H01

12.1

Describe the given set with a single equation or with a pair of equations.

19. The plane perpendicular to the.

a. x-axis at $(3,0,0)$. b. y-axis at $(0,-1,0)$. c. z-axis at $(0,0,-2)$.

$x=3$ $\quad$ $y=-1$ $\quad$ $z=-2$.

20. The plane through the point $(3,-1,2)$ $\perp$ to the

a. x-axis. b. y-axis c. z-axis.

$x=3$. $\quad$ $y=-1$ $\quad$ $z=2$

23. The circle of radius 2 centered at $(0,2,0)$ and lying in the

a. xy-plane b. yz-plane c. plane $y=2$.

$x^2+(y-2)^2=4, z=0$ $\quad$ $(y-2)^2+z^2=4, x=0$ $\quad$ $x^2+z^2=4, y=2$.

26. The set of points in space equidistant (空間等距) from the origin and the point $(0,2,0)$.

$$\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}=\sqrt{(x-0)^2+(y-2)^2+(z-0)^2}$$

$$x^2+y^2+z^2=x^2+(y-2)^2+z^2$$

$$y^2=y^2-4y+4$$

$$y=1.$$

27. The circle in which the plane through the point $(1,1,3)$ perpendicular to the z-axis meets the sphere of radius 5 centered at the origin.

$x^2+y^2+z^2=25$, $\quad$ when $z=3 \Rightarrow x^2+y^2+3^2=25$

$$x^2+y^2=16$$

12.2.

Find the component form of the vector.

9. The vector $\overrightarrow{PQ}$, where $P=(1,3)$ and $Q=(2,-1)$

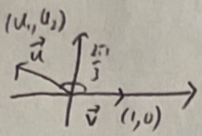
$$\overrightarrow{PQ}=(1,-4)$$

10. The vector $\overrightarrow{OP}$ where O is the origin and P is the midpoint of segment RS, where $R=(2,-1)$, $S=(-4,3)$

$$\overrightarrow{OP}=\left(\frac{2-4}{2}-0, \frac{-1+3}{2}-0\right)=(-1,1)$$

13. The unit vector that makes an angle $\theta = \frac{2\pi}{3}$ with the positive x-axis.

Let $\vec{u} = (u_1; u_2)$. , $\vec{v} = (1,0)$



$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$$

$$\cos\frac{2\pi}{3} = \frac{u_1}{1\cdot 1} \qquad \sqrt{u_1^2 + u_2^2} = 1$$

$$u_1 = -\frac{1}{2} \qquad \sqrt{(-\frac{1}{2})^2 + u_2^2} = 1 \qquad \vec{u} = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$$

$$u_2 = \frac{\sqrt{3}}{2}$$

Express each vector in the form $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$.

17. $\overrightarrow{P_1P_2}$ if P_1 is the point $(5,7,-1)$ and P_2 is the point $(2,9,-2)$.

$$\overrightarrow{P_1P_2} = (-3, 2, 1) = -3\vec{i} + 2\vec{j} + \vec{k}$$

18. $\overrightarrow{P_1P_2}$ if P_1 is the point $(1,2,0)$ and P_2 is the point $(-3,0,5)$.

$$\overrightarrow{P_1P_2} = (-4, -2, 5) = -4\vec{i} - 2\vec{j} + 5\vec{k}$$

41. Linear combination.

Let $\vec{u} = 2\vec{i} + \vec{j}$, $\vec{v} = \vec{i} + \vec{j}$, and $\vec{w} = \vec{i} - \vec{j}$. Find scalars a and b such that $\vec{u} = a\vec{v} + b\vec{w}$

$$a\vec{v} + b\vec{w} = (a+b)\vec{i} + (a-b)\vec{j}$$
$$\vec{u} = 2\vec{i} + \vec{j}$$
$$\Rightarrow \begin{cases} a+b = 2 \\ a-b = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{3}{2} \\ b = \frac{1}{2} \end{cases}$$

Let $\vec{u} = \vec{i} - 2\vec{j}$, $\vec{v} = 2\vec{i} + 3\vec{j}$, and $\vec{w} = \vec{i} + \vec{j}$. Write $\vec{u} = \vec{u_1} + \vec{u_2}$, where $\vec{u_1}$ is parallel to $\vec{v}$ and $\vec{u_2}$ is parallel to $\vec{w}$

