电磁场方程与公式

（考试时会提供）

第 1 章 静电场

$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{F/m}$

$$\begin{cases}\nabla\cdot\boldsymbol{D}=\rho\\ \nabla\times\boldsymbol{E}=0\\ \boldsymbol{D}=\varepsilon\boldsymbol{E}\end{cases}\quad \begin{cases}\oiint_s \boldsymbol{D}\cdot\mathrm{d}\boldsymbol{S}=q\\ \oint_l \boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}=0\\ \boldsymbol{D}=\varepsilon\boldsymbol{E}\end{cases}\quad \boldsymbol{E}(\boldsymbol{r})=\frac{q}{4\pi\varepsilon_0|\boldsymbol{r}-\boldsymbol{r}'|^2}\boldsymbol{e}_R\quad \boldsymbol{E}(\boldsymbol{r})=\int\frac{\boldsymbol{e}_R\mathrm{d}q}{4\pi\varepsilon_0|\boldsymbol{r}-\boldsymbol{r}'|^2}$$

$$\boldsymbol{E}=-\nabla\varphi\quad \varphi_a=\int_a^\infty \boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}\quad \varphi=\frac{q}{4\pi\varepsilon_0 r}\quad U_{ab}=\varphi_a-\varphi_b=\int_a^b\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}\quad \varphi(r)=\frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$\boldsymbol{P}=\lim_{\Delta V\to0}\frac{\sum\boldsymbol{p}}{\Delta V}\quad \rho_{ps}=\boldsymbol{P}\cdot\boldsymbol{e}_n\quad \rho_p=-\nabla\cdot\boldsymbol{P}\quad \boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P}\quad \boldsymbol{P}=\varepsilon_0\chi\boldsymbol{E}\quad 1+\chi=\varepsilon_r\quad \boldsymbol{D}=\varepsilon_r\varepsilon_0\boldsymbol{E}$$

$$D_{2n}-D_{1n}=\rho_s\quad E_{2t}=E_{1t}\quad \frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\varepsilon_1}{\varepsilon_2}\quad \nabla^2\varphi=-\rho/\varepsilon\quad \varepsilon_1\frac{\partial\varphi_1}{\partial n}-\varepsilon_2\frac{\partial\varphi_2}{\partial n}=\rho_s\quad \varphi_1=\varphi_2\quad -\iint_S\varepsilon\frac{\mathrm{d}\varphi}{\mathrm{d}n}\mathrm{d}S=q$$

$\boldsymbol{n}$ 为外法向

$$q'=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}q\quad q''=\frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}q\quad q'=-\frac{R}{d}q，b=\frac{R^2}{d}\quad a_i^2+b^2=h_i^2\quad \varphi_p=\frac{\tau}{2\pi\varepsilon}\ln\frac{r^-}{r^+}$$

$$C=\frac{Q}{U}\quad \begin{aligned}q_1&=C_{10}U_{10}+C_{12}U_{12}+C_{13}U_{13}+\cdots\cdots+C_{1k}U_{1k}+\cdots+C_{1n}U_{1n}\\ q_2&=C_{20}U_{20}+C_{21}U_{21}+C_{23}U_{23}+\cdots\cdots+C_{2k}U_{2k}+\cdots+C_{2n}U_{2n}\\ &\cdots\cdots\cdots\end{aligned}\quad W_e=\frac{1}{2}CU^2\quad \boldsymbol{f}=q\boldsymbol{E}\quad \boldsymbol{f}=\int\boldsymbol{E}\mathrm{d}q$$

$$W_e=\frac{1}{2}\sum_{k=1}^n\varphi_k q_k\quad W_e=\frac{1}{2}\int_V\rho\varphi\mathrm{d}V+\frac{1}{2}\int_S\rho_s\varphi\mathrm{d}S\quad W_e=\frac{1}{2}\int_V\boldsymbol{D}\cdot\boldsymbol{E}\mathrm{d}V\quad f=-\frac{\partial W_e}{\partial g}\bigg|_{q_k=const}\quad f=\frac{\partial W_e}{\partial g}\bigg|_{\varphi_k=const}$$

