第九章

9.17. 已知：$T_1 = 250K$, $T_2 = 310K$, $P_1 = P_2$, $V_1 = V_2$

求：T

解：$\because P_1V_1 = \nu_1 RT_1$，$P_2V_2 = \nu_2 RT_2$

$\therefore \nu_1 RT_1 = \nu_2 RT_2$

$\nu_2 = \nu_1 \frac{T_1}{T_2}$

注意到气体混合前后内能相等，$E_0 = E$

混合前：$E_0 = E_1 + E_2 = \frac{3}{2}\nu_1 RT_1 + \frac{5}{2}\nu_2 RT_2 = 4R\nu_1 T_1$,

$E = E_1 + E_2 = \frac{3}{2}\nu_1 RT + \frac{5}{2}\nu_2 RT = \frac{RT}{2}(3\nu_1 + 5\nu_1 \frac{T_1}{T_2})$

$E_0 = E$

$4R\nu_1 T_1 = \frac{1}{2}RT(3\nu_1 + 5\nu_1 \frac{T_1}{T_2})$

$T = \frac{8\nu_1 T_1}{3\nu_1 + 5\nu_1 \frac{T_1}{T_2}}$

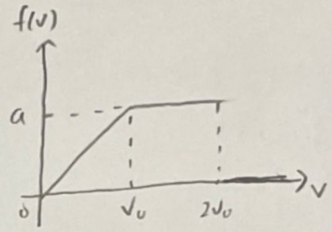
$T = \frac{8T_1}{3 + 5\frac{T_1}{T_2}}$

$T = \frac{8 \times 250}{3 + 5 \times \frac{250}{310}} = 284K$

9.18. 已知：$f(v) = av/v_0$ $(0 \le v \le v_0)$，$f(v) = a$ $(v_0 \le v \le 2v_0)$，$f(v) = 0$ $(v > 2v_0)$

求：a, ΔN, $\bar{v}$

(1). 分布曲线如图，由归一化知



$\int_0^\infty f(v)dv = 1$

$\Rightarrow \int_0^{v_0} \frac{av}{v_0} dv + \int_{v_0}^{2v_0} a\,dv = 1$

$\Rightarrow \frac{1}{2}av_0 + a(2v_0 - v_0) = \frac{3}{2}av_0 = 1$

$a = \frac{2}{3v_0}$

(2) 速率大于v_0的粒子数

$\Delta N = \int_{v_0}^\infty dN = \int_{v_0}^\infty Nf(v)dv = \int_{v_0}^{2v_0} Na\,dv = Nav_0 = \frac{2}{3}N$

速率小于v_0的粒子数

$(\Delta N)' = N - \Delta N = N - \frac{2}{3}N = \frac{1}{3}N$.

(3) $\bar{v} = \int_0^\infty vf(v)dv = \int_0^{v_0} v \cdot \frac{av}{v_0} dv + \int_{v_0}^{2v_0} va\,dv = \int_0^{v_0} \frac{2v^2}{3v_0^2} dv + \int_{v_0}^{2v_0} \frac{2v}{3v_0} dv$

$= \left[\frac{2v^3}{9v_0^2}\right]_0^{v_0} + \left[\frac{v^2}{3v_0}\right]_{v_0}^{2v_0}$

$= \frac{2v_0^3}{9v_0^2} + \frac{4v_0^2}{3v_0} - \frac{v_0^2}{3v_0}$

$= \frac{11}{9}v_0$

9.19. 已知：$T_1 = 2\times10^6 K$，$T_2 = 2.7K$，$T_3 = 2.4\times10^{-11} K$

求：v_{rms_1}，v_{rms_2}，v_{rms_3}

解：$v_{rms_1} = \sqrt{\frac{3kT}{m_e}} = \sqrt{\frac{3\times1.38\times10^{-23}\times2\times10^6}{9.1\times10^{-31}}} = 9.54\times10^6 m/s$.

$v_{rms_2} = \sqrt{\frac{3RT}{M_m}} = \sqrt{\frac{3\times8.31\times2.7}{1\times10^{-3}}} = 259 m/s$.

$v_{rms_3} = \sqrt{\frac{3RT}{M_{Na}}} = \sqrt{\frac{3\times8.31\times2.4\times10^{-11}}{23\times10^{-3}}} = 1.61\times10^{-4} m/s$

9.26. 已知：$V=20L$，$m=1.1kg$，$T=13°C=286K$，$a=3.64\times10^5 Pa\cdot L^2/mol^2$，$b=0.0427L/mol$.

求：P.

