(A)

6.A. 17. $\oint_C f\nabla g\cdot d\vec{r} = \iint_S \nabla\times(f\nabla g)\cdot\vec{n}\,d\sigma$

$= \iint_S (f\nabla\times\nabla g + \nabla f\times\nabla g)\cdot\vec{n}\,d\sigma$

$= \iint_S [(f)(0) + \nabla f\times\nabla g]\cdot\vec{n}\,d\sigma$

$= \iint_S (\nabla f\times\nabla g)\,\vec{n}\,d\sigma$

21. $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k} \Rightarrow \nabla\cdot\vec{r} = 1+1+1 = 3 \Rightarrow \iiint_D \nabla\cdot\vec{r}\,dV$

$= 3\iiint_D dV = 3V$

$V = \frac{1}{3}\iiint_D \nabla\cdot\vec{r}\,dV. = \frac{1}{3}\iint_S \vec{r}\cdot\vec{n}\,d\sigma$, by the Divergence

(B).

1.1. 13. $a_n = (-1)^{n+1}, n=1,2\ldots$ 14. $a_n = (-1)^n, n=1,2,\ldots$ 15. $a_n = (-1)^{n+1}n^2, n=1,2\ldots$

18. $a_n = n-4, n=1,2\ldots$ 19. $a_n = 4n-3, n=1,2\ldots$ 20. $a_n = 4n-2, n=1,2,\ldots$

21. $a_n = \frac{1+(-1)^{n+1}}{2}, n=1,2$ 22. $a_n = \frac{n-\frac{1}{2}+(-1)^n(\frac{1}{2})}{2}$ $n=1,2$

(C)

23 $\lim_{n\to\infty} 2+(0.1)^n = 2.$ 24. $\lim_{n\to\infty}\frac{n+(-1)^n}{n} = 1+\frac{(-1)^n}{n} = 1$ 27. $\lim_{n\to\infty}\frac{1-5n^4}{n^4+8n^3} = -5$

40 $\lim_{n\to\infty} n\pi\cos(n\pi)$ not exist. 29. $\lim_{n\to\infty}\frac{n^2-2n+1}{n-1} = \infty$ 42 $\lim_{n\to\infty}\frac{\sin^2 n}{2^n} = 0.$ Since $0\le\frac{\sin^2 n}{2^n}\le\frac{1}{2^n}$

55. $\lim_{n\to\infty}\frac{\ln n}{n^{\frac{1}{n}}} = \frac{\lim_{n\to\infty}\ln n}{\lim_{n\to\infty}n^{\frac{1}{n}}} = \frac{\infty}{1} = \infty$ 56. $\lim_{n\to\infty}(\ln n-\ln(n+1)) = \ln(\lim_{n\to\infty}\frac{n}{n+1}) = \ln 1 = 0.$

(D).

117. Given an $\epsilon>0$, by definition of convergence there corresponds an N such that for all $n>N$. $|L_1-a_n|<\epsilon$ and $|L_2-a_n|<\epsilon$. Now $|L_2-L_1| = |L_2-a_n+a_n-L_1| \le |L_2-a_n|+|a_n-L_1| < \epsilon+\epsilon = 2\epsilon$ $|L_2-L_1|<2\epsilon$ says that the difference between two fixed value is values is smaller than any positive numbe 2ϵ. The only nonnegative number smaller tha every positiv number is 0 so $|L_1-L_2|=0$ or $L_1=L_2$.

118. Let $k(n)$ and $i(n)$ be two order-preserving funtion whose domains are the set of positive integers and whose ranges are a subset of the positive integers. Consider the two $a_{k(n)}$ and $a_{i(n)}$ where $a_{k(n)}\to L_1$, $a_{i(n)}\to L_2$ and $L_1\ne L_2$. Thus $|a_{k(n)}-a_{i(n)}|\to|L_2-L_1|>0$. So there does not exist N s.t for all $m,n>N$. $\Rightarrow |a_m-a_n|<\epsilon$. So, the $\{a_n\}$ is not convergent.

