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Homework 5 Solutions

2.5.6 (a) $AB = AC \rightsquigarrow A^{-1}(AB) = A^{-1}(AC)$

$\rightsquigarrow (A^{-1}A)B = (A^{-1}A)C \rightsquigarrow IB = IC \rightsquigarrow B = C$

if A is invertible.

(b) Here, A is <u>not</u> invertible. In general, $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

$= \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix}$. For example, we would get $AB = 0$ if

$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has $a = -c$, $b = -d$: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} a & b \\ -a & -b \end{bmatrix}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Infinitely many choices for a, b.

For example:

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}}_{B} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}}_{C} \longrightarrow$ You could pick two possible choices for B and C

2.5.11 Many possible solutions. Here are some simple ones:

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ are invertible, but $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not.

(b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are not invertible, but $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is.

2.5.21 Invertible

$\begin{bmatrix}1&1\\0&1\end{bmatrix}, \begin{bmatrix}1&1\\1&0\end{bmatrix}, \begin{bmatrix}1&0\\1&1\end{bmatrix}, \begin{bmatrix}0&1\\1&1\end{bmatrix}, \begin{bmatrix}1&0\\0&1\end{bmatrix}, \begin{bmatrix}0&1\\1&0\end{bmatrix}$

6 are invertible

Non-invertible

$\begin{bmatrix}1&1\\1&1\end{bmatrix}, \begin{bmatrix}1&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\1&1\end{bmatrix}, \begin{bmatrix}1&0\\1&0\end{bmatrix}, \begin{bmatrix}0&1\\0&1\end{bmatrix},$

$\begin{bmatrix}0&1\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}0&0\\1&0\end{bmatrix}, \begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&0\end{bmatrix}$

2.5.25

$$[A \mid I] = \left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\1&2&1&0&1&0\\1&1&2&0&0&1\end{array}\right] \xrightarrow[\text{Row 3}-\frac{1}{2}\text{Row 1}]{\text{Row 2}-\frac{1}{2}\text{Row 1}} \left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\0&3/2&1/2&-1/2&1&0\\0&1/2&3/2&-1/2&0&1\end{array}\right]$$

$$\xrightarrow{\text{Row 3}-\frac{1}{3}\text{Row 2}} \left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\0&3/2&1/2&-1/2&1&0\\0&0&4/3&-1/3&-1/3&1\end{array}\right] \xrightarrow[\frac{3}{4}\text{Row 3}]{\frac{1}{2}\text{Row 1},\ \frac{2}{3}\text{Row 2}} \left[\begin{array}{ccc|ccc}1&1/2&1/2&1/2&0&0\\0&1&1/3&-1/3&2/3&0\\0&0&1&-1/4&-1/4&3/4\end{array}\right]$$

$$\xrightarrow[\text{Row 2}-\frac{1}{3}\text{Row 3}]{\text{Row 1}-\frac{1}{2}\text{Row 3}} \left[\begin{array}{ccc|ccc}1&1/2&0&5/8&1/8&-3/8\\0&1&0&-1/4&3/4&-1/4\\0&0&1&-1/4&-1/4&3/4\end{array}\right] \xrightarrow{\text{Row 1}-\frac{1}{2}\text{Row 2}} \left[\begin{array}{ccc|ccc}1&0&0&3/4&-1/4&-1/4\\0&1&0&-1/4&3/4&-1/4\\0&0&1&-1/4&-1/4&3/4\end{array}\right]$$

(the right-hand block is labelled A^{-1})

$$\text{So } A^{-1} = \frac{1}{4}\begin{bmatrix}3&-1&-1\\-1&3&-1\\-1&-1&3\end{bmatrix}$$

$$[B \mid I] = \left[\begin{array}{ccc|ccc}2&-1&-1&1&0&0\\-1&2&-1&0&1&0\\-1&-1&2&0&0&1\end{array}\right] \xrightarrow[\text{Row 3}+\frac{1}{2}\text{Row 1}]{\text{Row 2}+\frac{1}{2}\text{Row 1}} \left[\begin{array}{ccc|ccc}2&-1&-1&1&0&0\\0&3/2&-3/2&1/2&1&0\\0&-3/2&3/2&1/2&0&1\end{array}\right]$$

$$\xrightarrow{\text{Row 3}+\text{Row 2}} \left[\begin{array}{ccc|ccc}2&-1&-1&1&0&0\\0&3/2&-3/2&1/2&1&0\\0&0&0&1&1&1\end{array}\right]$$

Row of 0's ~~mo~~ means that we cannot eliminate all the way to I, which means that B is not invertible.

