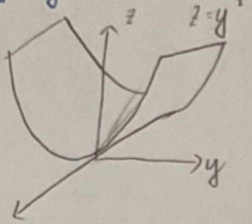
14.1

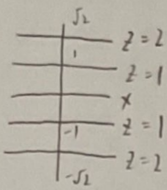
(a) by sketching the surface $z = f(x,y)$ and (b) by draw an assortment of level curves in the function's domain. Label each level curve with its function value.

19. $f(x,y) = y^2$

(a)

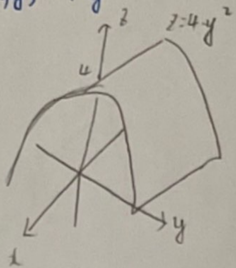


(b)

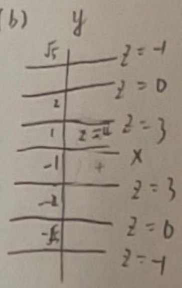


20. $f(x,y) = 4 - y^2$

(a)

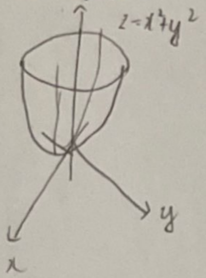


(b)

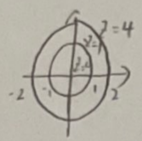


21. $f(x,y) = x^2 + y^2$

(a)

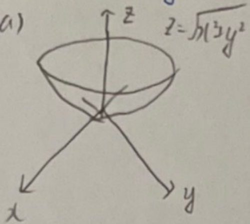


(b)

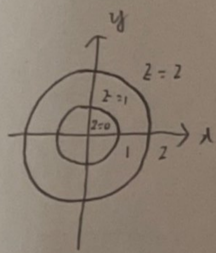


22. $f(x,y) = \sqrt{x^2 + y^2}$

(a)

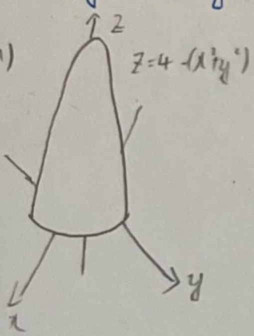


(b)

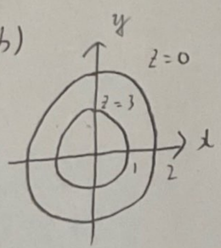


24. $f(x,y) = 4 - x^2 - y^2$

(a)



(b)



45. Does the function $f(x,y,z) = xyz$ have a max value on the line $x = 20 - t$, $y = t$, $z = 20$?

$f(x,y,z) = (20-t)(t)(20) = 400t - 20t^2$

$f'(x,y,z) = 400 - 40t$

Let $f'(x,y,z) = 0$

$400 - 40t = 0$

$t = 10$

$f''(x,y,z) = -40 < 0.$

Therefore, $f(x,y,z)$ has maximum value at $t = 10$

$x = 20 - 10 = 10$

$y = 10$

$z = 20.$

$f(x,y,z)_{max} = f(10,10,20)$

$= 10 \times 10 \times 20$

$= 2000$

46. Does the function $f(x,y,z) = xy - z$ have a minimum on the line $x = t-1$, $y = t-2$, $z = t+7$?

$f(x,y,z) = (t-1)(t-2) - (t+7) = t^2 - 4t - 5$

$f'(x,y,z) = 2t - 4$

Let $f'(x,y,z) = 0$

$2t - 4 = 0$

$t = 2.$

$f''(x,y,z) = 2 > 0.$

Therefore, $f(x,y,z)$ has minimum value at $t = 2.$

$x = 2 - 1 = 1$

$y = 2 - 2 = 0$

$z = 2 + 7 = 9$

$f(x,y,z)_{min} = f(1,0,9)$

$= 1 \times 0 - 9$

$= -9$

14.2

1. $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2}$

$= \frac{3\times0^2-0^2+5}{0^2+0^2+2}$

$= \frac{5}{2}$

2. $\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}}$

$= \frac{0}{\sqrt{4}}$

$= 0$

3. $\lim_{(x,y)\to(3,4)} \sqrt{x^2+y^2-1}$

$= \sqrt{3^2+4^2-1}$

$= 2\sqrt{6}$

4. $\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2$

$= \left(\frac{1}{2}+\frac{1}{-3}\right)^2$

$= \frac{1}{36}$

5. $\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec x \tan y$

$= \sec 0 \tan\frac{\pi}{4}$

$= 1\times1$

$= 1$

13. $\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-2xy+y^2}{x-y}$

$= \lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{(x-y)^2}{x-y}$

$= 1-1$

$= 0$

14. $\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^2-y^2}{x-y}$

$= \lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{(x-y)(x+y)}{x-y}$

$= 1+1$

$= 2$

15. $\lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy-y-2x+2}{x-1}$

$= \lim_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{y(x-1)-2(x-1)}{x-1}$