第 2 章 恒定电流场

$$R=\frac{U}{I}$$

$$\begin{cases}\nabla\cdot\boldsymbol{J}=0\\ \nabla\times\boldsymbol{E}=0\\ \boldsymbol{J}=\sigma\boldsymbol{E}\end{cases}\quad \begin{cases}\oiint_S\boldsymbol{J}\cdot\mathrm{d}\boldsymbol{S}=0\\ \oint_l\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l}=0\\ \boldsymbol{J}=\sigma\boldsymbol{E}\end{cases}\quad \oiint_S\boldsymbol{J}\cdot\mathrm{d}\boldsymbol{S}=\sum I$$

包含电极时

$$\boldsymbol{E}=-\nabla\varphi\quad \nabla^2\varphi=0\quad E_{1t}=E_{2t}\quad \varphi_1=\varphi_2\quad J_{1n}=J_{2n}\quad \sigma_1\frac{\partial\varphi_1}{\partial n}=\sigma_2\frac{\partial\varphi_2}{\partial n}$$

$$I'=\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}I\quad I''=\frac{2\sigma_2}{\sigma_1+\sigma_2}I$$

$$p'=\boldsymbol{E}\cdot\boldsymbol{J}\quad P=\iiint_V p'\mathrm{d}V=\iiint_V\boldsymbol{E}\cdot\boldsymbol{J}\mathrm{d}V\quad P=-\oiint_S(\varphi\boldsymbol{J})\cdot\mathrm{d}\boldsymbol{S}$$

第 3 章 恒定磁场

源成立条件：$\nabla\cdot\boldsymbol{J}=0$

$$\begin{cases}\nabla\times\boldsymbol{H}=\boldsymbol{J}\\ \nabla\cdot\boldsymbol{B}=0\\ \boldsymbol{B}=\mu\boldsymbol{H}\end{cases}\quad \begin{cases}\oint_l\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{l}=i\\ \oiint_s\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}=0\\ \boldsymbol{B}=\mu\boldsymbol{H}\end{cases}\quad \boldsymbol{B}=\frac{\mu_0}{4\pi}\int_{V'}\frac{\boldsymbol{J}\times\boldsymbol{r}^0}{r^2}\mathrm{d}V'\quad \boldsymbol{B}=\frac{\mu_0}{4\pi}\int_l\frac{I\mathrm{d}\boldsymbol{l}\times\boldsymbol{r}^0}{r^2}$$

$$\mathrm{d}\boldsymbol{f}=I\mathrm{d}\boldsymbol{l}\times\boldsymbol{B}\quad \mu_0=4\pi\times10^{-7}H/m\quad \Phi=\iint_S\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}$$

$$\boldsymbol{M}=\lim_{\Delta V\to0}\frac{\sum\boldsymbol{m}}{\Delta V}\quad \boldsymbol{J}_m=\nabla\times\boldsymbol{M}\quad \boldsymbol{K}_m=\boldsymbol{M}\times\boldsymbol{n}\quad \boldsymbol{H}=\frac{\boldsymbol{B}}{\mu_0}-\boldsymbol{M}\quad \boldsymbol{M}=\chi_m\boldsymbol{H}\quad 1+\chi_m=\mu_r\quad \boldsymbol{B}=\mu_r\mu_0\boldsymbol{H}$$

$$B_{1n}=B_{2n}\quad H_{1t}-H_{2t}=J_{su}\quad \frac{\tan\alpha_1}{\tan\alpha_2}=\frac{\mu_1}{\mu_2}\quad \boldsymbol{B}=\nabla\times\boldsymbol{A}\quad \nabla^2\boldsymbol{A}=-\mu\boldsymbol{J}\quad \nabla\cdot\boldsymbol{A}=0\quad \boldsymbol{A}_1=\boldsymbol{A}_2\quad \frac{1}{\mu_1}(\nabla\times\boldsymbol{A}_1)_t-\frac{1}{\mu_2}(\nabla\times\boldsymbol{A}_2)_t=K_u$$