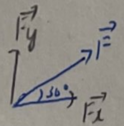
$$\vec{u_1} = a(2\vec{i} + 3\vec{j}) \qquad \vec{u} = \vec{u_1} + \vec{u_2}$$
$$\vec{u_2} = b(\vec{i} + \vec{j}) \qquad \vec{i} - 2\vec{j} = a(2\vec{i} + 3\vec{j}) + b(\vec{i} + \vec{j})$$
$$\vec{i} - 2\vec{j} = (2a+b)\vec{i} + (3a+b)\vec{j}$$
$$\Rightarrow \begin{cases} 2a+b = 1 \\ 3a+b = -2 \end{cases} \Rightarrow \begin{cases} a = -3 \\ b = 5 \end{cases}$$

$$\therefore \vec{u} = \vec{u_1} + \vec{u_2}$$
$$\vec{u} = -3\vec{v} + 5\vec{w}$$

43. Force vector.

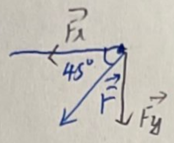
Pulling on a suitcase with a force F whose magnitude is $|F| = 10$ lb. Find the $\vec{i}$ and $\vec{j}$ components of $\vec{F}$.

$$|F_x| = |F|\cos\theta \qquad |F_y| = |F|\sin\theta$$
$$= 10 \times \cos 30^\circ \qquad = 10 \times \sin 30^\circ$$
$$= 5\sqrt{3}\ (\text{lb}) \qquad = 5\ (\text{lb})$$
$$\vec{F_x} = 5\sqrt{3}\,\vec{i} \qquad \vec{F_y} = 5\vec{j}$$



44. Force vector.

$|\vec{F}| = 12$ on a kite and makes a $45°$ angle with the horizontal. Find the horizontal and vertical components of $\vec{F}$



$|F_x| = |F|\cos\theta$
$= 12 \times \cos 45°$
$= 6\sqrt{2}$ (lb).

$|F_y| = |F|\sin\theta$
$= 12 \times \sin 45°$
$= 6\sqrt{2}$ (lb).

$\vec{F_x} = -6\sqrt{2}\vec{i}$

$\vec{F_y} = -6\sqrt{2}\vec{j}$

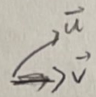
12.3

Find a. $\vec{v}\cdot\vec{u}$, $|\vec{v}|$, $|\vec{u}|$.

b. the cosine of the angle betwee $\vec{v}$ and $\vec{u}$

c. the scalar component of $\vec{u}$ in the direction of $\vec{v}$

d the vector $\text{proj}_v \vec{u}$.



1. $\vec{v} = 2\vec{i} - 4\vec{j} + \sqrt{5}\vec{k}$, $\vec{u} = -2\vec{i} + 4\vec{j} - \sqrt{5}\vec{k}$

$\vec{v}\cdot\vec{u} = 2\times(-2) + (-4)\times 4 + \sqrt{5}\times\sqrt{5} = -4 - 16 + 5 = -25$.

$|\vec{v}| = \sqrt{2^2 + (-4)^2 + (\sqrt{5})^2} = \sqrt{4+16+5} = 5$.

$|\vec{u}| = \sqrt{(-2)^2 + 4^2 + (-\sqrt{5})^2} = \sqrt{4+16+5} = 5$.

$\cos\theta = \frac{-25}{5\times 5} = -1$

$|\vec{u}|\cos\theta = 5 \cdot -1 = -5$.

$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{-25}{25} \times (2\vec{i} - 4\vec{j} + \sqrt{5}\vec{k})$

$= -2\vec{i} + 4\vec{j} - \sqrt{5}\vec{k}$

2. $\vec{v} = \frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}$, $\vec{u} = 5\vec{i} + 12\vec{j}$.

$\vec{v}\cdot\vec{u} = 3$

$|\vec{v}| = \sqrt{(\frac{3}{5})^2 + 0^2 + (\frac{4}{5})^2} = 1$.

$|\vec{u}| = \sqrt{5^2 + 12^2 + 0} = 13$.

$\cos\theta = \frac{3}{13}$

$|\vec{u}|\cos\theta = 13\frac{3}{13} = 3$

$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{3}{1^2} \times (\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k})$

$= \frac{9}{5}\vec{i} + \frac{12}{5}\vec{k}$

3. $\vec{v} = 10\vec{i} + 11\vec{j} - 2\vec{k}$, $\vec{u} = 3\vec{j} + 4\vec{k}$.

