12.5

Find parametric equations for the lines

1. The line through the point $P(3,-4,-1)$ parallel to the vector $\vec{i}+\vec{j}+\vec{k}$

$$x=3+t, \quad y=-4+t, \quad z=-1+t$$

2. The line through $P(1,2,-1)$ and $Q(-1,0,1)$

$$\overrightarrow{PQ}=-2\vec{i}-2\vec{j}+2\vec{k}$$

$$x=1-2t, \quad y=2-2t, \quad z=-1+2t$$

8. The line through $(2,4,5)$ $\perp$ to the plane. $3x+7y-5z=21$

$$\vec{n}=3\vec{i}+7\vec{j}-5\vec{k}$$

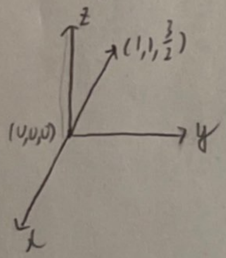
$$x=2+3t, \quad y=4+7t, \quad z=5-5t$$

Find parametrization for the line segments joining the points. Draw coordinate axes and sketch each segment.

13. $(0,0,0)$, $(1,1,\frac{3}{2})$

$$\overrightarrow{PQ}=\vec{i}+\vec{j}+\tfrac{3}{2}\vec{k}$$

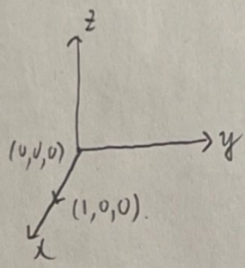
$x=t, \ y=t, \ z=\frac{3}{2}t$ where $0\le t\le 1$



14. $(0,0,0)$, $(1,0,0)$.

$$\overrightarrow{PQ}=\vec{i}$$

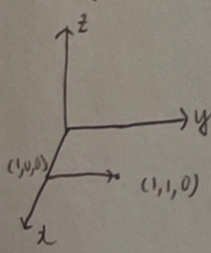
$x=t, \ y=0, \ z=0$, where $0\le t\le 1$



15. $(1,0,0)$, $(1,1,0)$

$$\overrightarrow{PQ}=\vec{j}$$

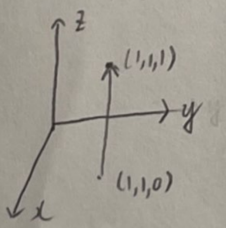
$x=1, \ y=t, \ z=0$, where $0\le t\le 1$



16. $(1,1,0)$, $(1,1,1)$.

$$\overrightarrow{PQ}=\vec{k}$$

$x=1; \ y=1, \ z=t.$, where $0\le t\le 1$



Find equations for the planes:

21. The plane through $P_0(0,2,-1)$ normal to $\vec{n}=3\vec{i}-2\vec{j}-\vec{k}$.

$$3(x-0)-2(y-2)-(z+1)=0$$

$$3x-2y-z=-3$$

22. The plane through $(1,-1,3)$ parallel to the plane. $3x+y+z=7$.

$$3(x-1)+(y+1)+(z-3)=0$$

$$3x+y+z=5$$

23. The plane through $\underset{A}{(1,1,-1)}$, $\underset{B}{(2,0,2)}$ and $\underset{C}{(0,-2,1)}$

$$\overrightarrow{AB}=(1,-1,3), \quad \overrightarrow{AC}=(-1,-3,2)$$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -1 & 3\\ -1 & -3 & 2\end{vmatrix}=7\vec{i}-5\vec{j}-4\vec{k}$$

$$7(x-1)-5(y-1)-4(z+1)=0$$

$$7x-5y-4z=6$$

29. Find the plane determined by the intersecting lines.

$L1: x=-1+t, \; y=2+t, \; z=1-t \quad -\infty<t<\infty$

$L2: x=1-4s, \; y=1+2s, \; z=2-2s \quad -\infty<s<\infty$

$\vec{v}_1=(1,1,-1), \; \vec{v}_2=(-4,2,-2)$

$\vec{n}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & -1\\ -4 & 2 & -2\end{vmatrix}=6\vec{j}+6\vec{k}$

$0(x+1)+6(y-2)+6(z-1)=0$

$6y-6z=18$

$y-6=3.$

31. Find a plane through $P_0(2,1,-1)$ and $\perp$ to the line of intersection of the planes $2x+y-z=3$, $x+2y+z=2$.

$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 1 & -1\\ 1 & 2 & 1\end{vmatrix}=3\vec{i}-3\vec{j}+3\vec{k}$

$3(x-2)-3(y-1)+3(z+1)=0$

$3x-3y+3z=0$

$x-y+z=0.$

Find the distance from the point to the line.

