第三章

3.1 已知：$F=-kx$，$x=A\cos\omega t$，$t=0\sim t=\frac{\pi}{2\omega}$

求：I

解：$I=\int_{t_0}^{t'} F\,dt$

$$I=\int_0^{\frac{\pi}{2\omega}} -kx\,dt$$

$$I=-k\int_0^{\frac{\pi}{2\omega}} x\,dt$$

$$I=-k\int_0^{\frac{\pi}{2\omega}} A\cos\omega t\,dt$$

$$I=-kA\left(\frac{1}{\omega}\sin\omega t\right)\Big|_0^{\frac{\pi}{2\omega}}$$

$$I=-\frac{kA}{\omega}$$

3.3 已知：$m=150kg$，$h=5m$，$t=0.3s$

求：F

解：$F-mg=ma$

$$F=m(a+g)$$

$$F=m\left(\frac{\Delta v}{\Delta t}+g\right)$$

$$F=m\left(\frac{0-(-v)}{\Delta t}+g\right)$$

$\frac{1}{2}mv^2=mgh \Rightarrow v=\sqrt{2gh}$

$$F=m\left(\frac{\sqrt{2gh}}{\Delta t}+g\right)$$

$$F=150\times\left(\frac{\sqrt{2\times9.8\times5}}{0.3}+9.8\right)$$

$$F=6419.75(N)$$

3.13 在太空静止的单级火箭，点火后，其质量的减少与初质量之比为多大时，它喷出的废气将是静止的

已知：火箭点火时质量为M_i，初速为$v_i=0$，喷出废气静止时速度$v_f=u$，u是喷气速率

解：

$$v_f-v_i=u\ln\frac{M_i}{M_f}$$

$$v_f-0=u\ln\frac{M_i}{M_f}$$

$$v_f=u\ln\frac{M_i}{M_f}$$

$$\frac{\Delta M}{M_i}=\frac{M_i-M_f}{M_i}=1-\frac{1}{e}=0.632$$

$$v_f=u=u\ln\frac{M_i}{M_f}$$

$$1=\ln\frac{M_i}{M_f}$$

$$\frac{M_i}{M_f}=e$$

3.15 已知 $v = 210\,m/s$, $\frac{dm_1}{dt} = 75kg/s$, $\frac{dM}{dt} = -3kg/s$, $u = 490\,m/s$.

求: F

解: 根据动量守恒:

$$dm_1 \times 0 + Mv = (dm_1 - dM)(v-u) + (M+dM)(v+dv)$$

$$\cancel{Mv} = dm_1 v - dm_1 u - \cancel{dMv} + dMu + \cancel{Mv} + Mdv + \cancel{dMv} + dMdv$$

$$-Mdv = dMu - dm_1(v-u)$$

$$Mdv = dm_1(u-v) - dMu$$

$$F = M\frac{dv}{dt} = (u-v)\frac{dm_1}{dt} - u\frac{dM}{dt}$$

$$F = (490-210) \times 75 - 490 \times (-3)$$

$$F = 22470\,(N)$$

3.22. 已知 $h_1 = 205.5 \times 10^3\,m$, $h_2 = 35835.7 \times 10^3\,m$, $v_1 = 10.2\,km/s$, $R_{地} = 6378\,km$.

求 v_2, T.

解: 根据角动量守恒:

$$r_1 \times mv_1 = r_2 \times mv_2$$

$$v_2 = \frac{r_1 v_1}{r_2}$$

$$v_2 = \frac{6378+205.5}{6378+35835.7} \times 10.2$$

$$v_2 = 1.59\,(km/s)$$

$$T = \frac{S}{\frac{ds}{dt}} = \frac{\pi(r_1+r_2)\sqrt{r_1 r_2}}{v_1 r_1}$$

$$= \frac{\pi[(6378+205.5)+(6378+35835.7)]\sqrt{(6378+205.5)\times(6378+35835.7)}}{10.2\times(6378+205.5)}$$

$$= 38057\,(s)$$

$$= 10.57\,(h)$$

3.24. 已知：质量为m，长度为a，角速度w

求：L, V, L_1, L_2, w'

解：

(1) $2mL = m(a-L)$

$$L = \frac{a}{3}$$

根据动量守恒：

$$3mV = mv_0 + m(-v_0) + m\cdot 0$$

$$V = 0$$

(2) $L = mv\cdot r$

$$L_1 = \left(m\times\frac{wa}{2}\times\frac{a}{3}\right)+\left(m\times\frac{wa}{2}\times\frac{2a}{3}\right)+\left(m\frac{a}{3}\times 0\right)$$

$$L_1 = \frac{3mwa^2}{6}$$

$$L_1 = \frac{1}{2}mwa^2$$

$L_2 = L_1 = \frac{1}{2}mwa^2$ — 无外力作用，碰撞前后

(3) $L_2 = 2m\frac{aw'}{3}\times\frac{a}{3} + m\frac{2aw'}{3}\times\frac{2a}{3} = \frac{1}{2}mwa^2$

$$\frac{6mw'a^2}{9} = \frac{1}{2}mwa^2$$

$$\frac{2w'}{3} = \frac{1}{2}w$$

$$w' = \frac{3}{4}w$$