HW6

Find equations for the (a) tangent plane and (b) normal line at the point P_0 on the given surface.

1. $x^2+y^2+z^2=3$, $P_0(1,1,1)$

$\nabla f = 2x\vec{i}+2y\vec{j}+2z\vec{k}$

$\nabla f(1,1,1) = 2\vec{i}+2\vec{j}+2\vec{k}$

Tangent plane:

$2(x-1)+2(y-1)+2(z-1)=0$

Normal line:

$x=1+2t,\ y=1+2t,\ z=1+2t.$

2. $x^2+y^2-z^2=18$, $P_0(3,5,-4)$

$\nabla f = 2x\vec{i}+2y\vec{j}-2z\vec{k}$

$\nabla f(3,5,-4) = 6\vec{i}+10\vec{j}+8\vec{k}$

Tangent plane:

$6(x-3)+10(y-5)+8(z+4)=0$

Normal line:

$x=3+6t,\ y=5+10t,\ z=-4+8t$

3. $2z-x^2=0$, $P_0(2,0,2)$

$\nabla f = -2x\vec{i}+2\vec{k}$

$\nabla f(2,0,2) = -4\vec{i}+2\vec{k}$

Tangent plane:

$-4(x-2)+2(z-2)=0$

Normal line:

$x=2-4t,\ y=0,\ z=2+2t$

Find parametric equations for the line tangent to the curve of intersection of the surfaces at the give point.

13. Surfaces: $\overset{f}{x+y^2+2z=4}$, $\overset{g}{x=1}$

Point: $(1,1,1)$

$\nabla f = \vec{i}+2y\vec{j}+2\vec{k}$ $\quad \nabla g = \vec{i}$

$\nabla f(1,1,1) = \vec{i}+2\vec{j}+2\vec{k}$ $\quad \nabla g(1,1,1)=\vec{i}$

$\vec{v} = \nabla f\times\nabla g$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 2\vec{j}-2\vec{k}$

Tangent line:

$x=1,\ y=1+2t,\ z=1-2t.$

14. Surfaces: $xyz=1$, $x^2+2y^2+3z^2=6$.

Point: $(1,1,1)$.

$\nabla f = yz\vec{i}+xz\vec{j}+xy\vec{k}$ $\quad \nabla g = 2x\vec{i}+4y\vec{j}+6z\vec{k}$

$\nabla f(1,1,1) = \vec{i}+\vec{j}+\vec{k}$ $\quad \nabla g(1,1,1) = 2\vec{i}+4\vec{j}+6\vec{k}$

$\vec{v} = \nabla f\times\nabla g$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = 2\vec{i}-4\vec{j}+2\vec{k}$

Tangent line:

$x=1+2t,\ y=1-4t,\ z=1+2$

19. By about how much will $f(x,y,z)=\ln\sqrt{x^2+y^2+z^2}$ change if the point $\overset{P(x,y,z)}{\text{moves}}$ from $P_0(3,4,12)$ a distance of $ds=0.1$ unit in the direction of $3\vec{i}+6\vec{j}-2\vec{k}$?

$\nabla f = \left(\frac{x}{x^2+y^2+z^2}\right)\vec{i}+\left(\frac{y}{x^2+y^2+z^2}\right)\vec{j}+\left(\frac{z}{x^2+y^2+z^2}\right)\vec{k}$

$\nabla f(3,4,12) = \frac{3}{169}\vec{i}+\frac{4}{169}\vec{j}+\frac{12}{169}\vec{k}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{3}{7}\vec{i}+\frac{6}{7}\vec{j}-\frac{2}{7}\vec{k}$

$df = (\nabla f\cdot u)\cdot ds$ $\quad -\frac{9}{1183}$

$= 0.0008.$

20. $f(x,y,z)=e^x\cos yz$. $ds=0.1$ unit, $\overset{P_0(0,0,0)}{}$ direction $2\vec{i}+2\vec{j}-2\vec{k}$.

$\nabla f = e^x\cos yz\,\vec{i} - ze^x\sin yz\,\vec{j} - ye^x\sin yz\,\vec{k}$

$\nabla f(0,0,0) = \vec{i}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{3}}\vec{i}+\frac{1}{\sqrt{3}}\vec{j}-\frac{1}{\sqrt{3}}\vec{k}$

$df = (\nabla f\cdot u)\cdot ds$

$= 0.0577.$

21. $g(x,y,z)=x+x\cos z-y\sin z+y$, $P_0(2,-1,0)$, $P_1(0,1,2)$, $ds=0.2$ unit.

