

1-2 一半径为 1cm 的球面上均匀分布有电荷,总电量为 q ,处于空气中。空气的击穿电场强度为 30kV/cm。问当 q 增至多大时,空气局部电场将达击穿电场强度?

分别计算球面内电场 E_1 与球面外电场 E_2

由对称性知, E_1, E_2 只与点到球心的距离 r 有关

$$\therefore \vec{E}_1 = E_{1r}(r) \cdot \vec{r}, \quad \vec{E}_2 = E_{2r}(r) \cdot \vec{r}$$

由高斯定理:

$$\oint_S \vec{E}_1 \cdot d\vec{s} = 0$$

$$\therefore E_{1r}(r) = 0 \quad (0 \leq r < 1\text{cm})$$

$$\oint_S \vec{E}_2 \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$E_{2r}(r) = \frac{q}{4\pi r^2 \epsilon_0} \quad (r > 1\text{cm})$$

即在球面外表面的电场强度最大

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

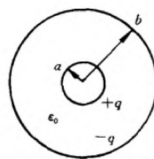
$$q = E \cdot 4\pi \epsilon_0 r^2$$

$$q = 30\text{kV/cm} \cdot 4\pi \cdot \epsilon_0 \cdot (1\text{cm})^2$$

$$q = 3.337 \times 10^{-8} \text{C}$$

当 q 达到 $3.337 \times 10^{-8} \text{C}$ 时可能会击穿空气

1-8 圆球形电容器,如图题 1-8 所示。内球半径为 a ,带有电荷 $+q$; 外球内半径为 b ,带有电荷 $-q$ 。今取无穷远处为电位参考点,设其电位为零。计算各处($r < a, a \leq r \leq b, b \leq r$)的电位。



图题 1-8

$$\varphi = \int_p^\infty \vec{E} \cdot d\vec{l} = \int_p^\infty E dr$$

$$\textcircled{1} \quad r < a$$

$$E = 0$$

$$\varphi = \varphi_a = \frac{q}{4\pi \epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$\textcircled{2} \quad a \leq r \leq b$$

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

$$\varphi = \int_r^b E dr + \varphi_b = \frac{q}{4\pi \epsilon_0} \left(\frac{1}{r} - \frac{1}{b} \right)$$

$$\textcircled{3} \quad b \leq r$$

$$E = 0$$

$$\varphi = 0$$

1-9 空心导体球,如图题 1-9 所示,内半径为 a ,外半径为 b 。若球心置一点电荷 q ,计算各处($r < a, a \leq r \leq b, b < r$) 电位。

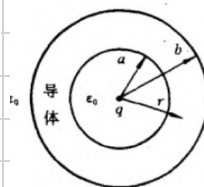


图 题 1-9

$$\varphi = \int_p^\infty \vec{E} d\vec{l} = \int_p^\infty E dr$$

① $r < a$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\begin{aligned}\varphi &= \int_r^a E dr + \frac{q}{4\pi\epsilon_0 b} \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{a} \right) + \frac{q}{4\pi\epsilon_0 b} \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{a} + \frac{1}{b} \right)\end{aligned}$$

② $a \leq r \leq b$

$$E = 0$$

$$\varphi = \frac{q}{4\pi\epsilon_0 b}$$

③ $b < r$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\begin{aligned}\varphi &= \int_r^\infty E dr \\ &= \frac{q}{4\pi\epsilon_0 r}\end{aligned}$$

1-12 定性地画出以下平行平面场(见图题 1-12)的场图(先画出等位线再画出电力线)。

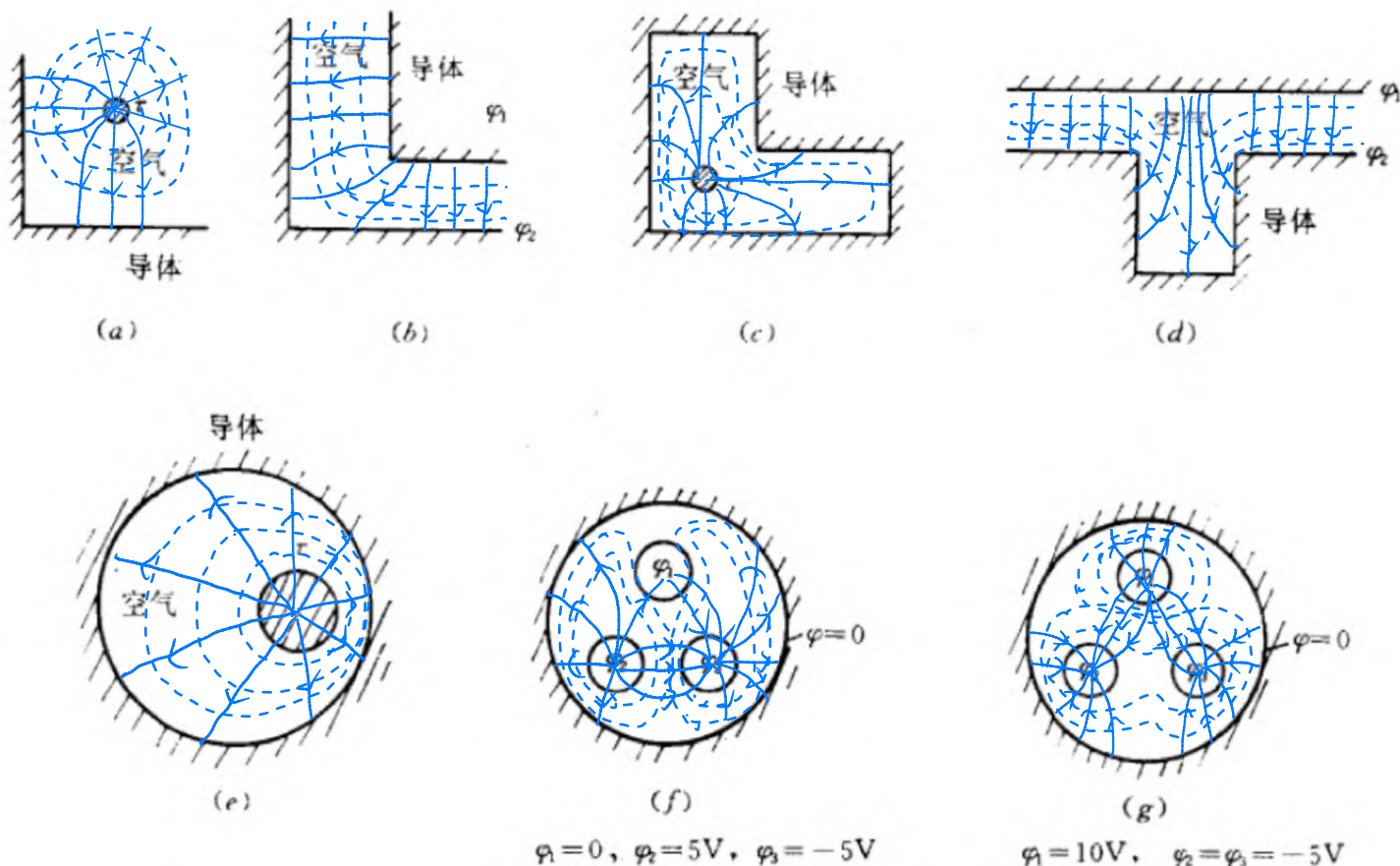


图 题 1-12