$$\boldsymbol{A}(r)=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(r')}{|r-r'|}\mathrm{d}V'\quad \boldsymbol{A}=\frac{\mu}{4\pi}\int_{l'}\frac{I}{R}\mathrm{d}\boldsymbol{l}'\quad B_x=\frac{\partial A_z}{\partial y}\quad B_y=-\frac{\partial A_z}{\partial x}\quad \frac{1}{\mu_1}\frac{\partial A_{z1}}{\partial n}=\frac{1}{\mu_2}\frac{\partial A_{z2}}{\partial n}$$

$$\Phi=\iint_S\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}=\oint_l\boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}\quad I'=\frac{\mu_2-\mu_1}{\mu_2+\mu_1}I,\quad I''=\frac{2\mu_1}{\mu_2+\mu_1}I\quad \boldsymbol{A}=\frac{\mu_0 I}{2\pi}\ln\frac{r^-}{r^+}\boldsymbol{e}_z$$

$$L=\frac{\Psi}{I}\quad \mathrm{d}\psi=N_{\mathrm{d}\Phi}\mathrm{d}\Phi=N_{\mathrm{d}\Phi}\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{S}\quad L_i=\frac{\mu_0}{8\pi}\quad L_o=\frac{\mu_0}{4\pi}\oint_{l_2}\oint_{l_1}\frac{\mathrm{d}\boldsymbol{l}_1\cdot\mathrm{d}\boldsymbol{l}_2}{r}\quad L=L_i+L_o\quad M_{12}=\frac{\psi_{12}}{I_2}\quad M_{12}=M_{21}$$

$$W_m=\underbrace{\frac{1}{2}\sum_{k=1}^n L_k I_k^2}_{\text{自有能}}+\underbrace{\frac{1}{2}\sum_{k=1}^n\sum_{j=1}^n M_{ij}I_iI_j}_{k\neq j,\text{互有能}}\quad M_{21}=\frac{\mu_o}{4\pi}\oint_{l_2}\oint_{l_1}\frac{\mathrm{d}\boldsymbol{l}_1\cdot\mathrm{d}\boldsymbol{l}_2}{r}$$

$$=\frac{1}{2}\sum_{k=1}^n\psi_{kk}I_k+\frac{1}{2}\underbrace{\sum_{k=1}^n\sum_{j=1}^n}_{k\neq j}\psi_{kj}I_j=\frac{1}{2}\sum_{k=1}^n I_k\psi_k\quad W_m=\frac{1}{2}\iiint_V\boldsymbol{A}\cdot\boldsymbol{J}\mathrm{d}V\quad W_m=\frac{1}{2}\int_V\boldsymbol{H}\cdot\boldsymbol{B}\mathrm{d}V$$

$$f=\frac{\partial W_m}{\partial g}\bigg|_{I_k=\text{常量}}\quad f=-\frac{\partial W_m}{\partial g}\bigg|_{\psi_k=\text{常量}}$$

第 5 章 时变电磁场与准静态场

$$e=\oint_l \boldsymbol{E}_i\cdot \mathrm{d}\boldsymbol{l}=-\frac{\mathrm{d}\Phi}{\mathrm{d}t}=\iint_S(-\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t})\cdot\mathrm{d}\boldsymbol{S}\qquad e=\oint_l \boldsymbol{E}_i\cdot\mathrm{d}\boldsymbol{l}=\oint_l(\boldsymbol{v}\times\boldsymbol{B})\cdot\mathrm{d}\boldsymbol{l}$$

$$\boldsymbol{E}=\boldsymbol{E}_i+\boldsymbol{E}_c$$

$$\nabla\times\boldsymbol{E}_i=-\frac{\partial\boldsymbol{B}}{\partial t}+\nabla\times(\boldsymbol{v}\times\boldsymbol{B})\qquad \boldsymbol{E}_i=\boldsymbol{v}\times\boldsymbol{B}$$

$$\oint_l\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{l}=\iint_S(\boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t})\cdot\mathrm{d}\boldsymbol{S}$$

$$\Updownarrow$$

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}$$

$$\oiint_S\boldsymbol{D}\cdot\mathrm{d}\boldsymbol{S}=\iiint_V\rho\mathrm{d}V$$

$$\Updownarrow$$

$$\nabla\cdot\boldsymbol{D}=\rho$$

$$\nabla\times\boldsymbol{H}=\boldsymbol{J}+\frac{\partial\boldsymbol{D}}{\partial t}\quad(1)\qquad \boldsymbol{B}=\mu\boldsymbol{H}$$