$\vec{v}\cdot\vec{u} = 33 - 8 = 25$.

$|\vec{v}| = \sqrt{100+121+4} = 15$

$|\vec{u}| = \sqrt{3^2+4^2} = 5$

$\cos\theta = \frac{25}{15\times 5} = \frac{1}{3}$

$|\vec{u}|\cos\theta = \frac{5}{3}$

$\text{proj}_{\vec{v}}\vec{u} = \frac{25}{15^2} \cdot (10\vec{i} + 11\vec{j} - 2\vec{k})$

$= \frac{10}{9}\vec{i} + \frac{11}{9}\vec{j} - \frac{2}{9}\vec{k}$

Write $\vec u$ as the sum of a vector parallel to $\vec v$ and a vector orthogonato $\vec v$

17. $\vec u = 3\vec j + 4\vec k$, $\vec v = \vec i + \vec j$

$\vec u = \text{proj}_{\vec v}\vec u + (\vec u - \text{proj}_{\vec v}\vec u)$

$= \frac{\vec u\cdot\vec v}{|\vec v|^2}\vec v + \left(\vec u - \frac{\vec u\cdot\vec v}{|\vec v|^2}\vec v\right)$

$= \frac{3}{2}\vec v + \vec u - \frac{3}{2}\vec v$

$= \left(\frac{3}{2}\vec i + \frac{3}{2}\vec j\right) + \left[(3\vec j + 4\vec k) - \left(\frac{3}{2}\vec i + \frac{3}{2}\vec j\right)\right]$

$= \left(\frac{3}{2}\vec i + \frac{3}{2}\vec j\right) + \left(-\frac{3}{2}\vec i + \frac{3}{2}\vec j + 4\vec k\right)$

18. $\vec u = \vec j + \vec k$, $\vec v = \vec i + \vec j$

$\vec u = \text{proj}_{\vec v}\vec u + (\vec u - \text{proj}_{\vec v}\vec u)$.

$= \frac{\vec u\cdot\vec v}{|\vec v|^2}\vec v + \left(\vec u - \frac{\vec u\cdot\vec v}{|\vec v|^2}\vec v\right)$

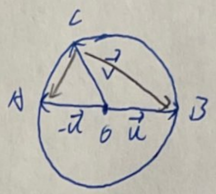
$= \frac{1}{2}\vec v + \left(\vec u - \frac{1}{2}\vec v\right)$.

$= \left(\frac{1}{2}\vec i + \frac{1}{2}\vec j\right) + \left[(\vec j + \vec k) - \left(\frac{1}{2}\vec i + \frac{1}{2}\vec j\right)\right]$

$= \left(\frac{1}{2}\vec i + \frac{1}{2}\vec j\right) + \left(-\frac{1}{2}\vec i + \frac{1}{2}\vec j + \vec k\right)$

22. Orthogonality on a circle.

Suppose that AB is the diameter of a circle with center O and that C is a point on one of the two arcs joining A and B. Show that $\vec{CA}$ and $\vec{CB}$ are orthogonal.



$\vec{CB} = \vec{OB} - \vec{OC}$

$\vec{CA} = \vec{OA} - \vec{OC}$

$\vec{CA}\cdot\vec{CB} = (\vec{OA} - \vec{OC})(\vec{OB} - \vec{OC})$

$= (-\vec u - \vec v)\cdot(\vec u - \vec v)$

$= -|\vec u|^2 + \vec u\cdot\vec v - \vec u\cdot\vec v + |\vec v|^2$

$= -|\vec u|^2 + |\vec v|^2$.

Since $|\vec u| = |\vec v|$, $\vec{CA}\cdot\vec{CB} = 0$, so that $\vec{CA}$ and $\vec{CB}$ are orthogonal.

菱形

Diagonals of a rhombus.

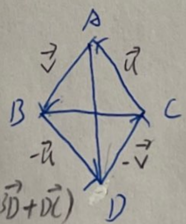
23 Show that the diagonals of a rhombus are perpendicular.

