

7.2. 已知: $x=0.1$ 时, $y=0.05 \sin(1-4t)$, $u=0.8 \text{ m/s}$.

求: 波函数.

解: $y = 0.05 \sin(1-4t)$

$\therefore \omega = 4 \text{ s}^{-1}$

$$\Delta\varphi = \frac{2\pi}{\lambda} (x-0.1) = \frac{\omega}{u} (x-0.1) = \frac{4}{0.8} (x-0.1) = 5x - 0.5$$

波函数: ① $y = 0.05 \sin(1-4t + \Delta\varphi)$
 $= 0.05 \sin(1-4t + 5x - 0.5)$
 $= 0.05 \sin(4t - 5x + \pi - 0.5)$

② $y = 0.05 \sin(1-4t - \Delta\varphi)$
 $= 0.05 \sin(-4t - 5x + 1.5)$
 $= 0.05 \sin(4t + 5x + \pi - 1.5)$

7.5. 已知: $u=0.08 \text{ m/s}$, $\lambda=0.4 \text{ m}$, $A=0.04 \text{ m}$

求: 波函数, $t=\frac{T}{8}$ 时波形曲线.

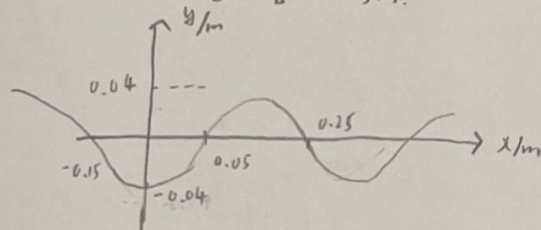
解: 以 \cos 表示函数; 当 $t=0$ 且 $x=0$ 时, $y=0$, 则 $\varphi=\frac{\pi}{2}$, 有

$$y = A \cos\left[2\pi\left(\nu t - \frac{x}{\lambda}\right) + \varphi\right] = A \cos\left[2\pi\left(\frac{u}{\lambda} t - \frac{x}{\lambda}\right) + \varphi\right]$$

$$= 0.04 \cos\left[2\pi\left(\frac{0.08}{0.4} t - \frac{x}{0.4}\right) + \frac{\pi}{2}\right]$$

$$= 0.04 \cos\left(0.4\pi t - 5\pi x + \frac{\pi}{2}\right)$$

将波向右移 $\frac{\lambda}{8} = \frac{0.4}{8} = 0.05 \text{ m}$.



7.6. 已知: 波函数 $y = A \cos \pi(4t + 2x)$.

求: $t=4.2 \text{ s}$ 时, 波峰表达式, 离原点最近的波峰, 何时通过原点, 波形曲线

解: $\pi(4t + 2x) = 2\pi n$

$\Rightarrow \pi(4 \times 4.2 + 2x) = 2\pi n$

$x = n - 8.4, n \in \mathbb{Z}$

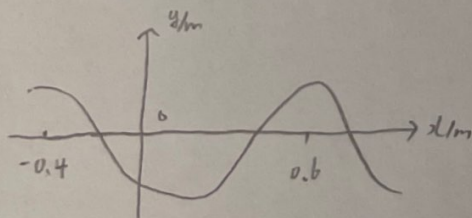
要使 $|x|$ 最小, 可取 $n=8$, 此时 $x=-0.4 \text{ m}$, 该通过时间 t_0 .

$\pi(4t_0 + 2x_0) = 2\pi n$

$\Rightarrow \pi(4t_0 + 2x(0.4)) = 2\pi \times 8$

$\Rightarrow t_0 = 4 \text{ s}$

由波函数知: $2 = \frac{2\pi}{\lambda} \Rightarrow \lambda = 1 \text{ m}$.