2.5.3 $[A \mid I] = \left[\begin{array}{cccc|cccc} 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\text{Row 3+Row 4}]{\text{Row 1+Row 4, Row 2-Row 4}}$

$$\left[\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\text{Row 2+Row 3}]{\text{Row 1-Row 3}} \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\text{Row 1+Row 2}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & \end{array}\underbrace{\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}}_{A^{-1}}\right] \qquad \text{So } A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5x5 case: $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ Guess $A^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Multiply to confirm: $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1+1 & 1-1 & -1+1 & 1-1 \\ 0 & 1 & -1+1 & 1-1 & -1+1 \\ 0 & 0 & 1 & -1+1 & 1-1 \\ 0 & 0 & 0 & 1 & -1+1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{I} \checkmark$

Now solve $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$: $\vec{x} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix}$

2.5.39 $A = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -b & 0 \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $[A \mid I] = \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -b & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -c & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$$\xrightarrow{\text{Row3} + c\text{Row4}} \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -b & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{Row2} + b\text{Row3}} \left[\begin{array}{cccc|cccc} 1 & -a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & b & bc \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\text{Row1} + a\text{Row2}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & a & ab & abc \\ 0 & 1 & 0 & 0 & 0 & 1 & b & bc \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

This is A^{-1}

2.6.6 $$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{bmatrix} \xrightarrow{\text{Row 2} - 2\text{Row 1}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 0 \end{bmatrix} \xrightarrow{\text{Row 3} - 2\text{Row 2}} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{bmatrix}}_{U}$$

$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

So $E_{32}E_{21}A = U \rightsquigarrow A = \underbrace{E_{21}^{-1}E_{32}^{-1}}_{L} U$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

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2.6.8 $A = L = \begin{bmatrix}1&0&0\\a&1&0\\b&c&1\end{bmatrix} \xrightarrow{E_{21}=\begin{bmatrix}1&0&0\\-a&1&0\\0&0&1\end{bmatrix}} \xrightarrow{E_{31}=\begin{bmatrix}1&0&0\\0&1&0\\-b&0&1\end{bmatrix}} \xrightarrow{E_{32}=\begin{bmatrix}1&0&0\\0&1&0\\0&-c&1\end{bmatrix}} I = U$

$$E_{32}E_{31}E_{21} = \begin{bmatrix}1&0&0\\0&1&0\\0&-c&1\end{bmatrix}\underbrace{\begin{bmatrix}1&0&0\\0&1&0\\-b&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\-a&1&0\\0&0&1\end{bmatrix}}_{\begin{bmatrix}1&0&0\\-a&1&0\\-b&0&1\end{bmatrix}} = \begin{bmatrix}1&0&0\\-a&1&0\\ac-b&-c&1\end{bmatrix} = E$$

a, b, c are "mixed up"

$$E_{21}^{-1}E_{31}^{-1}E_{32}^{-1} = \underbrace{\begin{bmatrix}1&0&0\\a&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\b&0&1\end{bmatrix}}_{\begin{bmatrix}1&0&0\\a&1&0\\b&0&1\end{bmatrix}}\begin{bmatrix}1&0&0\\0&1&0\\0&c&1\end{bmatrix} = \underbrace{\begin{bmatrix}1&0&0\\a&1&0\\b&c&1\end{bmatrix}}_{}$$

This is L back again.

2.6.13 $\begin{bmatrix}a&a&a&a\\a&b&b&b\\a&b&c&c\\a&b&c&d\end{bmatrix} \xrightarrow[\text{Row 4}-\boxed{1}\text{Row 1}]{\text{Row 2}-\boxed{1}\text{Row 1},\ \text{Row 3}-\boxed{1}\text{Row 1}} \begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&b-a&c-a&c-a\\0&b-a&c-a&d-a\end{bmatrix} \xrightarrow[\text{Row}-\boxed{1}\text{Row 2}]{\text{Row 3}-\boxed{1}\text{Row 2}}$

All lower-left entries of L are $\boxed{1}$'s.

$$\begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&c-b&d-b\end{bmatrix} \xrightarrow{\text{Row 4}-\boxed{1}\text{Row 3}} \underbrace{\begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&0&d-c\end{bmatrix}}_{U}$$

$$A = \begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&0&d-c\end{bmatrix}$$

For A^{-1} to exist, need $a \neq 0$, $b-a \neq 0$, $c-b \neq 0$, $d-c \neq 0$.