$\nabla g = (1+\cos z)\vec{i}+(1-\sin z)\vec{j}+(-\sin z-y\cos z)\vec{k}$

$\nabla g(2,-1,0) = 2\vec{i}+\vec{j}+\vec{k}$

$\overrightarrow{P_0P_1} = -2\vec{i}+2\vec{j}+2\vec{k}$

$\vec{u} = -\frac{1}{\sqrt{3}}\vec{i}+\frac{1}{\sqrt{3}}\vec{j}+\frac{1}{\sqrt{3}}\vec{k}$

$df = (\nabla f\cdot u)\cdot ds$

$= 0$

24. Changing temperature along a space curve.

$T(x,y,z)=2x^2-xyz$, $x=2t^2$, $y=3t$, $z=-t^2$

a. How fast ~ $P(8,6,-4)$?

$\nabla T=(4x-yz)\vec{i}-xz\vec{j}-xy\vec{k}$ $\nabla T_{(8,6,-4)}=56\vec{i}+32\vec{j}-48\vec{k}$

$r(t)=2t^2\vec{i}+3t\vec{j}-t^2\vec{k}$

$\Rightarrow$ $t=2$ at the point $(8,6,-4)$

$v(t)=4t\vec{i}+3\vec{j}-2t\vec{k}$

$v(2)=8\vec{i}+3\vec{j}-4\vec{k}$

$\vec{u}=\frac{\vec{v}}{|\vec{v}|}=\frac{8}{\sqrt{89}}\vec{i}+\frac{3}{\sqrt{89}}\vec{j}-\frac{4}{\sqrt{89}}\vec{k}$

$\nabla T\cdot\vec{u}=\frac{8}{\sqrt{89}}\times 56+\frac{3}{\sqrt{89}}\times 32+\frac{-4}{\sqrt{89}}\times(-48)$

$=\frac{736}{\sqrt{89}}$ °C/m.

b. per second at P?

$(\nabla T\cdot u)\cdot|\vec{v}|$

$=\left(\frac{736}{\sqrt{89}}\right)\sqrt{89}$

$=736$°C/sec.

$\frac{dT}{dt}=\frac{\partial T}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial T}{\partial y}\frac{dy}{dt}$

$=\nabla T\cdot\vec{v}$

$=\nabla T\cdot\vec{u}|\vec{v}|$

Find the linearization $L(x,y)$ of the function at each point.

25. $f(x,y)=x^2+y^2+1$ at a. $(0,0)$, b $(1,1)$

$f(0,0)=1$

$f_x(0,0)=2x|_{(0,0)}=0$

$f_y(0,0)=2y|_{(0,0)}=0$.

$L(x,y)=1+0(x-0)+0(y-0)$

$=1$

$f(1,1)=3$.

$f_x(1,1)=2$

$f_y(1,1)=2$.

$L(x,y)=3+2(x-1)+2(y-1)$

$=2x+2y-1$

26 $f(x,y)=(x+y+2)^2$ at a $(0,0)$ b, $(1,2)$

$f(0,0)=4$

$f_x(0,0)=2(x+y+2)|_{(0,0)}=0$

$f_y(0,0)=2(x+y+2)|_{(0,0)}=0$.

$L(x,y)=4$

$f(1,2)=25$

$f_x(1,2)=10$

$f_y(1,2)=10$.

$L(x,y)=25+10(x-1)+10(y-2)$

$=10x+10y-5$.

27. $f(x,y)=3x-4y+5$ at $a(0,0)$, $b(1,1)$.

$f(0,0)=5$

$f_x(0,0)=3$

$f_y(0,0)=-4$

$L(x,y)=5+3x-4y$

$f(1,1)=4$.

$f_x(1,1)=3$

$f_y(1,1)=-4$

$L(x,y)=4+3(x-1)-4(y-1)$

$=3x-4y+5$.

41. $f(x,y,z)=e^x+\cos(y+z)$ at a. $(0,0,0)$, b $(0,\frac{\pi}{2},0)$ c. $(0,\frac{\pi}{4},\frac{\pi}{4})$

$f(0,0,0)=1+1=2$.

