H10

A

16.A

17. If $f(x, y, z)$ and $g(x, y, z)$ are continuously differentiable scalar functions defined over the oriented surface S with boundary curve C, prove that

$$\iint_S (\nabla f \times \nabla g) \cdot \mathbf{n}\, d\sigma = \oint_C f\, \nabla g \cdot d\mathbf{r}.$$

21. Show that the volume V of a region D in space enclosed by the oriented surface S with outward normal $\mathbf{n}$ satisfies the identity

$$V = \frac{1}{3} \iint_S \mathbf{r} \cdot \mathbf{n}\, d\sigma,$$

where $\mathbf{r}$ is the position vector of the point (x, y, z) in D.

B

11.1

Finding a Sequence's Formula

In Exercises 13–22, find a formula for the nth term of the sequence.

13. The sequence $1, -1, 1, -1, 1, \ldots$ 1's with alternating signs

14. The sequence $-1, 1, -1, 1, -1, \ldots$ 1's with alternating signs

15. The sequence $1, -4, 9, -16, 25, \ldots$ Squares of the positive integers; with alternating signs

18. The sequence $-3, -2, -1, 0, 1, \ldots$ Integers beginning with -3

19. The sequence $1, 5, 9, 13, 17, \ldots$ Every other odd positive integer

20. The sequence $2, 6, 10, 14, 18, \ldots$ Every other even positive integer

21. The sequence $1, 0, 1, 0, 1, \ldots$ Alternating 1's and 0's

22. The sequence $0, 1, 1, 2, 2, 3, 3, 4, \ldots$ Each positive integer repeated

C

Finding Limits

Which of the sequences $\{a_n\}$ in Exercises 23–84 converge, and which diverge? Find the limit of each convergent sequence.

23. $a_n = 2 + (0.1)^n$

24. $a_n = \dfrac{n + (-1)^n}{n}$

27. $a_n = \dfrac{1 - 5n^4}{n^4 + 8n^3}$

40. $a_n = n\pi \cos(n\pi)$

29. $a_n = \dfrac{n^2 - 2n + 1}{n - 1}$

42. $a_n = \dfrac{\sin^2 n}{2^n}$

55. $a_n = \dfrac{\ln n}{n^{1/n}}$

56. $a_n = \ln n - \ln(n + 1)$

D

117. Uniqueness of limits Prove that limits of sequences are unique. That is, show that if L_1 and L_2 are numbers such that $a_n \to L_1$ and $a_n \to L_2$, then $L_1 = L_2$.

118. Limits and subsequences If the terms of one sequence appear in another sequence in their given order, we call the first sequence a **subsequence** of the second. Prove that if two subsequences of a sequence $\{a_n\}$ have different limits $L_1 \neq L_2$, then $\{a_n\}$ diverges.

119. For a sequence $\{a_n\}$ the terms of even index are denoted by a_{2k} and the terms of odd index by a_{2k+1}. Prove that if $a_{2k} \to L$ and $a_{2k+1} \to L$, then $a_n \to L$.

E

11.2

Series with Geometric Terms

In Exercises 7–14, write out the first few terms of each series to show how the series starts. Then find the sum of the series.

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{4^n}$

Telescoping Series

Use partial fractions to find the sum of each series in Exercises 15–22.

15. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(4n - 3)(4n + 1)}$

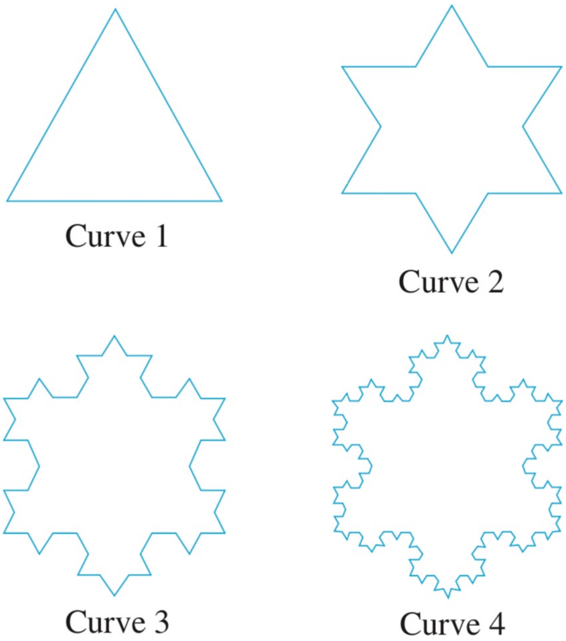
16. $\displaystyle\sum_{n=1}^{\infty} \frac{6}{(2n - 1)(2n + 1)}$

F

77. Helga von Koch's snowflake curve Helga von Koch's snowflake is a curve of infinite length that encloses a region of finite area. To see why this is so, suppose the curve is generated by starting with an equilateral triangle whose sides have length 1.

a. Find the length L_n of the nth curve C_n and show that $\lim_{n\to\infty} L_n = \infty$.

b. Find the area A_n of the region enclosed by C_n and calculate $\lim_{n\to\infty} A_n$.



G

Convergence or Divergence

Which series in Exercises 23–40 converge, and which diverge? Give reasons for your answers. If a series converges, find its sum.

27. $\displaystyle\sum_{n=0}^{\infty} \cos n\pi$

28. $\displaystyle\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

33. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$

35. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{1000^n}$

H

11.7

Intervals of Convergence

In Exercises 1–32, **(a)** find the series' radius and interval of convergence. For what values of x does the series converge **(b)** absolutely, **(c)** conditionally?

1. $\displaystyle\sum_{n=0}^{\infty} x^n$

2. $\displaystyle\sum_{n=0}^{\infty} (x+5)^n$

3. $\displaystyle\sum_{n=0}^{\infty} (-1)^n(4x+1)^n$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

I

39. For what values of x does the series

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 + \cdots + \left(-\frac{1}{2}\right)^n (x-3)^n + \cdots$$

converge? What is its sum? What series do you get if you differentiate the given series term by term? For what values of x does the new series converge? What is its sum?

40. If you integrate the series in Exercise 39 term by term, what new series do you get? For what values of x does the new series converge, and what is another name for its sum?

J

11.11

Establish the results in Exercises 9–13, where p and q are positive integers.

9. $\displaystyle\int_0^{2\pi} \cos px\, dx = 0$ for all p.

10. $\displaystyle\int_0^{2\pi} \sin px\, dx = 0$ for all p.

11. $\displaystyle\int_0^{2\pi} \cos px \cos qx\, dx = \begin{cases} 0, & \text{if } p \neq q \\ \pi, & \text{if } p = q \end{cases}.$

(*Hint:* $\cos A \cos B = (1/2)[\cos(A+B) + \cos(A-B)]$.)

12. $\displaystyle\int_0^{2\pi} \sin px \sin qx\, dx = \begin{cases} 0, & \text{if } p \neq q \\ \pi, & \text{if } p = q \end{cases}.$

(*Hint:* $\sin A \sin B = (1/2)[\cos(A-B) - \cos(A+B)]$.)