33. $S(0,0,12)$: $x=4t, \; y=-2t, \; z=2t.$

$P(0,0,0). \; \vec{v}=(4,-2,2)$

$\vec{PS}=(0,0,12)$

$\vec{PS}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & 12\\ 4 & -2 & 2\end{vmatrix}=24\vec{i}-48\vec{j}$

$=24(\vec{i}-2\vec{j})$

$d=\frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$

$d=\frac{24\sqrt{1^2+(-2)^2}}{\sqrt{4^2+(-2)^2+2^2}}$

$d=2\sqrt{30}$

34. $S(0,0,0)$: $x=5+3t, \; y=5+4t, \; z=-3-5t.$

$P=(5,5,-3) \; \vec{v}=(3,4,-5).$

$\vec{PS}=(-5,-5,3)$

$\vec{PS}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -5 & -5 & 3\\ 3 & 4 & -5\end{vmatrix}=-13\vec{i}-16\vec{j}-5\vec{k}$

$d=\frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$

$=\frac{\sqrt{13^2+(-16)^2+(-5)^2}}{\sqrt{3^2+4^2+(-5)^2}}$

$=\frac{\sqrt{450}}{\sqrt{50}}$

$=3.$

35. $S(2,1,3)$: $x=2+2t, \; y=1+6t, \; z=3.$

$P=(2,1,3) \; \vec{v}=(2,6,0)$

$\vec{PS}=(0,0,0)$

$\vec{PS}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & 0 & 0\\ 2 & 6 & 0\end{vmatrix}=0$

$d=\frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$

$d=0$

Find the distance from the point to the plane

39. $S(2,-3,4)$ $x+2y+2z=13.$

$P=(13,0,0)$ ← let $y,z=0.$

$\vec{n}=(1,2,2)$

$\vec{PS}=(-11,-3,4)$

$d=\left|\vec{PS}\cdot\frac{\vec{n}}{|\vec{n}|}\right|$

$=\left|(-11,-3,4)\cdot\frac{(1,2,2)}{3}\right|$

$=\left|\frac{-11-6+8}{3}\right|$

$=3$

40. $S(0,0,0)$, $3x+2y+6z=6.$

$P=(0,0,1) \quad \vec{n}=(3,2,6).$

$\vec{PS}=(0,0,-1)$

$d=\left|\vec{PS}\cdot\frac{\vec{n}}{|\vec{n}|}\right|$

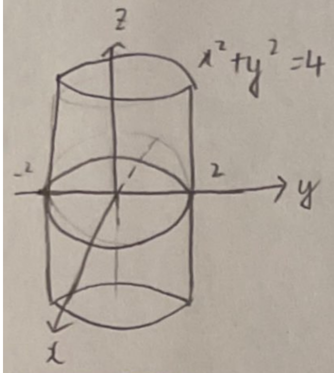
$=\left|\frac{6}{\sqrt{49}}\right|$

$=\frac{6}{7}$

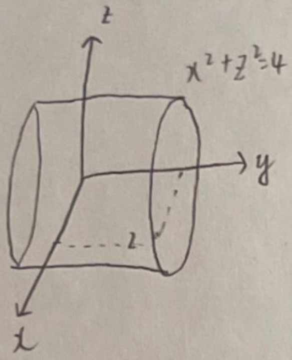
12.6

CYLINDERS

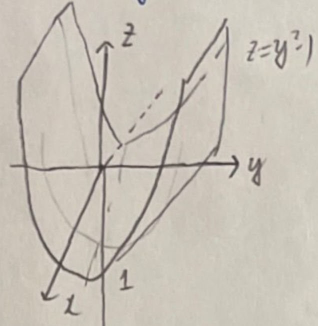
13. $x^2+y^2=4$.