$f_x(0,0,0)=e^x|_{(0,0,0)}=1$

$f_y(0,0,0)=-\sin(y+z)|_{(0,0,0)}=0$

$f_z(0,0,0)=-\sin(y+z)|_{(0,0,0)}=0$.

$L(x,y,z)=2+x$

$f(0,\frac{\pi}{2},0)=1$

$f_x(0,\frac{\pi}{2},0)=1$

$f_y(0,\frac{\pi}{2},0)=-1$

$f_z(0,\frac{\pi}{2},0)=-1$.

$L(x,y,z)=1+x-(y-\frac{\pi}{2})-z$

$=x-y-z+1+\frac{\pi}{2}$

$f(0,\frac{\pi}{4},\frac{\pi}{4})=1$

$f_x(0,\frac{\pi}{4},\frac{\pi}{4})=1$

$f_y(0,\frac{\pi}{4},\frac{\pi}{4})=-1$

$f_z(0,\frac{\pi}{4},\frac{\pi}{4})=-1$

$L(x,y,z)=1+x-(y-\frac{\pi}{4})-(z-\frac{\pi}{4})$ $=x-y-z+1+\frac{\pi}{2}$

42 $f(x,y,z) = \tan^{-1}(xyz)$ at $a(1,0,0)$ $b(1,1,0)$ $c(1,1,1)$

$f(1,0,0) = 0.$

$f_x(1,0,0) = \dfrac{yz}{(xyz)^2+1}\Big|_{(1,0,0)} = 0$

$f_y(1,0,0) = \dfrac{xz}{(xyz)^2+1}\Big|_{(1,0,0)} = 0$

$f_z(1,0,0) = \dfrac{xy}{(xyz)^2+1}\Big|_{(1,0,0)} = 0$

$L(x,y,z) = 0.$

$f(1,1,0) = 0.$

$f_x(1,1,0) = 0$

$f_y(1,1,0) = 0$

$f_z(1,1,0) = 1$

$L(x,y,z) = z$

$f(1,1,1) = \frac{\pi}{4}$

$f_x(1,1,1) = \frac{1}{2}$

$f_y(1,1,1) = \frac{1}{2}$

$f_z(1,1,1) = \frac{1}{2}$

$L(x,y,z) = \frac{\pi}{4} + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) + \frac{1}{2}(z-1)$

$= \frac{1}{2}x + \frac{1}{2}y + \frac{1}{2}z - \frac{3}{2} + \frac{\pi}{4}$

14.7

Find all local maxima, local minima, saddle points.

1. $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4.$

$f_x = 2x + y + 3 = 0$

$f_y = x + 2y - 3 = 0.$

$\begin{cases} 2x+y+3=0 \\ x+2y-3=0. \end{cases}$

$\Rightarrow \begin{cases} x=-3 \\ y=3. \end{cases}$

$f_{xx} = 2,\ f_{yy} = 2,\ f_{xy} = 1$

$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3 > 0.$, and $f_{xx} > 0$

$f(-3,3) = 9-9+9+9-9+4$

$= 5.$ is the local minima

28. $f(x,y) = \frac{1}{x} + xy + \frac{1}{y}$

$f_x = -\frac{1}{x^2} + y = 0.$ $\quad f_{xx} = \frac{2}{x^3}\Big|_{(1,1)} = 2$

$f_y = -\frac{1}{y^2} + x = 0$ $\quad f_{yy} = \frac{2}{y^3}\Big|_{(1,1)} = 2$

$f_{xy} = 1$

$\begin{cases} -\frac{1}{x^2}+y=0 \\ -\frac{1}{y^2}+x=0 \end{cases}$

$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3 > 0$ and $f_{xx} > 0$

$\Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$

$f(1,1) = 1+1+1 = 3$ is the local minima

30. $f(x,y) = e^{2x}\cos y.$

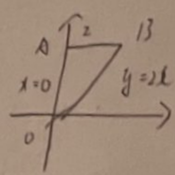
$f_x = 2e^{2x}\cos y = 0$

$f_y = -e^{2x}\sin y = 0.$

$\begin{cases} 2e^{2x}\cos y = 0 \\ -e^{2x}\sin y = 0 \end{cases}$

no solutions $\Rightarrow$ no local minima, local maxima, saddle point.

31. $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1.$, closed triangular plate bounded by lines $x=0, y=2, y=2x$, in the first quadrant.



① OA.

