{uu}\cdot u_y+g_{uv}\cdot v_y)+x(g_{vu}\cdot u_y+g_{vv}\cdot v_y)$

$=-g_u+y^2g_{uu}-xy\,g_{uv}-xy\,g_{vu}+x^2g_{vv}$

$z_{xx}+z_{yy}=(x^2+y^2)g_{uu}+(x^2+y^2)g_{vv}$

$=(x^2+y^2)(g_{uu}+g_{vv})=0.$

G. 1.



$f(x,y)=x^2-y.\quad f_x=2x,\ f_y=-1$

$\nabla f=\dfrac{\partial f}{\partial x}i+\dfrac{\partial f}{\partial y}j=2x\,i-j.$

$\nabla f\big|_{(\sqrt{2},1)}=2\sqrt{2}\,i-j.$

The tangent is the line $2\sqrt{2}(x-\sqrt{2})-(y-1)=0$

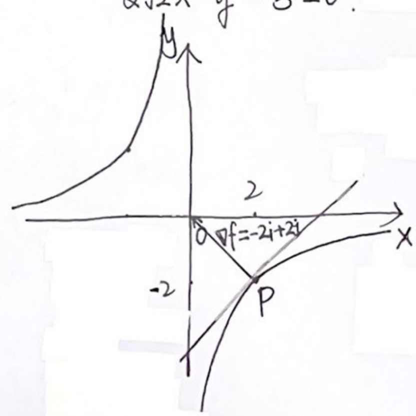
$2\sqrt{2}x-y-3=0.$

2. $f(x,y)=xy,\quad f_x=y,\ f_y=x.$

$\nabla f=y\,i+x\,j\qquad \nabla f\big|_{(2,-2)}=-2i+2j.$

The tangent is the line $-2(x-2)+2(y+2)=0$

$x-y-4=0$



H. 1. $\underline{r}'(t) = (e^t\cos t - e^t\sin t,\ e^t\sin t + e^t\cos t,\ e^t)$

$|\underline{r}'(t)| = \sqrt{e^{2t}\cos^2 t + e^{2t}\sin^2 t + e^{2t}\sin^2 t + e^{2t}\cos^2 t + e^{2t}}$

$= \sqrt{3}\, e^t$

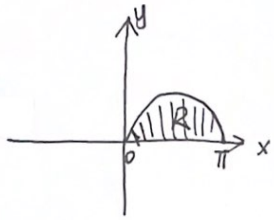
$S = \int_{-\ln 4}^{0} \sqrt{3} e^t\, dt = \sqrt{3} e^t \Big|_{-\ln 4}^{0} = \sqrt{3}\left(1 - \frac{1}{4}\right) = \frac{3\sqrt{3}}{4}.$

2. $\underline{r}'(t) = (2, 3, -6)$

$|\underline{r}'(t)| = \sqrt{4+9+36} = 7$

$s = \int_{-1}^{0} 7\, dt = 7.$

I. 1.



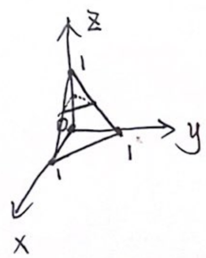
$\int_R dA = \int_0^{\pi} \sin x\, dx = -\cos x \Big|_0^{\pi} = 2.$

$\int_R x\, dA = \int_0^{\pi} x\sin x\, dx = -\int_0^{\pi} x\, d\cos x = -x\cos x \Big|_0^{\pi} + \int_0^{\pi} \cos x\, dx = \pi.$

$\int_R y\, dA = \int_0^{\pi} \frac{1}{2}\sin^2 x\, dx = \int_0^{\pi} \frac{1-\cos 2x}{4}\, dx = \left(\frac{x}{4} - \frac{\sin 2x}{8}\right)\Big|_0^{\pi} = \frac{\pi}{4}$

The centroid of the region R is $\left(\frac{\pi}{2}, \frac{\pi}{8}\right)$.