2.6.16 $L\vec{c}=\vec{b}$:

$$\begin{aligned} c_1 &= 4 \\ c_1+c_2 &= 5 \to c_2 = 5-c_1 = 1 \\ c_1+c_2+c_3 &= 6 \to c_3 = 6-c_1-c_2 = 1 \end{aligned}$$

$$\to \vec{c}=\begin{bmatrix}4\\1\\1\end{bmatrix}. \qquad U\vec{x}=\vec{c}: \quad \begin{aligned} x_1+x_2+x_3 &= 4 \to x_1 = 4-x_2-x_3 = 3 \\ x_2+x_3 &= 1 \to x_2 = 1-x_3 = 0 \\ x_3 &= 1 \end{aligned}$$

$$\to \vec{x}=\begin{bmatrix}3\\0\\1\end{bmatrix}, \quad A \text{ was } LU = \begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix} = \begin{bmatrix}1&1&1\\1&2&2\\1&2&3\end{bmatrix}$$

Graded Problem 1. $[A \mid I] = \left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\1&2&2&2&0&1&0&0\\1&2&3&3&0&0&1&0\\1&2&3&4&0&0&0&1\end{array}\right] \xrightarrow[\text{Row 4}-\text{Row 2}]{\text{Row 2}-\text{Row 1},\ \text{Row 3}-\text{Row 1}}$

$$\left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&1&2&2&-1&0&1&0\\0&1&2&3&-1&0&0&1\end{array}\right] \xrightarrow[\text{Row 4}-\text{Row 2}]{\text{Row 3}-\text{Row 2}} \left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&1&1&0&-1&1&0\\0&0&1&2&0&-1&0&1\end{array}\right] \xrightarrow[-\text{Row 3}]{\text{Row 4}}$$

$$\left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&1&1&-1&1&0&0\\0&0&1&1&0&-1&1&0\\0&0&0&1&0&0&-1&1\end{array}\right] \xrightarrow[\text{Row 3}-\text{Row 4}]{\text{Row 1}-\text{Row 4},\ \text{Row 2}-\text{Row 4}} \left[\begin{array}{cccc|cccc}1&1&1&0&1&0&1&-1\\0&1&1&0&-1&1&1&-1\\0&0&1&0&0&-1&2&-1\\0&0&0&1&0&0&-1&1\end{array}\right] \xrightarrow[\text{Row 2}-\text{Row 3}]{\text{Row 1}-\text{Row 3}}$$

$$\left[\begin{array}{cccc|cccc}1&1&0&0&1&1&-1&0\\0&1&0&0&-1&2&-1&0\\0&0&1&0&0&-1&2&-1\\0&0&0&1&0&0&-1&1\end{array}\right] \xrightarrow{\text{Row 1}-\text{Row 2}} \left[\begin{array}{cccc|cccc}1&0&0&0&2&-1&0&0\\0&1&0&0&-1&2&-1&0\\0&0&1&0&0&-1&2&-1\\0&0&0&1&0&0&-1&1\end{array}\right]$$

$$\underbrace{\begin{matrix}2&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&1\end{matrix}}_{\text{This is } A^{-1}}$$

Solve $A\vec{x}=\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$: $\vec{x}=A^{-1}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\begin{bmatrix}2&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&1\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\begin{bmatrix}2\\-1\\-1\\1\end{bmatrix}$

Graded Problem 2

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \xrightarrow[\text{Row4}-\boxed{1}\text{Row1}]{\substack{\text{Row2}-\boxed{1}\text{Row1} \\ \text{Row3}-\boxed{1}\text{Row1}}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$$\xrightarrow{\text{Row3}-\boxed{1}\text{Row2}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix} \xrightarrow[-\boxed{-1}]{\text{Row 4}+\text{Row 3}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

From coefficients: $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{bmatrix}$

First solve $L\vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

$$\begin{aligned} y_1 &= 1 \\ y_1 + y_2 &= 2 \rightarrow y_2 = 2-1 = 1 \\ y_1 + y_2 + y_3 &= 3 \rightarrow y_3 = 3-1-1 = 1 \\ y_1 - y_3 + y_4 &= 4 \rightarrow y_4 = 4-1+1 = 4 \end{aligned}$$

Now solve $U\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 4 \end{bmatrix}$:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \rightarrow x_1 = 1 - 0 - (-\tfrac{1}{2}) - (-1) = 5/2 \\ -2x_2 - 2x_3 &= 1 \rightarrow x_2 = -\tfrac{1}{2}\left(1 + 2(-\tfrac{1}{2})\right) = 0 \\ 2x_3 - 2x_4 &= 1 \rightarrow x_3 = \tfrac{1}{2}\left(1 + 2(-1)\right) = -\tfrac{1}{2} \\ -4x_4 &= 4 \rightarrow x_4 = -1 \end{aligned}$$

So $\vec{x}_4 = \begin{bmatrix} 5/2 \\ 0 \\ -1/2 \\ -1 \end{bmatrix}$