Let $\vec{AB} = \vec v$, $\vec{DC} = -\vec v$,

Diagonals:

$\vec{AD} = \vec{AB} + \vec{BD}$

$\vec{BC} = \vec{BD} + \vec{DC}$



$\vec{AD}\cdot\vec{BC} = (\vec{AB} + \vec{BD})\cdot(\vec{BD} + \vec{DC})$

$= (\vec v - \vec u)\cdot(-\vec u - \vec v)$

$= -\vec u\vec v - |\vec v|^2 + |\vec u|^2 + \vec u\vec v$

$= -|\vec v|^2 + |\vec u|^2$

By definition.

$|\vec v| = |\vec u|$.

So that:

$\vec{AD}\cdot\vec{BC} = 0$, the diagonals of a rhombus are perpendicular.

24. Perpendicular diagonals.

Show that squares are the only rectangles with perpendicular diagonals.

Let $\vec{AB} = \vec u$, $\vec{CD} = -\vec u$,

$\vec{BC} = \vec v$, $\vec{DA} = -\vec v$

Diagonals:

$\vec{AC} = \vec{AB} + \vec{BC}$

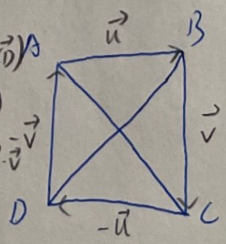
$\vec{DB} = \vec{BC} + \vec{CD}$

$\vec{AC}\cdot\vec{DB} = (\vec{AB} + \vec{BC})\cdot(\vec{BC} + \vec{CD})$

$= (\vec u + \vec v)\cdot(\vec v - \vec u)$

$= \vec u\vec v - |\vec u|^2 + |\vec v|^2 - \vec u\cdot\vec v$

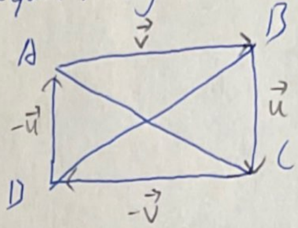
$= -|\vec u|^2 + |\vec v|^2$



$\vec{AC}\perp\vec{DB}$ if and only if $\vec{AC}\cdot\vec{DB} = 0$, which means $|\vec u| = |\vec v|$, so squares are the only rectangles with perpendicular diagonals.

25. When parallelograms are rectangles

Prove that a parallelogram is a rectangle if and only if its diagonals are equal in length.



Let $\vec{AB} = \vec{v} = (v_1\vec{i} + v_2\vec{j})$, $\vec{BC} = \vec{u} = (u_1\vec{i} + u_2\vec{j})$.

$\vec{CD} = -\vec{v} = (-v_1\vec{i} - v_2\vec{j})$, $\vec{DA} = -\vec{u} = (-u_1\vec{i} - u_2\vec{j})$.

Diagonals: $\vec{AC} = \vec{AB} + \vec{BC} = (v_1\vec{i} + v_2\vec{j}) + (u_1\vec{i} + u_2\vec{j})$

$\vec{BD} = \vec{BC} + \vec{CD} = (u_1\vec{i} + u_2\vec{j}) + (-v_1\vec{i} - v_2\vec{j})$.

If $|\vec{AC}| = |\vec{BD}|$

$$|(v_1+u_1)\vec{i} + (v_2+u_2)\vec{j}| = |(v_1-u_1)\vec{i} + (v_2-u_2)\vec{j}|$$

$$\sqrt{(v_1+u_1)^2 + (v_2+u_2)^2} = \sqrt{(v_1-u_1)^2 + (v_2-u_2)^2}$$

$$v_1u_1 + v_2u_2 = 0.$$

$$\Rightarrow (v_1\vec{i} + v_2\vec{j}) \cdot (u_1\vec{i} + u_2\vec{j}) = 0.$$

$\vec{AB} \cdot \vec{BC} = 0$ which means the parallelogram must be a rectangle.

29. a. Cauchy-Schwartz inequality.

Use the fact that $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$ to show that the inequality $|\vec{u} \cdot \vec{v}| \le |\vec{u}||\vec{v}|$ hold for any vector $\vec{u}$ and $\vec{v}$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta.$$

$$|\vec{u} \cdot \vec{v}| = |\vec{u}||\vec{v}||\cos\theta|$$

$$|\vec{u} \cdot \vec{v}| \le |\vec{u}||\vec{v}|$$

$\}$ Since $|\cos\theta| \le 1$.

b. Under what circumstances, if any, does $|\vec{u} \cdot \vec{v}|$ equal $|\vec{u}||\vec{v}|$?