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t}\quad(2)\qquad \boldsymbol{D}=\varepsilon\boldsymbol{E}$$

$$\nabla\cdot\boldsymbol{B}=0\quad(3)\qquad \boldsymbol{J}=\sigma(\boldsymbol{E}+\boldsymbol{E}_e)$$

$$\nabla\cdot\boldsymbol{D}=\rho\quad(4)\qquad i=\iint_S\boldsymbol{J}\cdot\mathrm{d}\boldsymbol{S}$$

$$\nabla\cdot\boldsymbol{J}=-\frac{\partial\rho}{\partial t}\quad(5)$$

电准静态场是忽略$\partial\boldsymbol{B}/\partial t$
磁准静态场是忽略$\partial\boldsymbol{D}/\partial t$
对涡流问题有扩散方程：

$$(\nabla^2-\mu\sigma\frac{\partial}{\partial t})\begin{Bmatrix}\boldsymbol{H}\\ \boldsymbol{E}\end{Bmatrix}=0$$

$$(\nabla^2-j\omega\mu\sigma)\begin{Bmatrix}\dot{\boldsymbol{H}}\\ \dot{\boldsymbol{E}}\end{Bmatrix}=0$$

$$J_{1n}+\frac{\partial D_{1n}}{\partial t}=J_{2n}+\frac{\partial D_{2n}}{\partial t}$$

$$\dot{H}_z(x)=\dot{C}_1\mathrm{e}^{-\Gamma x}+\dot{C}_2\mathrm{e}^{\Gamma x}$$

$$\Gamma=\sqrt{j\omega\mu\sigma}$$

第 6 章 电磁波

$$(\nabla^2-\mu\sigma\frac{\partial}{\partial t}-\mu\varepsilon\frac{\partial^2}{\partial t^2})\begin{Bmatrix}\boldsymbol{H}\\ \boldsymbol{E}\end{Bmatrix}=0$$ 广义波动方程。在三参数都存在的连续媒质中。

$$(\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2})\begin{Bmatrix}\boldsymbol{H}\\ \boldsymbol{E}\end{Bmatrix}=0$$ 波动方程。在没有电荷的非导电连续媒质中。

一维波动方程的解为：$f(x-vt)$ 和 $f(x+vt)$或$f(t-\frac{x}{v})$和 $f(t+\frac{x}{v})$ 非导电媒质中波速 $v=1/\sqrt{\mu\varepsilon}$

$$-\oiint_S(\boldsymbol{E}\times\boldsymbol{H})\cdot\mathrm{d}\boldsymbol{S}=\frac{\partial W}{\partial t}+P_R-P_e=\frac{\partial W}{\partial t}+\iiint_{V_J}J^2/\sigma\mathrm{d}V-\iiint_{V_e}\boldsymbol{E}_e\cdot\boldsymbol{J}\,\mathrm{d}V\qquad \boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}\qquad \frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}$$

$$-\oiint_S(\dot{\boldsymbol{E}}\times\dot{\boldsymbol{H}}^*)\cdot\mathrm{d}\boldsymbol{S}=j\omega\iiint_V(\mu H^2-\varepsilon E^2)\mathrm{d}V+\iiint_V\frac{J^2}{\sigma}\mathrm{d}V-\iiint_V\dot{\boldsymbol{E}}_e\cdot\boldsymbol{J}^*\mathrm{d}V\qquad \tilde{\boldsymbol{S}}=\dot{\boldsymbol{E}}\times\dot{\boldsymbol{H}}^*\qquad \sin\theta_c=\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}$$