119. $a_{2k}\to L \Leftrightarrow$ given an $\epsilon>0$ there correspond an N_1 s.t. $[2k>N_1 \Rightarrow |a_{2k}-L|<\epsilon]$ $a_{2k+1}\to L \Leftrightarrow [2k+1>N_2 \Rightarrow |a_{2k+1}-L|<\epsilon]$. Let $N=\max\{N_1,N_2\}$. Then $n>N \Rightarrow |a_n-L|<\epsilon$ whether n is even or odd, and hence $a_n\to L$.

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E. 11.2. 7. $1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\dots$

Sum $= \frac{1}{1-(-\frac{1}{4})} = \frac{4}{5}$

8. $\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\dots$ $\sum_{n=2} \frac{1}{4^n}$ $\sum_{n=1} \frac{1}{4^{n+1}}$ $\frac{1}{4} \cdot \frac{1}{4^n}$

Sum $= \frac{(\frac{1}{16})}{1-\frac{1}{4}} = \frac{1}{12}$

15. $\frac{4}{(4n-3)(4n+1)} = \frac{1}{4n-3} - \frac{1}{4n+1}$ $S_n = (1-\frac{1}{5})+(\frac{1}{5}-\frac{1}{9})+(\frac{1}{9}-\frac{1}{13})+\dots+(\frac{1}{4n-7}-\frac{1}{4n-3})+(\frac{1}{4n-3}-\frac{1}{4n+1})$

$= 1-\frac{1}{4n+1}$ $= \lim_{n\to\infty} S_n = \lim_{n\to\infty}(1-\frac{1}{4n+1}) = 1$

16. $\frac{6}{(2n-1)(2n+1)} = \frac{3}{2n-1} - \frac{1}{2n+1}$ $\sum_{n=1}^{k} \frac{6}{(2n-1)(2n+1)} = 3\sum_{n=1}(\frac{1}{2n-1}-\frac{1}{2n+1}) = 3(1-\frac{1}{3}+\frac{1}{3}-\dots\frac{1}{2k-1}-\frac{1}{2k+1})$

$= 3(1-\frac{1}{2k+1})$

Sum $= \lim_{k\to\infty} 3(1-\frac{1}{2k+1}) = 3$

F. 77. (a) $L_1 = 3, L_2 = 3(\frac{4}{3}), L_3 = 3(\frac{4}{3})^2, \dots L_n = 3(\frac{4}{3})^{n-1}$

$\Downarrow$

$\lim_{n\to\infty} L_n = \lim_{n\to\infty} 3(\frac{4}{3})^{n-1} = \infty$

(b). $A_1 = \frac{\sqrt{3}}{4}$

$A_3 = \frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{12}+\frac{\sqrt{3}}{27}$

$A_2 = A_1 + 3(\frac{\sqrt{3}}{4})(\frac{1}{3})^2 = \frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{12}$

$A_4 = A_3 + 3(4)^2(\frac{\sqrt{3}}{4})(\frac{1}{3^3})^2 \dots$

$A_n = \frac{\sqrt{3}}{4} + \sum_{k=2}^{n} 3(4)^{k-2}(\frac{\sqrt{3}}{4})(\frac{1}{3^2})^{k-1} = \frac{\sqrt{3}}{4} + 3\sqrt{3}(\sum_{k=2}^{n} \frac{4^{k-3}}{9^{k-1}})$

$\lim_{n\to\infty} = \frac{2\sqrt{3}}{5}$

G. 27. $\lim_{n\to\infty} \cos(n\pi) = \lim_{n\to\infty}(-1)^n \neq 0$. diverges

28. $\lim_{n\to\infty} \frac{\cos n\pi}{5^n} = \lim_{n\to\infty} \frac{(-1)^n}{5^n} = \frac{1}{1-(-\frac{1}{5})} = \frac{5}{6}$

33. sum $= \frac{1}{1-\frac{1}{3}} - \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

35. $\lim_{n\to\infty} \frac{n!}{1000^n} = \infty$ diverges

H 11.7.

1. (a) radius: 1, interval of convergence $-1<x<1$
(b) the interval of absolute convergence is $-1<x<1$
(c) no values.

2. (a) radius: 1, the interval of convergence $-6<x<-4$
(b) the interval of absolute convergence is $-6<x<-4$
(c) no values.