$|\vec{u}\vec{v}|$ equals $|\vec{u}||\vec{v}|$ when $\cos\theta = 1$, $\vec{u}$ and $\vec{v}$ are both 0. or when $\theta = \{0 | \pi + k\pi, k \in \mathbb{Z}\}$

12.4

Find the length and direction of $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

1. $\vec{u} = 2\vec{i} - 2\vec{j} - \vec{k}$, $\vec{v} = \vec{i} - \vec{k}$

$\vec{u} \times \vec{v}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & -1 \\ 1 & 0 & -1 \end{vmatrix} = 2\vec{i} + \vec{j} + 2\vec{k}$$

length: $\sqrt{2^2 + 1^2 + 2^2}$

$= 3$

Direction: $\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$

$\vec{v} \times \vec{u} = -\vec{u} \times \vec{v}$.

length $= 3$.

Direction: $-\frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$

2. $\vec{u} = 2\vec{i} + 3\vec{j}$, $\vec{v} = -\vec{i} + \vec{j}$

$\vec{u} \times \vec{v}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ -1 & 1 & 0 \end{vmatrix} = 5\vec{k}$$

length: $\sqrt{5^2}$
$= 5$

Direction: $\vec{k}$

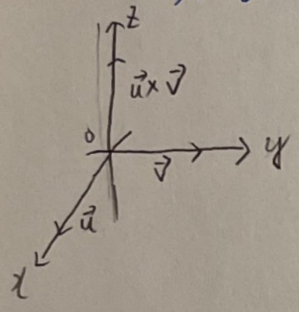
$\vec{v} \times \vec{u} = -\vec{u} \times \vec{v}$

length: 5

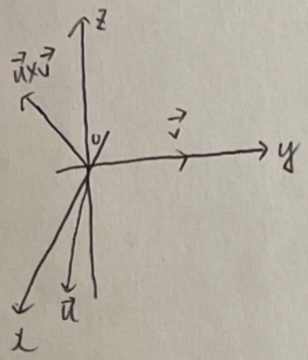
Direction: $\vec{k}$

Sketch the coordinate axes and then include the vector $\vec{u}, \vec{v}$ and $\vec{u} \times \vec{v}$ as vectors starting at the origin.

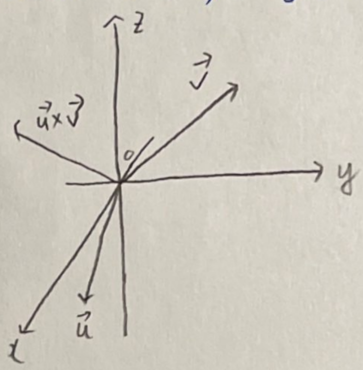
9. $\vec{u} = \vec{i}$, $\vec{v} = \vec{j}$



10. $\vec{u} = \vec{i} - \vec{k}$, $\vec{v} = \vec{j}$



11. $\vec{u} = \vec{i} - \vec{k}$, $\vec{v} = \vec{j} + \vec{k}$



23. Parallel and perpendicular vectors.

Let $\vec{u} = 5\vec{i} - \vec{j} + \vec{k}$, $\vec{v} = \vec{j} - 5\vec{k}$, $\vec{w} = -15\vec{i} + 3\vec{j} - 3\vec{k}$

(a) perpendicular?

$\vec{u} \cdot \vec{v} = 0 - 1 - 5 = -6$

$\vec{u} \cdot \vec{w} = -75 - 3 - 3 = -81$

$\vec{v} \cdot \vec{w} = 0 + 3 + 15 = 18.$

no vectors are perpendicular.

(b) Parallel?

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -1 & 1 \\ 0 & 1 & -5 \end{vmatrix} = 4\vec{i} + 25\vec{j} + 5\vec{k}$$

$$\vec{u} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & -1 & 1 \\ -15 & 3 & -3 \end{vmatrix} = 0 + 0 + 0 = 0.$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -5 \\ -15 & 3 & -3 \end{vmatrix} = 5\vec{i} - 75\vec{j} + 15\vec{k}.$$

So $\vec{u} \parallel \vec{w}$

24. Parallel and perpendicular vectors.

Let $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$, $\vec{v} = -\vec{i} + \vec{j} + \vec{k}$, $\vec{w} = \vec{i} + \vec{k}$ $\vec{r} = -\frac{\pi}{2}\vec{i} - \pi\vec{j} + \frac{\pi}{2}\vec{k}$

(a) perpendicular?