7.12. 已知: $v_A = v_B = 100 \text{ Hz}$, $\varphi_A - \varphi_B = \pi$, $u_A = u_B = 400 \text{ m/s}$, $l = 30 \text{ m}$

求: 静止点的位置.

$$\begin{aligned} \text{解: } \Delta\varphi &= (\varphi_A - \frac{2\pi}{\lambda}x) - (\varphi_B - \frac{2\pi}{\lambda}(l-x)) \\ &= \varphi_A - \varphi_B + \frac{2\pi v}{u}(l-x) \\ &= \pi + \frac{2\pi \times 100}{400} \times (30-x) \\ &= (16-x)\pi \end{aligned}$$

静止时有: $\Delta\varphi = (2n+1)\pi$

$$(16-x)\pi = (2n+1)\pi$$

$$x = 15 - 2n$$

由于 $n \in \mathbb{Z}$, $0 \leq x \leq 30$, 故 $x = 1, 3, 5, \dots, 27, 29$

7.14. 已知: A, v, u , 原点 O 处为平衡位置.

求: 波函数, 反射波的波函数, 两波叠加.

解: 在原点的表达式为 $y_0 = A \cos(2\pi vt - \frac{\pi}{2})$

$$\text{故波函数为 } y_1 = A \cos(vt - \frac{2\pi v}{u}x - \frac{\pi}{2}), \quad 0 \leq x \leq \frac{3u}{4v}$$

$$\begin{aligned} \text{反射波 } y_r &= A \cos[2\pi vt - \frac{2\pi v}{u} \cdot \frac{3u}{4v} - \frac{\pi}{2} + \pi - \frac{2\pi v}{u}(\frac{3u}{4v} - x)] \\ &= A \cos(2\pi vt + \frac{2\pi v}{u}x - \frac{5\pi}{2}) \\ &= A \cos(2\pi vt + \frac{2\pi v}{u}x - \frac{\pi}{2}) \quad (x \leq \frac{3u}{4v}) \end{aligned}$$

当反射面上的点与波相加静止, 其他点需满足 $\frac{2\pi}{\lambda}(x - \frac{3}{4}\lambda) = n\pi$ 且 $0 \leq x \leq \frac{3u}{4v}$

故取 $n=1$, $x = \frac{1}{4}\lambda$ (或 $\frac{u}{4v}$)

7.15. 已知: $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$, $u = 5.74 \times 10^3 \text{ m/s}$.

求: v .

解: $\lambda = 2d$.

$$v = \frac{u}{\lambda} = \frac{u}{2d} = \frac{5.74 \times 10^3}{2 \times 2 \times 10^{-3}} = 1.44 \times 10^6 \text{ Hz}$$

7.16. 已知: $L_1 = 115 \text{ dB}$, $L_2 = 141 \text{ dB}$.

求: I_1, I_2 .

$$\text{解: } I_1 = I_0 10^{\frac{L_1}{10}} = 10^{-12} \times 10^{\frac{115}{10}} = 0.316 \text{ W/m}^2$$

$$I_2 = I_0 10^{\frac{L_2}{10}} = 10^{-12} \times 10^{\frac{141}{10}} = 126 \text{ W/m}^2$$

7.20. 已知: $v_R = -80 \text{ km/h} = -22.2 \text{ m/s}$, $v_S = 120 \text{ km/h} = 33.3 \text{ m/s}$, $\nu_S = 400 \text{ Hz}$, $u = 300 \text{ m/s}$.

求: ν_R

$$\text{解: } \nu_R = \frac{u + v_R}{u - v_S} \cdot \nu_S = \frac{330 + (-22.2)}{330 - 33.3} \times 400 = 415 \text{ Hz}$$

7.21. 已知: $\lambda = 120 \text{ m}$, $T = 10 \text{ s}$, $v = 24 \text{ m/s}$.

求: ν_1, ν_2, T_1, T_2 .

$$\text{解: } u = \frac{\lambda}{T} = \frac{120}{10} = 12 \text{ m/s}$$

$$\nu_1 = \frac{u + v}{\lambda} = \frac{12 + 24}{120} = 0.3 \text{ Hz}$$

$$\nu_2 = \frac{v - u}{\lambda} = \frac{24 - 12}{120} = 0.1 \text{ Hz}$$

$$T_1 = \frac{1}{\nu_1} = \frac{1}{0.3} = 3.33 \text{ s}$$

$$T_2 = \frac{1}{\nu_2} = \frac{1}{0.1} = 10 \text{ s}$$

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