2.



Let D be tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

$\int_D dV = \int_0^1 dz \iint_{R_z} dx\,dy$, where R_z is the triangular region with vertices $(0,0,z)$, $(1-z,0,z)$, $(0,1-z,z)$.

$= \int_0^1 dz \int_0^{1-z} dx \int_0^{1-z-x} dy = \int_0^1 dz \int_0^{1-z} 1-z-x\, dx = \frac{1}{2}\int_0^1 (1-z)^2 dz = \frac{1}{6}.$

$\int_D x\, dV = \int_0^1 dz \int_0^{1-z} x\,dx \int_0^{1-z-x} dy = \frac{1}{6}\int_0^1 (1-z)^3\, dz = \frac{1}{24}.$

$\int_D y\, dV = \int_0^1 dz \int_0^{1-z} y\, dy \int_0^{1-z-y} dx = \frac{1}{24}.$

$\int_D z\, dV = \int_0^1 z\, dz \int_0^{1-z} dx \int_0^{1-z-x} dy = \frac{1}{24}.$

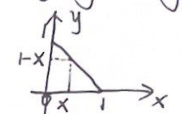
$(\bar{x}, \bar{y}, \bar{z}) = \left(\int_D x\,dV,\ \int_D y\,dV,\ \int_D z\,dV\right) / \int_D dV = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right).$

$\Rightarrow$ The centroid of the tetrahedron is $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$.

J. 1. $\int_{-1}^{0}\int_{-1}^{1} x+y+1\,dx\,dy = \int_{-1}^{0}\left[\frac{1}{2}x^2+yx+x\right]_{-1}^{1} dy = \int_{-1}^{0} 2y+2\,dy = 1.$

2. $\int_{\pi}^{2\pi}\int_{0}^{\pi} \sin x+\cos y\,dx\,dy = \int_{\pi}^{2\pi}\left[-\cos x + x\cos y\right]_{0}^{\pi} dy = \int_{\pi}^{2\pi} 2+\pi\cos y\,dy = \left[2y+\pi\sin y\right]\Big|_{\pi}^{2\pi} = 2\pi$

3. Let R be the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$.

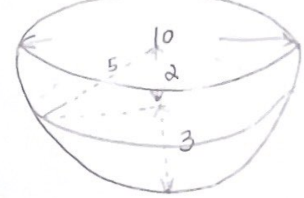


$$\int_R f(x,y)\,dA = \int_R x^2+y^2\,dA = \int_0^1\int_0^{1-x} x^2+y^2\,dy\,dx = \int_0^1\left[x^2y+\frac{1}{3}y^3\right]_0^{1-x} dx = \int_0^1 x^2(1-x)+\frac{1}{3}(1-x)^3$$

$$=\left[-\frac{1}{4}x^4+\frac{1}{3}x^3\right]_0^1-\frac{1}{12}\left[(x-1)^4\right]_0^1$$

$$=\frac{1}{6}.$$

4. Let D be the space filled with water.



$$\int_D dV = \int_0^3 dz \iint_{R_z} dA.$$

The radius of R_z is $\sqrt{25-(5-z)^2}$.

So $\iint_{R_z} dA = \pi\cdot\left[25-(5-z)^2\right] = \pi(10z-z^2)$.

$$\Rightarrow \int_D dV = \int_0^3 \pi(10z-z^2)\,dz = \pi\left[5z^2-\frac{z^3}{3}\right]_0^3 = 36\pi$$

So the volume of water in the bowl is $36\pi\ cm^3$.

5. $\iint_R f(x)g(y)\,dA = \int_c^d\int_a^b f(x)g(y)\,dx\,dy = \int_c^d g(y)\left(\int_a^b f(x)dx\right)dy = \left(\int_a^b f(x)dx\right)\left(\int_c^d g(y)dy\right)$

since $\int_a^b f(x)dx$ is a constant.