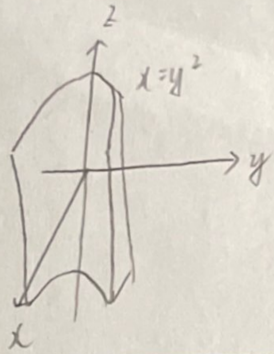
14. $x^2+z^2=4$



15 $z=y^2-1$

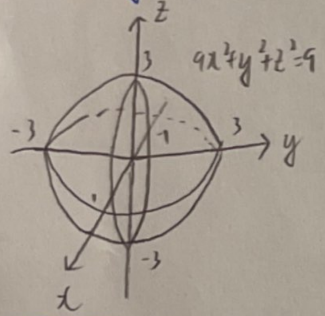


16. $x=y^2$

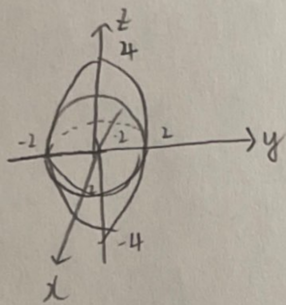


ELLIPSOIDS

21. $9x^2+y^2+z^2=9$

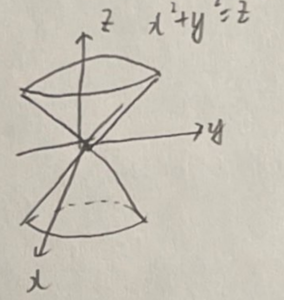


22. $4x^2+4y^2+z^2=16$.



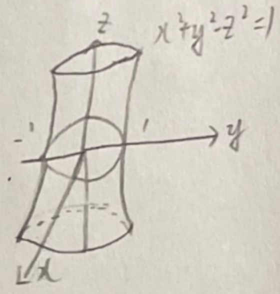
CONES

31. $x^2+y^2=z^2$



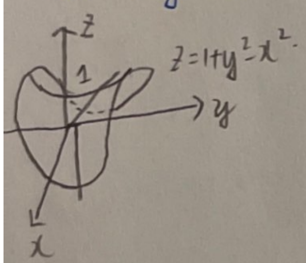
HYPERBOLOIDS

35. $x^2+y^2-z^2=1$

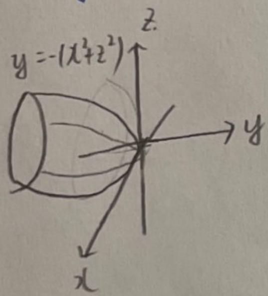


ASSORTED

47. $z=1+y^2-x^2$



49. $y=-(x^2+z^2)$



13.1

Find $\vec{v}$ and $\vec{a}$.

5. Motion on the circle $x^2+y^2=1$

$\vec{r}(t)=(\sin t)\vec{i}+(\cos t)\vec{j}$ $\quad t=\frac{\pi}{4}$ and $\frac{\pi}{2}$

$\vec{v}=\frac{d\vec{r}}{dt}=(\cos t)\vec{i}-(\sin t)\vec{j}$

$\vec{a}=\frac{d\vec{v}}{dt}=-(\sin t)\vec{i}-(\cos t)\vec{j}$

$\vec{v}(\frac{\pi}{4})=\frac{\sqrt{2}}{2}\vec{i}-\frac{\sqrt{2}}{2}\vec{j}$ $\quad \vec{a}(\frac{\pi}{4})=-\frac{\sqrt{2}}{2}\vec{i}-\frac{\sqrt{2}}{2}\vec{j}$

$\vec{v}(\frac{\pi}{2})=-\vec{j}$ $\quad a(\frac{\pi}{2})=-\vec{i}$

6 Motion on the circle $x^2+y^2=16$.

$\vec{r}(t)=(4\cos\frac{t}{2})\vec{i}+(4\sin\frac{t}{2})\vec{j}$ $\quad t=\pi$ and $\frac{3\pi}{2}$

$\vec{v}=(-2\sin\frac{t}{2})\vec{i}+(2\cos\frac{t}{2})\vec{j}$

$\vec{a}=(-\cos\frac{t}{2})\vec{i}-(\sin\frac{t}{2})\vec{j}$

$\vec{v}(\pi)=-2\vec{i}$ $\quad \vec{a}(\pi)=-\vec{j}$

$\vec{v}(\frac{3\pi}{2})=-\sqrt{2}\vec{i}-\sqrt{2}\vec{j}$ $\quad \vec{a}(\frac{3\pi}{2})=\frac{\sqrt{2}}{2}\vec{i}-\frac{\sqrt{2}}{2}\vec{j}$