$f(x,y) = f(0,y) = y^2 - 4y + 1 \quad 0 \le y \le 2.$

$f'(0,y) = 2y - 4 = 0 \Rightarrow y = 2.$

$f(0,0) = 1$ and $f(0,2) = -3.$

② AB

$f(x,y) = f(x,2) = 2x^2 - 4x - 3, \quad 0 \le x \le 1$

$f'(x,2) = 4x - 4 = 0 \Rightarrow x = 1.$

$f(0,2) = -3$ and $f(1,2) = -5$

③ OB

$f(x,y) = f(x,2x) = 6x^2 - 12x + 1 \quad 0 \le x \le 1$

$f'(x,2x) = 12x - 12 = 0 \Rightarrow x = 1$ and $y = 2.$

but $(1,2)$ is not an interior point of OB

④ $f_x(x,y) = 4x - 4 = 0$

$f_y(x,y) = 2y - 4 = 0$

$\Rightarrow \begin{cases} x=1 \\ y=2. \end{cases}$

but $(1,2)$ is not an interior point of the region.

Therefore,

the absolute maxima is 1 at $(0,0)$

the absolute minima is -5 at $(1,2)$

37. $f(x,y)=(4x-x^2)\cos y$ on the plate $1\le x\le 3$, $-\frac{\pi}{4}\le y\le\frac{\pi}{4}$

① AB

$f(x,y)=f(1,y)=3\cos y$, $-\frac{\pi}{4}\le y\le\frac{\pi}{4}$

$f'(1,y)=-3\sin y=0 \Rightarrow y=0$ and $x=1$

$f(1,0)=3$, $f(1,-\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$ and $f(1,\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$

② CD

$f(x,y)=f(3,y)=3\cos y$ on $-\frac{\pi}{4}\le y\le\frac{\pi}{4}$

$f'(3,y)=-3\sin y=0\Rightarrow y=0$ and $x=3$.

$f(3,0)=3$, $f(3,-\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$ and $f(3,\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$

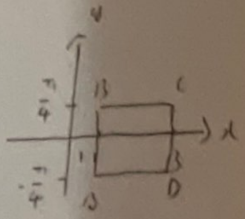
③ BC.

$f(x,y)=f(x,\frac{\pi}{4})=\frac{\sqrt{2}}{2}(4x-x^2)$ on $1\le x\le 3$

$f'(x,\frac{\pi}{4})=\sqrt{2}(2-x)=0\Rightarrow x=2$ and $y=\frac{\pi}{4}$

$f(2,\frac{\pi}{4})=2\sqrt{2}$.

$f(1,\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$, $f(3,\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$



④ AD.

$f(x,y)=f(x,-\frac{\pi}{4})=\frac{\sqrt{2}}{2}(4x-x^2)$ on $1\le x\le 3$

$f'(x,-\frac{\pi}{4})=\sqrt{2}(2-x)=0\Rightarrow x=2$ and $y=-\frac{\pi}{4}$.

$f(2,-\frac{\pi}{4})=2\sqrt{2}$, $f(1,-\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$, $f(3,-\frac{\pi}{4})=\frac{3\sqrt{2}}{2}$

⑤ For interior points of the region.

$f_x(x,y)=(4-2x)\cos y=0$

$f_y(x,y)=-(4x-x^2)\sin y=0$

$\Rightarrow\begin{cases}x=2\\ y=0.\end{cases}$

$f(2,0)=4$

Therefore:

absolute maximum is 4 at $(2,0)$

absolute minimum is $\frac{3\sqrt{2}}{2}$ at $(3,-\frac{\pi}{4})$, $(3,\frac{\pi}{4})$, $(1,-\frac{\pi}{4})$ and $(1,\frac{\pi}{4})$

42. Find critical point of $f(x,y)=xy+2x-\ln x^2y^2$ in the open first quadrant $(x>0, y>0)$ and show that f takes on a minimum there.

$f_x(x,y)=y+2-\frac{2}{x}=0$

$f_y(x,y)=x-\frac{1}{y}=0$

$\begin{cases}y+2-\frac{2}{x}=0\\ x-\frac{1}{y}=0\end{cases}$

$\Rightarrow\begin{cases}x=\frac{1}{2}\\ y=2.\end{cases}$

$f_{xx}(\frac{1}{2},2)=\frac{2}{x^2}\Big|_{(\frac{1}{2},2)}=8.$

$f_{yy}(\frac{1}{2},2)=\frac{1}{y^2}\Big|_{(\frac{1}{2},2)}=\frac{1}{4}$

$f_{xy}(\frac{1}{2},2)=1$

$\begin{vmatrix}8 & 1\\ 1 & \frac{1}{4}\end{vmatrix}=4-1=3>0$ and $f_{xx}>0$.