$\vec{u} \cdot \vec{v} = -1 + 2 - 1 = 0.$ So $\vec{u} \perp \vec{v}$,

$\vec{u} \cdot \vec{w} = 1 + 0 - 1 = 0.$ $\vec{u} \perp \vec{w}$

$\vec{u} \cdot \vec{r} = -\frac{\pi}{2} - 2\pi - \frac{\pi}{2} = -3\pi$ $\vec{v} \perp \vec{w}$

$\vec{v} \cdot \vec{w} = -1 + 0 + 1 = 0.$ $\vec{v} \perp \vec{r}$

$\vec{v} \cdot \vec{r} = \frac{\pi}{2} - \pi + \frac{\pi}{2} = 0$ $\vec{w} \perp \vec{r}$

$\vec{w} \cdot \vec{r} = -\frac{\pi}{2} + \frac{\pi}{2} = 0.$

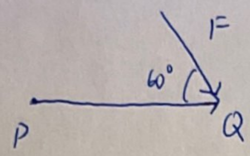
(b) parallel?

$$\vec{u} \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -\frac{\pi}{2} & -\pi & \frac{\pi}{2} \end{vmatrix} = 0 + 0 + 0.$$

So $\vec{u} \parallel \vec{r}$

Find the magnitude of the torque exerted by $\vec{F}$ on the bolt at P if $|\vec{PQ}| = 8$ in. and $|\vec{F}| = 30$ lb.

25. $|\vec{PQ} \times \vec{F}|$

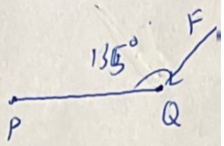


$= |\vec{PQ}||\vec{F}| \sin 60°$

$= 8 \times 30 \times \frac{\sqrt{3}}{2}$

$= 120\sqrt{3}$ (lb)

$|\vec{PQ} \times \vec{F}|$



$= |\vec{PQ}||\vec{F}| \sin 135°$

$= 8 \times 30 \times \frac{\sqrt{2}}{2}$

$= 120\sqrt{2}$ (lb)

43. Triangle area.

Find a formula for the area of the triangle in xy-plane with vertices at $(0,0)$, (a_1, a_2) and (b_1, b_2).

Let $\vec{A} = a_1\vec{i} + a_2\vec{j}$, $\vec{B} = b_1\vec{i} + b_2\vec{j}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

area is $\frac{1}{2}|\vec{A} \times \vec{B}|$

$= \pm\frac{1}{2}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

Since area must be nonnegative number, formula for area is $\frac{1}{2}\left|\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right|$

Find a concise (精简) formula for the area of a triangle with vertices (顶点) (a_1, a_2), (b_1, b_2) and (c_1, c_2)

Let $\vec{A} = a_1\vec{i} + a_2\vec{j}$, $\vec{B} = b_1\vec{i} + b_2\vec{j}$, $\vec{C} = c_1\vec{i} + c_2\vec{j}$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 - a_1 & b_2 - a_2 & 0 \\ c_1 - a_1 & c_2 - a_2 & 0 \end{vmatrix} = \begin{vmatrix} b_1 - a_1 & b_2 - a_2 \\ c_1 - a_2 & c_2 - a_2 \end{vmatrix} \vec{k}$$

area is $\frac{1}{2}|\vec{AB} \times \vec{AC}|$

$= \frac{1}{2}\begin{vmatrix} b_1 - a_1 & b_2 - a_2 \\ c_1 - a_1 & c_2 - a_2 \end{vmatrix}$

$= \frac{1}{2}|(b_1 - a_1)(c_2 - a_2) - (b_2 - a_2)(c_1 - a_1)|$

$= \frac{1}{2}|b_1c_2 - b_1a_2 - a_1c_2 + \cancel{a_1a_2} - b_2c_1 + b_2a_1 + a_2c_1 - \cancel{a_1a_2}|$

$= \frac{1}{2}|a_1(b_2 - c_2) + a_2(c_1 - b_1) + b_1c_2 - b_2c_1|$

$= \pm\frac{1}{2}\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix}$

Since area must be nonnegative number, formula for area is $\frac{1}{2}\left|\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix}\right|$