7. Motion on the cycloid $x=t-\sin t$, $y=1-\cos t$.

$\vec{r}(t)=(t-\sin t)\vec{i}+(1-\cos t)\vec{j}$ $\quad t=\pi$ and $\frac{3\pi}{2}$

$\vec{v}=(1-\cos t)\vec{i}+(\sin t)\vec{j}$

$\vec{a}=(\sin t)\vec{i}-(\cos t)\vec{j}$

$\vec{v}(\pi)=2\vec{i}$ $\quad \vec{a}(\pi)=\vec{j}$

$\vec{v}(\frac{3\pi}{2})=\vec{i}-\vec{j}$ $\quad \vec{a}(\frac{3\pi}{2})=-\vec{i}$

Find $\vec{v}$, $\vec{a}$, speed and direction, write $\vec{v}$ as the product of its speed and direction.

9. $\vec{r}(t)=(t+1)\vec{i}+(t^2-1)\vec{j}+2t\vec{k}$ $\quad t=1$.

$\vec{v}=\vec{i}+2t\vec{j}+2\vec{k}$

$\vec{a}=2\vec{j}$

speed: $|\vec{v}(1)|=\sqrt{1^2+(2\times1)^2+2^2}=3$.

Direction: $\frac{\vec{v}(1)}{|\vec{v}(1)|}=\frac{1}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k}$

$\vec{v}(1)=|\vec{v}(1)|\cdot\frac{\vec{v}(1)}{|\vec{v}(1)|}=3(\frac{1}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{2}{3}\vec{k})$

10. $\vec{r}(t)=(1+t)\vec{i}+\frac{t^2}{\sqrt{2}}\vec{j}+\frac{t^3}{3}\vec{k}$, $t=1$

$\vec{v}=\vec{i}+\sqrt{2}t\vec{j}+t^2\vec{k}$

$\vec{a}=\sqrt{2}\vec{j}+2t\vec{k}$

Speed: $|\vec{v}(1)|=\sqrt{1^2+(\sqrt{2}\times1)^2+(1^2)^2}=2$.

Direction: $\frac{\vec{v}(1)}{|\vec{v}(1)|}=\frac{1}{2}\vec{i}+\frac{\sqrt{2}}{2}\vec{j}+\frac{1}{2}\vec{k}$

$\vec{v}(1)=|\vec{v}(1)|\cdot\frac{\vec{v}(1)}{|\vec{v}(1)|}=2(\frac{1}{2}\vec{i}+\frac{\sqrt{2}}{2}\vec{j}+\frac{1}{2}\vec{k})$

21. $\int_0^1 [t^3\vec{i}+7\vec{j}+(t+1)\vec{k}\,dt$

$=[(\frac{1}{4}t^4)\vec{i}+(7t)\vec{j}+(\frac{1}{2}t^2+t)\vec{k}]\Big|_0^1$

$=\frac{1}{4}\vec{i}+7\vec{j}+\frac{3}{2}\vec{k}$

22. $\int_1^2 [(6-6t)\vec{i}+3\sqrt{t}\vec{j}+(\frac{4}{t^2})\vec{k}]dt.$

$=[(6t-3t^2)\vec{i}+(2t^{\frac{3}{2}})\vec{j}-(4t^{-1})\vec{k}]\Big|_1^2$

$=(6-3)\vec{i}+(4\sqrt{2}-2)\vec{j}+(-2+4)\vec{k}$

$=3\vec{i}+(4\sqrt{2}-2)\vec{j}+2\vec{k}$

23. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [(\sin t)\vec{i}+(1+\cos t)\vec{j}+(\sec^2 t)\vec{k}]dt$

$=[(-\cos t)\vec{i}+(t+\sin t)\vec{j}+(\tan t)\vec{k}]\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$=(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2})\vec{i}+(\frac{\pi}{4}+\frac{\sqrt{2}}{2}+\frac{\pi}{4}+\frac{\sqrt{2}}{2})\vec{j}+(1+1)\vec{k}$