$f(\frac{1}{2},2)=\frac{1}{2}\times 2+2\times\frac{1}{2}-\ln\frac{1^2}{2}2^2$

$=1+1+\ln 2$

$=2+\ln 2$ is the local minimum

44. The discriminant $f_{xx}f_{yy}-f_{xy}^2$ to Determine.

c. $f(x,y)=xy^2$

Neither since $f(x,y)<0$

for $x<0$ and $f(x,y)>0$ for $x>0$.

e $f(x,y)=x^3y^3$

Neither since $f(x,y)<0$ for $x<0$ and $y>0$,

but $f(x,y)>0$ for $x>0$ and $y>0$

50. Find the point on the graph of $z=x^2+y^2+10$ nearest the plane $x+2y-z=0$.

Let $w=z-x^2-y^2-10$, then $\nabla w=-2x\vec{i}-2y\vec{j}+\vec{k}$ is normal to $z=x^2+y^2+10$

∇w is parallel to $\vec{i}+2\vec{j}-\vec{k}$ if $x=\frac{1}{2}$, $y=1$.

Thus, the point $(\frac{1}{2},1,\frac{1}{4}+1+10)$ is the point.

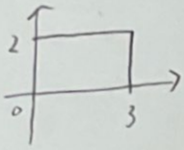
15.1

1. $\int_0^3\int_0^2 (4-y^2)\,dy\,dx$

$= \int_0^3 (4y - \frac{1}{3}y^3)\Big|_0^2 dx$

$= \int_0^3 \frac{16}{3} dx$

$= 16$



2. $\int_0^3\int_{-2}^0 (x^2y - 2xy)\,dy\,dx$

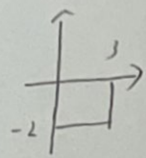
$= \int_0^3 (\frac{x^2y^2}{2} - xy^2)\Big|_{-2}^0 dx$

$= \int_0^3 (-2x^2 + 4x)\,dx$

$= (-\frac{2}{3}x^3 + 2x^2)\Big|_0^3$

$= -18 + 18$

$= 0$



5. $\int_0^\pi\int_0^x x\sin y\,dy\,dx$

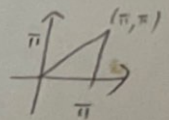
$= \int_0^\pi -x\cos y\Big|_0^x dx$

$= \int_0^\pi (-x\cos x + x)\,dx$

$= \left(\frac{1}{2}x^2 - (\cos x + x\sin x)\right)\Big|_0^\pi$

$= \left(\frac{\pi^2}{2} - (-1)\right) - \left[0 - (1)\right]$

$= \frac{\pi^2}{2} + 2$



$\boxed{\int u\,dv = uv - \int v\,du}$

$u = -x \quad dv = \cos x\,dx$

$du = -dx \quad v = \sin x$

$-x\sin x - \int \sin x \cdot -dx$

$-x\sin x - \cos x$

6. $\int_0^\pi\int_0^{\sin x} y\,dy\,dx$

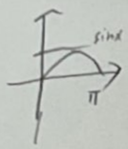
$= \int_0^\pi \frac{y^2}{2}\Big|_0^{\sin x} dx$

$= \int_0^\pi \frac{\sin^2 x}{2} dx$

$= \frac{1}{4}\int_0^\pi (1-\cos 2x)\,dx$

$= \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]\Big|_0^\pi$

$= \frac{\pi}{4}$



11. $f(x,y) = \frac{x}{y}$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$, $x = 2$.

$\int_1^2\int_x^{2x} \frac{x}{y}\,dy\,dx$

$= \int_1^2 x\ln y\Big|_x^{2x} dx$

$= \int_1^2 (x\ln 2x - x\ln x)\,dx$

$= \ln 2\int_1^2 x\,dx$

$= \ln 2\,\frac{x^2}{2}\Big|_1^2$

$= \frac{3}{2}\ln 2$

60. Evaluate the integral.

$\int_0^2 (\tan^{-1}\pi x - \tan^{-1}x)\,dx$

$= \int_0^2\int_x^{\pi x} \frac{1}{1+y^2}\,dy\,dx$

$= \int_0^2\int_{\frac{y}{\pi}}^{y} \frac{1}{1+y^2}\,dx\,dy + \int_2^{2\pi}\int_{\frac{y}{\pi}}^{2} \frac{1}{1+y^2}\,dx\,dy$