解：根据范德瓦耳斯方程.

$$(P+\frac{a}{V_m^2})\cdot(V_m-b) = RT$$

$$(P+\frac{m^2a}{(MV)^2})(\frac{MV}{m}-b) = RT$$

$$(P+\frac{1.1^2\times3.64\times10^5\times10^{-6}}{(44\times10^{-3}\times20\times10^{-3})^2})\times(\frac{44\times10^{-3}\times20\times10^{-3}}{1.1}-0.0427\times10^{-3}) = 8.31\times286$$

$$P = 2.57\times10^6 Pa.$$

理想气体方程：$P = \frac{mRT}{MV} = \frac{1.1\times8.31\times286}{44\times10^{-3}\times20\times10^{-3}} = 2.97\times10^6 Pa$.

内压强：$P_{in} = \frac{a}{V_m^2} = \frac{m^2a}{(MV)^2} = \frac{1.1^2\times3.64\times10^5\times10^{-6}}{(44\times10^{-3}\times20\times10^{-3})^2} = 5.69\times10^5 Pa$

9.28. 已知：$\eta = 1.89\times10^{-5} Pa\cdot s$，$M = 0.004kg/mol$，$\bar{v} = 1.2\times10^3 m/s$.

求：$\bar{\lambda}$，d.

解：$\bar{\lambda} = \frac{3\eta}{mn\bar{v}} = \frac{3\eta}{\rho\bar{v}} = \frac{3\times1.89\times10^{-5}}{\frac{0.004}{22.4\times10^{-3}}\times1.2\times10^3} = 2.65\times10^{-7} m$.

由 $\bar{\lambda} = \frac{kT}{\sqrt{2}\pi d^2 p}$ 知 $d = \sqrt{\frac{kT}{\sqrt{2}\pi p\bar{\lambda}}} = \sqrt{\frac{1.38\times10^{-23}\times273}{\sqrt{2}\pi\times1.01\times10^5\times2.65\times10^{-7}}} = 1.78\times10^{-10} m$.

9.29. 已知：$l = 0.4cm = 4\times10^{-3} m$，$T = 27° = 300K$，$d = 3.7\times10^{-10} m$.

求：p

解：由 $K = \frac{1}{3}mn\bar{v}\bar{\lambda}C_v = \frac{1}{3}mn\bar{v}\cdot\frac{1}{\sqrt{2}\pi d^2 n} = \frac{m\bar{v}}{3\sqrt{2}\pi d^2}$ 知 K 与 n，p 无关

当 $\bar{\lambda} = l$ 时，$K = \frac{1}{3}mn\bar{v}lC_v$，要使 K 变小，则要 n 减小，又 $p = nkT$，知 p 减小

要使 k 最大，则 p 最小，此时 $\bar{\lambda} = \frac{kT}{\sqrt{2}\pi d^2 p}$ 最大，值为 l，故

$$p = \frac{kT}{\sqrt{2}\pi d^2 l} = \frac{1.38\times10^{-23}\times300}{\sqrt{2}\pi\times(3.7\times10^{-10})^2\times4\times10^{-3}} = 1.7 Pa.$$

9.30 已知：扩散系数D，水蒸气密度ρ，远处密度ρ_∞，水密度ρ_w.

求：(1) w (2) t.

解：(1) 水蒸气沿球面扩散，设此球半径为r，有

$$\frac{dM}{dt} = -D\frac{d\rho}{dr}\cdot ds = -D\cdot\frac{d\rho}{dr}\cdot 4\pi r^2$$

而每个球面上的水蒸气密度应相等，故 $\frac{d\rho}{dr}r^2 = C$.

$$\Rightarrow \int_{\rho}^{\rho_\infty} d\rho = C\int_R^{\infty}\frac{dr}{r^2} \Rightarrow \rho_\infty - \rho = \frac{C}{R} \Rightarrow C = R(\rho_\infty - \rho)$$

因此 $w = \frac{dM}{dt} = -D4\pi\cdot\frac{d\rho}{dr}r^2 = -D\cdot 4\pi C = -D\cdot 4\pi R(\rho_\infty - \rho)$

(2) dt 时间内蒸发量：$-dM = 4\pi DR(\rho_\infty - \rho)dt$，$R$为变量.

又 $M = \frac{4}{3}\pi R^3\cdot\rho_\infty \Rightarrow dM = 4\pi R^2\rho_w dR$.

故 $-4\pi R^2\rho_w dR = 4\pi DR(\rho_\infty - \rho)dt$.

$$\Rightarrow -\int_R^0 R\,dR = \frac{D(\rho_\infty - \rho)}{\rho_w}\cdot\int dt.$$

$$\frac{1}{2}R^2 = \frac{D(\rho_\infty - \rho)}{\rho}t$$

$$t = \frac{R^2\rho_w}{2D(\rho_\infty - \rho)}$$