$= 1-2$

$= -1$

16. $\lim_{\substack{(x,y)\to(2,-4)\\ y\neq-4,\ x\neq x^2}} \frac{y+4}{x^2y-xy+4x^2-4x}$

$= \lim_{\substack{(x,y)\to(2,-4)\\ y\neq-4,\ x\neq x^2}} \frac{y+4}{x^2(y+4)-x(y+4)}$

$= \frac{1}{2^2-2}$

$= \frac{1}{2}$

17. $\lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$= \lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \left[\frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}{\sqrt{x}-\sqrt{y}}+2\right]$

$= \sqrt{0}+\sqrt{0}+2$

At what points (x,y) in the plane are the functions in. continuous?

27. a. $f(x,y)=\sin(x+y)$.

$\mathbb{R}$

b. $f(x,y)=\ln(x^2+y^2)$.

$x\neq0,\ y\neq0$

28. a. $f(x,y)=\frac{x+y}{x-y}$

$x\neq y$

b. $f(x,y)=\frac{y}{x^2+1}$

$\mathbb{R}$

31. a. $f(x,y,z)=x^2+y^2-2z^2$

$\mathbb{R}$

b. $f(x,y,z)=\sqrt{x^2+y^2-1}$

$x^2+y^2\neq1$

32. a. $f(x,y,z)=\ln xyz$

$xyz>0$

b. $f(x,y,z)=e^{x+y}\cos z$.

$\mathbb{R}$

50. Continuous extension

Define $f(0,0)$ in a way that extends $f(x,y)=xy\frac{x^2-y^2}{x^2+y^2}$ to be continuous at the origin.

$|xy(x^2-y^2)| = |xy||x^2-y^2| \le |x||y||x^2+y^2|$

$\Rightarrow \sqrt{x^2}\sqrt{y^2}|x^2+y^2| \le \sqrt{x^2+y^2}\sqrt{x^2+y^2}|x^2+y^2|$

$\Rightarrow \left|\frac{xy(x^2-y^2)}{x^2+y^2}\right| \le \frac{(x^2+y^2)^2}{x^2+y^2} = x^2+y^2$

$\Rightarrow -(x^2+y^2) \le \frac{xy(x^2-y^2)}{x^2+y^2} \le (x^2+y^2)$

By Sandwich Law.

$\lim_{(x,y)\to(0,0)} -(x^2+y^2) = \lim_{(x,y)\to(0,0)} x^2+y^2 = 0$.

So $\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$.

Sine $f(x,y)$ is continuous at $(0,0)$

$f(0,0) = \lim_{(x,y)\to(0,0)} f(x,y)$

$= 0$

By considering different paths of approach show that the functions have no limit as $(x,y)\to(0,0)$.

35 $f(x,y) = -\frac{x}{\sqrt{x^2+y^2}}$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=x\\ x>0}} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to0^+} -\frac{x}{\sqrt{x^2+x^2}} = \lim_{x\to0^+} -\frac{x}{\sqrt{2}|x|} = \lim_{x\to0^+} -\frac{x}{\sqrt{2}x} = -\frac{1}{\sqrt{2}}$$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=x\\ x<0}} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to0^-} -\frac{x}{\sqrt{x^2+x^2}} = \lim_{x\to0^-} -\frac{x}{\sqrt{2}|x|} = \lim_{x\to0^-} \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$$

36 $f(x,y) = \frac{x^4}{x^4+y^2}$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=0}} \frac{x^4}{x^4+y^2} = \lim_{x\to0} \frac{x^4}{x^4+0} = 1 \quad , \quad \lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=x^2}} \frac{x^4}{x^4+y^2} = \lim_{x\to0} \frac{x^4}{x^4+(x^2)^2} = \frac{1}{2}$$

37. $f(x,y) = \frac{x^4-y^2}{x^4+y^2}$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=0}} \frac{x^4-y^2}{x^4+y^2} = \lim_{y\to0} \frac{0-y^2}{0+y^2} = -1 \quad , \quad \lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=0}} \frac{x^4-y^2}{x^4+y^2} = \lim_{x\to0} \frac{x^4-0}{x^4+0} = 1$$

38 $f(x,y) = \frac{xy}{|xy|}$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=kx\\ k\neq0}} \frac{xy}{|xy|} = \lim_{x\to0} \frac{x(kx)}{|x(kx)|} = \lim_{x\to0} \frac{kx^2}{x^2|k|} = \frac{k}{|k|}.$$

if $k>0$, limit $=1$

if $k<0$, limit $=-1$

39. $g(x,y) = \frac{x-y}{x+y}$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=0}} \frac{x-y}{x+y} = \lim_{x\to0} \frac{x-0}{x+0} = 1 \quad , \quad \lim_{\substack{(x,y)\to(0,0)\\ \text{along } x=0}} = \lim_{y\to0} \frac{0-y}{0+y} = -1$$