$= \int_0^2 \frac{1-\frac{1}{\pi}}{1+y^2}\,y\,dy + \int_2^{2\pi} \frac{2-\frac{y}{\pi}}{1+y^2}\,dy$

$= \left(\frac{\pi-1}{2\pi}\right)\left[\ln(1+y^2)\right]_0^2 + \left[2\tan^{-1}y + \frac{1}{2\pi}\ln(1+y^2)\right]_2^{2\pi}$

$= 2\tan^{-1}2\pi - 2\tan^{-1}2 - \frac{1}{2\pi}\ln(1+4\pi^2) + \frac{\ln 5}{2}$

61. Maximizing a double integral

What region R in the x-y-plane maximizes the value of

$$\iint_R (4-x^2-2y^2)\,dA$$

These criteria are met by the point (x,y) s.t $4-x^2-2y^2 \geq 0$, which is the ellipse $x^2+2y^2=4$ with its interior.

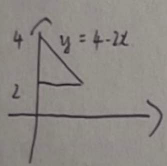
62. Minimizing a double integral.

$$\iint_R (x^2+y^2-9)\,dA.$$

These criteria are met by the point (x,y) s.t $x^2+y^2-9 \leq 0$, which is the closed disk of radius 3 centered at the origin.

21. $\int_0^1 \int_2^{4-2x} dy\,dx.$

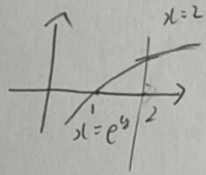
$$\int_2^4 \int_0^{\frac{4-y}{2}} dx\,dy$$



题图

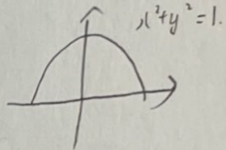
26. $\int_0^{\ln 2} \int_{e^x}^{2} dy\,dx$

$$\int_1^2 \int_0^{\ln y} dx\,dy$$

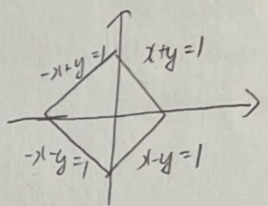


29. $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} 3y\,dy\,dx$$



39. $\iint_R (y-2x^2)\,dA$. where R is the region bounded by the square $|x|+|y|=1$



$$= \int_{-1}^0 \int_{-x-1}^{x+1} (y-2x^2)\,dy\,dx + \int_0^1 \int_{x-1}^{1-x} (y-2x^2)\,dy\,dx.$$

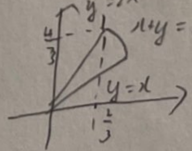
$$= \int_{-1}^0 \left[\tfrac{1}{2}y^2 - 2x^2y\right]_{-x-1}^{x+1} dx + \int_0^1 \left[\tfrac{1}{2}y^2-2x^2y\right]_{x-1}^{1-x} dx$$

$$= -4\int_{-1}^0 (x^3+x^2)\,dx + 4\int_0^1 (x^3-x^2)\,dx.$$

$$= -4\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_{-1}^0 + 4\left[\frac{x^4}{4}-\frac{x^3}{3}\right]_0^1$$

$$= -\frac{2}{3}$$

40. $\iint_R xy\,dA$. by the lines $y=x$, $y=2x$, and $x+y=2$.



$$= \int_0^{\frac{2}{3}} \int_x^{2x} xy\,dy\,dx + \int_{\frac{2}{3}}^1 \int_x^{2-x} xy\,dy\,dx$$ 上 - 下

$$= \int_0^{\frac{2}{3}} \left[\tfrac{1}{2}xy^2\right]_x^{2x} dx + \int_{\frac{2}{3}}^1 \left[\tfrac{1}{2}xy^2\right]_x^{2-x} dx$$

$$= \int_0^{\frac{2}{3}} \frac{3}{2}x^3\,dx + \int_{\frac{2}{3}}^1 (2x-x^2)\,dx.$$

$$= \frac{3}{8}x^4\Big|_0^{\frac{2}{3}} + \left(x^2-\frac{2}{3}x^3\right]_{\frac{2}{3}}^1$$

$$= \frac{13}{81}$$