$=(\frac{\pi}{2}+\sqrt{2})\vec{j}+2\vec{k}$

Solve the initial value problem in 21-28 for $\vec{r}$ as a vector function of t

27. Differential equation: $\frac{d\vec{r}}{dt}=-t\vec{i}-t\vec{j}-t\vec{k}$

Initial condition: $\vec{r}(0)=\vec{i}+2\vec{j}+3\vec{k}$

$\vec{r}(t)=\int[(-t)\vec{i}-(t)\vec{j}-(t)\vec{k}]dt$

$=(-\frac{1}{2}t^2)\vec{i}-(\frac{1}{2}t^2)\vec{j}-(\frac{1}{2}t^2)\vec{k}+C$

$(-\frac{1}{2}0^2)\vec{i}-(\frac{1}{2}0^2)\vec{j}-(\frac{1}{2}0^2)\vec{k}+C=\vec{i}+2\vec{j}+3\vec{k}$

$C=\vec{i}+2\vec{j}+3\vec{k}$

$\vec{r}(t)=(-\frac{1}{2}t^2+1)\vec{i}-(\frac{1}{2}t^2+2)\vec{j}-(\frac{1}{2}t^2+3)\vec{k}$

28. Differential equation: $\frac{d\vec{r}}{dt}=(180t)\vec{i}+(180t-16t^2)\vec{j}$

Initial condition: $\vec{r}(0)=100\vec{j}$

$\vec{r}(t)=\int[(180t)\vec{i}+(180t-16t^2)\vec{j}]dt$

$=(90t^2)\vec{i}+(90t^2-\frac{16}{3}t^3)+C$

$(90\times0^2)\vec{i}+(90\times0^2-\frac{16}{3}\times0^3)+C=100\vec{j}$

$C=100\vec{j}$

$\vec{r}(t)=(90t^2)\vec{i}+(90t^2-\frac{16}{3}t^3+100)\vec{j}$

37. i Does the particle have constant speed? If so, what is its constant speed?
ii Is the particle's acceleration vector always $\perp$ to its velocity vector?
iii Does the particle move clockwise or counterclockwise around the circle.
iv. Does the particle begin at the point (1,0)?

a $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j}$, $t \geq 0$

$\vec{v}(t) = (\sin t)\vec{i} + (\cos t)\vec{j}$

$\vec{a}(t) = (-\cos t)\vec{i} + (-\sin t)\vec{j}$

i $|\vec{v}| = \sqrt{(-\sin^2 t) + \cos^2 t} = 1 \Rightarrow$ constant speed

ii $\vec{v} \cdot \vec{a} = [-\sin t \cdot (-\cos t)] + [\cos t \cdot (-\sin t)] = 0 \Rightarrow$ Yes

iii counterclockwise movement.

iv $\vec{r}(0) = (1,0) \Rightarrow$ Yes

b. $\vec{r}(t) = \cos(2t)\vec{i} + \sin(2t)\vec{j}$, $t \geq 0$.

$\vec{v}(t) = (-2\sin 2t)\vec{i} + (2\cos 2t)\vec{j}$

$\vec{a}(t) = (-4\cos 2t)\vec{i} - (4\sin 2t)\vec{j}$

i $|\vec{v}| = \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2} = 2 \Rightarrow$ constant speed

ii $\vec{v} \cdot \vec{a} = 8\sin 2t\cos 2t - 8\cos 2t\sin 2t = 0 \Rightarrow$ Yes

iii counterclockwise movement.

iv $\vec{r}(0) = (1,0) \Rightarrow$ Yes

c. $\vec{r}(t) = \cos(t-\frac{\pi}{2})\vec{i} + \sin(t-\frac{\pi}{2})\vec{j}$, $t \geq 0$.

$\vec{v}(t) = -\sin(t-\frac{\pi}{2})\vec{i} + \cos(t-\frac{\pi}{2})\vec{j}$

$\vec{a}(t) = -\cos(t-\frac{\pi}{2})\vec{i} - \sin(t-\frac{\pi}{2})\vec{j}$

i $|\vec{v}| = \sqrt{(-\sin(t-\frac{\pi}{2}))^2 + (\cos(t-\frac{\pi}{2}))^2} = 1 \Rightarrow$ constant speed.

ii $\vec{v} \cdot \vec{a} = \sin(t-\frac{\pi}{2})\cos(t-\frac{\pi}{2}) - \cos(t-\frac{\pi}{2})\sin(t-\frac{\pi}{2}) = 0 \Rightarrow$ Yes

iii counterclockwise movement.

iv $\vec{r}(0) = (0,-1) \Rightarrow$ No

38. Show that the vector-valued function

$\vec{r}(t) = (2\vec{i} + 2\vec{j} + \vec{k}) + \cos t(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}) + \sin t(\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k})$ describes the motion of a particle moving in the circle of radius 1 centered at the point (2,2,1) and lying in the plane $x+y-2z=2$.