14.3

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

1. $f(x,y) = 2x^2-3y-4$

$\frac{\partial f}{\partial x} = 4x \qquad \frac{\partial f}{\partial y} = -3$

2. $f(x,y) = x^2-xy+y^2$

$\frac{\partial f}{\partial x} = 2x-y \qquad \frac{\partial f}{\partial y} = -x+2y$

3. $f(x,y) = (x^2-1)(y+2)$

$\frac{\partial f}{\partial x} = 2x(y+2) \qquad \frac{\partial f}{\partial y} = x^2-1$

4. $f(x,y) = 5xy-7x^2-y^2+3x-6y+2.$

$\frac{\partial f}{\partial x} = 5y-14x+3 \qquad \frac{\partial f}{\partial y} = 5x-2y-6$

5. $f(x,y) = (xy-1)^2$

$\frac{\partial f}{\partial x} = 2y(xy-1) \qquad \frac{\partial f}{\partial y} = 2x(xy-1)$

6. $f(x,y) = (2x-3y)^3$

$\frac{\partial f}{\partial x} = 6(2x-3y)^2 \qquad \frac{\partial f}{\partial y} = -9(2x-3y)^2$

7. $f(x,y) = \sqrt{x^2+y^2}$

$\frac{\partial f}{\partial x} = \frac{y}{\sqrt{x^2+y^2}} \qquad \frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$

8. $f(x,y) = (x^3+\frac{y}{2})^{\frac{2}{3}}$

$$\frac{\partial f}{\partial x} = \frac{2}{3}(x^3+\frac{y}{2})^{-\frac{1}{3}}\cdot 3x^2 = \frac{2x^2}{\sqrt[3]{x^3+\frac{y}{2}}}$$

$$\frac{\partial f}{\partial x} = \frac{2}{3}(x^3+\frac{y}{2})^{-\frac{1}{3}}\cdot\frac{1}{2} = \frac{1}{3\sqrt[3]{x^3+\frac{y}{2}}}$$

9. $f(x,y) = \frac{1}{x+y}$

$\frac{\partial f}{\partial x} = -\frac{1}{(x+y)^2}$ $\quad \frac{\partial f}{\partial y} = -\frac{1}{(x+y)^2}$

10. $f(x,y) = \frac{x}{x^2+y^2}$

$\frac{\partial f}{\partial x} = \frac{x^2+y^2-2x^2}{(x^2+y)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$ $\quad \frac{\partial f}{\partial y} = \frac{0-2xy}{(x^2+y^2)^2} = -\frac{2xy}{(x^2+y^2)^2}$

Find all the second-order partial derivatives of the functions in

41 $f(x,y) = x+y+xy$.

$\frac{\partial f}{\partial x} = 1+y$ $\quad \frac{\partial f}{\partial y} = 1+x$

$\frac{\partial^2 f}{\partial x^2} = 0$ $\quad \frac{\partial^2 f}{\partial y^2} = 0$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 1$

42 $f(x,y) = \sin xy$

$\frac{\partial f}{\partial x} = y\cos xy$ $\quad \frac{\partial f}{\partial y} = x\cos xy$

$\frac{\partial^2 f}{\partial x^2} = -y^2 \sin xy$ $\quad \frac{\partial^2 f}{\partial y^2} = -x^2 \sin xy$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = \cos xy + y(-\sin xy \cdot x) = \cos xy - xy\sin xy$

43. $g(x,y) = x^2y + \cos y + y\sin x$.

$\frac{\partial g}{\partial x} = 2xy + y\cos x$ $\quad \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x$

$\frac{\partial^2 g}{\partial x^2} = 2y - y\sin x$ $\quad \frac{\partial^2 g}{\partial y^2} = -\cos y$

$\frac{\partial^2 g}{\partial x \partial y} = \frac{\partial^2 g}{\partial y \partial x} = 2x + \cos x$

44. $h(x,y) = xe^y + y + 1$

$\frac{\partial h}{\partial x} = e^y$ $\quad \frac{\partial h}{\partial y} = xe^y + 1$

$\frac{\partial^2 h}{\partial x^2} = 0$ $\quad \frac{\partial^2 h}{\partial y^2} = xe^y$

$\frac{\partial^2 h}{\partial x \partial y} = \frac{\partial^2 h}{\partial y \partial x} = e^y$

45 $r(x,y) = \ln(x+y)$

$\frac{\partial r}{\partial x} = \frac{1}{x+y}$ $\quad \frac{\partial r}{\partial y} = \frac{1}{x+y}$

$\frac{\partial^2 r}{\partial x^2} = -\frac{1}{(x+y)^2}$ $\quad \frac{\partial^2 r}{\partial y^2} = -\frac{1}{(x+y)^2}$

$\frac{\partial^2 r}{\partial x \partial y} = \frac{\partial^2 r}{\partial y \partial x} = -\frac{1}{(x+y)^2}$