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}$$

$$\boldsymbol{E}=-\frac{\partial\boldsymbol{A}}{\partial t}-\nabla\varphi$$

非导电媒质中

$$\begin{cases}\nabla^2\boldsymbol{A}-\varepsilon\mu\dfrac{\partial^2\boldsymbol{A}}{\partial t^2}-\mu\varepsilon\nabla\dfrac{\partial\varphi}{\partial t}=-\mu\boldsymbol{J}\\ \nabla^2\varphi=-\dfrac{\rho}{\varepsilon}\qquad 条件：\nabla\cdot\boldsymbol{A}=0\end{cases}$$

$$\begin{cases}\nabla^2\boldsymbol{A}-\mu\varepsilon\dfrac{\partial^2\boldsymbol{A}}{\partial t^2}=-\mu\boldsymbol{J}\\ \nabla^2\varphi-\mu\varepsilon\dfrac{\partial^2\varphi}{\partial t^2}=-\dfrac{\rho}{\varepsilon}\\ 条件：\nabla\cdot\boldsymbol{A}=-\mu\varepsilon\dfrac{\partial\varphi}{\partial t}\end{cases}$$ 达朗贝尔方程

正弦激励下无限大均匀介质中达朗贝尔方程的解：

$$\boldsymbol{A}(t,r)=\frac{\mu}{4\pi}\int_{V'}\frac{\boldsymbol{J}(r')\sin[\omega(t-\frac{r}{v})]}{r}\mathrm{d}V'\qquad \varphi(t,r)=\frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho(r')\sin[\omega(t-\frac{r}{v})]}{r}dV'$$

$$\dot{\boldsymbol{A}}=\frac{\mu}{4\pi}\int_{V'}\frac{\dot{\boldsymbol{J}}(r')\mathrm{e}^{-j\beta r}}{r}\mathrm{d}V'\qquad \dot{\varphi}=\frac{1}{4\pi\varepsilon}\int_{V'}\frac{\dot{\rho}(r')\mathrm{e}^{-j\beta r}}{r}\mathrm{d}V'$$

$$\begin{cases}\nabla^2\dot{\boldsymbol{A}}+\beta^2\dot{\boldsymbol{A}}=-\mu\dot{\boldsymbol{J}}\\ \nabla^2\dot{\varphi}+\beta^2\dot{\varphi}=-\dot{\rho}/\varepsilon\\ 条件：\nabla\cdot\dot{\boldsymbol{A}}=-j\omega\mu\varepsilon\dot{\varphi}\end{cases}$$

$$\beta=\frac{\omega}{v}\qquad v=1/\sqrt{\mu\varepsilon}$$

$$Z=\frac{E_\theta}{H_\alpha}=\sqrt{\frac{\mu}{\varepsilon}}\qquad Z_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377(欧)\qquad \boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}=E_\theta H_\alpha\boldsymbol{e}_r=\frac{E_\theta^2}{Z_0}\boldsymbol{e}_r$$

$$(\nabla^2-j\omega\mu\sigma+\omega^2\mu\varepsilon)\dot{\boldsymbol{E}}=\nabla^2\dot{\boldsymbol{E}}-\Gamma^2\dot{\boldsymbol{E}}=0\qquad \Gamma=\sqrt{-\omega^2\mu\varepsilon+j\omega\mu\sigma}=\alpha+j\beta$$ $\boldsymbol{H}$满足同样的方程

一维形式的解：$\dot{\boldsymbol{E}}(r)=\dot{\boldsymbol{E}}_0^+\mathrm{e}^{-\Gamma r}+\dot{\boldsymbol{E}}_0^-\mathrm{e}^{\Gamma r}=\dot{\boldsymbol{E}}_0^+\mathrm{e}^{-(\alpha+j\beta)r}+\dot{\boldsymbol{E}}_0^-\mathrm{e}^{(\alpha+j\beta)r}$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1]}\approx\sqrt{\frac{\omega\mu\sigma}{2}}\approx\beta\ ,\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}[\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1]},\ d=\frac{1}{\alpha}=\sqrt{\frac{2}{\omega\mu\sigma}}\qquad \lambda=Tv=\frac{v}{f}=\frac{2\pi}{\beta}$$