3. (a) $\frac{1}{4}$, $-\frac{1}{2}<x<0$
(b) $-\frac{1}{2}<x<0$
(c) no values.

4. (a) $\frac{1}{3}$, $\frac{1}{3} \le x<1$
(b) $\frac{1}{3}<x<1$
(c) $x=\frac{1}{3}$

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(I). 39. $\lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{2^{n+1}}\cdot\frac{2^n}{(x-3)^n}\right|<1 \Rightarrow |x-3|<2 \Rightarrow 1<x<5$, when $x=1$ we have $\sum_{n=1}^{\infty}1^n$ which diverges. when $x=5$ we have $\sum_{n=1}^{\infty}(-1)^n$ which also diverges; the interval of convergence is $1<x<5$; the sum of this convergent geometric series is $\frac{1}{1+\frac{x-3}{2}}=\frac{2}{x-1}$. If $f(x)=1-\frac{1}{2}(x-3)+\frac{1}{4}(x-3)^2+\ldots+(-\frac{1}{2})^n(x-3)^n$ $=\frac{2}{x-1}$ then $f'(x)=-\frac{1}{2}+\frac{1}{2}(x-3)+\ldots+(-\frac{1}{2})^n n(x-3)^{n-1}+\ldots$ is convergent when $1<x<5$, and diverges when $x=1$ or 5. The sum for $f'(x)$ is $\frac{-2}{(x-1)^2}$, the derivative of $\frac{2}{x-1}$

40. If $f(x)=1-\frac{1}{2}(x-3)+\frac{1}{4}(x-3)^2+\ldots+(-\frac{1}{2})^n(x-3)^n+\ldots=\frac{2}{x-1}$ then $\int f(x)\,dx = x-\frac{(x-3)^2}{4}+\frac{(x-3)^3}{12}+\ldots$ $+(-\frac{1}{2})^n\frac{(x-3)^{n+1}}{n+1}+\ldots$ At $x=1$ the series $\sum_{n=1}^{\infty}\frac{-2}{n+1}$ diverges; at $x=5$ the series $\sum_{n=1}^{\infty}\frac{(-1)^n 2}{n+1}$ converges. Therefore the interval of convergence is $1<x\le 5$ and the sum is $2\ln|x-1|+(3-\ln 4)$. since $\int\frac{2}{x-1}dx=$ $2\ln|x-1|+C$, where $C=3-\ln 4$ when $x=3$

(J). 11.11.

9. $\int_0^{2\pi}\cos px\,dx=\frac{1}{p}\sin px\Big|_0^{2\pi}=0$ if $p\neq 0$.

10. $\int_0^{2\pi}\sin px\,dx=-\frac{1}{p}\cos px\Big|_0^{2\pi}=-\frac{1}{p}[1-1]=0$ if $p\neq 0$.

11. $\int_0^{2\pi}\cos px\cos qx\,dx=\int_0^{2\pi}\frac{1}{2}[\cos(p+q)x+\cos(p-q)x]\,dx.=\frac{1}{2}\left[\frac{1}{p+q}\sin(p+q)x+\frac{1}{p-q}\sin(p-q)x\right]_0^{2\pi}$ if $p\neq q$

$\int_0^{2\pi}\cos px\cos qx\,dx=\int_0^{2\pi}\cos^2 px\,dx=\int_0^{2\pi}\frac{1}{2}(1+\cos 2px)\,dx=\frac{1}{2}(x+\frac{1}{2p}\sin 2px)\Big|_0^{2\pi}=\pi$ if $p=q$

12. $\int_0^{2\pi}\sin px\sin qx\,dx=\int_0^{2\pi}\frac{1}{2}[\cos(p-q)x-\cos(p+q)x]\,dx=\frac{1}{2}\left[\frac{1}{p-q}\sin(p-q)x-\frac{1}{p+q}\sin(p+q)x\right]_0^{2\pi}=0$ if $p\neq q$.

$\int_0^{2\pi}\sin px\sin qx\,dx=\int_0^{2\pi}\sin^2 px\,dx=\int_0^{2\pi}\frac{1}{2}(1-\cos 2px)\,dx=\frac{1}{2}(x-\frac{1}{2p}\sin 2px)\Big|_0^{2\pi}=\pi$

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