Let $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$, $\vec{v} = \frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}$, $\vec{w} = \frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$

So that $\vec{r}(t) = \vec{u} + \cos t\,\vec{v} + \sin t\,\vec{w}$

$\vec{n} = \vec{i} + \vec{j} - 2\vec{k}$

$\vec{v} \cdot \vec{n} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

$\vec{w} \cdot \vec{n} = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} = 0.$

Therefore $\vec{r}(t)$ is lying in the plane

which means. $\vec{v}$ and $\vec{w}$ are parallel to the plane., and (2,2,1) is in the plane $x+y-2z=2$.

Moreover, each t, $\cos t\,\vec{v} + \sin t\,\vec{w}$ is a unit vector.

Starting at the point $(2+\frac{1}{\sqrt{2}}, 2-\frac{1}{\sqrt{2}})$ the vector $\vec{r}(t)$ traces out a circle of radius 1 and center (2,2,1) in the plane $x+y-z=2$.

41、~~curve~~ $y^2=2x$. from left to right at a constant speed of 5 unit per second. Find $\vec{v}$ through the poin(2,2)

$y^2=2x$

$2y\frac{dy}{dx}=2$

$\frac{dy}{dx}=\frac{1}{y}$

$\frac{1}{y}\Big|_{(2,2)}$

$=\frac{1}{2}$

direction: $\vec{i}+\frac{1}{2}\vec{j}$

$\vec{v}=\frac{5}{\sqrt{1+\frac{1}{4}}}(\vec{i}+\frac{1}{2}\vec{j})$

$=2\sqrt{5}\vec{i}+\sqrt{5}\vec{j}$

43. $(\frac{y}{3})^2+(\frac{z}{2})^2=1$ in the yz-plane in such a way that its position at time t is. $\vec{r}(t)=(3\cos t)\vec{j}+(2\sin t)\vec{k}$

Find the maximum and minimum value of $|\vec{v}|$ and $|\vec{a}|$

$\vec{v}(t)=(-3\sin t)\vec{j}+(2\cos t)\vec{k}$

$\vec{a}(t)=(-3\cos t)\vec{j}-(2\sin t)\vec{k}$

$|\vec{v}|$:

$|\vec{v}|^2=(9\sin^2t)+(4\cos^2t)$

$\frac{d|\vec{v}|^2}{dt}=18\sin t\cos t-8\cos t\sin t$

$=10\sin t\cos t$

Let $\frac{d|\vec{v}|^2}{dt}=0$

$10\sin t\cos t=0$

$\sin t\cos t=0$

$\therefore t=0$ or $t=\pi$ or $t=\frac{\pi}{2}$ or $t=\frac{3\pi}{2}$

$|\vec{v(t)}|=\sqrt{9\sin^2t+4\cos^2t}$

$|\vec{v(0)}|=\sqrt{0+4}=2.$

$|\vec{v(\pi)}|=\sqrt{0+4}=2.$

$|\vec{v(\frac{\pi}{2})}|=\sqrt{9+0}=3.$

$|\vec{v(\frac{3\pi}{2})}|=\sqrt{9+0}=3.$

$\therefore|\vec{v}|_{max}=3$

$|\vec{v}|_{min}=2.$

$|\vec{a}|$:

$|\vec{a}|^2=(9\cos^2t)+(4\sin^2t)$

$\frac{d|\vec{a}|^2}{dt}=-18\cos t\sin t+8\sin t\cos t$

$=-10\sin t\cos t$

Let $\frac{d|\vec{a}|^2}{dt}=0$

$-10\sin t\cos t=0$

$\sin t\cos t=0.$

$\therefore t=0$ or $t=\pi$ or $t=\frac{\pi}{2}$ or $t=\frac{3\pi}{2}$

$|\vec{a(t)}|=\sqrt{9\cos^2t+4\sin^2t}$

$|\vec{a(0)}|=\sqrt{9+0}=3$

$|\vec{a(\pi)}|=\sqrt{9+0}=3.$

$|\vec{a(\frac{\pi}{2})}|=\sqrt{0+4}=2.$

$|\vec{a(\frac{3\pi}{2})}|=\sqrt{0+4}=2.$

$\therefore|\vec{a}|_{max}=3$

$|\vec{a}|_{min}=2.$